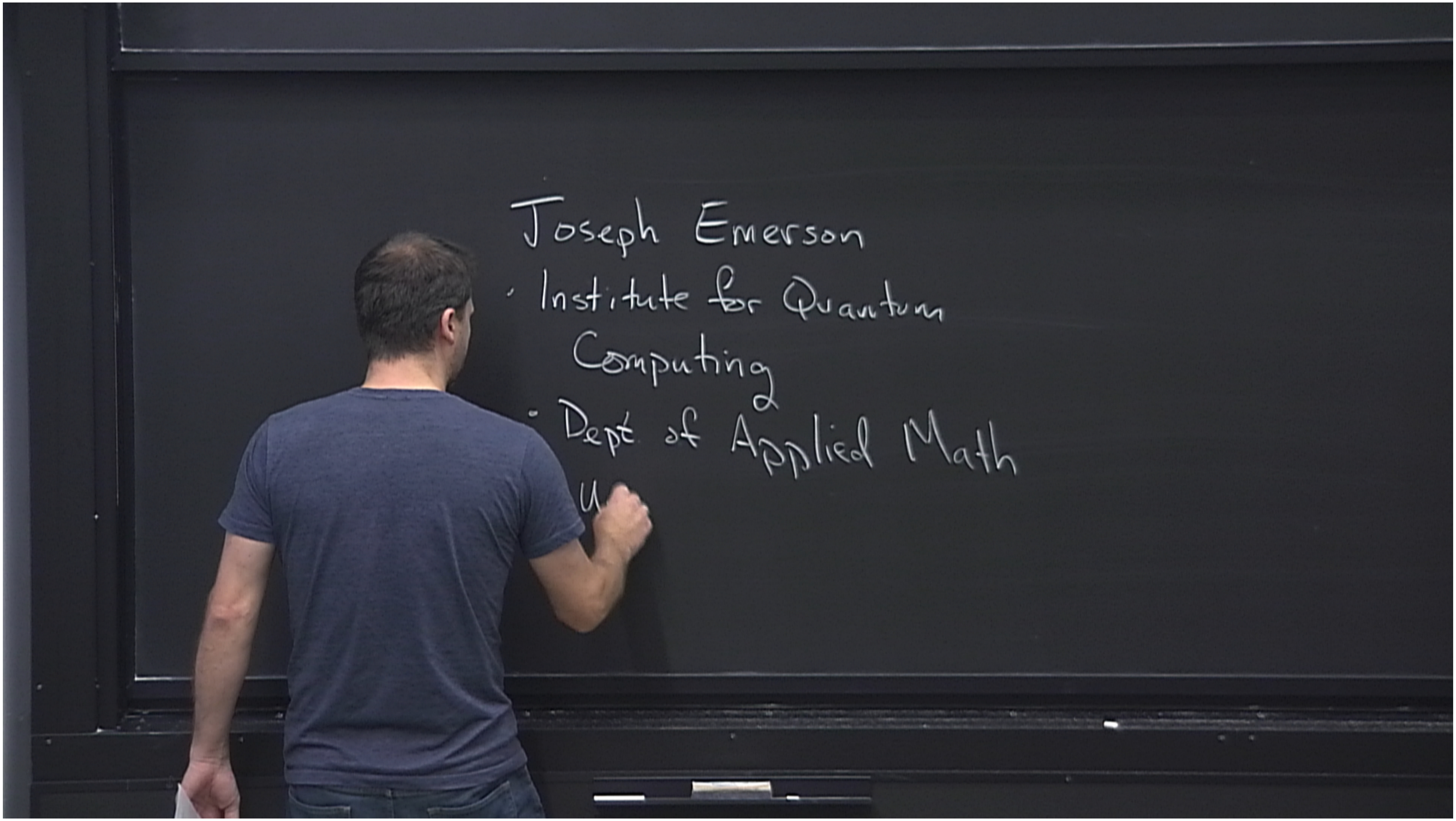
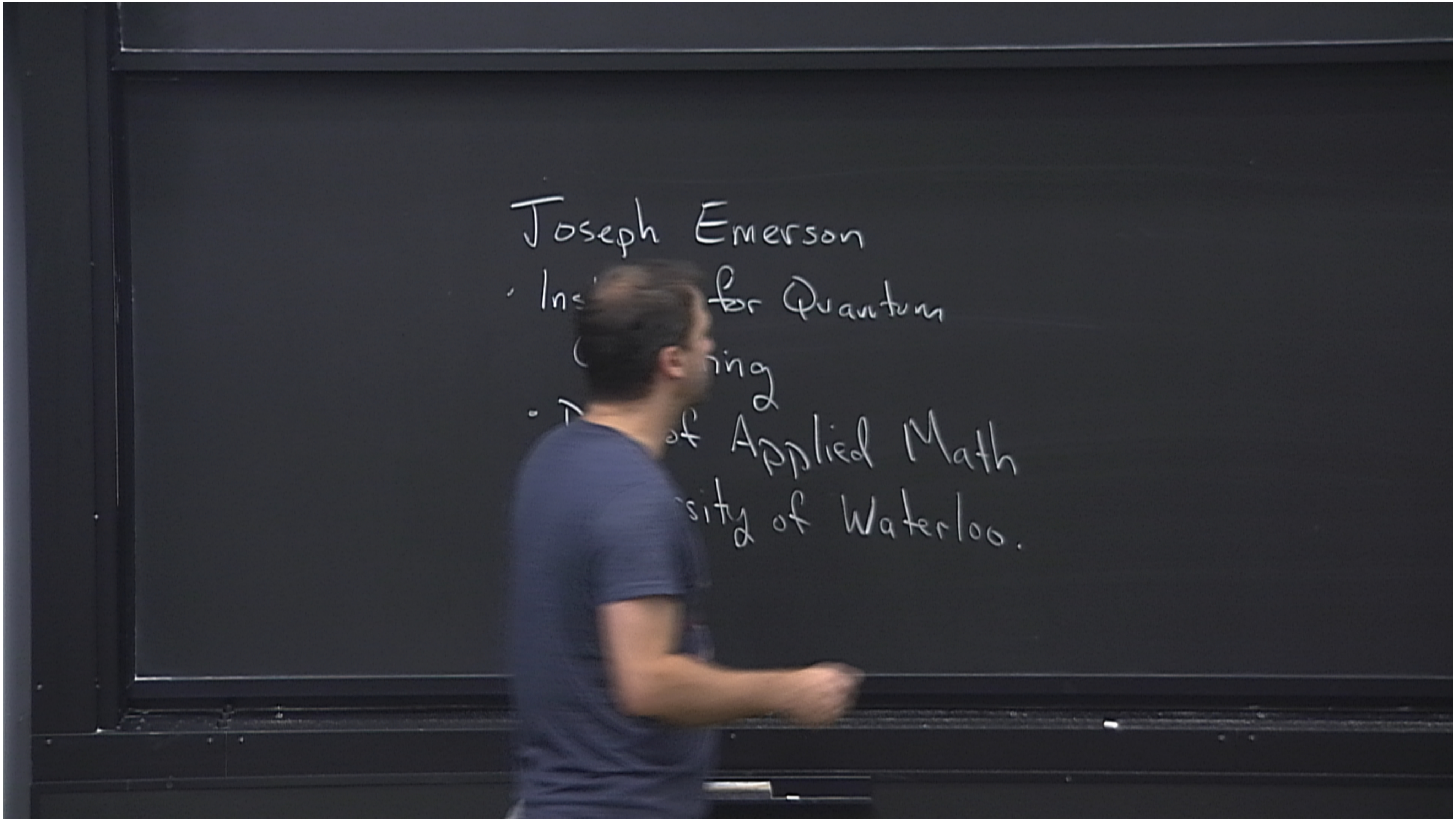
Title: 14/15 PSI - Quantum Theory-1

Date: Sep 08, 2014 10:45 AM

URL: http://pirsa.org/14090026

Abstract:


Joseph Emerson
· Institute for Quantum Computing
· Dept of Applied Math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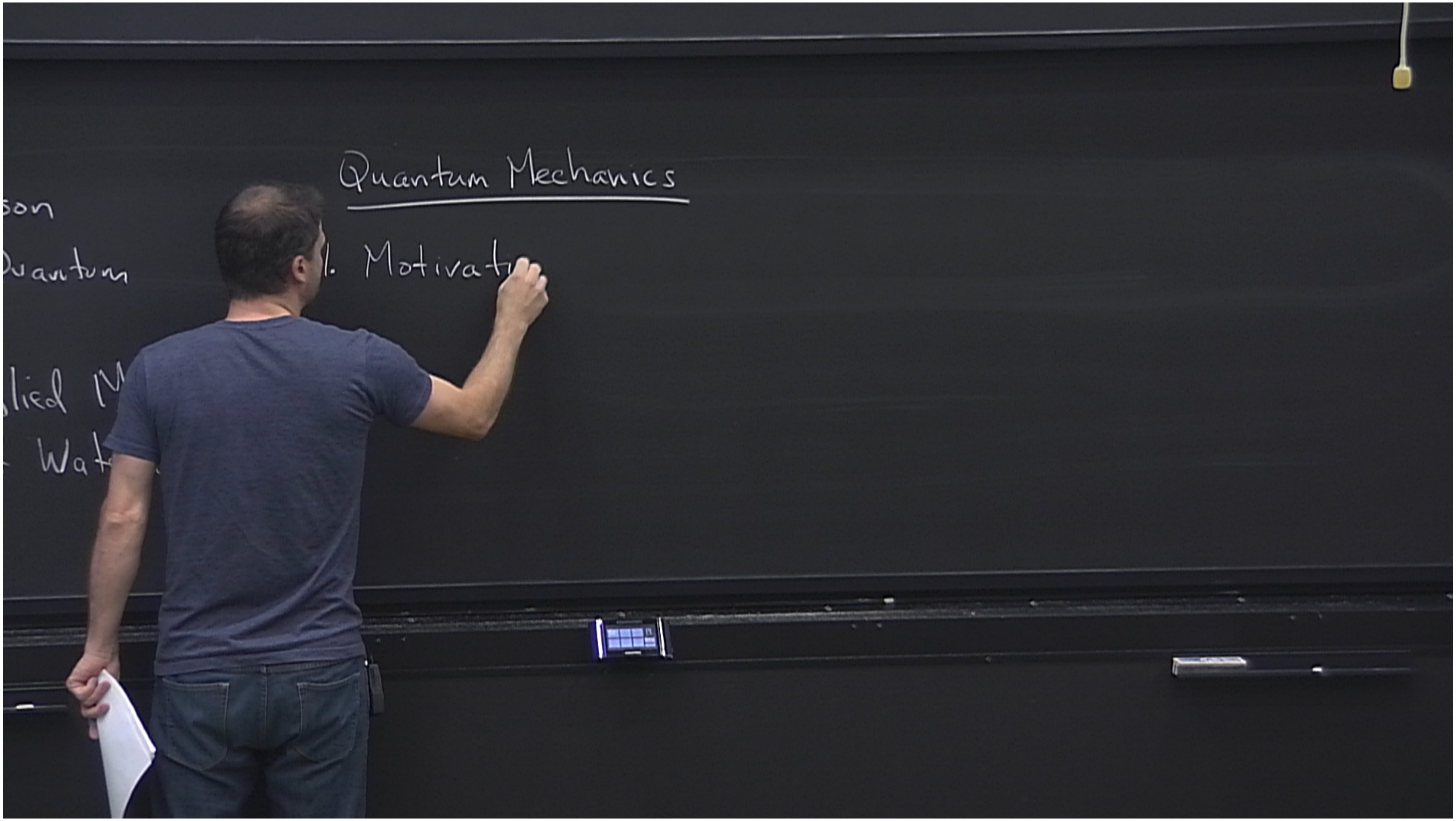
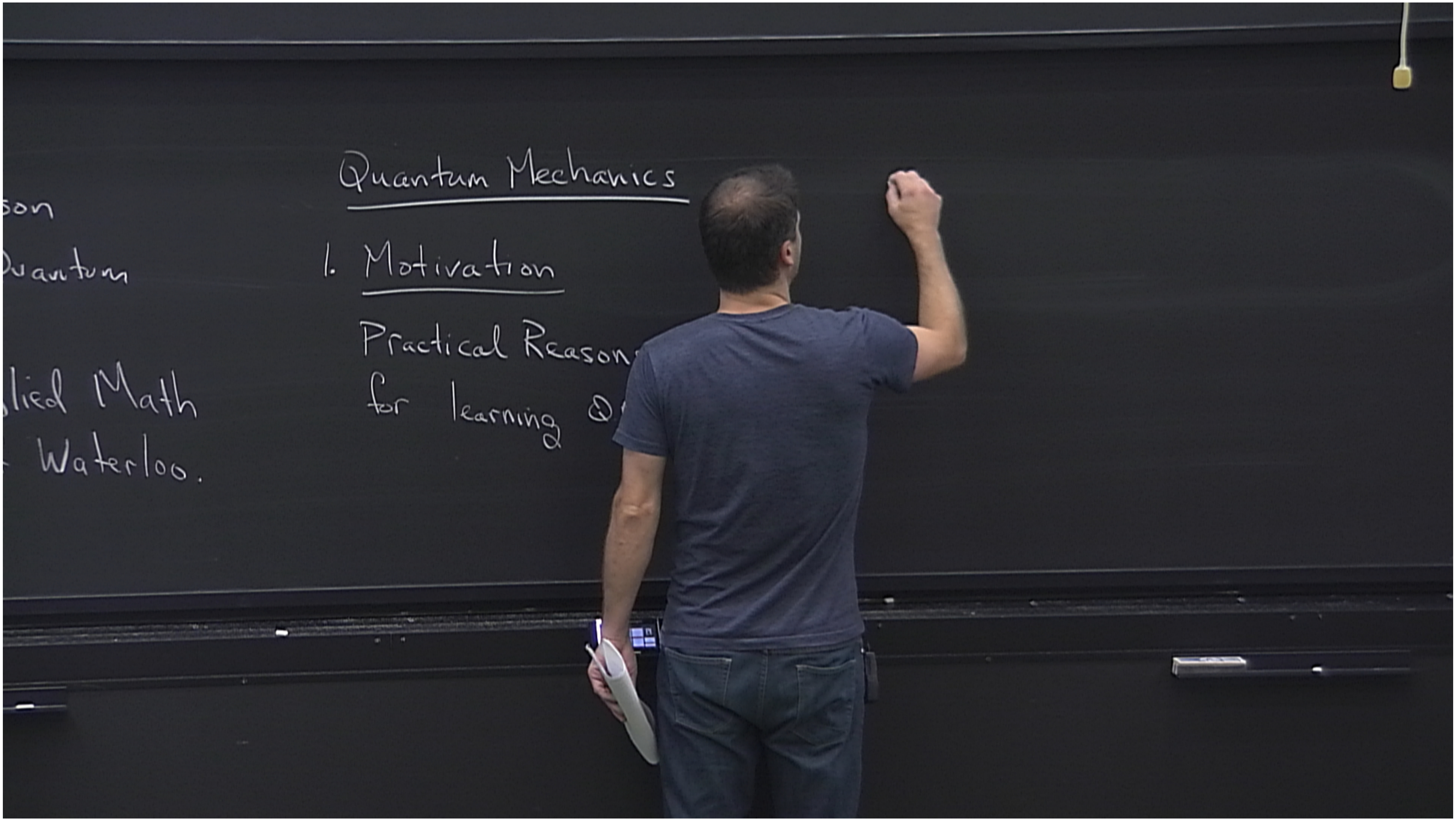

