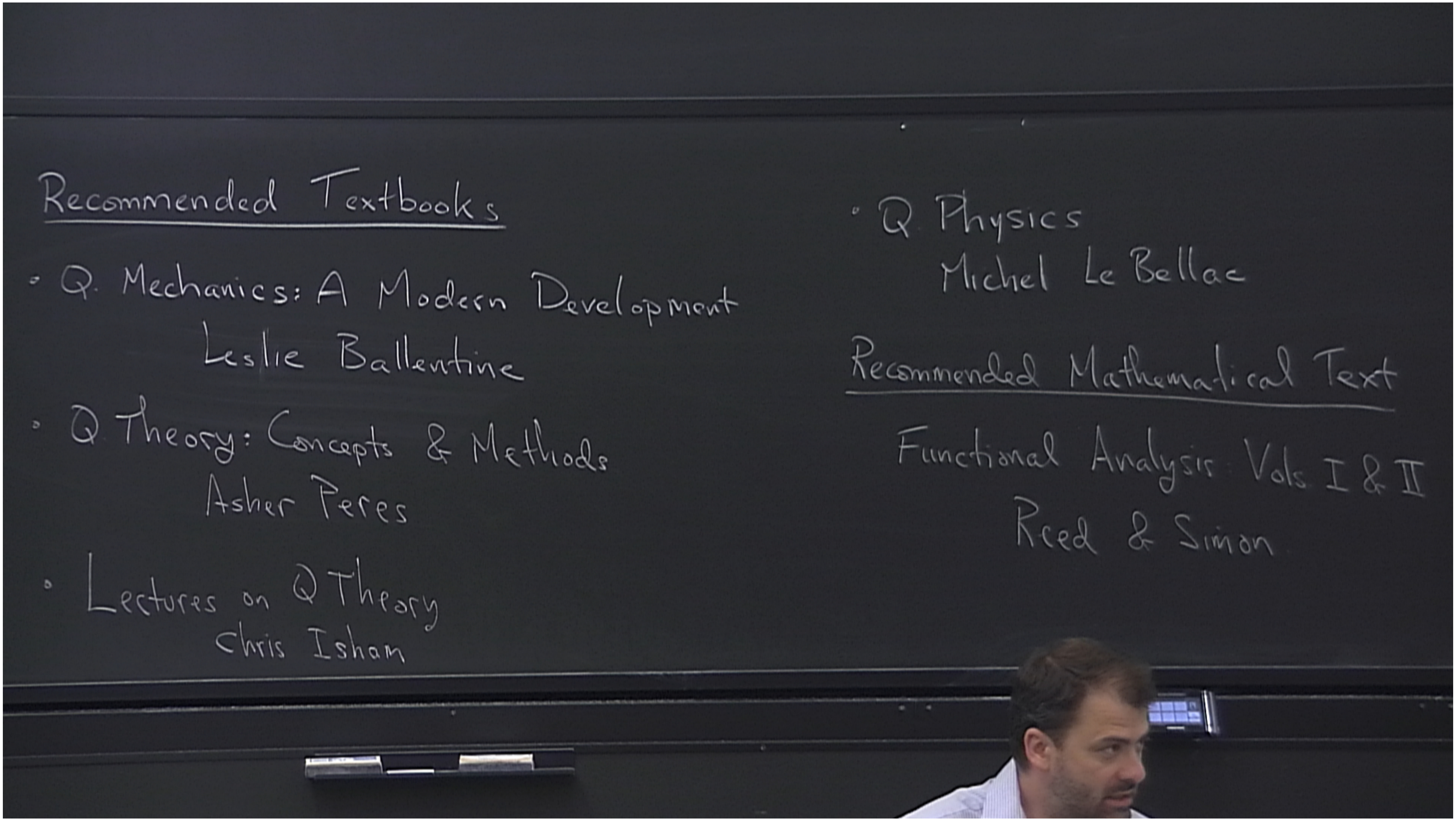
Title: 14/15 PSI Quantum Theory-2

Date: Sep 09, 2014 10:45 AM

URL: http://pirsa.org/14090043

Abstract:


Recommended Textbooks
Q. Mechanics: A Modern Development
Leslie Ballentine
Q Theory: Concepts & Methods
Asher Peres
Lectures on Q Theory
Chris Isham
Q Physics
Michel Le Bellac
Recommended Mathematical Text
Functional Analysis: Vols I & II
Reed & Simon

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hematical Text

alysis: Vols. I & II

& Simon

Recap

↗ Ideal case

Postulate 1: A system, or more precisely, a preparation procedure, is described by a Hilbert space vector.

Example of Hilbert Space

$L^2(\mathbb{R})$

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for 1 particle in 1 physical dim$^{\underline{n}}$

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$\hookrightarrow$ wavemechanics

with complex-valued functions

$\psi(x)$, satisfying $\int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = 1$

For 1 particle in 2D

$$\mathcal{H} = L^2(\mathbb{R}^2)$$

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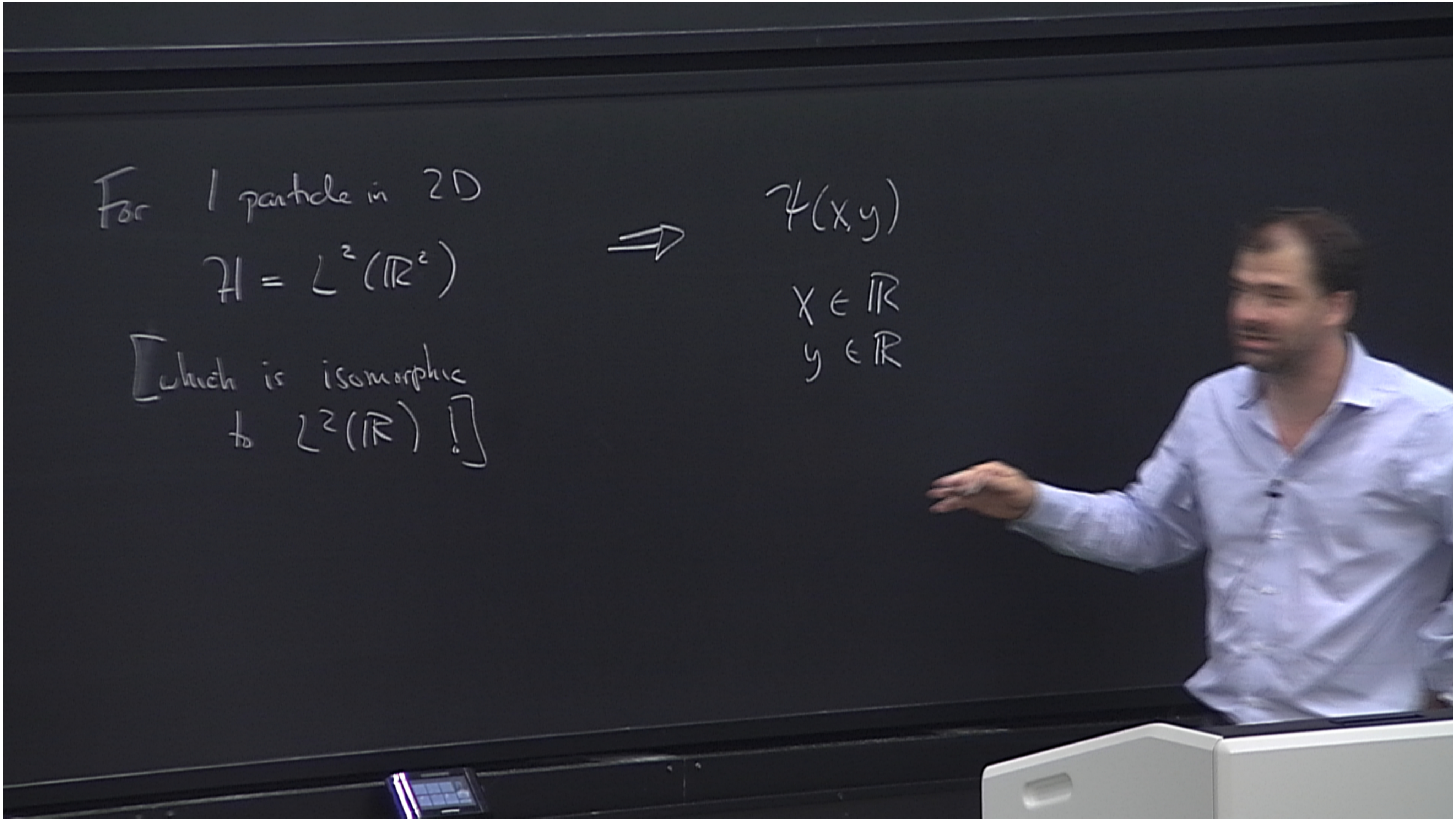
[which is isomorphic to $L^2(\mathbb{R})$!]


For 1 particle in 2D
$\mathcal{H} = L^2(\mathbb{R}^2)$
[which is isomorphic
to $L^2(\mathbb{R})$!]
$\Longrightarrow$
$\psi(x,y)$
$x \in \mathbb{R}$
$y \in \mathbb{R}$

For 1 particle in 2D

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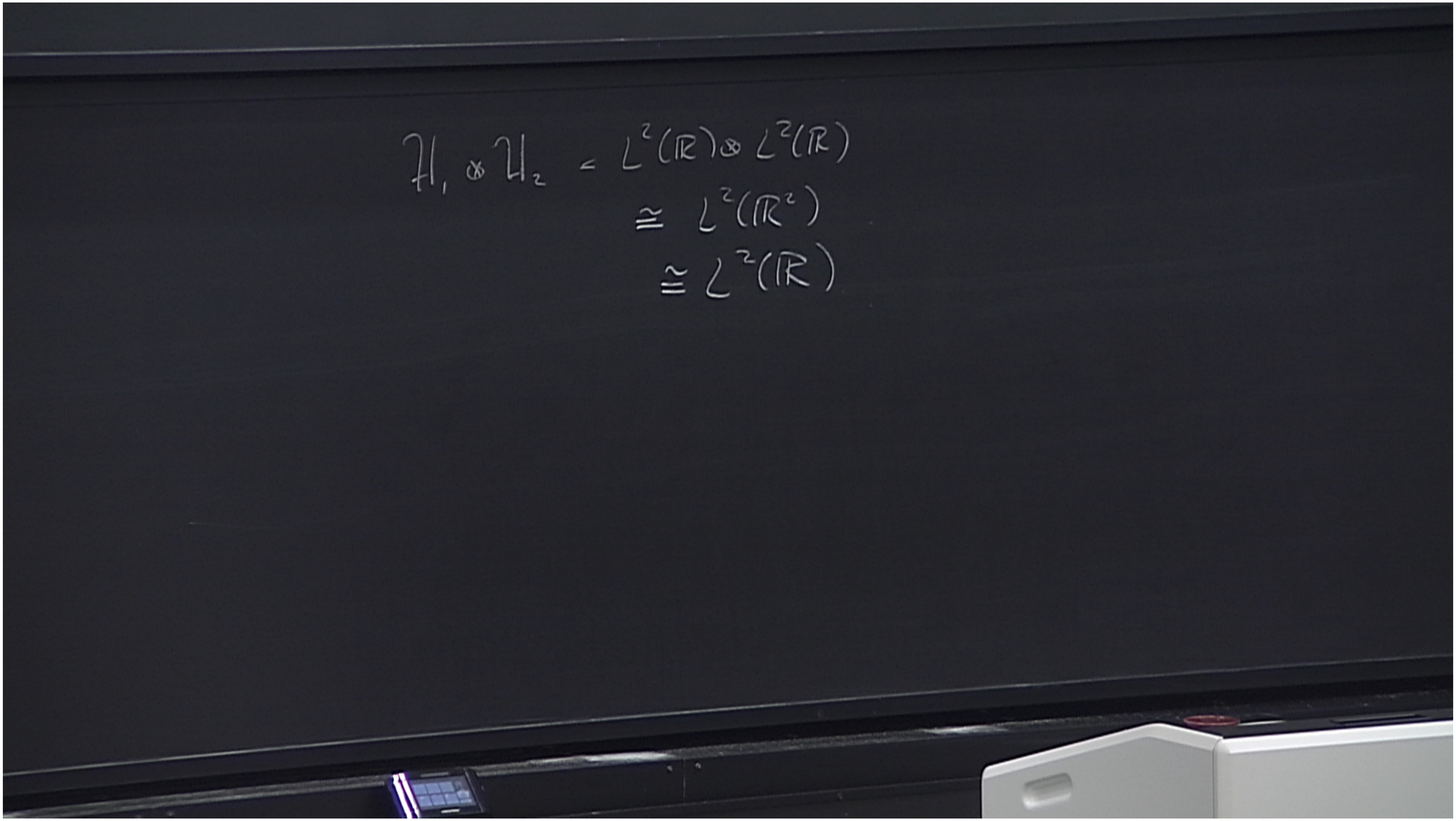
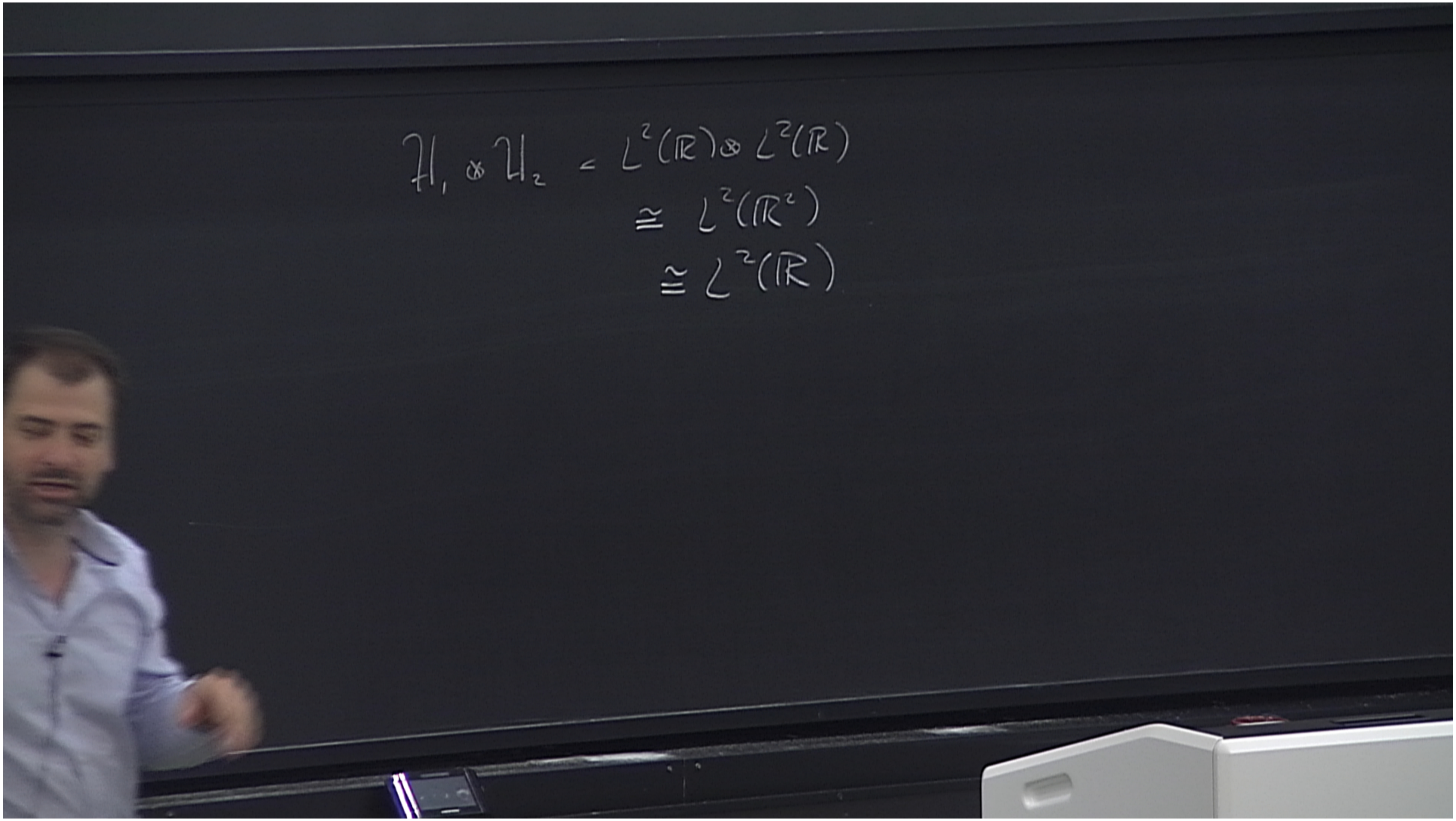
$y \in \mathbb{R}$

[which is isomorphic to $L^2(\mathbb{R})$!]

