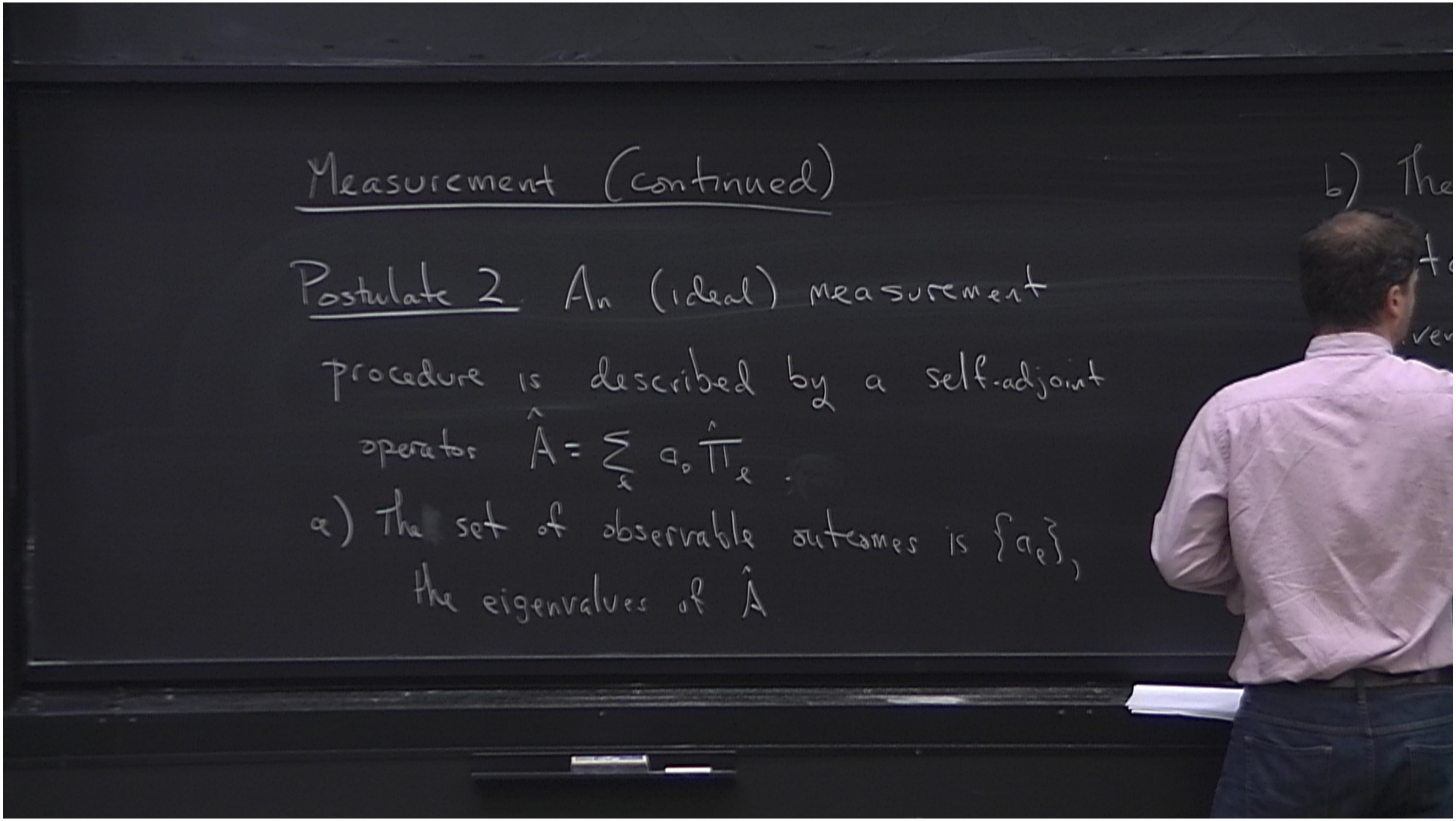
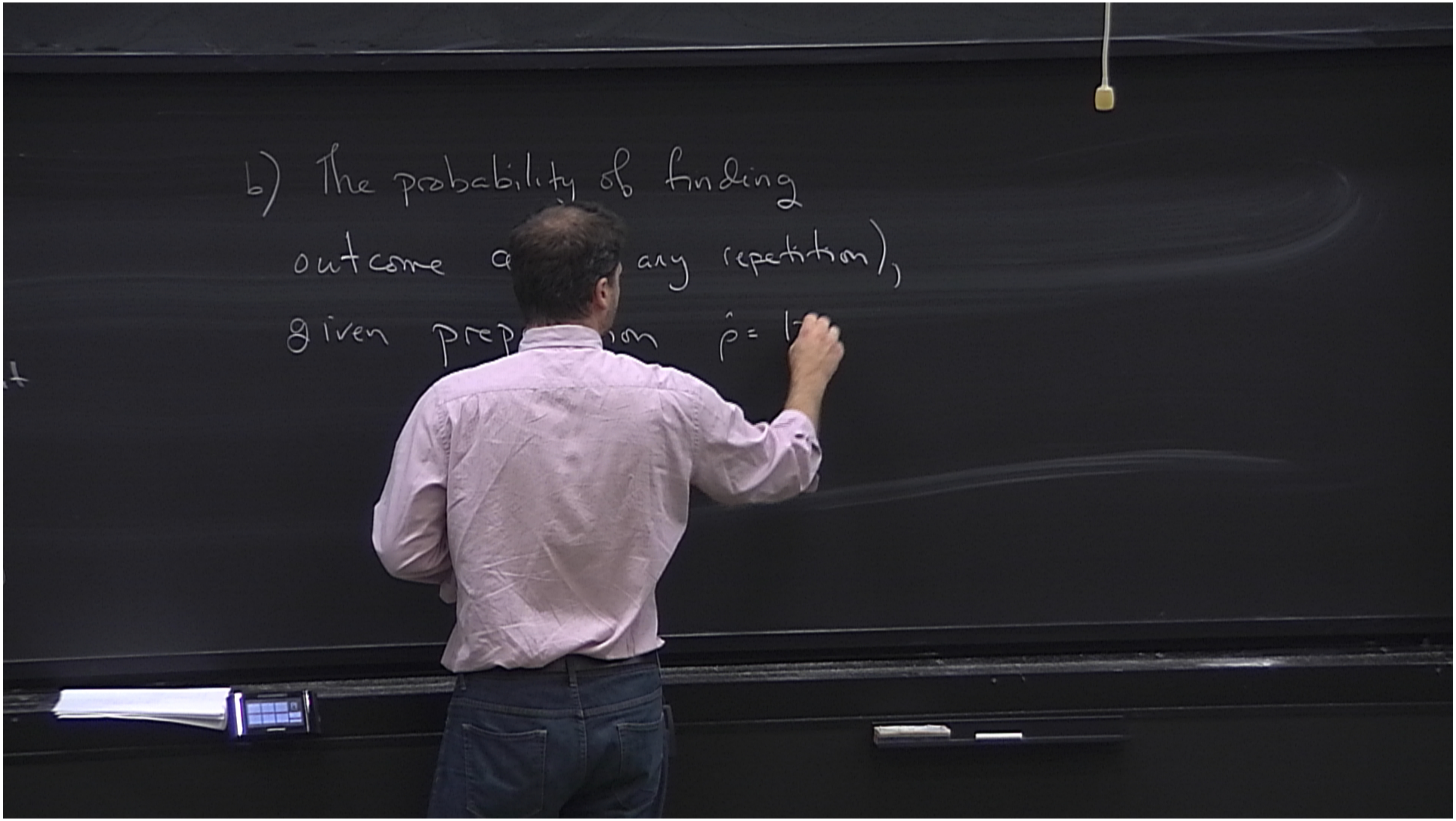
Title: 14/15 PSI Quantum Theory-3

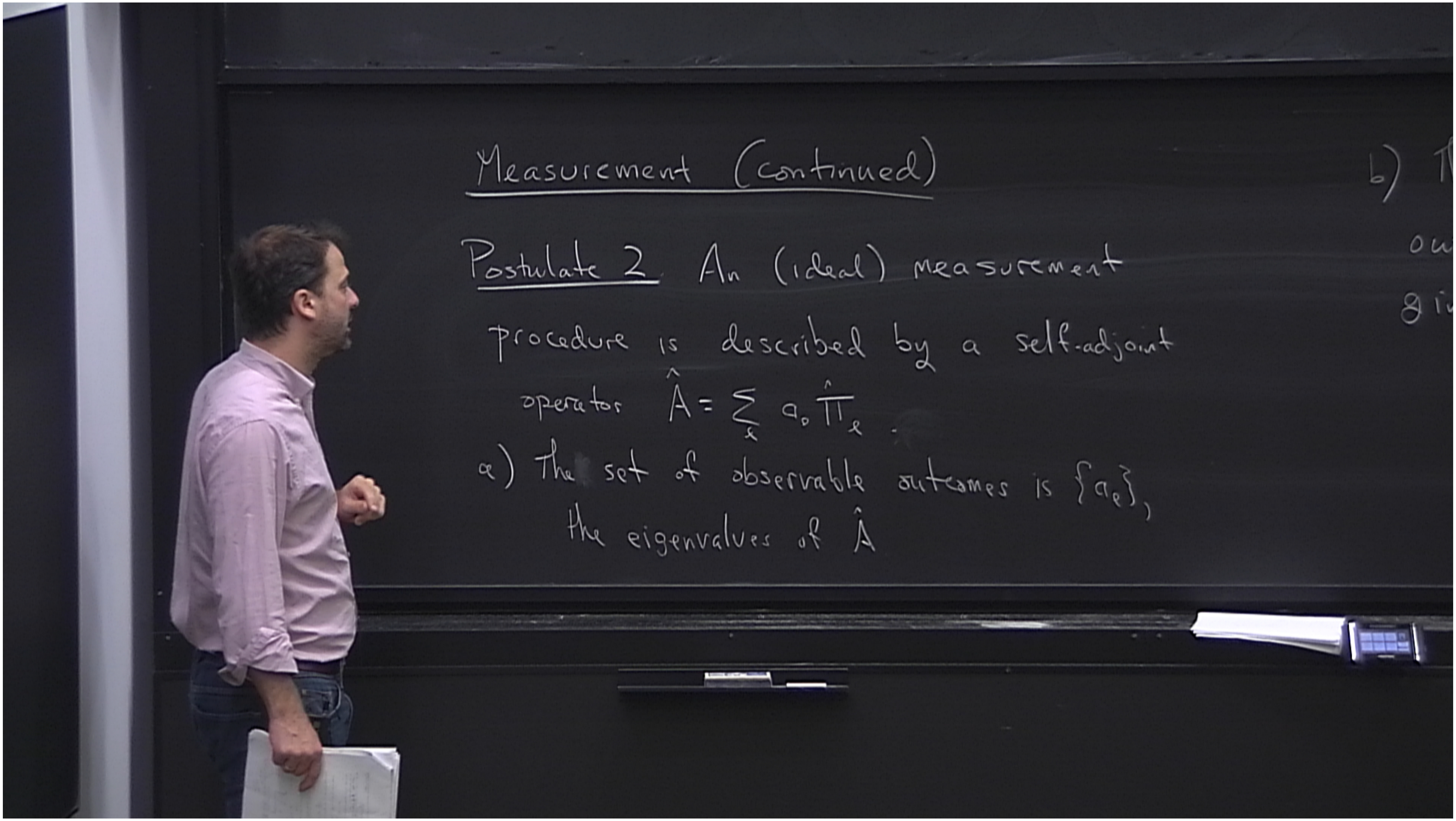
Date: Sep 10, 2014 10:45 AM

URL: http://pirsa.org/14090044

Abstract:


Measurement (continued)
Postulate 2 An (ideal) measurement procedure is described by a self-adjoint operator $\hat{A} = \sum_{\ell} a_{\ell} \hat{\Pi}_{\ell}$ .
a) The set of observable outcomes is $\{a_{\ell}\}$, the eigenvalues of $\hat{A}$
b) The


