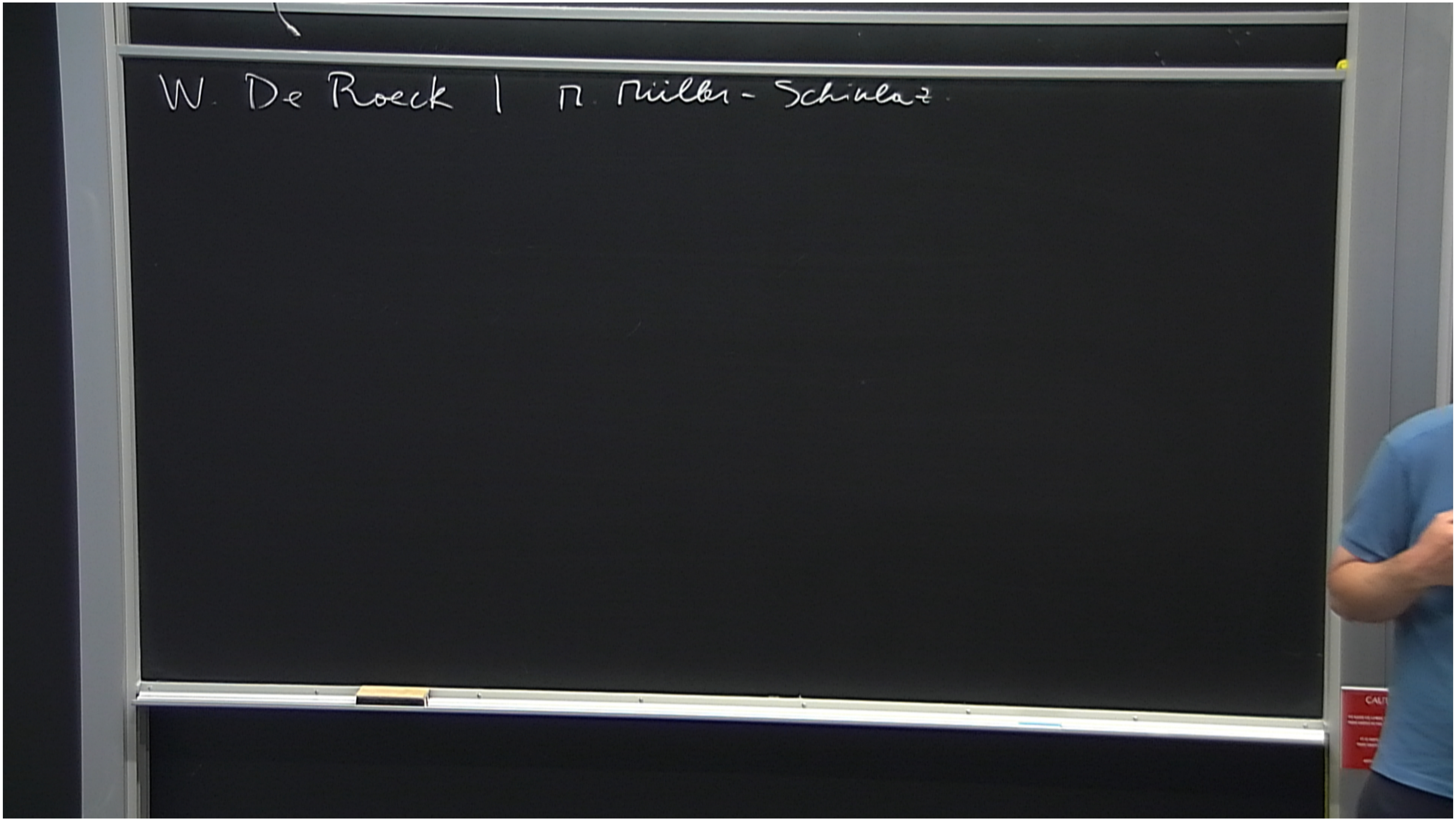
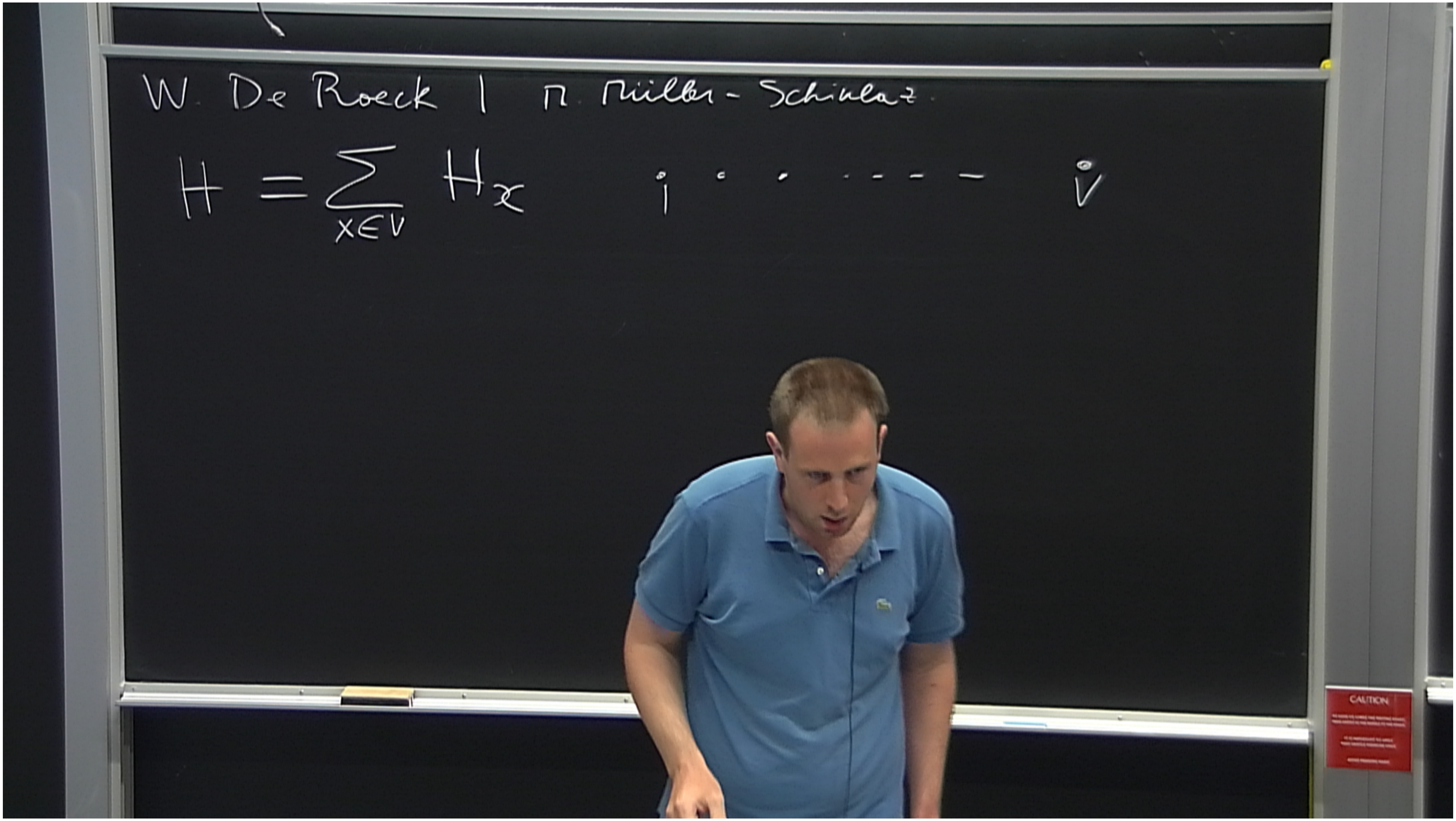
Title: Many-Body Localization in translation invariant systems?

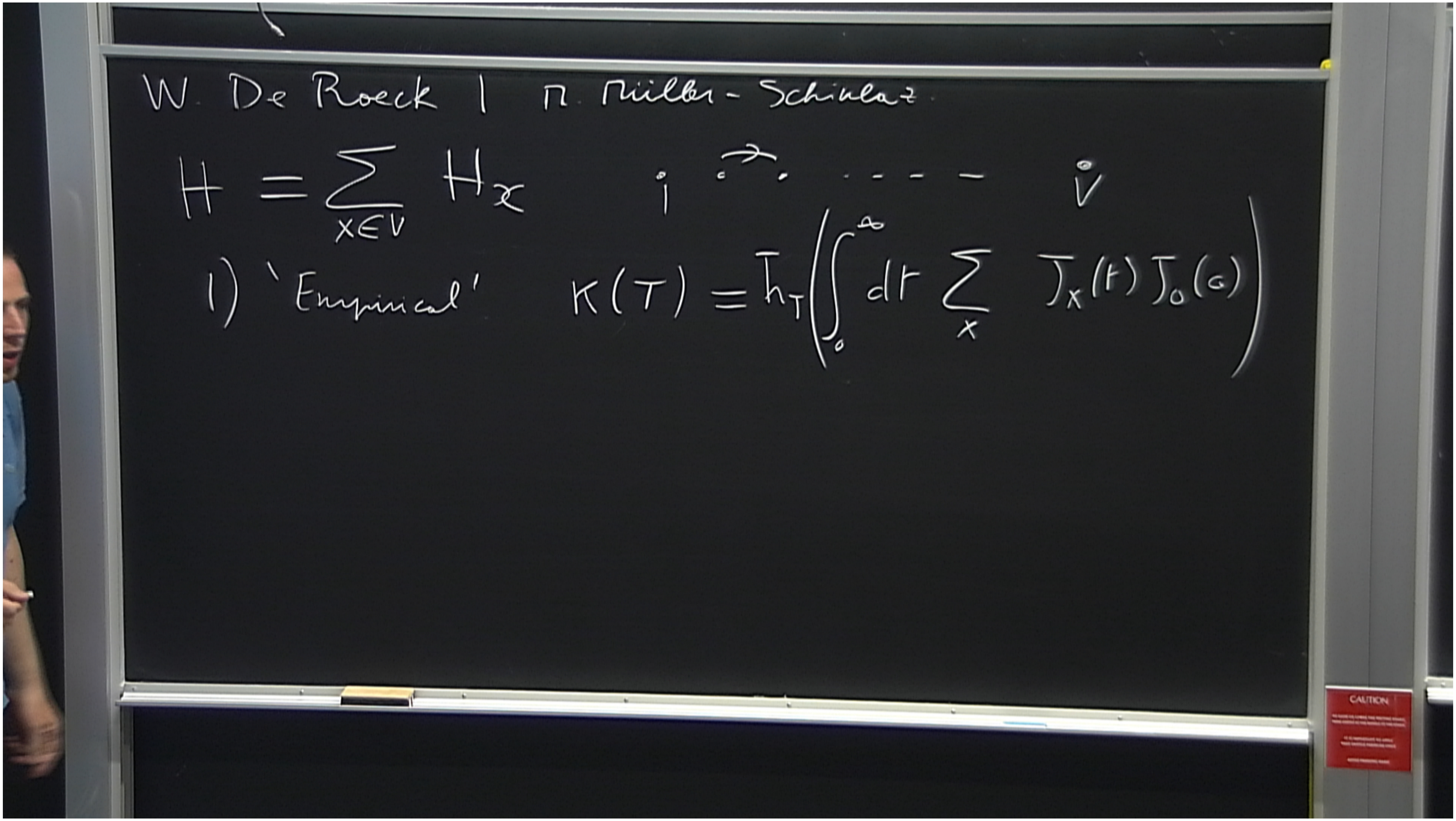
Date: Sep 09, 2014 03:30 PM

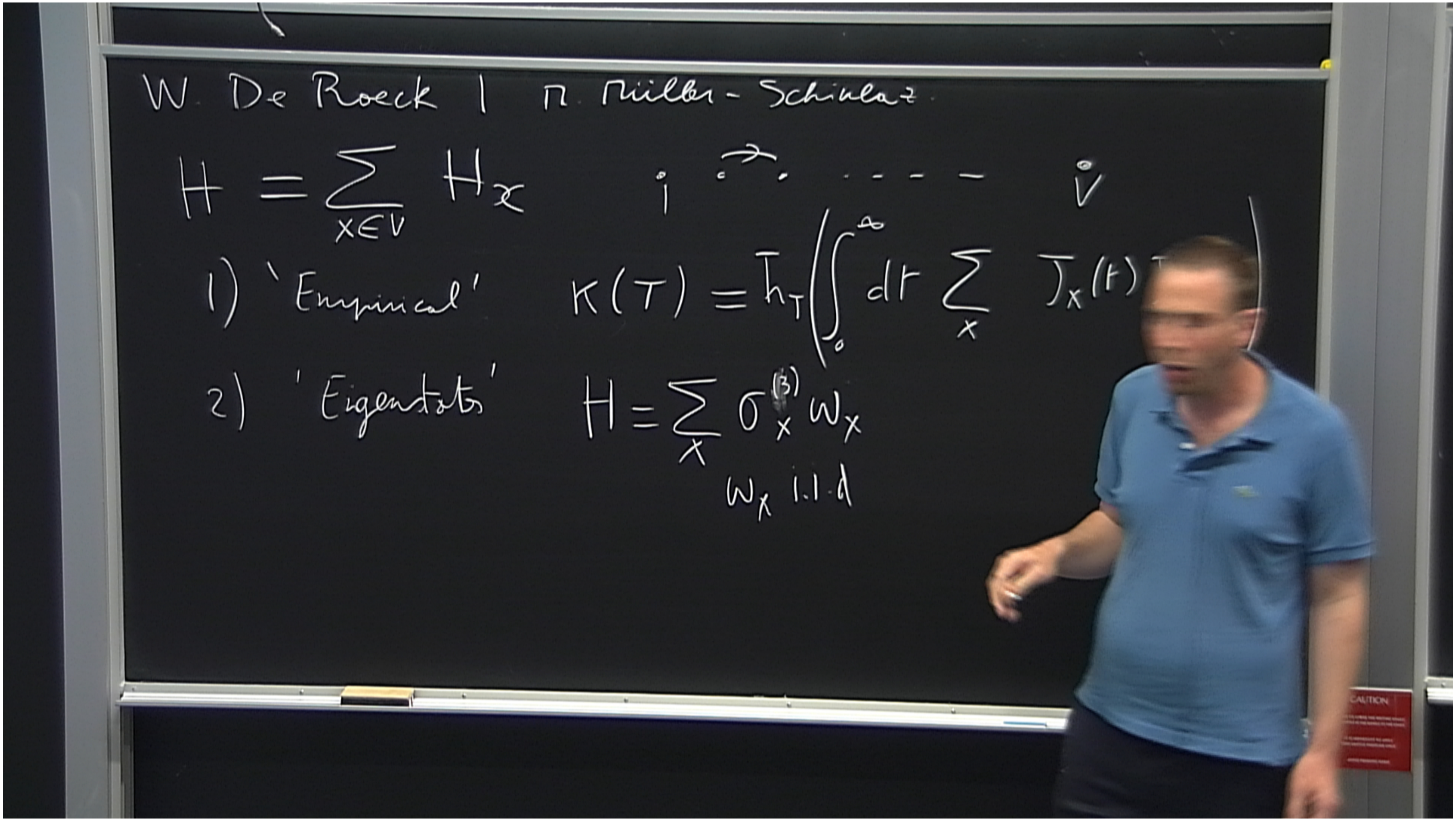
URL: http://pirsa.org/14090027

Abstract: Roughly speaking, Many-Body Localization (MBL) refers to the state of a material that fails to thermalize. Though MBL has mostly been studied in quenched disordered systems, several authors have recently proposed that this phase could be realized in clean (translation invariant) systems too. In this talk, I will discuss this idea and ask to which extent an MBL phase can indeed be expected in systems without quenched disorder. Hopefully, the discussion shed also some light on the localization-delocalization transition for more generic many-body systems. From joint work with W. De Roeck (Leuven), M. Mueller (Trieste), M. Schiulaz (Trieste).


W. De Roeck | M. Müller - Schiulaz


W. De Roeck
$H = \sum_{x \in V} H_x$


W. De Roeck
$H = \sum_{x \in V} H_x$
1) 'Empirical' $K(T) = \bar{h}_T\left(\int_0^\infty dt \sum_x J_x(t) J_0(0)\right)$
CAUTION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2) 'Eigenstates'
$H = \sum_x \sigma_x^{(3)} w_x$
w_x i.i.d

W. De Roeck | M. Müller - Schiulaz

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w_x i.i.d

W. De Roeck | M. Müller - Schinlaz

$$H = \sum_{x \in V} H_x \qquad i \quad \cdot \overset{\rightarrow}{\cdot} \quad \cdots \quad V$$

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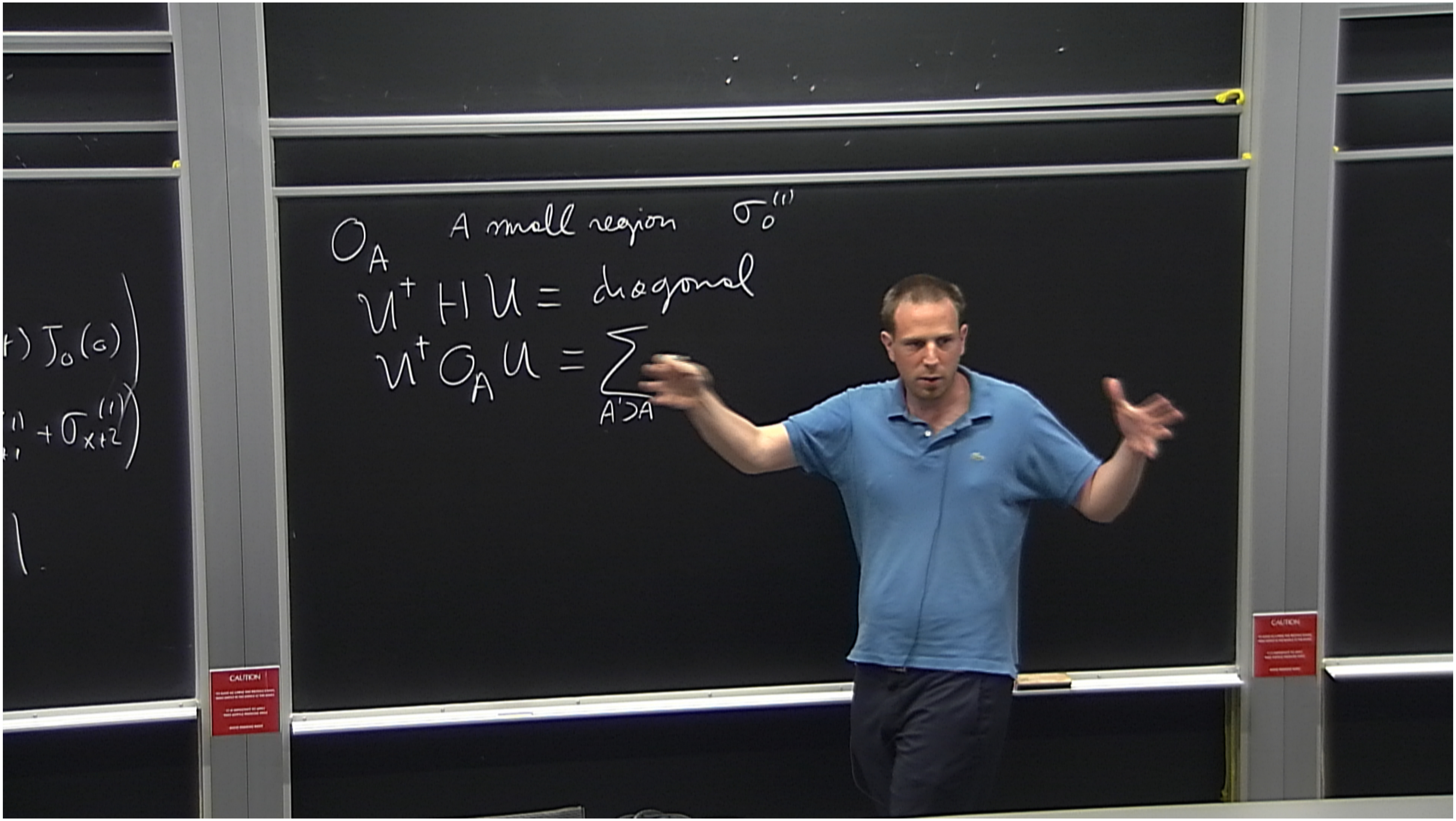
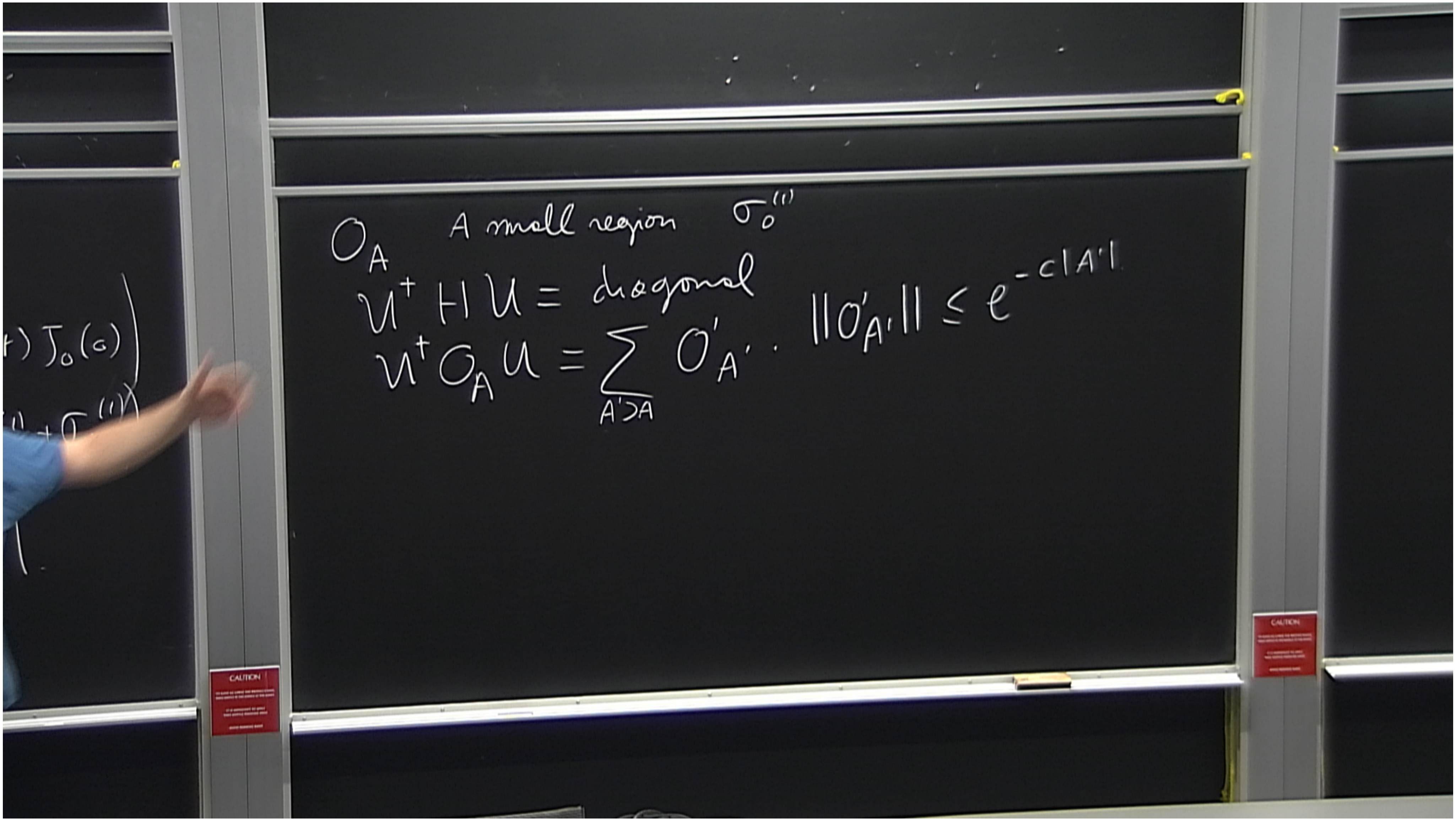
w_x i.i.d $\qquad J \ll 1$