For 2 particles in 1D

$$\mathcal{H} = L^2(\mathbb{R}^2) \Longrightarrow \psi(x_1, x_2)$$


$\mathcal{H}_1 \otimes \mathcal{H}_2 = L^2(\mathbb{R}) \otimes L^2(\mathbb{R})$
$\cong L^2(\mathbb{R}^2)$
$\cong L^2(\mathbb{R})$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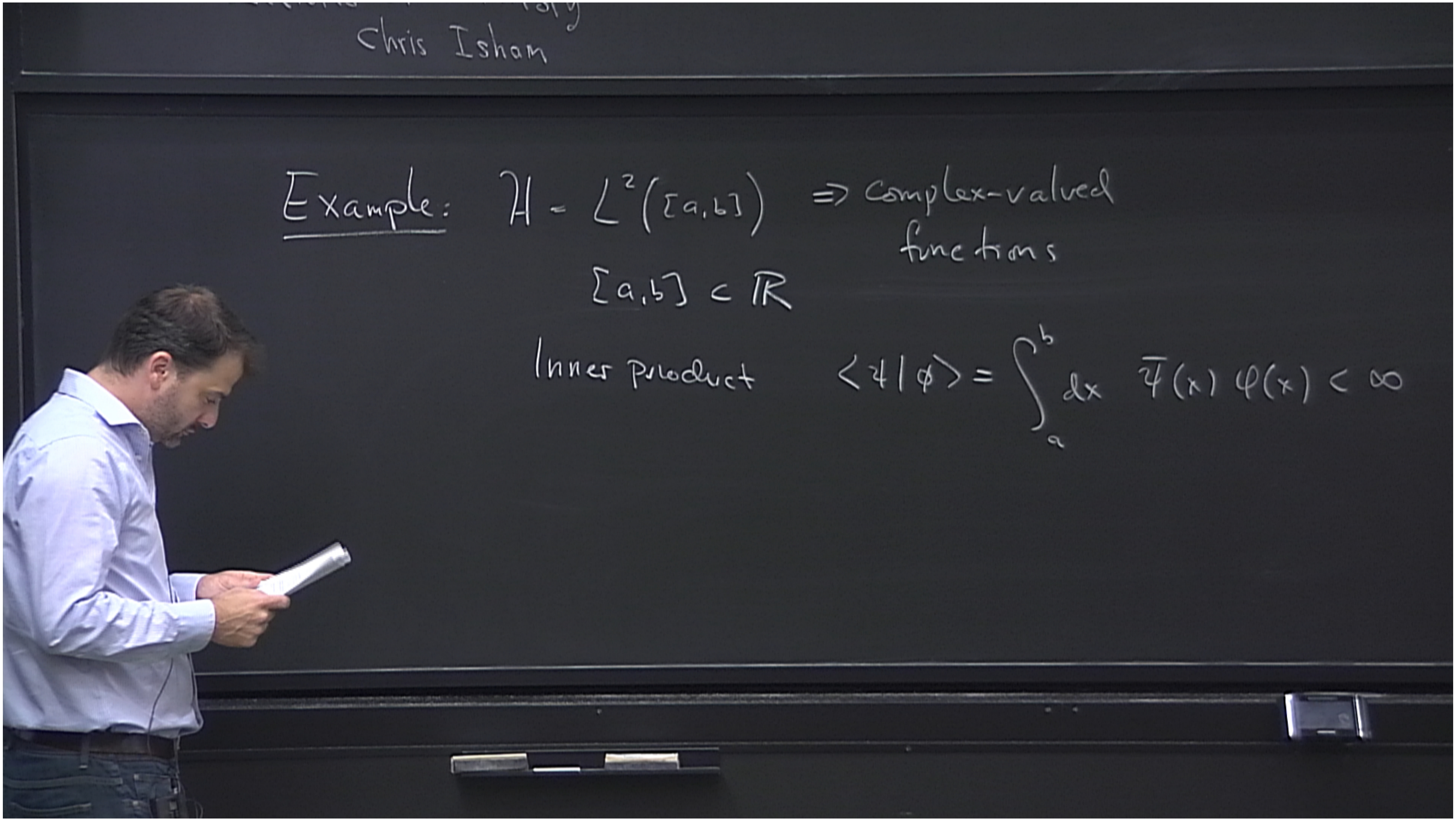
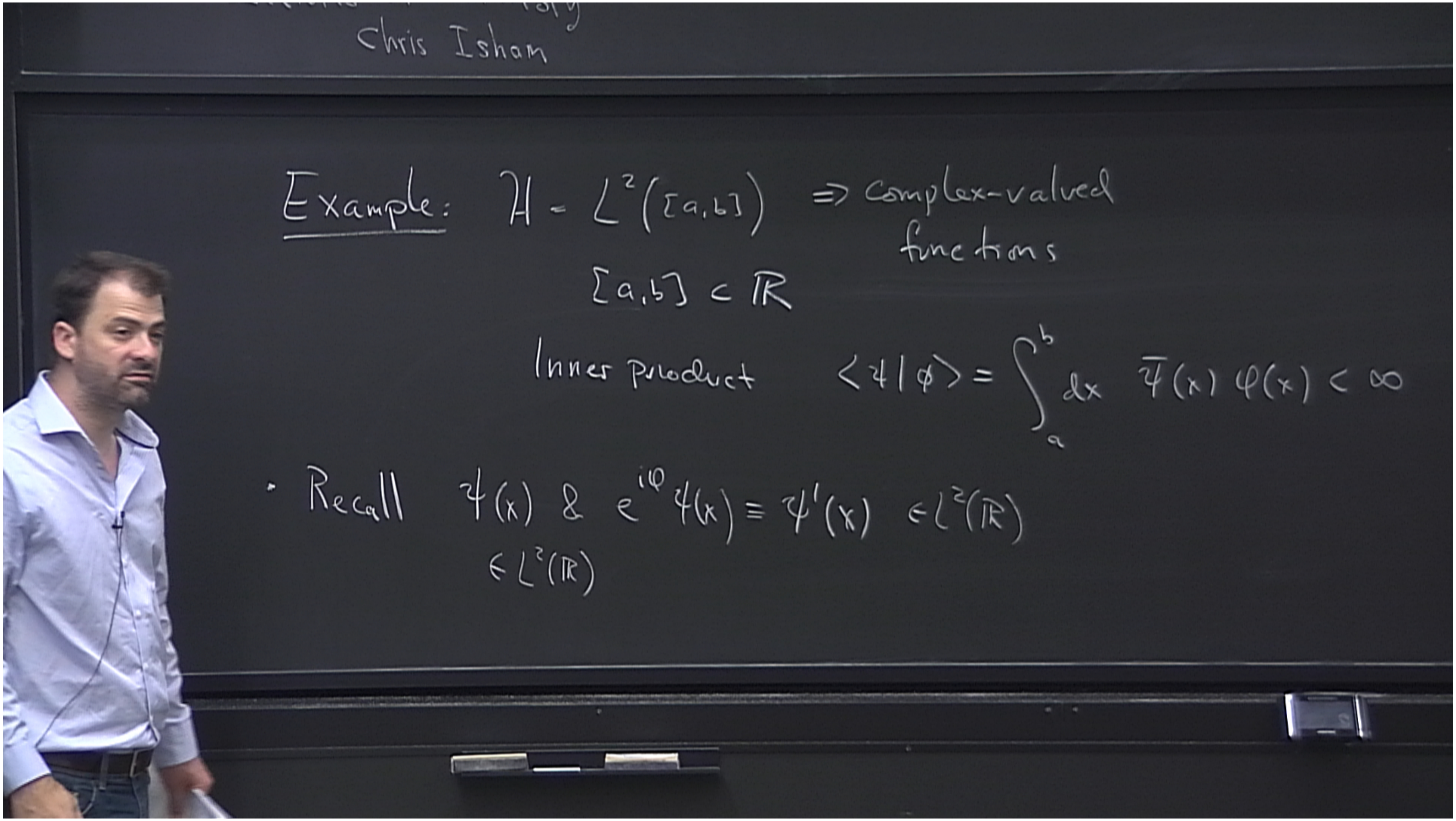