b) The probability of finding
outcome
any repetition),
given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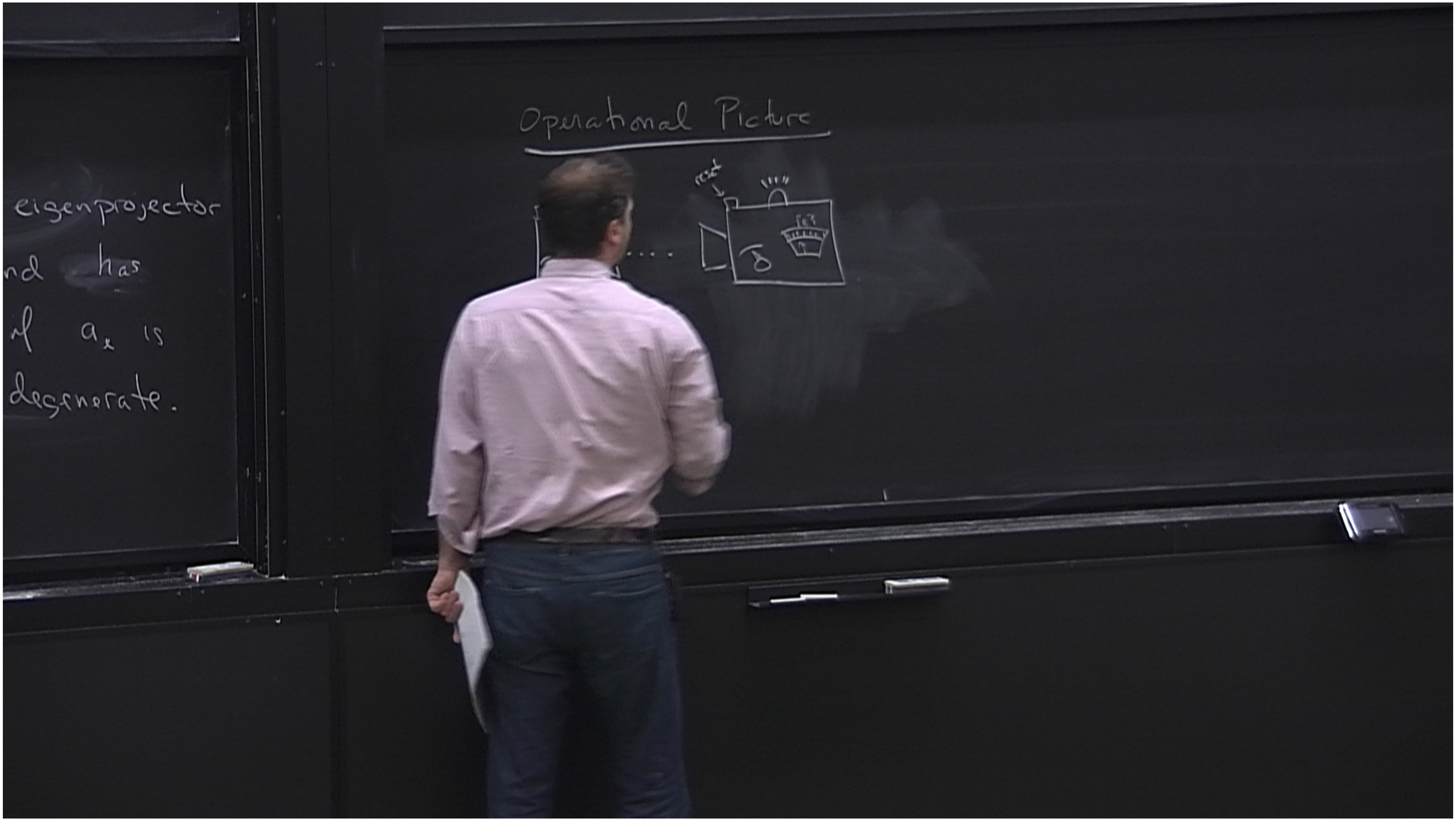
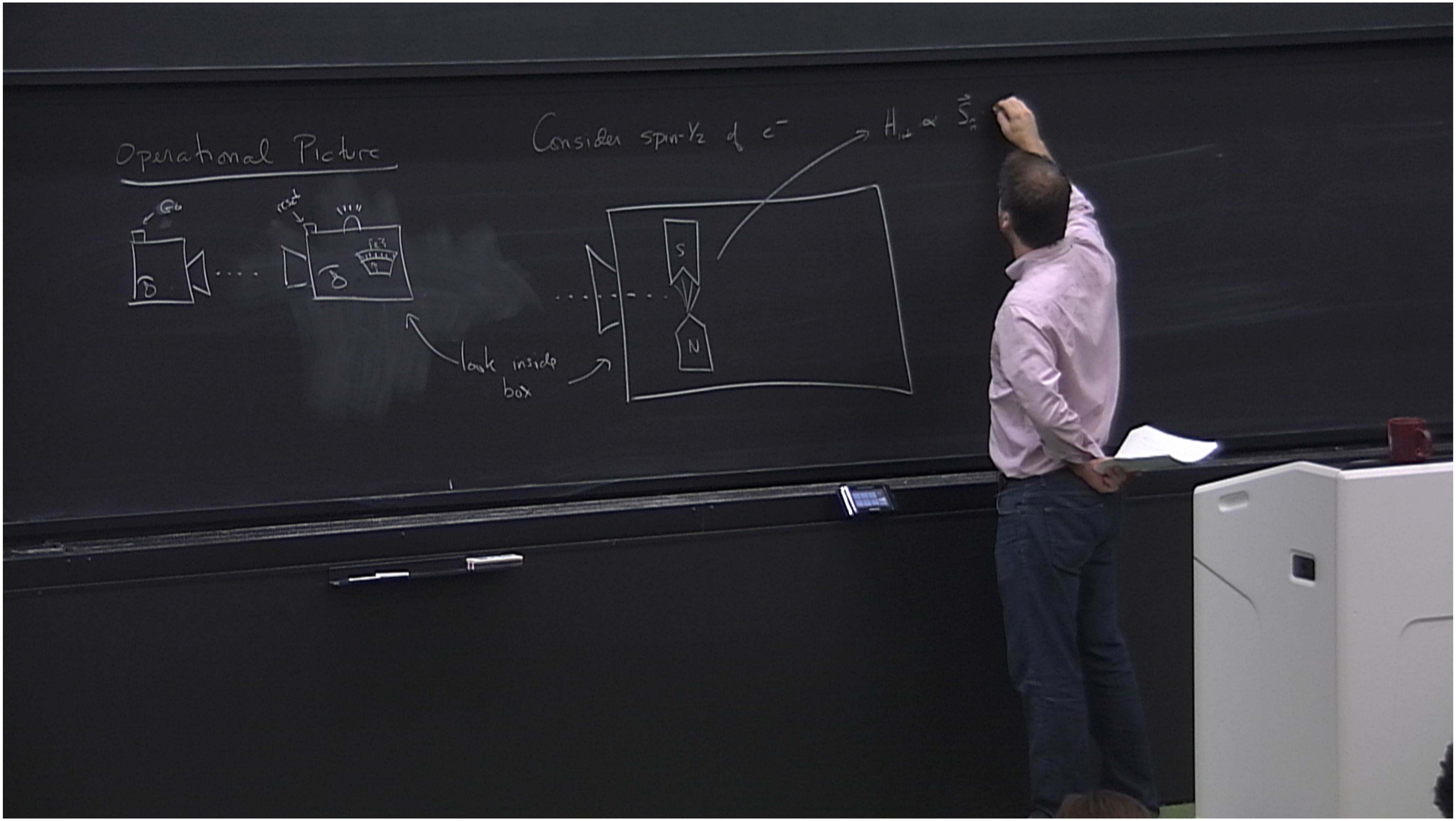
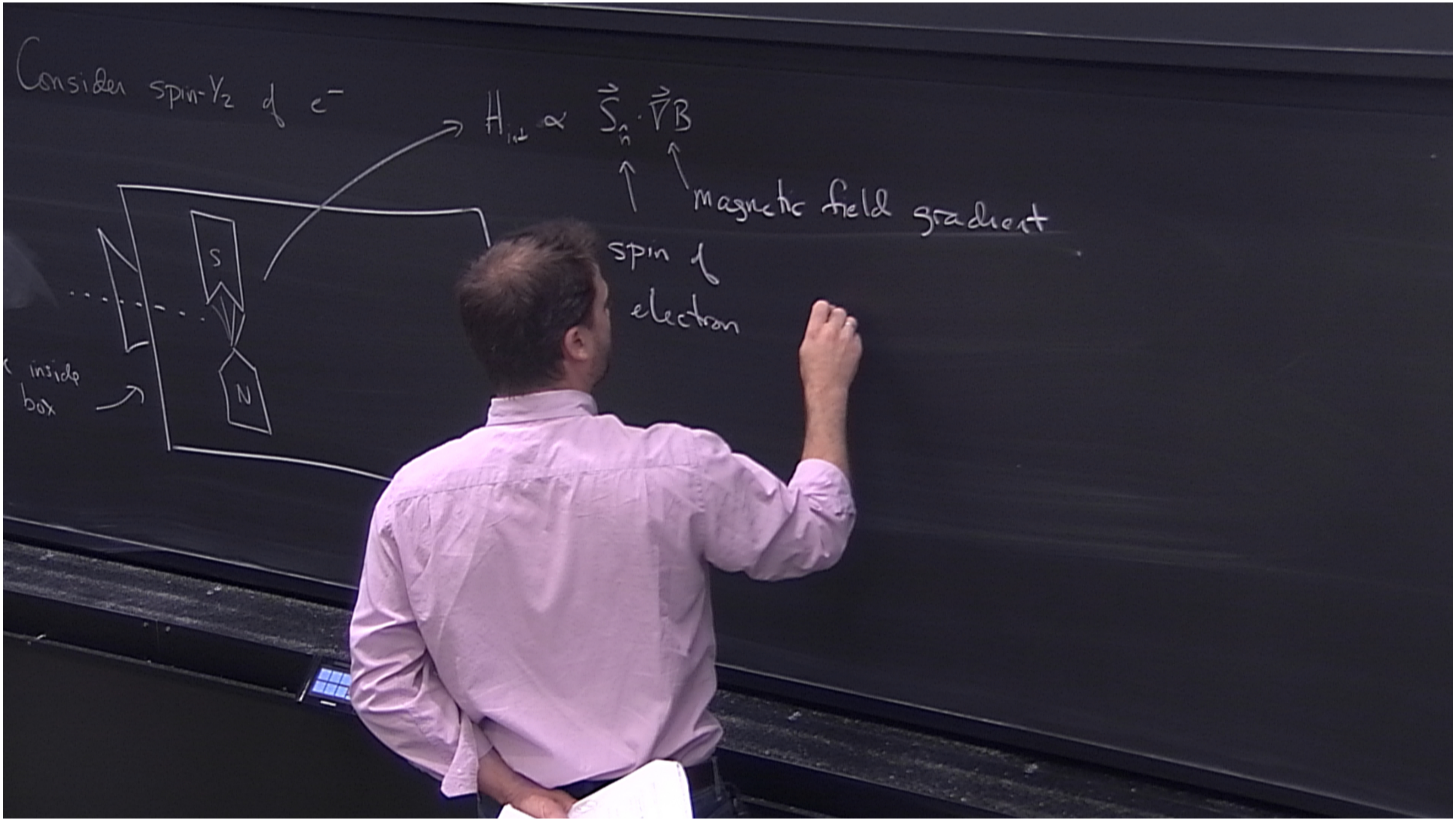
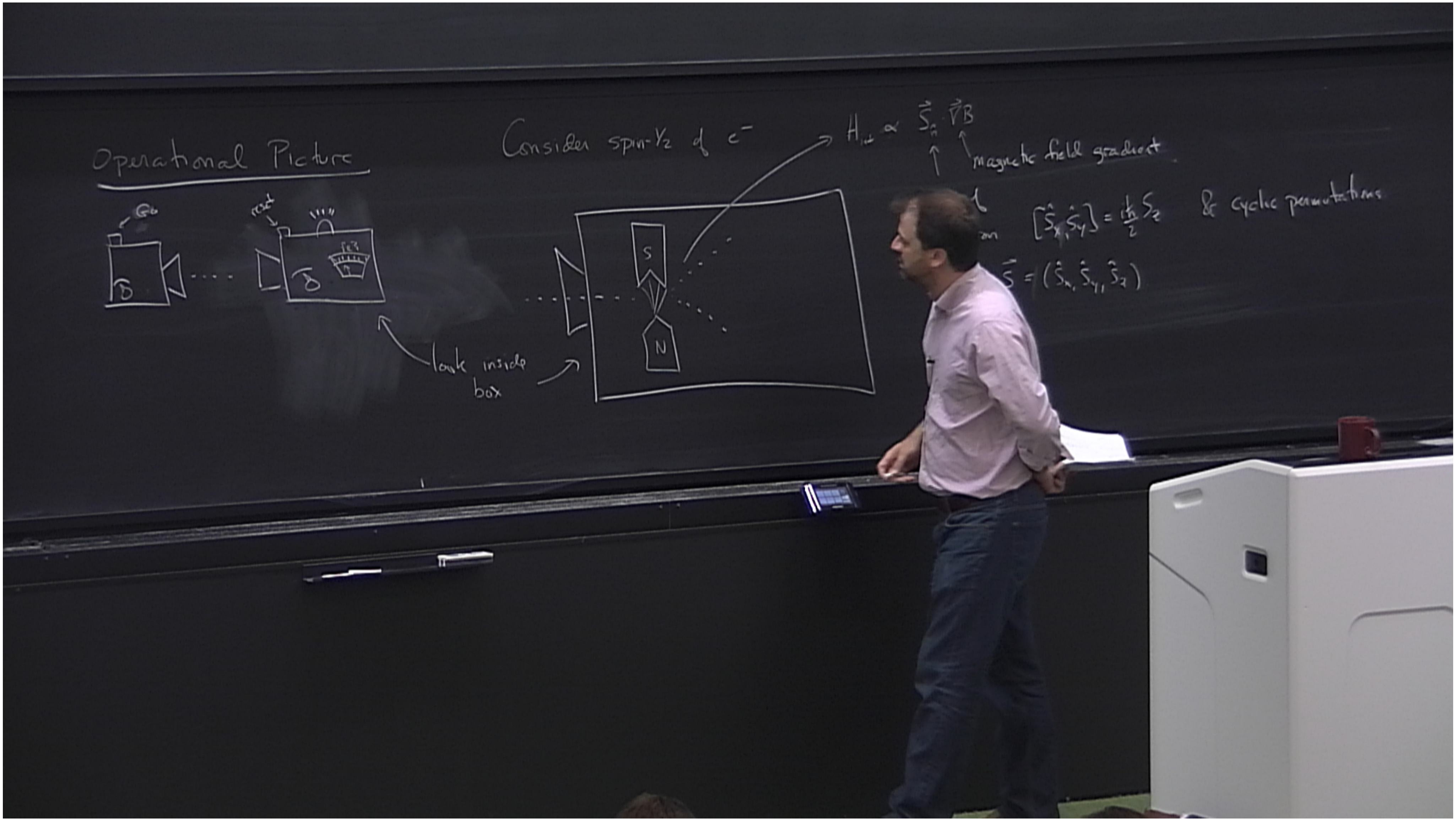
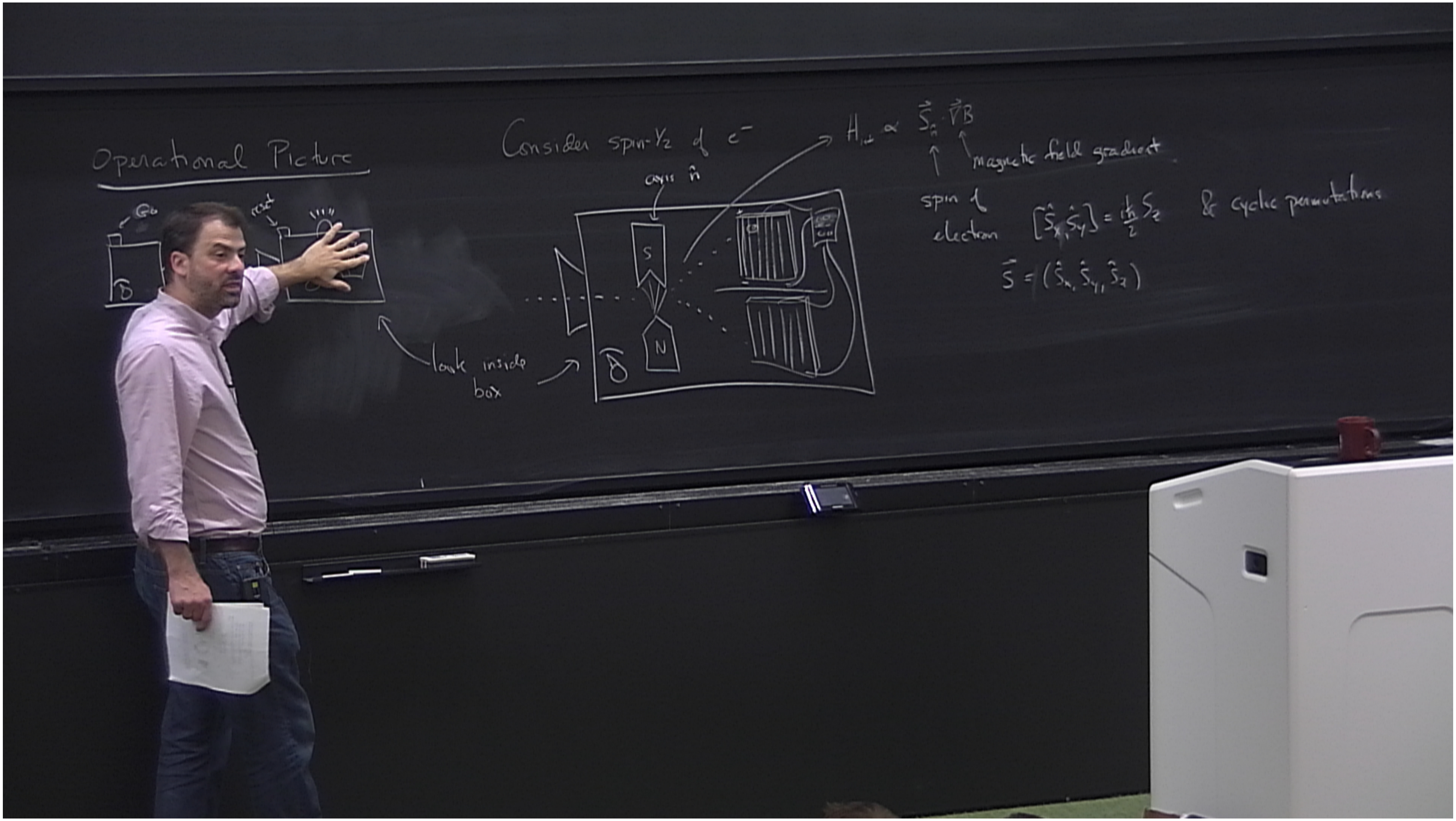

