Title: Irreversibility and Entanglement Spectrum Statistics in Quantum Circuits

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Abstract: We show that for a system evolving unitarily under a stochastic quantum circuit, the notions of irreversibility, universality of computation, and entanglement are closely related. As the state of the system evolves from an initial product state, it becomes increasingly entangled until entanglement reaches a maximum. We define irreversibility as the failure to find a circuit that disentangles a maximally entangled state. We show that irreversibility occurs when maximally entangled states are generated with a quantum circuit formed by gates from a universal quantum computation set. We find that irreversibility is also associated to a Wigner-Dyson statistics in the fluctuations of spacings between adjacent eigenvalues of the systemâ€™s reduced density matrix. In contrast, when the system is evolved with a non-universal set of gates, the statistics of the entanglement spacing deviates from Wigner-Dyson and the disentangling algorithm succeeds. We discuss how these findings open a new way to characterize non-integrability in quantum systems.

Irreversibility and Entanglement Spectrum Statistics in Quantum Circuits

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Outline
Motivation: irreversibility in quantum mechanics
Quantum entanglement and stochastic circuits
Disentangling algorithm: irreversibility & reversibility
Statistical fluctuations of entanglement
Discussion & summary

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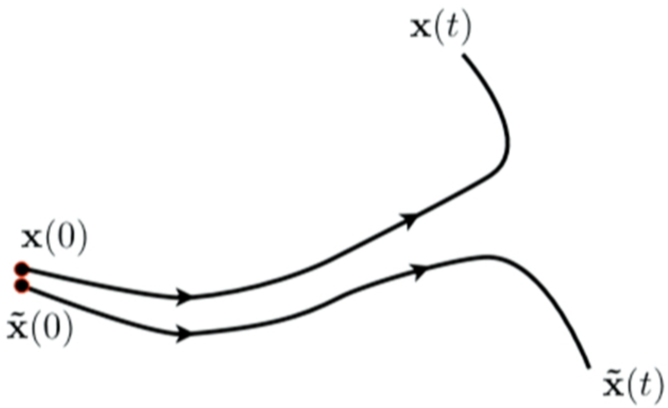
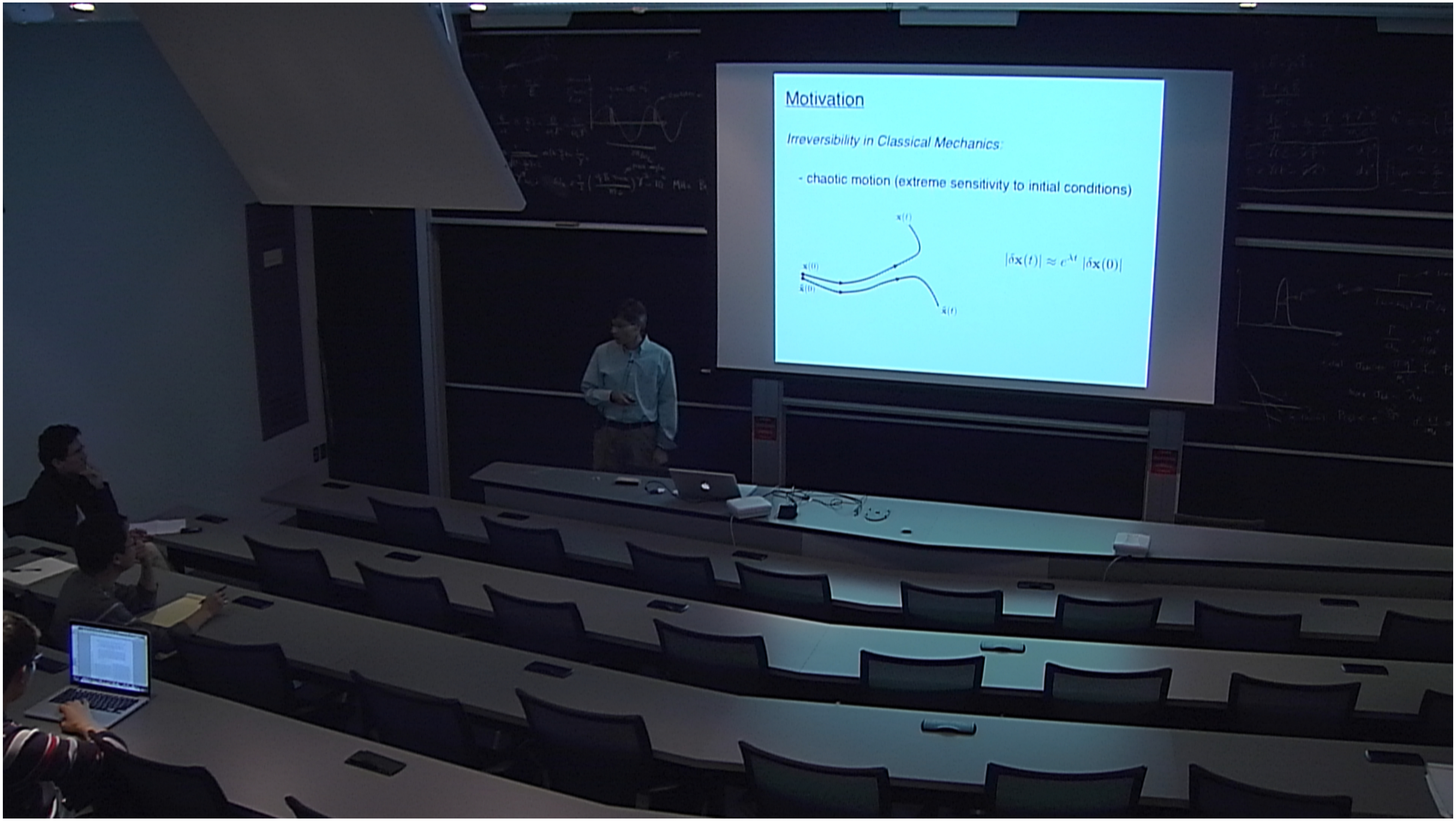
Irreversibility in Classical Mechanics:

