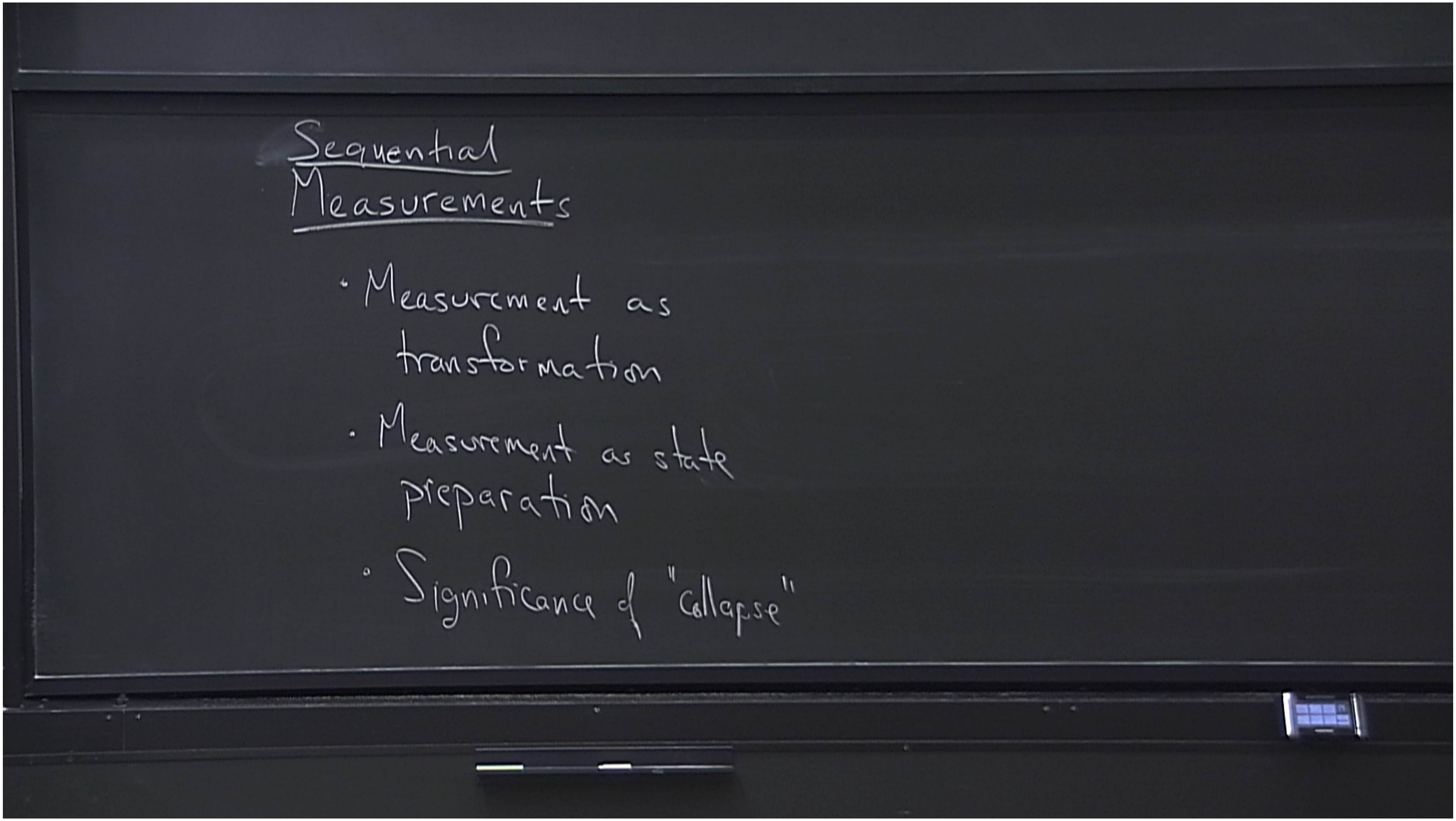
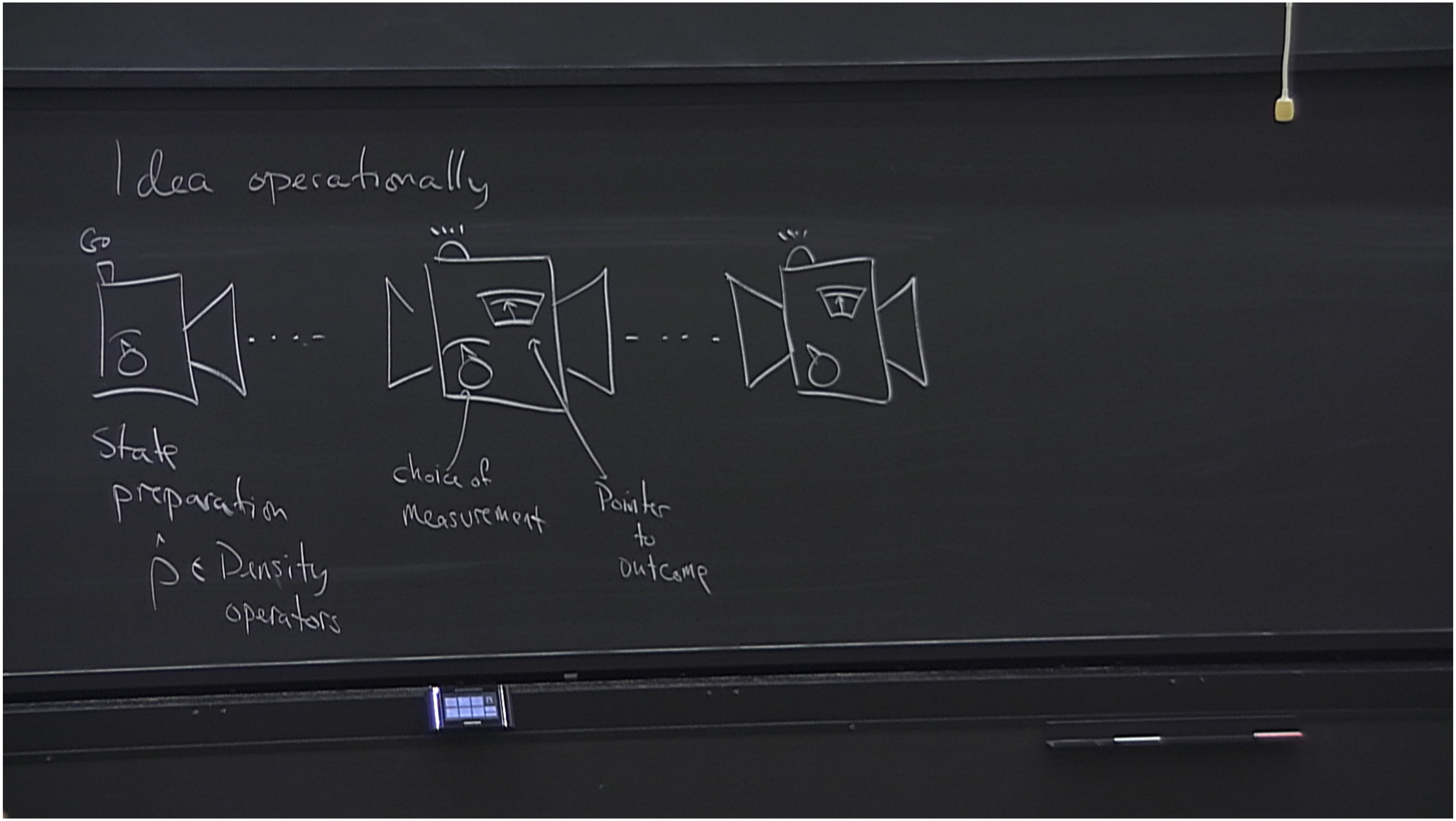
Title: 14/15 PSI Quantum Theory-8

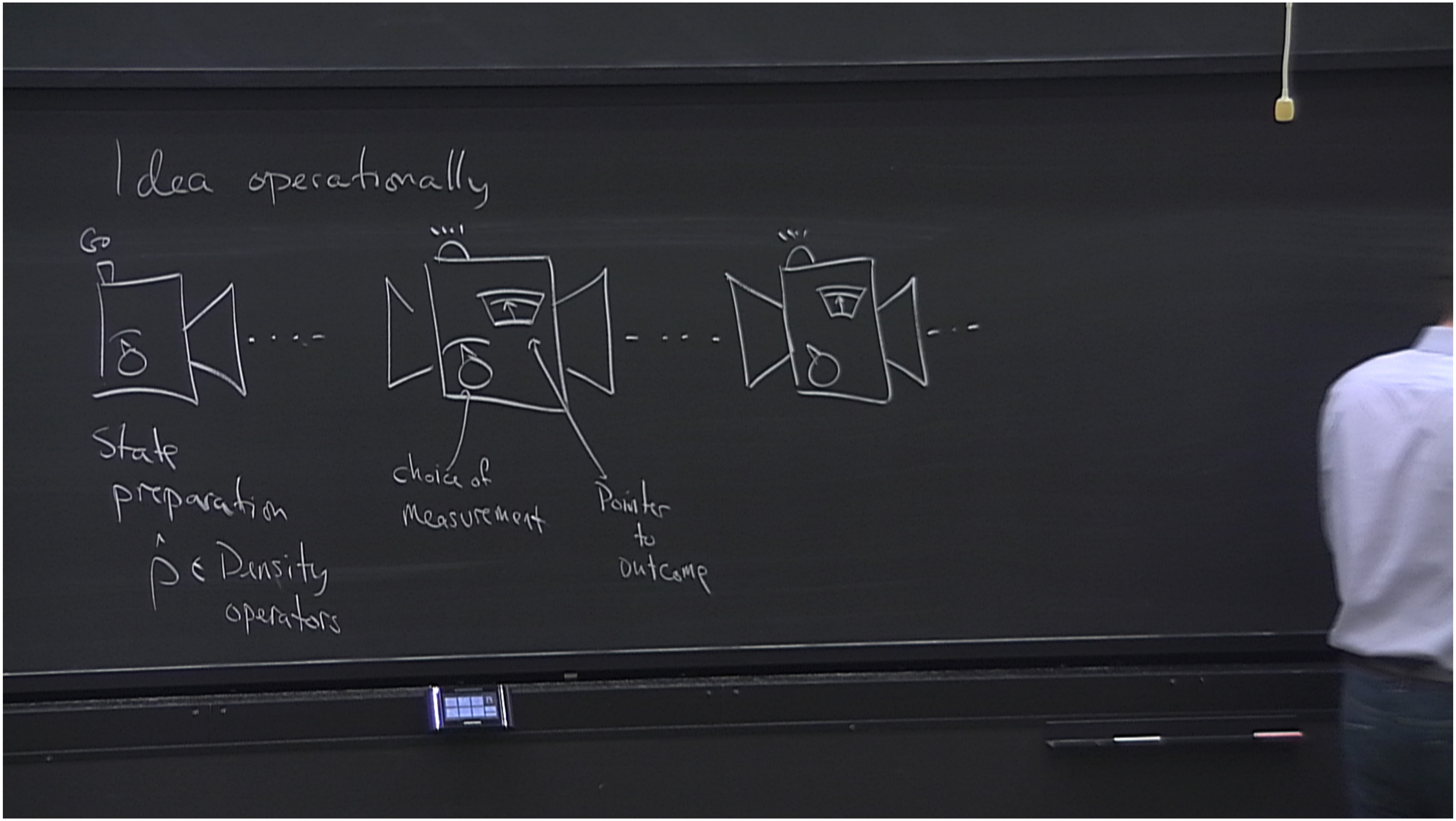
Date: Sep 17, 2014 10:45 AM

URL: http://pirsa.org/14090051

Abstract:


Sequential Measurements
• Measurement as transformation
• Measurement as state preparation
• Significance of "Collapse"