Operational Picture
reset
eigenprojector
nd has
degenerate.

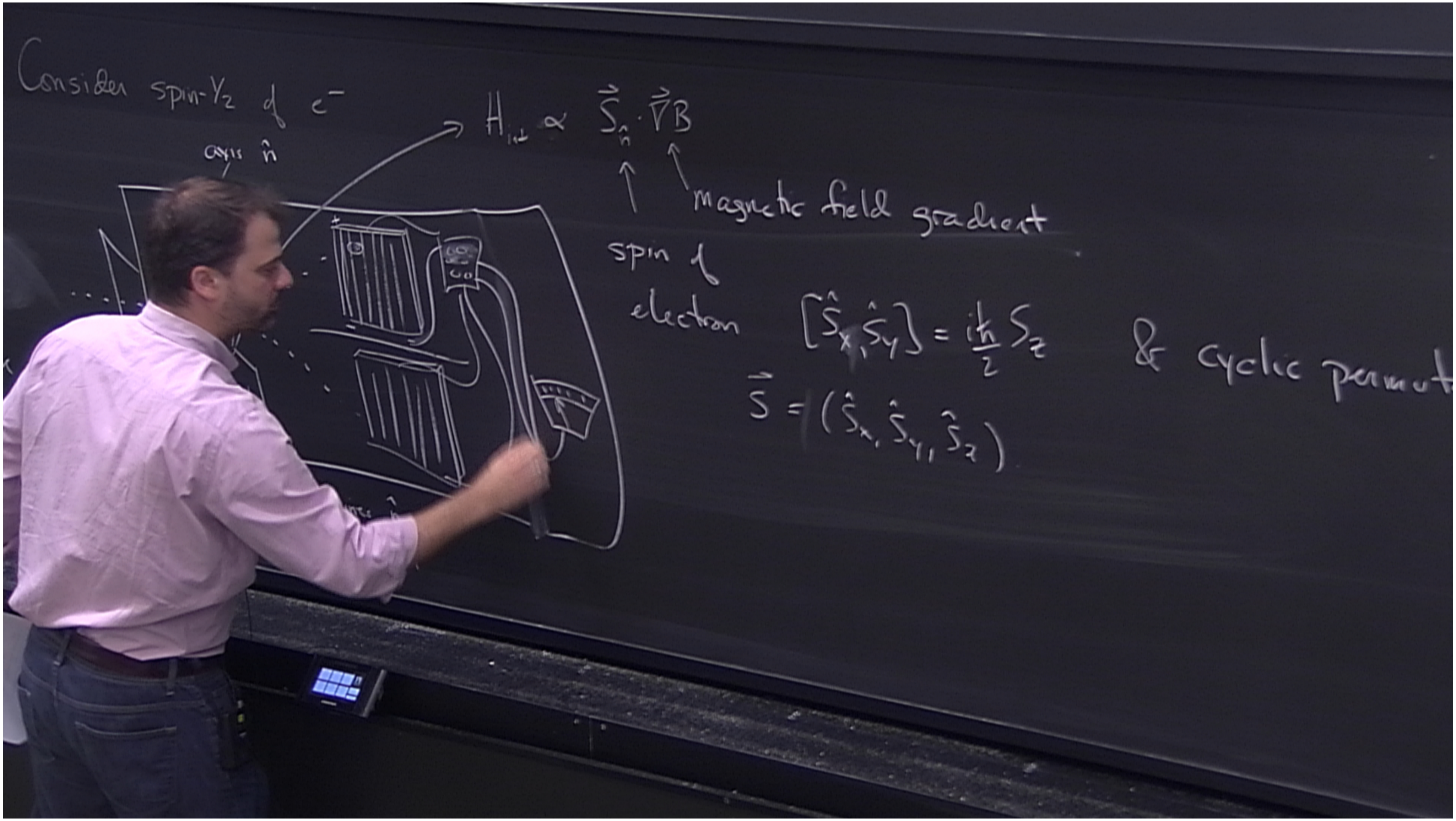

Operational Picture
look inside box
Consider spin-1/2 of e⁻
S
N

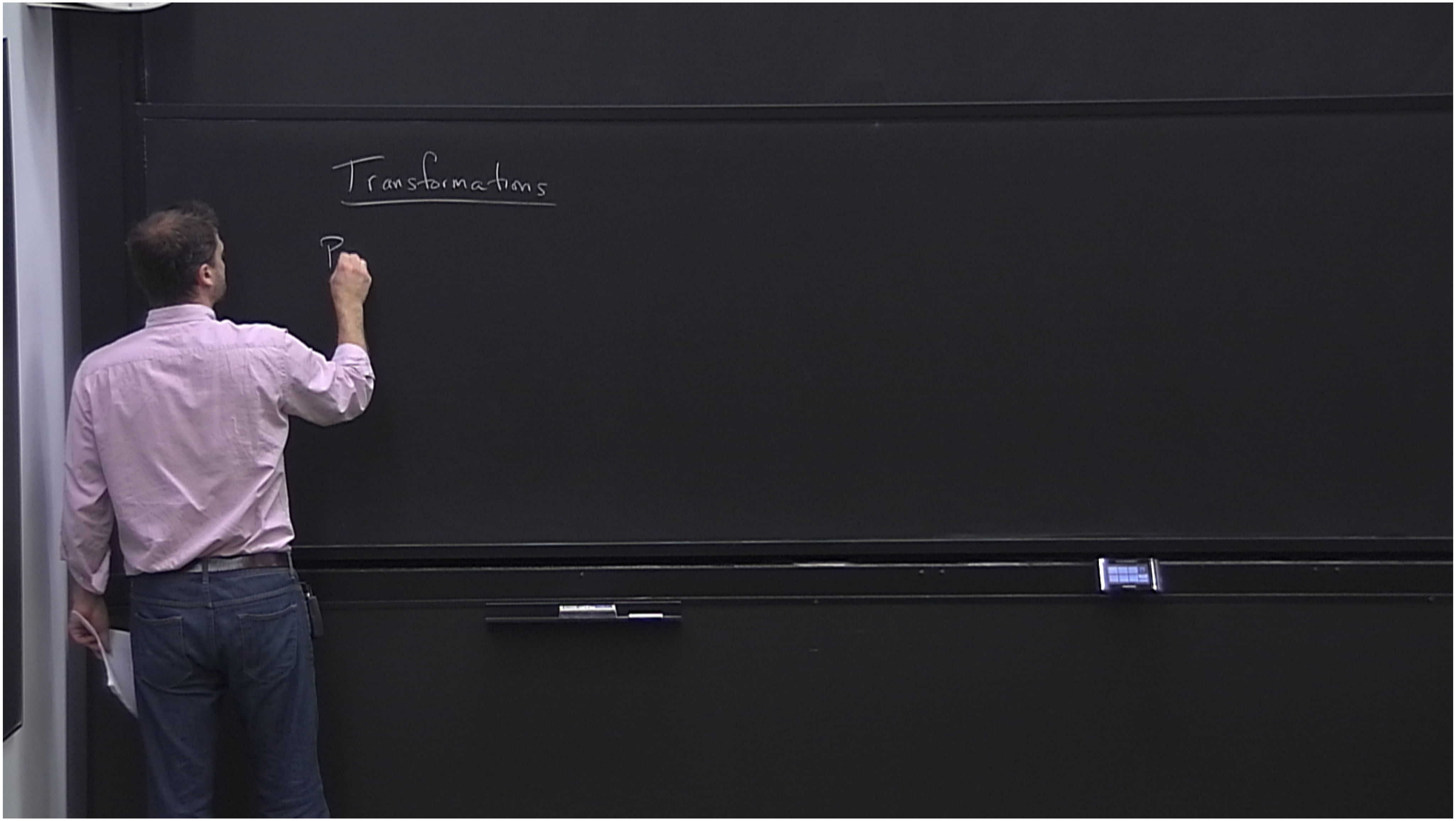

Consider spin-1/2 of $e^-$
$H_{int} \propto \vec{S}_{\hat{n}} \cdot \vec{\nabla} B$
magnetic field gradient
spin of electron
S
N
inside box

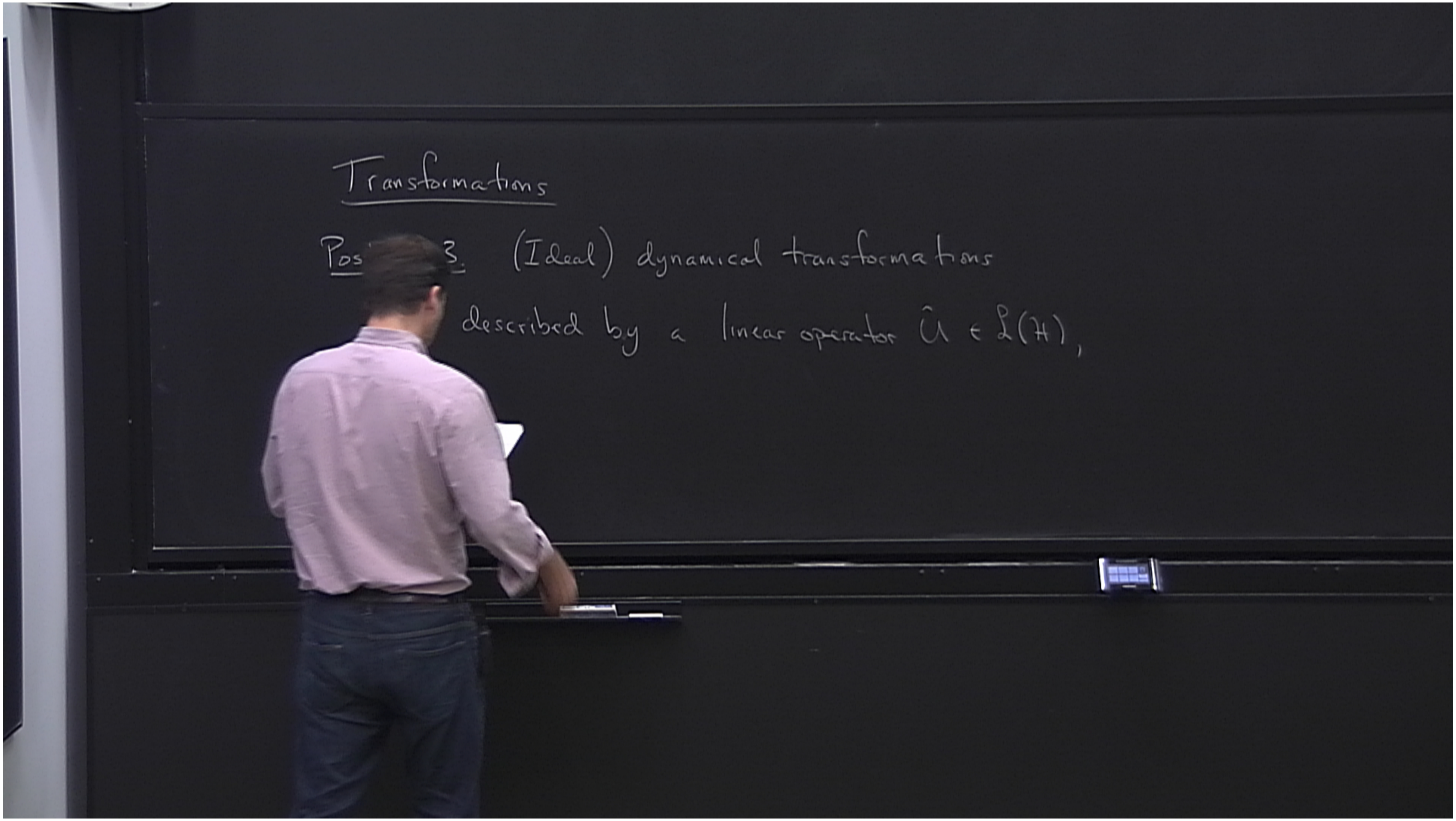

Operational Picture
Go
reset
look inside box
Consider spin-1/2 of e⁻
S
N
magnetic field gradient
& cyclic permutations