Quantum Mechanics


Quantum Mechanics
1. Motivation
Practical Reason
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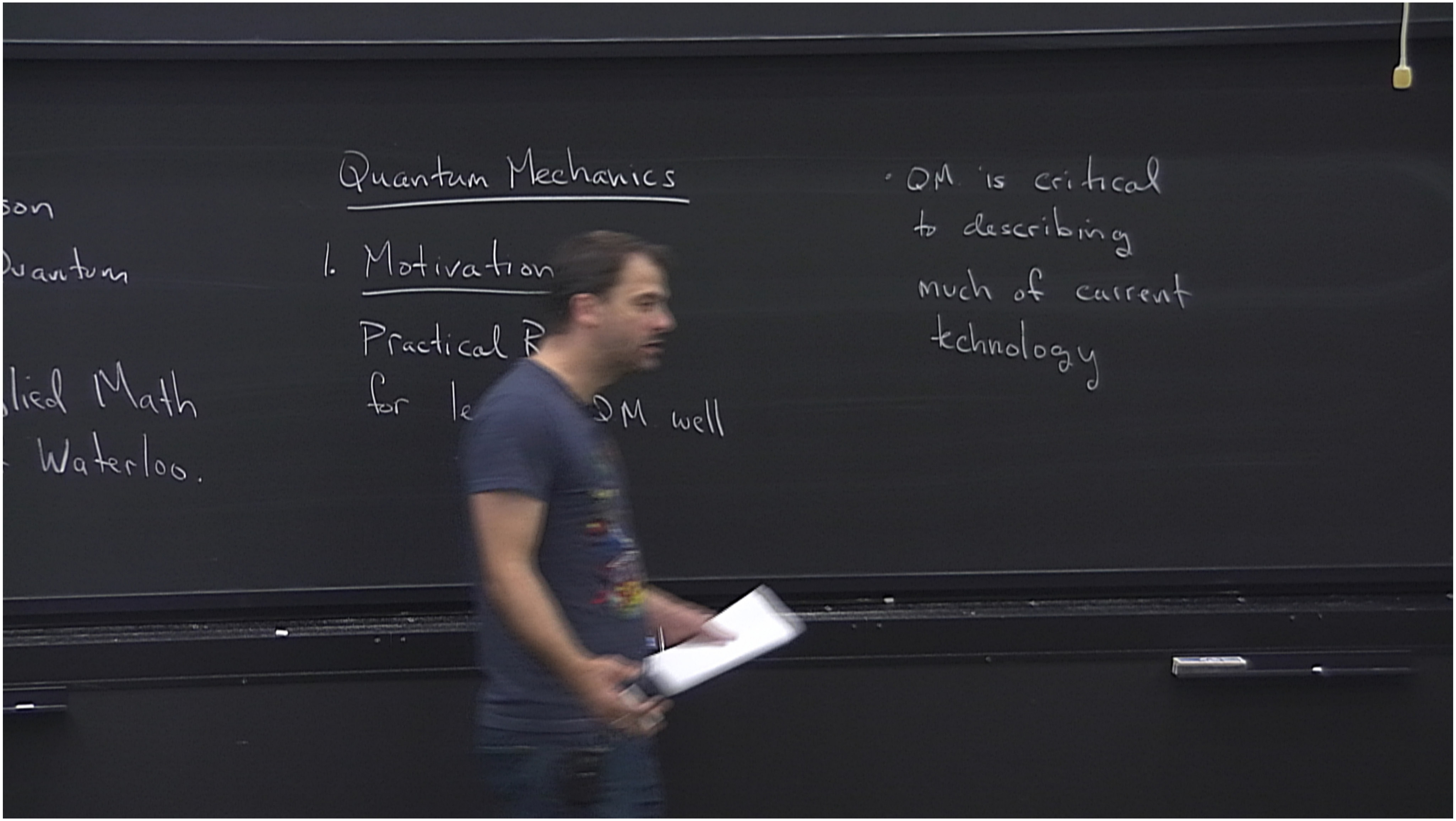