Idea operationally
Go
State preparation
$\hat{\rho} \in$ Density operators
choice of measurement
Pointer to outcome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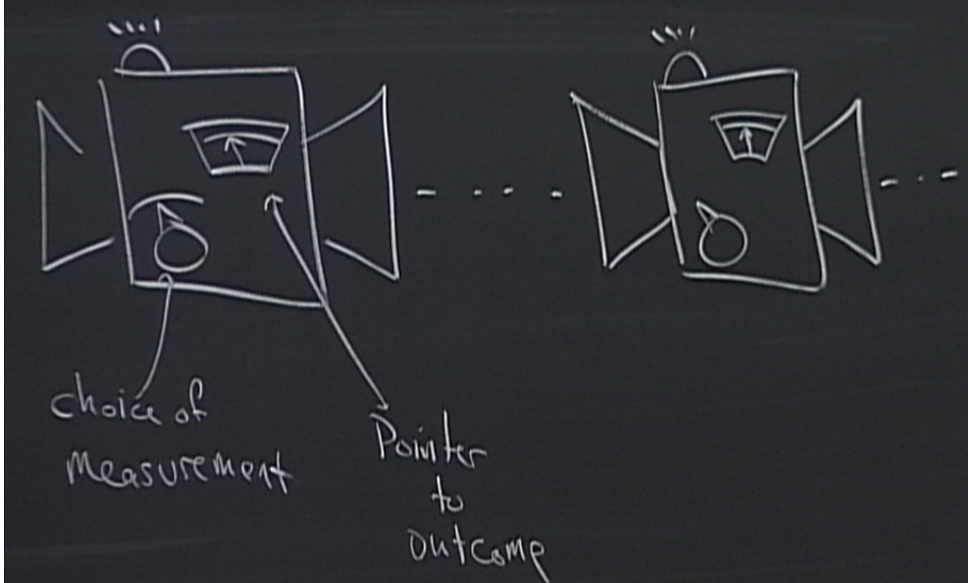


Von Neumann–Lüders rule

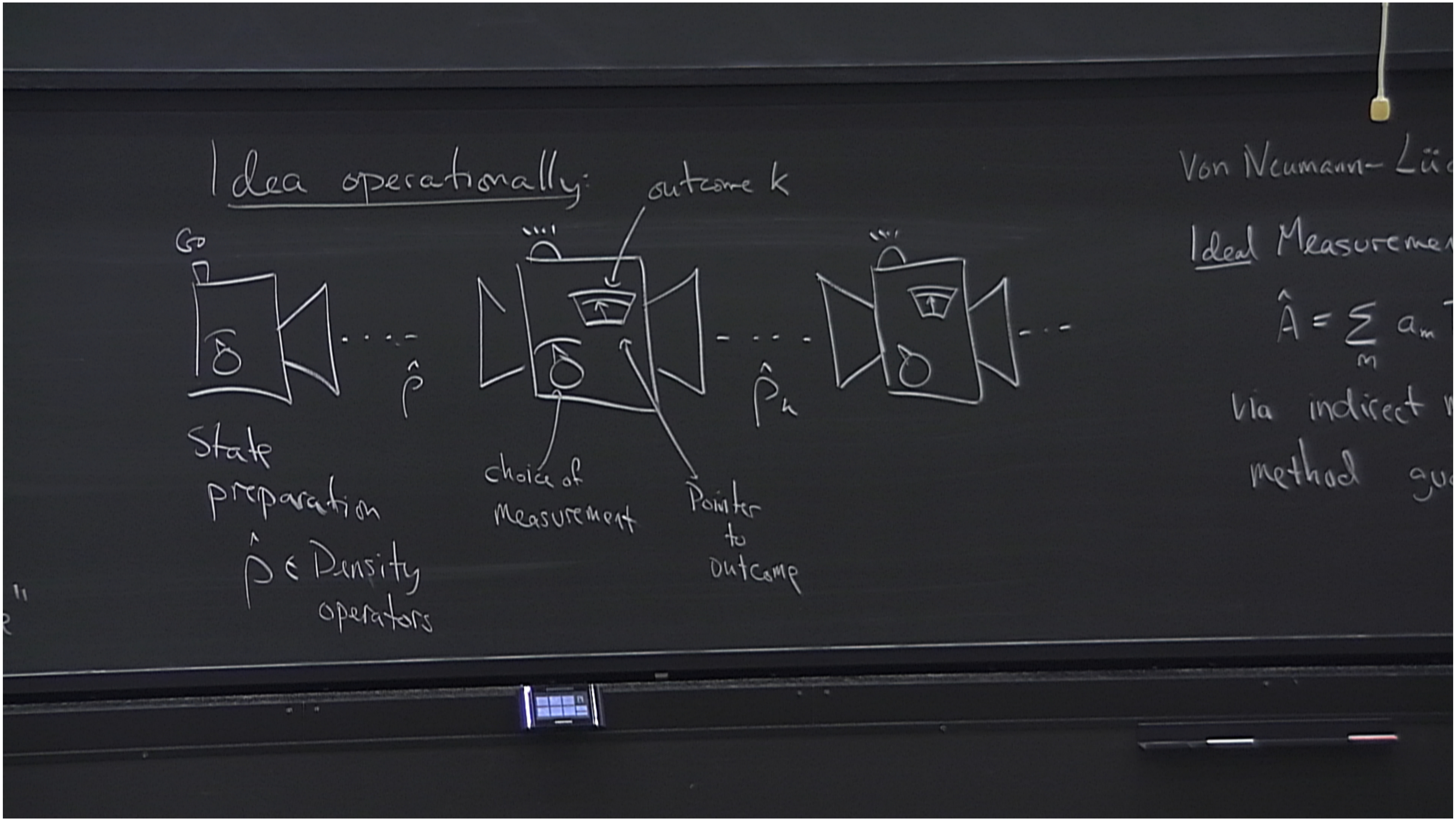
<u>Ideal</u> Measurement of

$$\hat{A} = \sum_m a_m \hat{\Pi}_m$$

via indirect measurement method guarantees that

given outcome k

$$\rho \rightarrow \rho_k$$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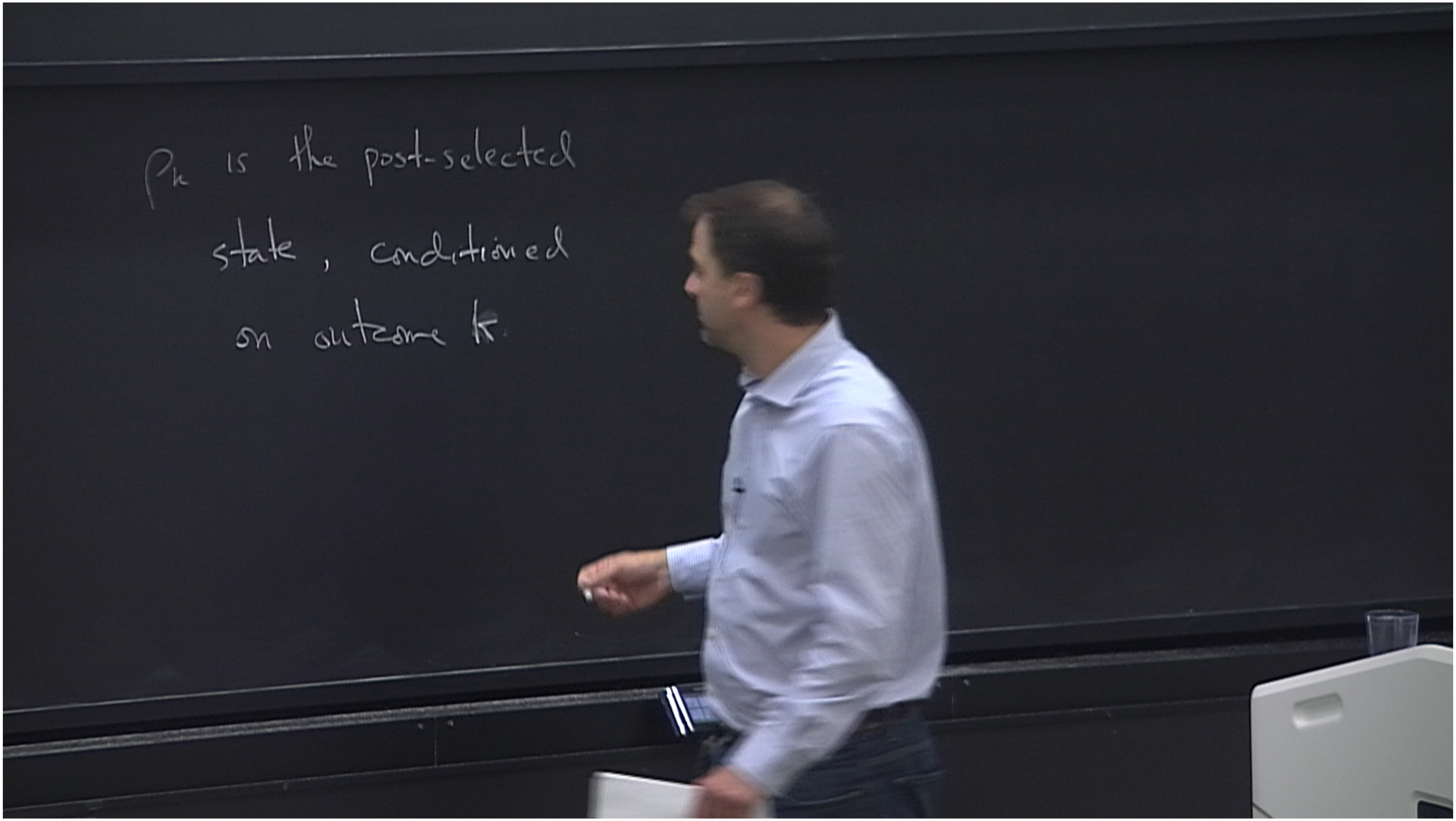
$\hat{\rho}_k$
Von Neumann- Lü
Ideal Measureme
$\hat{A} = \sum_m a_m$
via indirect
method

given outcome k

$$\rho \to \rho_k = \frac{\hat{\Pi}_k \hat{\rho} \hat{\Pi}_k}{\mathrm{Tr}(\hat{\rho} \hat{\Pi}_k)}$$

guarantees
that $\mathrm{tr}(\rho_k) = 1$.