O_A A small region $\sigma_0^{(1)}$
$U^+ H U$ = diagonal
$U^+ O_A U = \sum_{A' \supset A}$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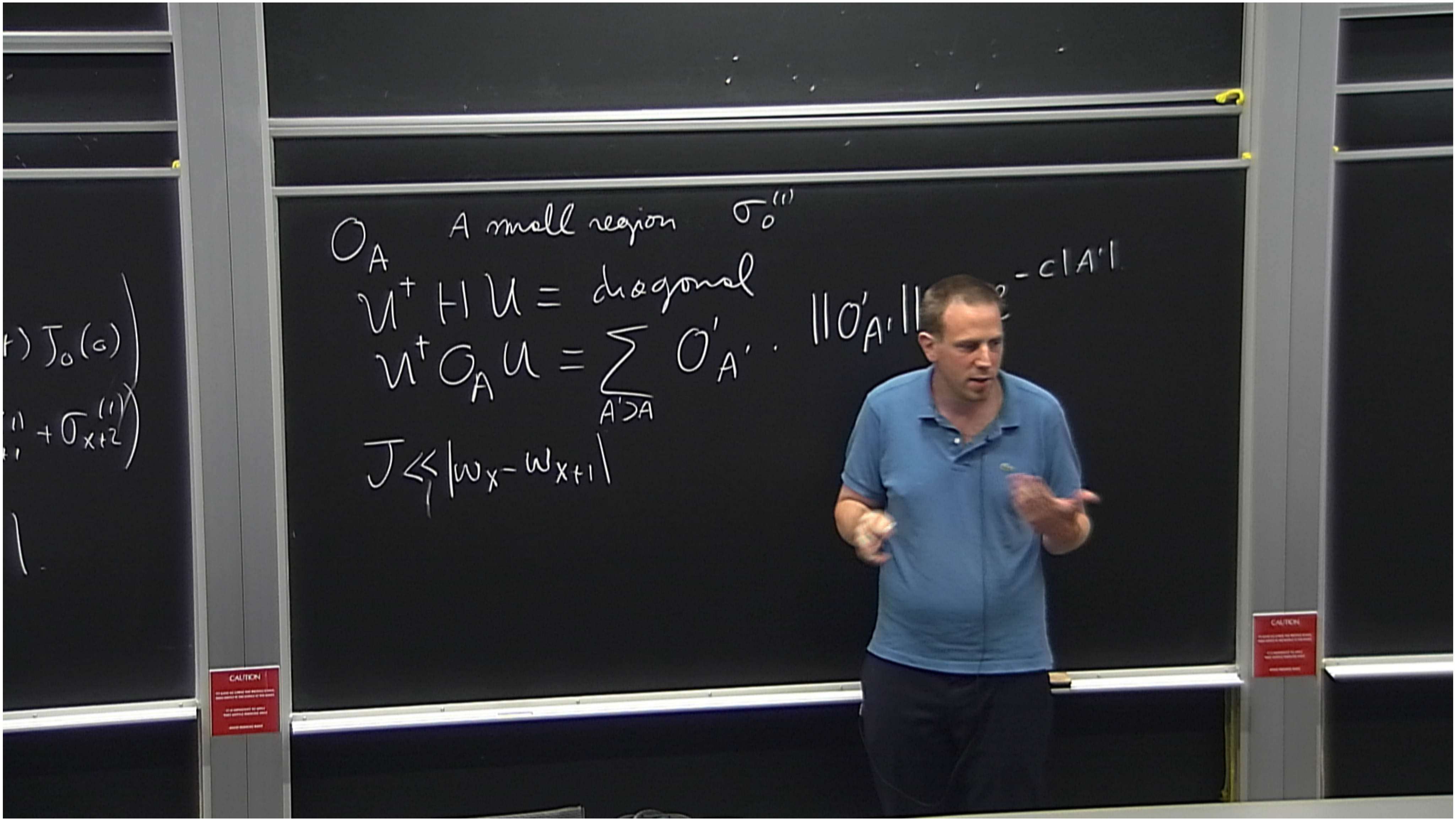
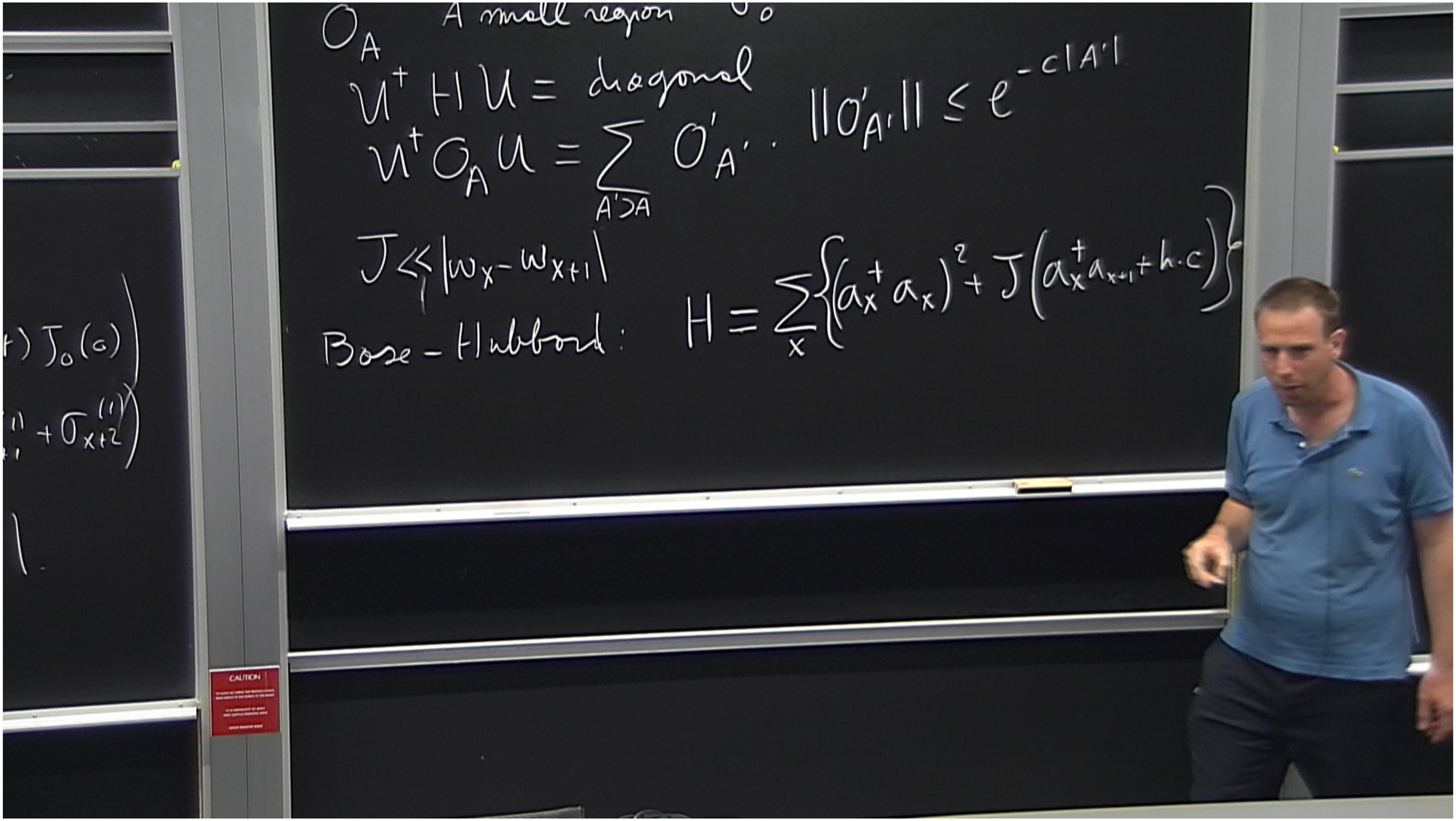
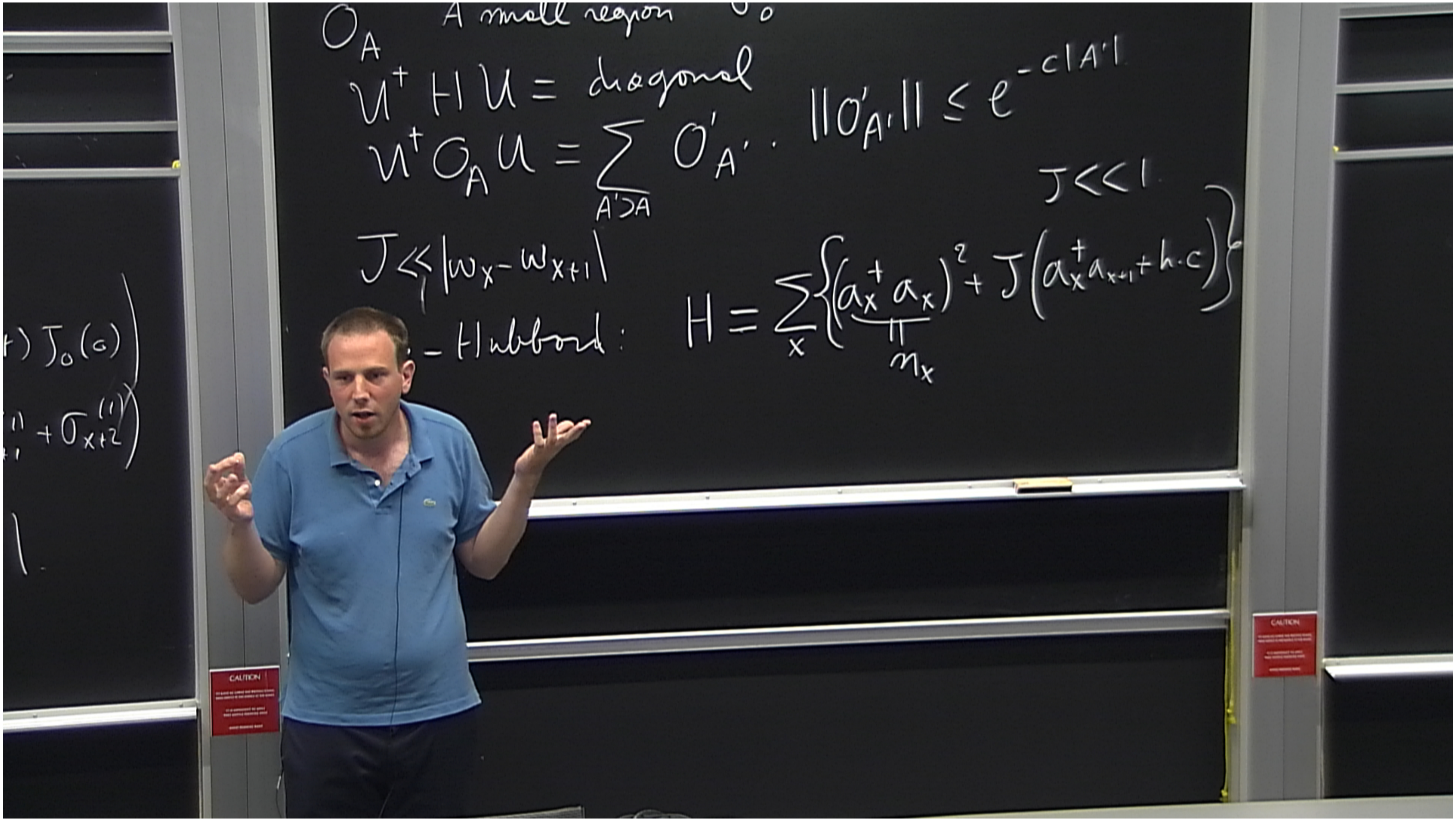
$U^+ O_A U = \sum_{A' \supset A} O'_{A'}$. $\|O'_{A'}\| \leq e^{-c|A'|}$


O_A   A small region   σ_0^(1)
U^+ H U = diagonal
U^+ O_A U = Σ_{A'⊃A} O'_{A'}
||O'_{A'}||
-c|A'|
J << |ω_x - ω_{x+1}|


O_A A small region
$U^\dagger H U =$ diagonal
$U^\dagger O_A U = \sum_{A' \supset A} O'_{A'}$. $\|O'_{A'}\| \le e^{-c|A'|}$
$J \ll |\omega_x - \omega_{x+1}|$
Bose-Hubbard: $H = \sum_x \{(a_x^\dagger a_x)^2 + J(a_x^\dagger a_{x+1} + h.c)\}$
CAUTION


O_A A small region
$U^\dagger H U$ = diagonal
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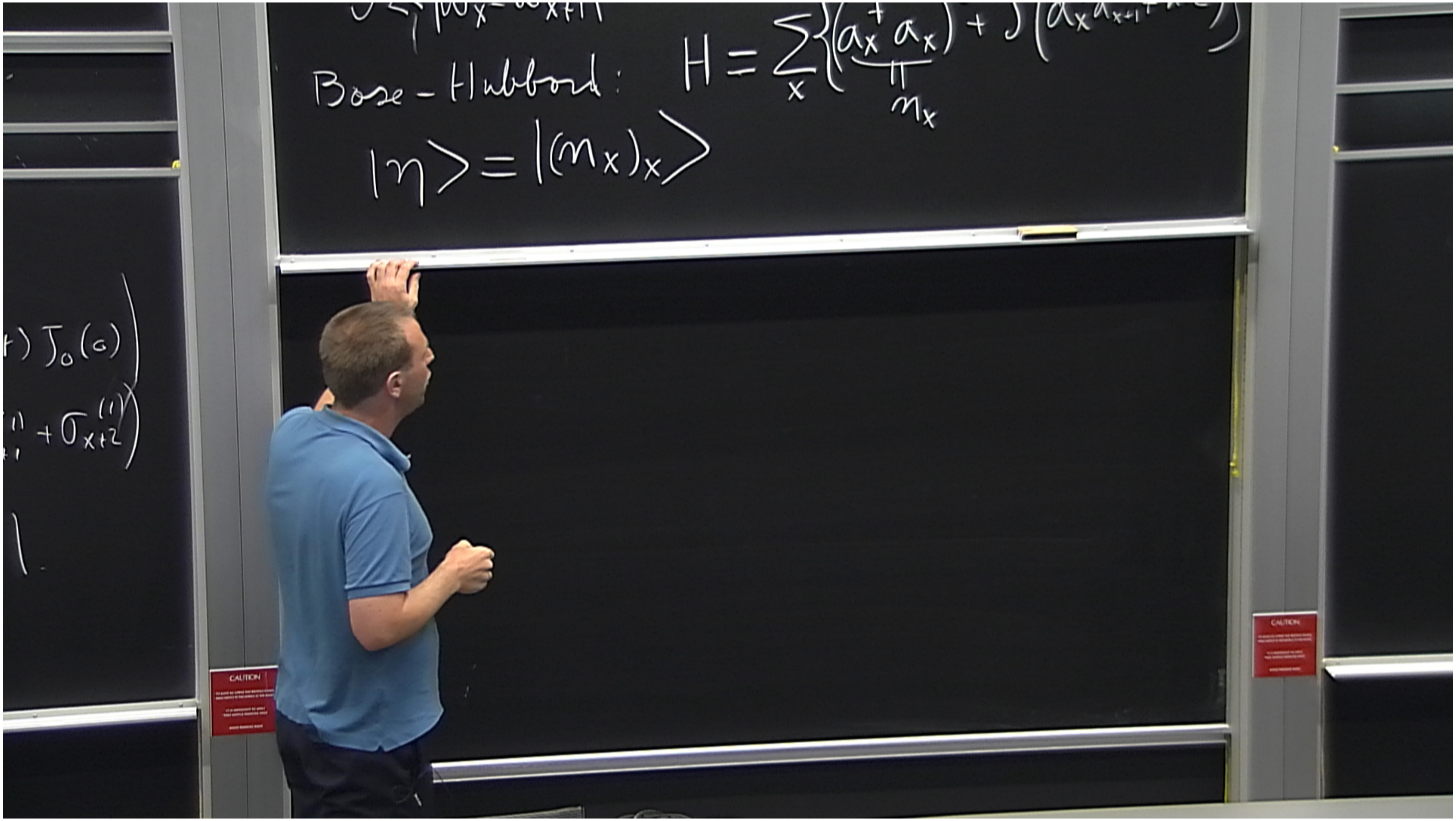
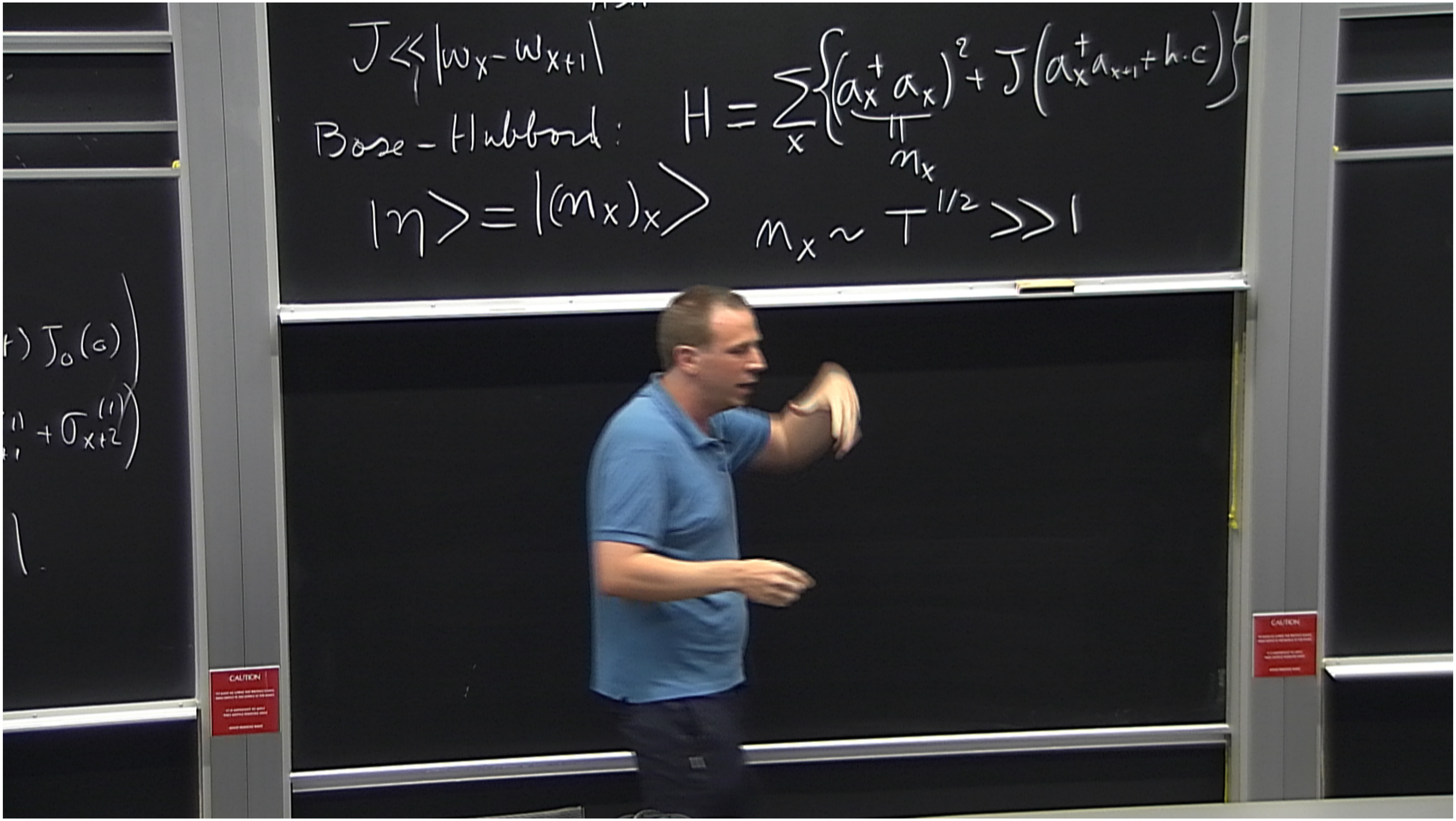
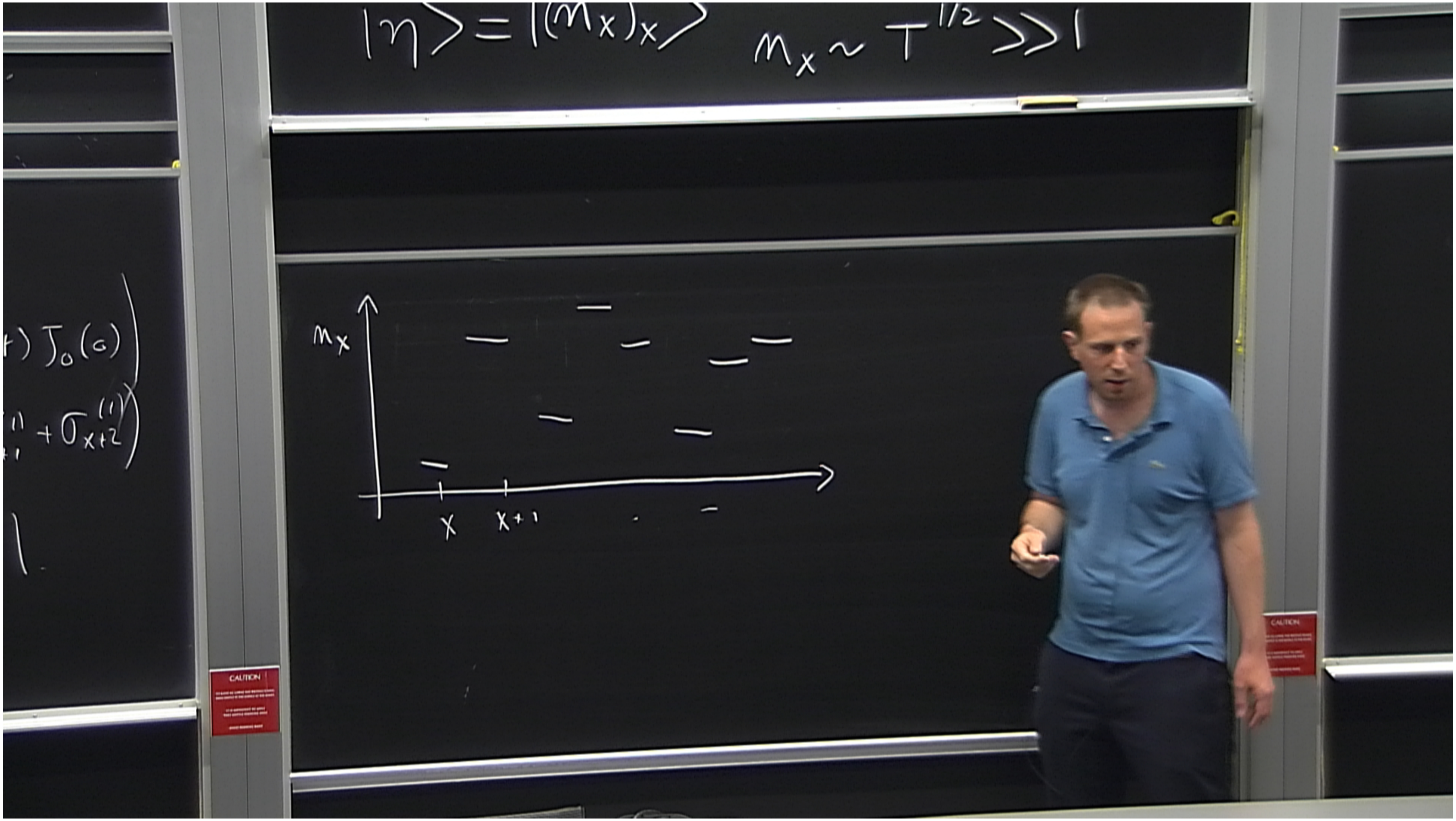
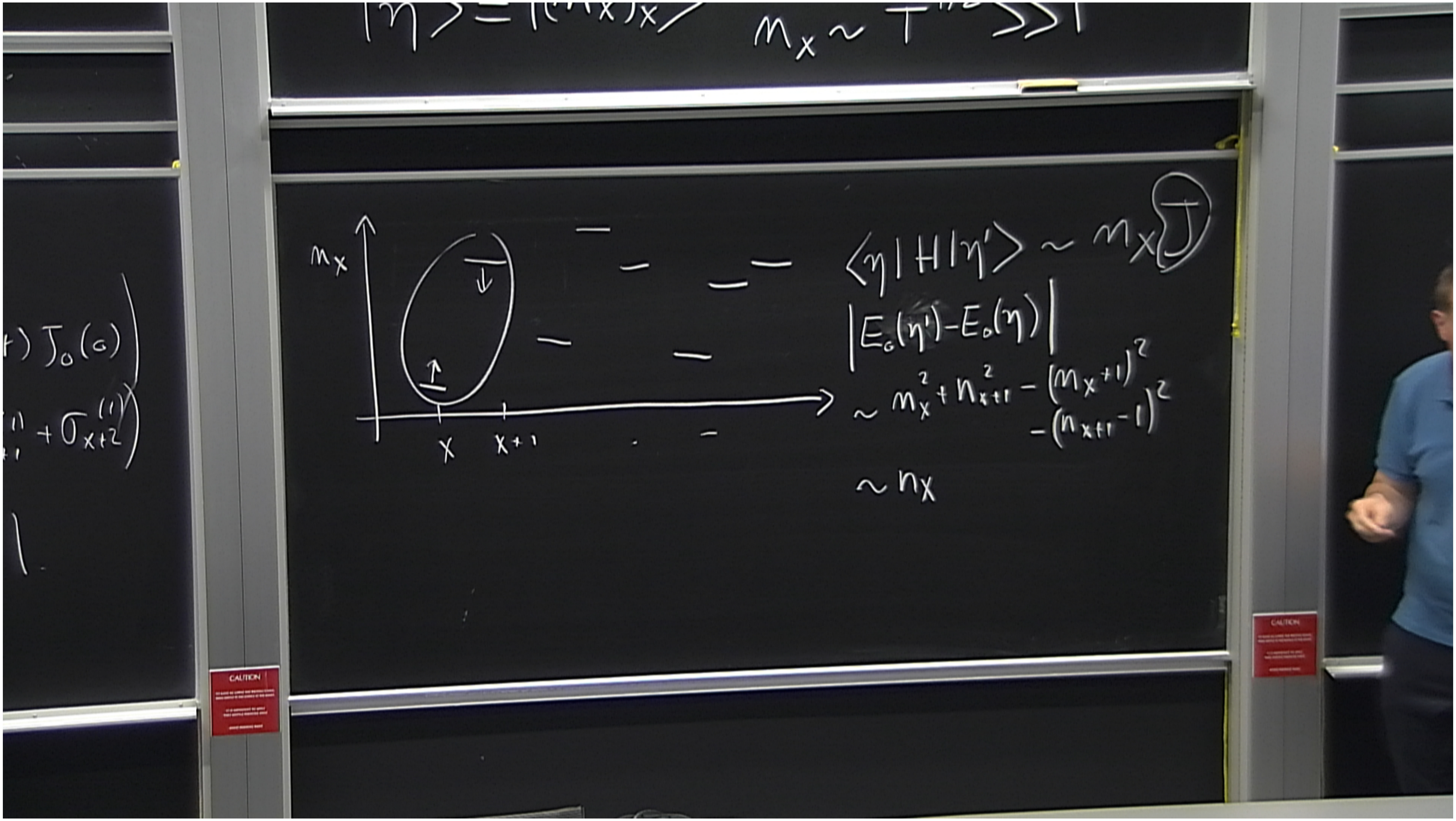
$J \ll 1$
$J \ll |w_x - w_{x+1}|$
-Hubbard: $H = \sum_x \{ (a_x^\dagger a_x)^2 + J(a_x^\dagger a_{x+1} + h.c) \}$
n_x