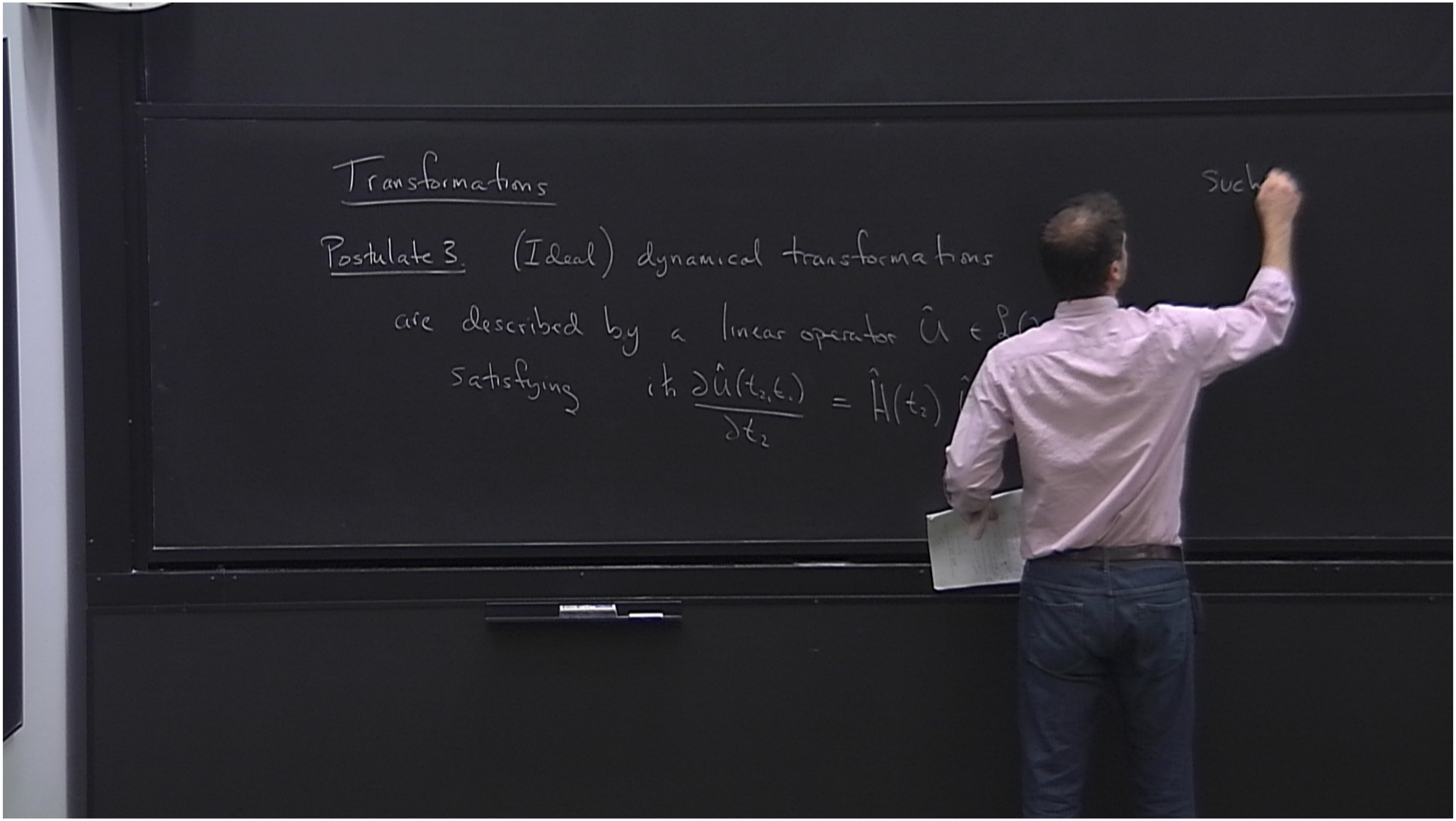


Operational Picture
Go
reset
look inside box
Consider spin-1/2 of e⁻
axis n̂
S
N
H_int ∝ S_n̂ · ∇B
magnetic field gradient
spin of electron
[Ŝ_x, Ŝ_y] = iħ/2 S_z
& cyclic permutations
S⃗ = (Ŝ_x, Ŝ_y, Ŝ_z)


Consider spin-$1/2$ of $e^-$
axis $\hat{n}$
$H_{int} \propto \vec{S}_{\hat{n}} \cdot \vec{\nabla} B$
magnetic field gradient
spin of electron
$[\hat{S}_x, \hat{S}_y] = i\frac{\hbar}{2} S_z$
& cyclic permut
$\vec{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$

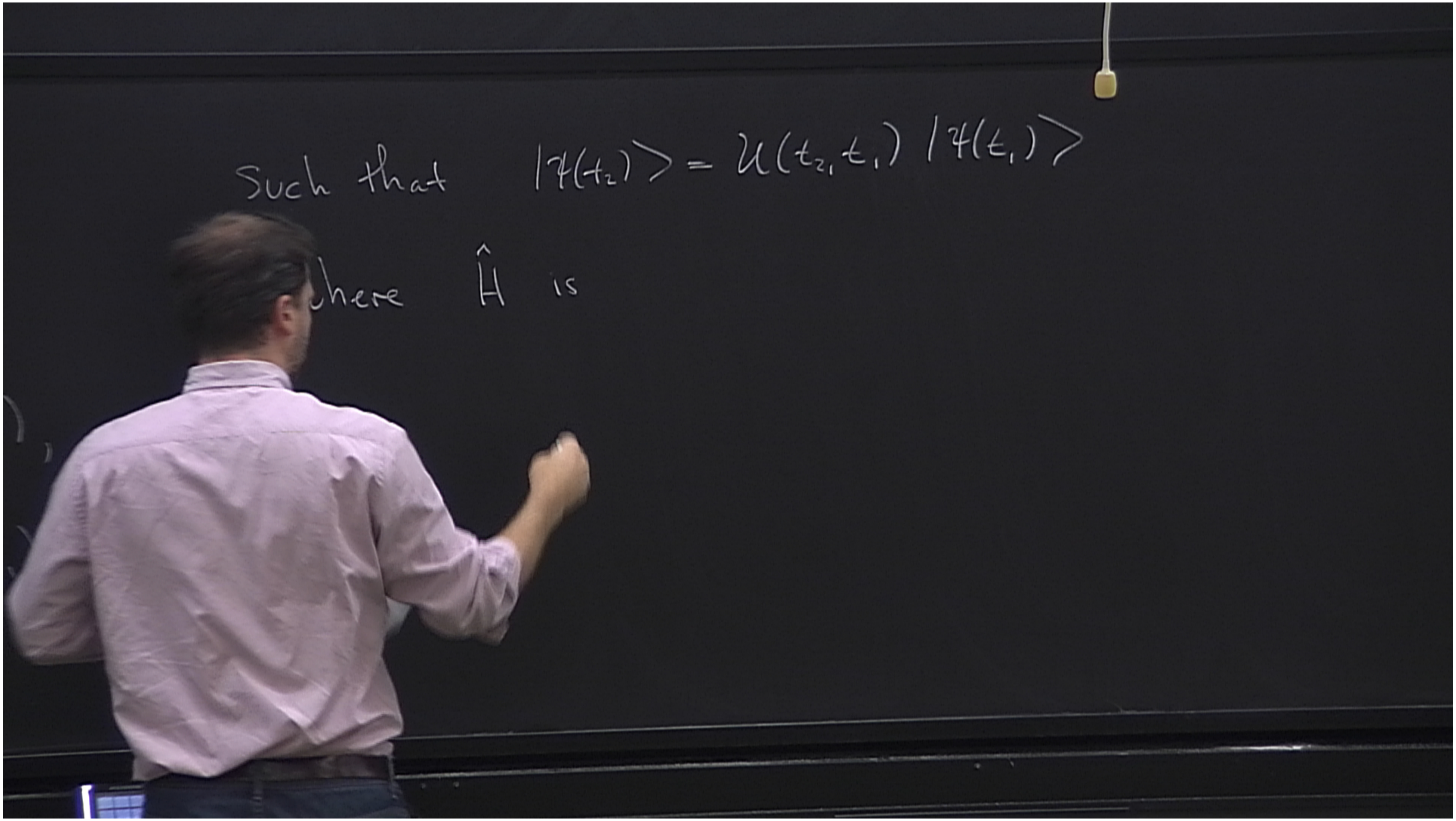

Transformations


Transformations
Pos 3. (Ideal) dynamical transformations
described by a linear operator $\hat{U} \in \mathcal{L}(\mathcal{H})$,


Transformations
Postulate 3. (Ideal) dynamical transformations
are described by a linear operator $\hat{U} \in \mathcal{L}($
satisfying $i\hbar \frac{\partial \hat{U}(t_2,t_1)}{\partial t_2} = \hat{H}(t_2)$
Such

## Transformations

**Postulate 3.** (Ideal) dynamical transformations are described by a linear operator $\hat{U} \in \mathcal{L}(\mathcal{H})$, satisfying
$$i\hbar \frac{\partial \hat{U}(t_2,t_1)}{\partial t_2} = \hat{H}(t_2)\,\hat{U}(t_2,t_1)$$


Such that $|\psi(t_2)\rangle = \mathcal{U}(t_2, t_1)\, |\psi(t_1)\rangle$
here $\hat{H}$ is

Such that $|\psi(t_2)\rangle = U(t_2, t_1)\,|\psi(t_1)\rangle$

where $\hat{H}(t)$ is the energy operator,

or Hamiltonian, $t \in \mathbb{R}$ is time, and $\hat{U}(t_1, t_1) = \hat{\mathbb{1}}$

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R ... $\mathcal{L}(\mathcal{H})$ denotes linear operators on $\mathcal{H}$

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<u>Remarks:</u>

- $\mathcal{L}(\mathcal{H})$ denotes linear operators on $\mathcal{H}$
- $\forall\, t_1, t_2$, $\mathcal{U}(t_1, t_2) : |\Psi(t_1)\rangle \longrightarrow |\Psi(t_2)\rangle$