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Quantum Mechanics
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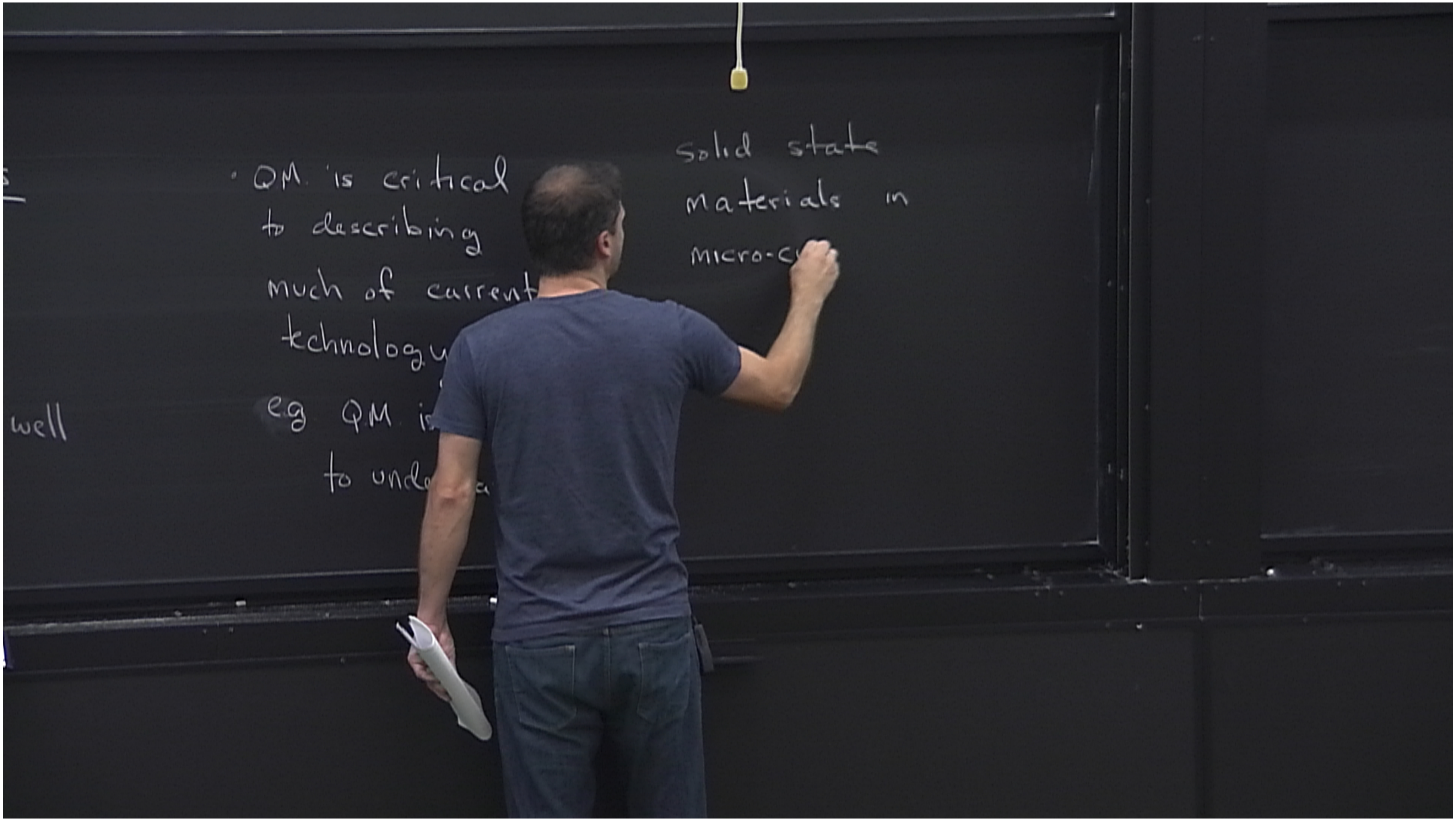
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eg QM
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solid state
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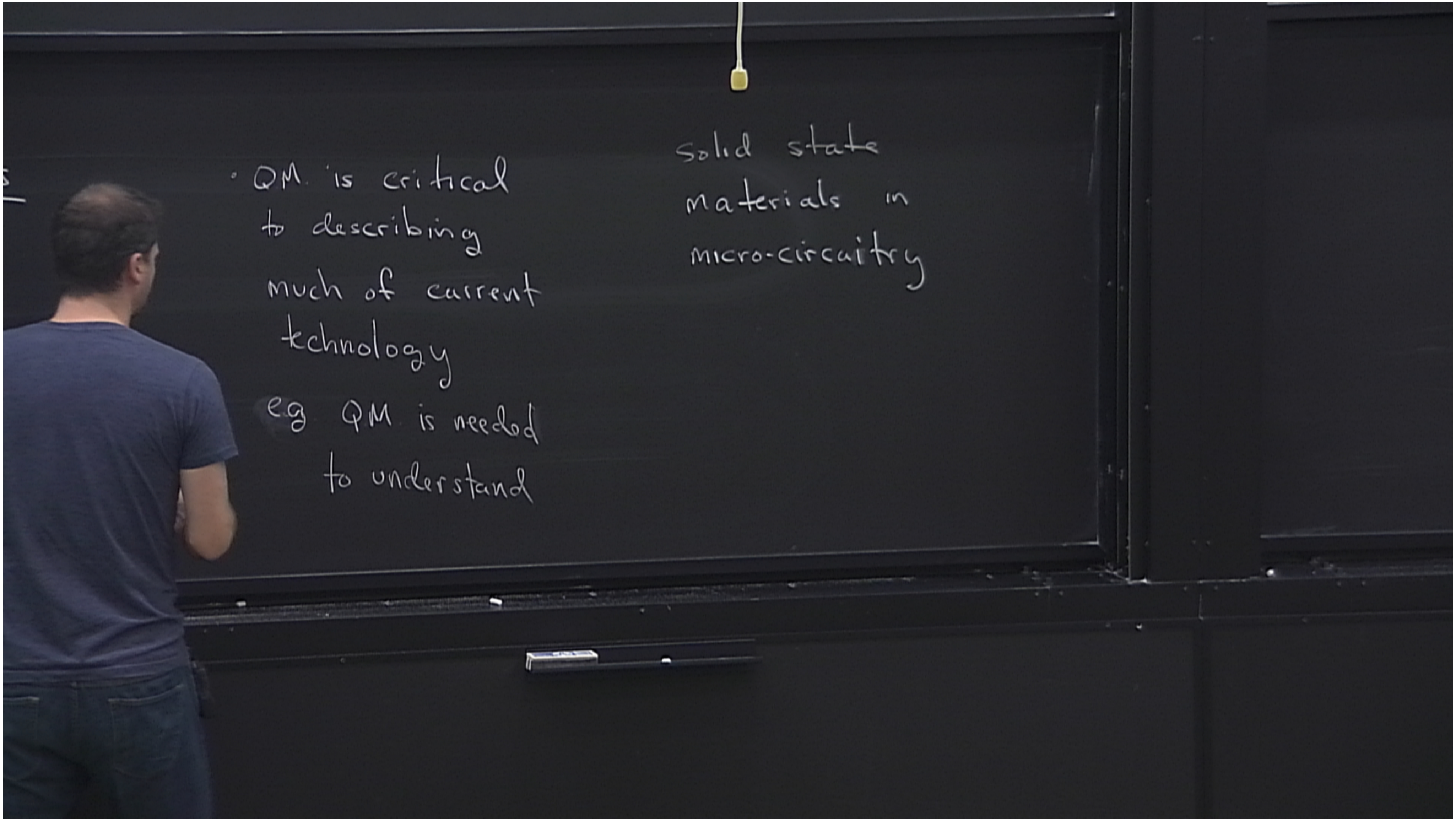
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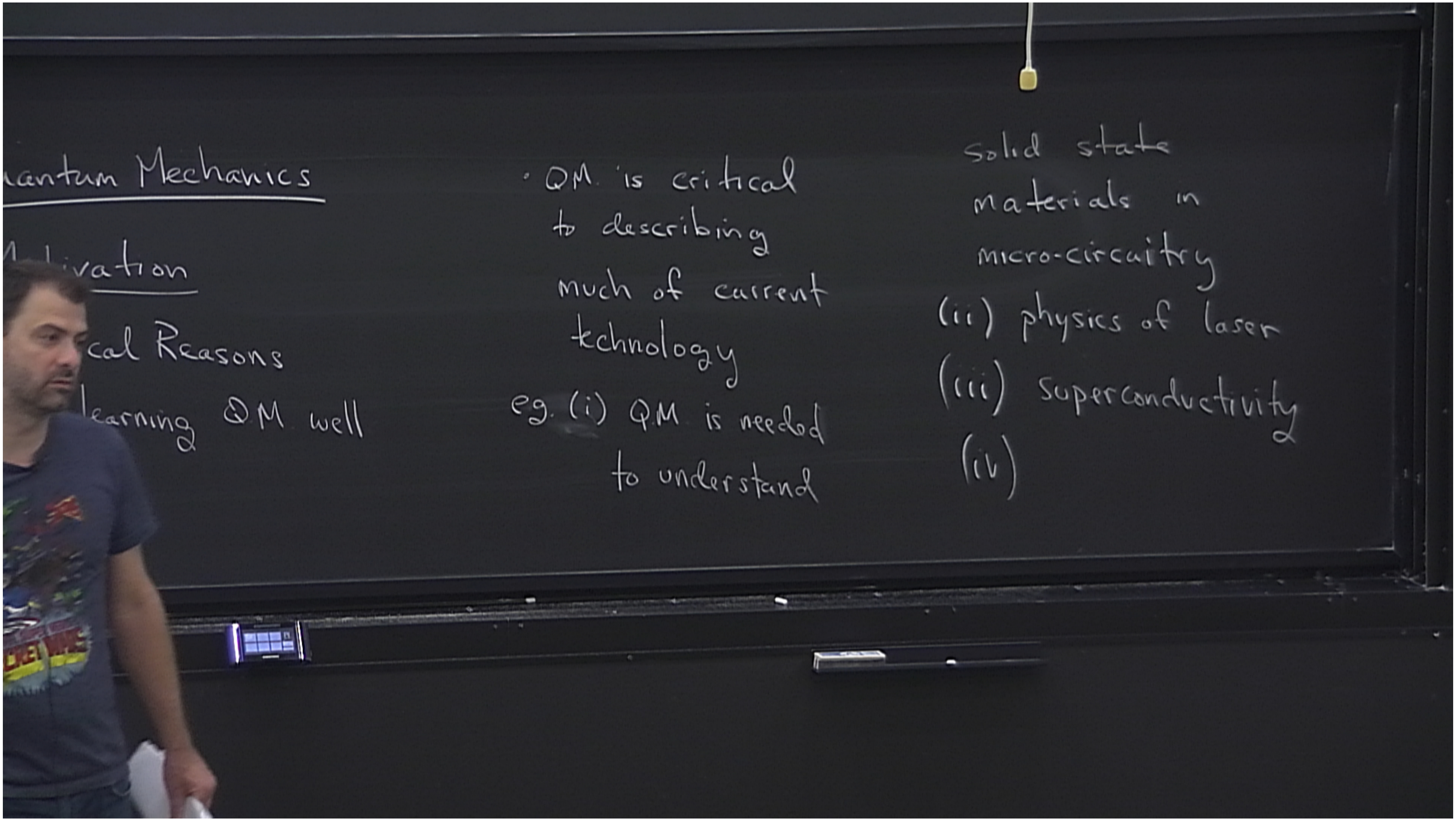