Chris Isham
Example: $\mathcal{H} = L^2([a,b])$ ⇒ complex-valued functions
$[a,b] \subset \mathbb{R}$
Inner product $\langle \psi | \phi \rangle = \int_a^b dx \; \bar{\psi}(x) \, \varphi(x) < \infty$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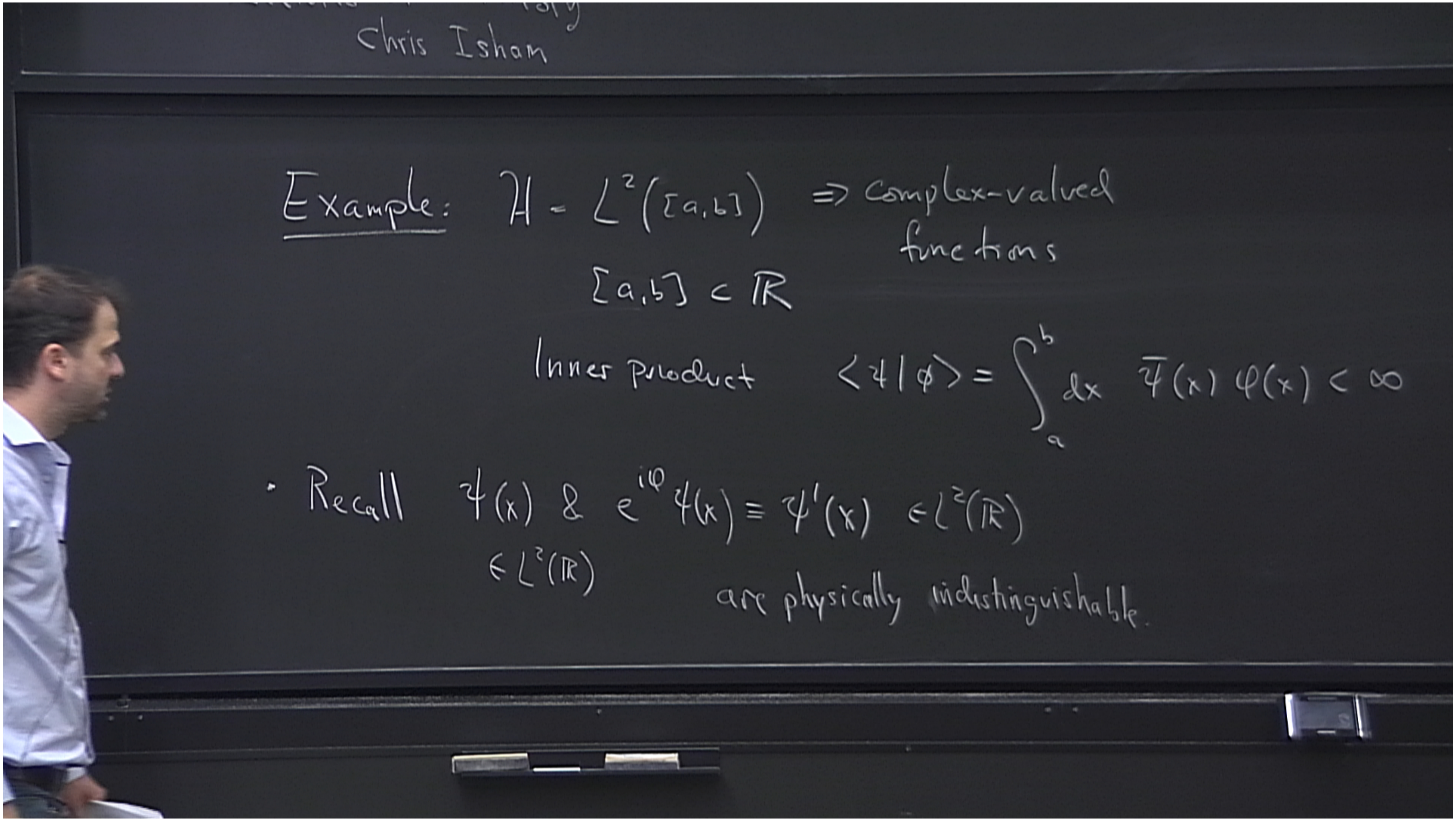
• Recall $\psi(x)$ & $e^{i\varphi}\psi(x) \equiv \psi'(x) \in L^2(\mathbb{R})$
$\in L^2(\mathbb{R})$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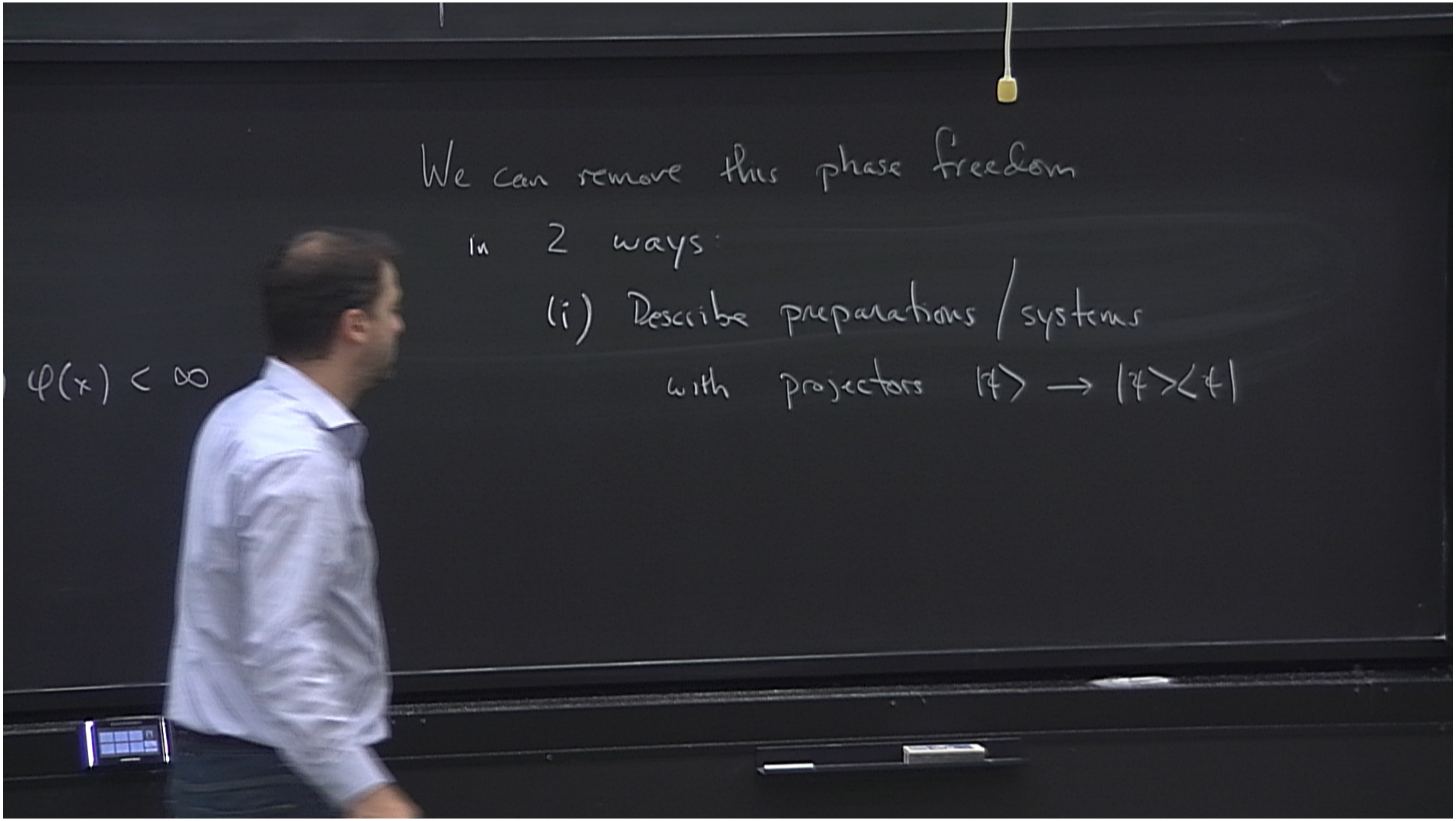
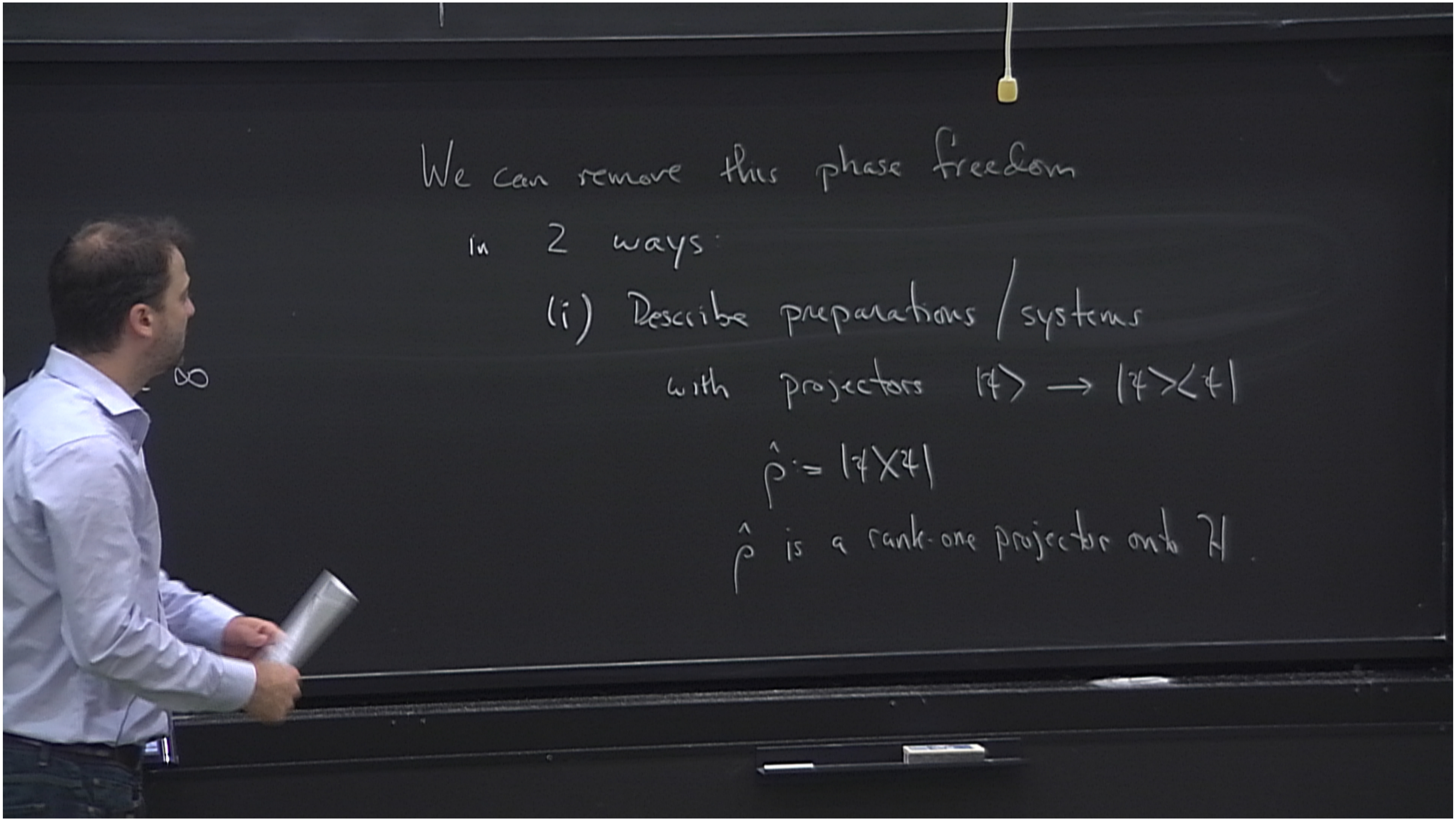
Example: $\mathcal{H} = \mathcal{L}^2([a,b])$ ⇒ complex-valued functions
$[a,b] \subset \mathbb{R}$
Inner product $\langle \psi | \phi \rangle = \int_a^b dx \; \overline{\psi}(x)\, \varphi(x) < \infty$
• Recall $\psi(x)$ & $e^{i\varphi}\psi(x) \equiv \psi'(x) \in \mathcal{L}^2(\mathbb{R})$
$\in \mathcal{L}^2(\mathbb{R})$
are physically indistinguishable.


We can remove this phase freedom
in 2 ways:
(i) Describe preparations / systems
with projectors $|\psi\rangle \longrightarrow |\psi\rangle\langle\psi|$
$\varphi(x) < \infty$


We can remove this phase freedom
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(i) Describe preparations / systems
with projectors $|\psi\rangle \to |\psi\rangle\langle\psi|$
$\hat{\rho} := |\psi\rangle\langle\psi|$
$\hat{\rho}$ is a rank-one projector onto $\mathcal{H}$.

Space of preparations

is complex projective

space $\mathbb{C}P^{d-1}$

associated with

pure states $|\psi\rangle \in \mathcal{H} = \mathbb{C}^d$

or equivalently

(ii) Space of preparations

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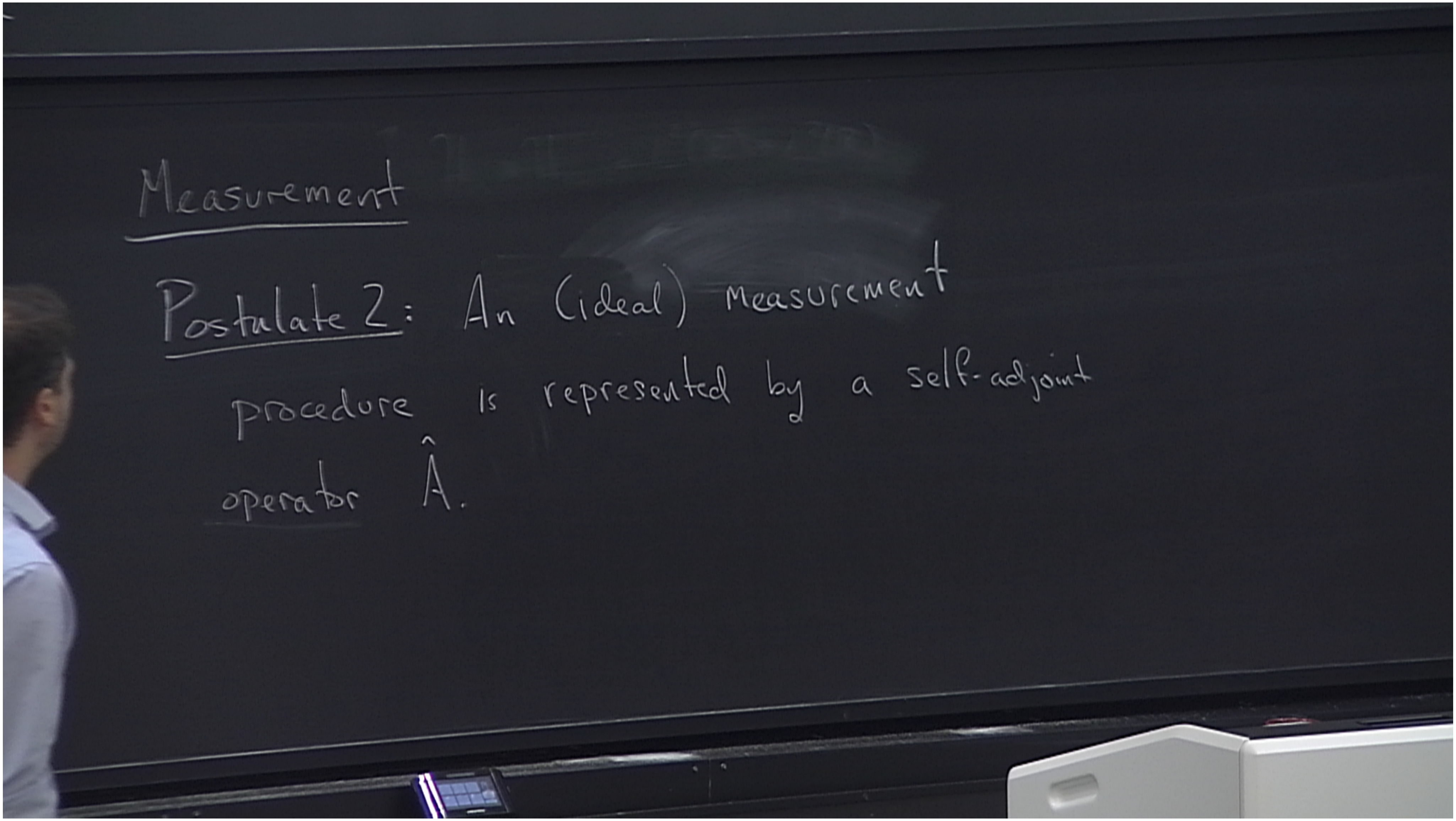
space $\mathbb{C}P^{d-1}$

associated with

pure states $|\psi\rangle \in \mathcal{H} = \mathbb{C}^d$


Measurement
Postulate 2: An (ideal) measurement
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Measurement