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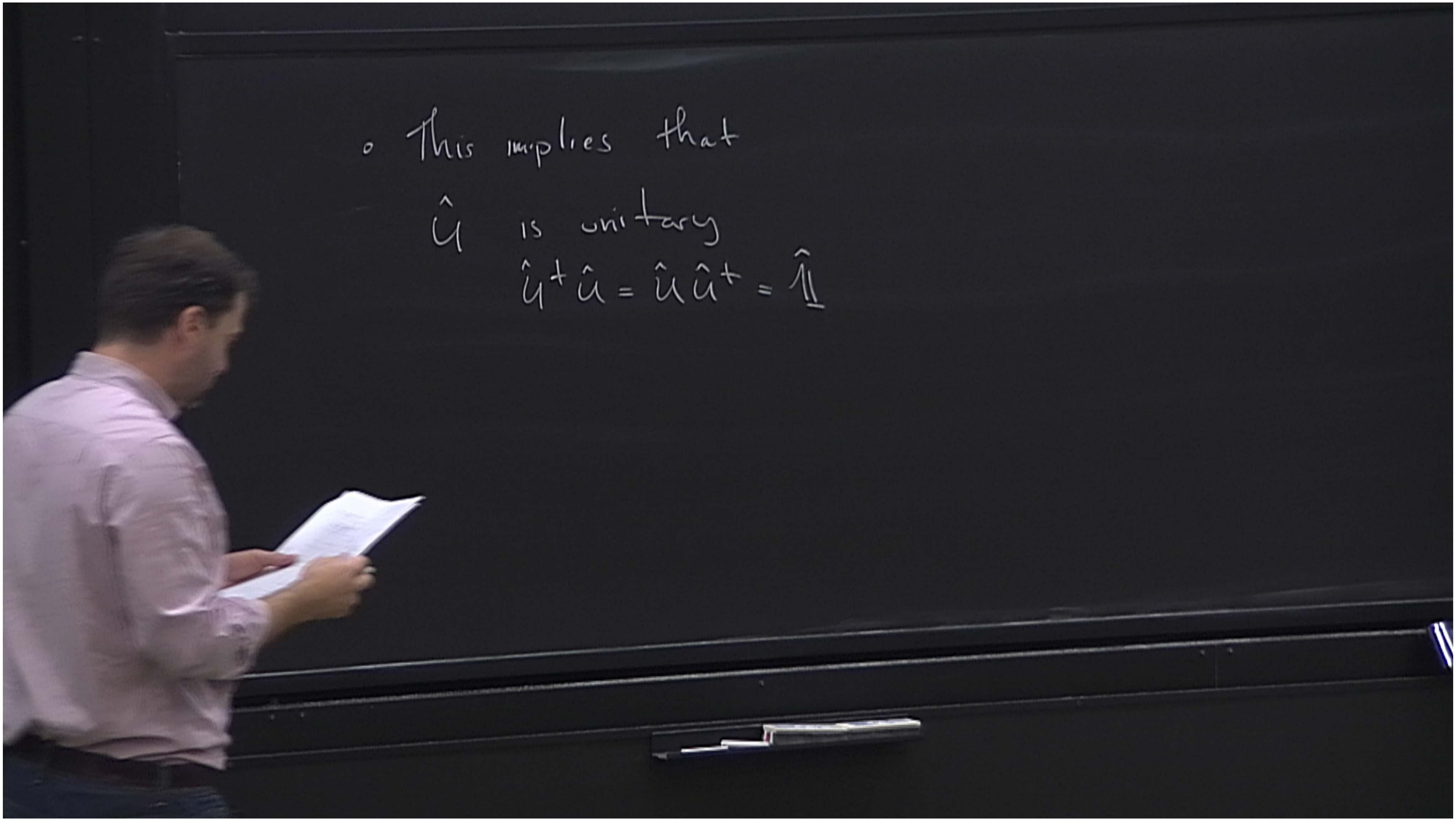
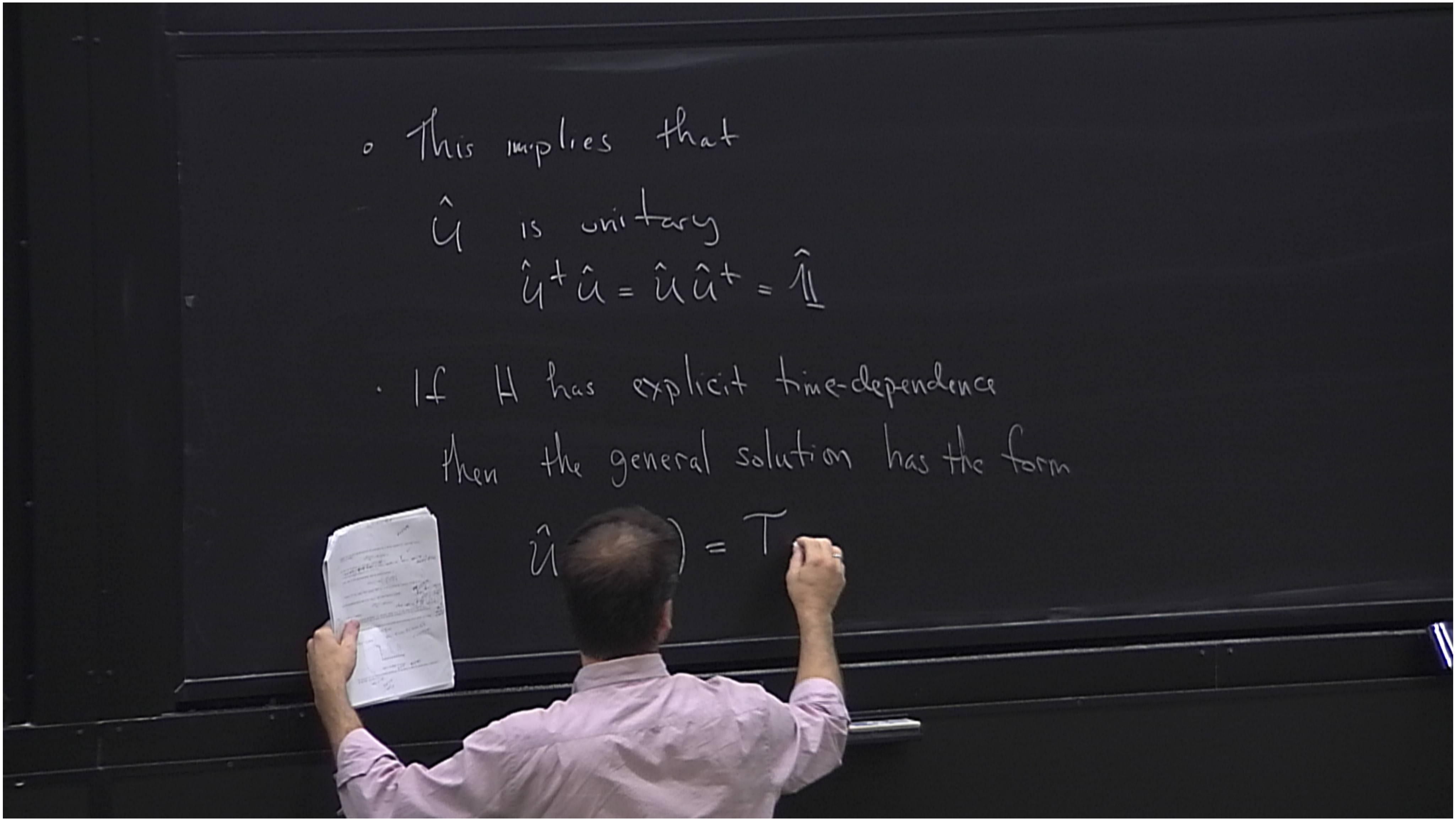

∘ This implies that
$\hat{u}$ is unitary
$\hat{u}^{+}\hat{u} = \hat{u}\hat{u}^{+} = \hat{\mathbb{1}}$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then the general solution has the form
$\hat{U}$ ) = T

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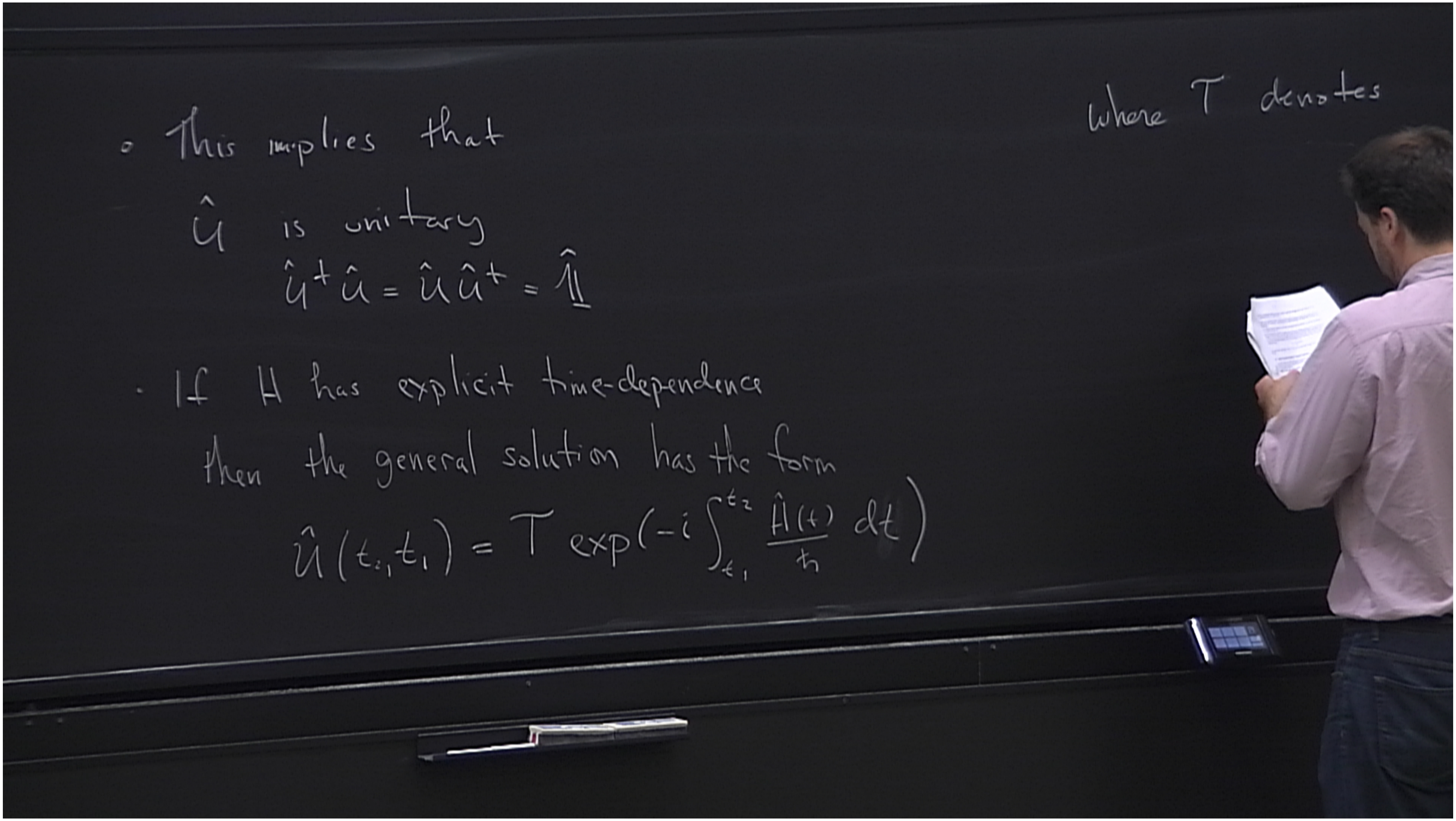
- If $H$ has explicit time-dependence then the general solution has the form

  $$\hat{U}(t_2, t_1) = T \exp\left(-i \int_{t_1}^{t_2} \frac{\hat{H}(t)}{\hbar}\, dt\right)$$



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$\hat{U}^\dagger \hat{U} = \hat{U}\hat{U}^\dagger = \hat{\mathbb{1}}$
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$\hat{U}(t_2,t_1) = T \exp\left(-i\int_{t_1}^{t_2} \frac{\hat{H}(t)}{\hbar}\, dt\right)$
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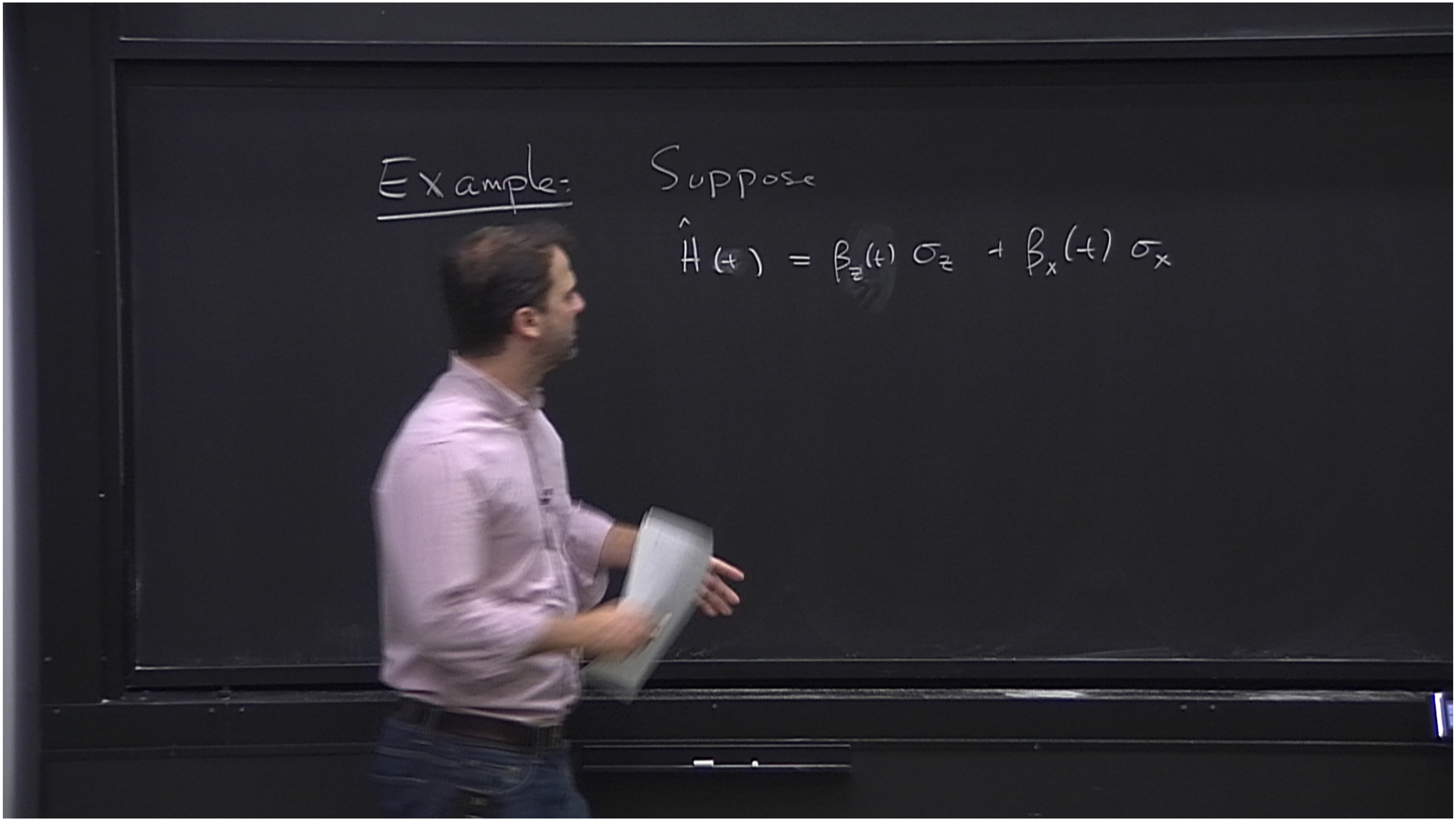
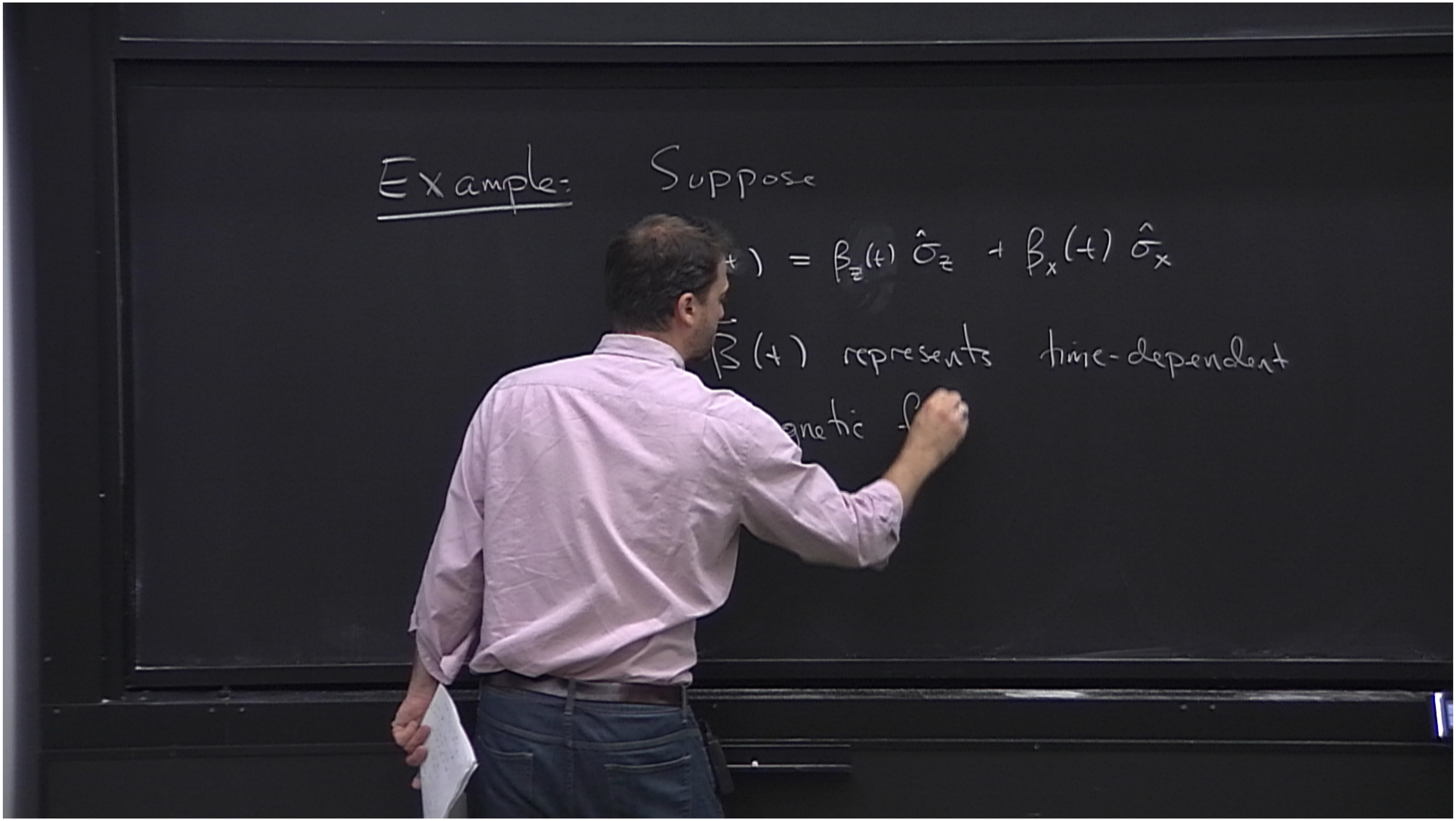
where $T$ denotes right-to-left