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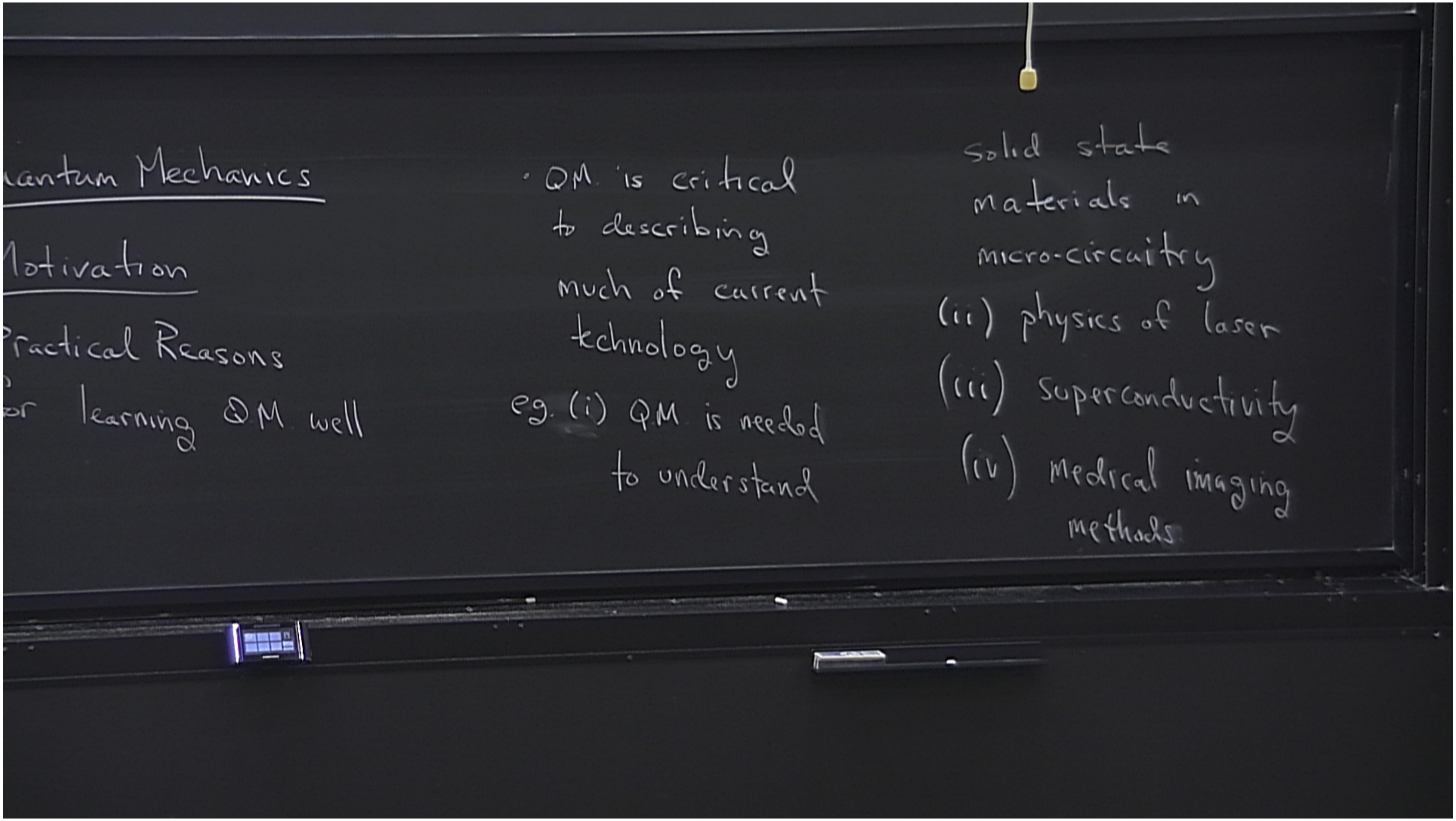
materials in
micro-circuitry
(ii) physics of laser
(iii) superconductivity
(iv)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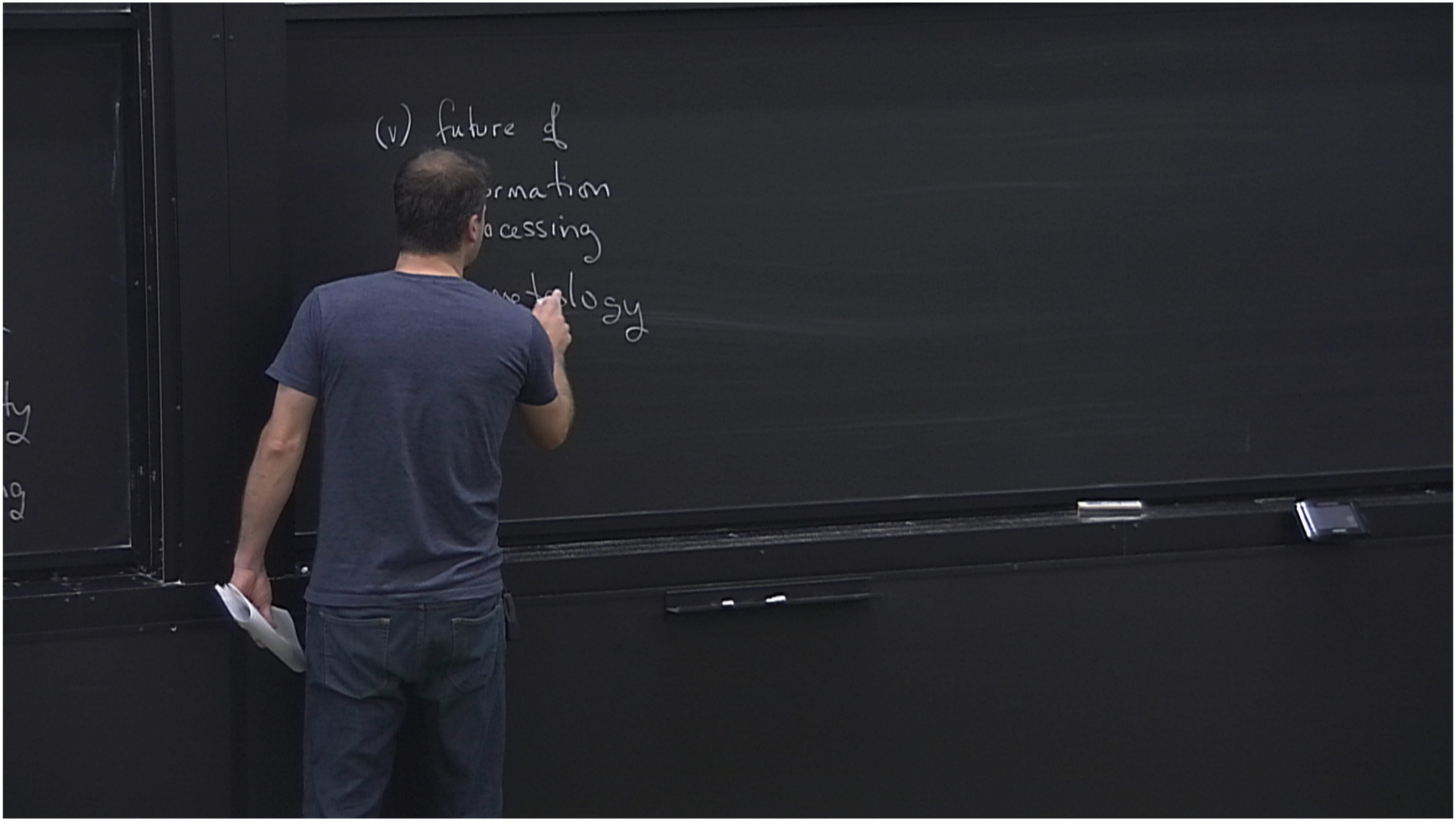
(iii) superconductivity
(iv) medical imaging methods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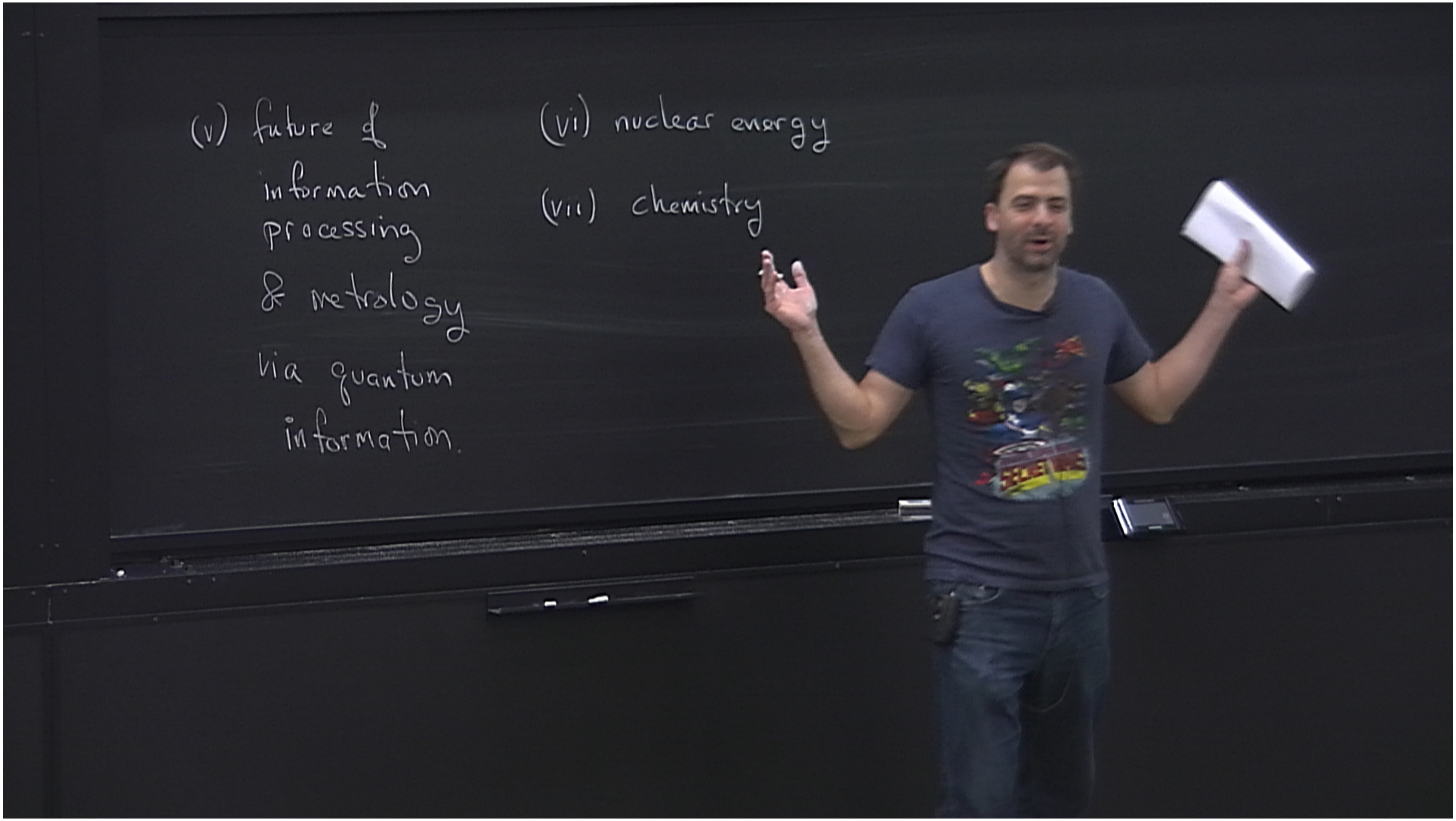
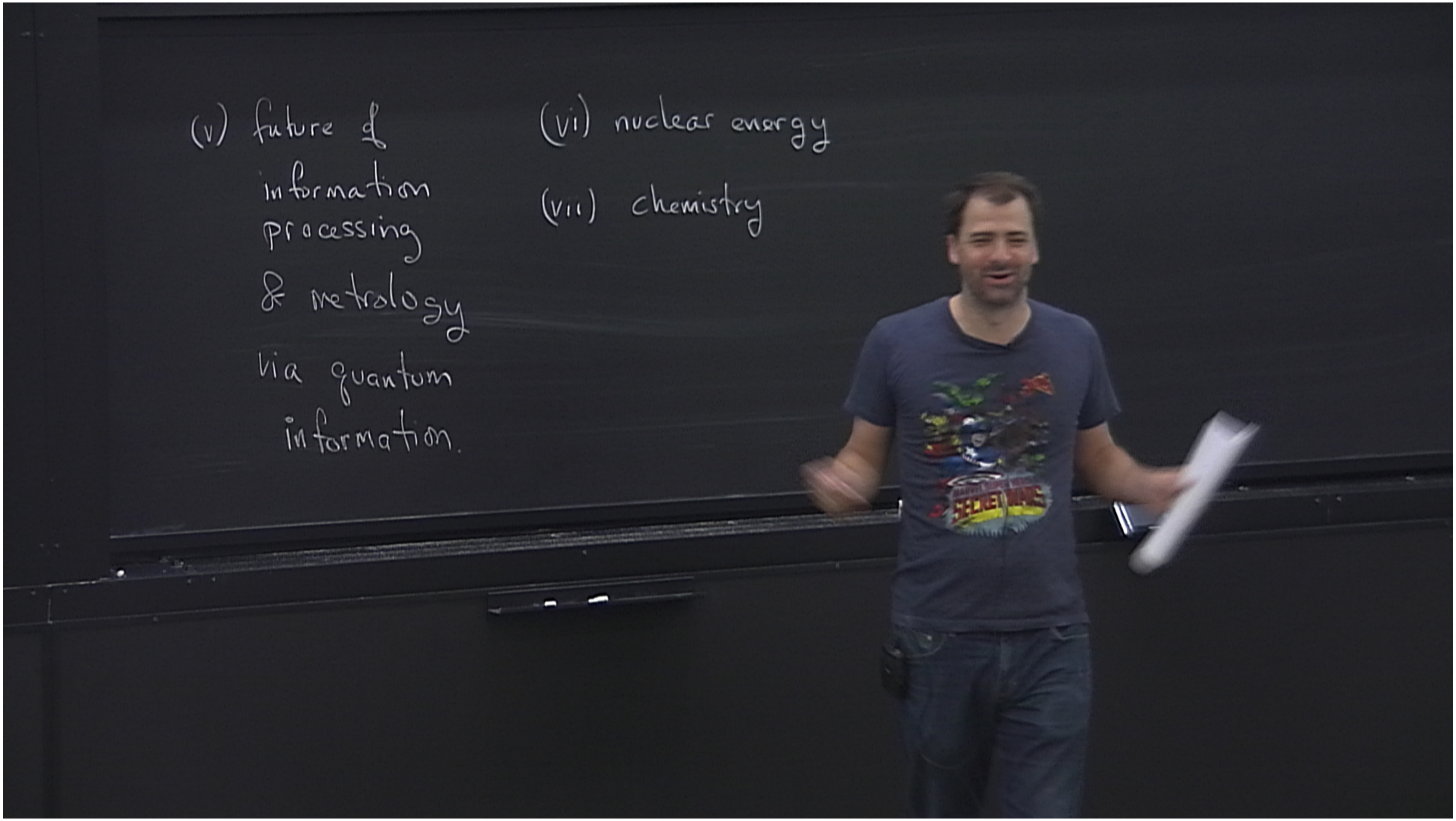