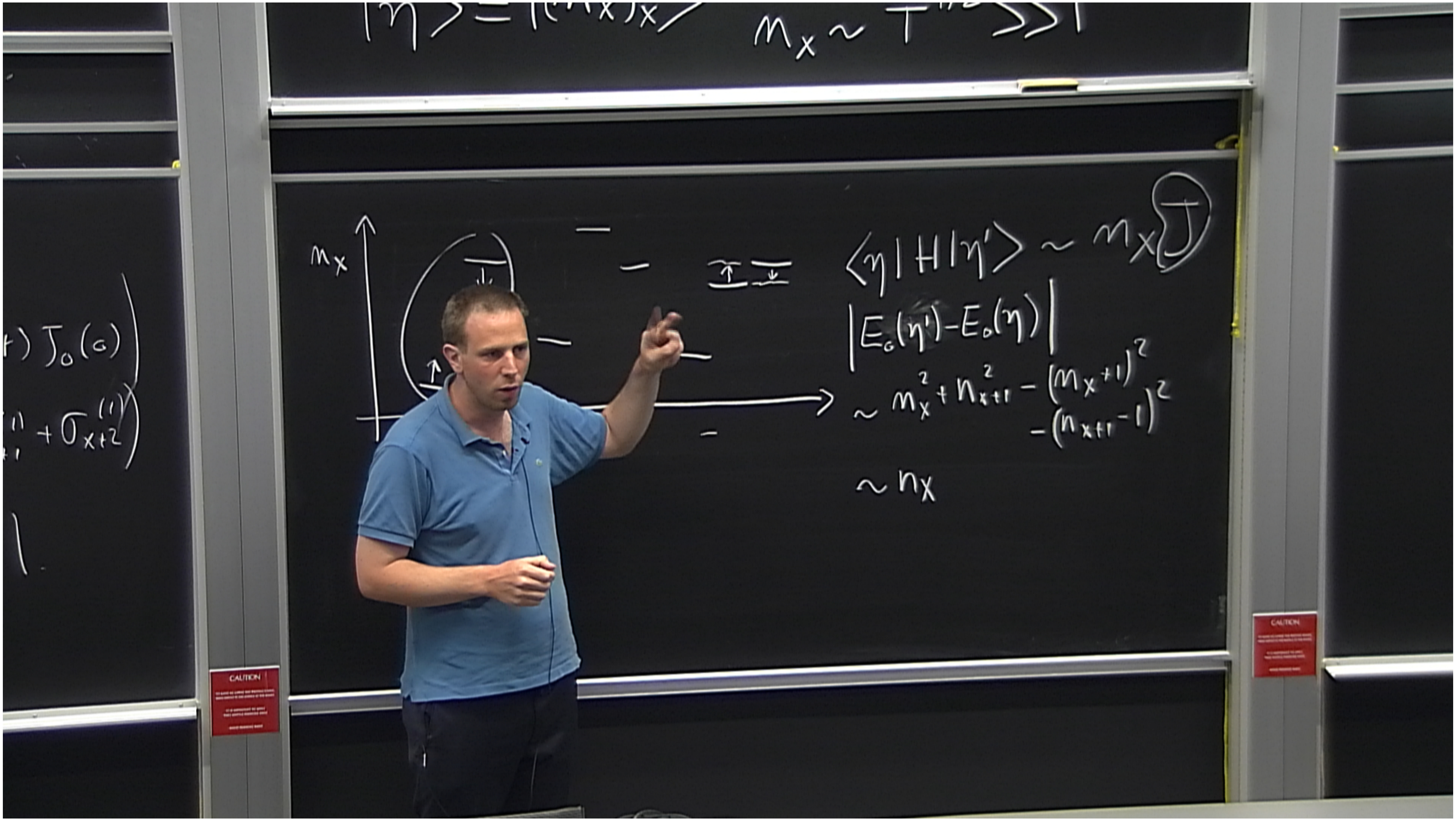

Bose-Hubbard:
|n> = |(n_x)_x>

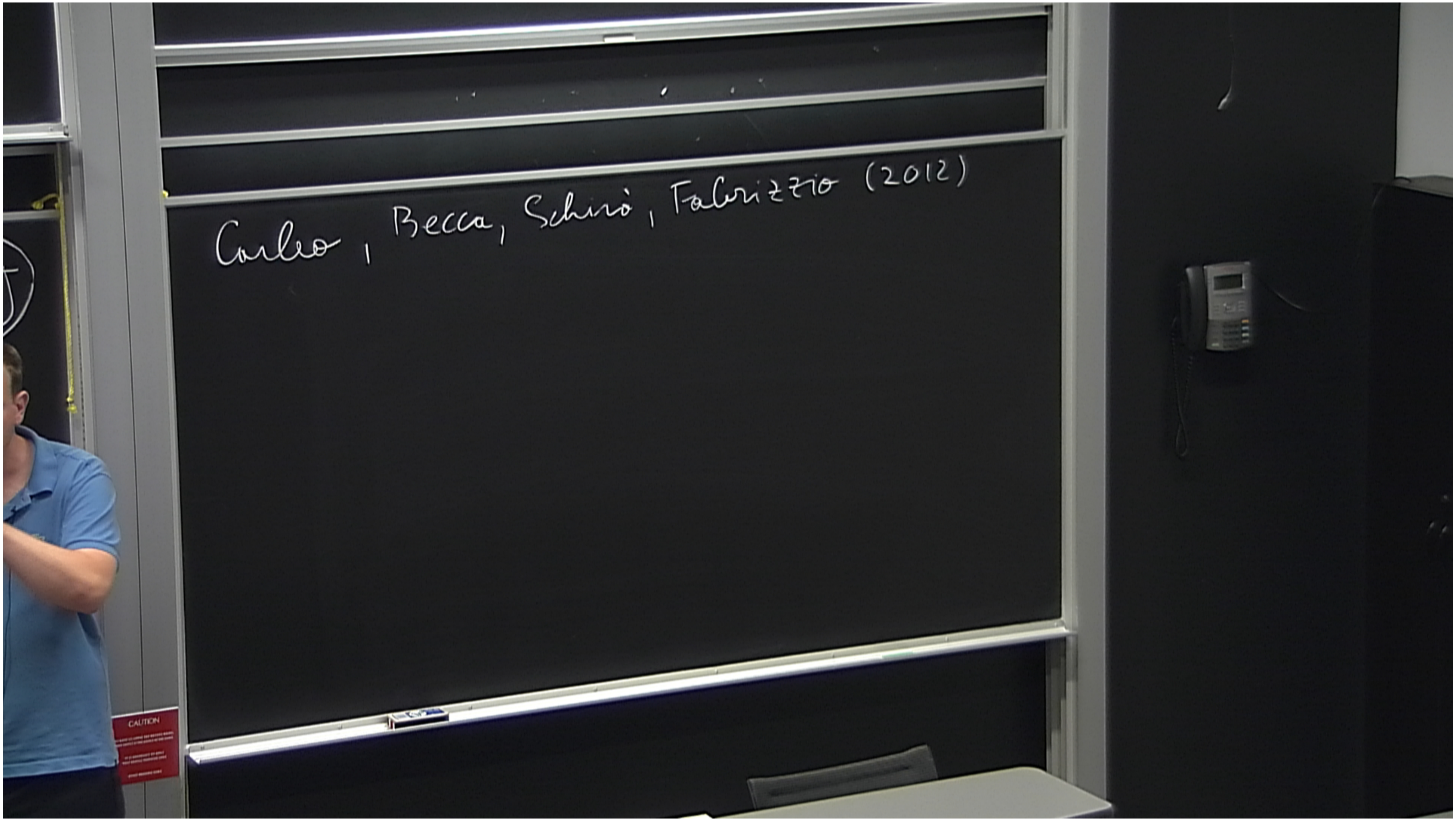

$J \ll |\omega_x - \omega_{x+1}|$
Bose-Hubbard: $H = \sum_x \{ (\underbrace{a_x^\dagger a_x}_{n_x})^2 + J(a_x^\dagger a_{x+1} + h.c) \}$
$|\eta\rangle = |(n_x)_x\rangle$
$n_x \sim T^{1/2} \gg 1$

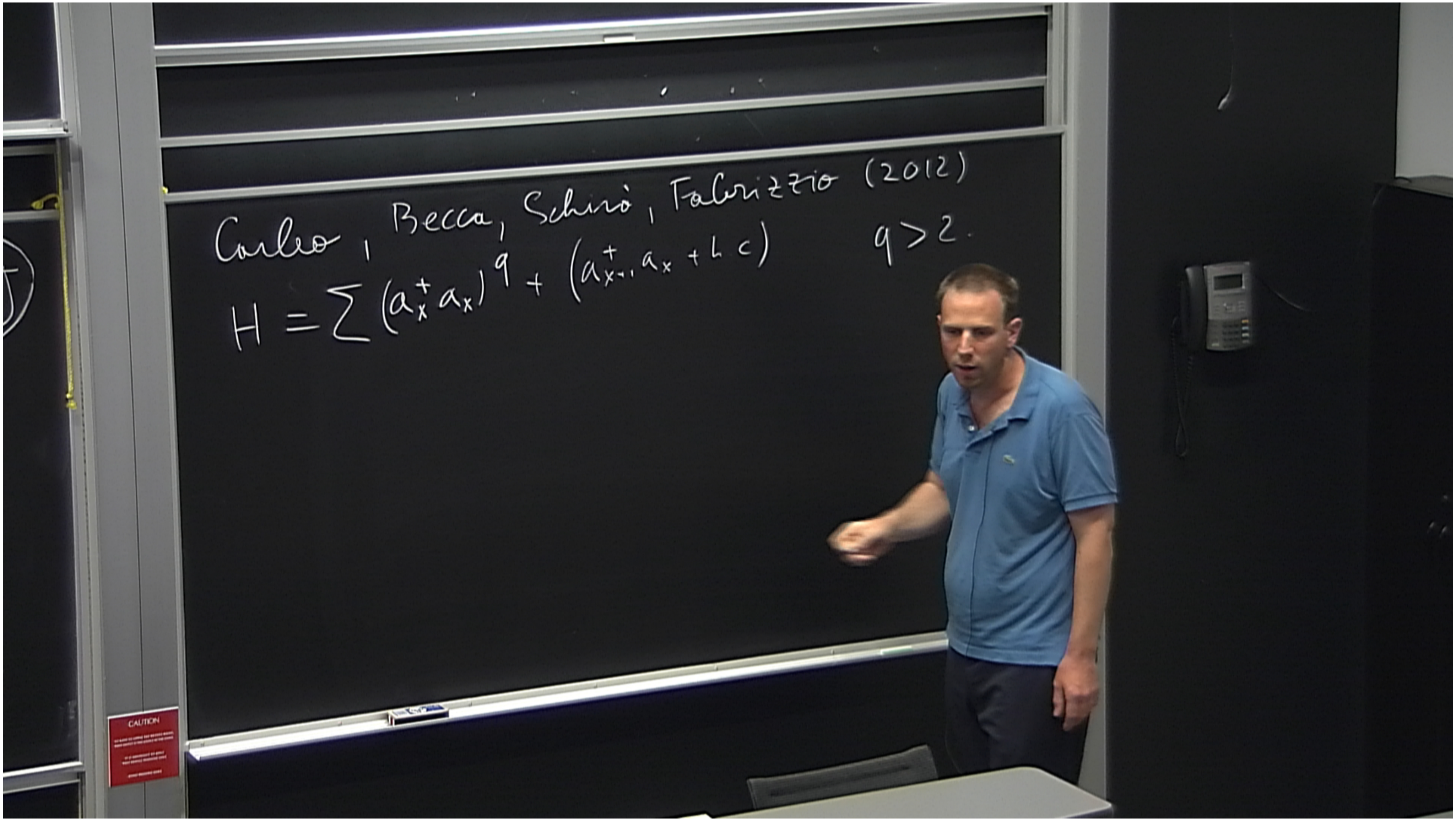


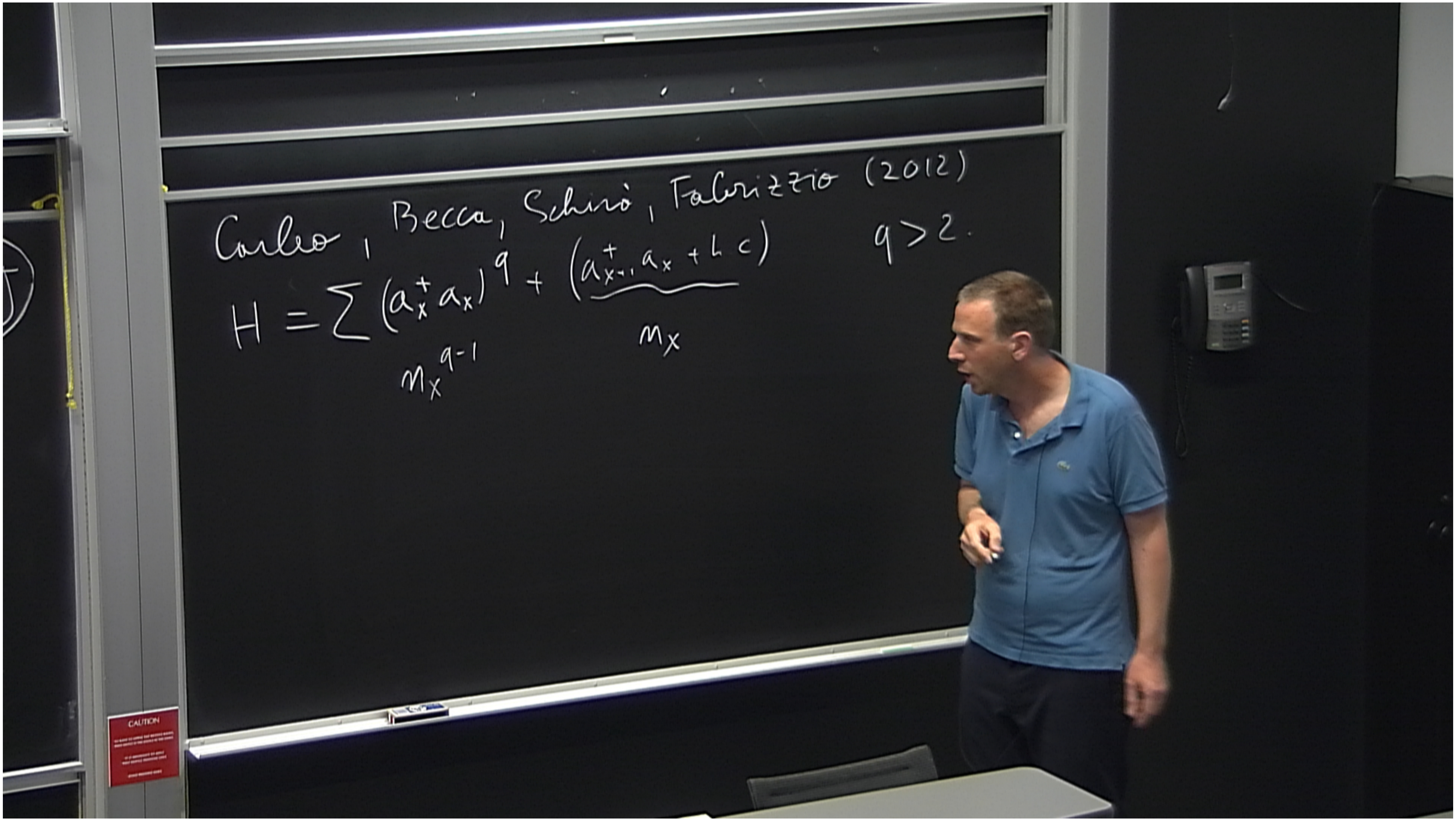
$|\eta\rangle = |(n_x)_x\rangle$ $n_x \sim T^{1/2} \gg 1$
n_x
x $x+1$
CAUTION


⟨η|H|η'⟩ ~ n_x J
|E₀(η') − E₀(η)|
~ n_x^2 + n_{x+1}^2 − (n_x+1)^2 − (n_{x+1}−1)^2
~ n_x
n_x
x
x+1
n_x ~ T


n_x
$\langle \eta | H | \eta' \rangle \sim n_x J$
$|E_0(\eta') - E_0(\eta)|$
$\sim n_x^2 + n_{x+1}^2 - (n_x+1)^2 - (n_{x+1}-1)^2$
$\sim n_x$
CAUTION


Carleo, Becca, Schirò, Fabrizzio (2012)


Carleo, Becca, Schirò, Fabrizzio (2012)
$H = \sum (a_x^+ a_x)^q + (a_{x+1}^+ a_x + h.c)$ $q > 2$.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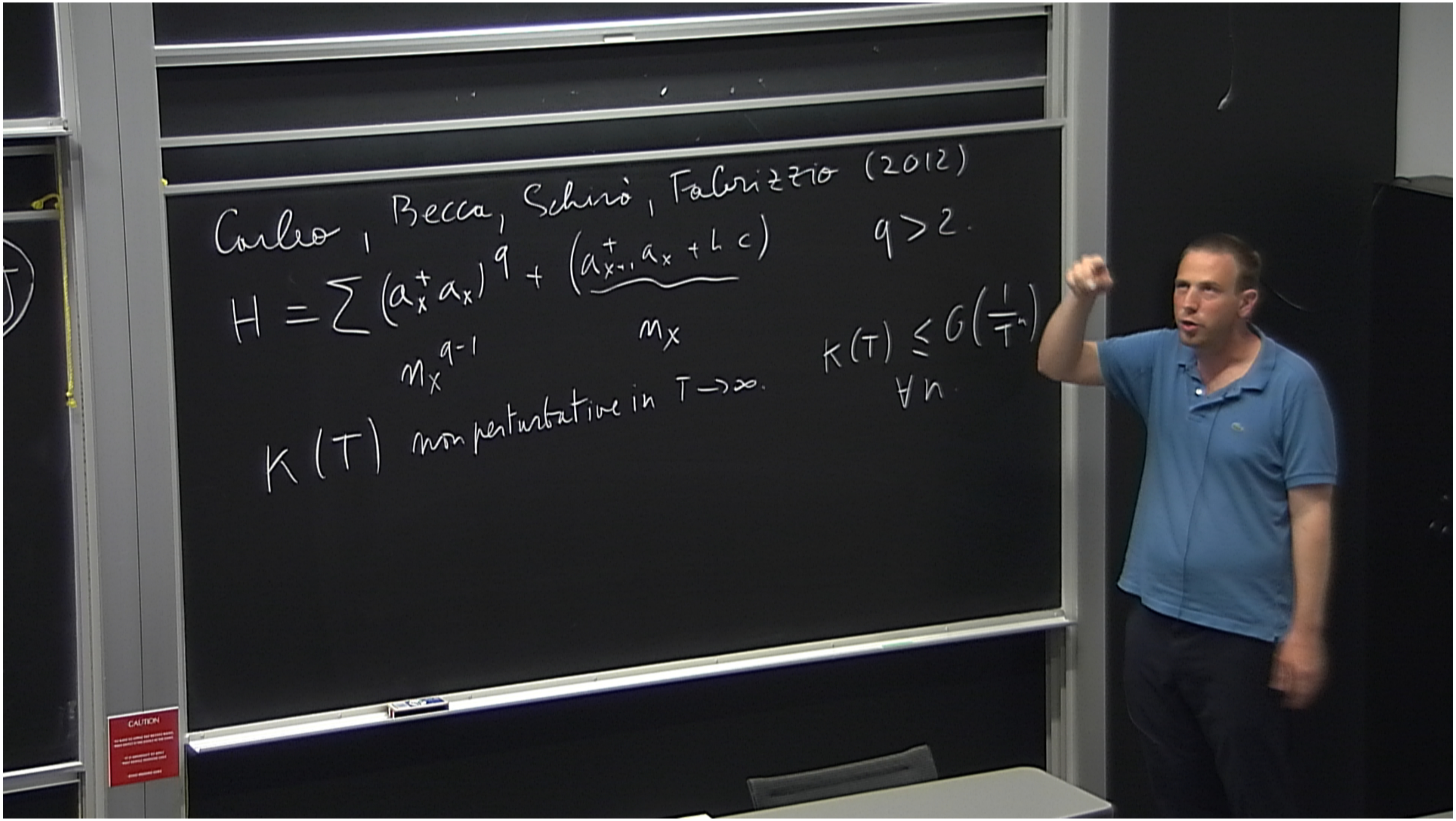
q > 2.
n_x^{q-1}
n_x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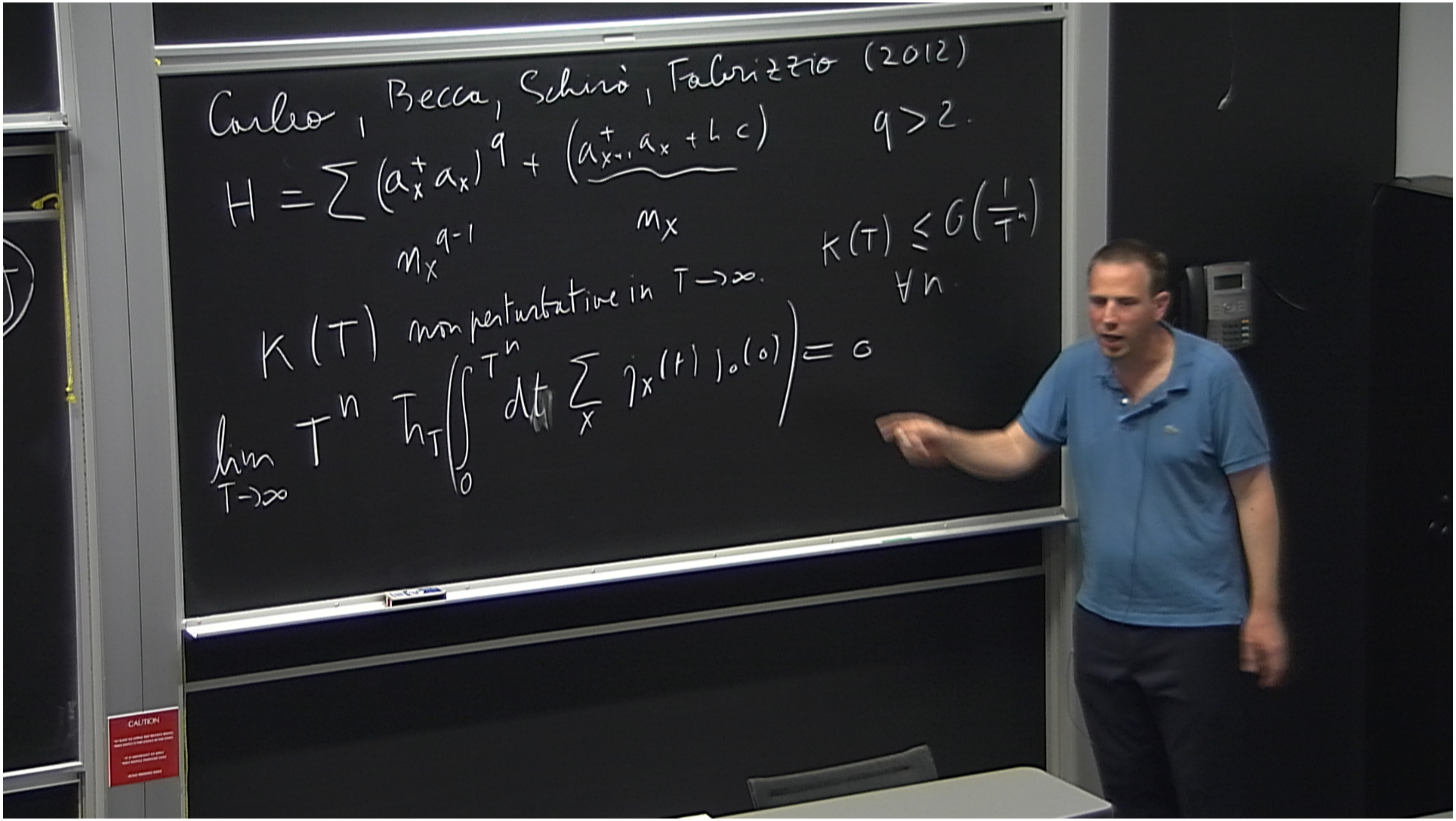
$K(T) \leq O(\frac{1}{T^n})$ $\forall n$
$K(T)$ nonperturbative in $T \to \infty$.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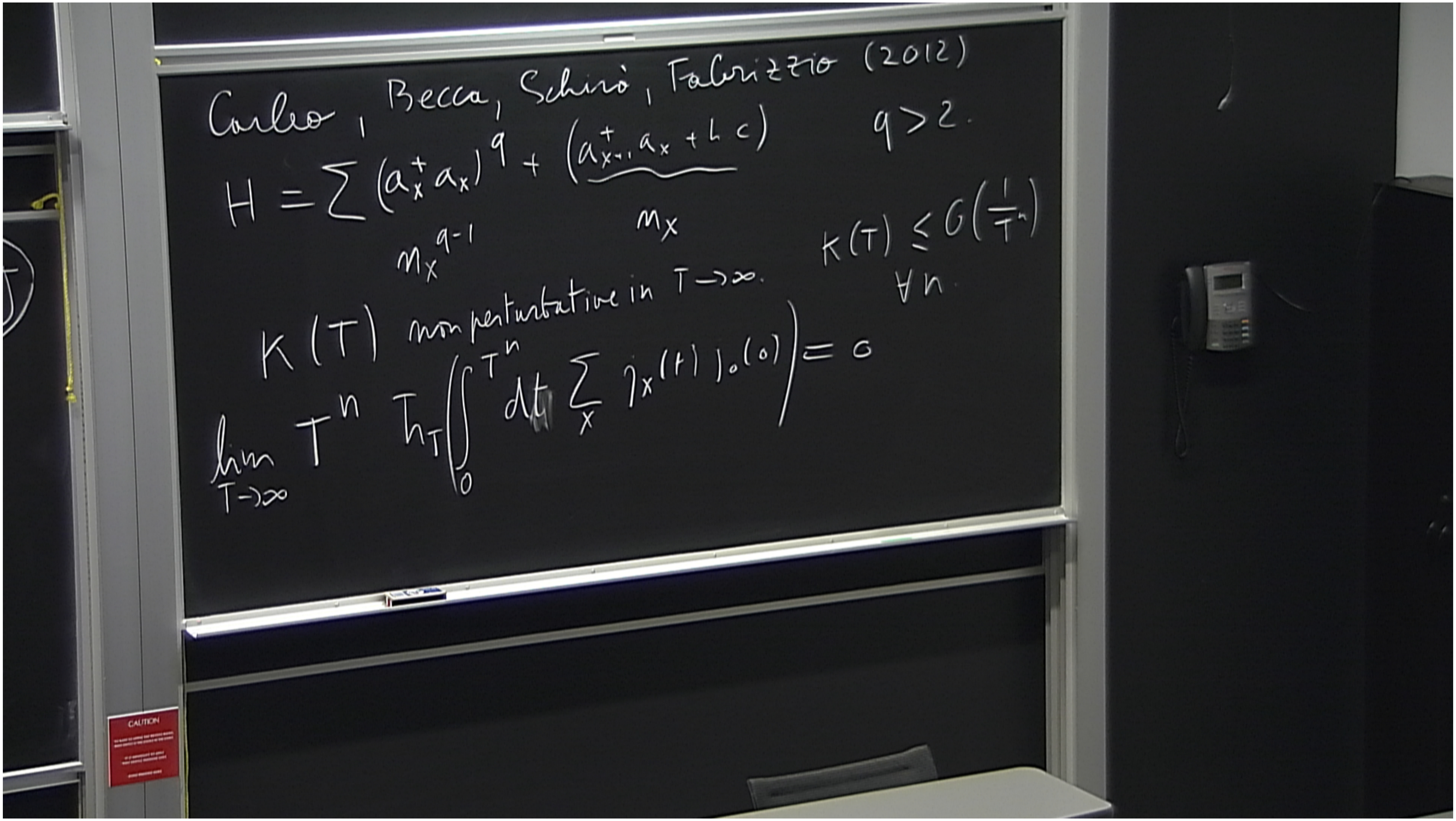
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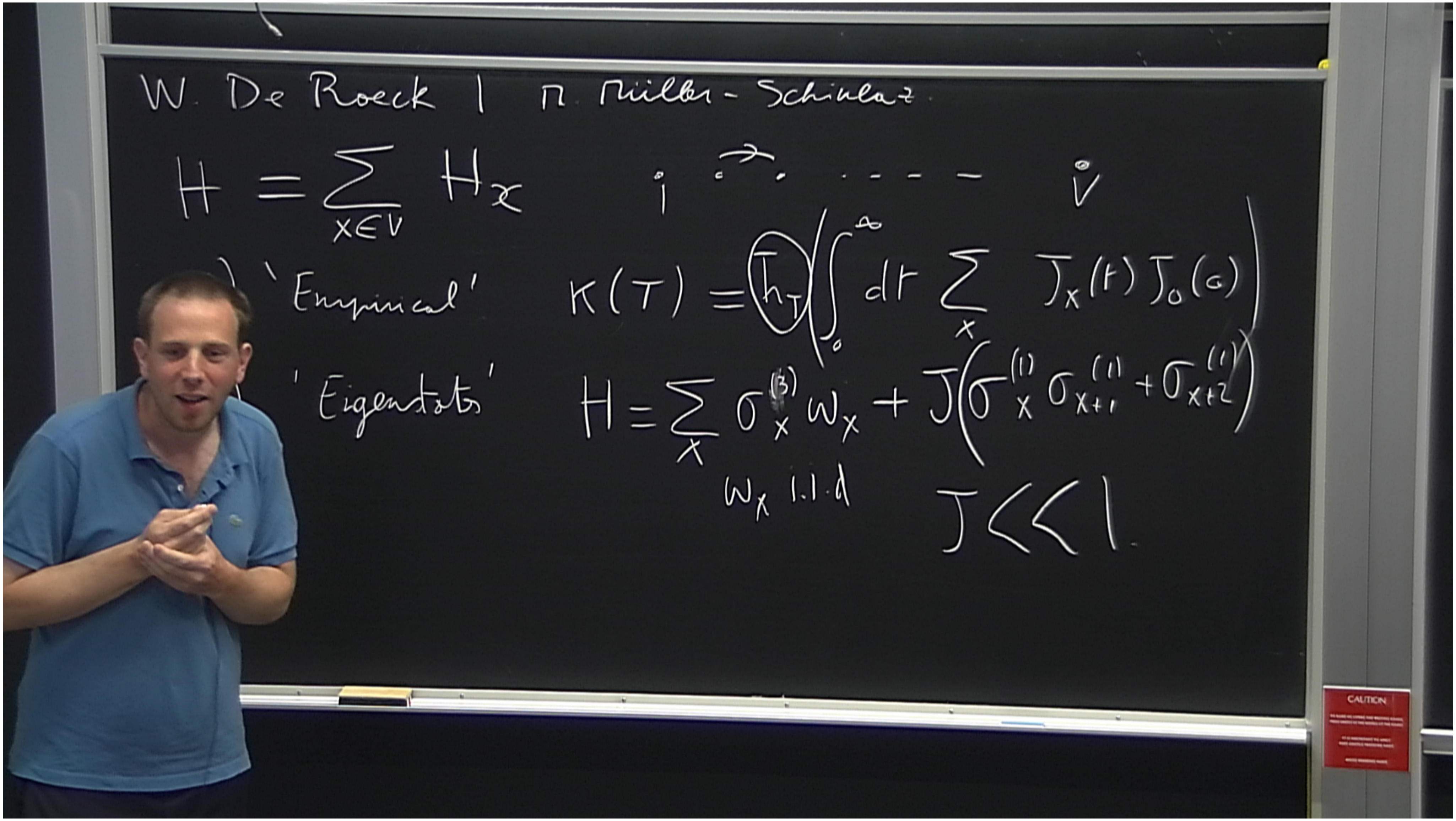
CAUTION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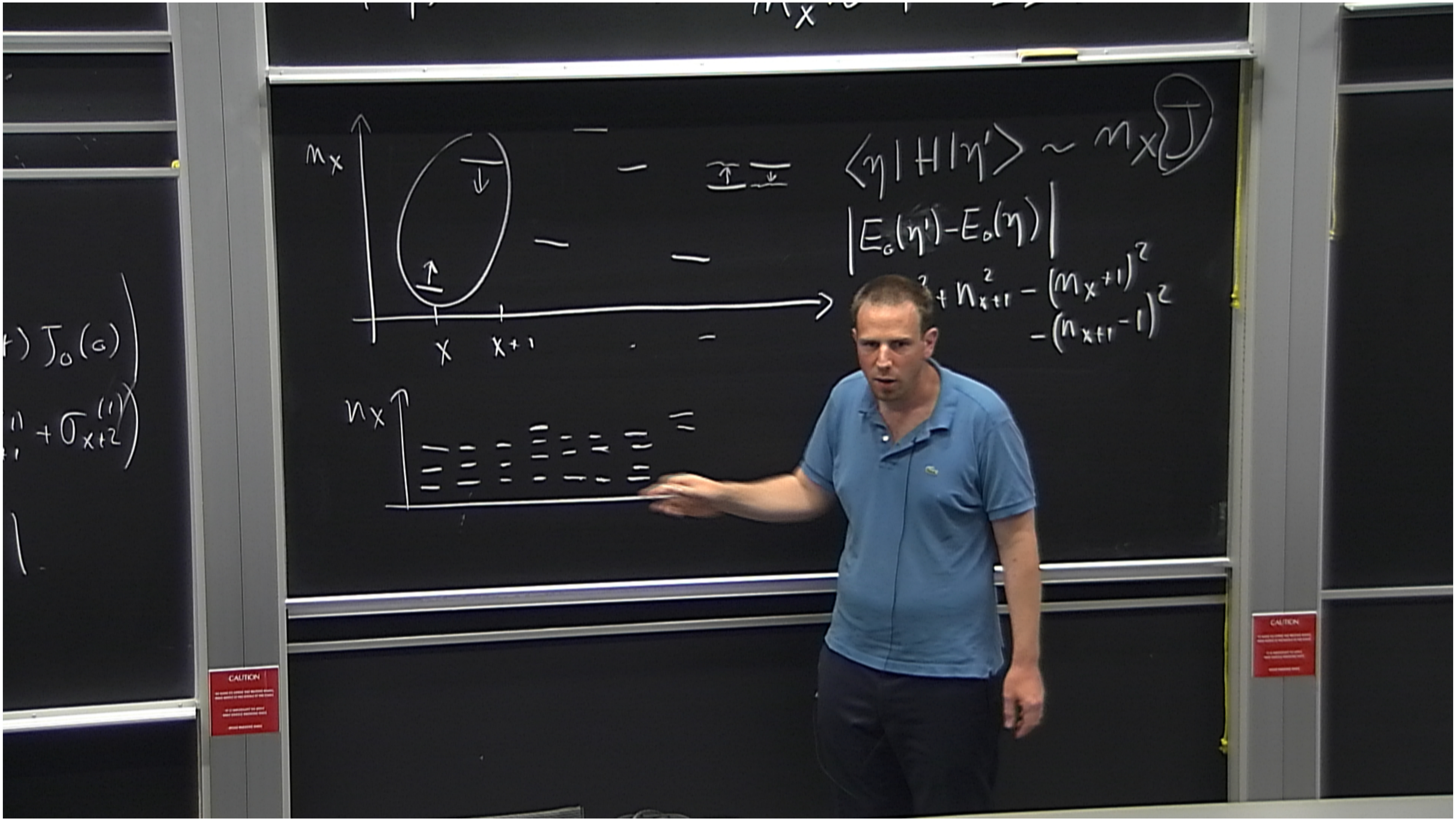