time-ordering.

$dt)$


Example:
Suppose
$\hat{H}(t) = \beta_z(t)\,\sigma_z + \beta_x(t)\,\sigma_x$


Example: Suppose
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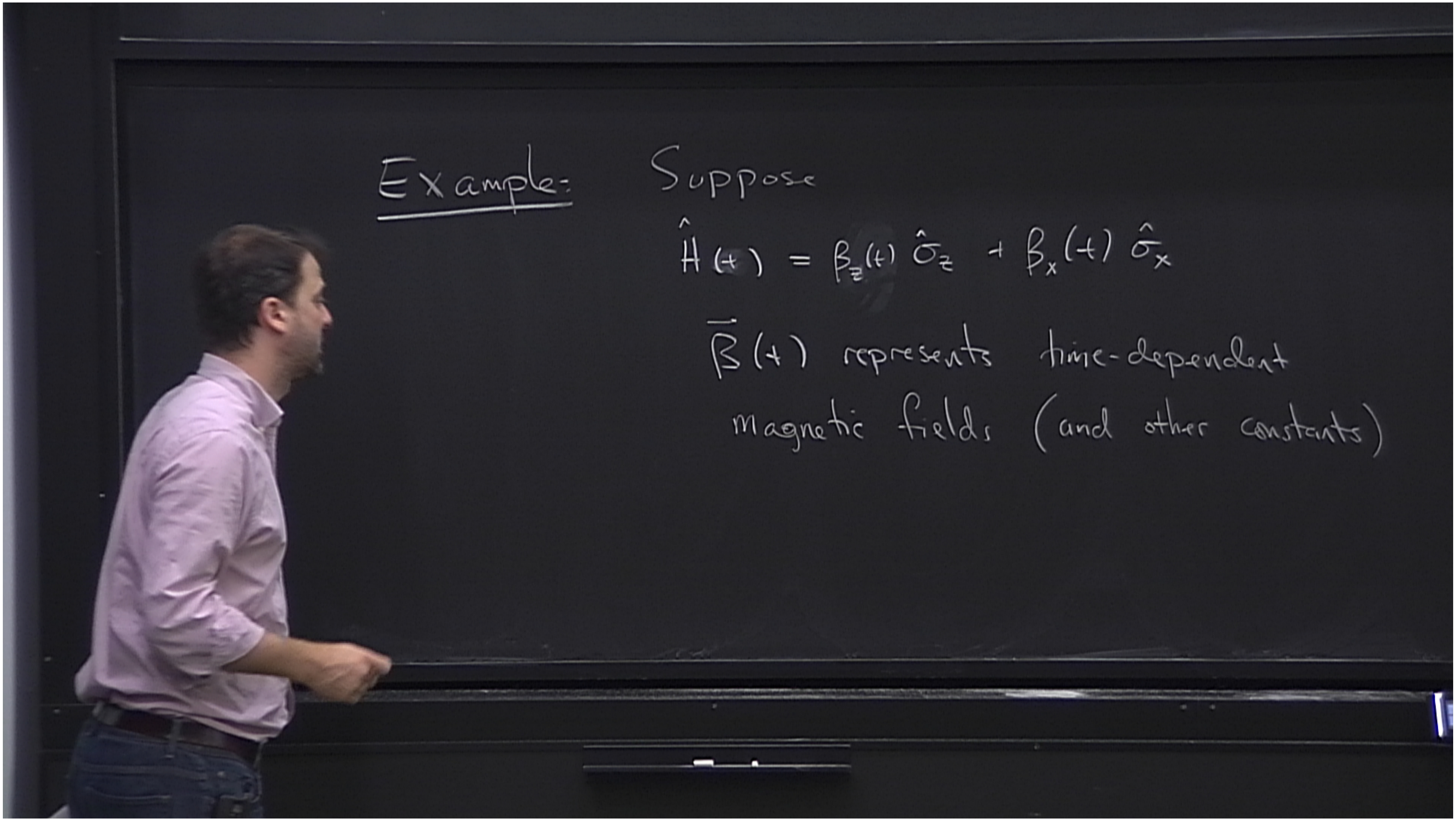
Example:
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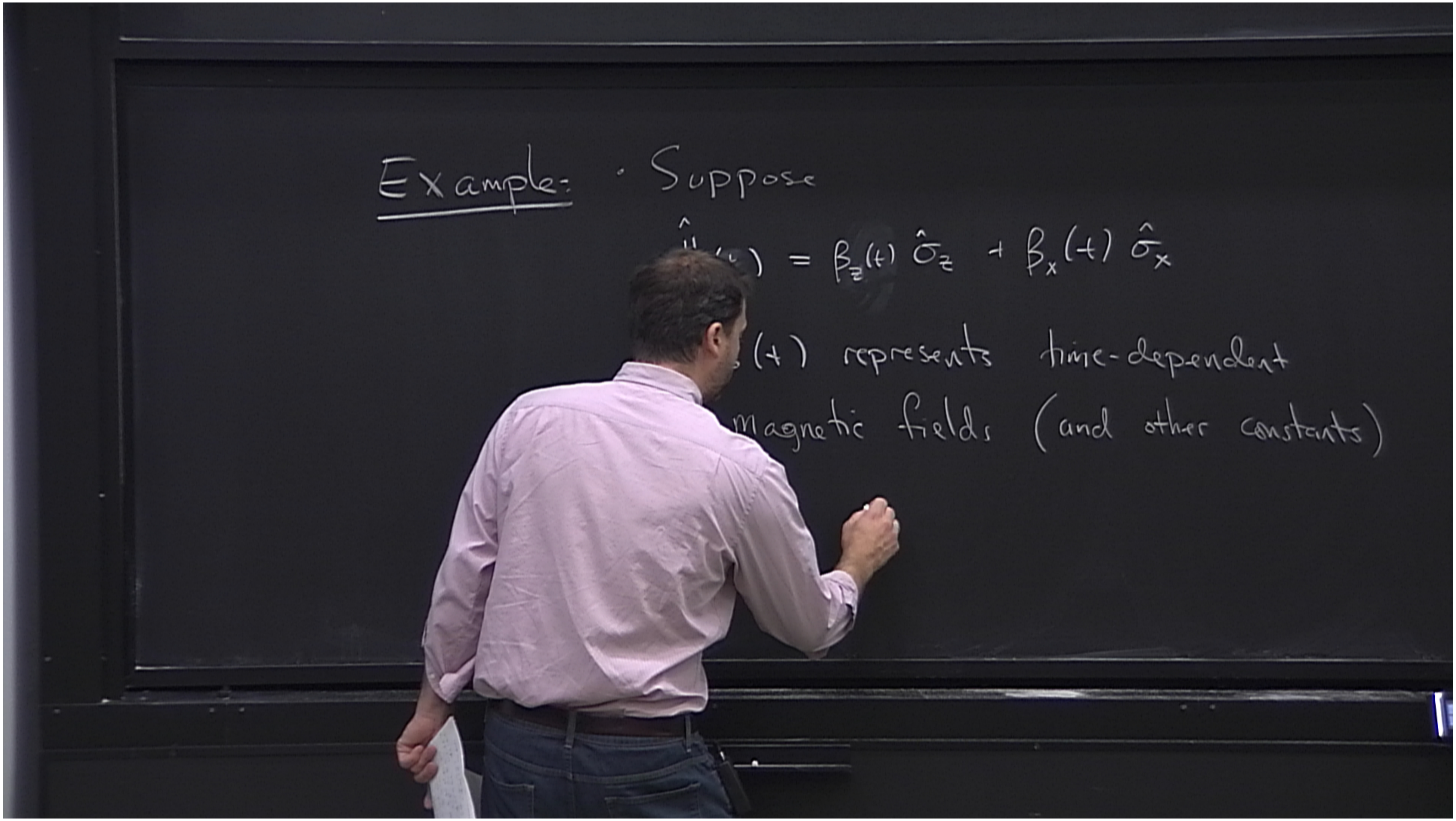
$\hat{H}(t) = \beta_z(t)\,\hat{\sigma}_z + \beta_x(t)\,\hat{\sigma}_x$
$\vec{\beta}(t)$ represents time-dependent
magnetic fields (and other constants)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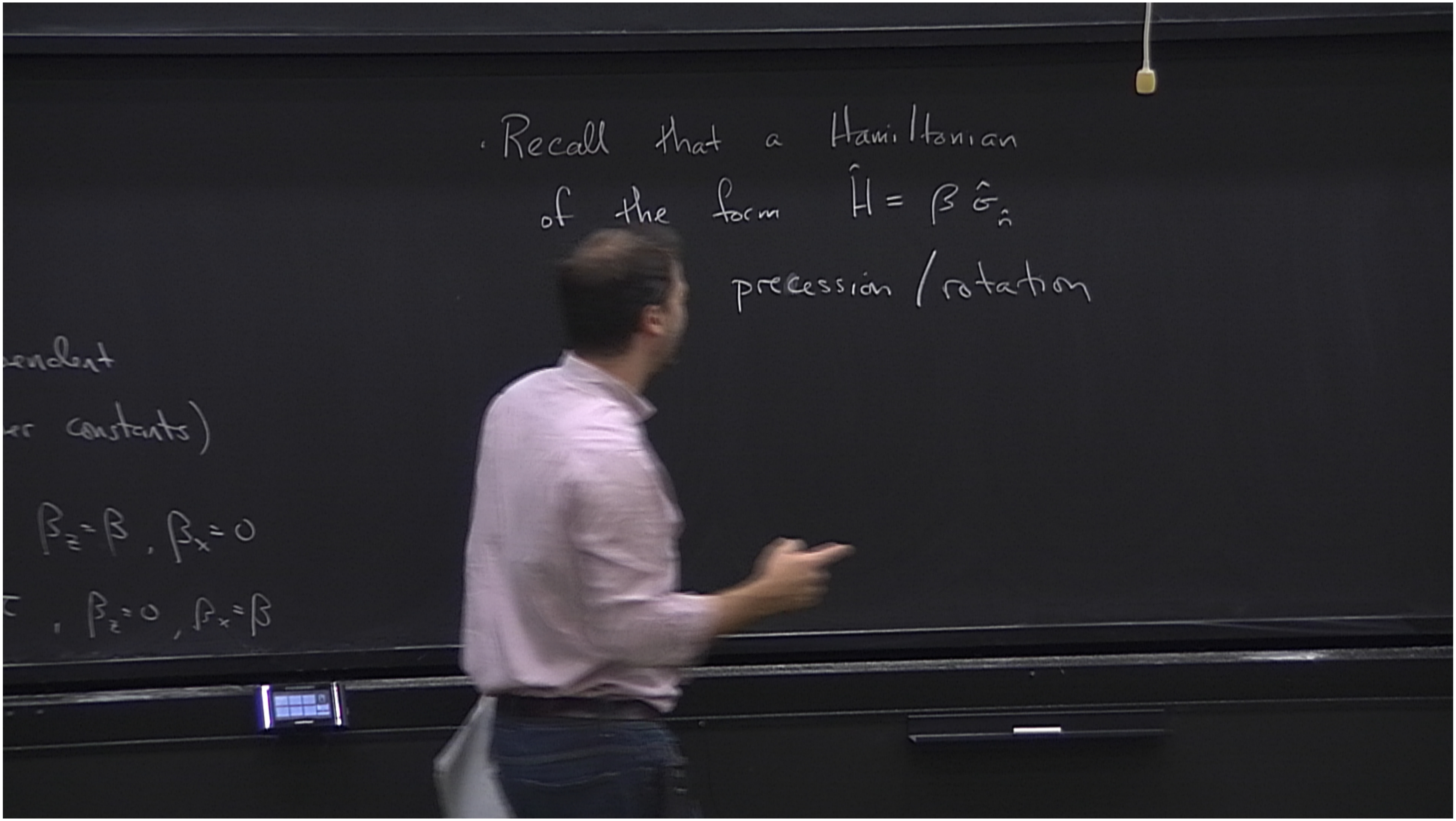
**Example:** · Suppose

$$\hat{H}(t) = \beta_z(t)\,\hat{\sigma}_z + \beta_x(t)\,\hat{\sigma}_x$$

$\vec{\beta}(t)$ represents time-dependent magnetic fields (and other constants)