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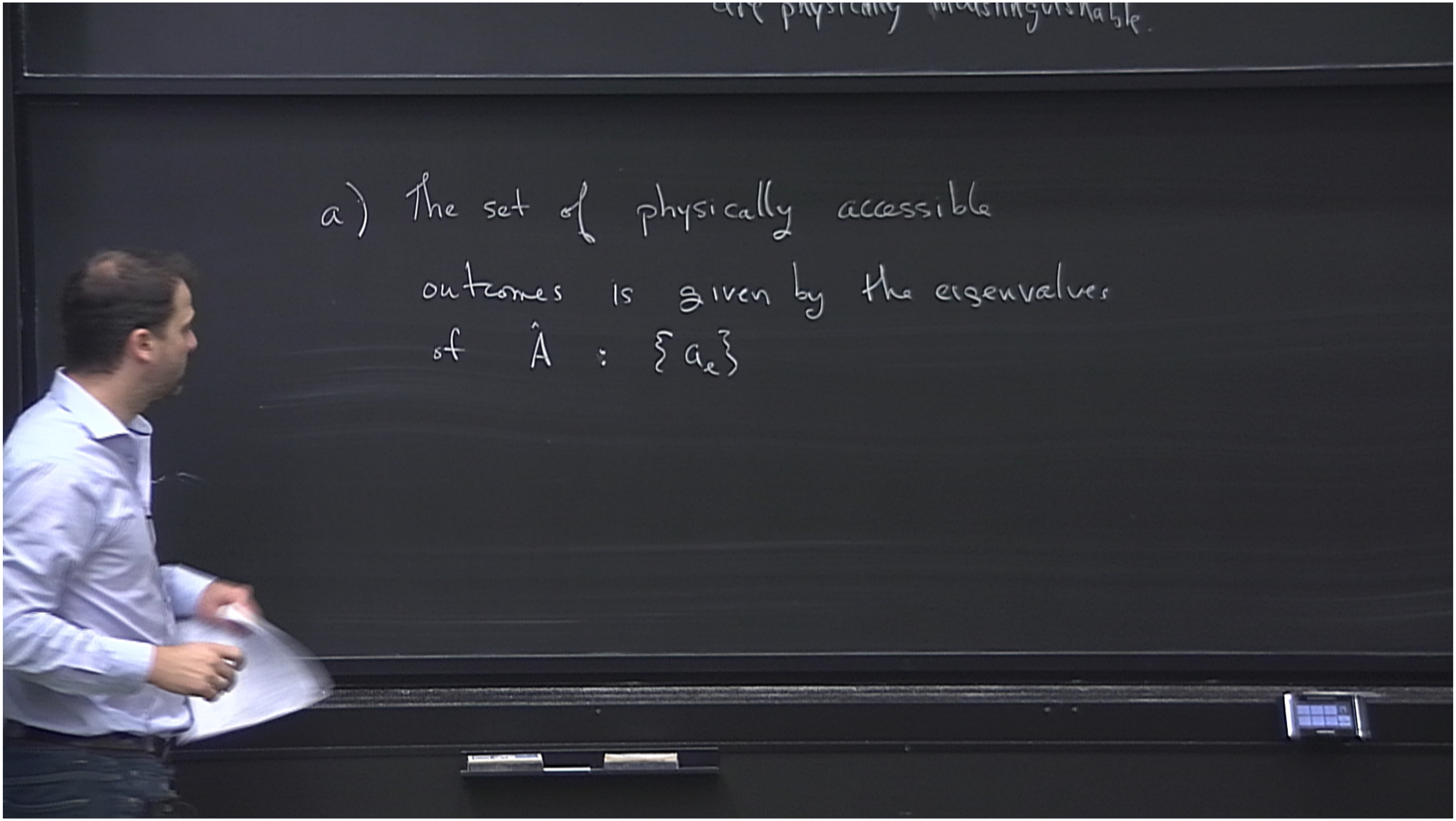
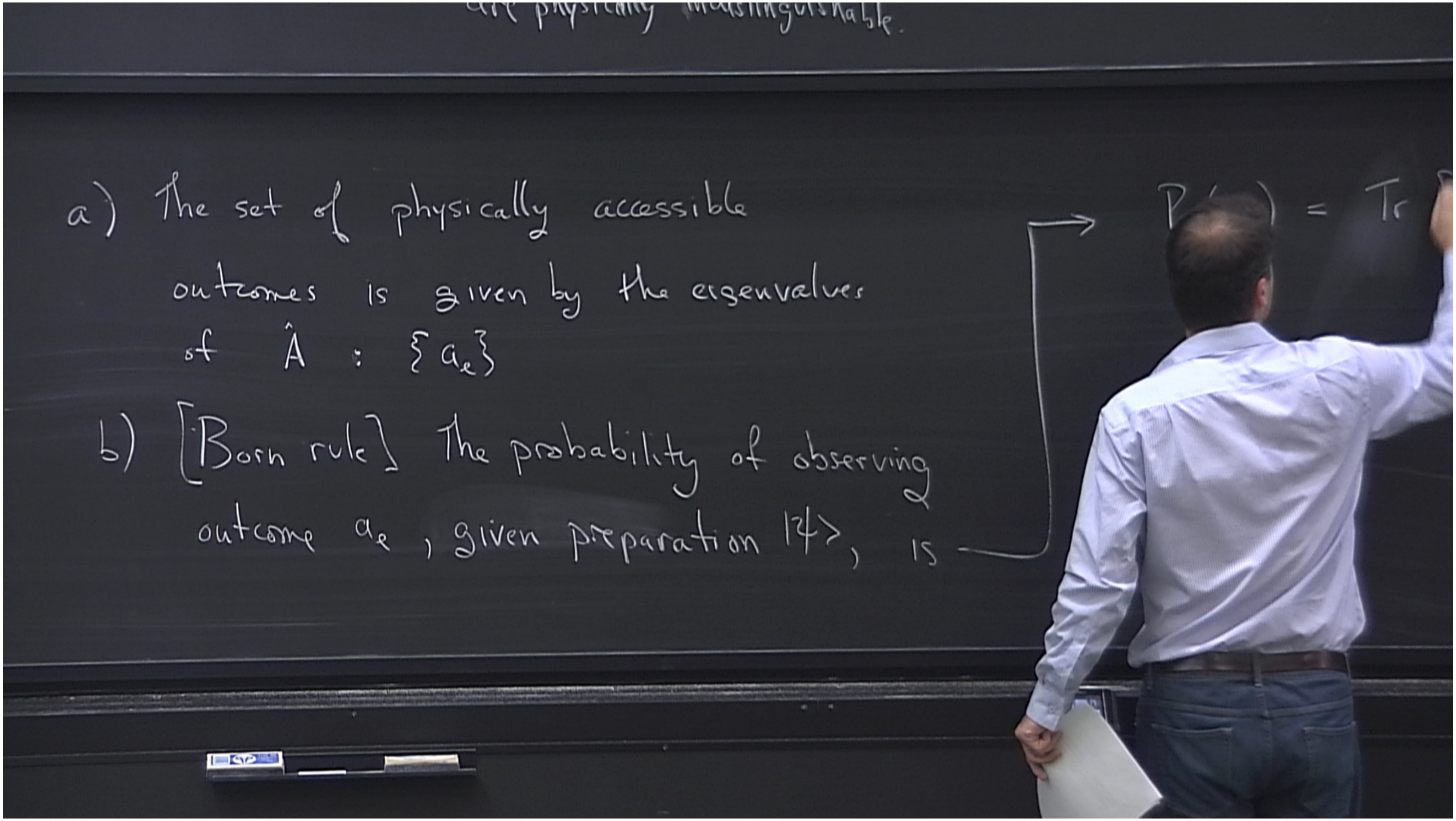

a) The set of physically accessible
outcomes is given by the eigenvalues
of $\hat{A}$: $\{a_e\}$


a) The set of physically accessible
outcomes is given by the eigenvalues
of Â : {a_ℓ}
b) [Born rule] The probability of observing
outcome a_ℓ , given preparation |ψ>, is
P( ) = Tr

$$\Pr(a_\ell) = \mathrm{Tr}\left[\underbrace{|\psi\rangle\langle\psi|}_{|\psi\rangle\langle\psi|}\hat{\Pi}_\ell\right]$$

Measurement

Postulate 2: An (ideal) measurement procedure is represented by a self-adjoint operator $\hat{A} = \sum_{e} a_e \hat{\Pi}_e$

$$\Pr(a_e) = \mathrm{Tr}\left[\underbrace{|\psi\rangle\langle\psi|}_{|\psi\rangle\langle\psi|} \hat{\Pi}_e \right].$$

- Let's unpack the math:

  Each $\hat{\Pi}_e$ is a projector onto $\mathcal{H}$

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  - Each $\hat{\Pi}_\ell$ is a projector onto $\mathcal{H}$; $\{\Pi_\ell\}$ are orthogonal.
  - Projectors satisfy $\hat{\Pi}_\ell \Pi$

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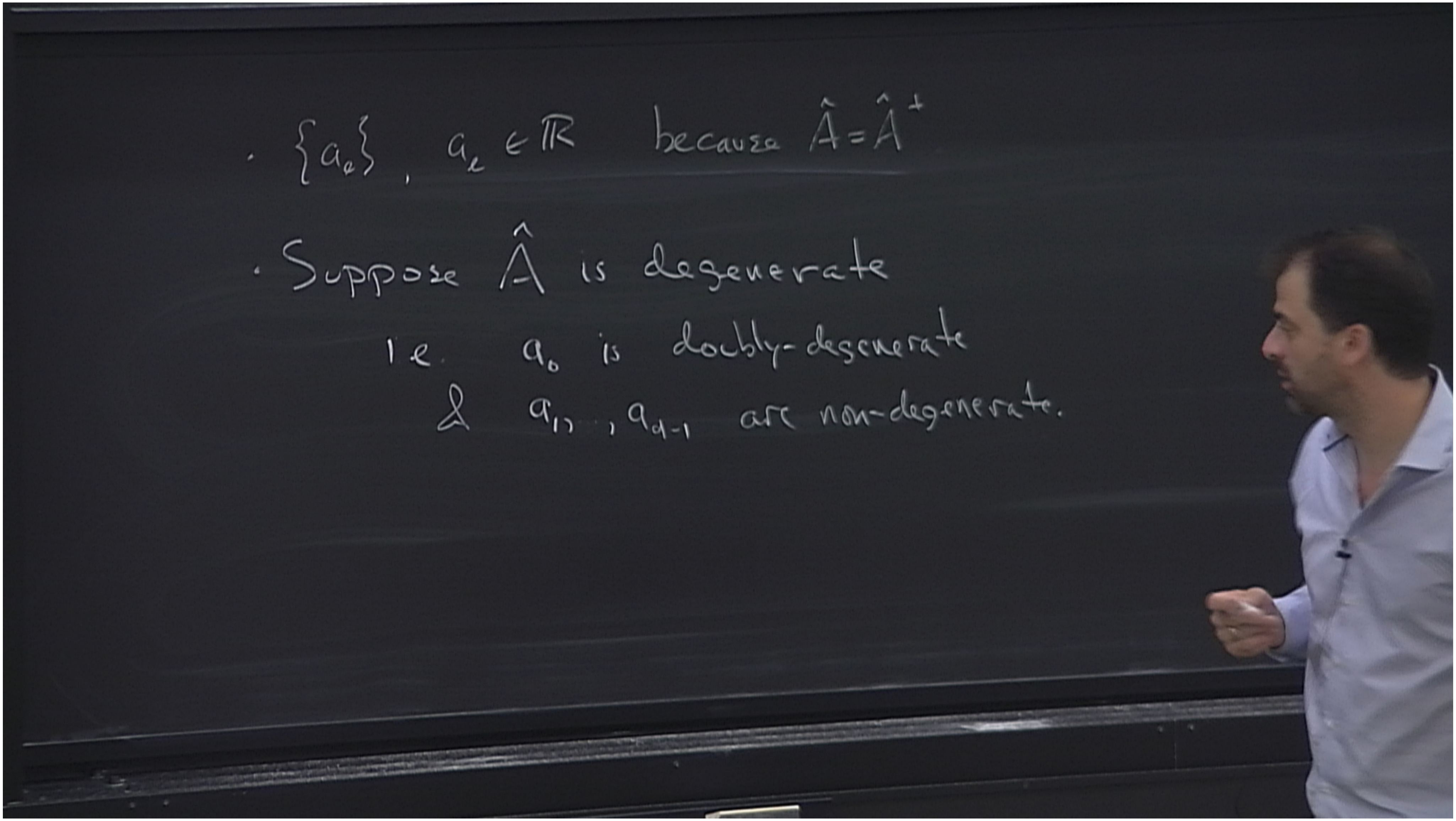
- $\{a_\ell\}$, $a_\ell \in \mathbb{R}$ because $\hat{A} = \hat{A}^\dagger$

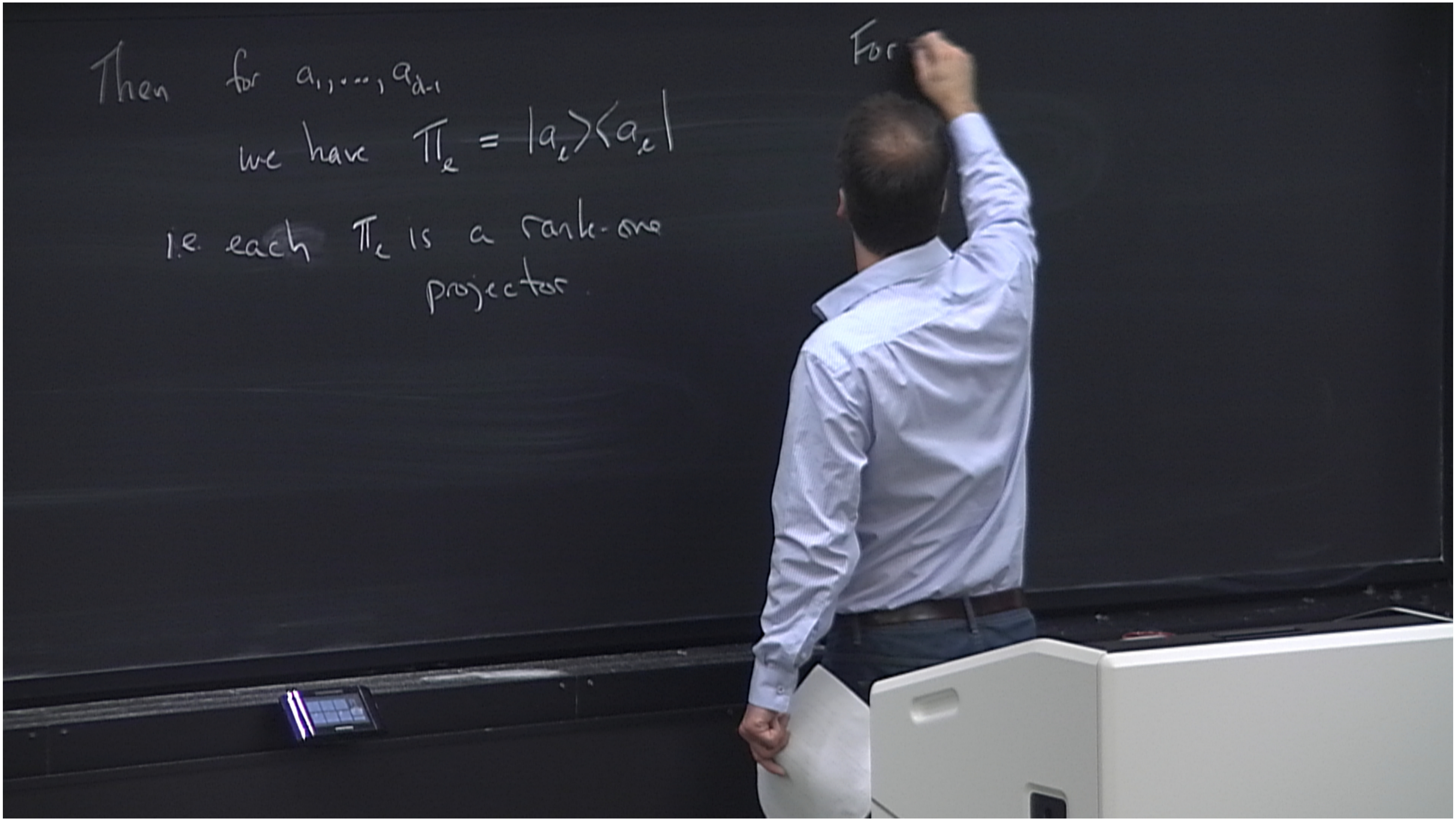


· $\{a_\ell\}$, $a_\ell \in \mathbb{R}$ because $\hat{A} = \hat{A}^\dagger$
· Suppose $\hat{A}$ is degenerate
i.e. a_0 is doubly-degenerate
& $a_1, \ldots, a_{d-1}$ are non-degenerate.