- chaotic motion (extreme sensitivity to initial conditions)

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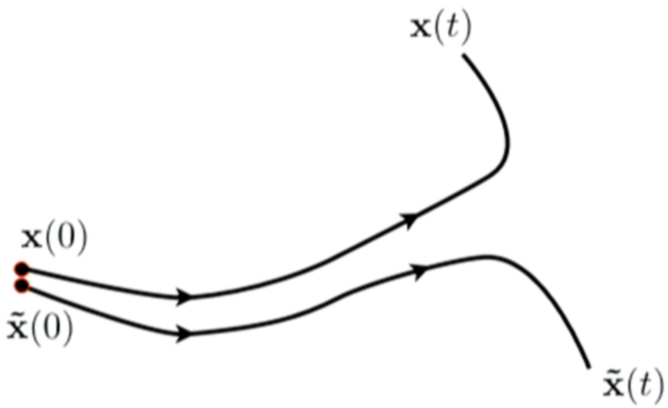

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Lyapunov exponent

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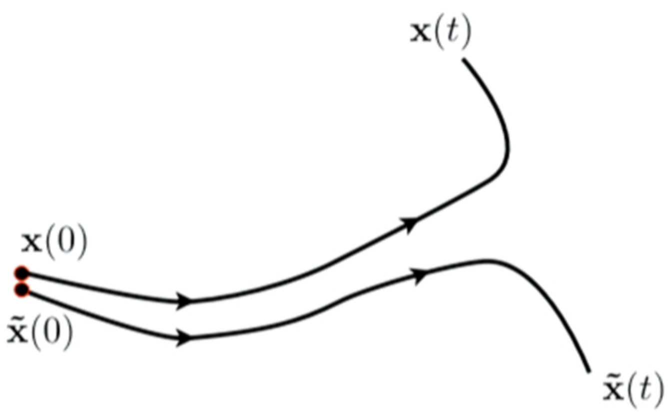

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Loschmidt echo: $\hat{H} \longrightarrow \hat{H} + \epsilon\,\hat{V}$ $\quad [\hat{H}, \hat{V}] \neq 0$

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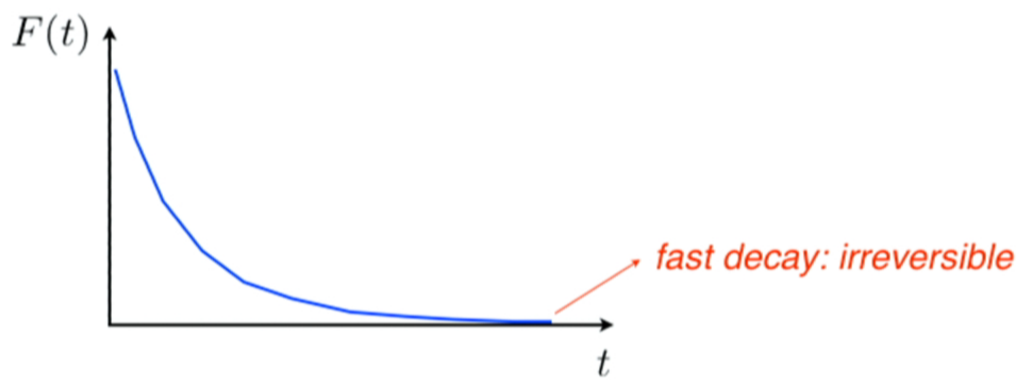