ρ_k is the post-selected
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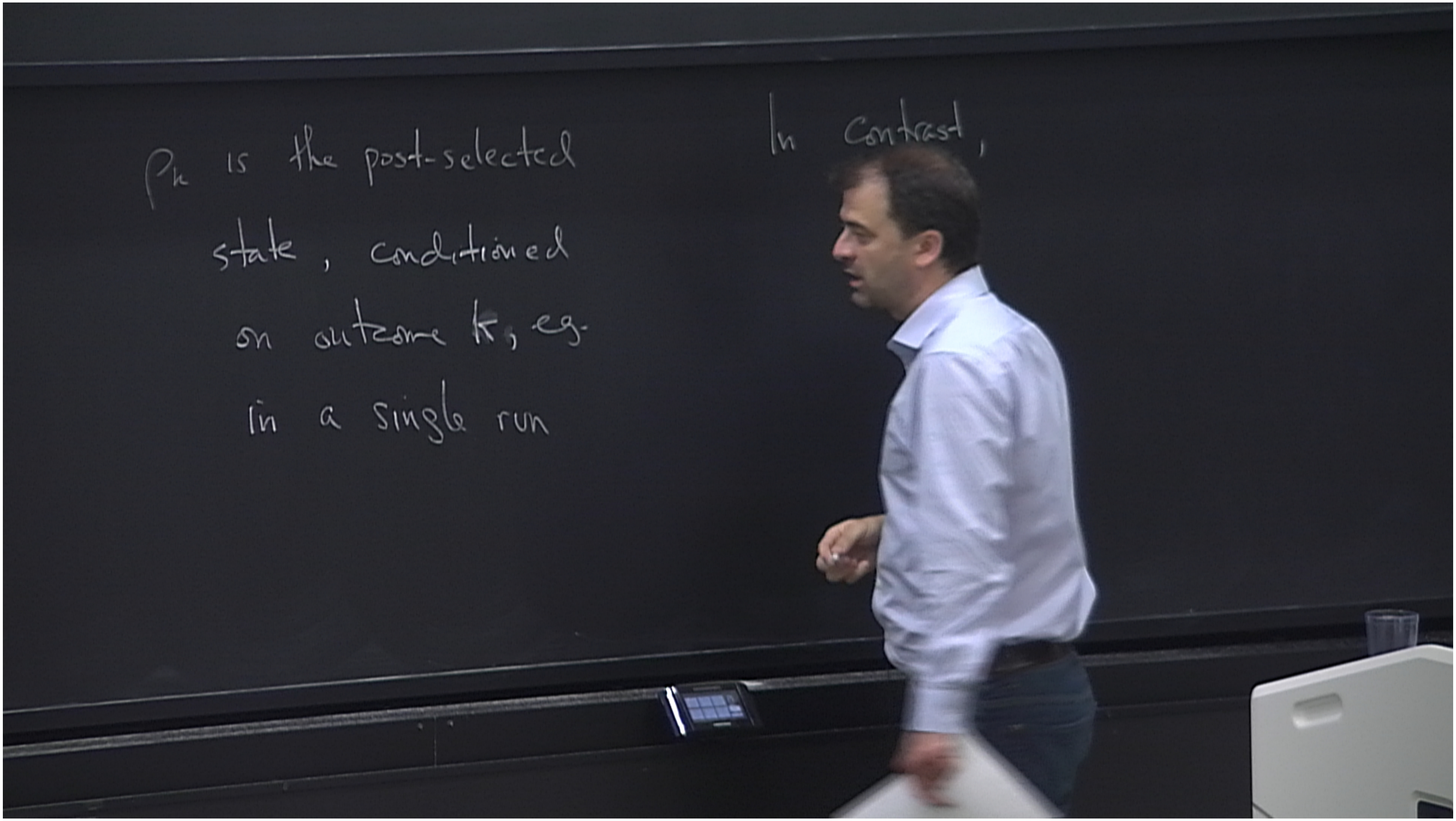
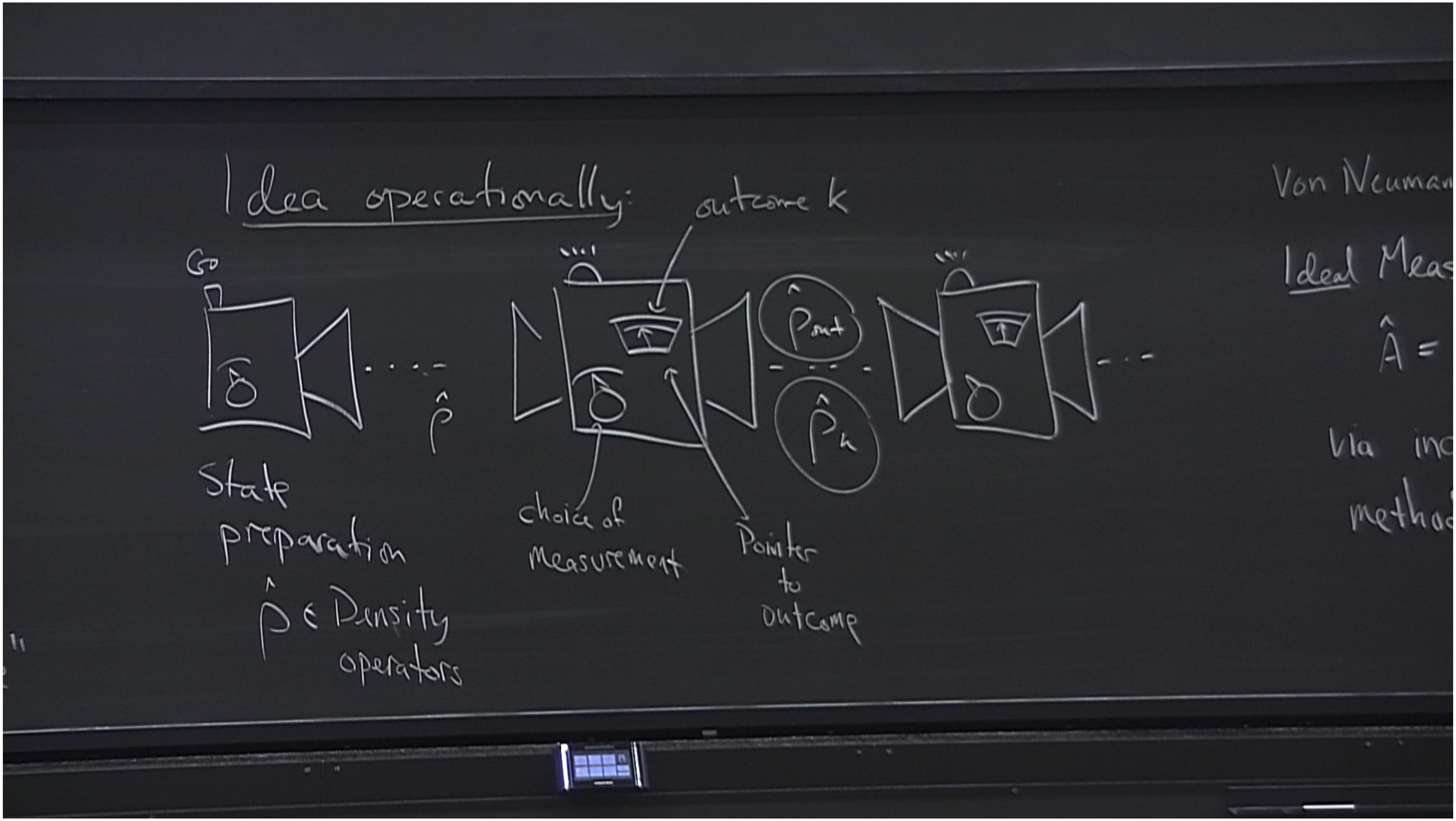
on outcome k.


ρ_k is the post-selected
state, conditioned
on outcome k, eg.
in a single run
In contrast,

In contrast, if I ignore outcome, or don't have access to outcome, or want to describe the ensemble of output systems (independent of outcome)

then $\rho_{out} = \sum_k Tr(\hat{\rho}\pi_k) \cdot \hat{\rho}_k$

$= \sum_k \hat{\pi}_k \hat{\rho} \hat{\pi}_k$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Von Neuman
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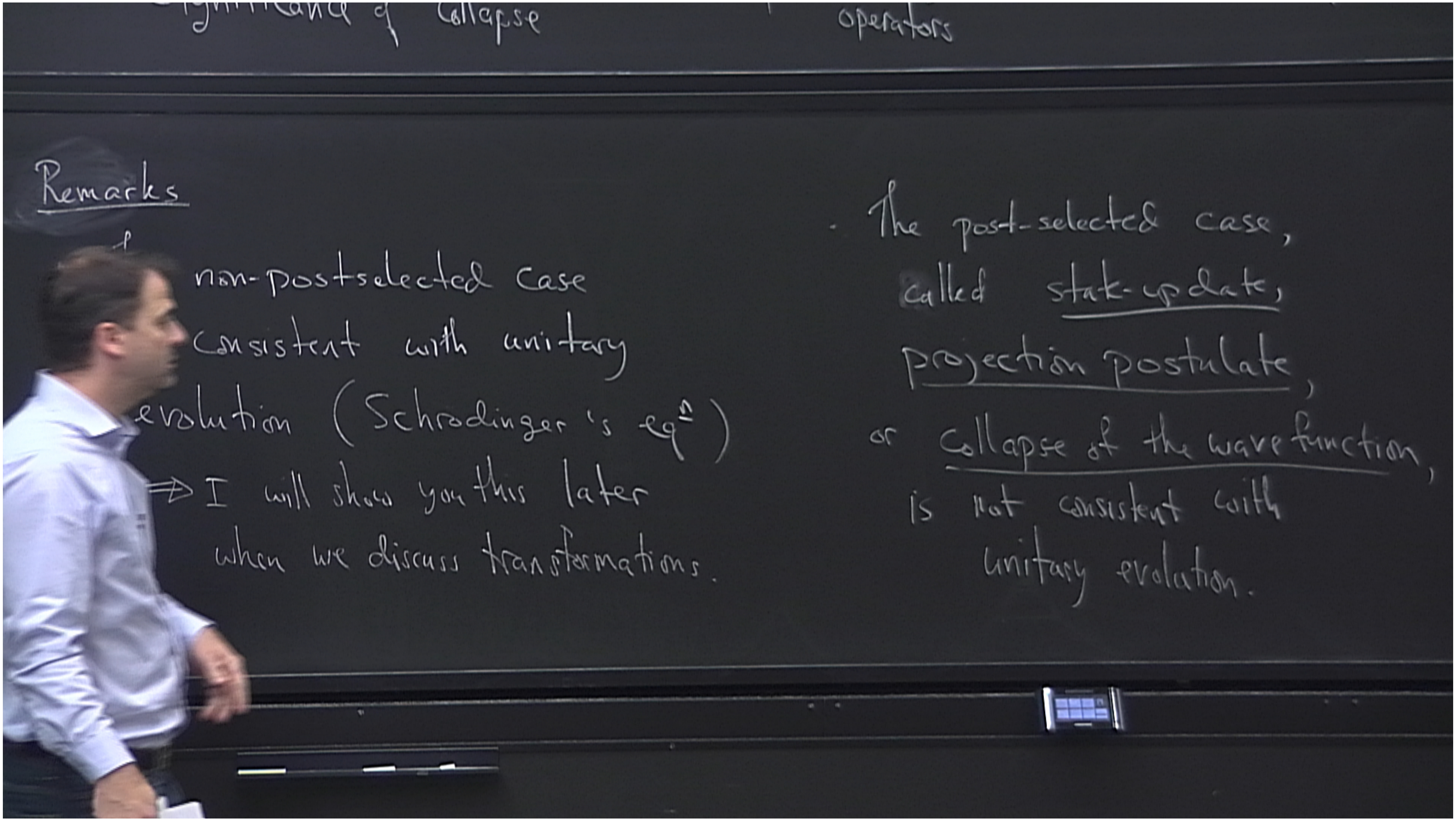
metho

Significance of collapse operators

Remarks

- The non-postselected case is consistent with unitary evolution (Schrodinger's eq$^{\underline{n}}$)
  $\Rightarrow$ I will show you this later when we discuss transformations.

- The post-selected case, called state-update, projection postulate, or collapse of the wavefunction, is not consistent with unitary evolution.


operators
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- State-update is natural if we think about / interpret q. states as descriptions of our knowledge rather than physical objects.

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stat-update,

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nitary evolution. (Measurement problem).