Then for $a_1, \ldots, a_{d-1}$

we have $\Pi_\ell = |a_\ell\rangle\langle a_\ell|$

i.e. Π_ℓ is a rank-one projector.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For

Then for $a_1, \ldots, a_{d-1}$

we have $\Pi_i = |a_i\rangle\langle a_i|$

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For a_0, we can pick some ON basis $\{|a_{0,1}\rangle, |a_{0,2}\rangle\}$

$\langle a_{0,1}|a_{0,2}\rangle = 0$.

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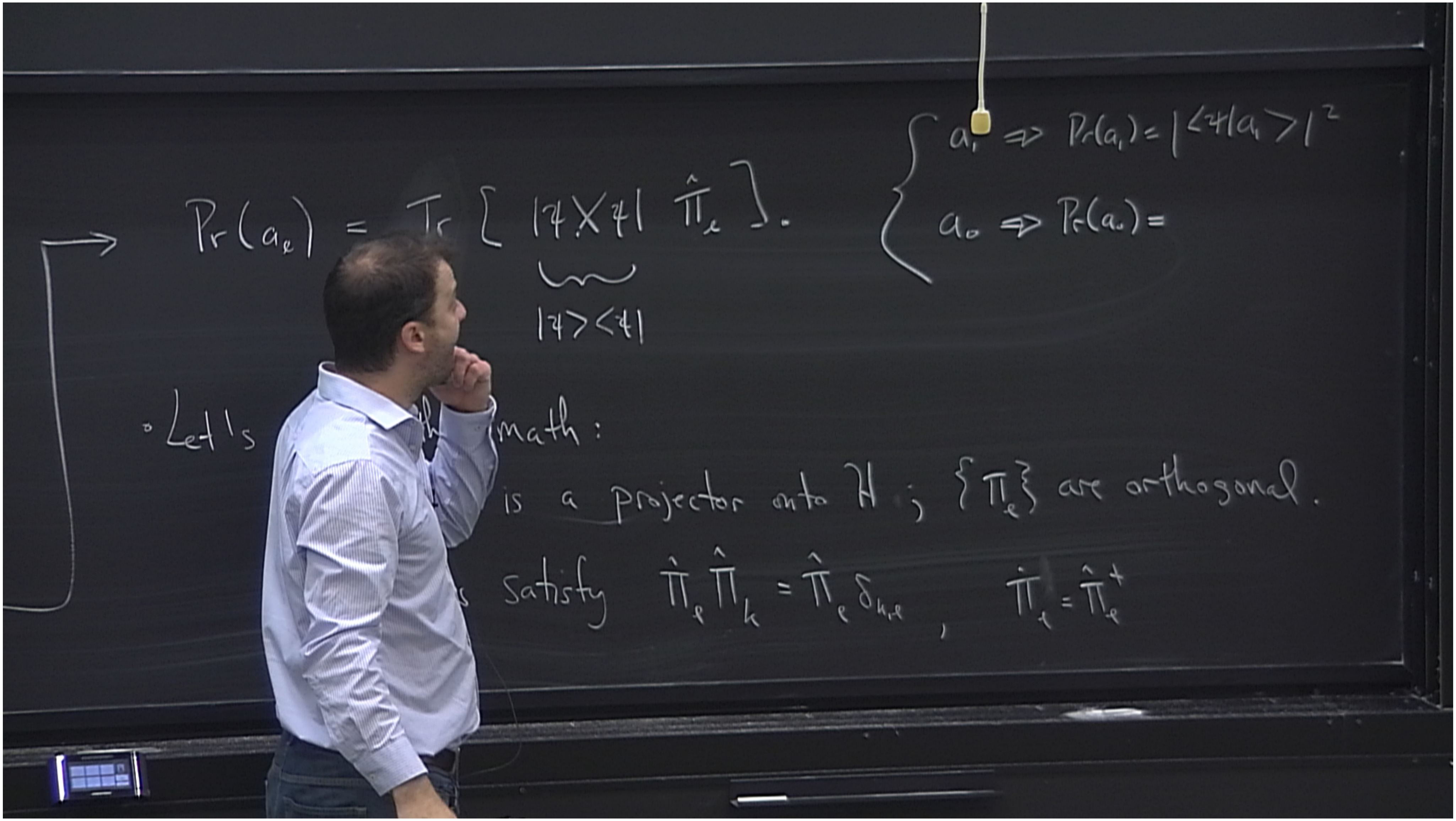
For a_0, we can pick some ON basis $\{|a_{0,1}\rangle, |a_{0,2}\rangle\}$

$$\langle a_{0,1}|a_{0,2}\rangle = 0$$

then $$\hat{\Pi}_0 = |a_{0,1}\rangle\langle a_{0,1}| + |a_{0,2}\rangle\langle a_{0,2}|$$

$\Rightarrow$ rank-2 projector.


$Pr(a_\ell) = Tr[\,|\psi\rangle\langle\psi|\,\hat{\Pi}_\ell\,]$.
$|\psi\rangle\langle\psi|$
$a_1 \Rightarrow Pr(a_1) = |\langle\psi|a_1\rangle|^2$
$a_0 \Rightarrow Pr(a_0) =$
Let's
math :
is a projector onto $\mathcal{H}$; $\{\Pi_\ell\}$ are orthogonal.
satisfy $\hat{\Pi}_\ell \hat{\Pi}_k = \hat{\Pi}_\ell \delta_{k,\ell}$, $\hat{\Pi}_\ell = \hat{\Pi}_\ell^\dagger$

$$\Pr(a_\ell) = \mathrm{Tr}\big[\, \underbrace{|\psi\rangle\langle\psi|}_{|\psi\rangle\langle\psi|}\, \hat{\Pi}_\ell \,\big].$$

$$\begin{cases} a_1 \Rightarrow \Pr(a_1) = |\langle\psi|a_1\rangle|^2 \\ a_0 \Rightarrow \Pr(a_0) = |\langle\psi|a_{0,1}\rangle|^2 + |\langle\psi|a_{0,2}\rangle|^2 \end{cases}$$

- Let's unpack the math:
  - Each $\hat{\Pi}_\ell$ is a projector onto $\mathcal{H}$; $\{\Pi_\ell\}$ are orthogonal.
  - Projectors satisfy $\hat{\Pi}_\ell \hat{\Pi}_k = \hat{\Pi}_\ell \delta_{k,\ell}$, $\hat{\Pi}_\ell = \hat{\Pi}_\ell^\dagger$

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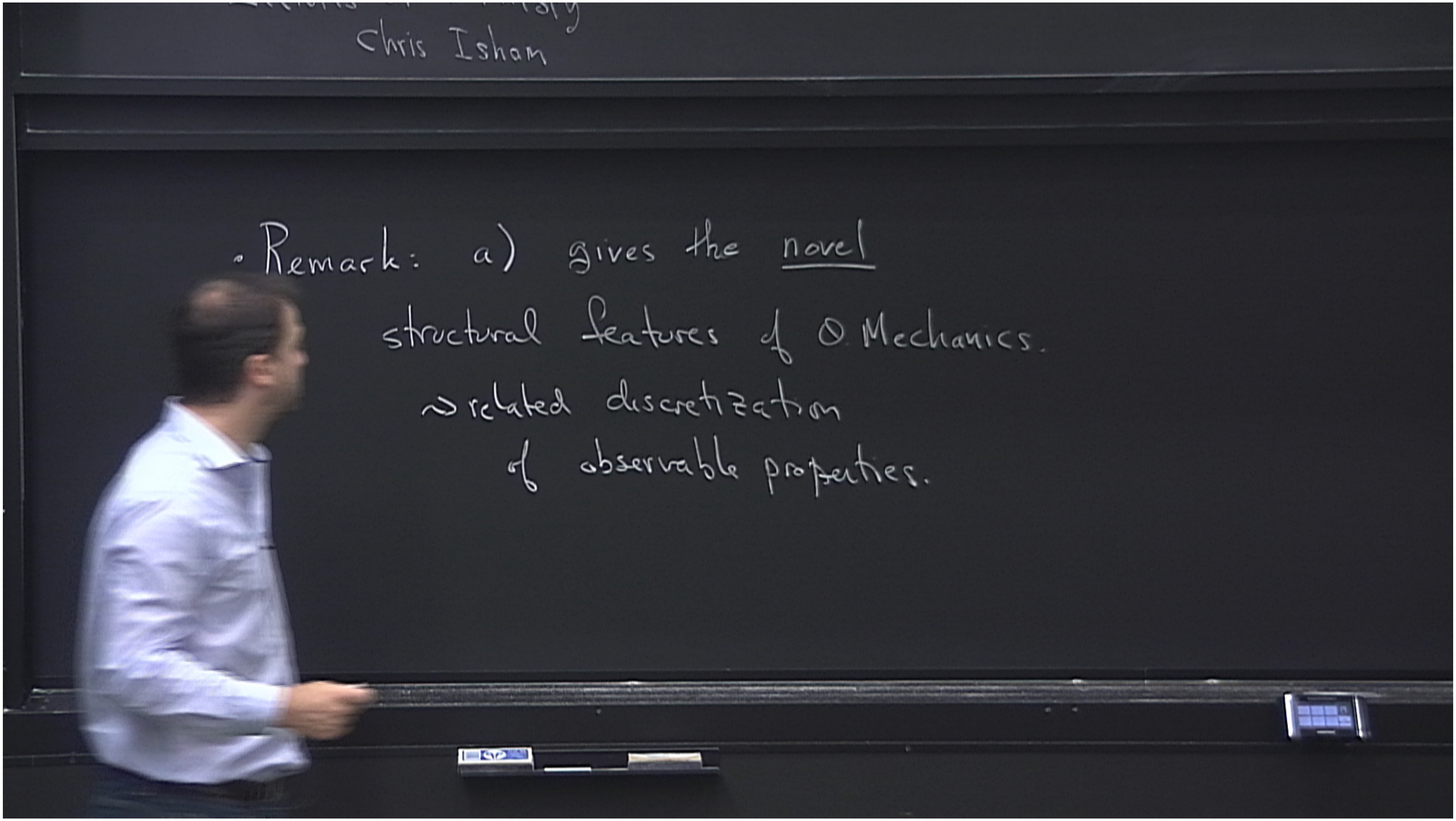
• Remark: a) gives the <u>novel</u> structural features of Q. Mechanics.


Chris Isham
· Remark: a) gives the novel
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⇝ related discretization
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b) prescribes the novel statistical aspects of q systems.

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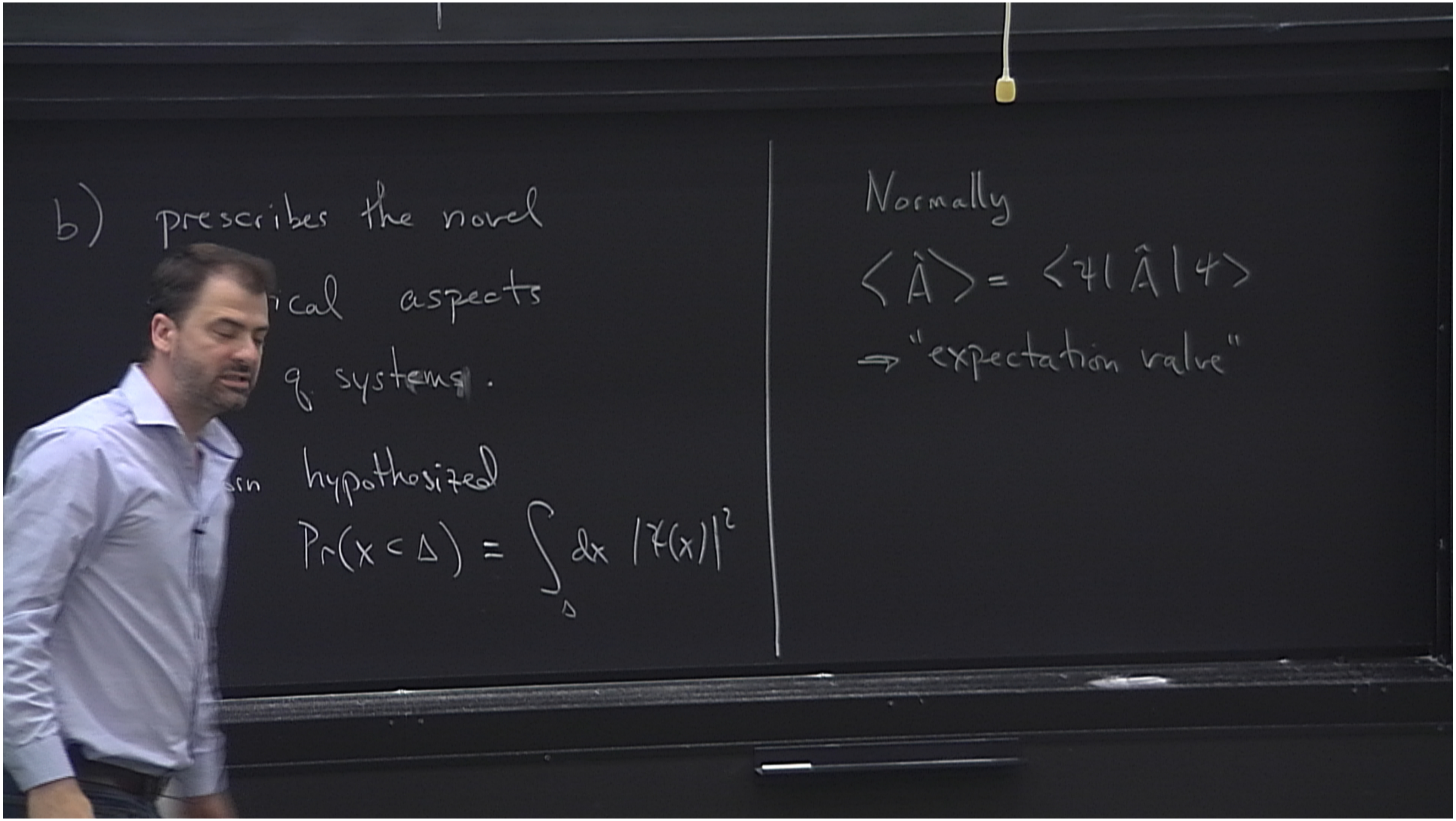
Note: Born hypothesized

$$\Pr(x \in \Delta) = \int_\Delta dx \, |\psi(x)|^2$$


b) prescribes the novel
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hypothesized
$Pr(x < \Delta) = \int_\Delta dx \, |\psi(x)|^2$
Normally
$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$
$\Rightarrow$ "expectation value"