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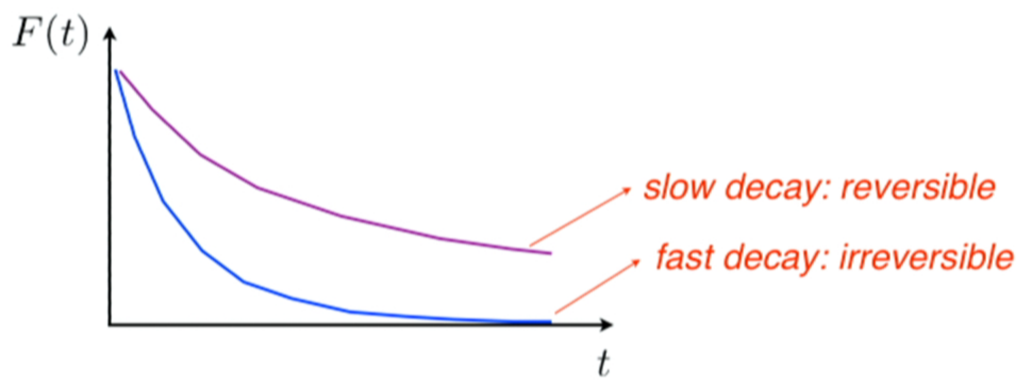

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  no clear-cut relationship between integrability and reversibility

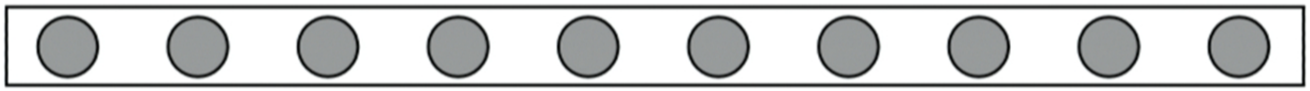
- *Our proposal*:

  Use entanglement to characterize irreversibility

  No need for a Hamiltonian (or even unitary) evolution

Model System

N qubit system: $\mathcal{H} = \otimes_{i=0}^{N-1} \mathcal{H}_i$ $\mathcal{H}_i = \mathrm{span}\{|0\rangle, |1\rangle\}_{\mathrm{C}^2}$



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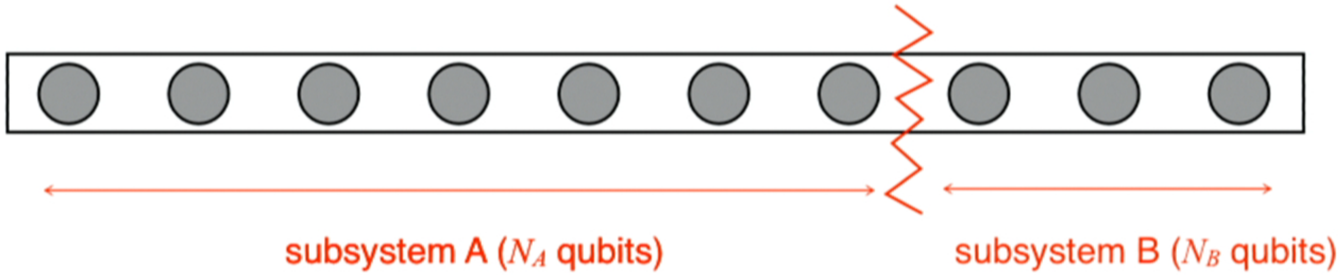

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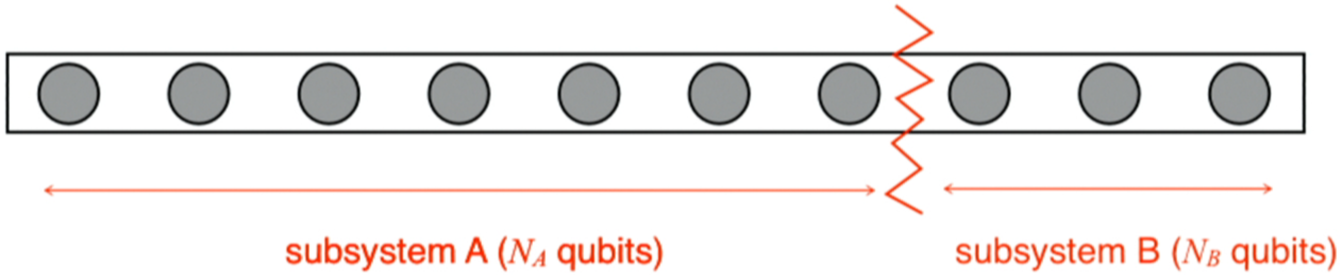

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$\rho_A = \mathrm{tr}_B[\rho]$ reduced density matrix

$\lfloor\!\!\rightarrow$ eigenvalues $\{\lambda_1, \lambda_2, \cdots, \lambda_{2^{N_A}}\}$

$$\begin{cases} \lambda_n \geq 0 \\ \sum_{n=1}^{2^{N_A}} \lambda_n = 1 \end{cases}$$