(v) future of information processing & metrology via quantum information.
(vi) nuclear energy
(vii) chemistry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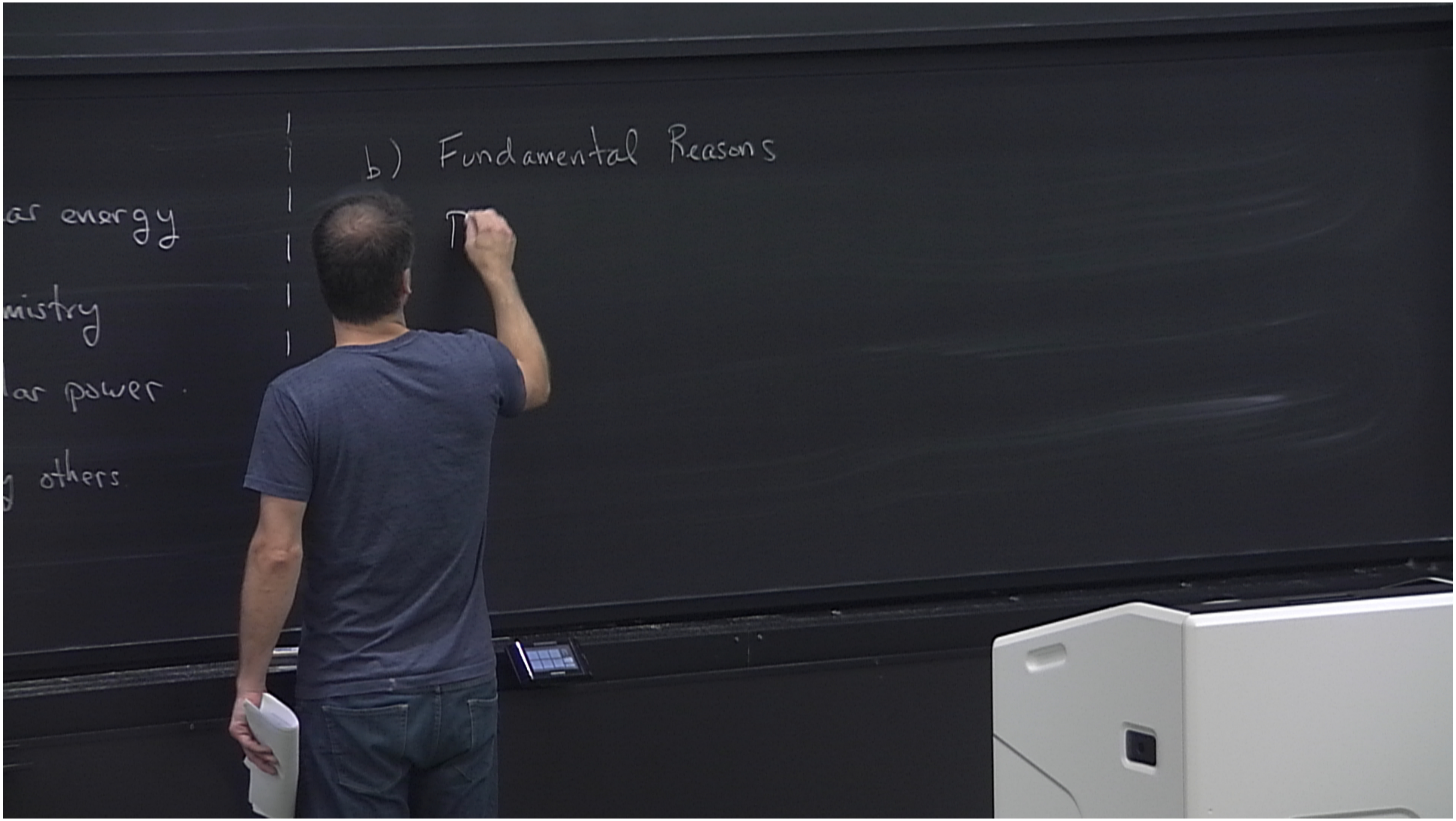
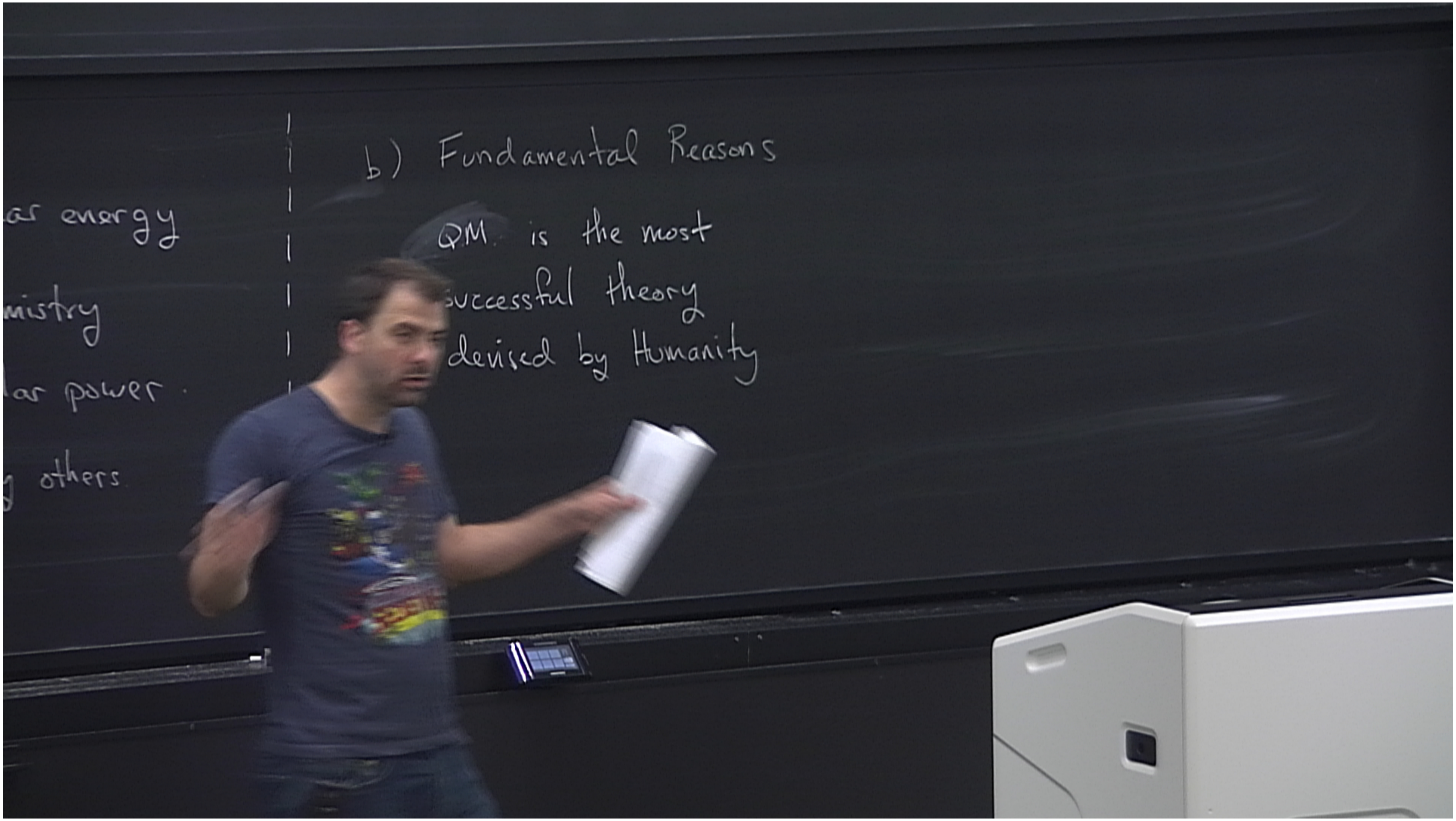