· Suppose from $t_1 = 0$ to $\frac{t_2}{2} = \tau$, $\beta_z = \beta$, $\beta_x = 0$

& from $\frac{t_2}{2} = \tau$ to $t_2 = 2\tau$, $\beta_z = 0$, $\beta_x = \beta$


· Recall that a Hamiltonian
of the form $\hat{H} = \beta \hat{\sigma}_{\hat{n}}$
precession / rotation
constants)
$\beta_z = \beta$ , $\beta_x = 0$
, $\beta_z = 0$ , $\beta_x = \beta$

endent

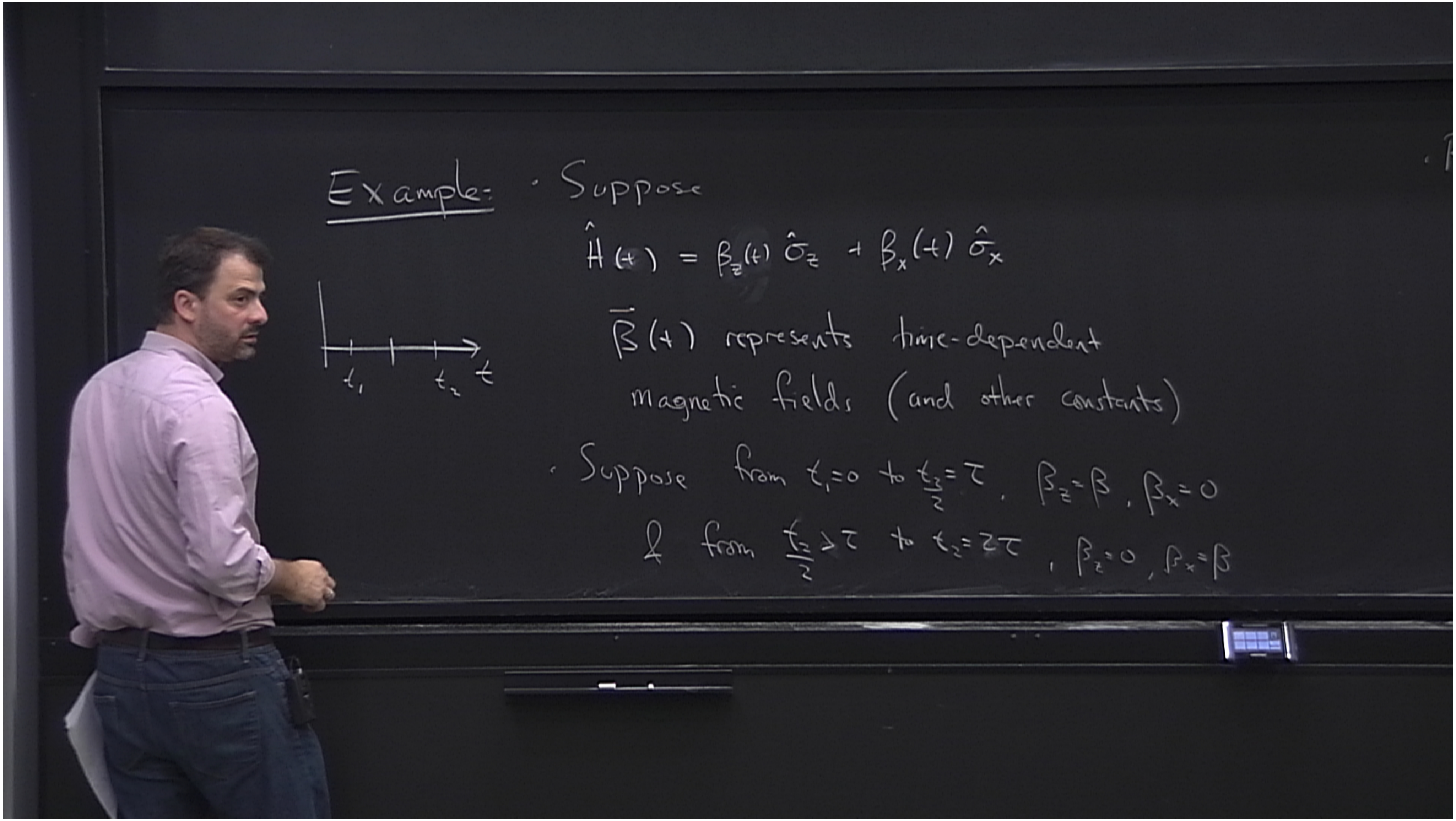
er constants)

$\beta_z = \beta$, $\beta_x = 0$

$\tau$, $\beta_z = 0$, $\beta_x = \beta$

· Recall that a Hamiltonian of the form $\hat{H}_0 = \beta \hat{\sigma}_{\hat{n}}$ induces precession/rotation about the $\hat{n}$ axis

$$U = \exp\left(-i \frac{\beta \hat{\sigma}_n}{\hbar}\left(\frac{\tau}{2}\right)\right)$$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$\vec{\beta}(t)$ represents time-dependent magnetic fields (and other constants)
· Suppose from $t_1=0$ to $\frac{t_2}{2}=\tau$, $\beta_z=\beta$, $\beta_x=0$
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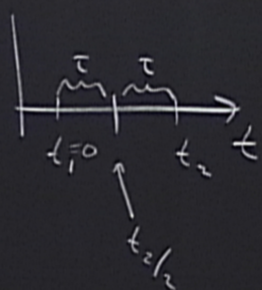
$t_1$
$t_2$
$t$

**Example:** • Suppose

$$\hat{H}(t) = \beta_z(t)\,\hat{\sigma}_z + \beta_x(t)\,\hat{\sigma}_x$$




$\vec{\beta}(t)$ represents time-dependent magnetic fields (and other constants)

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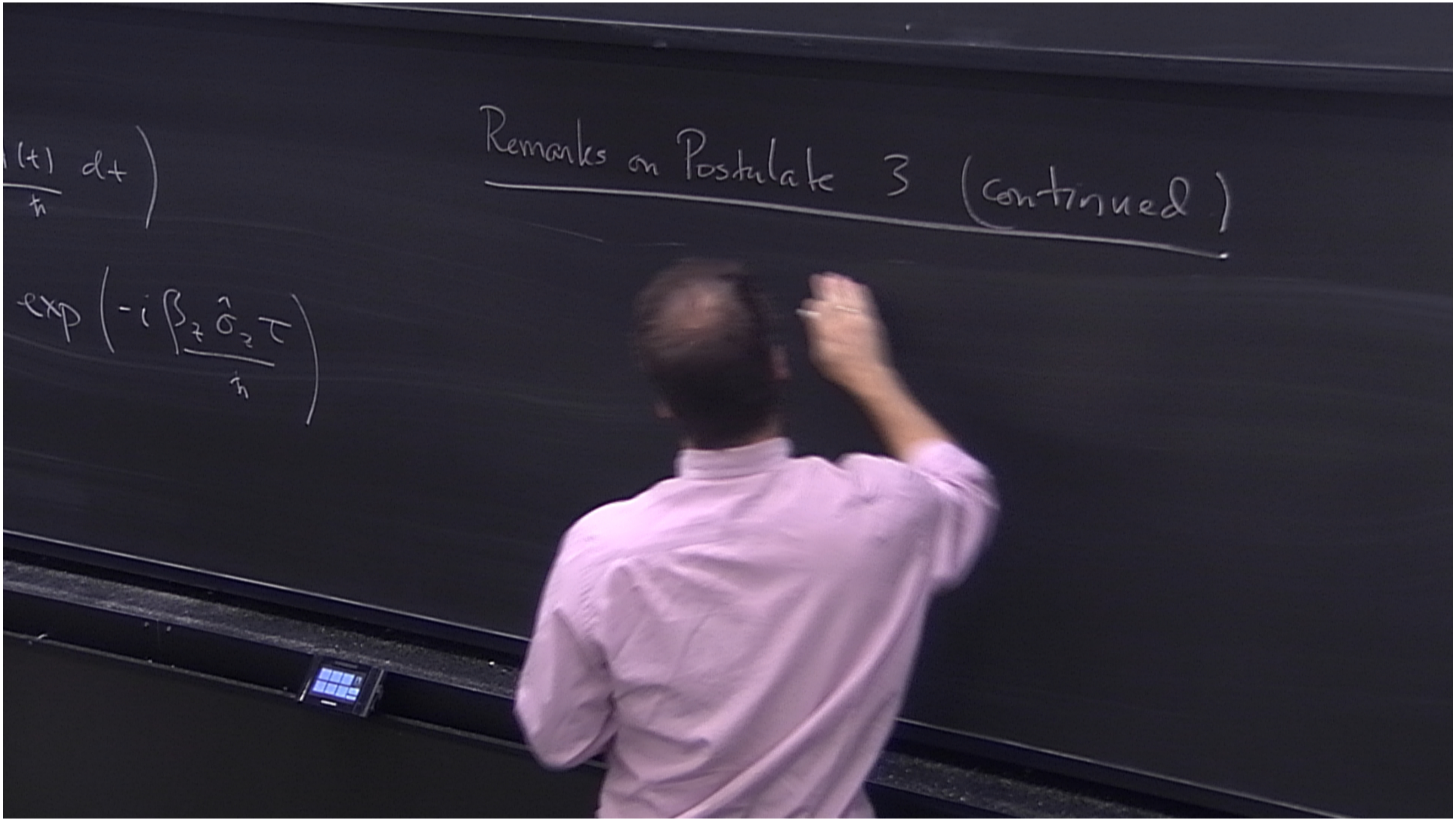
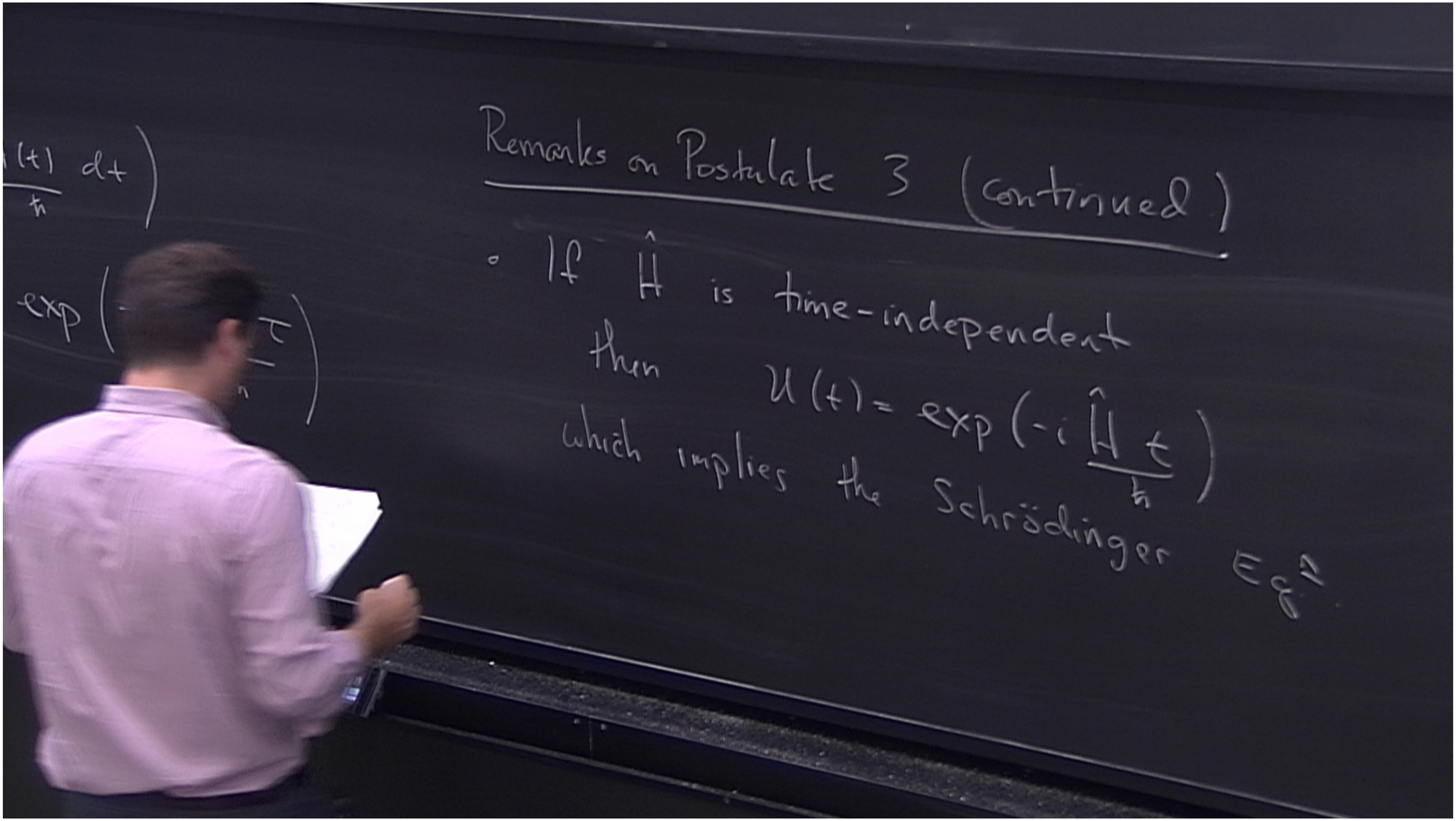
& from $\frac{t_2}{2}=\tau$ to $t_2=2\tau$, $\beta_z=0$, $\beta_x=\beta$