Entanglement measure

Rényi entropies

$(q = 0, 1, \ldots)$

$$S_q^A = -\frac{1}{1-q} \log_2 \sum_{n=1}^{2^{N_A}} \lambda_n^q$$

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$$S_0^A = \log_2 r$$ → rank of ρ_A

$$S_1^A = -\sum_{n=1}^{r} \lambda_n \log_2 \lambda_n$$ → von Neumann entropy

$$S_2^A = -\log_2 \sum_{n=1}^{r} \lambda_n^2$$

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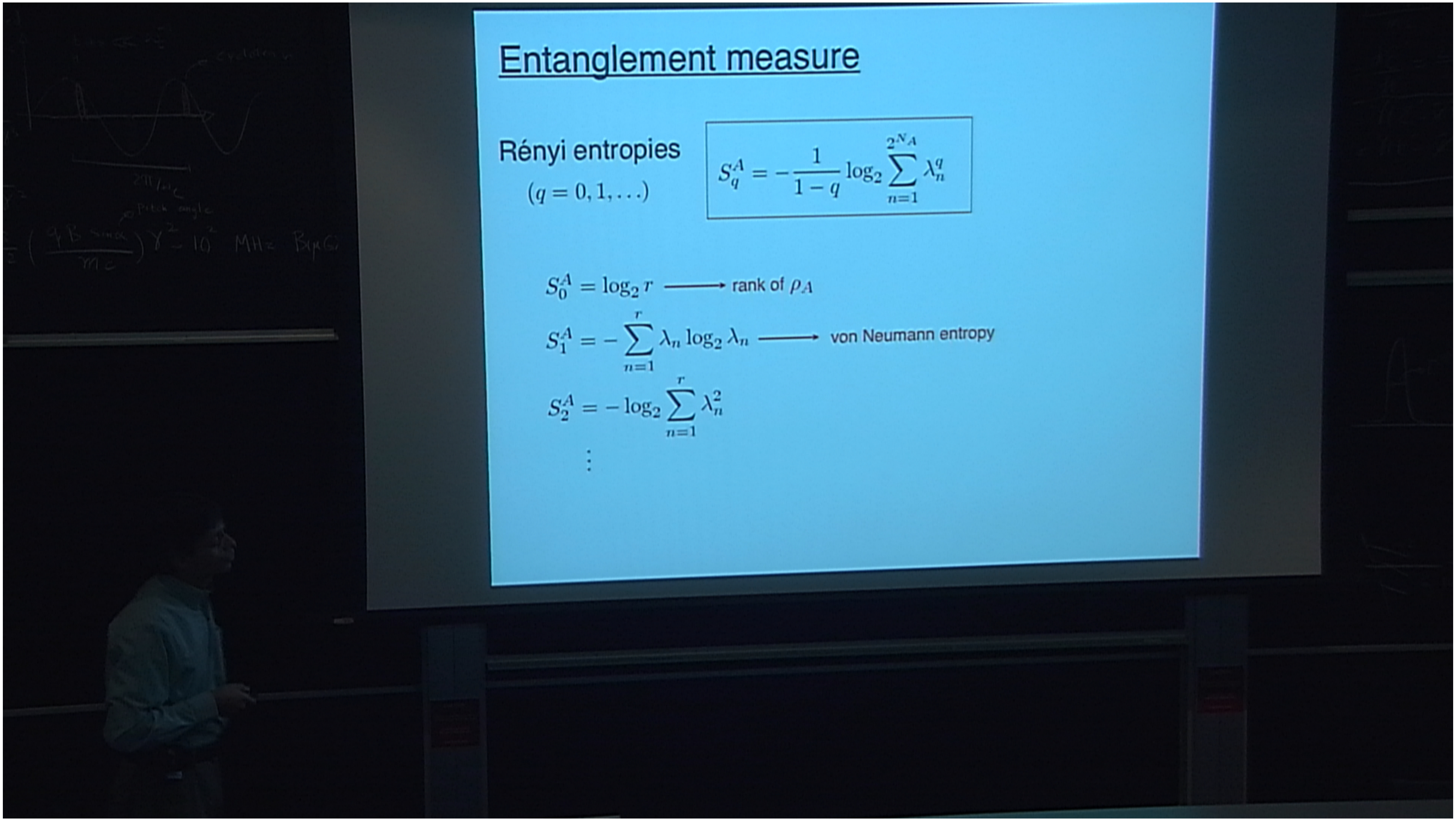
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hierarchy: $N_A \geq S_0^A \geq S_1^A \geq S_2^A \geq \cdots \geq 0$

Time evolution: stochastic circuit

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initial state (product) $|\Psi_0\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots |\psi_N\rangle$ ⟶ no entanglement

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random superposition $|\psi_n\rangle = \cos(\theta_n)|0\rangle + \sin(\theta_n)|1\rangle$

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Types of gate:

1-qubit	
NOT	Not
H	Hadamard
T S	phase gates

2-qubit	
CNOT	XOR

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Hadamard

$$|0\rangle \rightarrow \boxed{\texttt{H}} \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle \rightarrow \boxed{\texttt{H}} \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

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Notice:

{H,T,CNOT} ⟶ complete set for unitary evolution (universal quantum computation)