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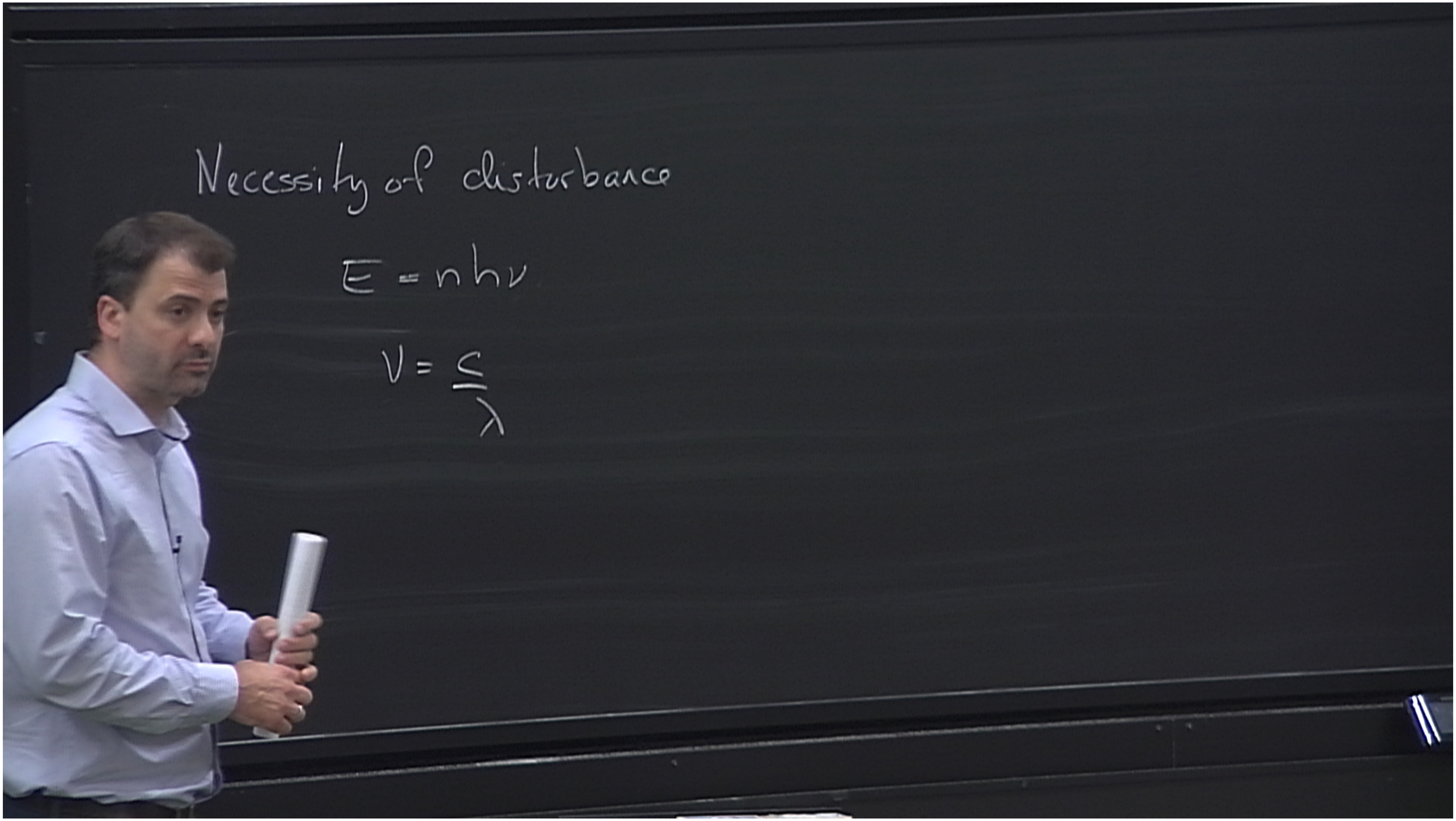
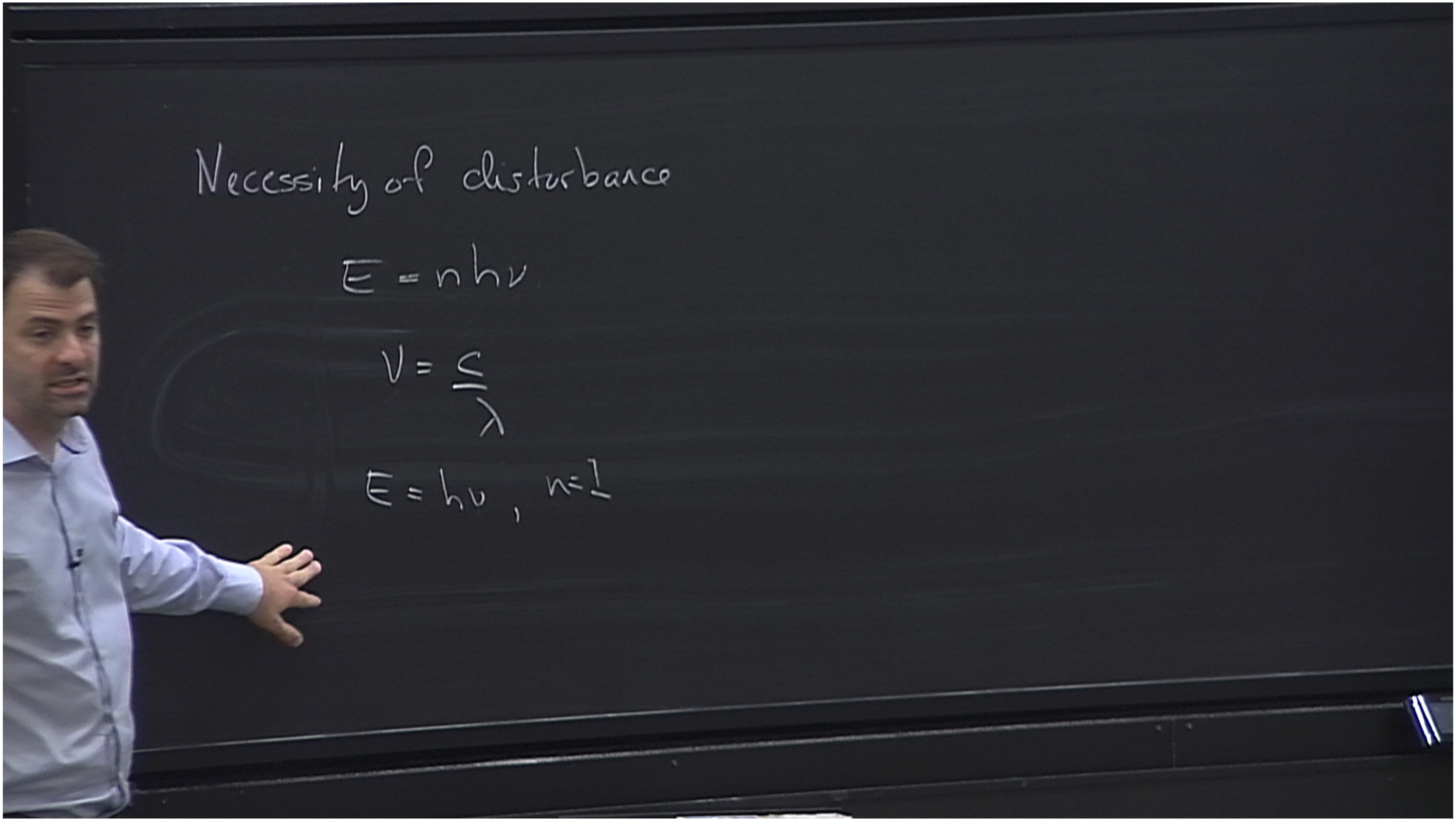
Necessity of disturbance

$$E = nh\nu$$


Necessity of disturbance
$E = nh\nu$
$\nu = \frac{c}{\lambda}$


Necessity of disturbance
$E = nh\nu$
$\nu = \frac{c}{\lambda}$
$E = h\nu \, , \; n=1$

Example

Measurement involving

2 projectors

$$P_1 = |\phi_1\rangle\langle\phi_1|$$

$$P_{1\perp} = |\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3|$$

Example

Measurement involving

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$$P_1 = |\phi_1\rangle\langle\phi_1|$$