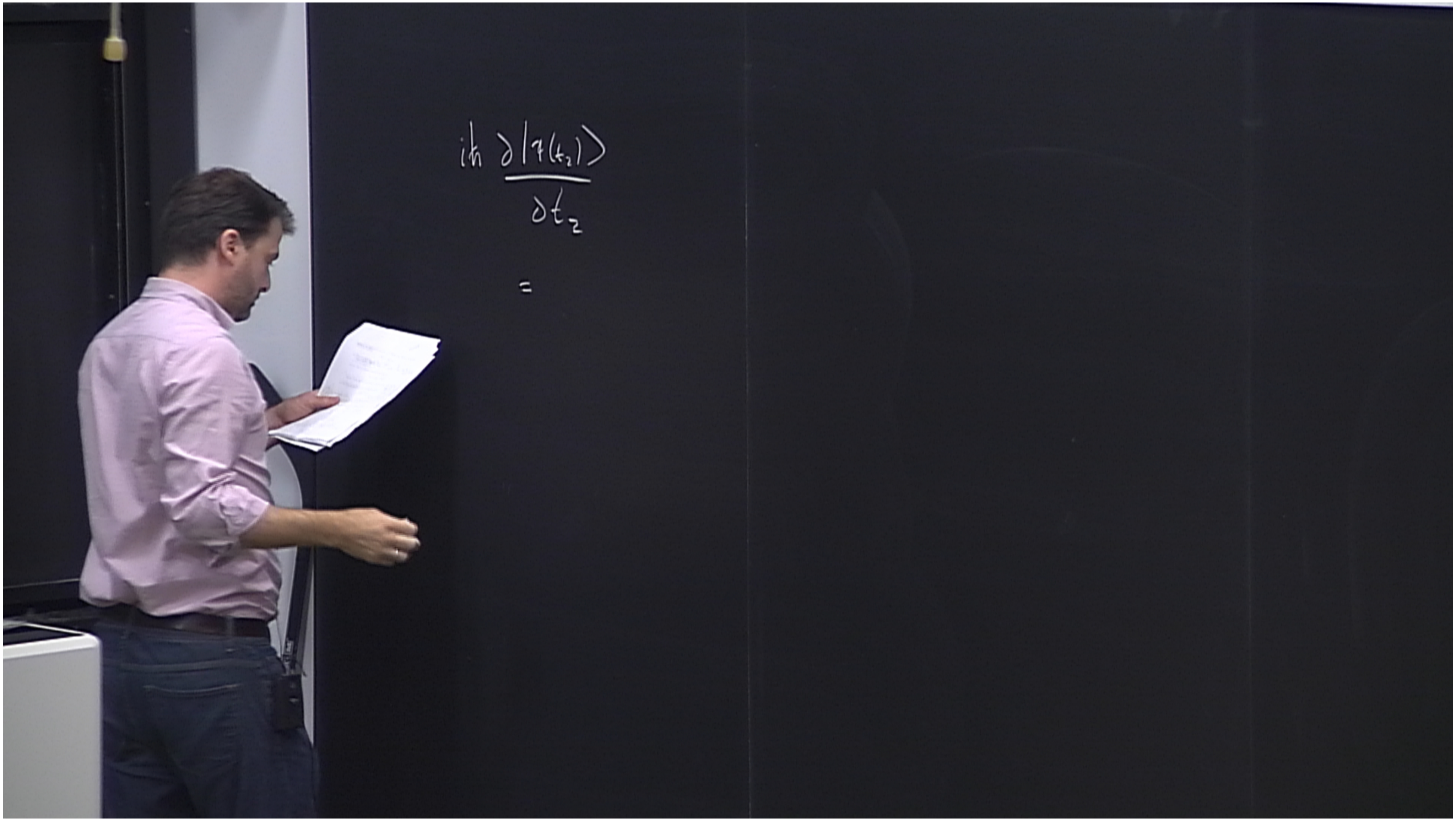
Then

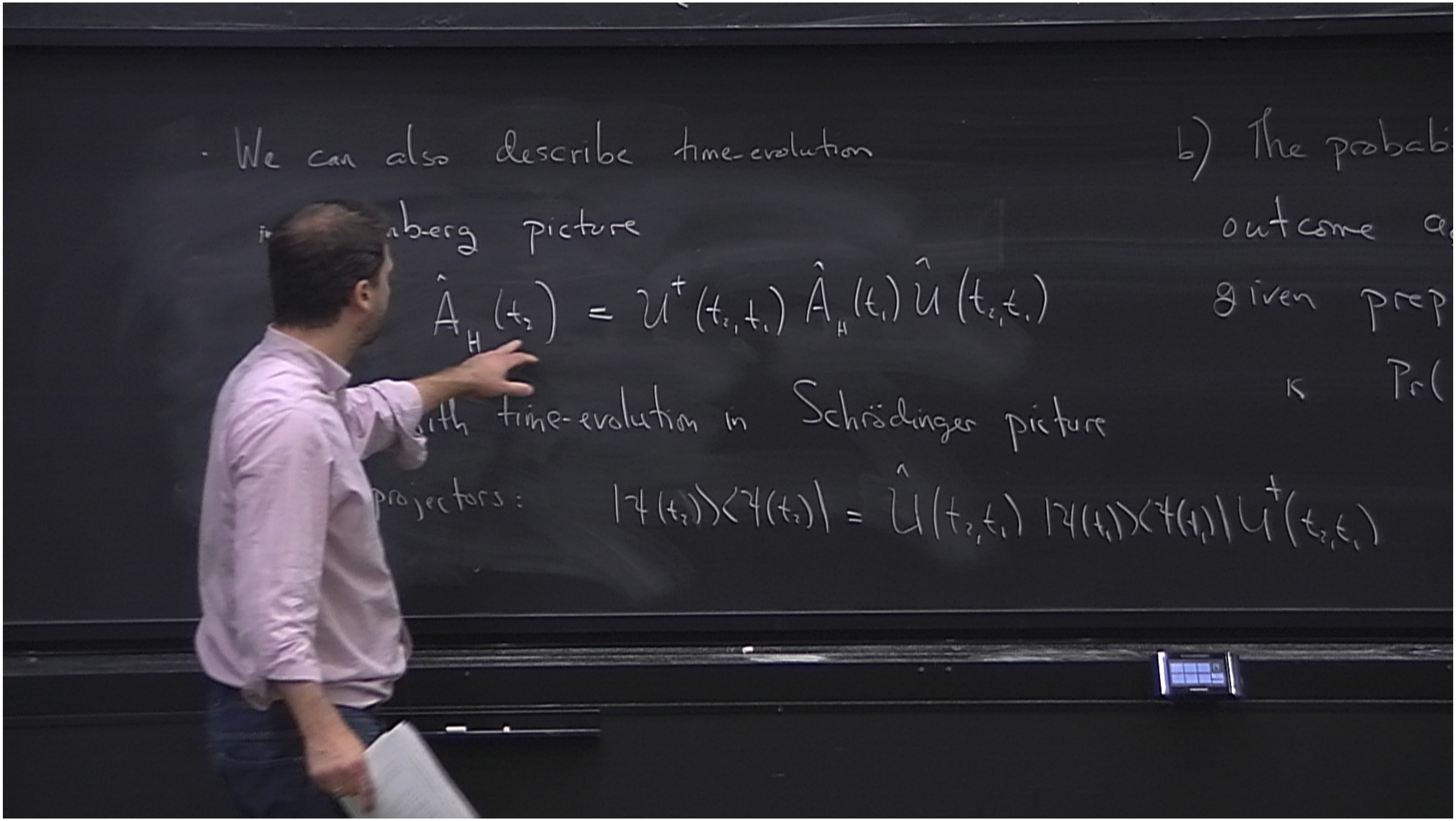
$$\hat{U}(t_2, t_1) = T \exp\left(-i \int_{t_1}^{t_2} \frac{H(t)}{\hbar}\, dt\right)$$

$$=$$


Remarks on Postulate 3 (continued)
exp(-i β_z σ̂_z τ / ħ)


Remarks on Postulate 3 (continued)
• If $\hat{H}$ is time-independent
then $\mathcal{U}(t) = \exp\left(-i \frac{\hat{H}\, t}{\hbar}\right)$
which implies the Schrödinger Eq$^{n}$.


$i\hbar \frac{\partial |\psi(t_2)\rangle}{\partial t_2}$
$=$



· We can also describe time-evolution
berg picture
$\hat{A}_H(t_2) = \mathcal{U}^\dagger(t_2,t_1)\,\hat{A}_H(t_1)\,\hat{\mathcal{U}}(t_2,t_1)$
time-evolution in Schrödinger picture
projectors: $|\psi(t_2)\rangle\langle\psi(t_2)| = \hat{\mathcal{U}}(t_2,t_1)\,|\psi(t_1)\rangle\langle\psi(t_1)|\,\mathcal{U}^\dagger(t_2,t_1)$
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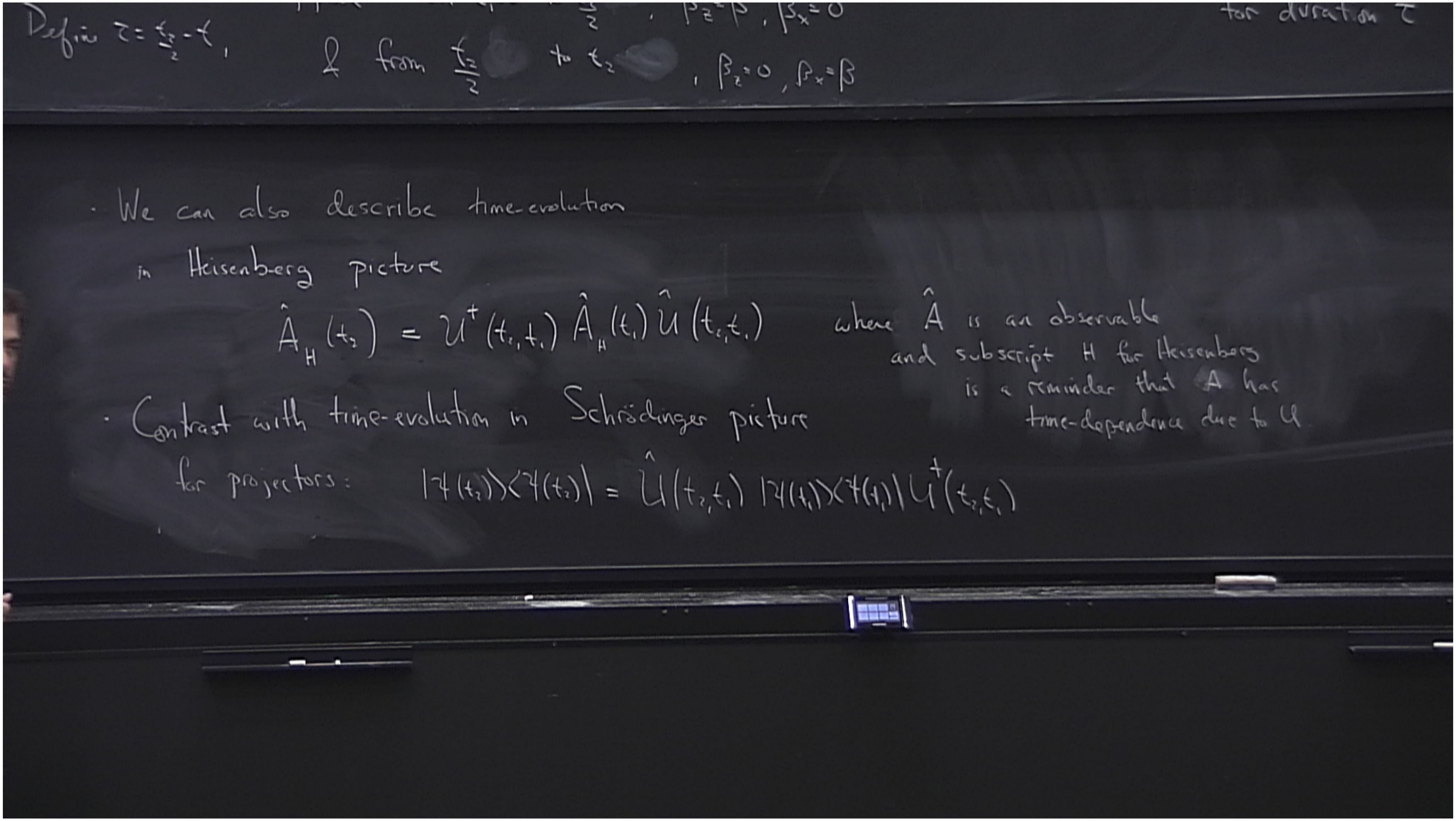

Define $\tau = \frac{t_2}{2} - t$,
for duration $\tau$
& from $\frac{t_2}{2}$ to $t_2$, $\beta_z = 0$, $\beta_x = \beta$
· We can also describe time-evolution
in Heisenberg picture
$\hat{A}_H(t_2) = \mathcal{U}^\dagger(t_2,t_1)\,\hat{A}_H(t_1)\,\hat{\mathcal{U}}(t_2,t_1)$
where $\hat{A}$ is an observable
and subscript H for Heisenberg
is a reminder that A has
time-dependence due to $\mathcal{U}$
· Contrast with time-evolution in Schrödinger picture
for projectors: $|\psi(t_2)\rangle\langle\psi(t_2)| = \hat{\mathcal{U}}(t_2,t_1)\,|\psi(t_1)\rangle\langle\psi(t_1)|\,\mathcal{U}^\dagger(t_2,t_1)$