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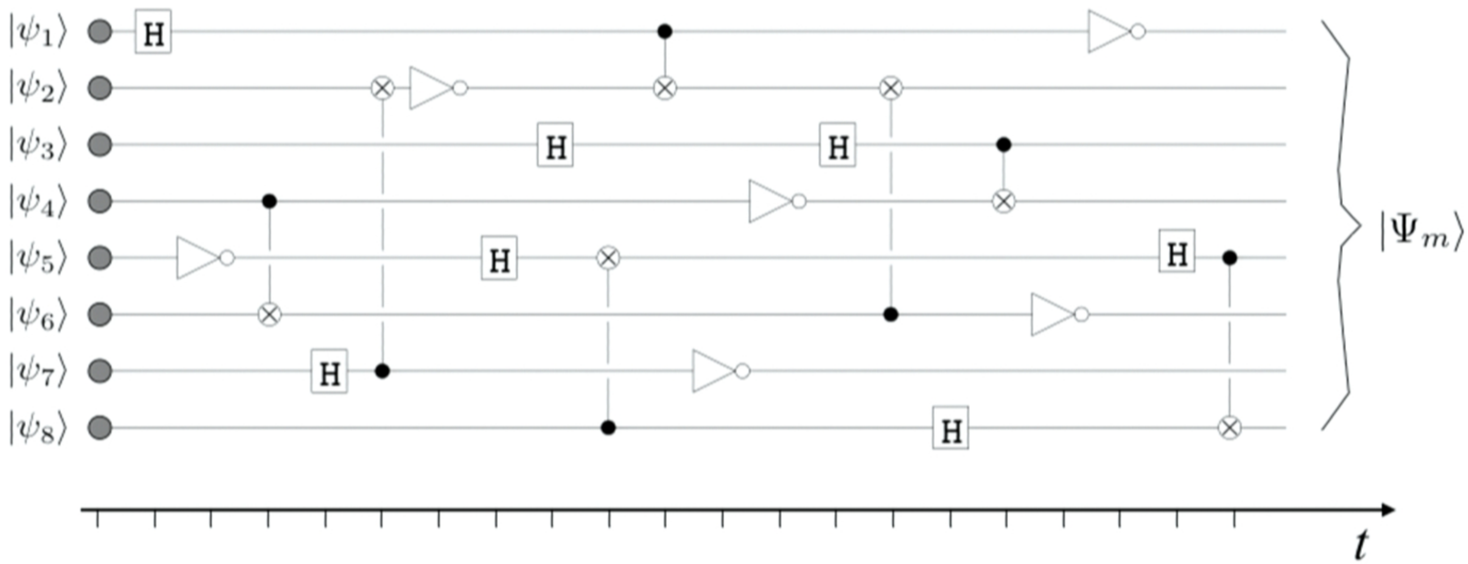
$\left\{\hat{U}_1, \hat{U}_2, \ldots, \hat{U}_m\right\}$ sequence of gates drawn randomly from a fixed set

e.g.: set {H,NOT,CNOT}

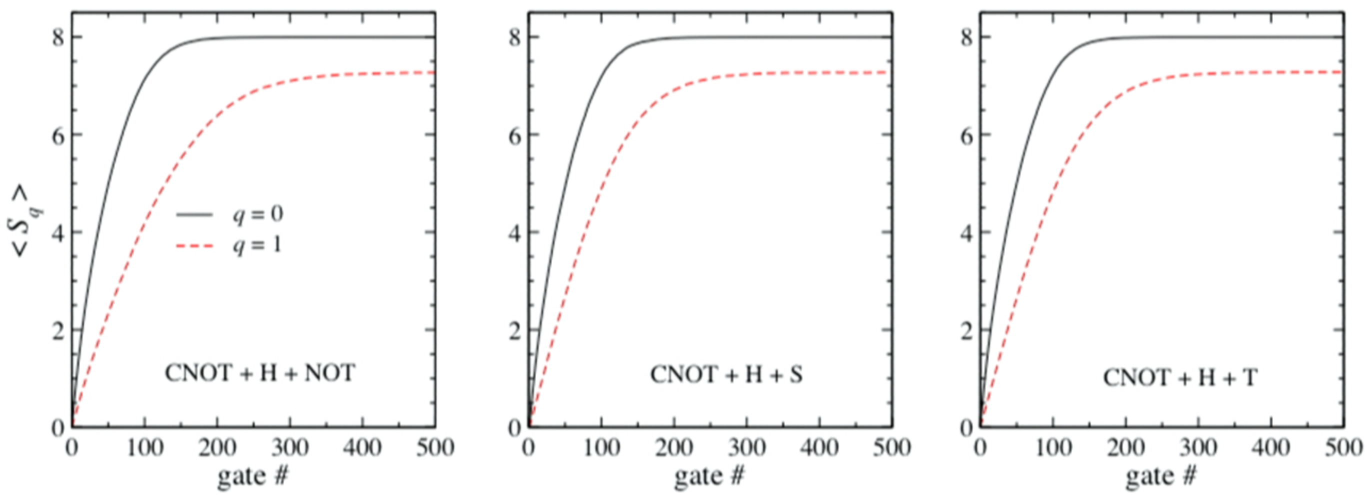


Evolution of the Rényi entropies (numerical work)

$N = 16$ $(N_A = N_B = 8)$

$m = 512$

5000 sample circuits



Disentangling algorithm

- Given a maximally entangled state $|\Psi_m\rangle$, find a circuit that brings the system back to a product state.