b) Fundamental Reasons
others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QM is the most
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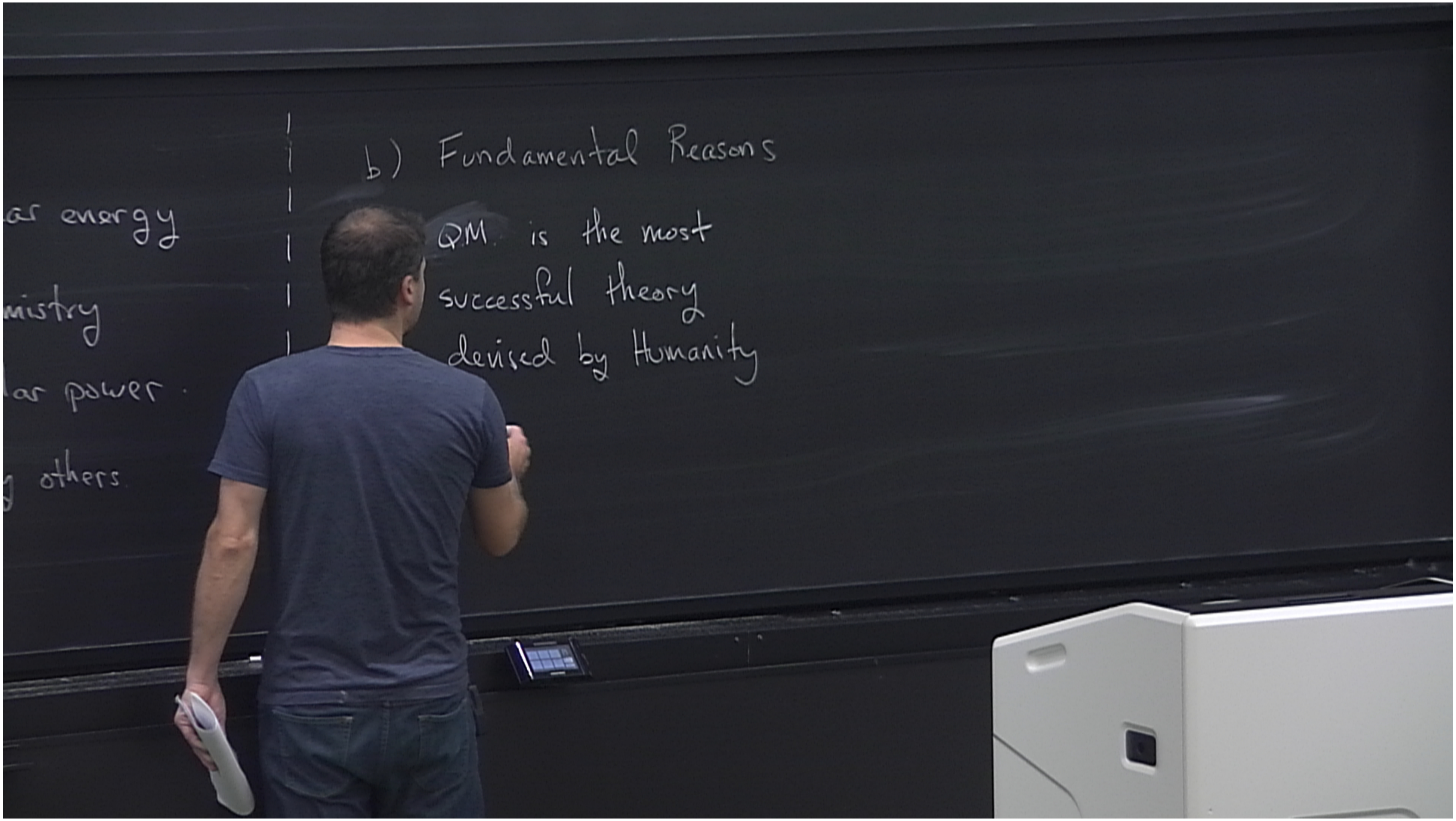
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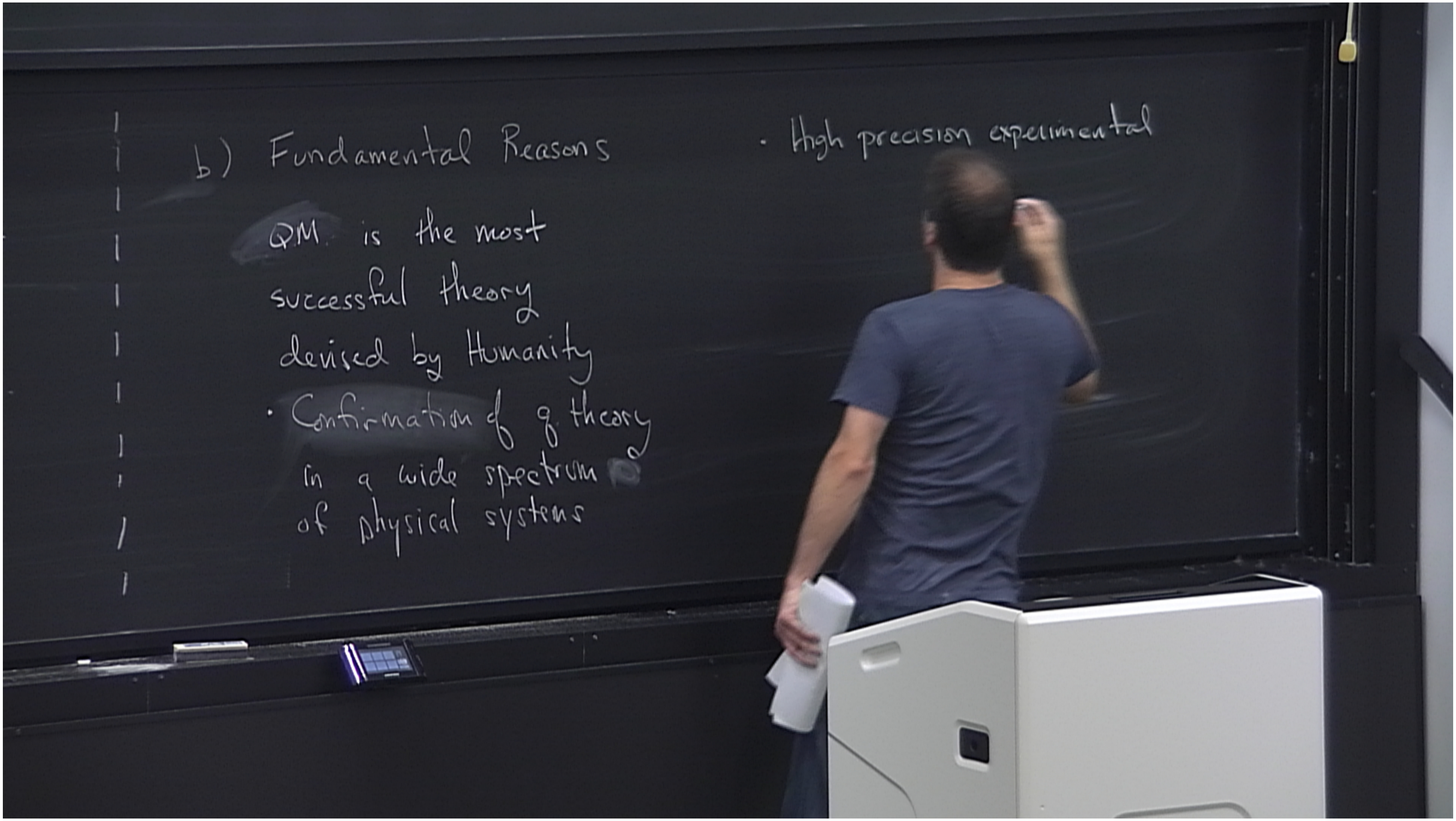
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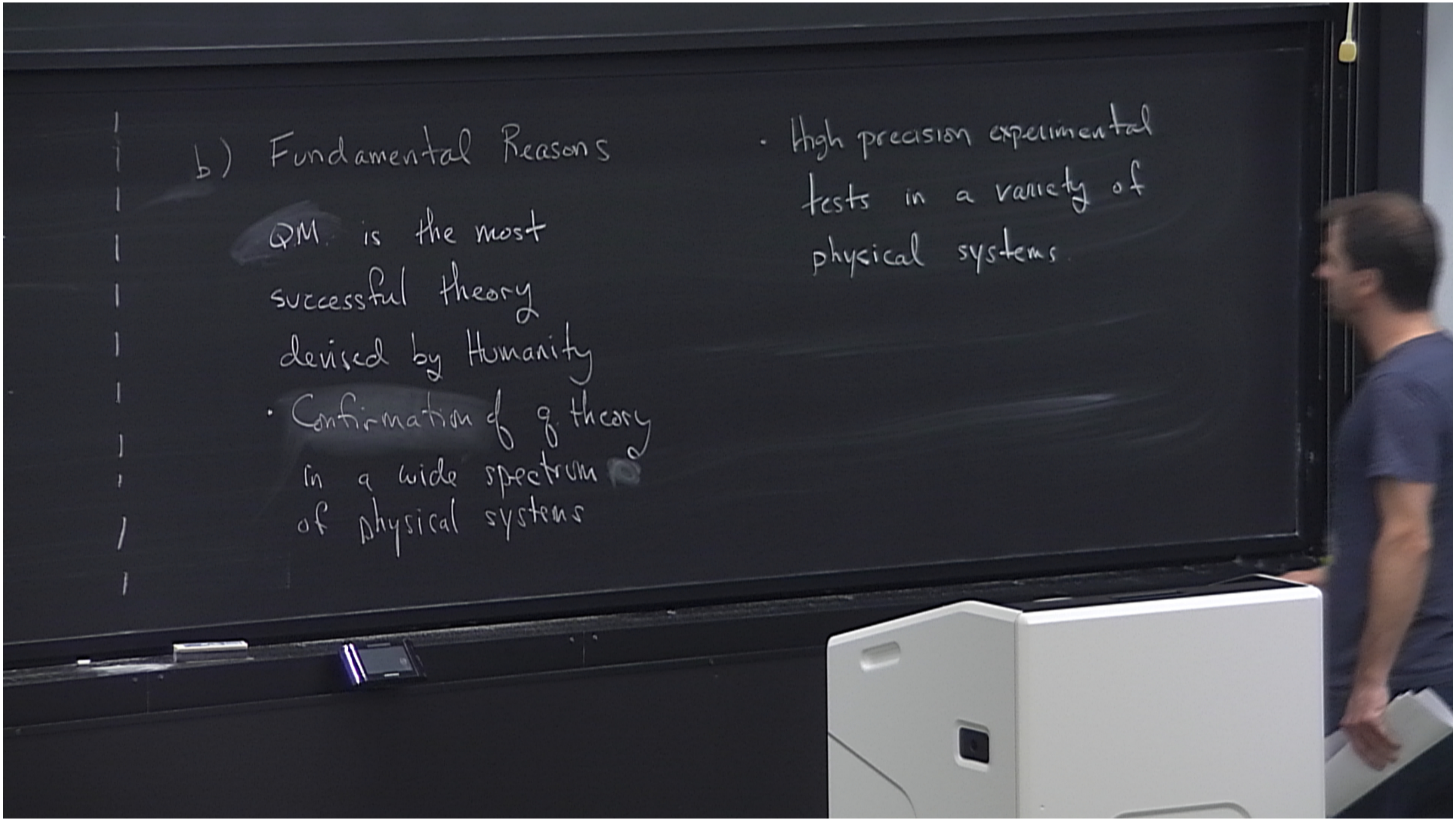
· High precision experimental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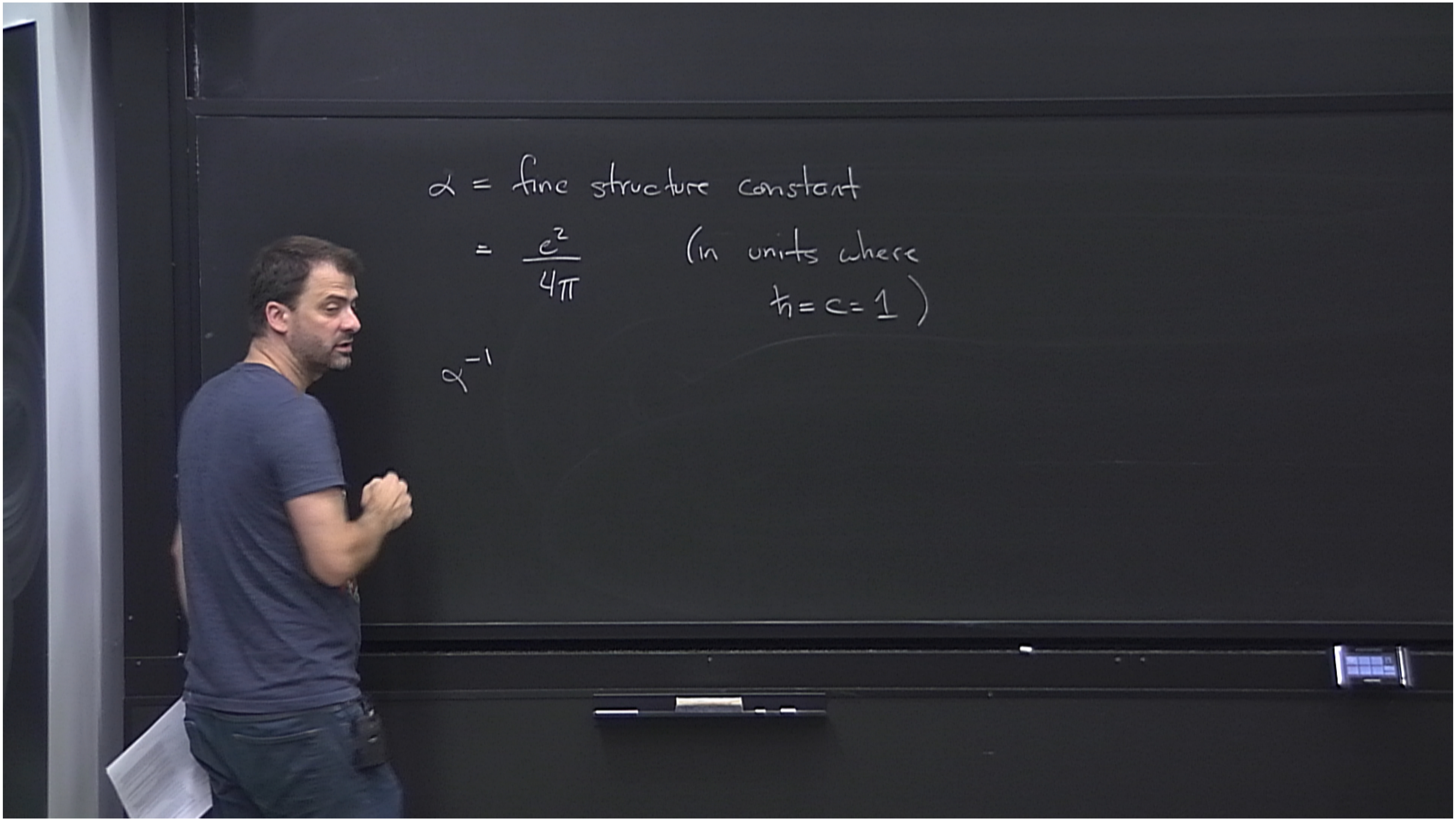
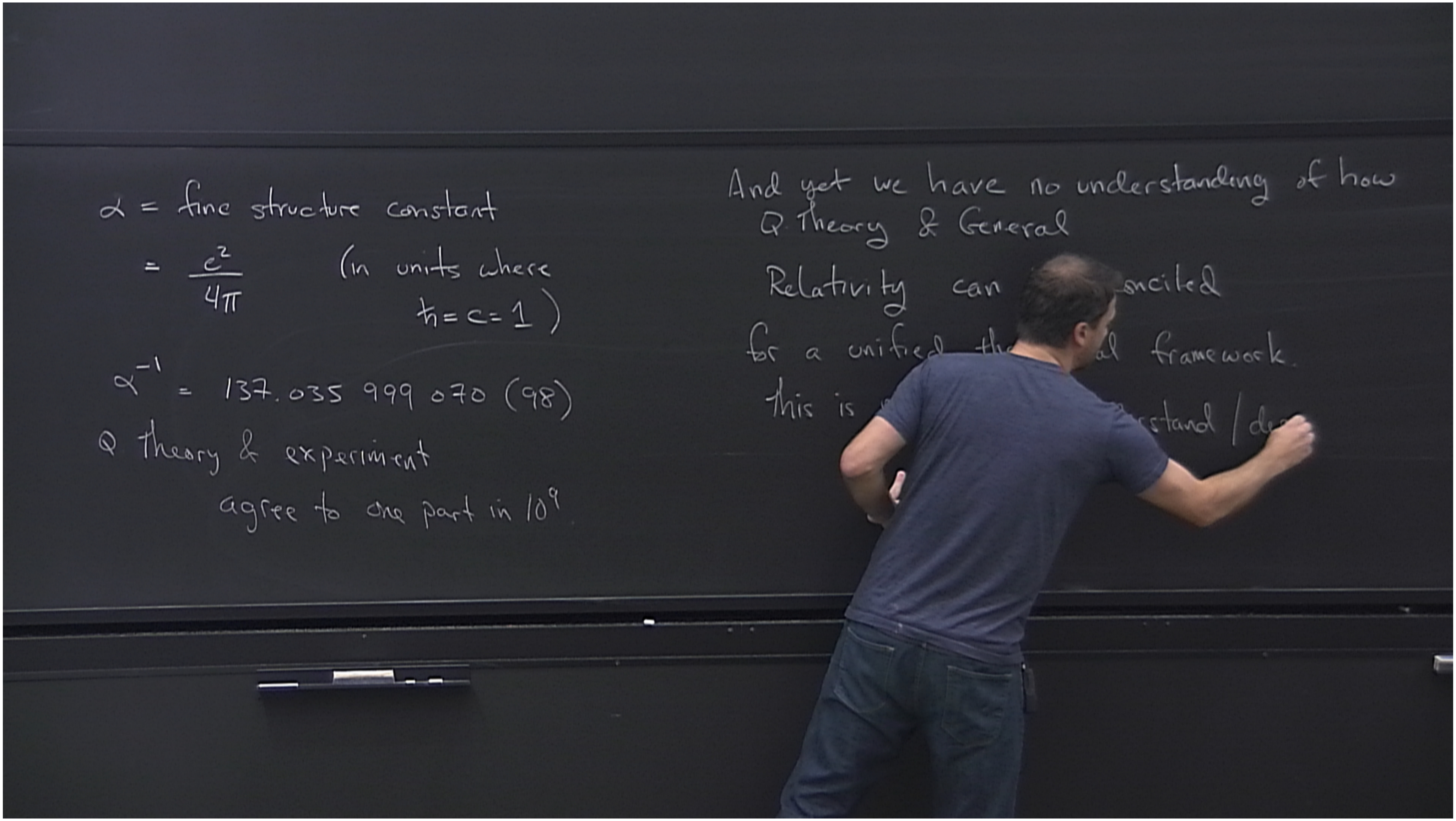
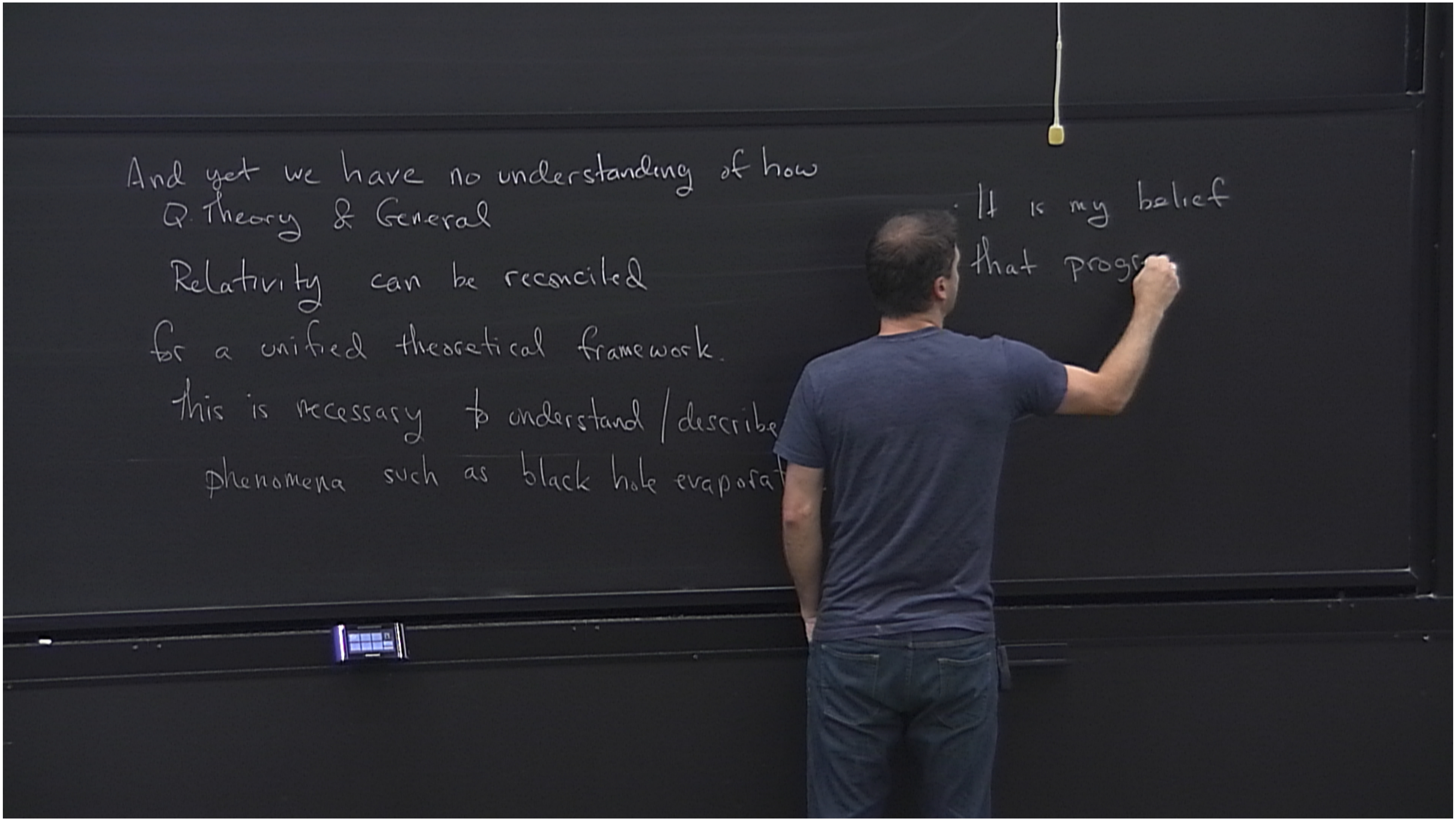
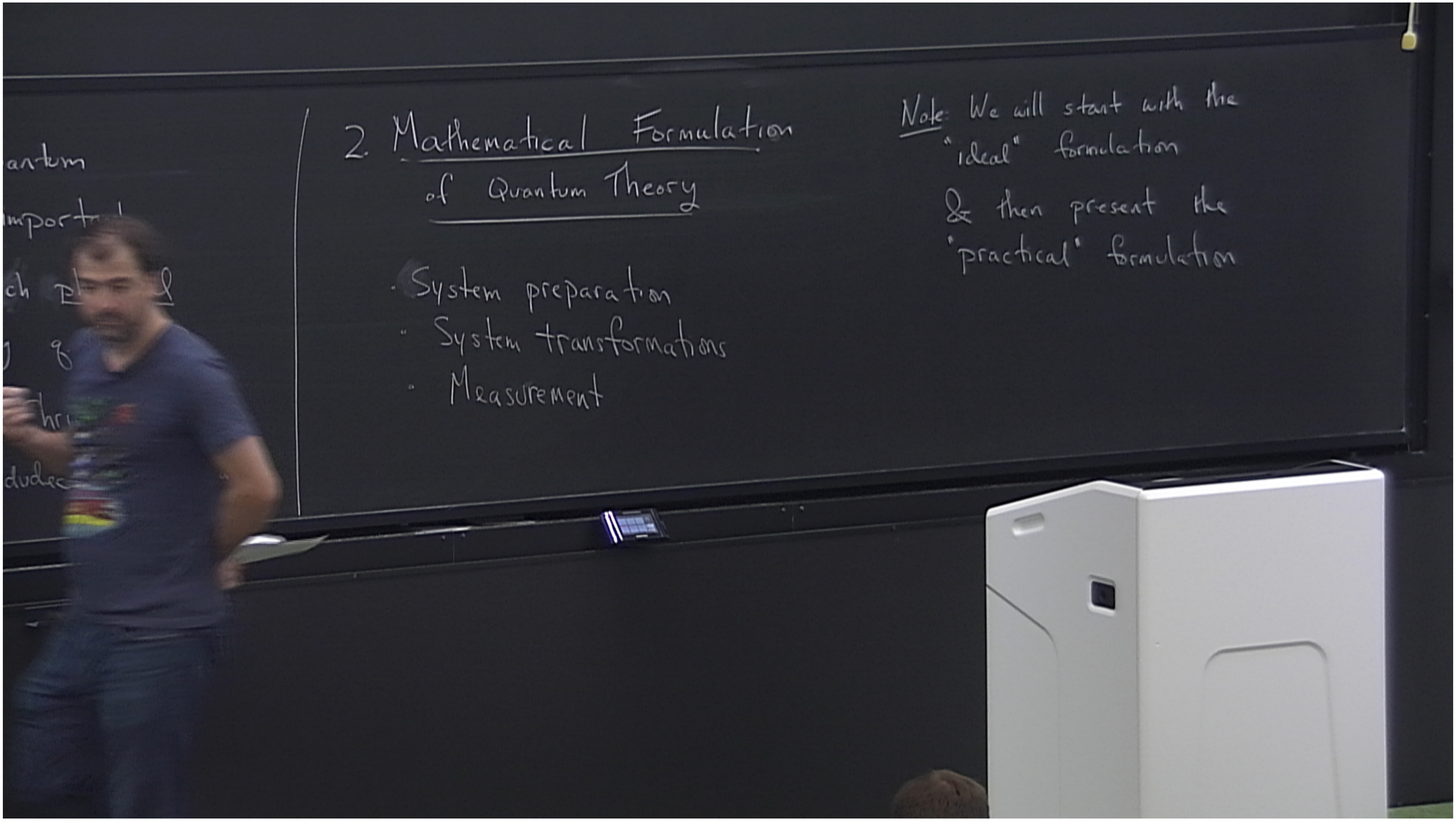