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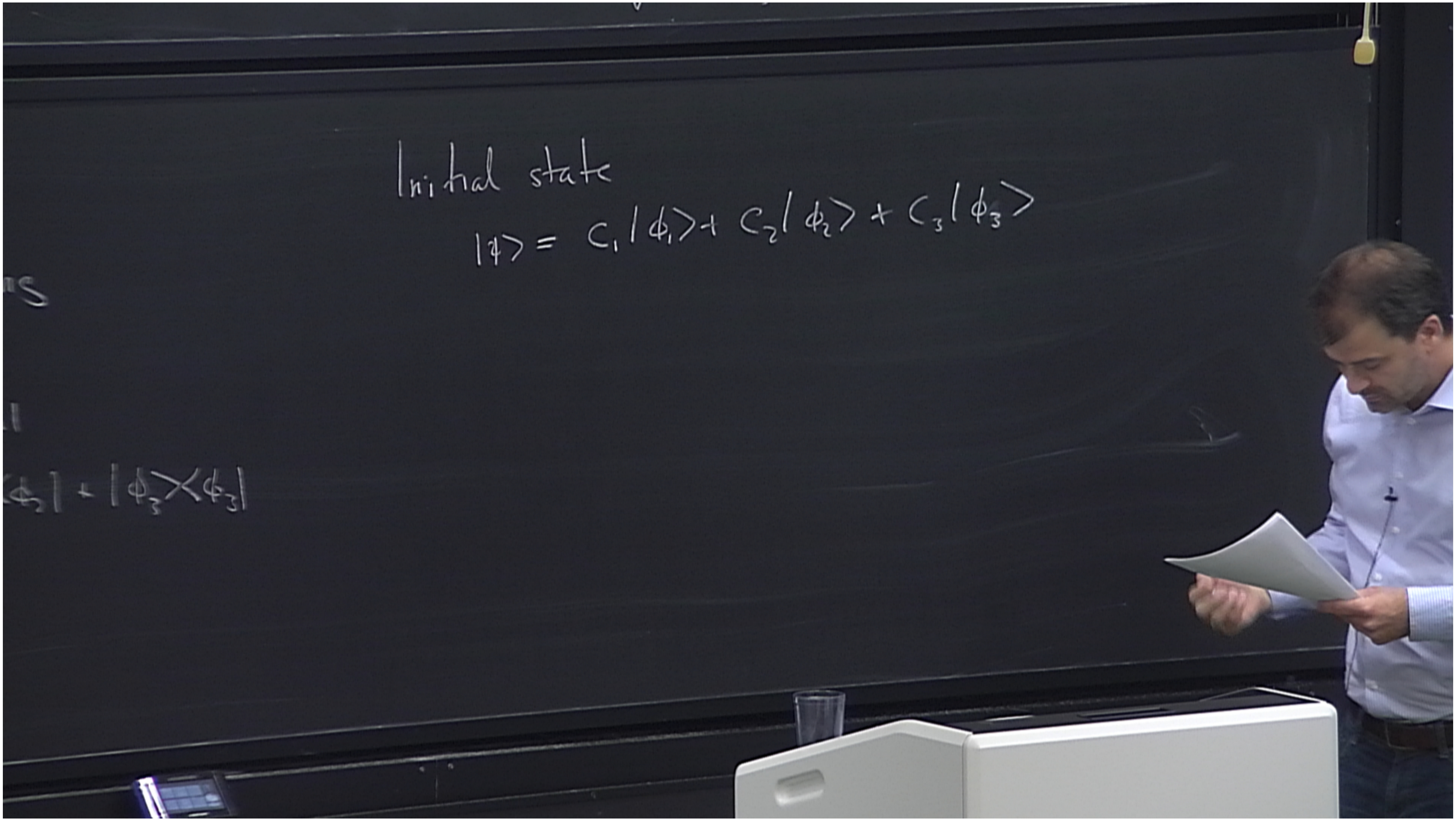
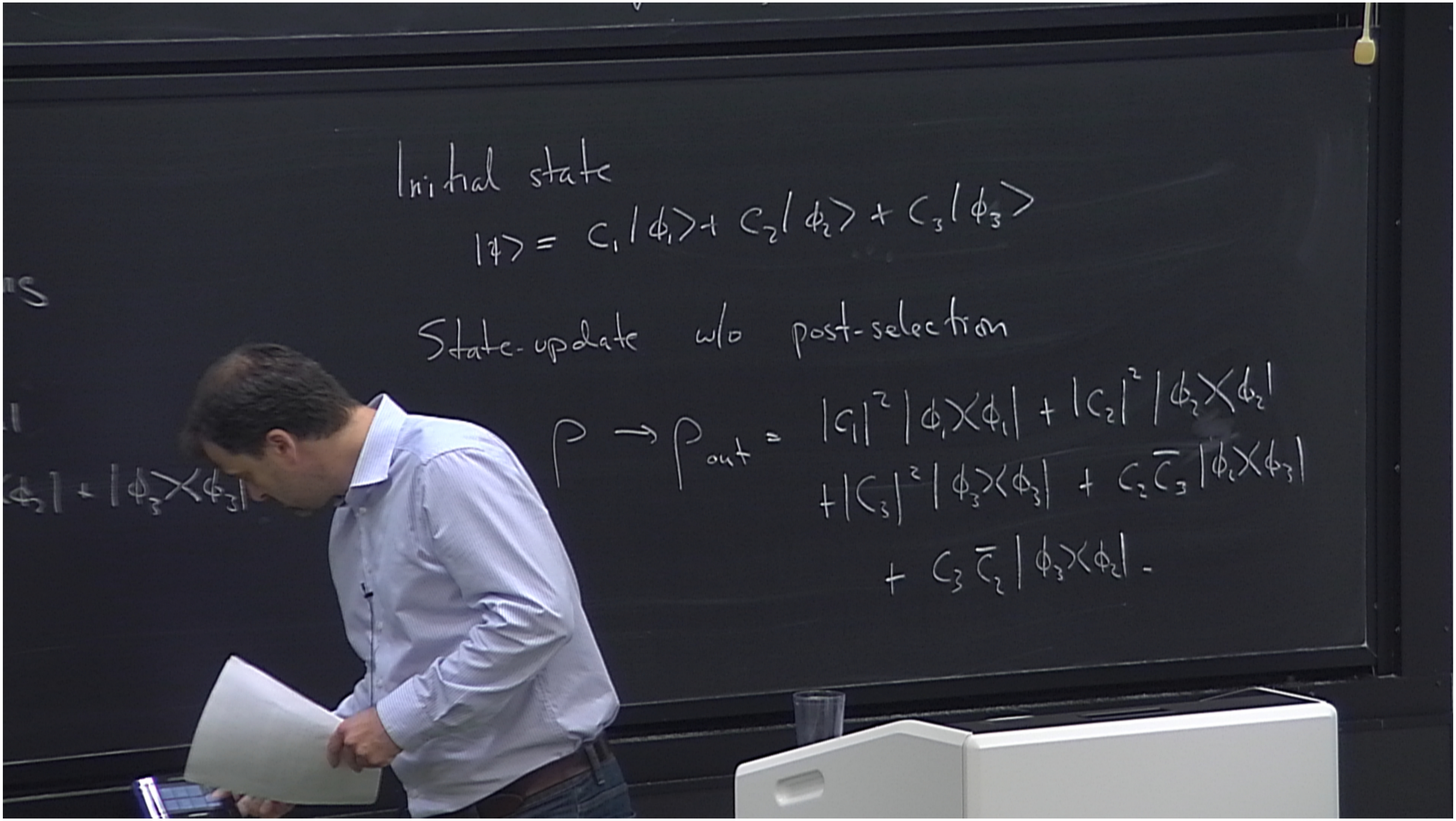
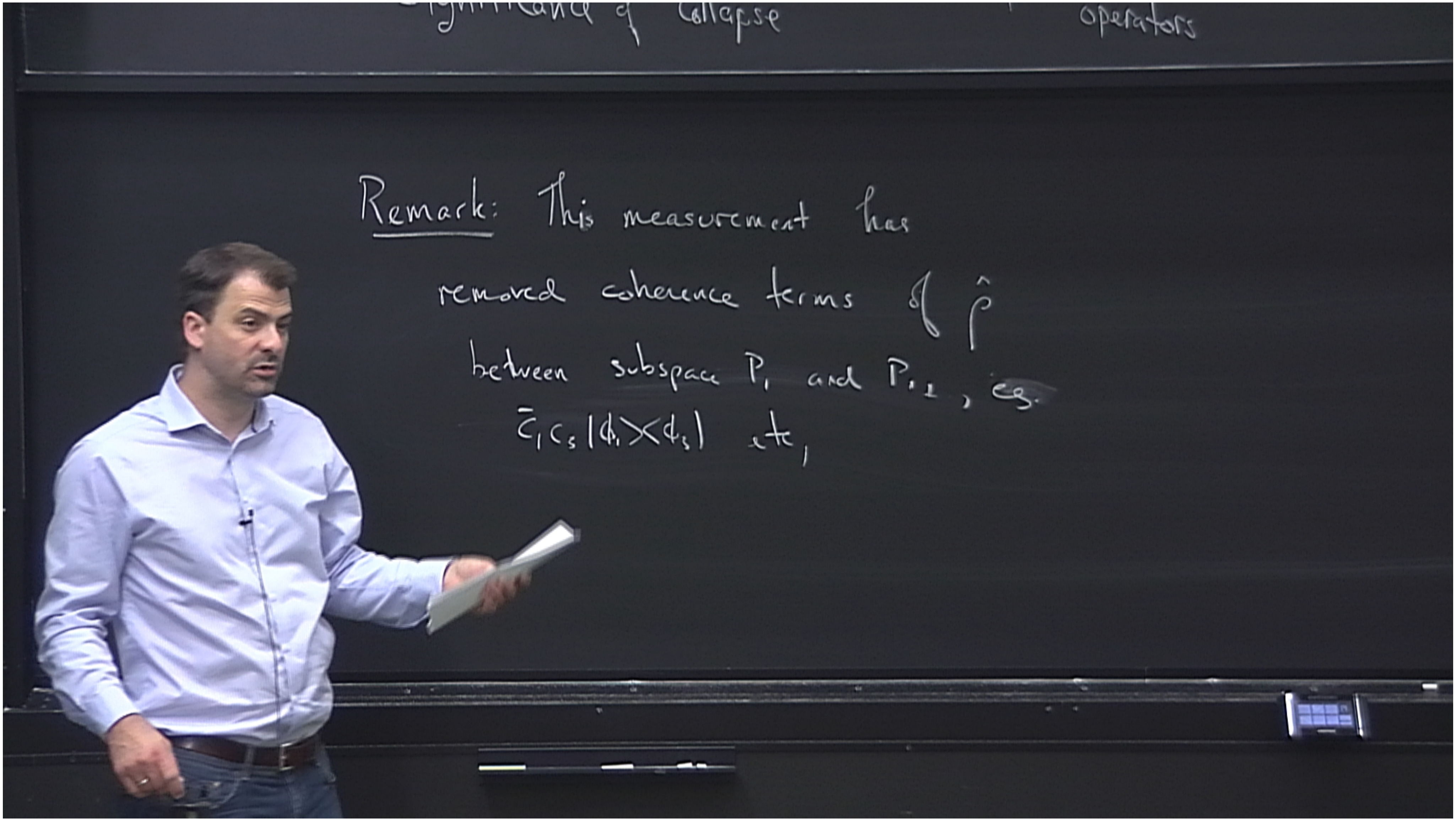
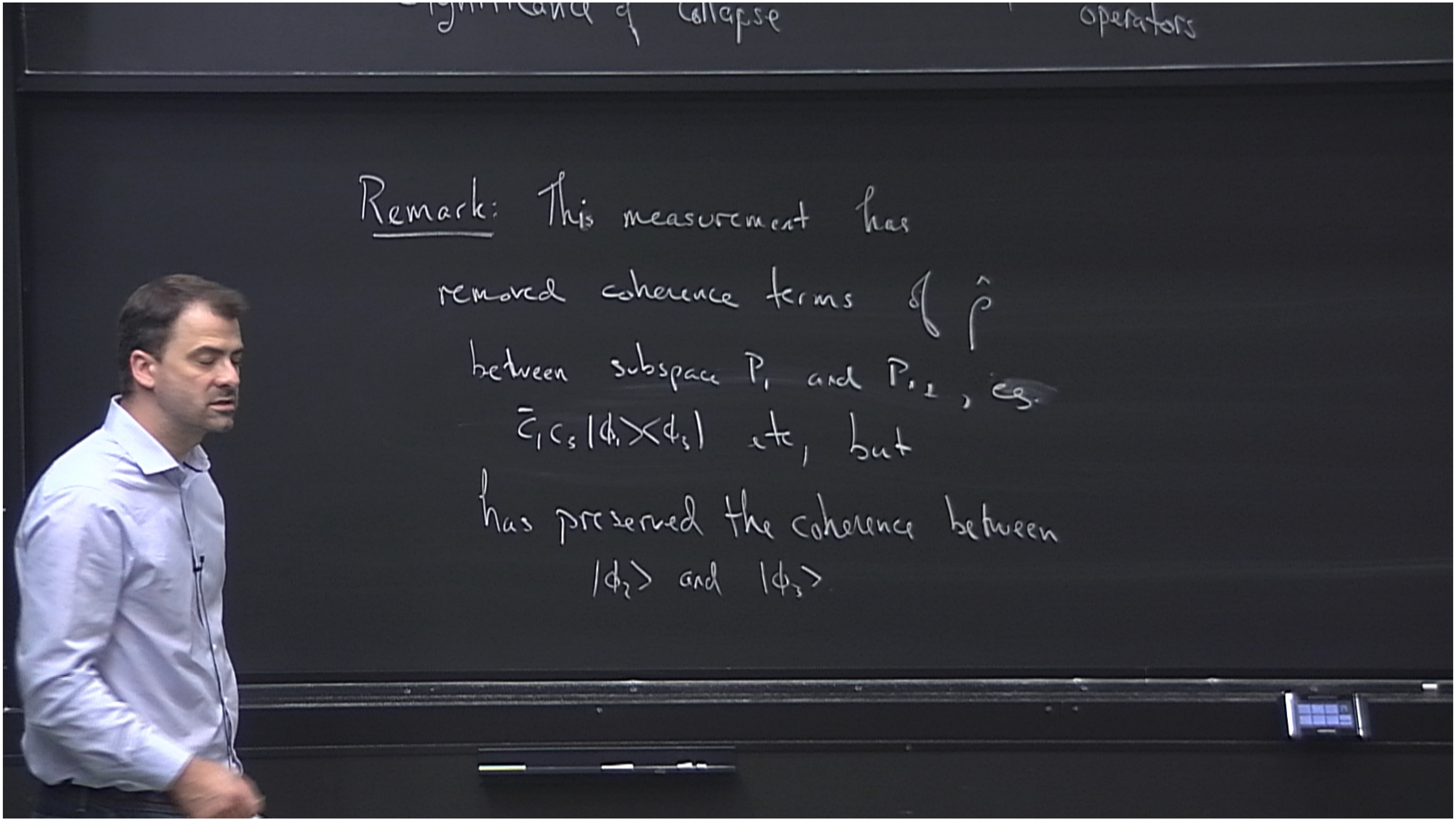
$$\{P_1, P_{1\perp}\}$$


Initial state
|ψ> = c1|φ1> + c2|φ2> + c3|φ3>


Initial state
$|\psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle + c_3|\phi_3\rangle$
State-update w/o post-selection
$\rho \rightarrow \rho_{out} = |c_1|^2|\phi_1\rangle\langle\phi_1| + |c_2|^2|\phi_2\rangle\langle\phi_2|$
$+|c_3|^2|\phi_3\rangle\langle\phi_3| + c_2\bar{c}_3|\phi_2\rangle\langle\phi_3|$
$+ c_3\bar{c}_2|\phi_3\rangle\langle\phi_2|.$


Collapse
operators
Remark: This measurement has
removed coherence terms of $\hat{\rho}$
between subspace P_1 and P_2, e.g.
$\bar{c}_1 c_s |\phi_1\rangle\langle\phi_s|$ etc,


of Collapse
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between subspace P_1 and P_2, e.g.
$\bar{c}_1 c_3 |\phi_1\rangle\langle\phi_3|$ etc, but
has preserved the coherence between
$|\phi_2\rangle$ and $|\phi_3\rangle$

- If we post-select on outcome 1 then

$$\rho \to \rho_1 = |\phi_1\rangle\langle\phi_1|$$

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↳ State-preparation of the pure state $|\phi_1\rangle$.