Metropolis algorithm:

- (1) Compute the entanglement entropies (all bipartitions)
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{CNOT,H,T} → never reversible

- For all these gate sets, the average entropies (S_0, S_1, S_2) reach the same maximum values.

- Entropies alone cannot distinguish reversible from irreversible unitary evolutions...

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$\rho_A = \mathrm{tr}_B \, |\Psi_m\rangle\langle\Psi_m| \longrightarrow$ eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_{2^{N_A}}\}$

$(N_A = N/2)$

- Average density: $\nu(\lambda) = \frac{1}{2^{N_A}} \left\langle \sum_{n=1}^{2^{N_A}} \delta(\lambda - \lambda_n) \right\rangle$

  (i) varies by orders of magnitude within the range $0 \leq \lambda \leq 1$

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- Better: distribution of ratios of consecutive level spacings

  (avoids spectrum unfolding)

$$s_n = \frac{\lambda_n - \lambda_{n+1}}{\lambda_{n-1} - \lambda_n} \longrightarrow P(s) = \frac{1}{2^{N_A}} \left\langle \sum_{n=1}^{2^{N_A}} \delta(s - s_n) \right\rangle$$

$$\lambda_1 \ge \lambda_2 \ge \lambda_3 \ge \cdots \Rightarrow 0 \le s_n < \infty$$

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(surmises derived by Atas *et al., 2013*)

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Wigner-Dyson ensembles

$\beta = 1$ GOE → real eigenvectors (TRS)

$\beta = 2$ GUE → complex eigenvectors (~~TRS~~)

$\beta = 4$ GSE → quaternion eigenvectors (TRS)

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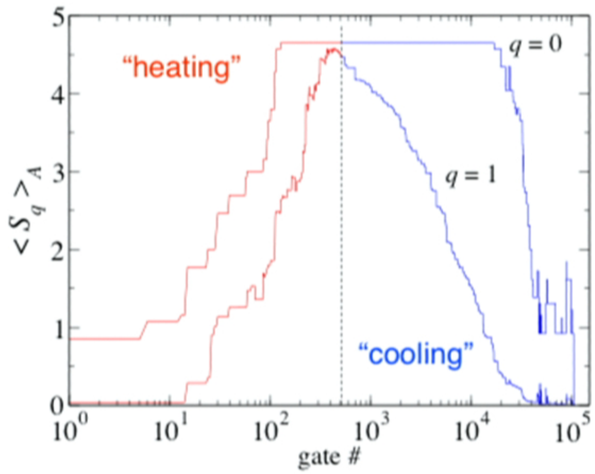

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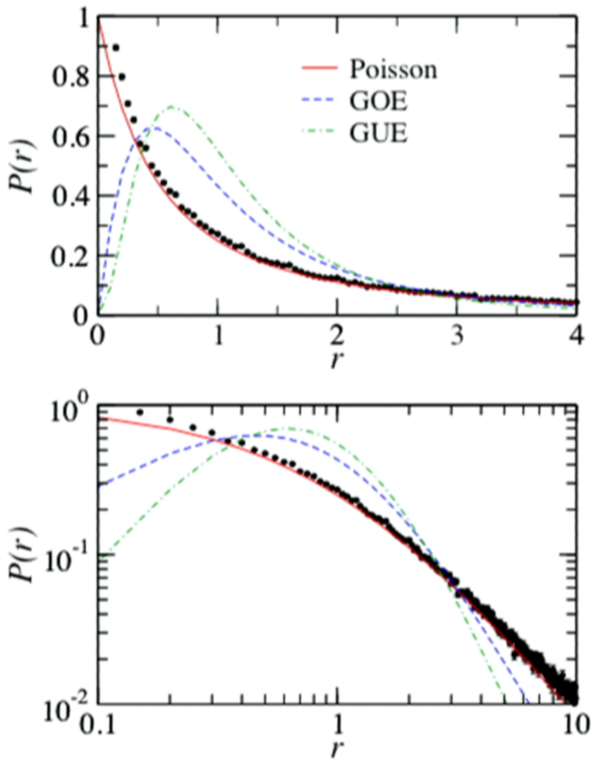
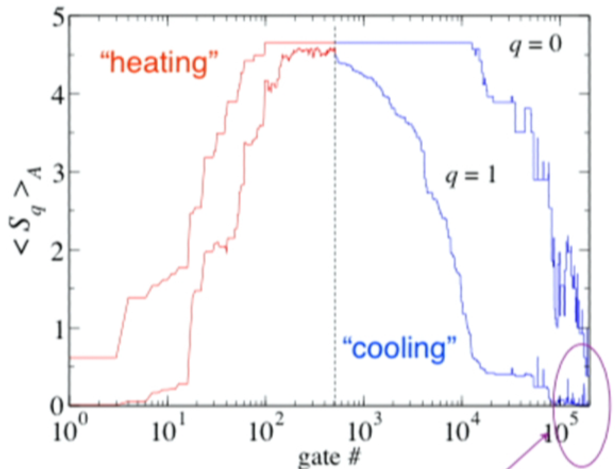
5000 samples, $N_A = N/2 = 8$