n_x
x
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n_x
CAUTION
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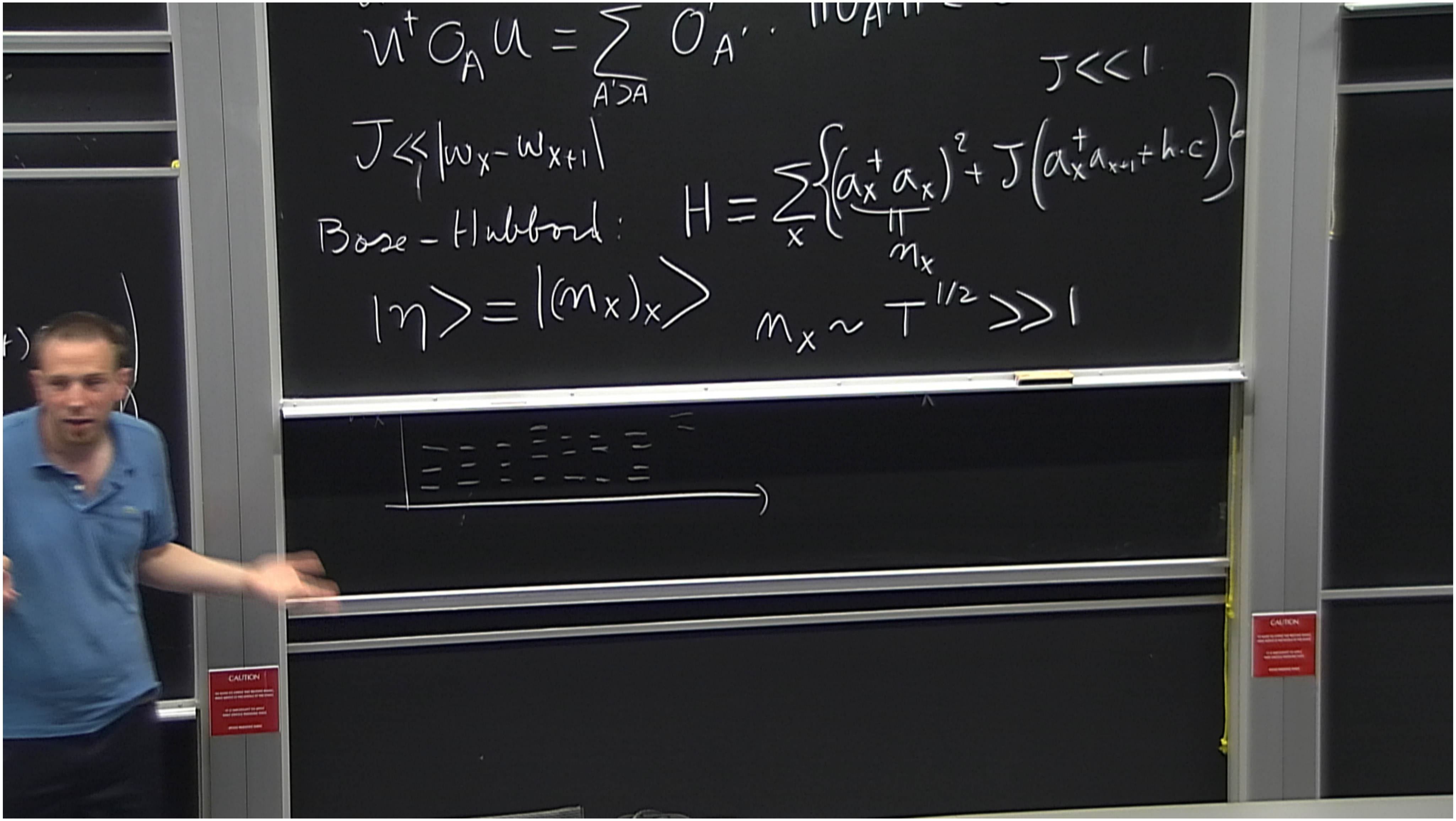
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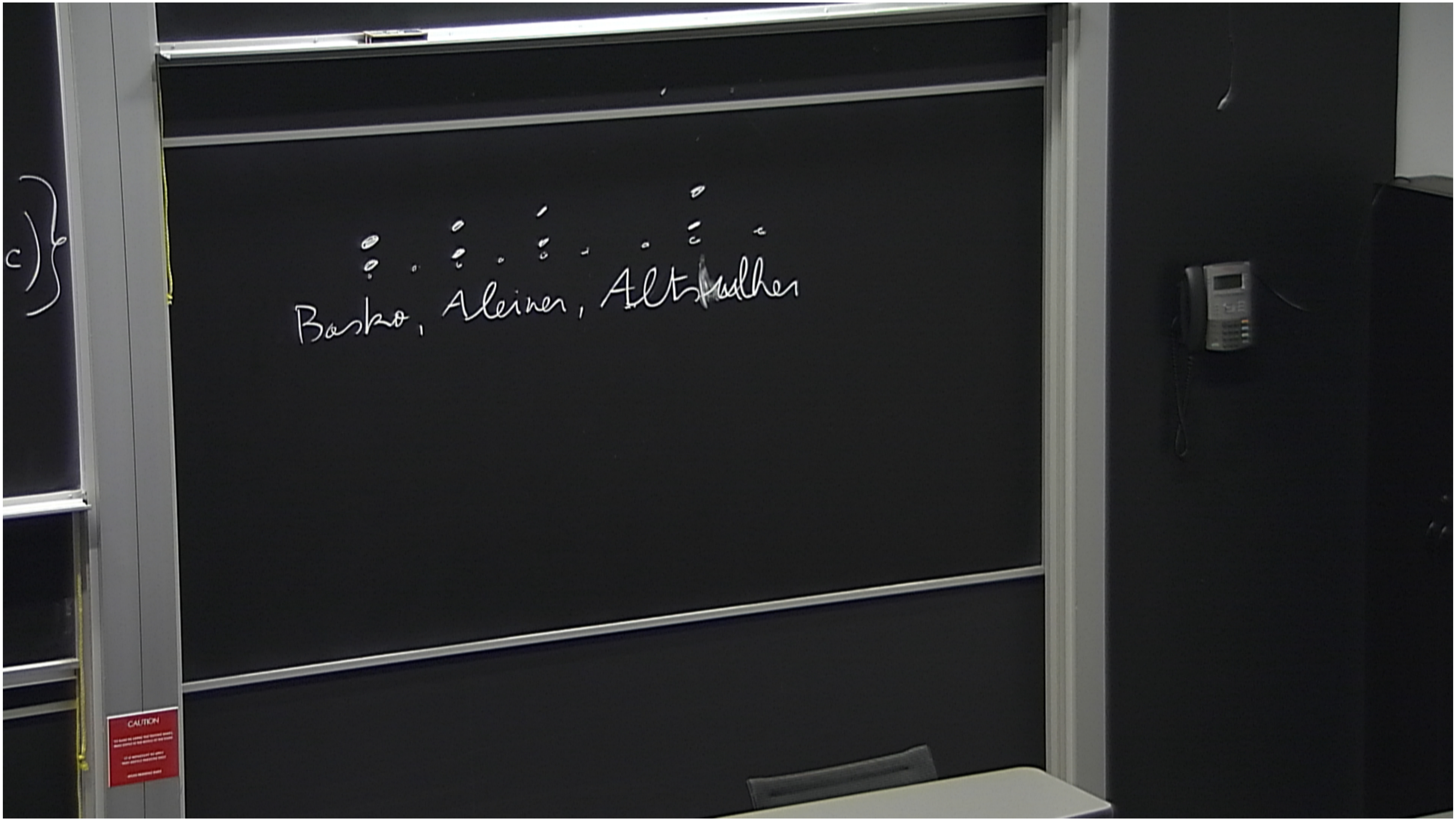
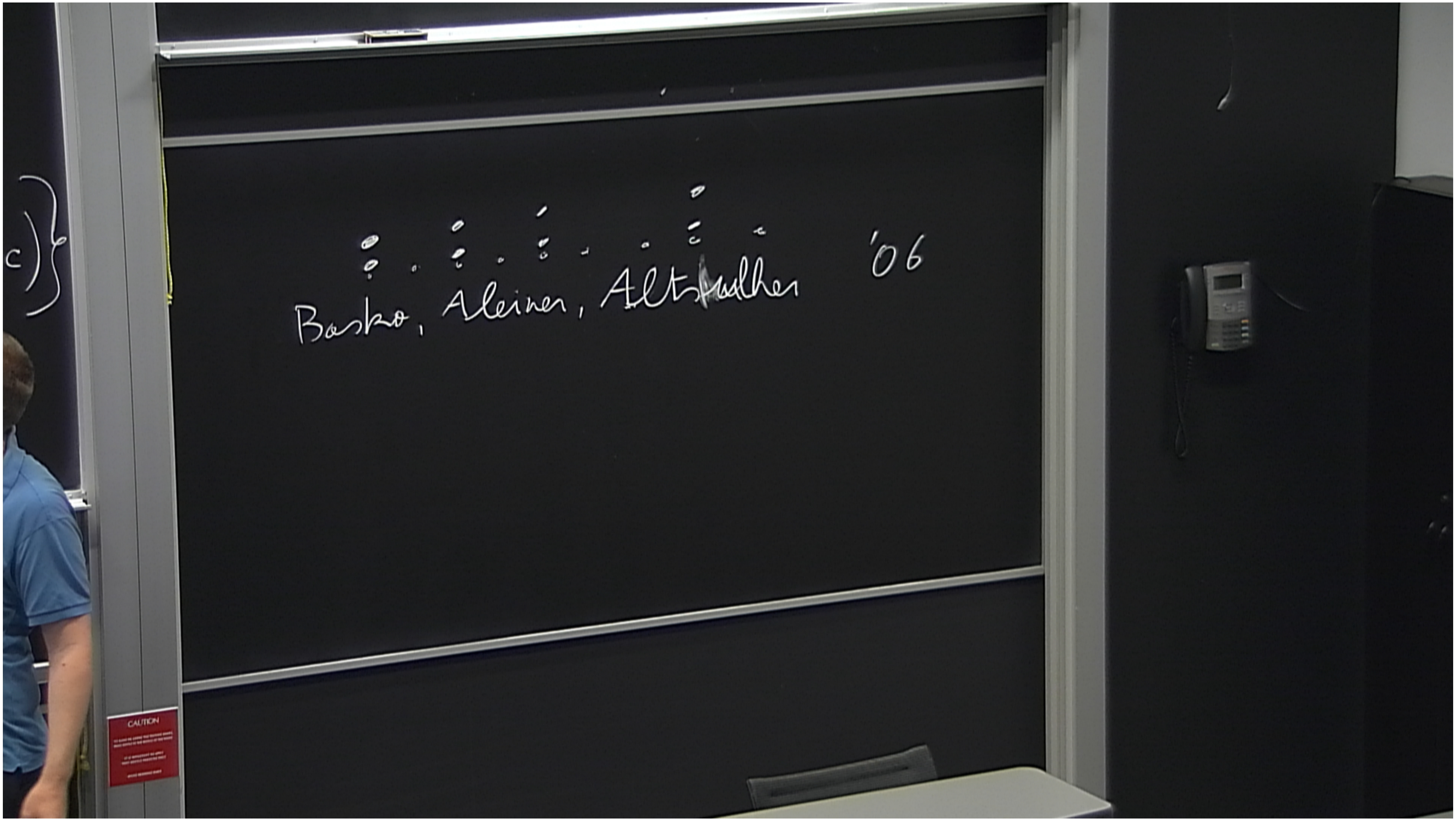
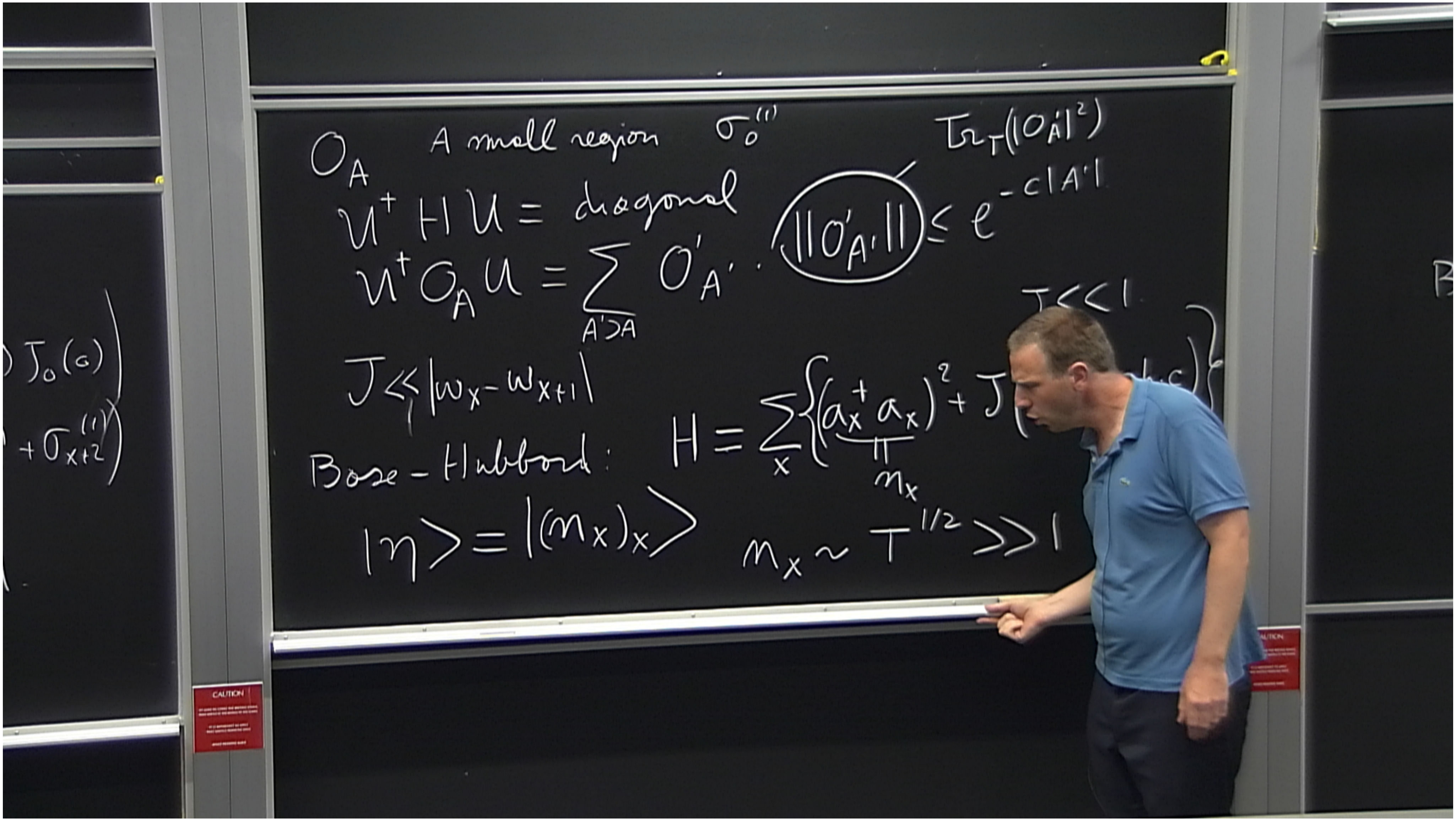

Basko, Aleiner, Altshuler


Basko, Aleiner, Altshuler '06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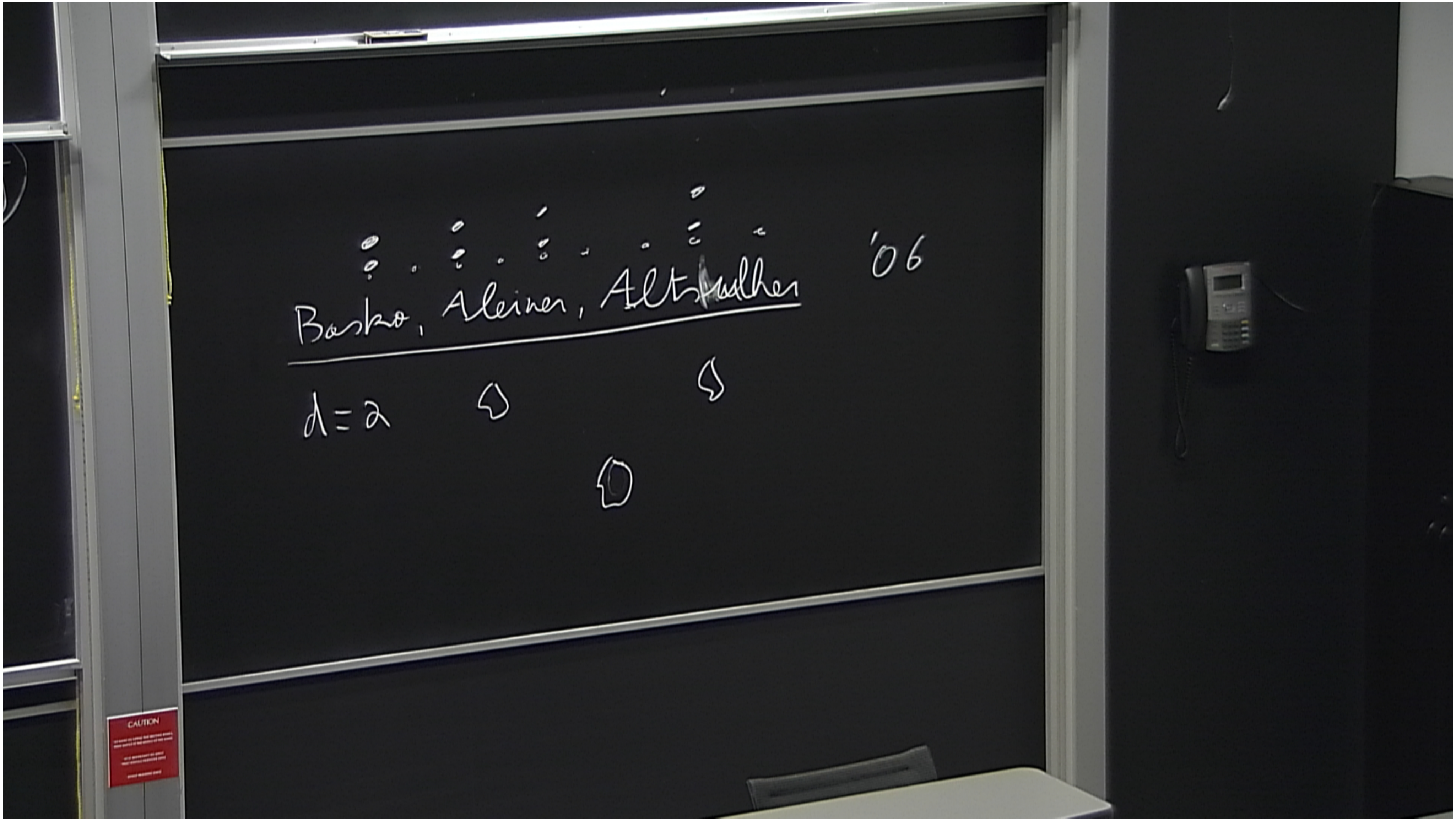
Basko, Aleiner, Altshuler '06
d = 2

W. De Roeck | M. Müller - Schiulaz.

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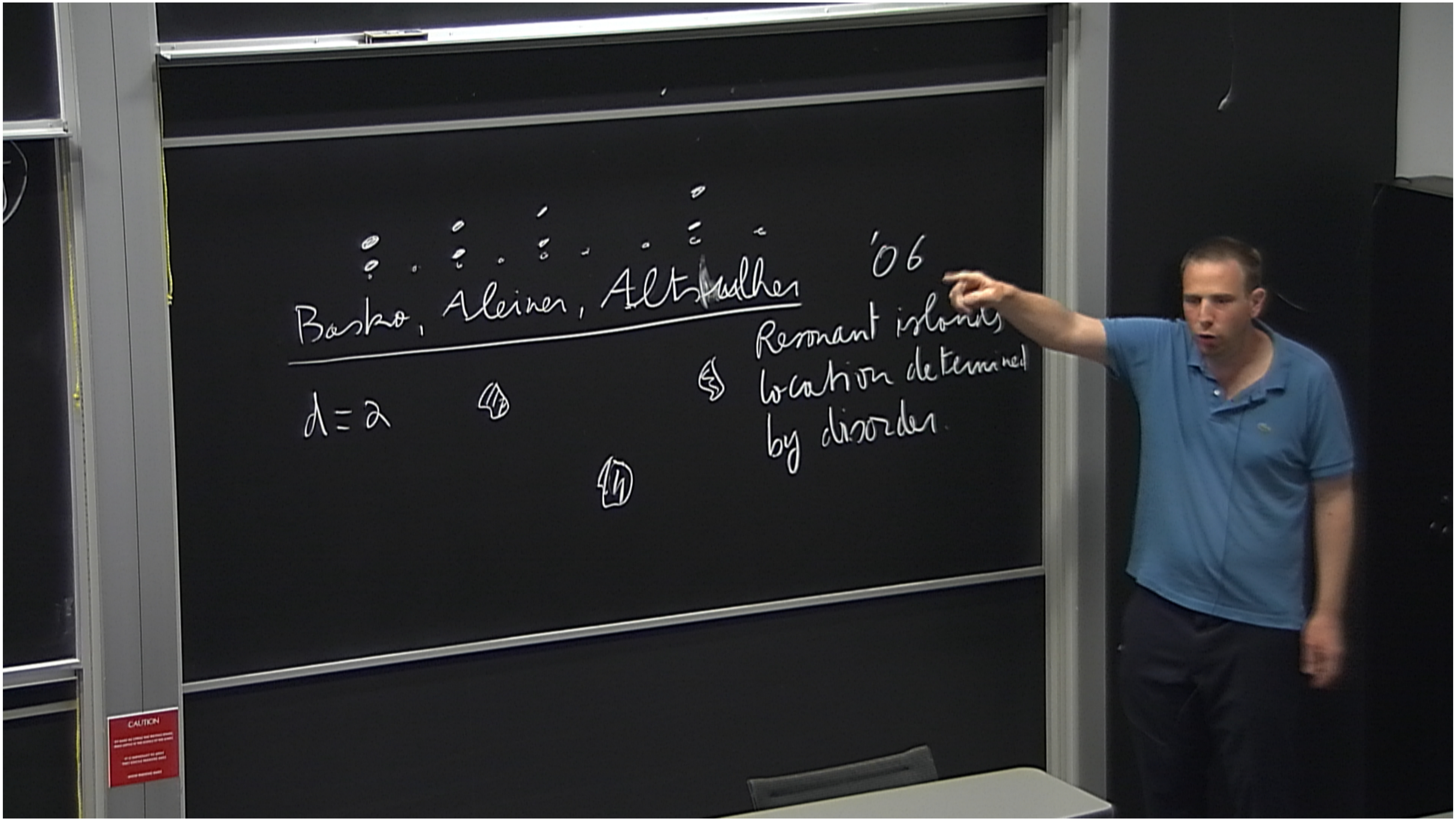
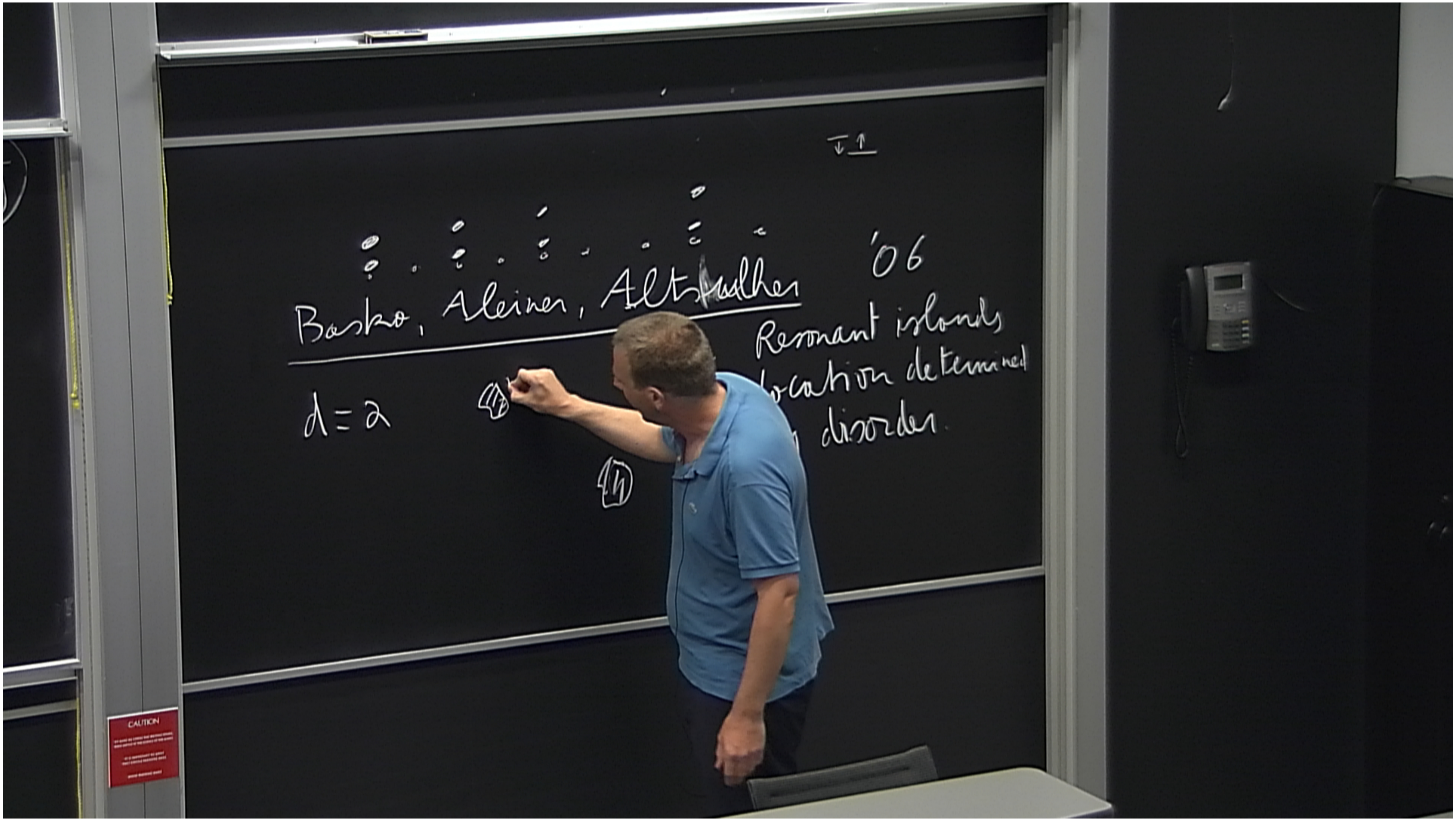

Basko, Aleiner, Altshuler '06
d = 2
Resonant islands
location determined
by disorder.