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<u>Note</u>: Born hypothesized

$$Pr(x \in \Delta) = \int_\Delta dx \, |\psi(x)|^2$$

Normally

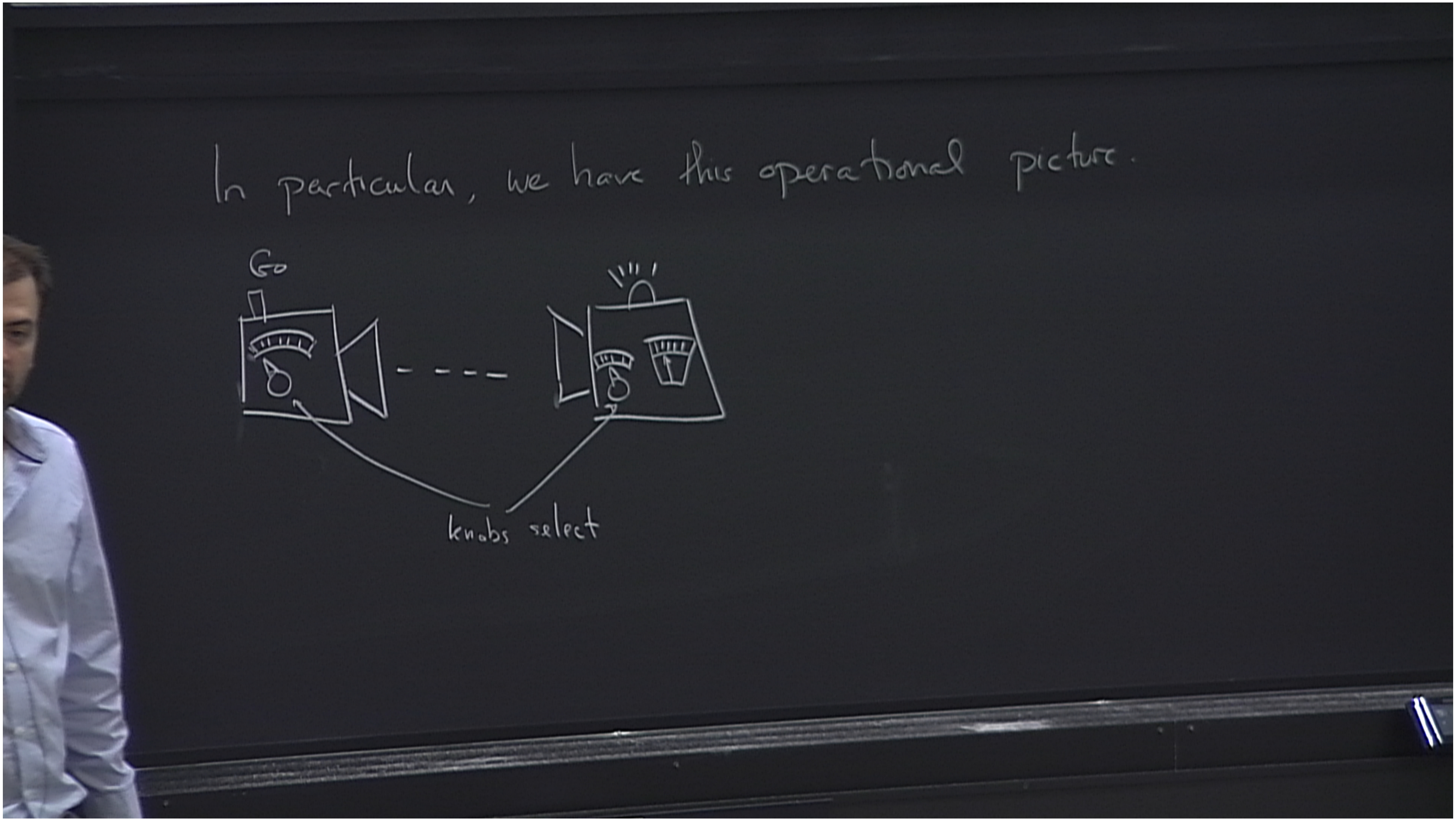
$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$$

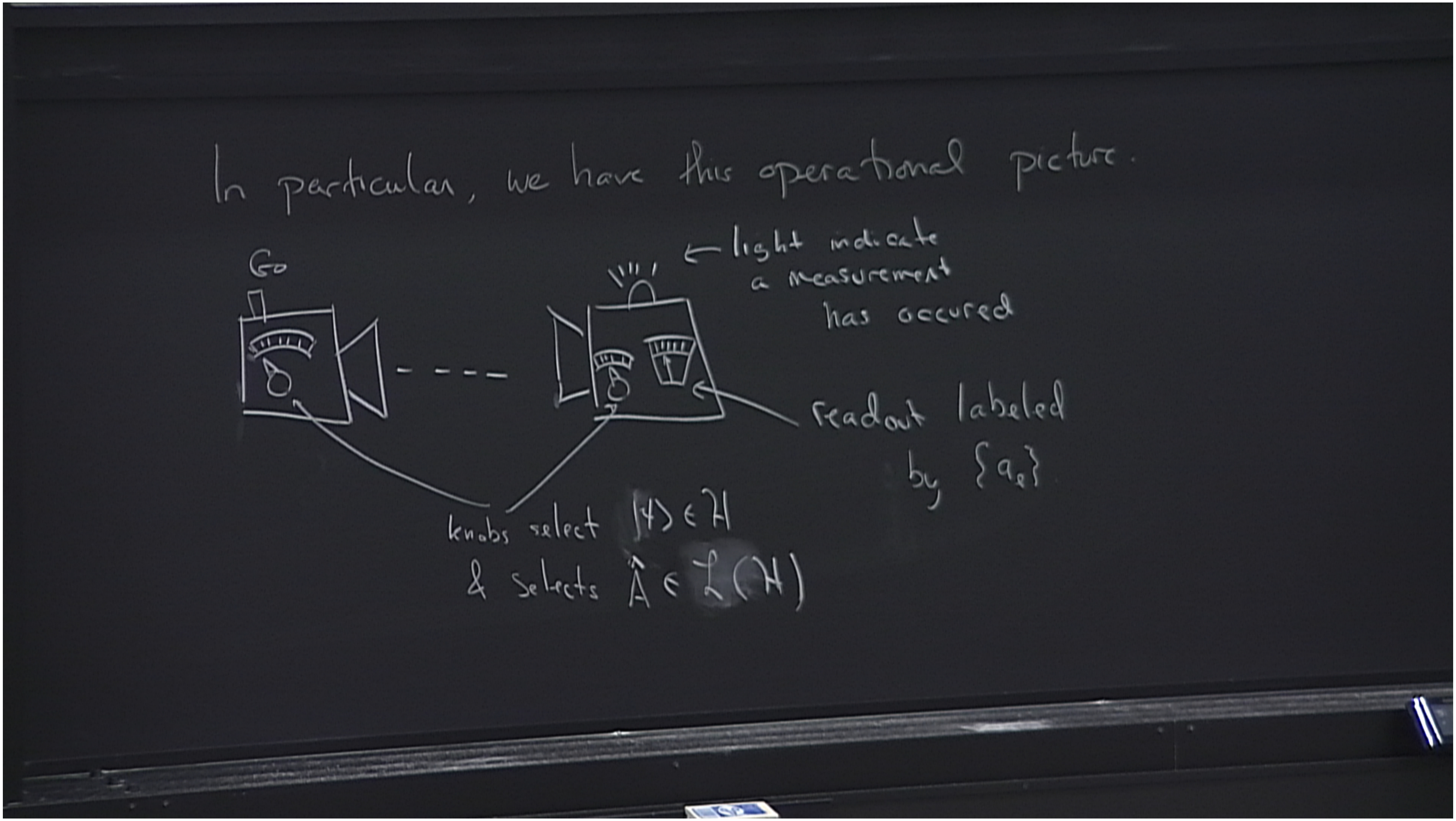
$\Rightarrow$ "expectation value"

We have $Pr(a_\ell) = Tr[\,|\psi\rangle\langle\psi|\,\hat{\Pi}_\ell\,]$

$$\hat{A} = \sum_\ell a_\ell \hat{\Pi}_\ell$$

$$\langle \psi | A | \psi \rangle = \sum_\ell a_\ell \, Pr(a_\ell)$$


In particular, we have this operational picture.
Go
knobs select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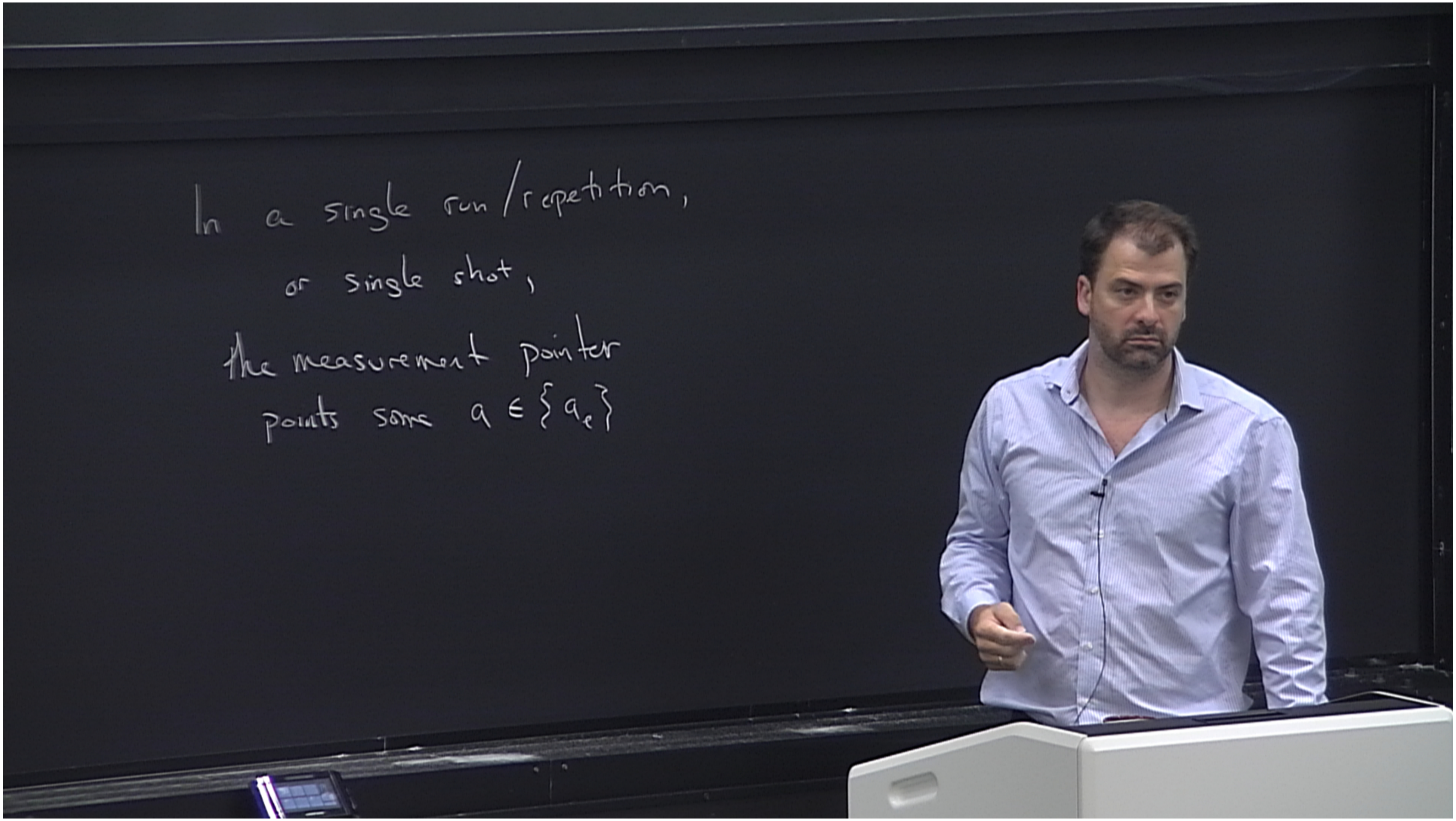
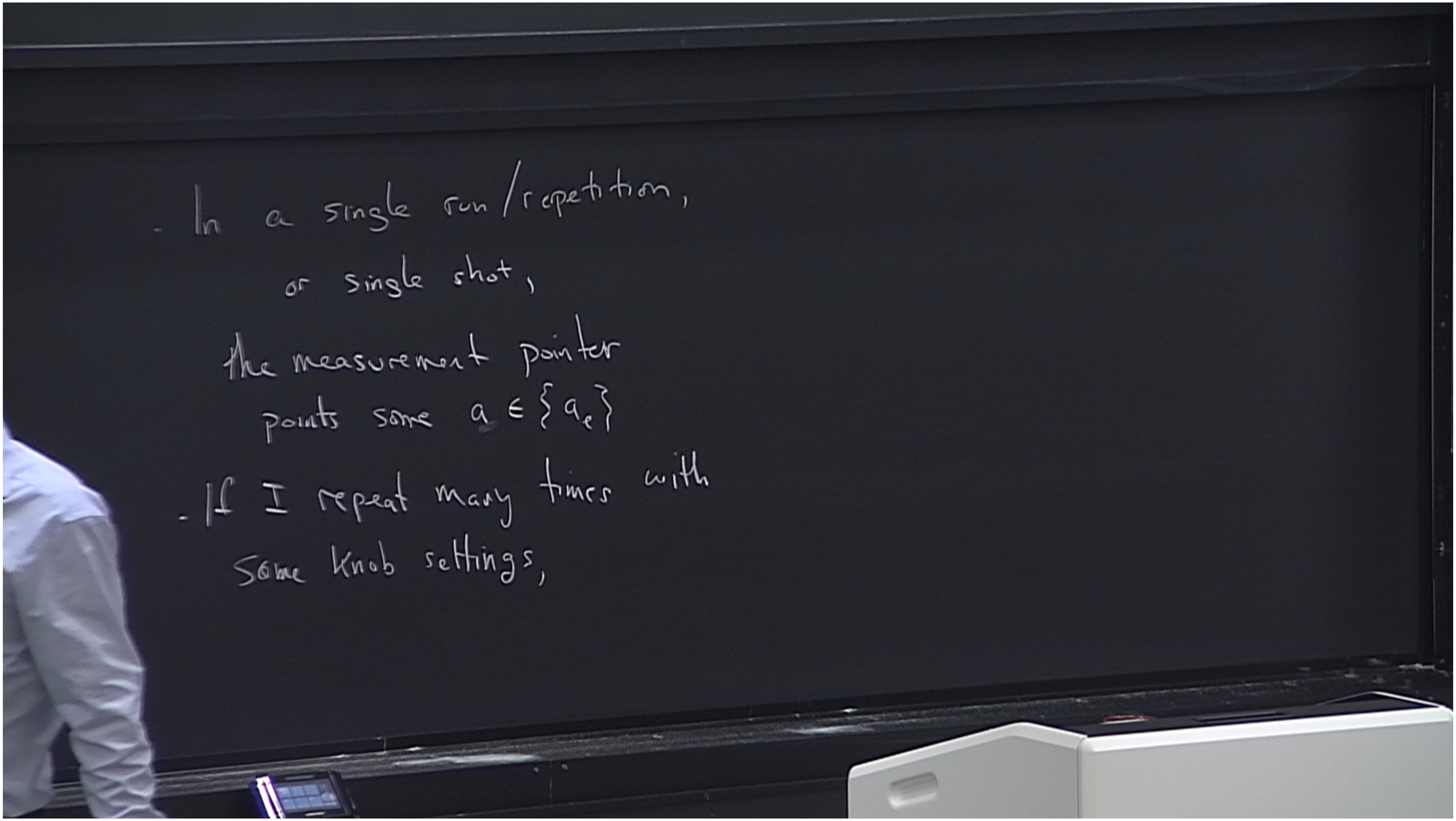
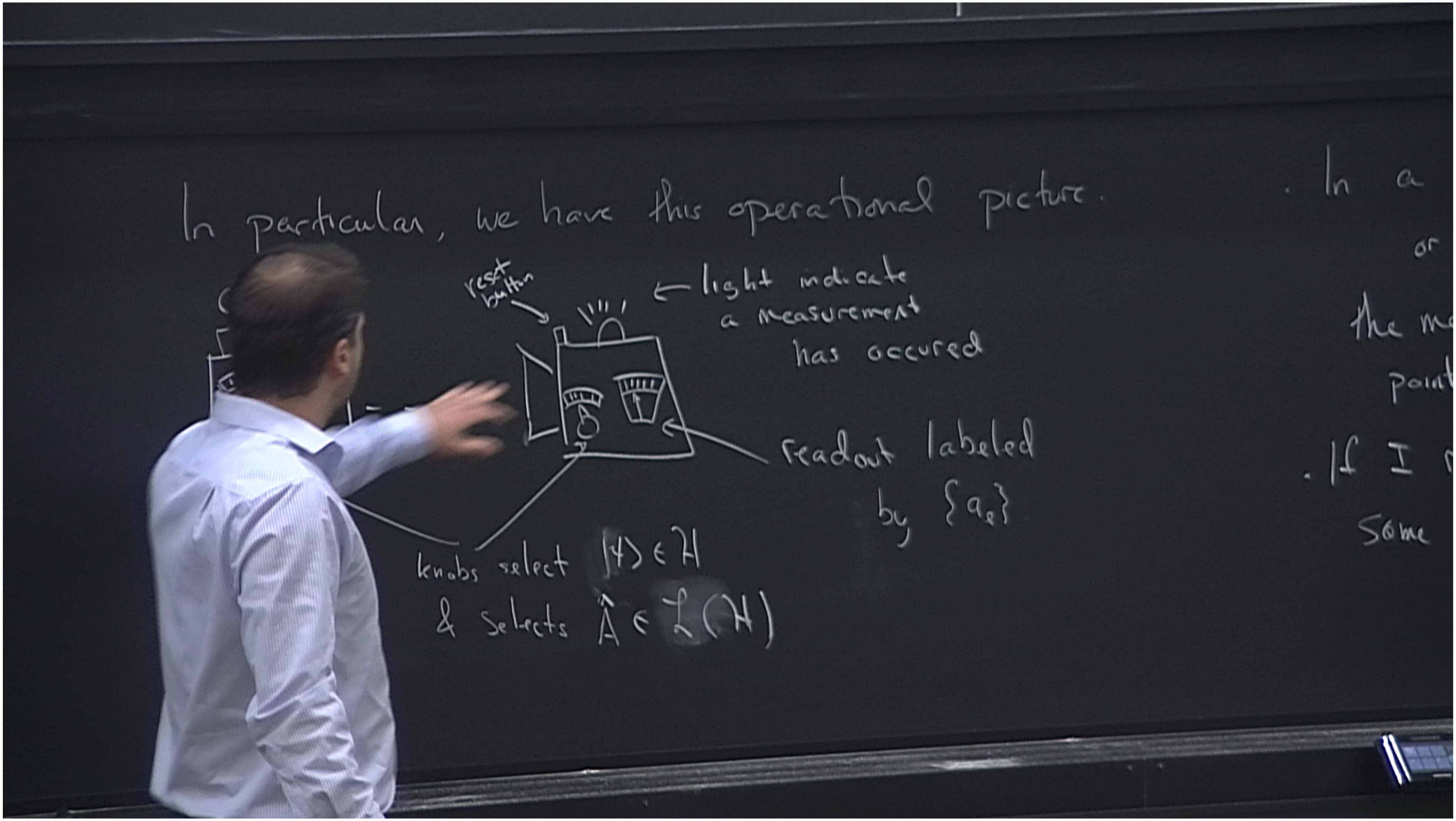
← light indicate a measurement has occured
readout labeled by $\{a_e\}$
knobs select $|\psi\rangle \in \mathcal{H}$
& selects $\hat{A} \in \mathcal{L}(\mathcal{H})$