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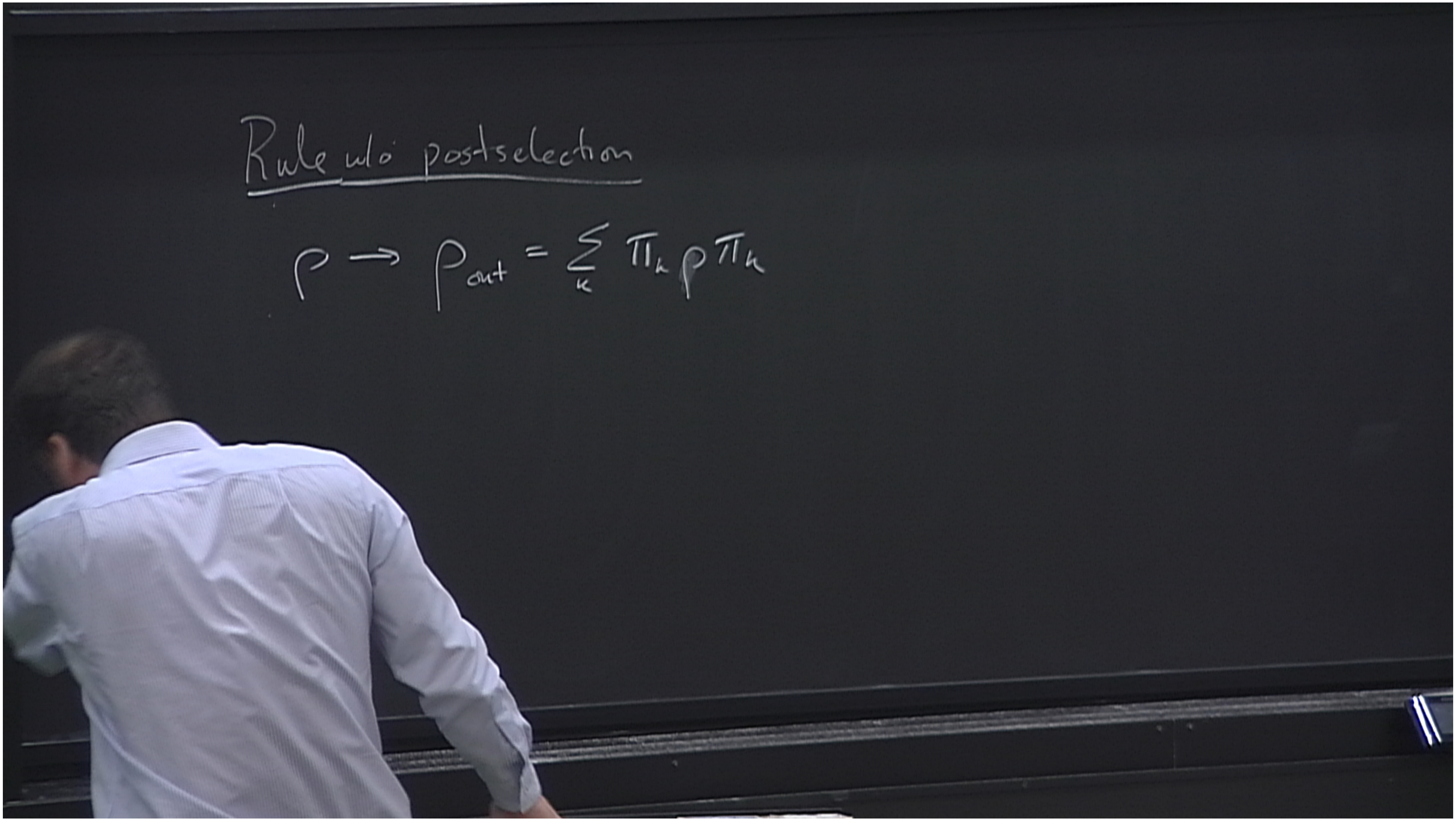
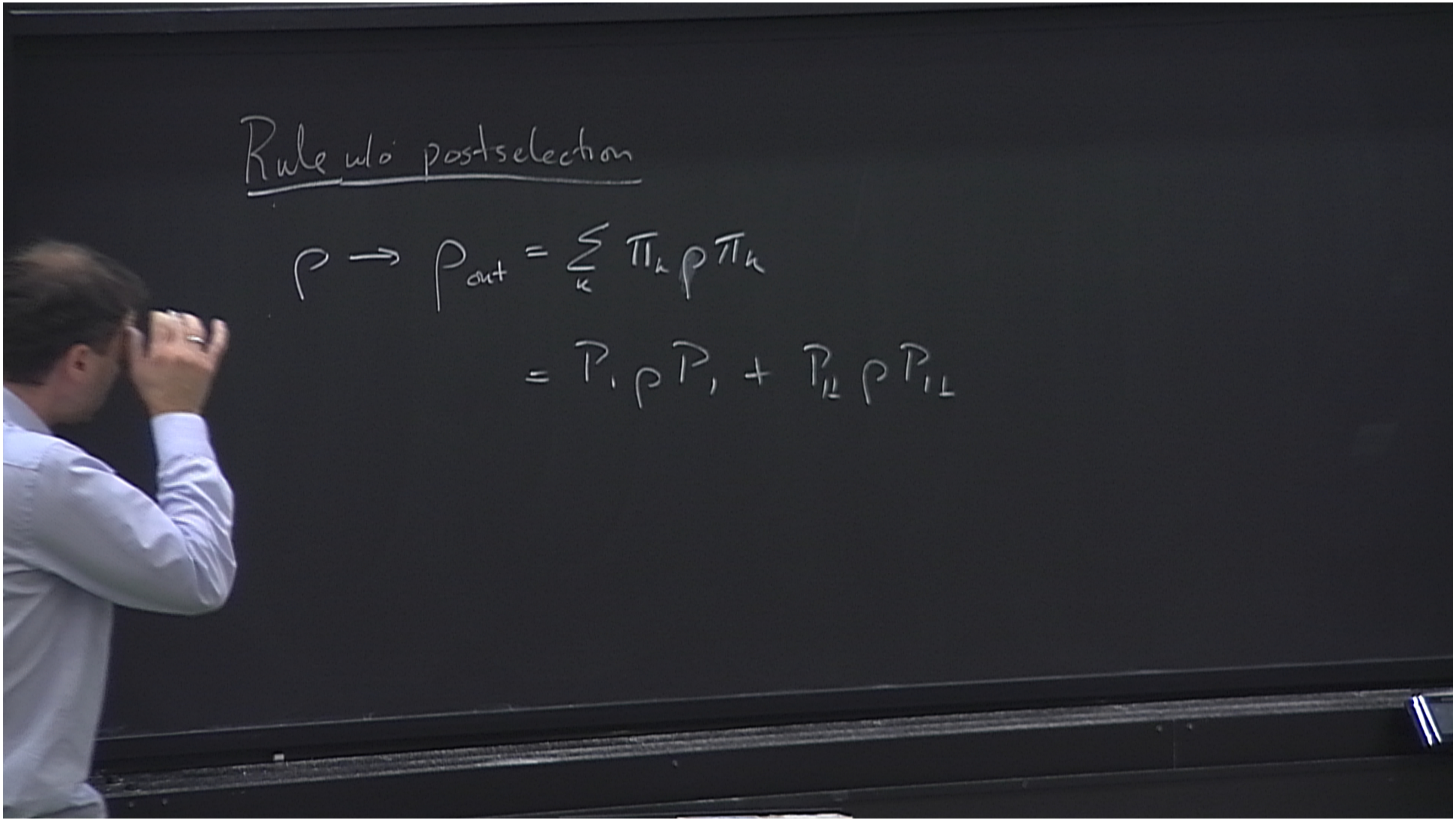
- If we postselect on $1_\perp$ outcome then

$$\rho \to \rho_{1_\perp} = |\psi\rangle\langle\psi|$$

where $|\psi\rangle = \dfrac{c_2|\phi_2\rangle + c_3|\phi_3\rangle}{\sqrt{|c_2|^2 + |c_3|^2}}$


Rule w/o postselection

$$\rho \to \rho_{out} = \sum_k \Pi_k \rho \Pi_k$$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$= P_1 \rho P_1 + P_{1\perp} \rho P_{1\perp}$

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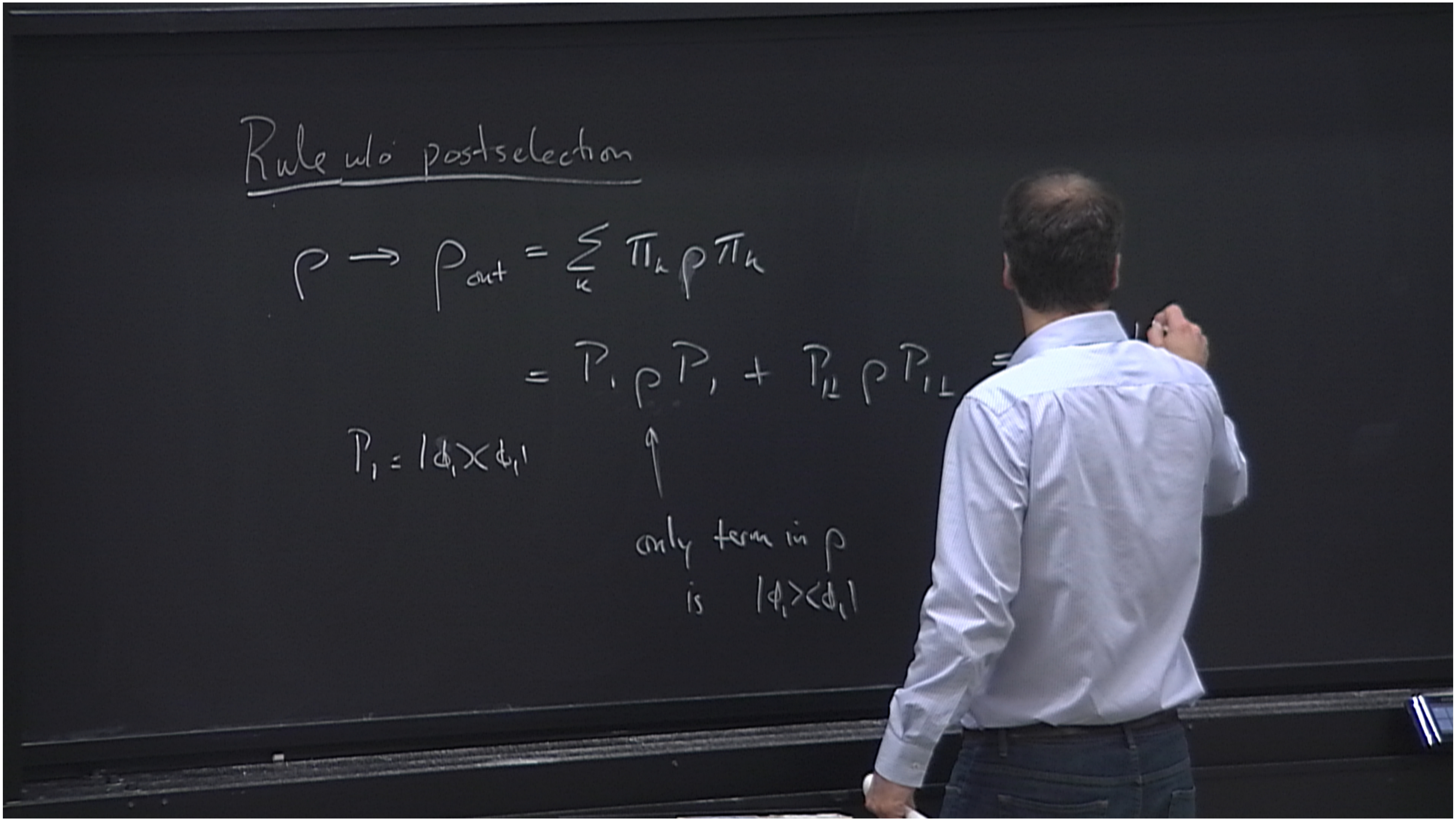
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$\rho \longrightarrow \rho_{out} = \sum_k \Pi_k \rho \Pi_k$
$= P_1 \rho P_1 + P_{1\perp} \rho P_{1\perp} =$
$P_1 = |\phi_1\rangle\langle\phi_1|$
only term in ρ
is $|\phi_1\rangle\langle\phi_1|$