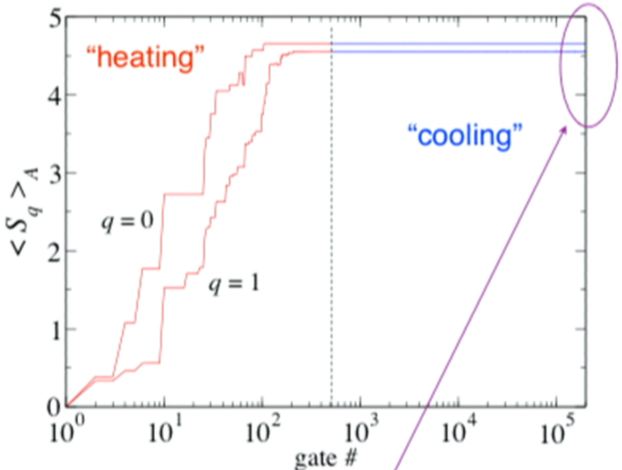


reversible

(100 cases checked)

Poisson statistics

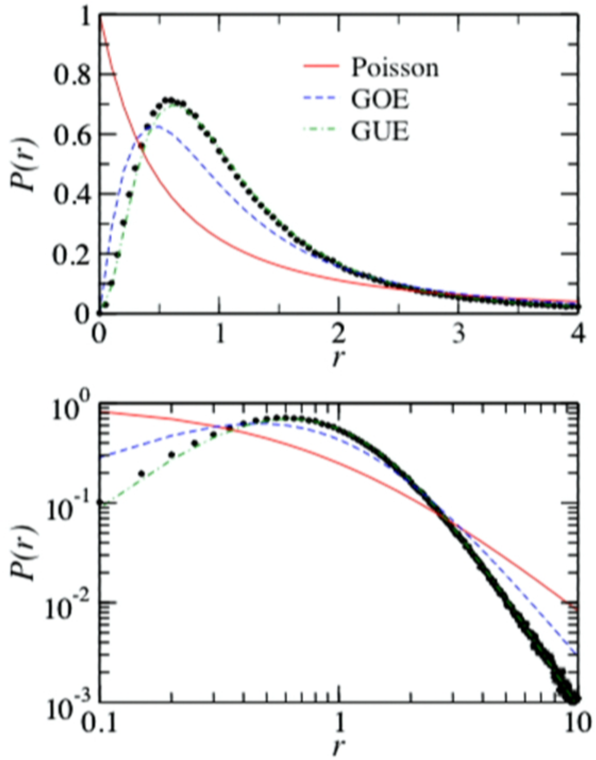
Results {CNOT,H,T}



irreversible

(100 cases checked)

5000 samples, $N_A = N/2 = 8$



Wigner-Dyson statistics

GUE symmetry class:

compatible with complex unitary gates

Back to integrability & reversibility

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- For Hamiltonian systems (constant H):

  • integrable systems: Poisson energy level statistics

Berry–Tabor conjecture (1977)

Additional evidence: {CNOT,NOT,Toffoli}

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permutation gates: complete set
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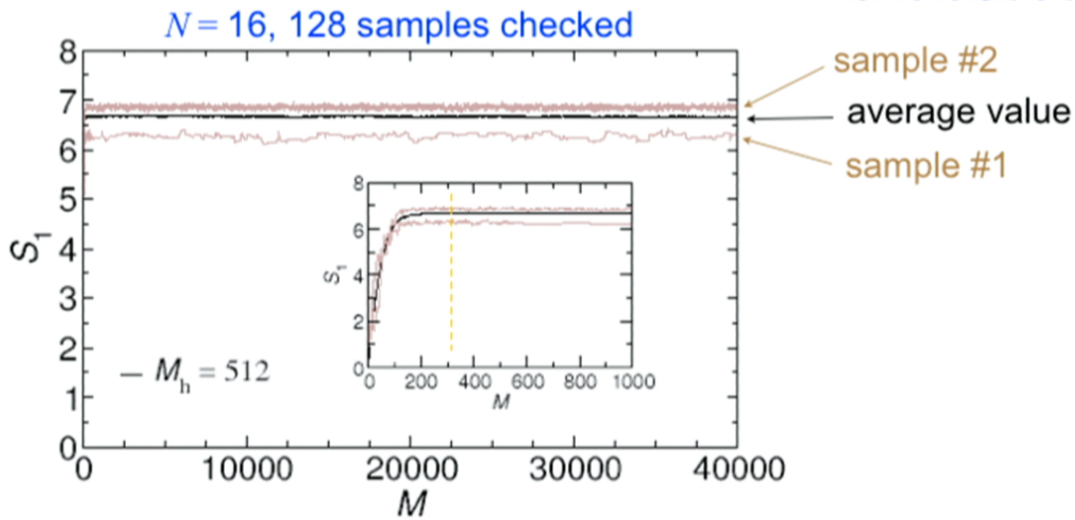