α = fine structure constant
$= \frac{e^2}{4\pi}$ (in units where $\hbar = c = 1$)
α^{-1}

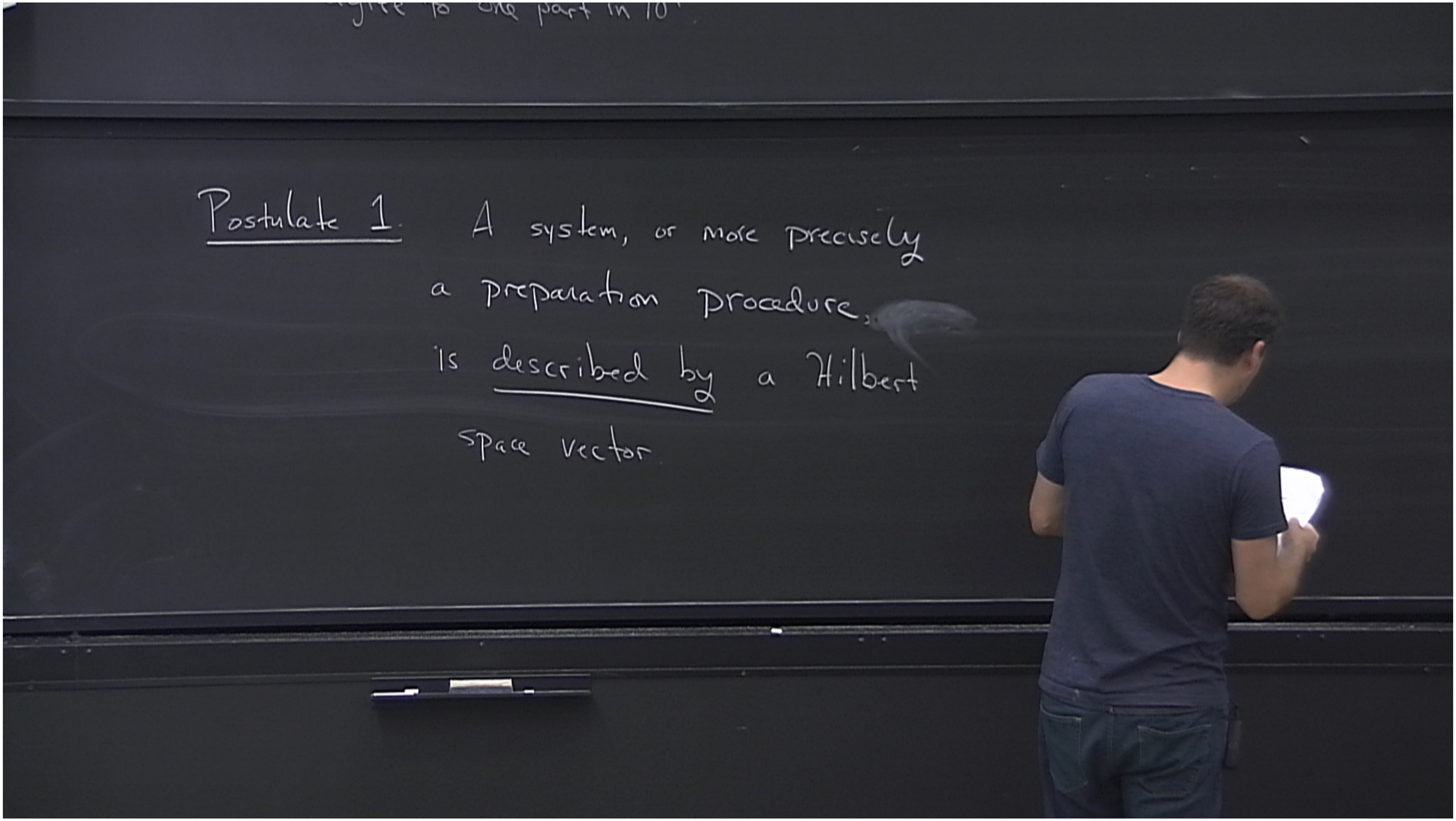

α = fine structure constant
$= \frac{e^2}{4\pi}$ (in units where $\hbar = c = 1$)
$\alpha^{-1} = 137.035\ 999\ 070\ (98)$
Q theory & experiment
agree to one part in 10^9.
And yet we have no understanding of how
Q. Theory & General
Relativity can
for a unified
framework.
this is