In a single run/repetition,
or single shot,
the measurement pointer
points some $a \in \{a_e\}$


· In a single run/repetition,
or single shot,
the measurement pointer
points some $a \in \{a_e\}$
· If I repeat many times with
some knob settings,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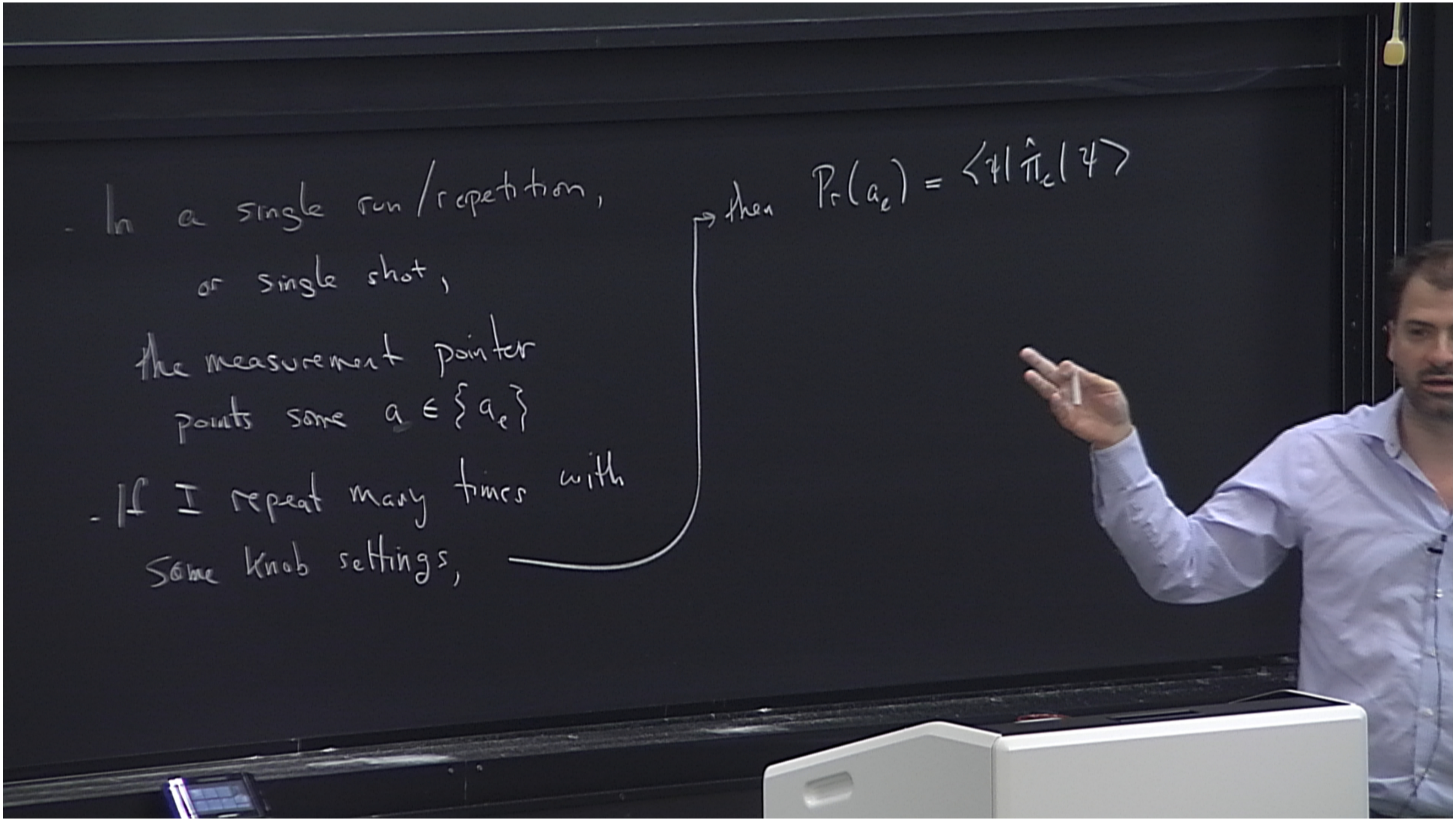
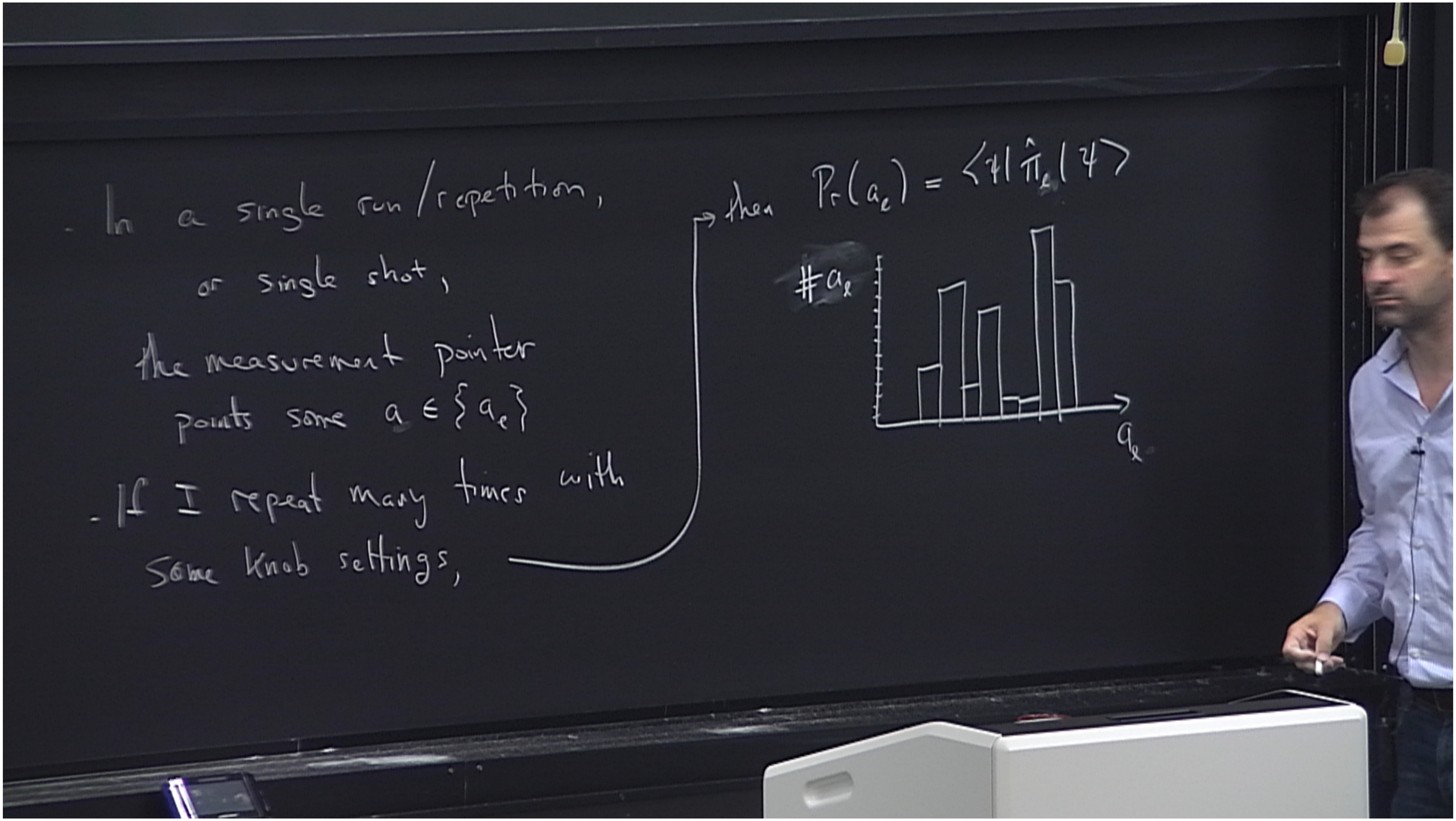
In a
or


- In a single run/repetition,
or single shot,
the measurement pointer
points some $a \in \{a_\ell\}$
- If I repeat many times with
some knob settings,
then $\Pr(a_\ell) = \langle\psi|\hat{\Pi}_\ell|\psi\rangle$