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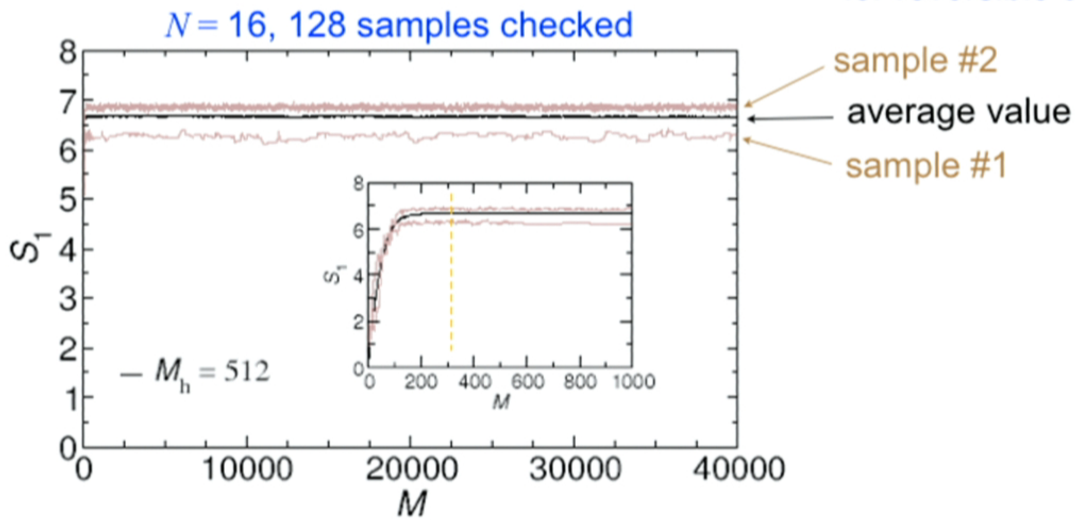

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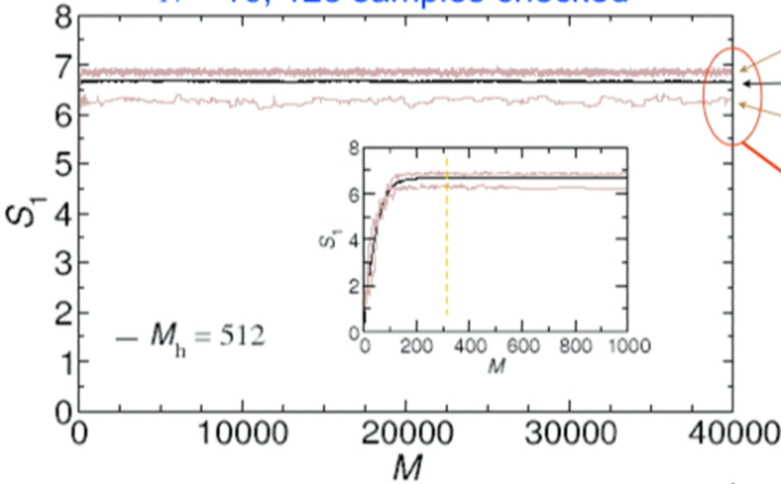
$N = 16$, 128 samples checked



sample #2

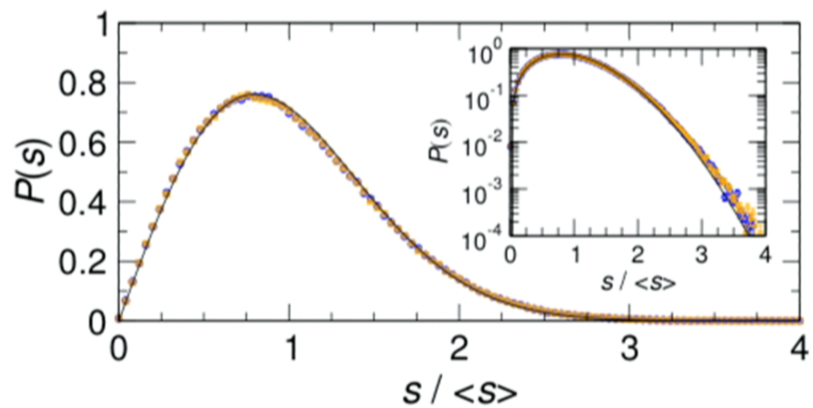
average value

sample #1

Irreversible

level spacing statistics of entanglement spectrum (after unfolding):

Wigner-Dyson GOE



Comments and thoughts

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THE END