Rule w/o postselection

$$\rho \longrightarrow \rho_{out} = \sum_k \Pi_k \rho \Pi_k$$

$$= P_1 \rho P_1 + P_{1\perp} \rho P_{1\perp} = |c_1|^2 |\phi_1\rangle\langle\phi_1| + $$

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Rule w/o postselection

$$P_{1\perp} = |\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3|$$

Consider $\bar{c}_1 c_3 |\phi_1\rangle\langle\phi_3|$

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$P_{1\perp} = |\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3|$

Consider $\cancel{\bar{c}_1 c_3 |\phi_1\rangle\langle\phi_3|} \to 0$

$\phi_3|$

$\cancel{\rangle\langle\phi_3|} \to 0$

Consider the term

$$\bar{c}_2 c_3 |\phi_2\rangle\langle\phi_3|$$

$$P_{1\perp} \bar{c}_2 c_3 |\phi_2\rangle\langle\phi_3| P_{1\perp}$$

$$= \bar{c}_2 c_3 \left(|\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3| \right) |\phi_2\rangle\langle\phi_3| \left(|\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3| \right)$$

$$= \bar{c}_2 c_3 |\phi_2\rangle\langle\phi_3|$$

State-update for Generalized Measurement

- Given some device performing POVM measurement $\{E_k\}$, where $Pr(k) = Tr(\rho E_k)$

State-update for Generalized Measurement

- Given some device performing POVM measurement $\{E_k\}$, where $Pr(k) = Tr(\rho E_k)$, the final state conditioned on outcome k, is undetermined

State-update for Generalized Measurement

- Given some device performing POVM measurement $\{E_k\}$, where $Pr(k) = Tr(\rho E_k)$, the final state conditioned on outcome k, is undetermined $\rightarrow$ What state ρ_k you get

depends on the measur

depends on how you do the measurement.

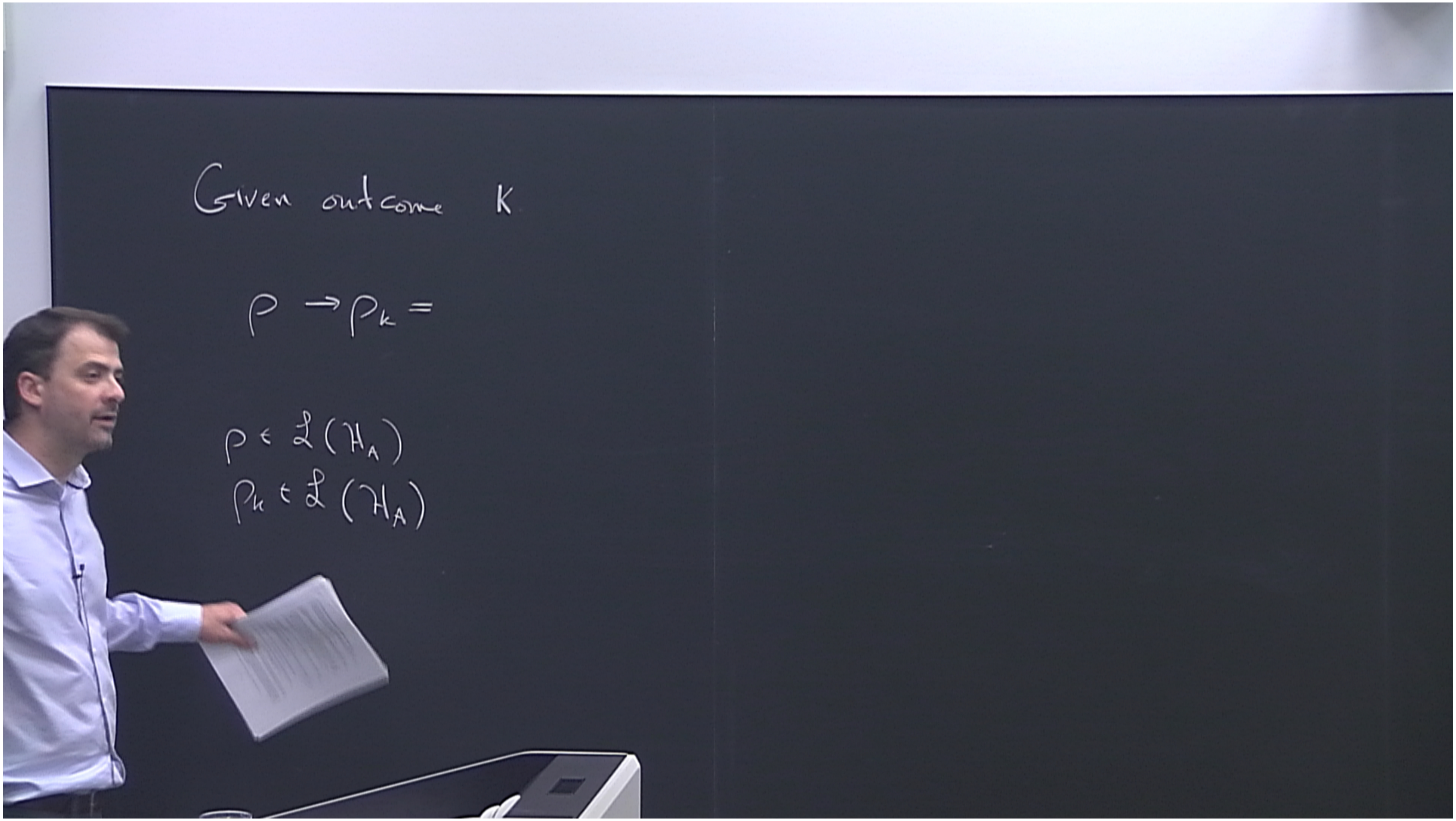
State-update in the indirect measurement model

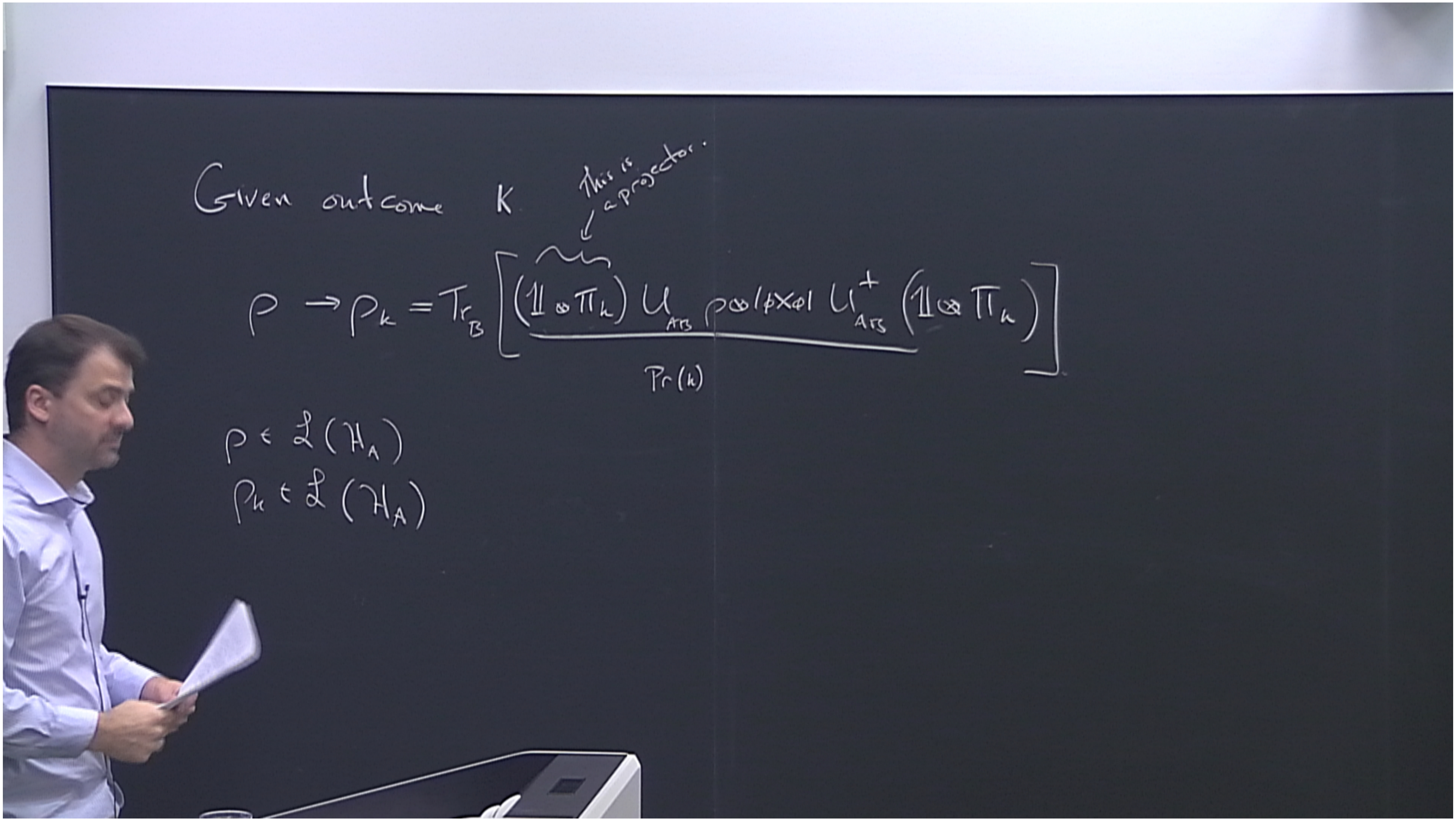
$$\rho \otimes |\phi\rangle\langle\phi|$$

depends on how you do the measurement.

State-update in the indirect measurement model

$$(\mathbb{1}\otimes\Pi_k)\,U_{AB}\,\rho\otimes|\phi\rangle\langle\phi|\,U_{AB}^{\dagger}\,(\mathbb{1}\otimes\Pi_k)$$


Given outcome K
$\rho \to \rho_k =$
$\rho \in \mathcal{L}(\mathcal{H}_A)$
$\rho_k \in \mathcal{L}(\mathcal{H}_A)$