Basko, Aleiner, Altshuler '06
Resonant islands
location determined
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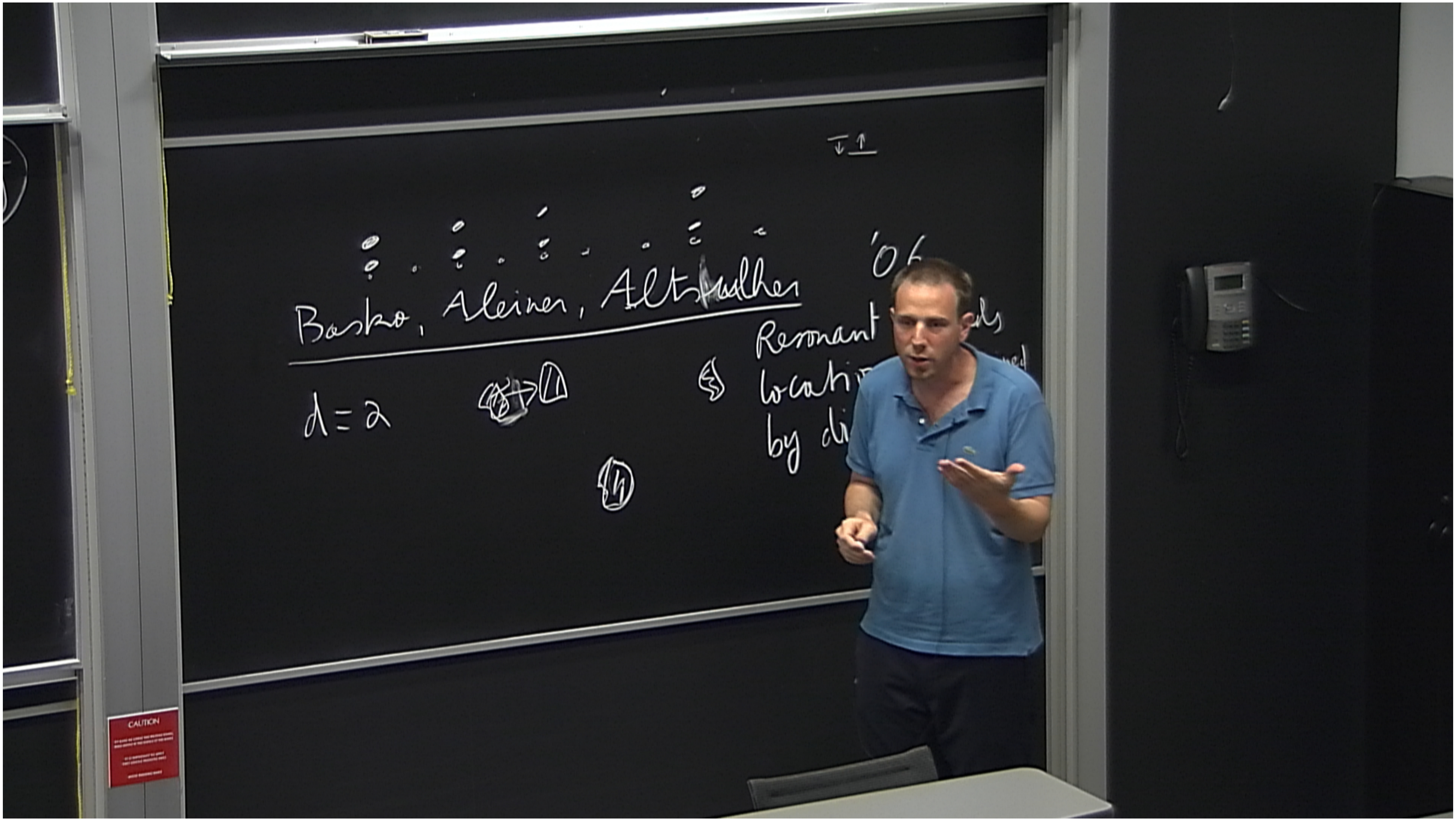
Basko, Aleiner, Altshuler
'06
d = 2
CAUTION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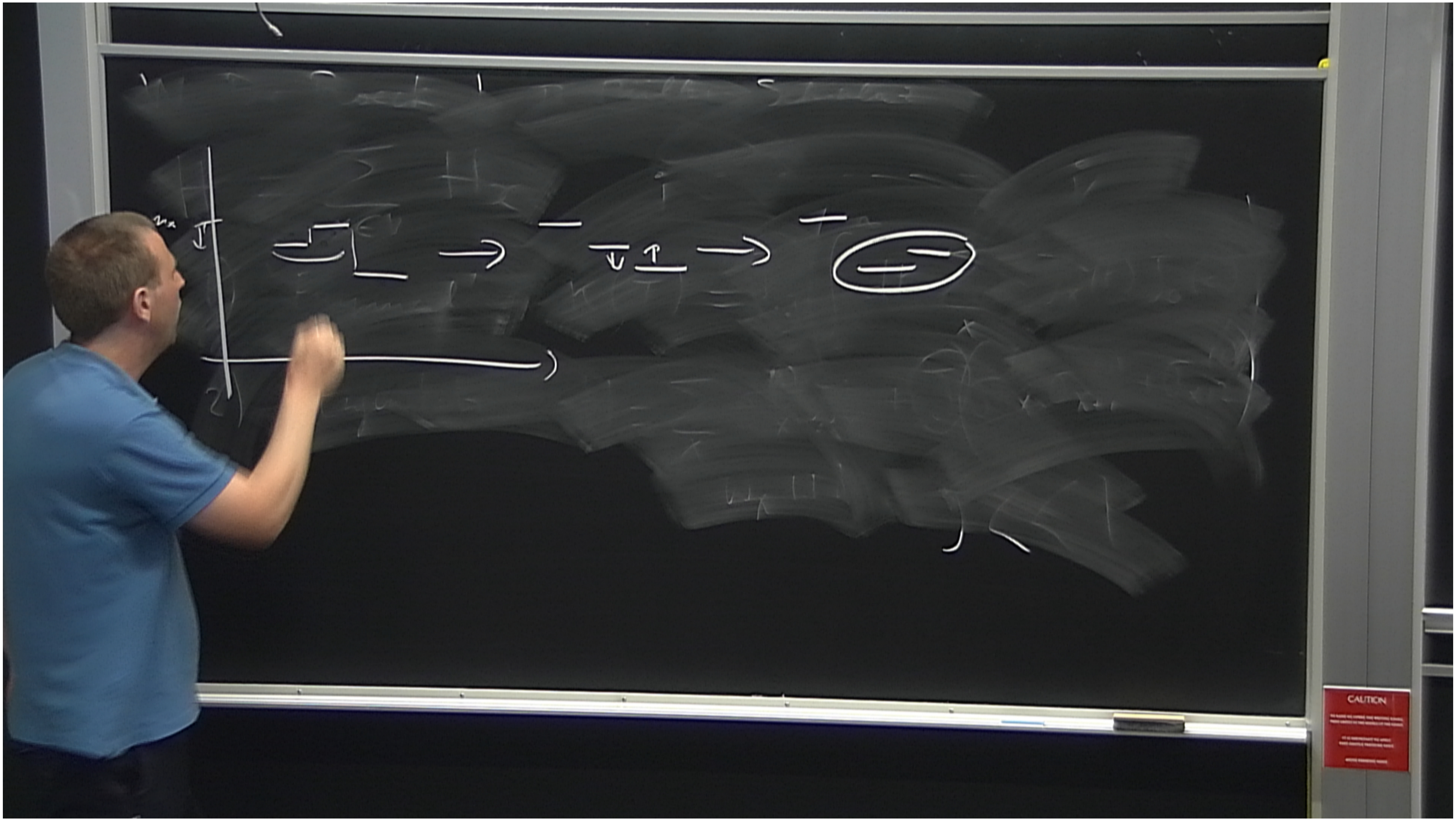
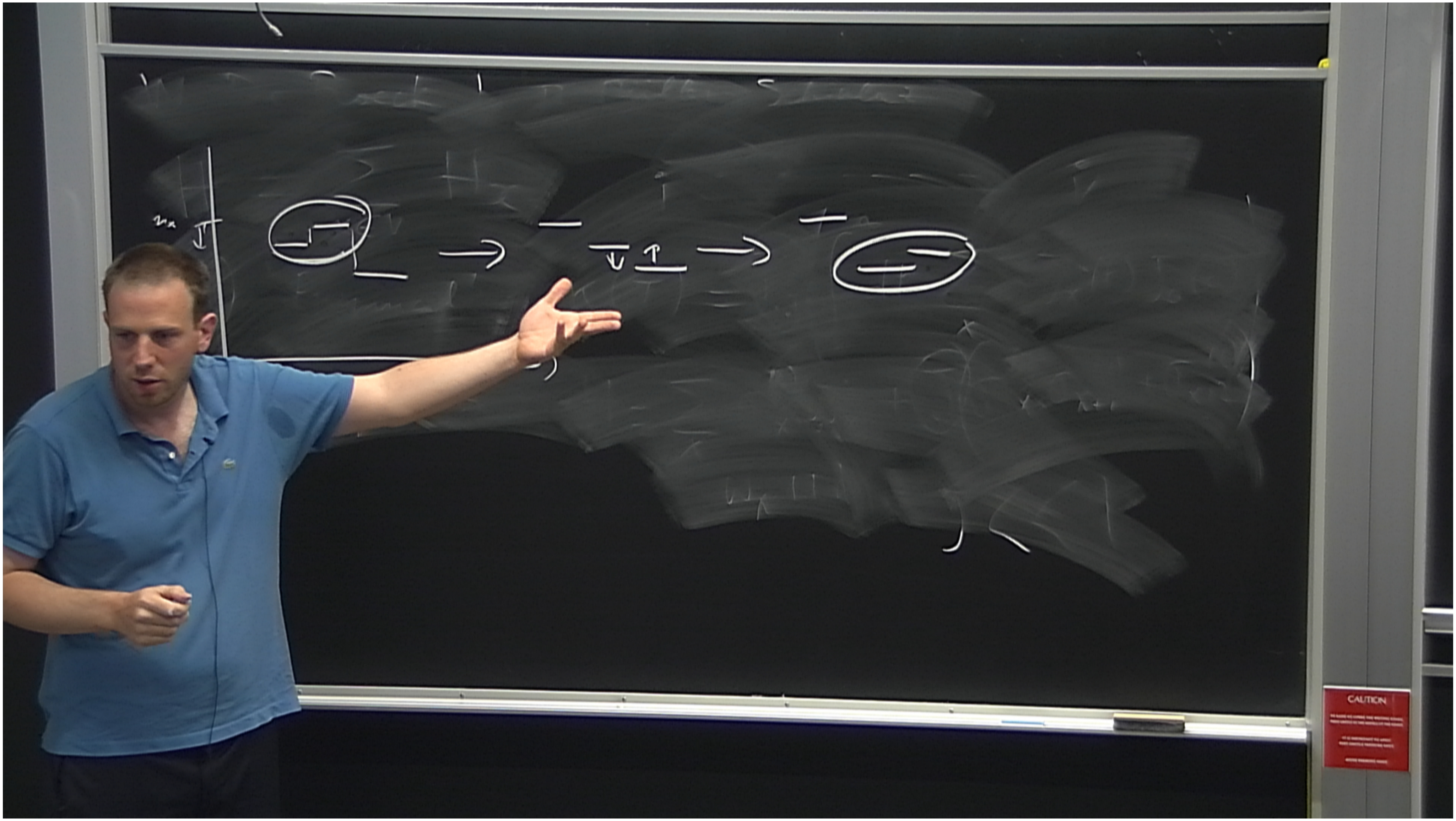
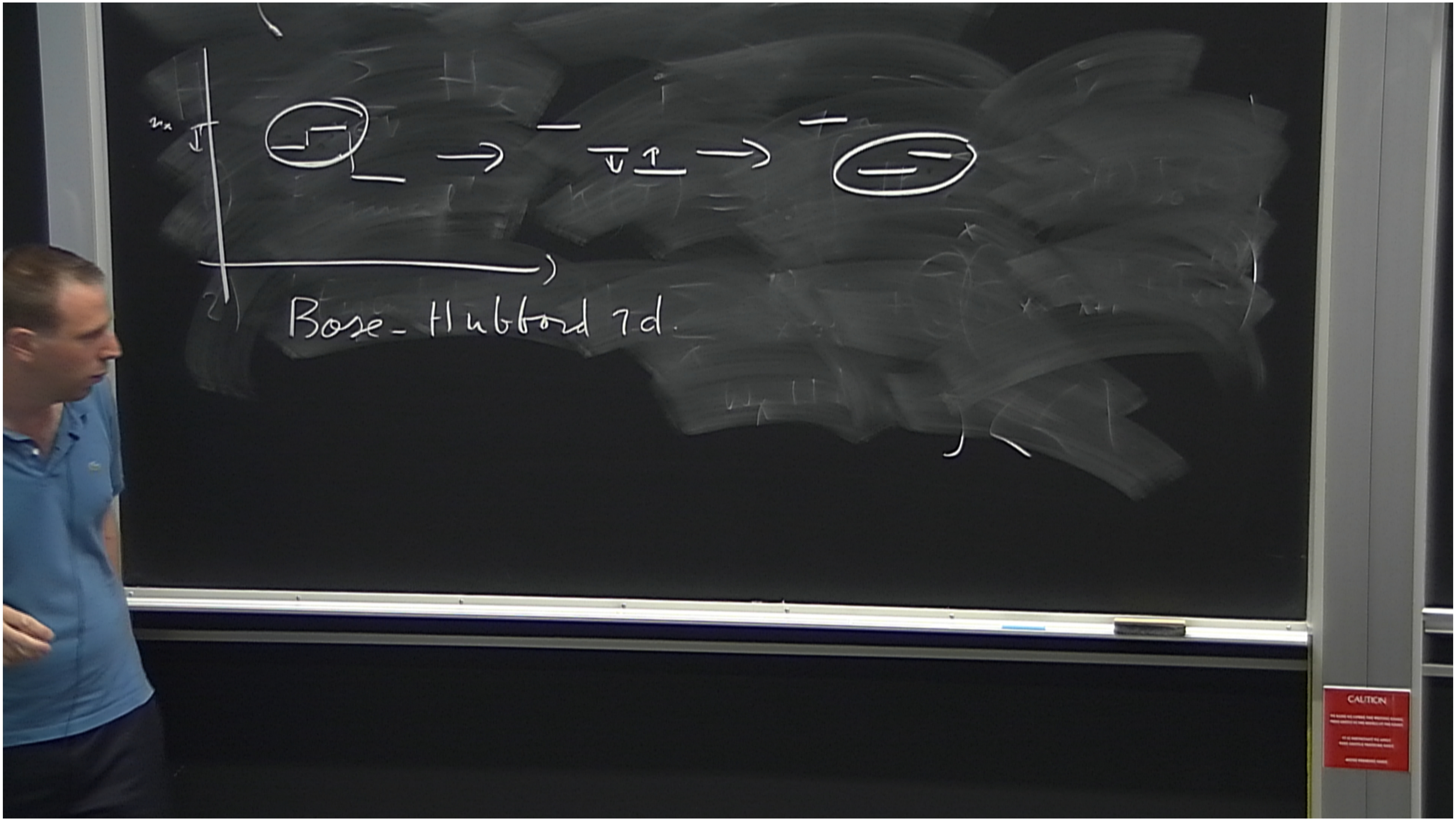
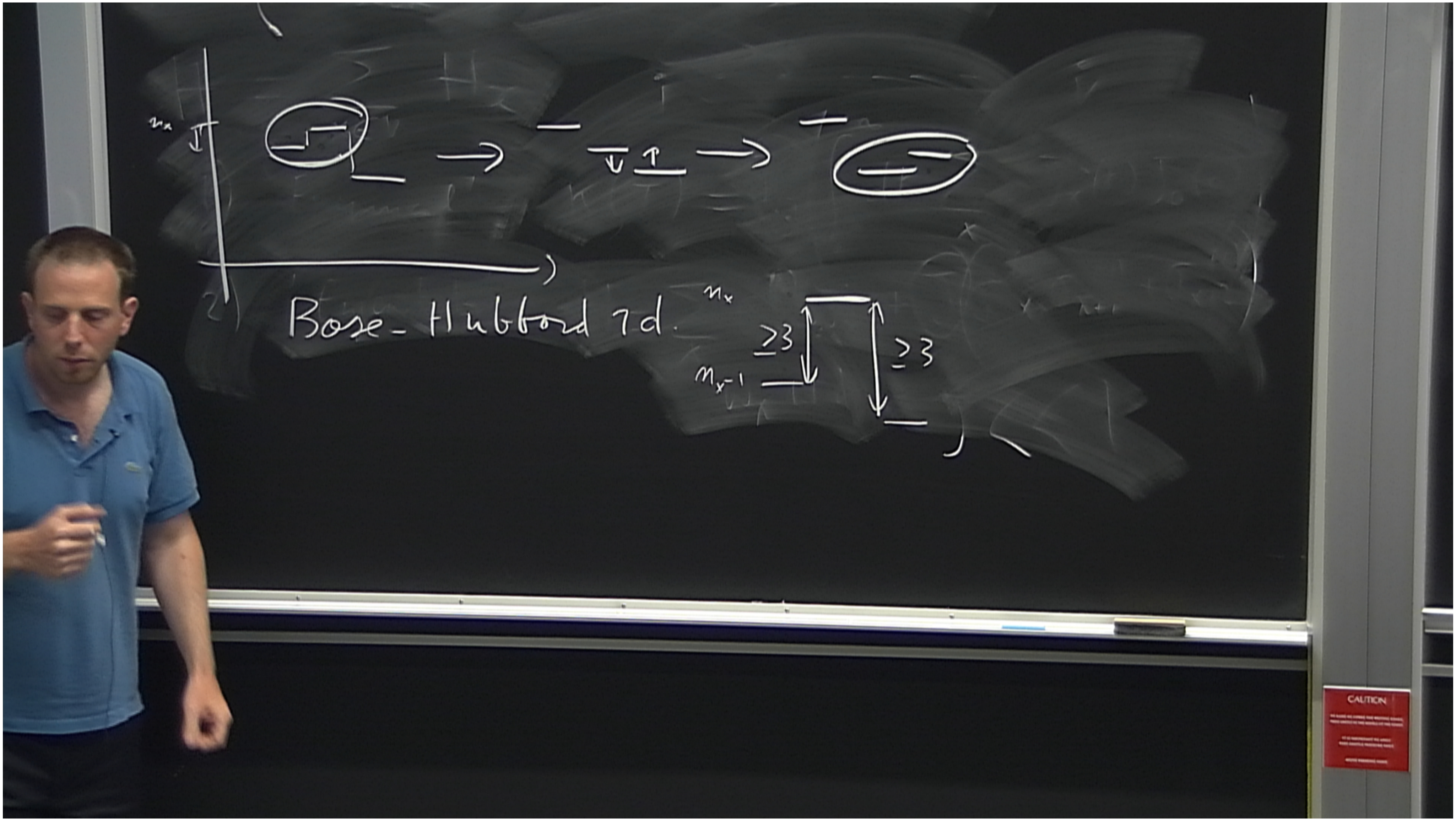
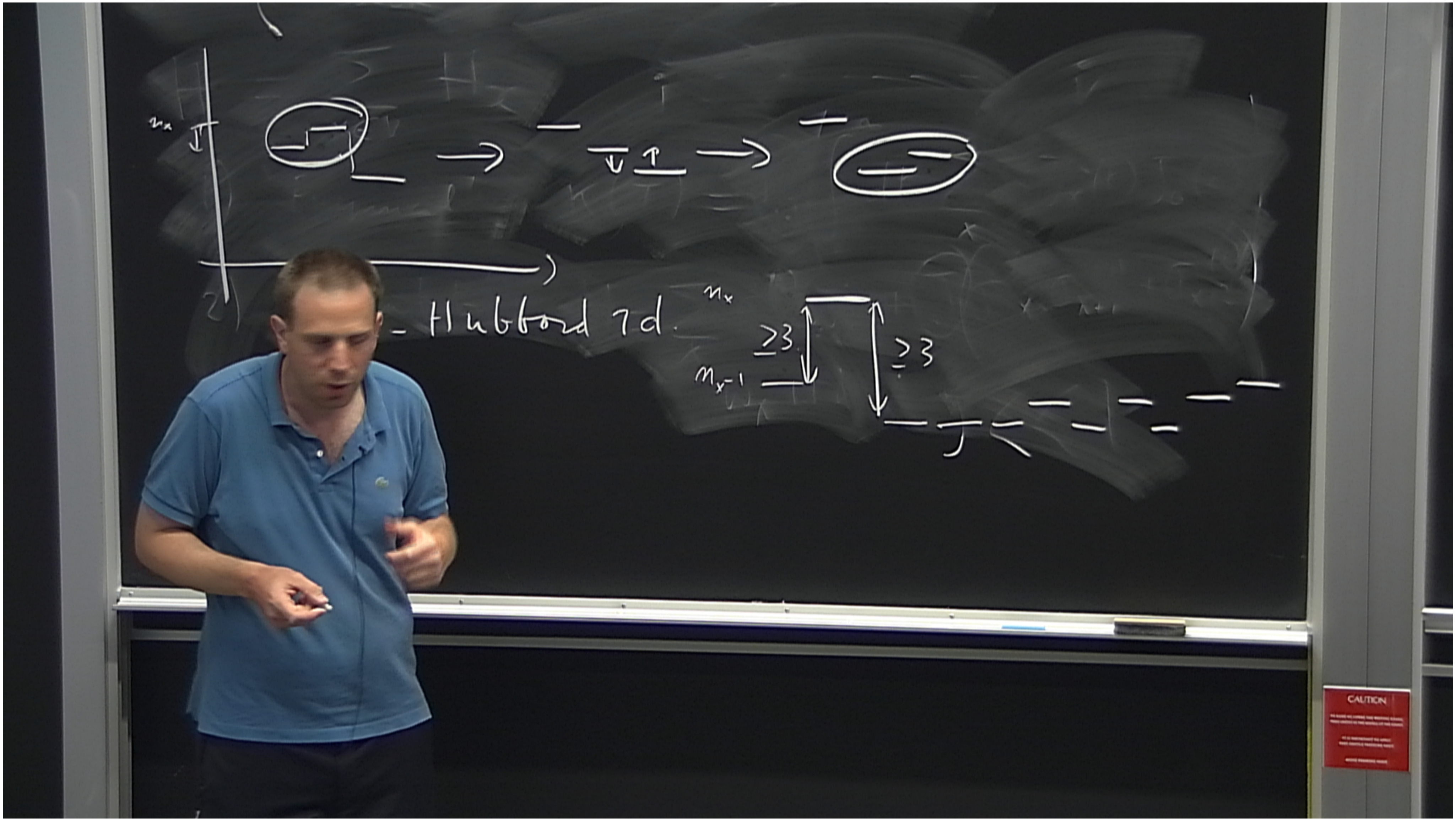