- In a single run/repetition,
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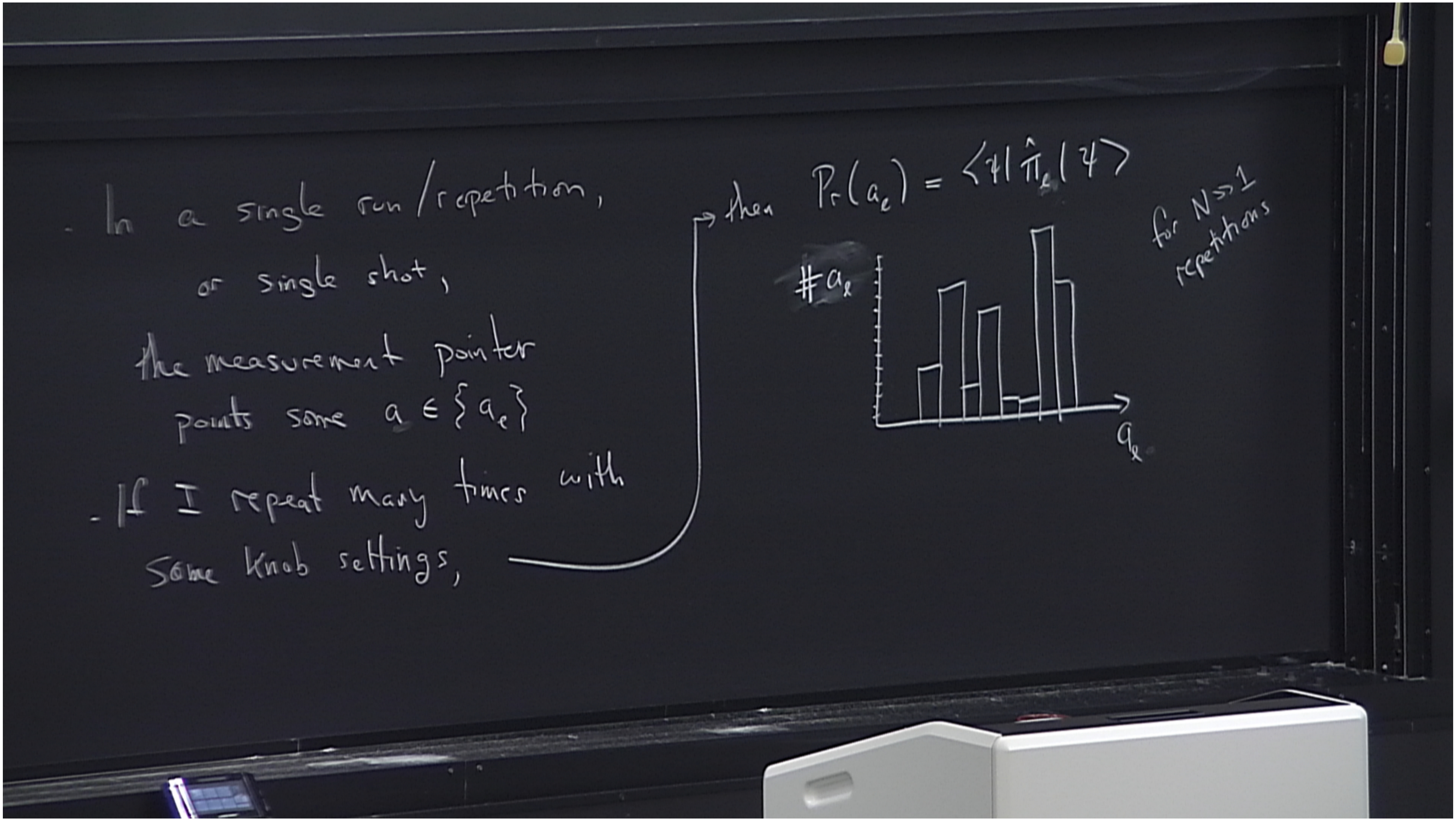
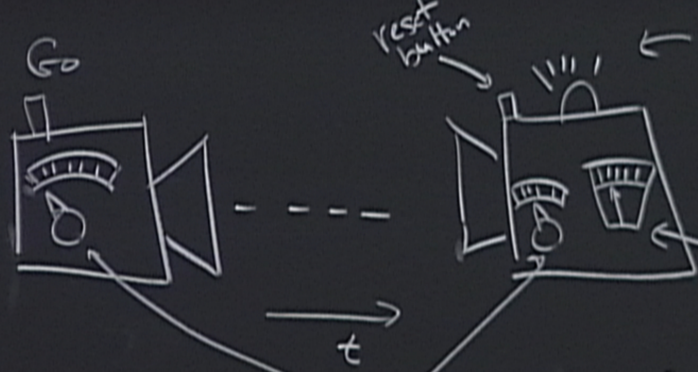
the measurement pointer
points some $a \in \{a_\ell\}$
- If I repeat many times with
some knob settings,
then $P_r(a_\ell) = \langle \psi | \hat{\pi}_\ell | \psi \rangle$
$\# a_\ell$
a_ℓ



. In a single run/repetition,
or single shot,
the measurement pointer
points some $a \in \{a_\ell\}$
. If I repeat many times with
some knob settings,
then $Pr(a_\ell) = \langle\psi|\hat{\pi}_\ell|\psi\rangle$
for $N >> 1$ repetitions
$\# a_\ell$
a_ℓ

In particular, we have this operational picture.



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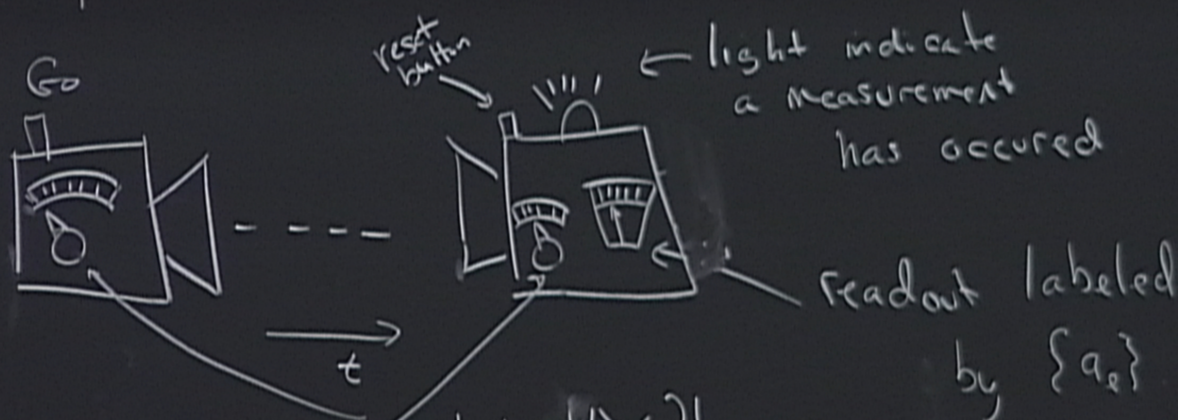

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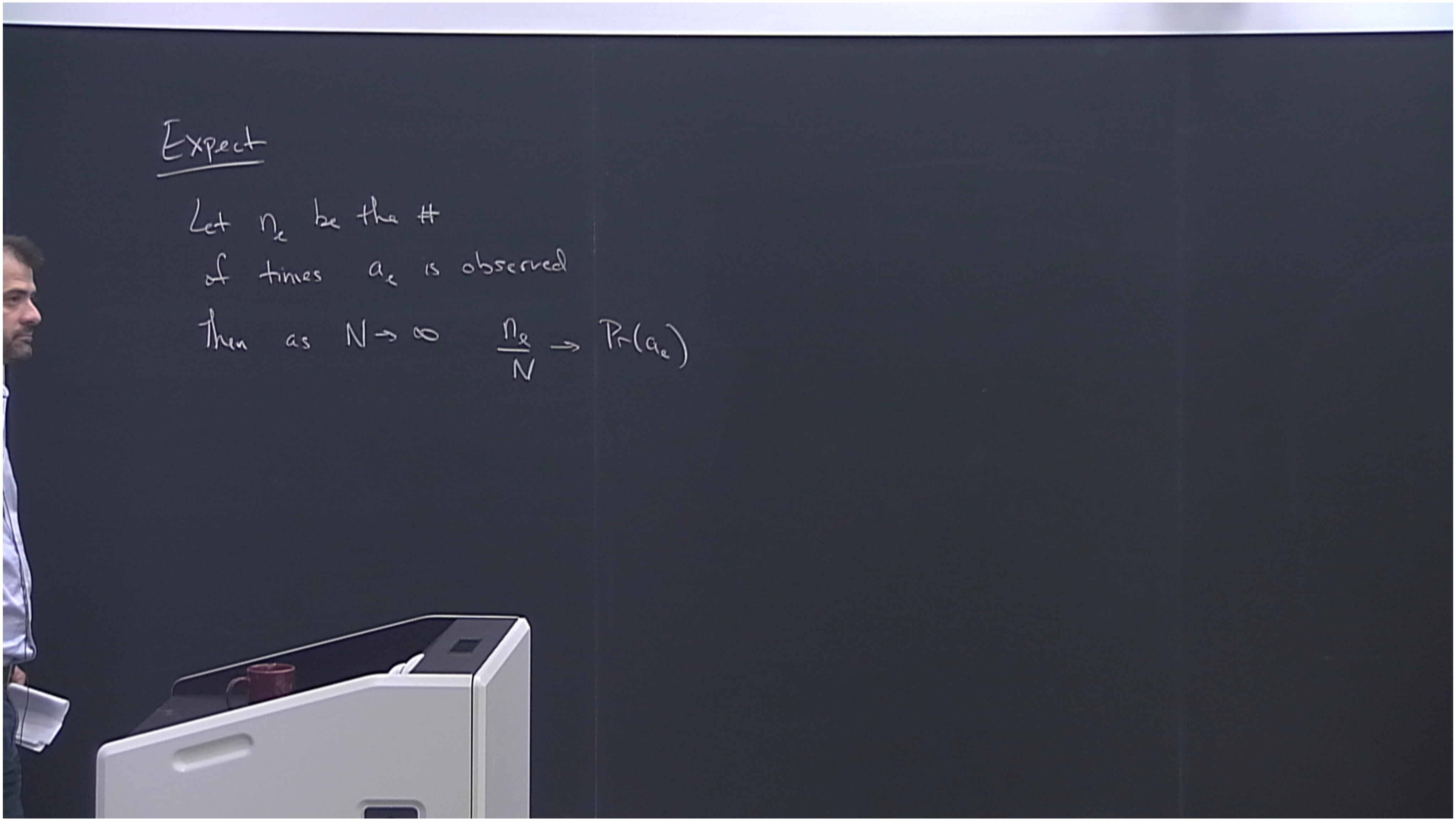
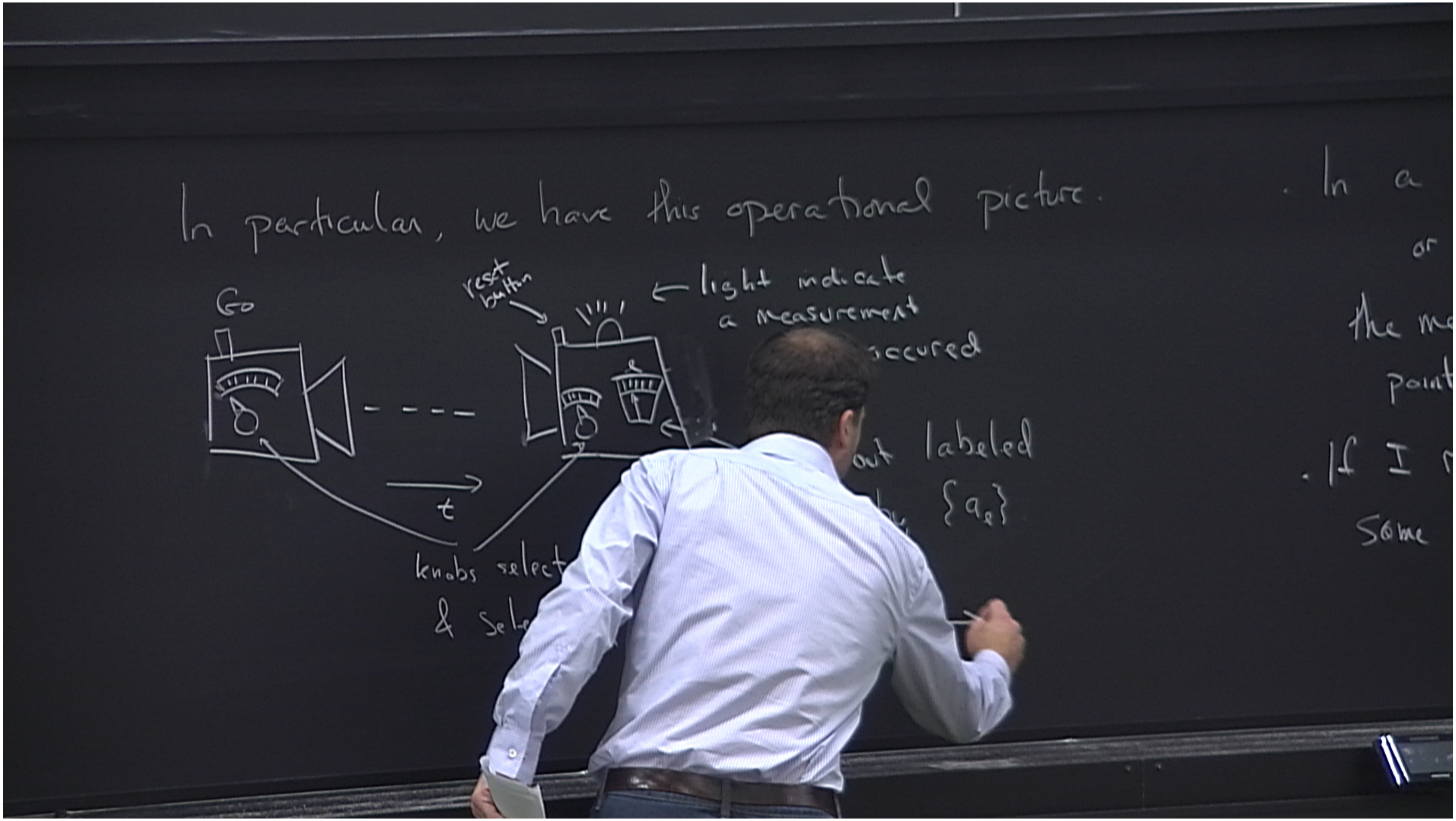

Expect
Let n_e be the #
of times a_e is observed
Then as $N \to \infty$ $\frac{n_e}{N} \to Pr(a_e)$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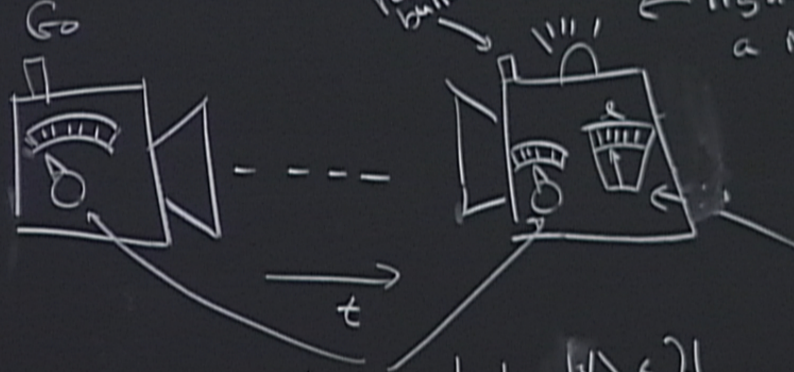

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Then as $N \to \infty$ $\quad \frac{n_e}{N} \to Pr(a_e)$