Given outcome K
this is a projector.
$\rho \to \rho_k = \mathrm{Tr}_B \left[\frac{(\mathbb{1} \otimes \Pi_k) U_{AB} \, \rho \otimes |\phi\rangle\langle\phi| \, U_{AB}^{\dagger}}{\Pr(k)} (\mathbb{1} \otimes \Pi_k) \right]$
$\rho \in \mathcal{L}(\mathcal{H}_A)$
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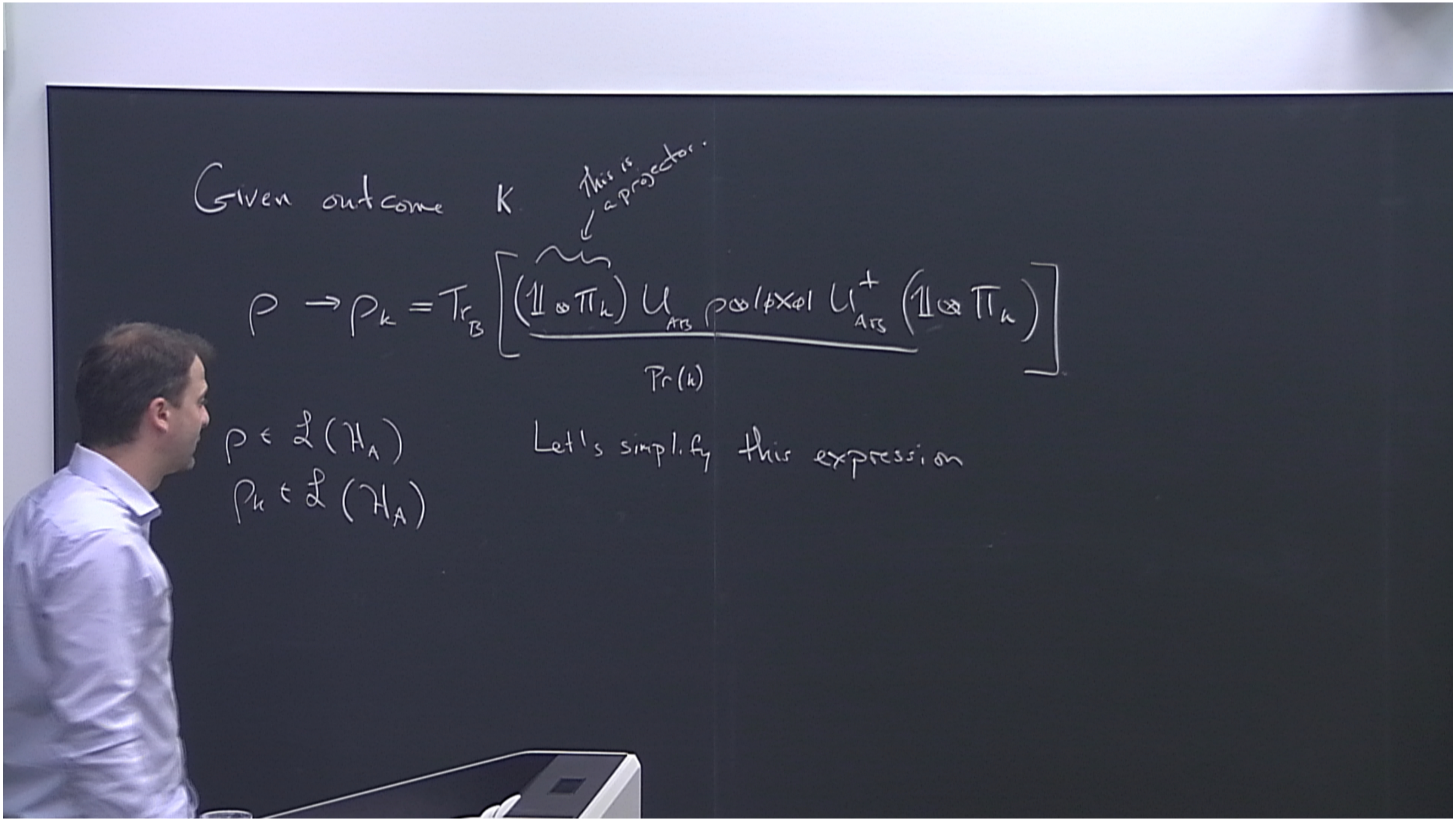
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$\rho \in \mathcal{L}(\mathcal{H}_A)$
$\rho_k \in \mathcal{L}(\mathcal{H}_A)$
Let's simplify this expression

Given outcome K

this is a projector.

$$\rho \to \rho_k = \mathrm{Tr}_B \left[\frac{(\mathbb{1} \otimes \Pi_k)\, U_{AB}\, \rho \otimes |\phi\rangle\langle\phi|\, U^{\dagger}_{AB}\, (\mathbb{1} \otimes \Pi_k)}{\Pr(k)} \right]$$

$\rho \in \mathcal{L}(\mathcal{H}_A)$

$\rho_k \in \mathcal{L}(\mathcal{H}_A)$

Let's simplify this expression

for simplicity choose $|\phi\rangle = |0\rangle$

For partial trace, use $|j\rangle$

where $\{|j\rangle\}$, $j \in \{0, 1, \ldots, \dim(\mathcal{H}_B) - 1\}$

Given outcome K

$$\rho \to \rho_k = \mathrm{Tr}_B\left[\frac{\overbrace{(\mathbb{1}\otimes\Pi_k)}^{\text{this is a projector.}}\, U_{AB}\, \rho\otimes|\phi\rangle\langle\phi|\, U_{AB}^{\dagger}}{Pr(k)}\,(\mathbb{1}\otimes\Pi_k)\right]$$

$\rho \in \mathcal{L}(\mathcal{H}_A)$

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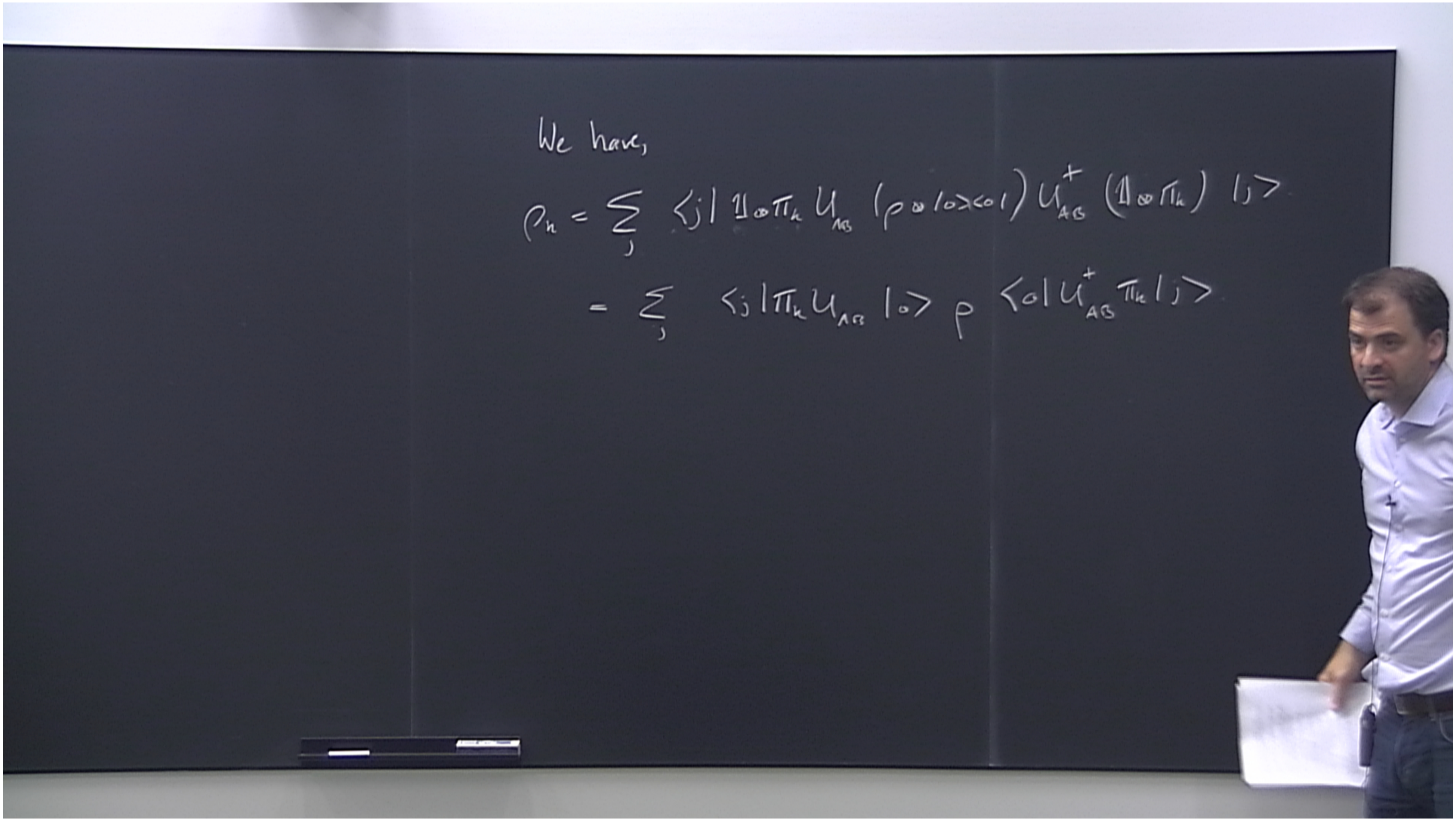
We have,

$$\rho_k = \sum_j \langle j| \, \mathbb{1}\otimes\Pi_k \, U_{AB} \, (\rho\otimes|0\rangle\langle 0|) \, U^{\dagger}_{AB} \, (\mathbb{1}\otimes\Pi_k) \, |j\rangle$$


We have,
$\rho_k = \sum_j \langle j | \mathbb{1} \otimes \Pi_k U_{AB} (\rho \otimes |0\rangle\langle 0|) U_{AB}^{\dagger} (\mathbb{1} \otimes \Pi_k) | j \rangle$
$= \sum_j \langle j | \Pi_k U_{AB} | 0 \rangle \rho \langle 0 | U_{AB}^{\dagger} \Pi_k | j \rangle$

We have,

$$Pr(k)\, \rho_k = \sum_j \langle j | \mathbb{1} \otimes \Pi_k U_{AB} (\rho \otimes |0\rangle\langle 0|) U_{AB}^{\dagger} (\mathbb{1} \otimes \Pi_k) | j \rangle$$

$$= \sum_j \langle j | \Pi_k U_{AB} | 0 \rangle \rho \langle 0 | U_{AB}^{\dagger} \Pi_k | j \rangle$$

Define. $M_{k,j} := \langle j | \Pi_k U_{AB} | 0 \rangle \in \mathcal{L}(\mathcal{H}_A)$

Then $\hat{\rho}_k = \sum_j \dfrac{M_{k,j}\, \rho\, M_{k,j}^{\dagger}}{Pr(k)}$, $\quad Pr(k) = Tr(E_k\, \rho)$

where $E_k = \sum_j M_{k,j}^{\dagger} M_{k,j}$.

Remark

If $U_{AB} \to (V \otimes \mathbb{1}_B) U_{AB}$

We have,

$$\Pr(k)\, \rho_k = \sum_j \langle j | \mathbb{1} \otimes \Pi_k U_{AB} (\rho \otimes |0\rangle\langle 0|) U_{AB}^{\dagger} (\mathbb{1} \otimes \Pi_k)$$

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Then $\hat{\rho}_k = \sum_j \dfrac{M_{k,j} \rho M_{k,j}^{\dagger}}{\Pr(k)}$, where $E_k = \sum_j$

$\Pr(k) = \mathrm{Tr}$

Remark

If $U_{AB} \to (V \otimes 1\!\!1_B)\, U_{AB}$

Then $\rho_k \to \rho_k' = V \rho_k V^\dagger$

but $Pr(k) = Tr(E_k' \rho)$

& $E_k' = E_k$

We have,

$$Pr(k)\, \rho_k = \sum_j \langle j |\, 1\!\!1 \otimes \Pi_k\, U_{AB} \left(\rho \otimes |0\rangle\langle 0| \right) U_{AB}^\dagger \left(1\!\!1 \otimes \Pi_k \right)$$

$$= \sum_j \langle j | \Pi_k U_{AB} | 0 \rangle\, \rho\, \langle 0 | U_{AB}^\dagger \Pi_k | j \rangle$$

Define $M_{k,j} := \langle j | \Pi_k U_{AB} | 0 \rangle \in \mathcal{L}(\mathcal{H}_A)$

Then $\hat{\rho}_k = \sum_j \frac{M_{k,j}\, \rho\, M_{k,j}^\dagger}{Pr(k)}$, where $E_k = \sum_j$

$Pr(k) = Tr$

__Remark__

If $U_{AB} \to (V \otimes \mathbb{1}_B)\, U_{AB}$

Then $\rho_k \to \rho_k' = V \rho_k V^+$

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We have,

$Pr(k)\, \rho_k = \sum_j \langle j | \mathbb{1} \otimes \Pi_k$

$= \sum_j \langle j | \Pi_k$

Define $M_{k,j} :=$

Then $\hat{\rho}_k = \sum_j$