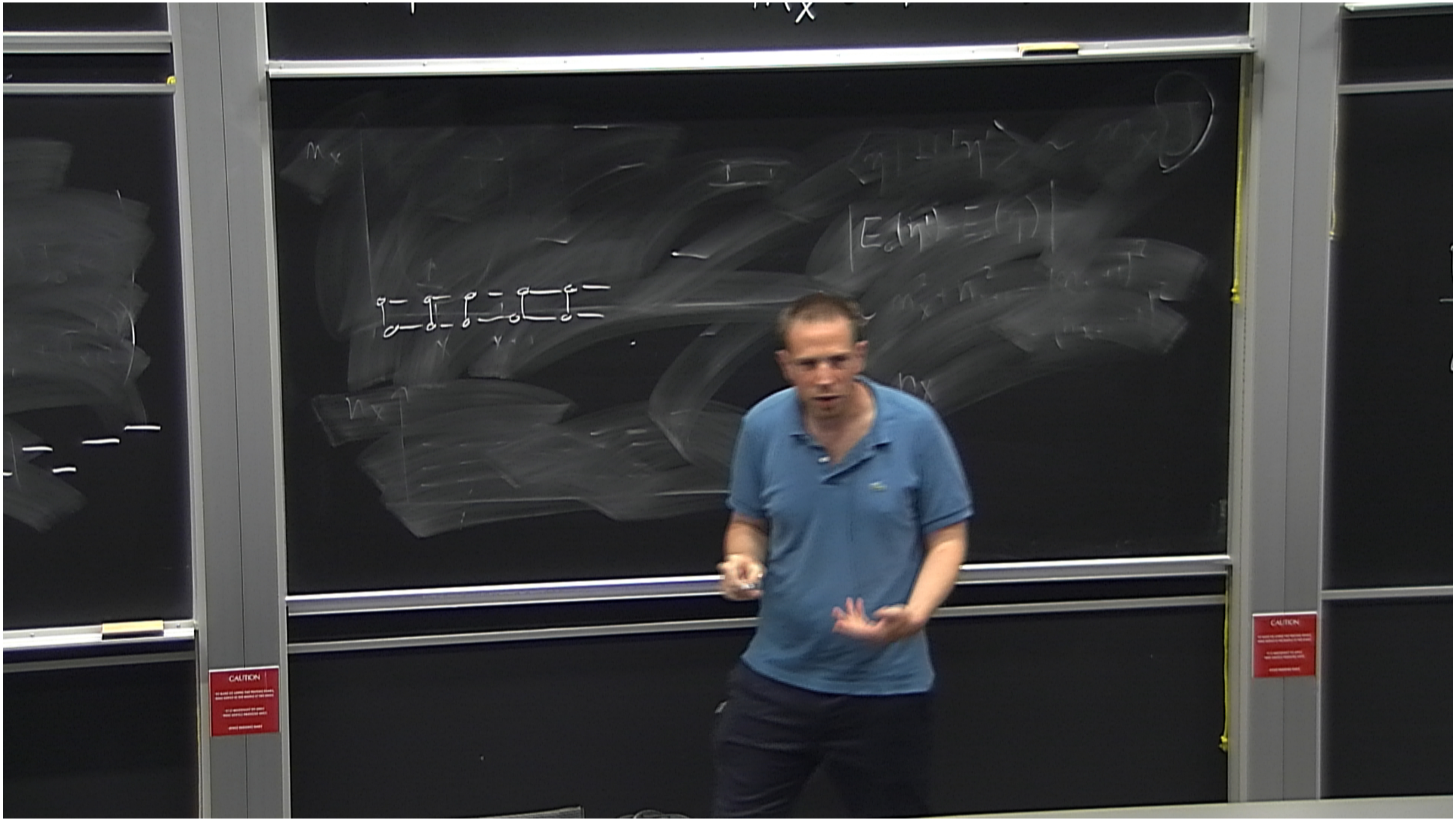


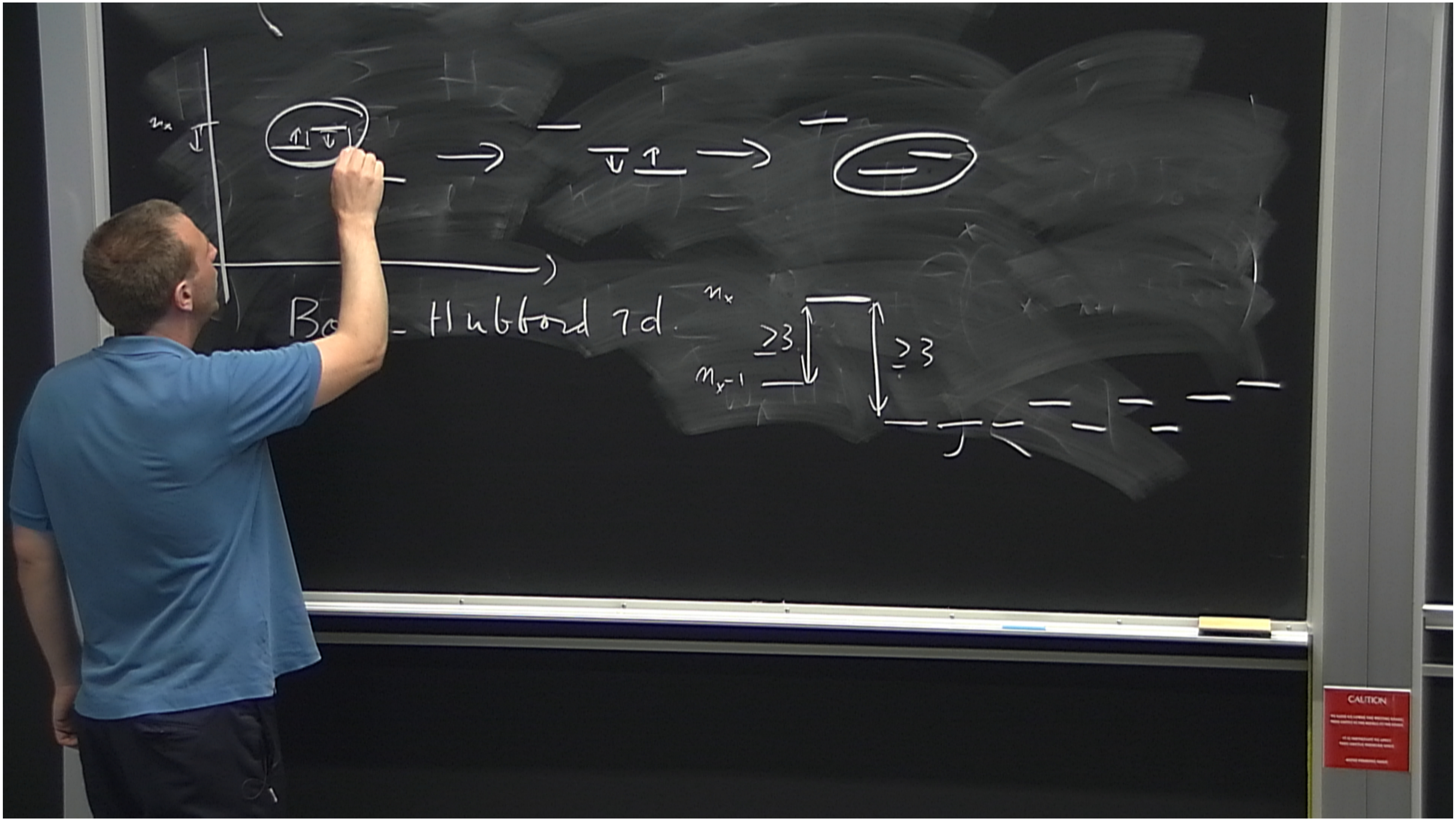

Bose-Hubbard 1d.


Bose- Hubbard 1d.
n_x
≥ 3
n_{x-1}
≥ 3
CAUTION

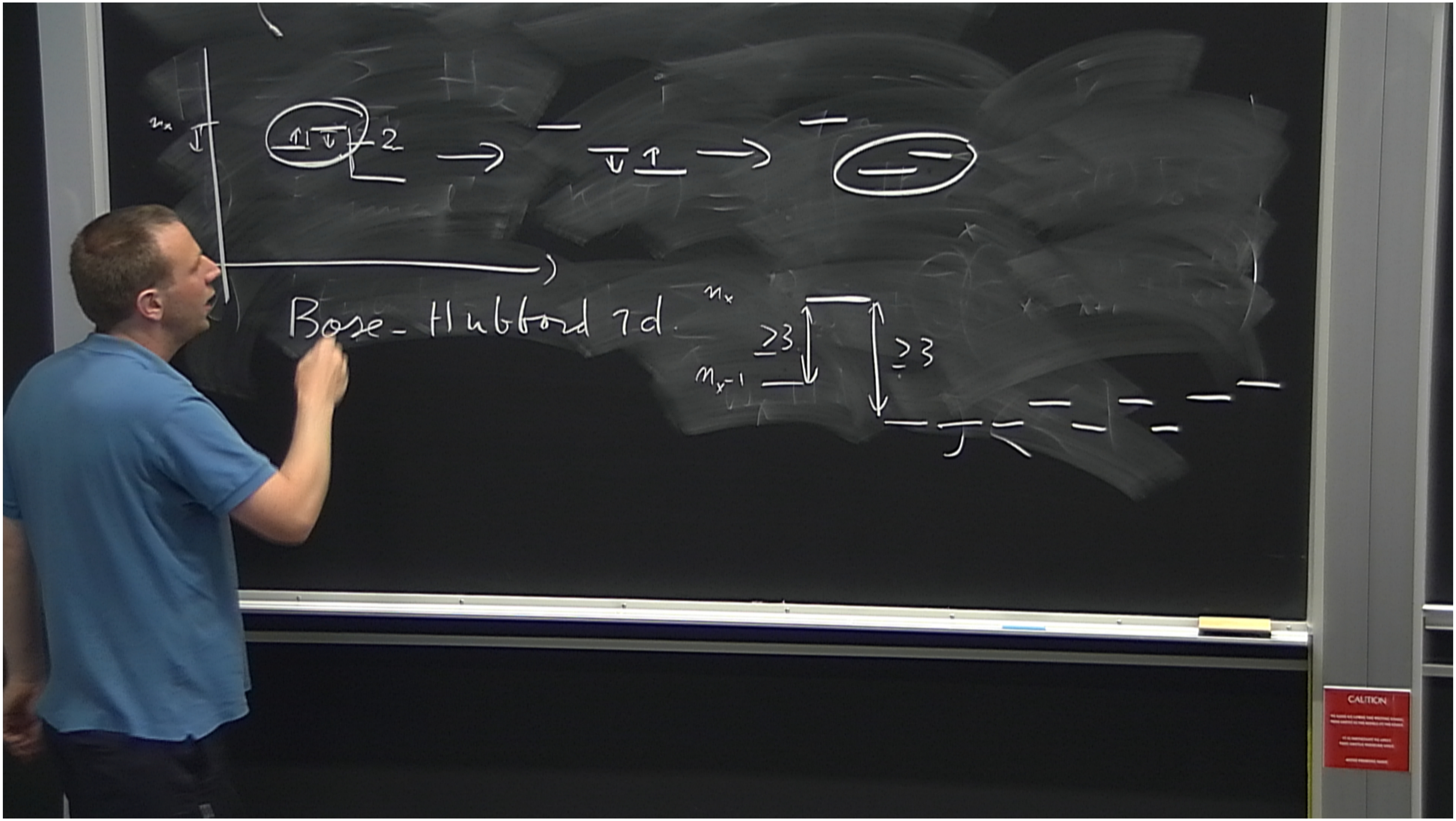


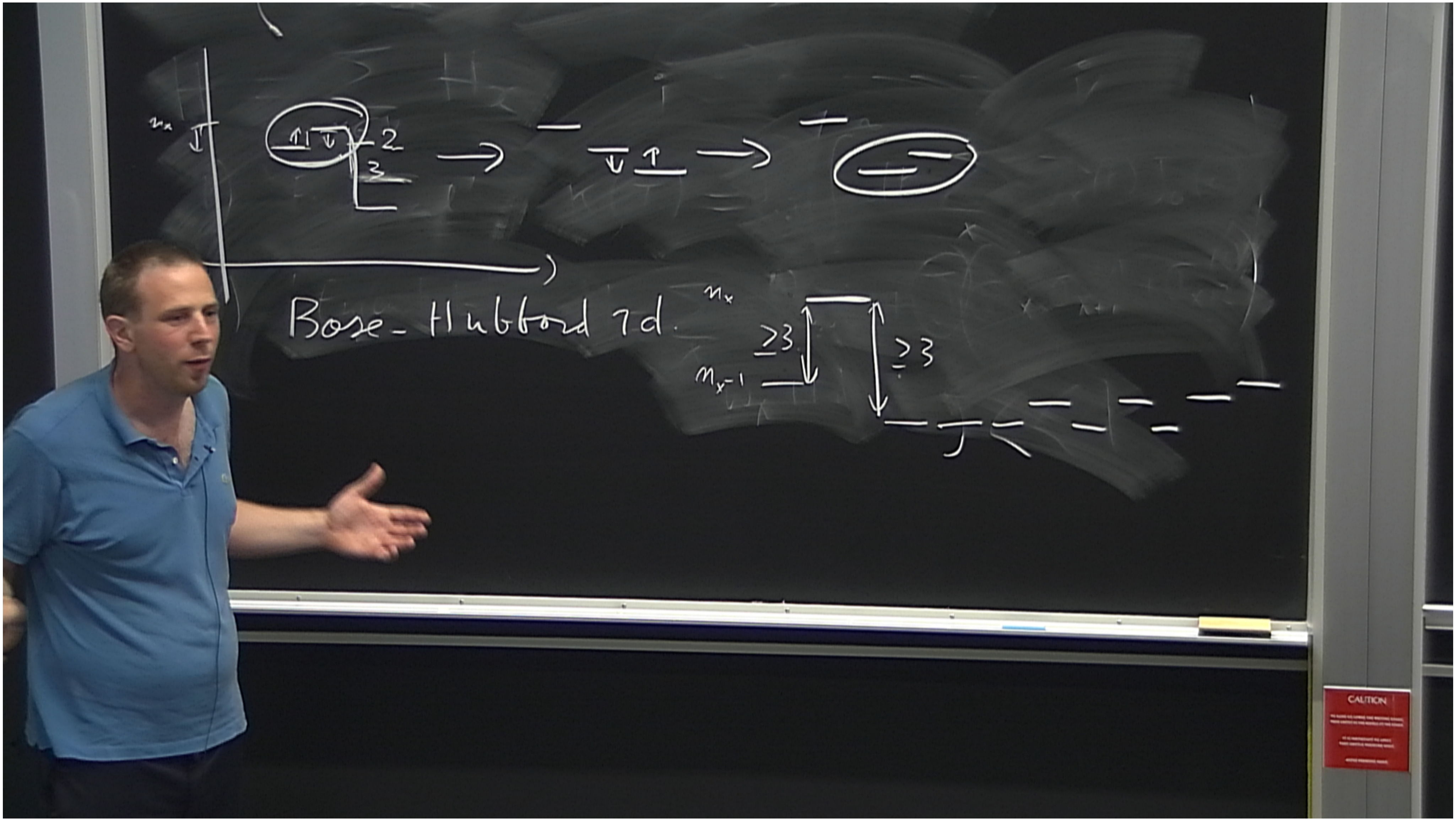
Hubbard
n_x
≥3
≥3
n_x-1

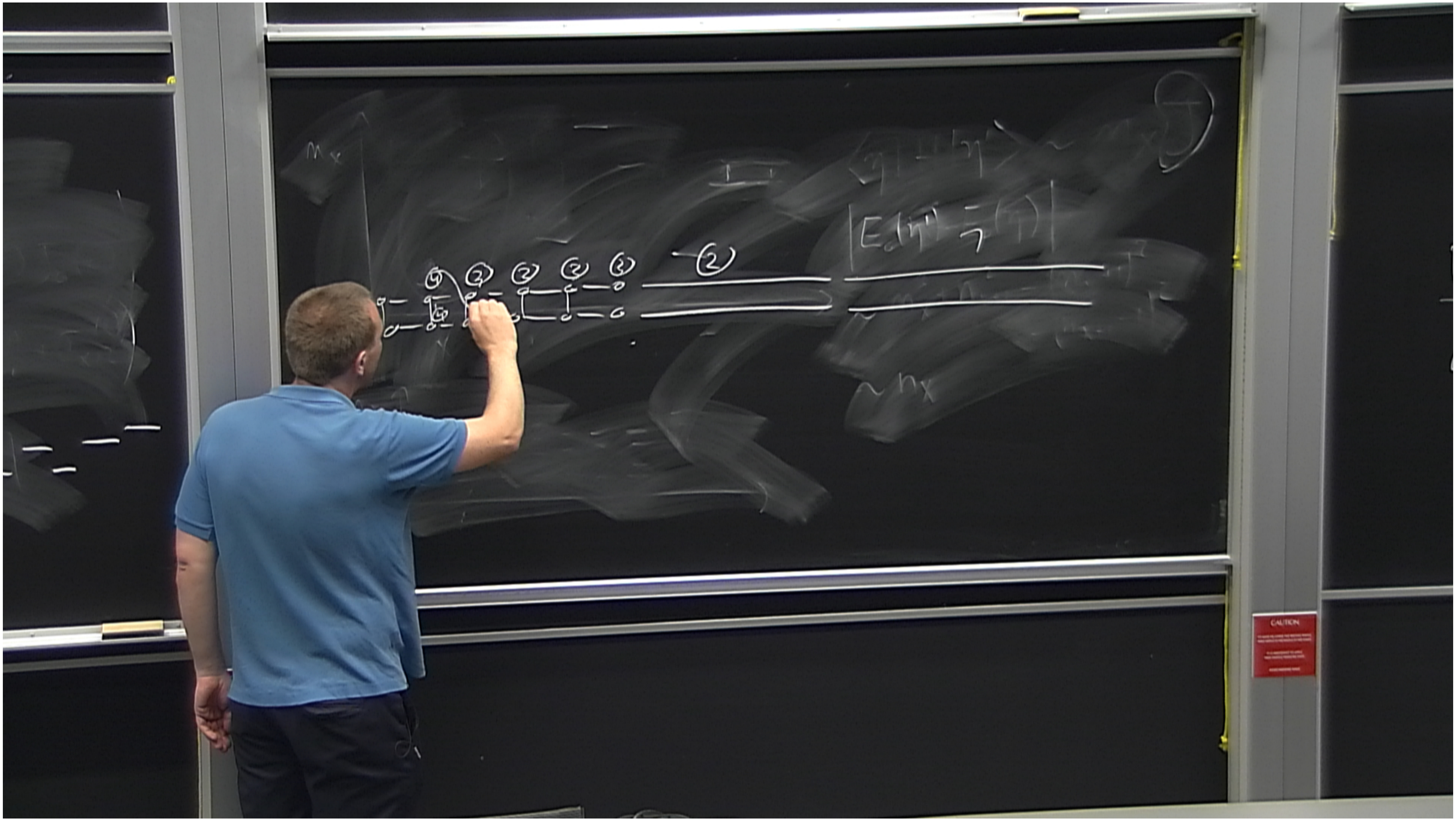


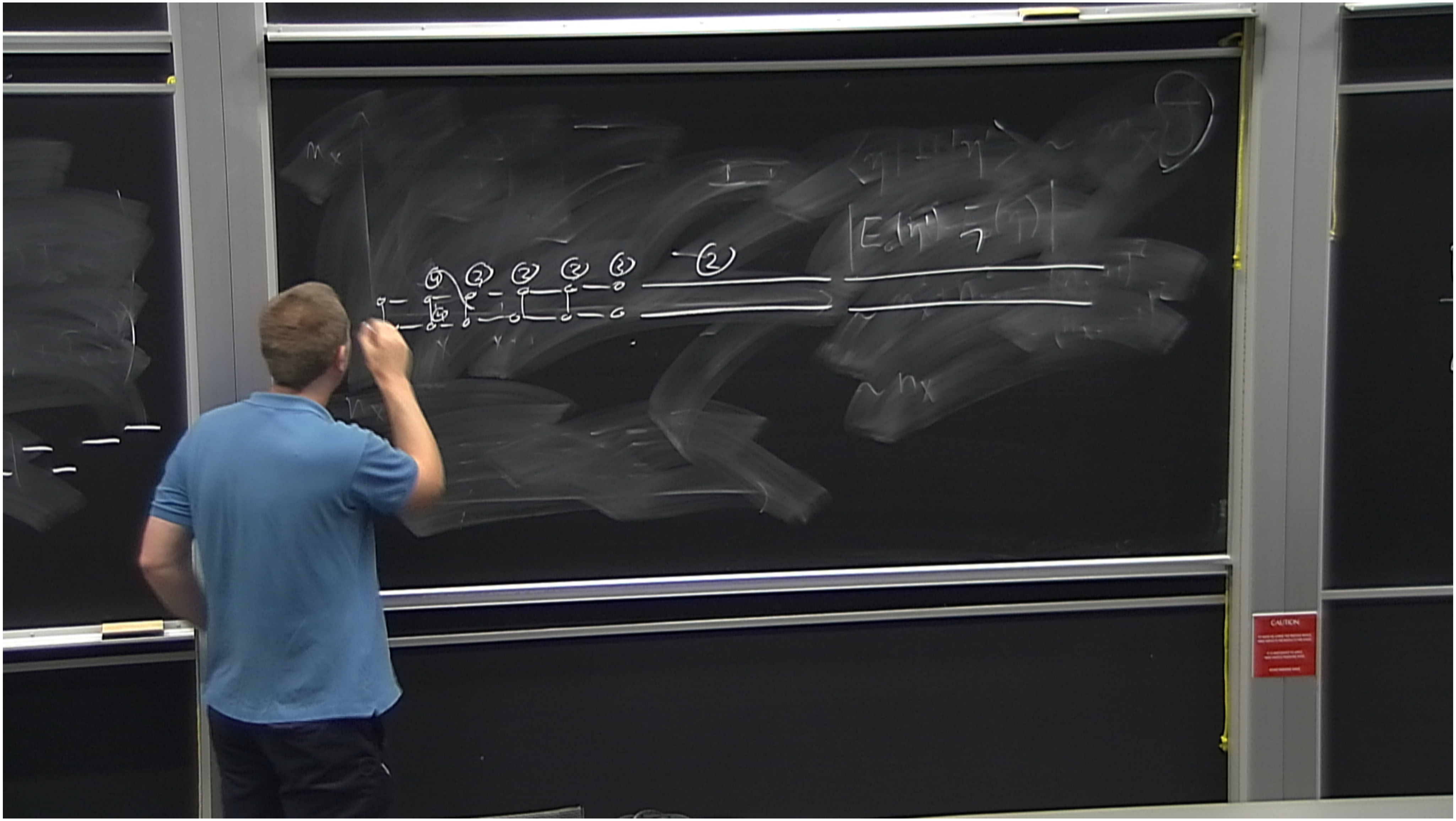


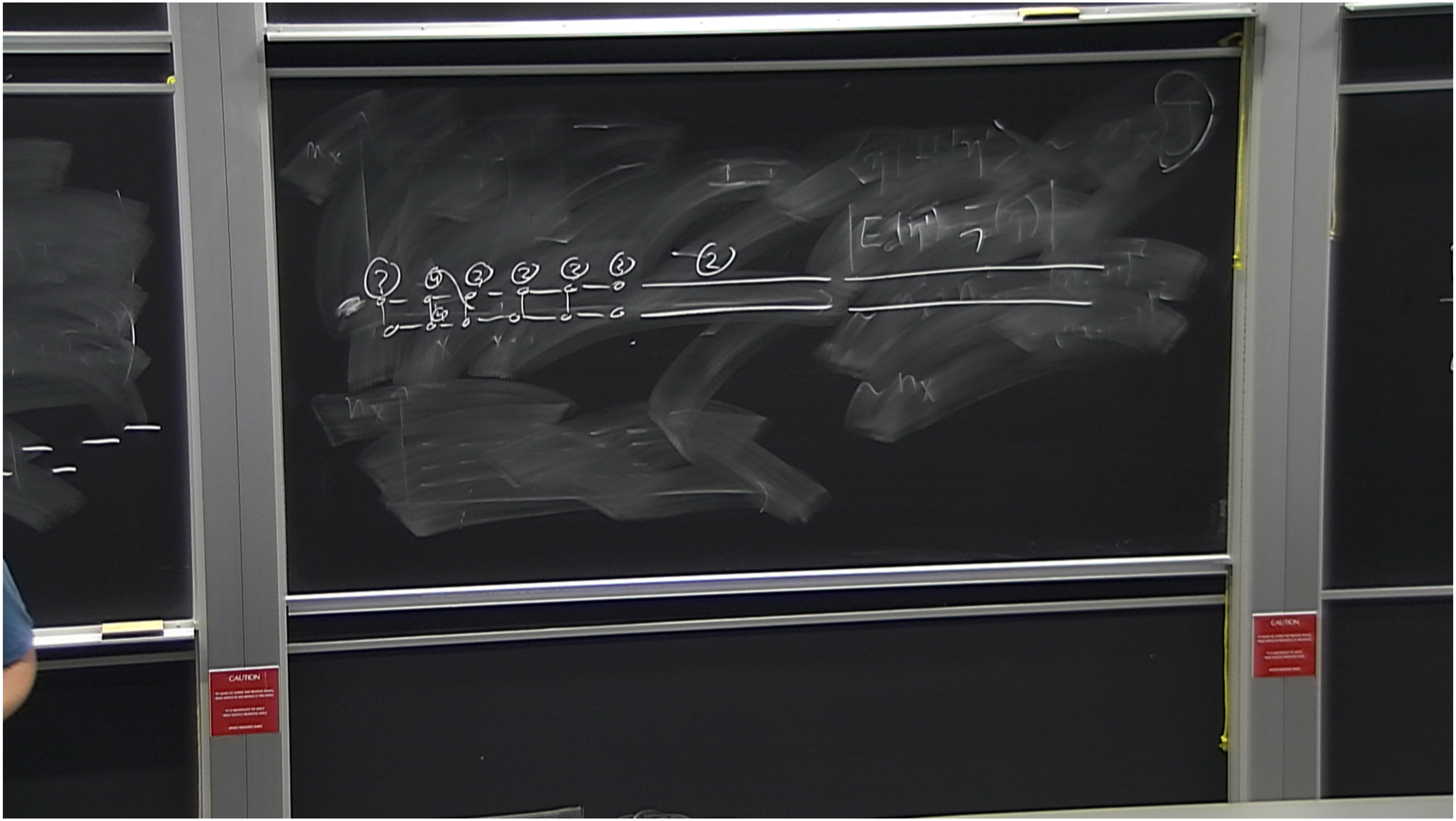
n_x
Bo... Hubbard 1d.
n_x
≥ 3
≥ 3
n_{x-1}

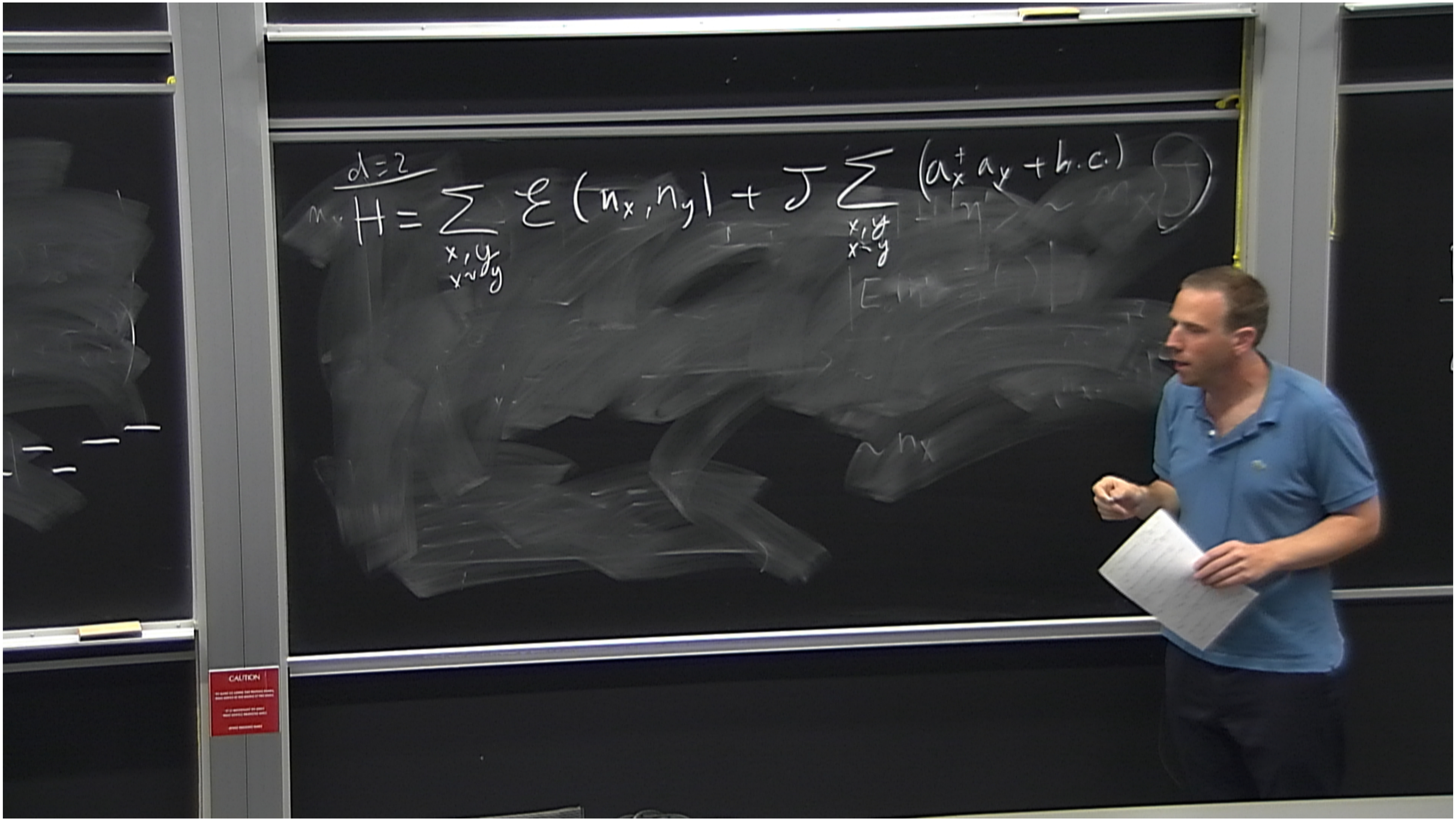

Bose-Hubbard 1d.
n_x
≥ 3
$n_x - 1$
≥ 3
2

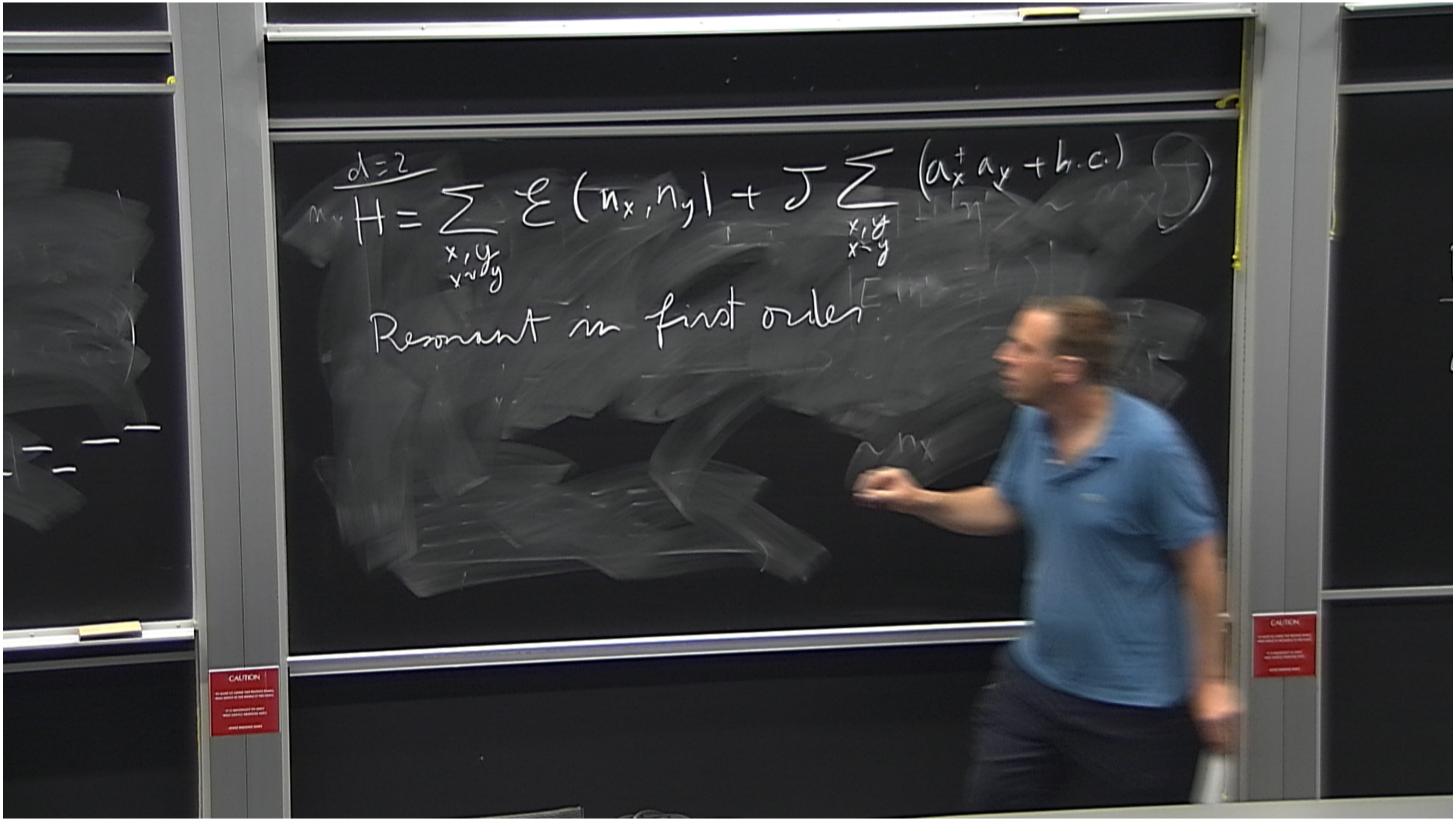

Bose- Hubbard 1d.
n_x
≥3
≥3
n_x-1
CAUTION

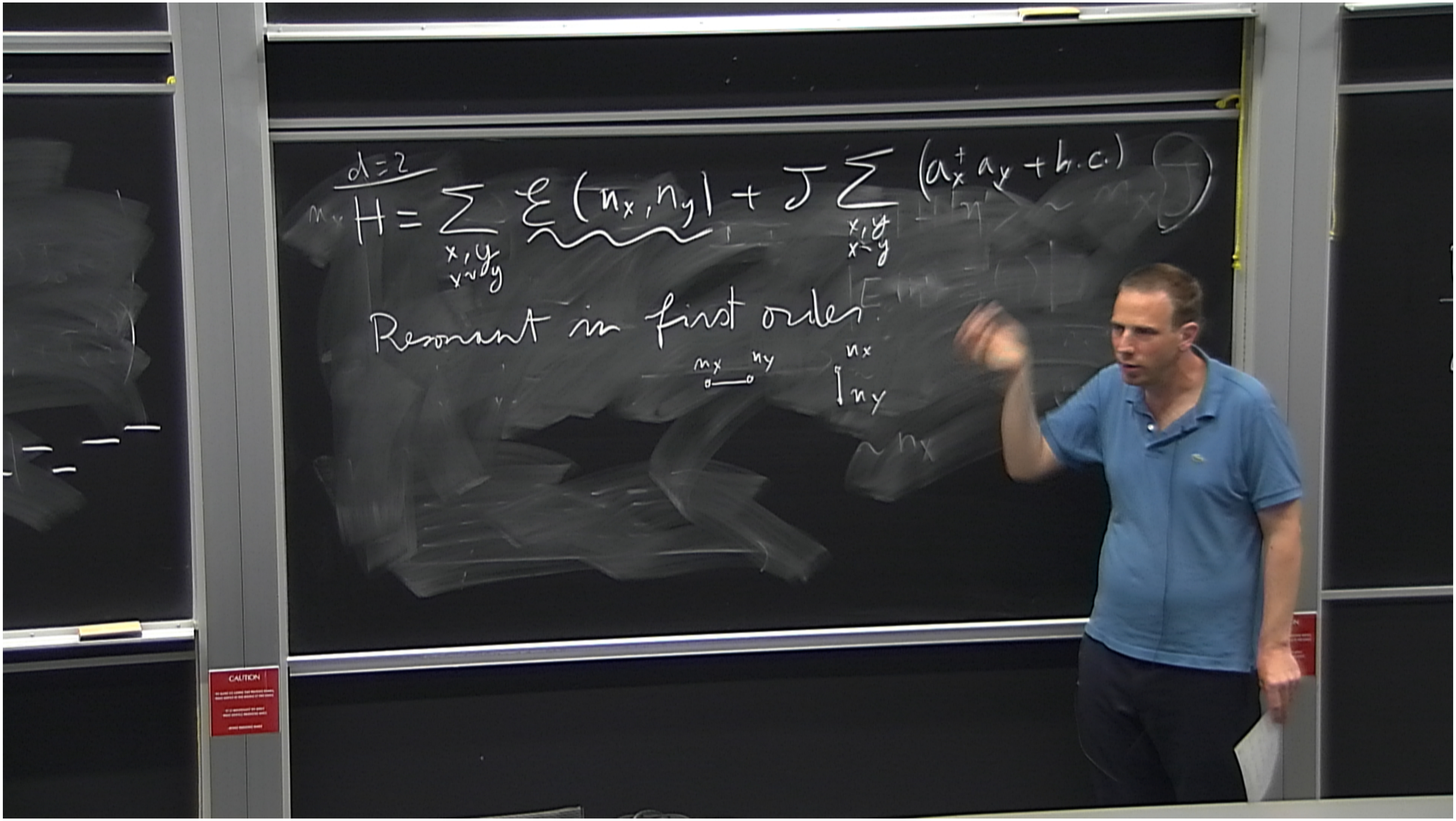


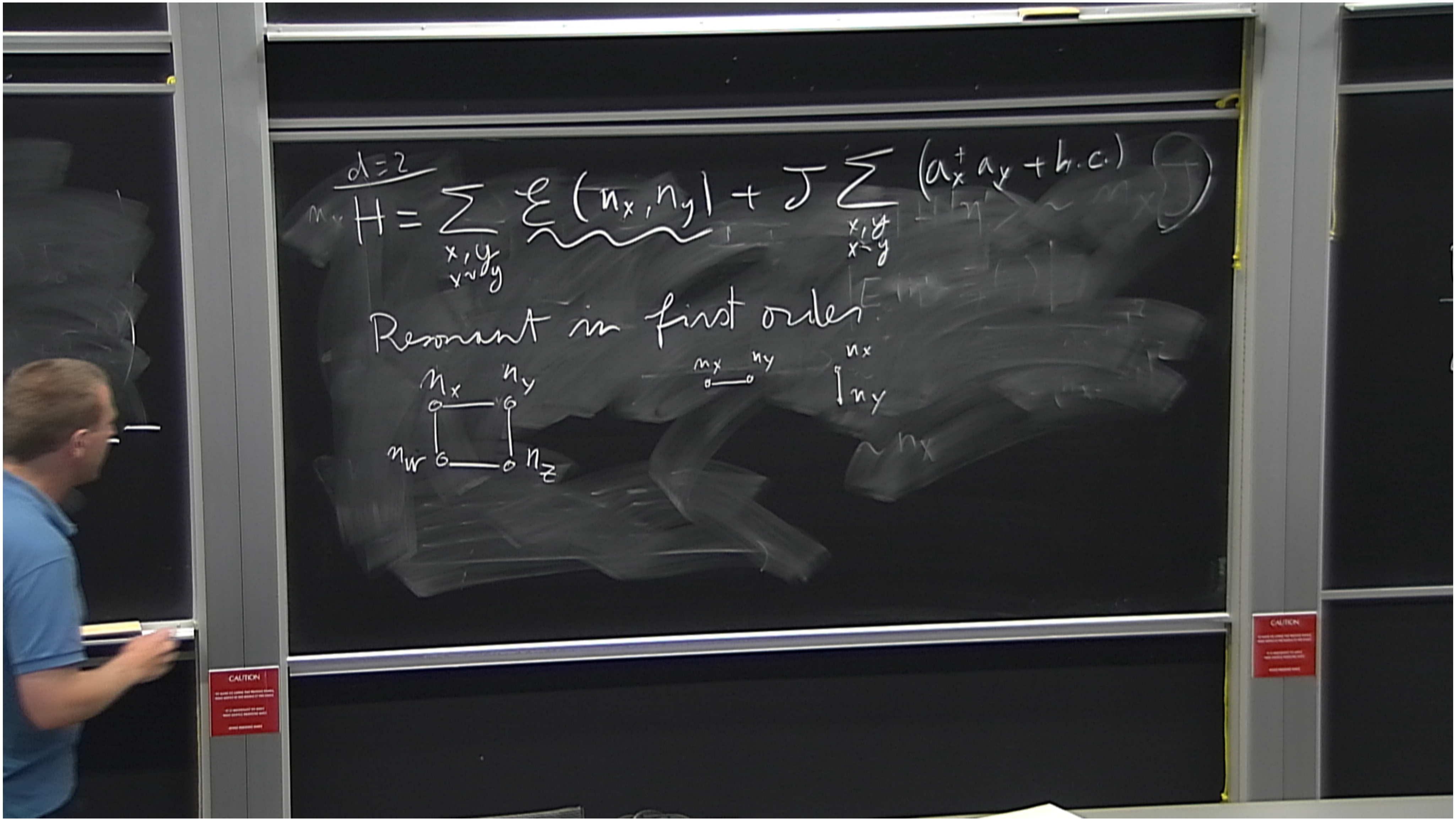


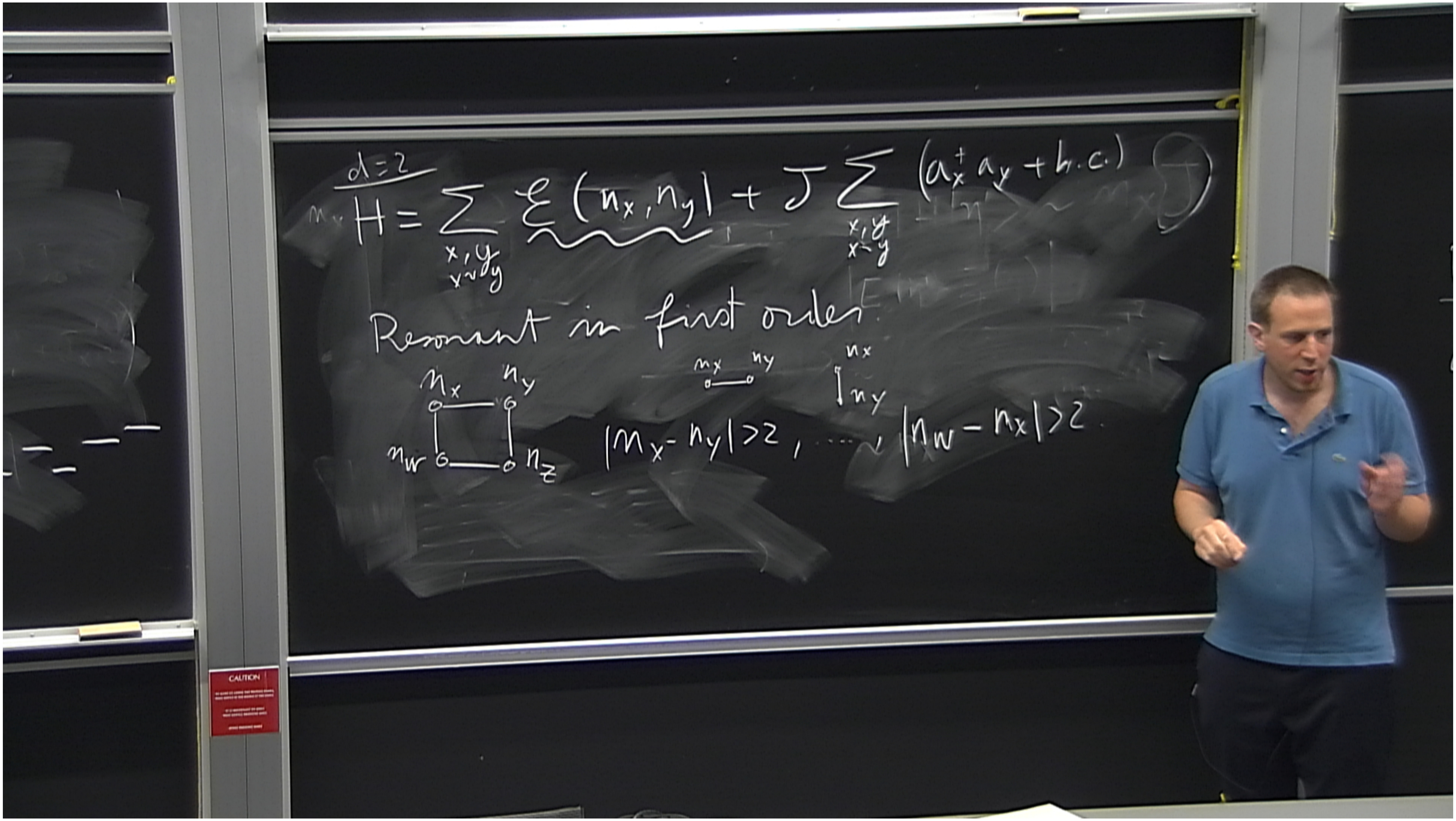


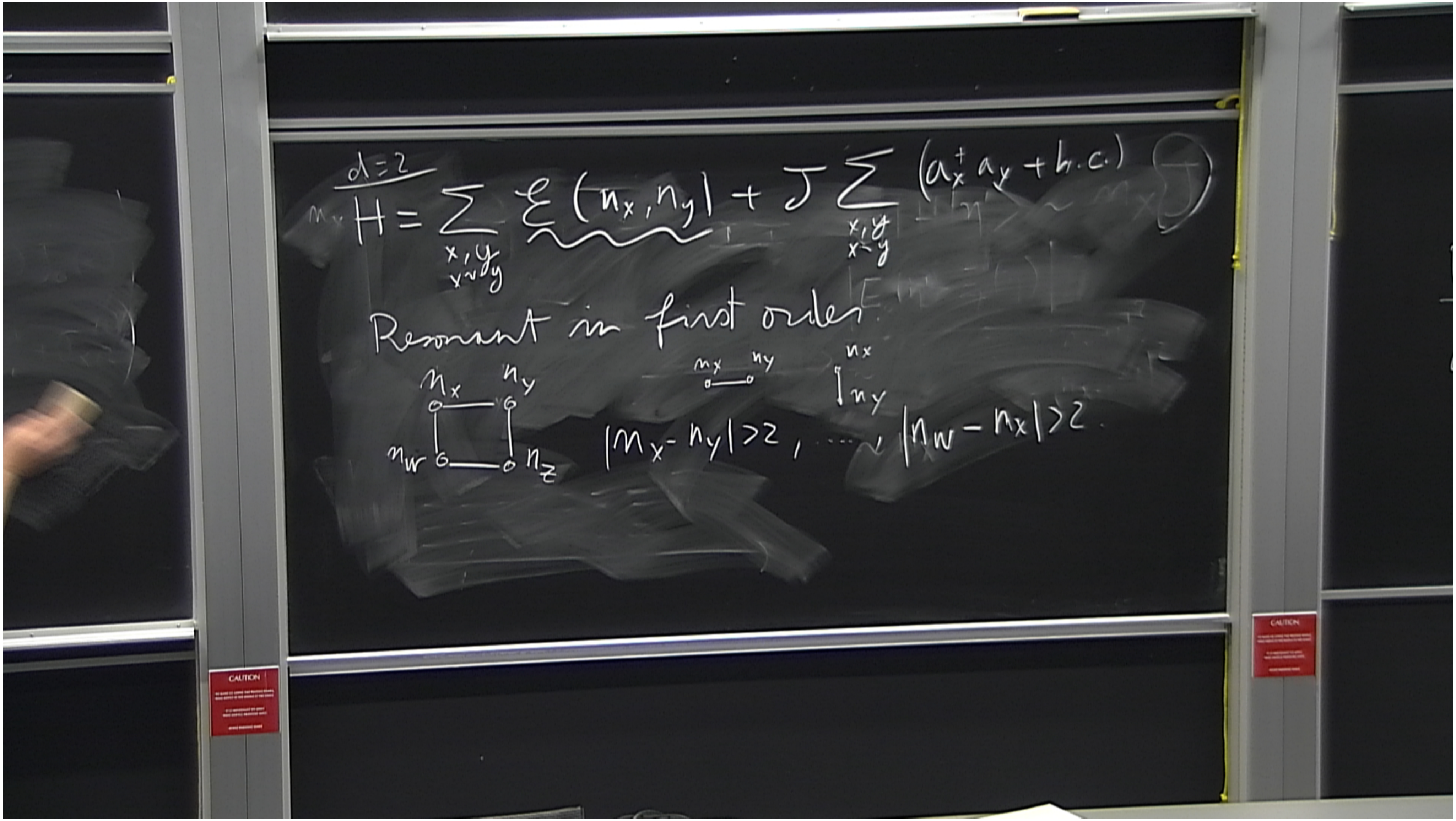

d=2
H = Σ_{x,y; x~y} ε(n_x, n_y) + J Σ_{x,y; x~y} (a_x^† a_y + h.c.)
CAUTION


d=2
H = Σ E(n_x, n_y) + J Σ (a_x^† a_y + h.c.)
x, y x~y
x, y x~y
Resonant in first order


d=2
$H = \sum_{x,y,\, x\sim y} \mathcal{E}(n_x, n_y) + J \sum_{x,y,\, x\sim y} (a_x^\dagger a_y + h.c.)$
Resonant in first order
n_x n_y
n_x
n_y

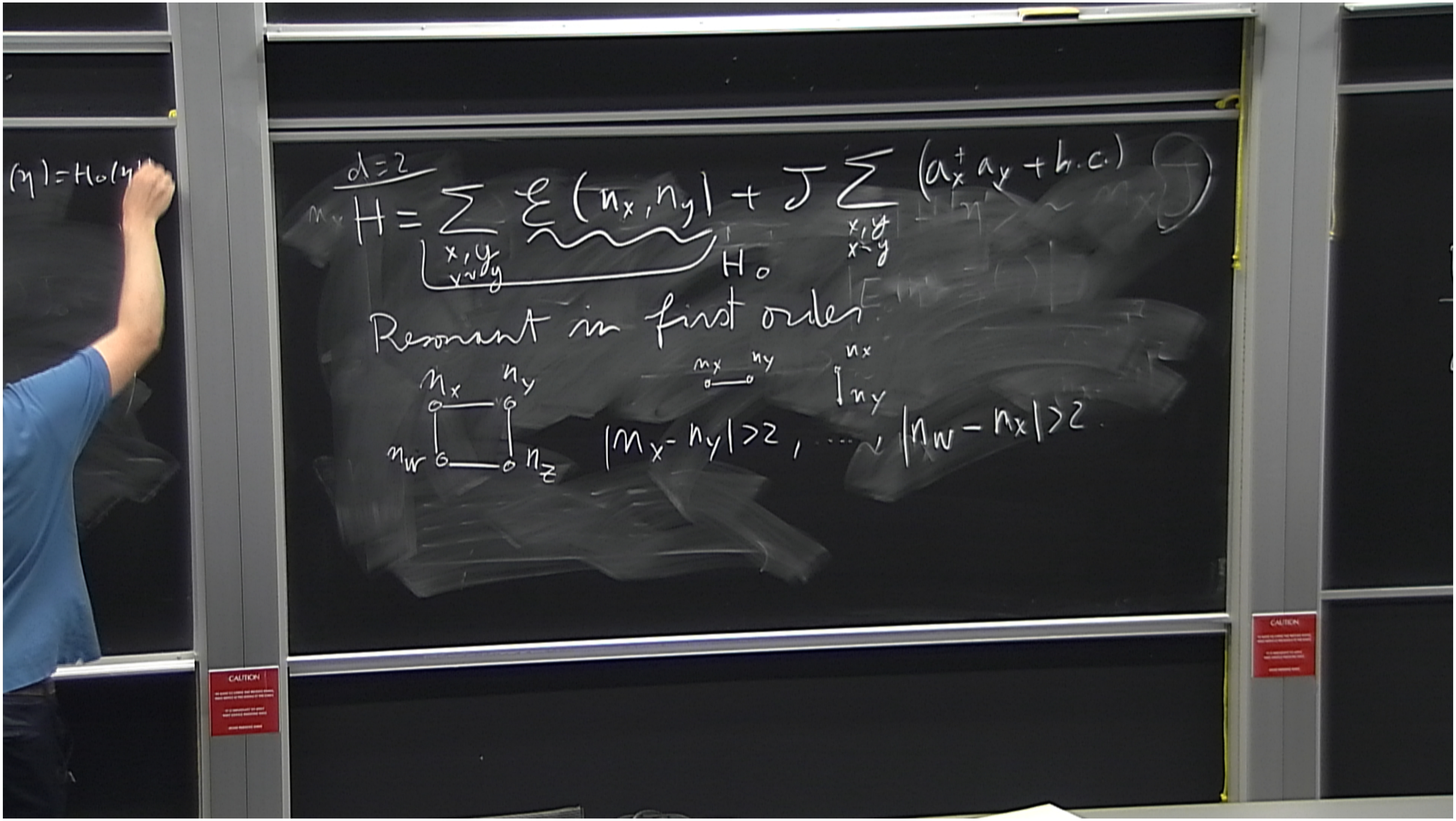

d=2
$H = \sum_{x,y,\ x\sim y} \mathcal{E}(n_x, n_y) + J \sum_{x,y,\ x\sim y} (a_x^\dagger a_y + h.c.)$
Resonant in first order
n_x n_y n_w n_z
n_x n_y
n_x n_y
$\sim n_x$

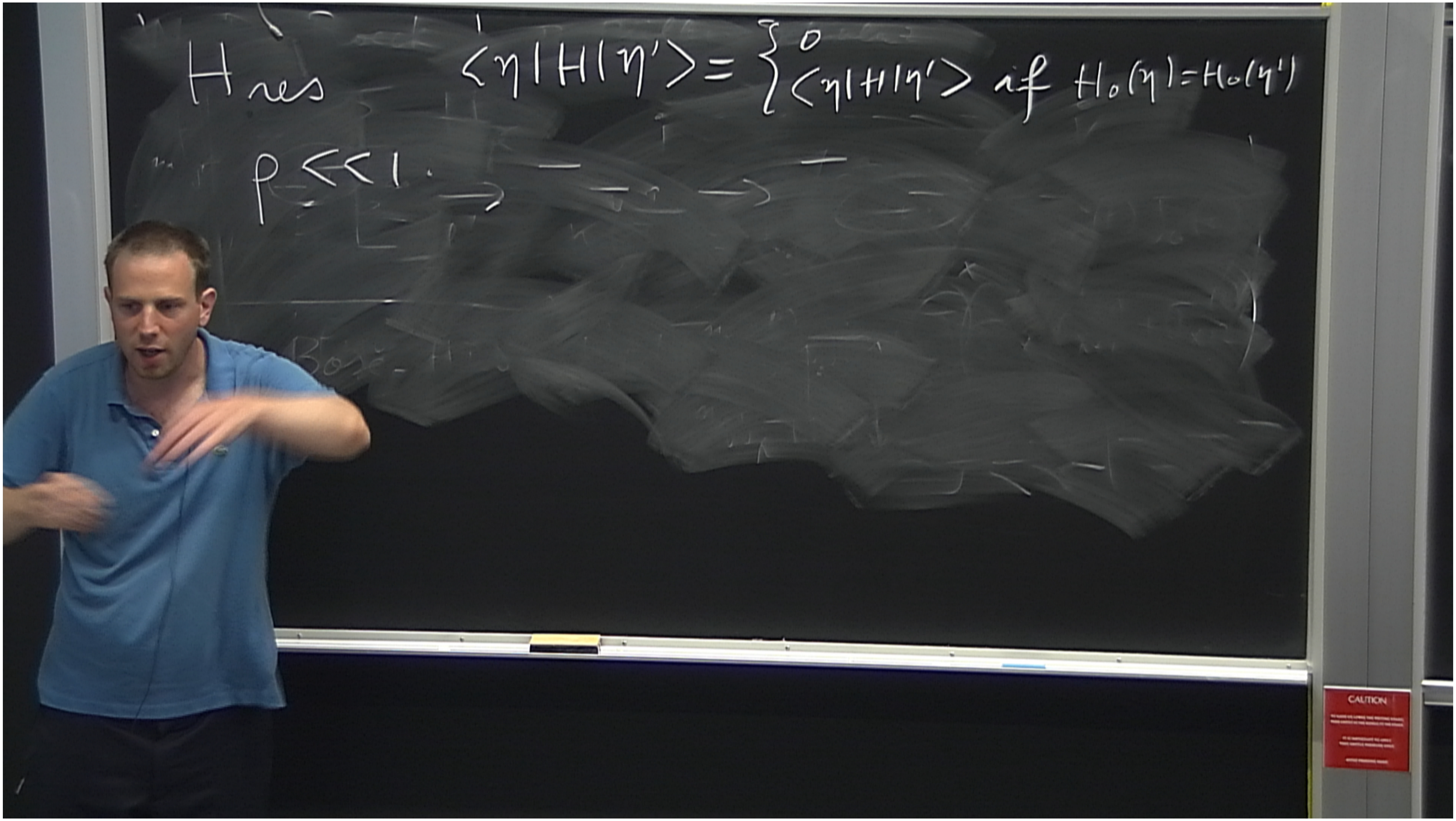

$d=2$
$H = \sum_{x,y \atop x\sim y} \mathcal{E}(n_x, n_y) + J \sum_{x,y \atop x\sim y} (a_x^\dagger a_y + h.c.)$
Resonant in first order
n_x n_y n_w n_z
$|n_x - n_y| > 2$, ..., $|n_w - n_x| > 2$


d=2
H = ∑ E(n_x, n_y) + J ∑ (a_x^† a_y + h.c.)
x,y x~y
Resonant in first order:
n_x n_y
n_w n_z
|n_x - n_y| > 2, ..., |n_w - n_x| > 2

H_{res}

$$\langle \eta | H | \eta' \rangle = \begin{cases} 0 \\ \langle \eta | H | \eta' \rangle \text{ if} \end{cases}$$


d=2
$H = \sum_{x,y;\, x\sim y} \mathcal{E}(n_x, n_y) + J \sum_{x,y;\, x\sim y} (a_x^\dagger a_y + h.c.)$
H_0
Resonant in first order
n_x n_y n_w n_z
$|n_x - n_y| > 2, \ldots, |n_w - n_x| > 2$

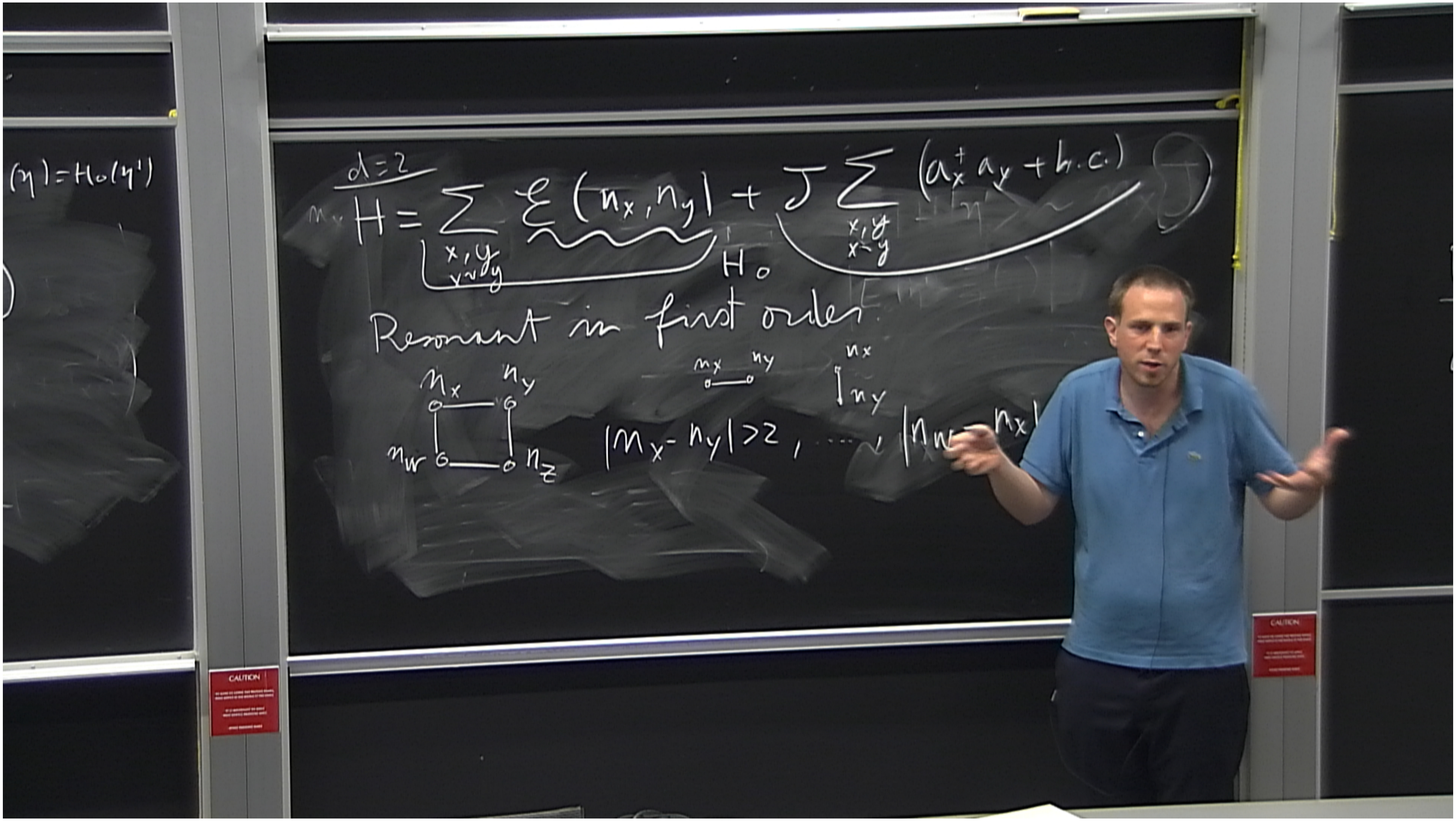

H res ⟨η|H|η'⟩ = { 0 ; ⟨η|H|η'⟩ if H₀(η)=H₀(η')
ρ << 1.

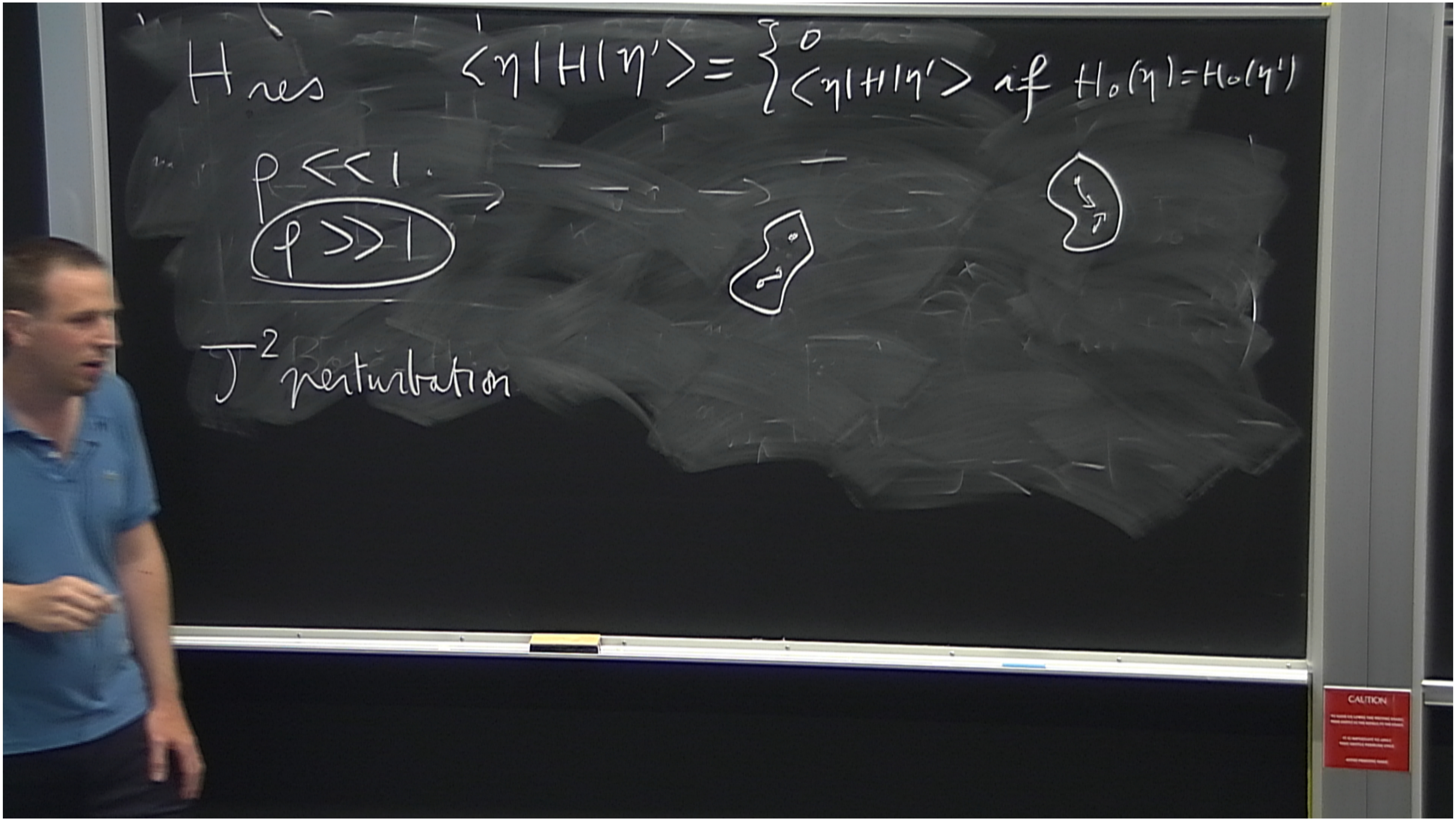

H res
⟨η|H|η'⟩ = { 0
⟨η|H|η'⟩ if H0(η) = H0(η')
ρ <<< 1.
ρ >>

$$H_{res} \quad \langle \eta | H | \eta' \rangle = \begin{cases} 0 \\ \langle \eta | H | \eta' \rangle \text{ if } H_0(\eta) = H_0(\eta') \end{cases}$$

$\rho \ll 1.$

$\rho \gg 1$


d=2
H = Σ_{x,y} ε(n_x, n_y) + J Σ_{x,y} (a_x^+ a_y + h.c.)
H_0
Resonant in first order:
|n_x - n_y| > 2



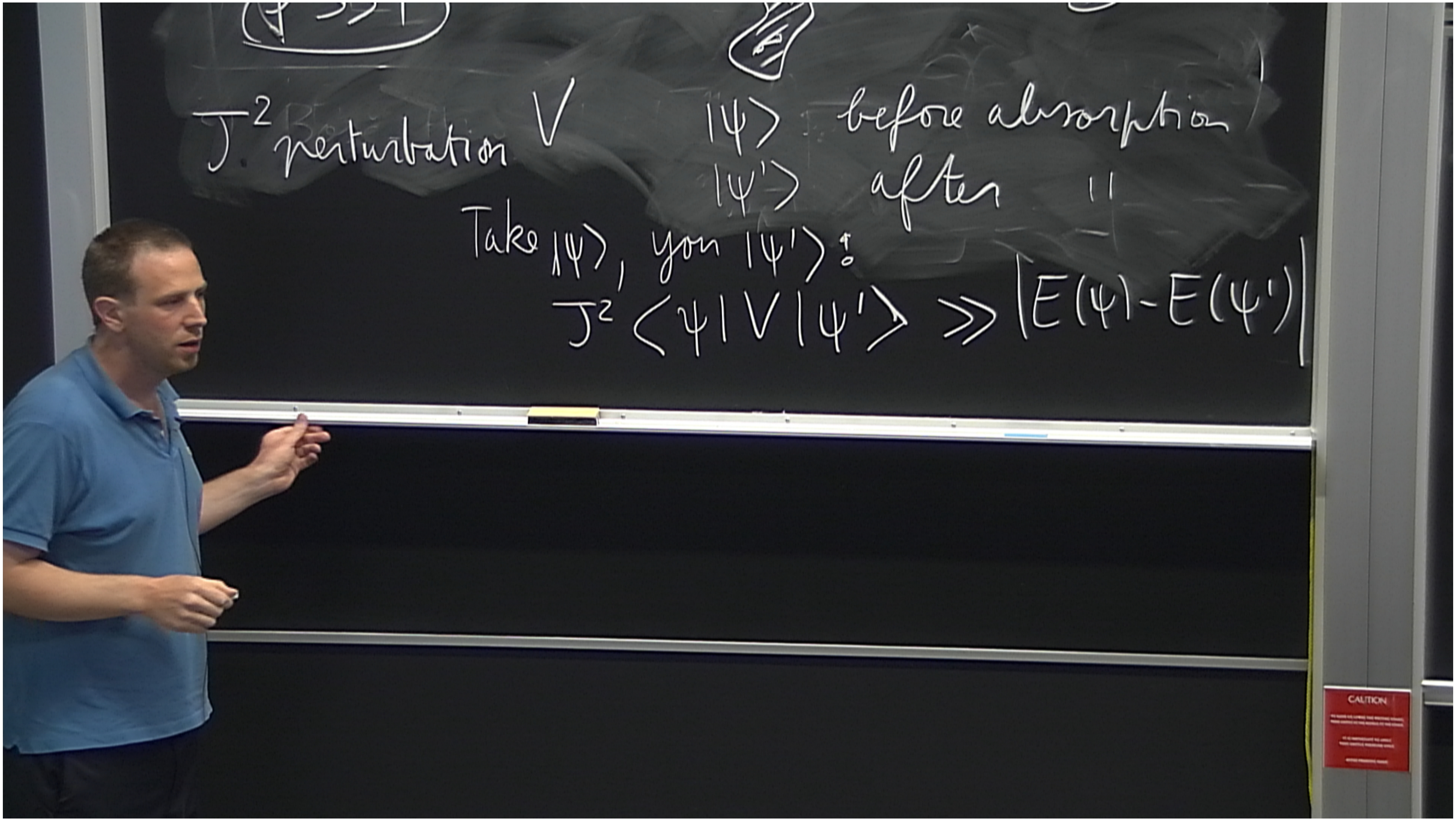
H_{res}
$\langle\eta|H|\eta'\rangle = \begin{cases} 0 \\ \langle\eta|H|\eta'\rangle \text{ if } H_0(\eta)=H_0(\eta') \end{cases}$
$\rho \ll 1$
$\rho \gg 1$
J^2 perturbation

H_{res} $\quad \langle \eta | H | \eta' \rangle = \begin{cases} 0 \\ \langle \eta | H | \eta' \rangle \text{ if } H_0(\eta) = H_0(\eta') \end{cases}$

$\rho \ll 1.$

$\rho \gg 1$

J^2 perturbation

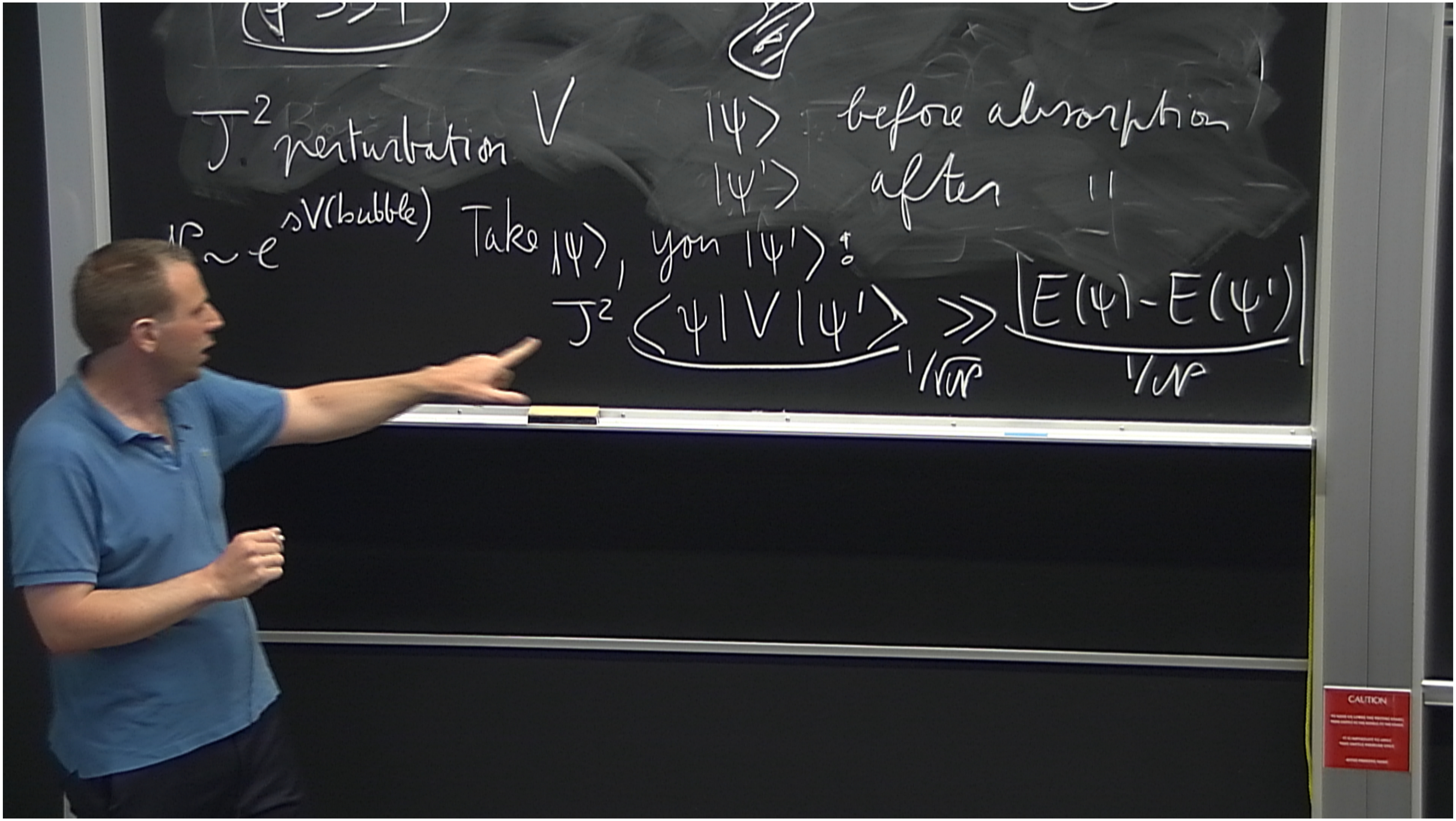

J^2 perturbation V
$|\psi\rangle$ before absorption
$|\psi'\rangle$ after "
Take $|\psi\rangle$, you $|\psi'\rangle$:
$J^2 \langle\psi|V|\psi'\rangle \gg |E(\psi) - E(\psi')|$
CAUTION

J^2 perturbation V $\quad |\psi\rangle$ before absorption

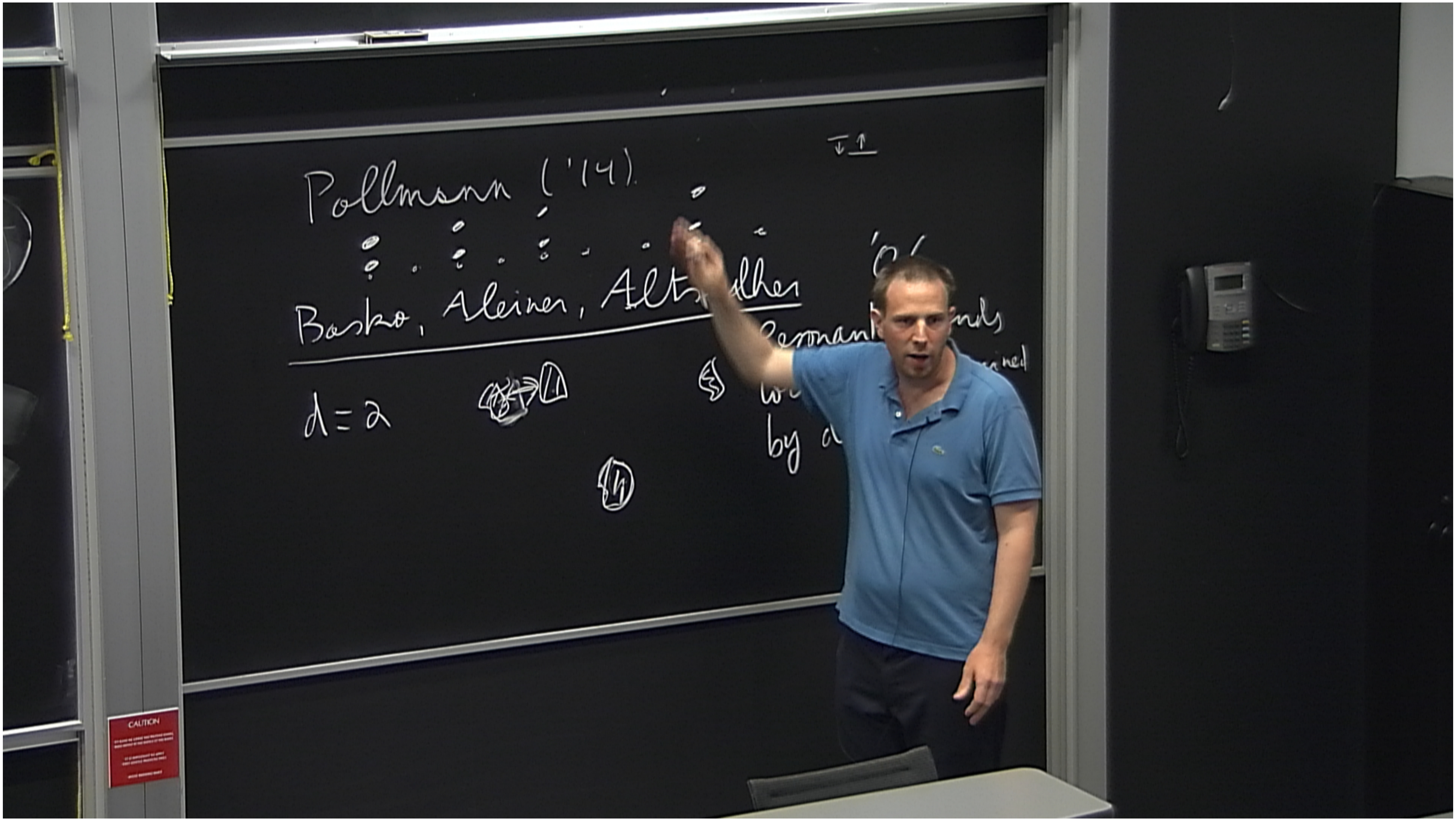
$|\psi'\rangle$ after "

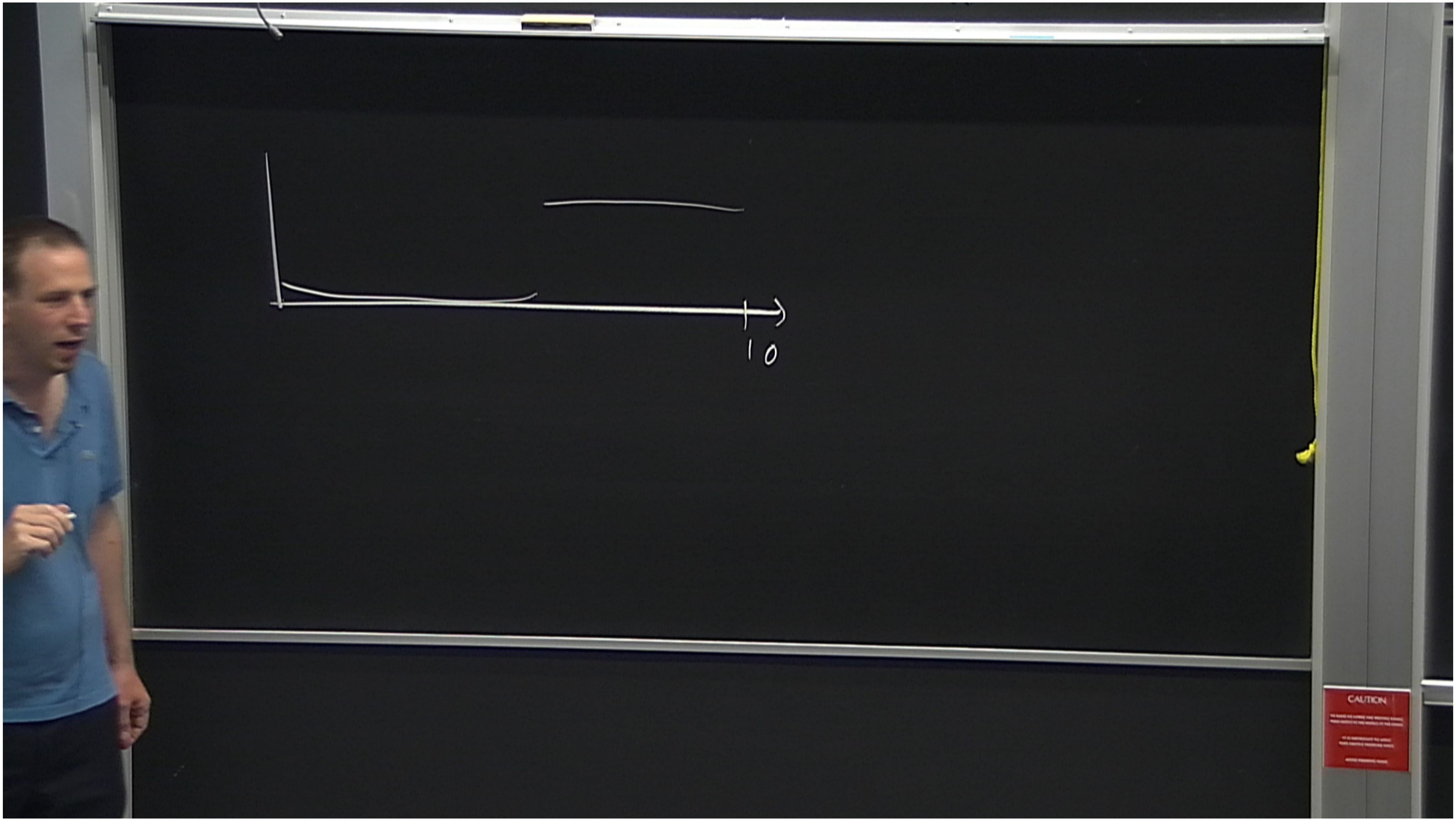
$\mathcal{N} \sim e^{sV(\text{bubble})}$ Take $|\psi\rangle$, you $|\psi'\rangle$:

$$J^2 \langle \psi | V | \psi' \rangle \gg \left| E(\psi) - E(\psi') \right|$$


J² perturbation V
|ψ⟩ before absorption
|ψ'⟩ after ''
Γ ~ e^{sV(bubble)}
Take |ψ⟩, you |ψ'⟩:
J² ⟨ψ|V|ψ'⟩ ≫ |E(ψ) - E(ψ')|
1/√N
1/N


|ψ'> after
N ~ e^sV(bubble) Take |ψ>, you |ψ'>:
J² <ψ|V|ψ'> >> |E(ψ)-E(ψ')|
1/√N
1/N
CAUTION


Pollmann ('14)
Basko, Aleiner,
d = 2
by
CAUTION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