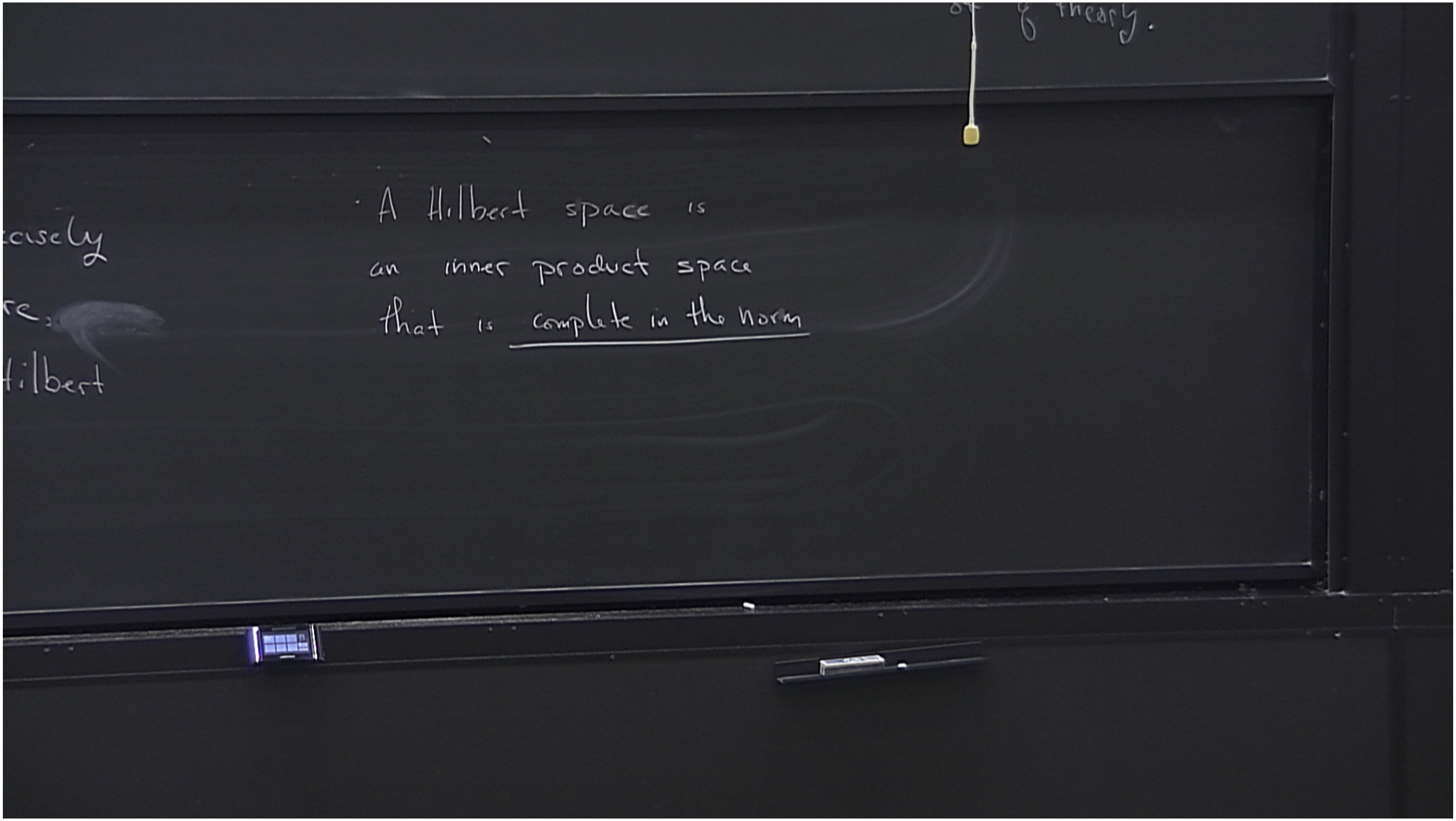

And yet we have no understanding of how
Q. Theory & General
Relativity can be reconciled
for a unified theoretical framework.
this is necessary to understand / describe
phenomena such as black hole evaporat
It is my belief
that progr

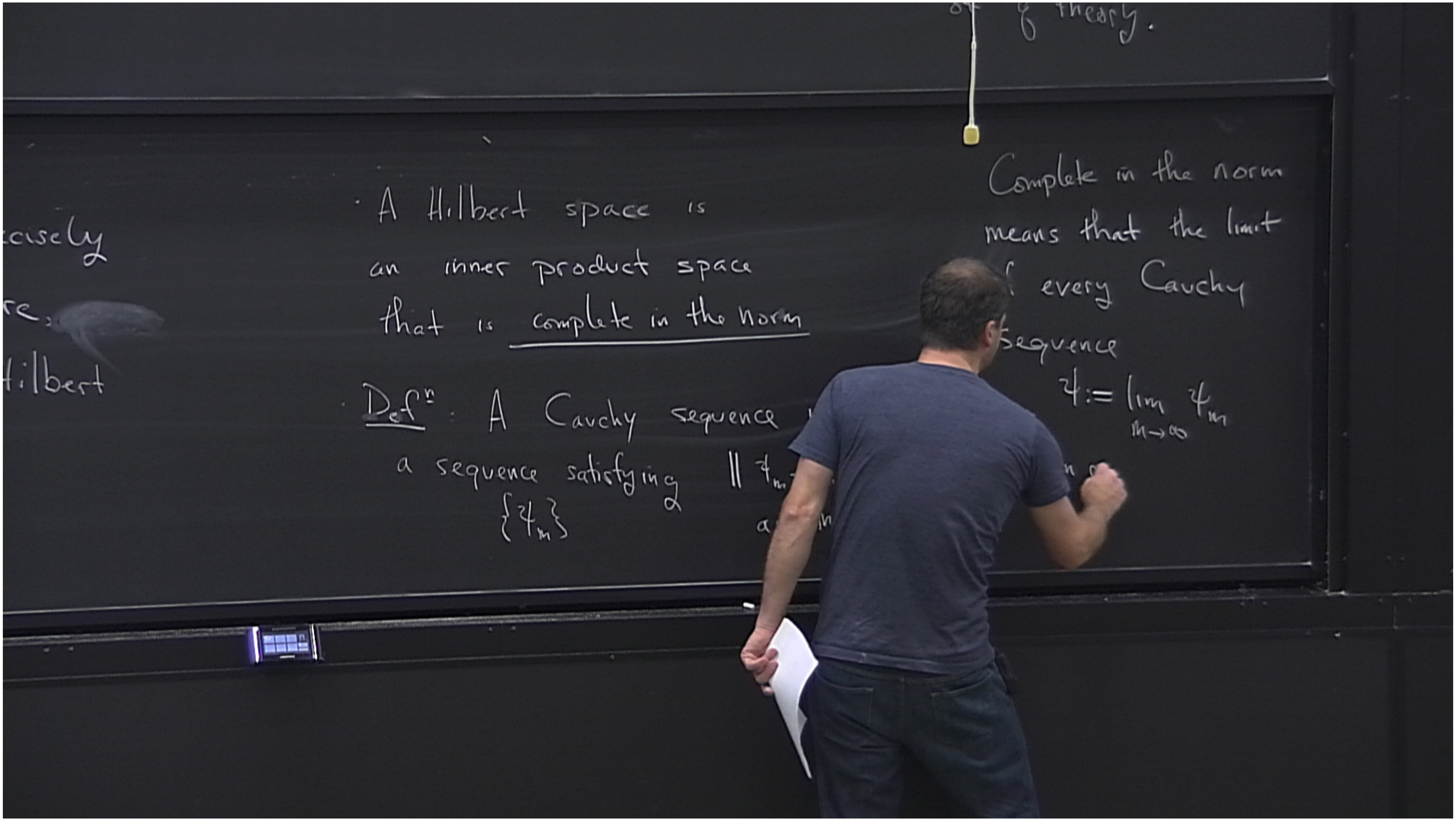
· This is why quantum foundations is important.

For example, which physical principles imply q theory.

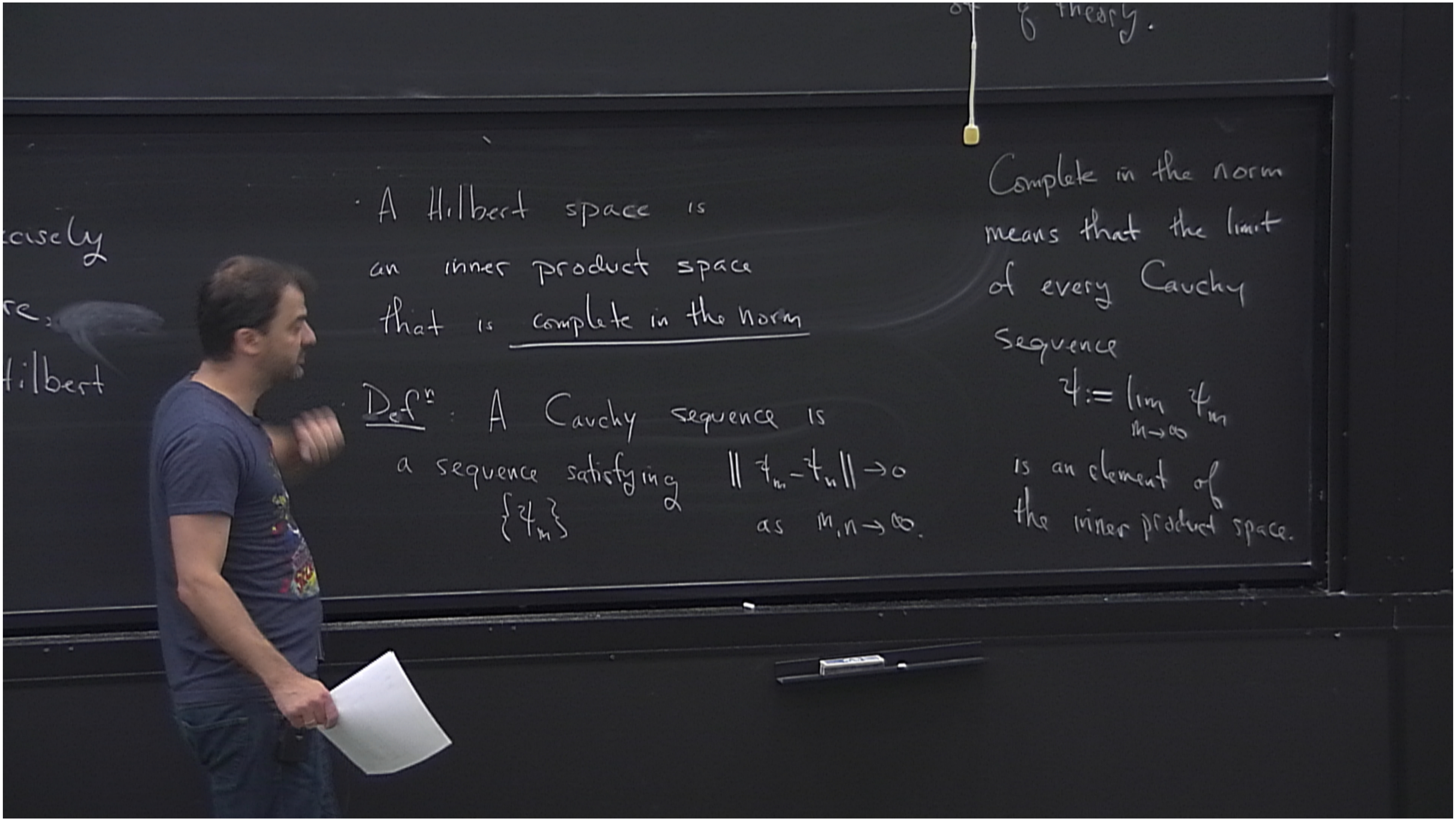

2. Mathematical Formulation of Quantum Theory
· System preparation
· System transformations
· Measurement
Note: We will start with the "ideal" formulation
& then present the "practical" formulation

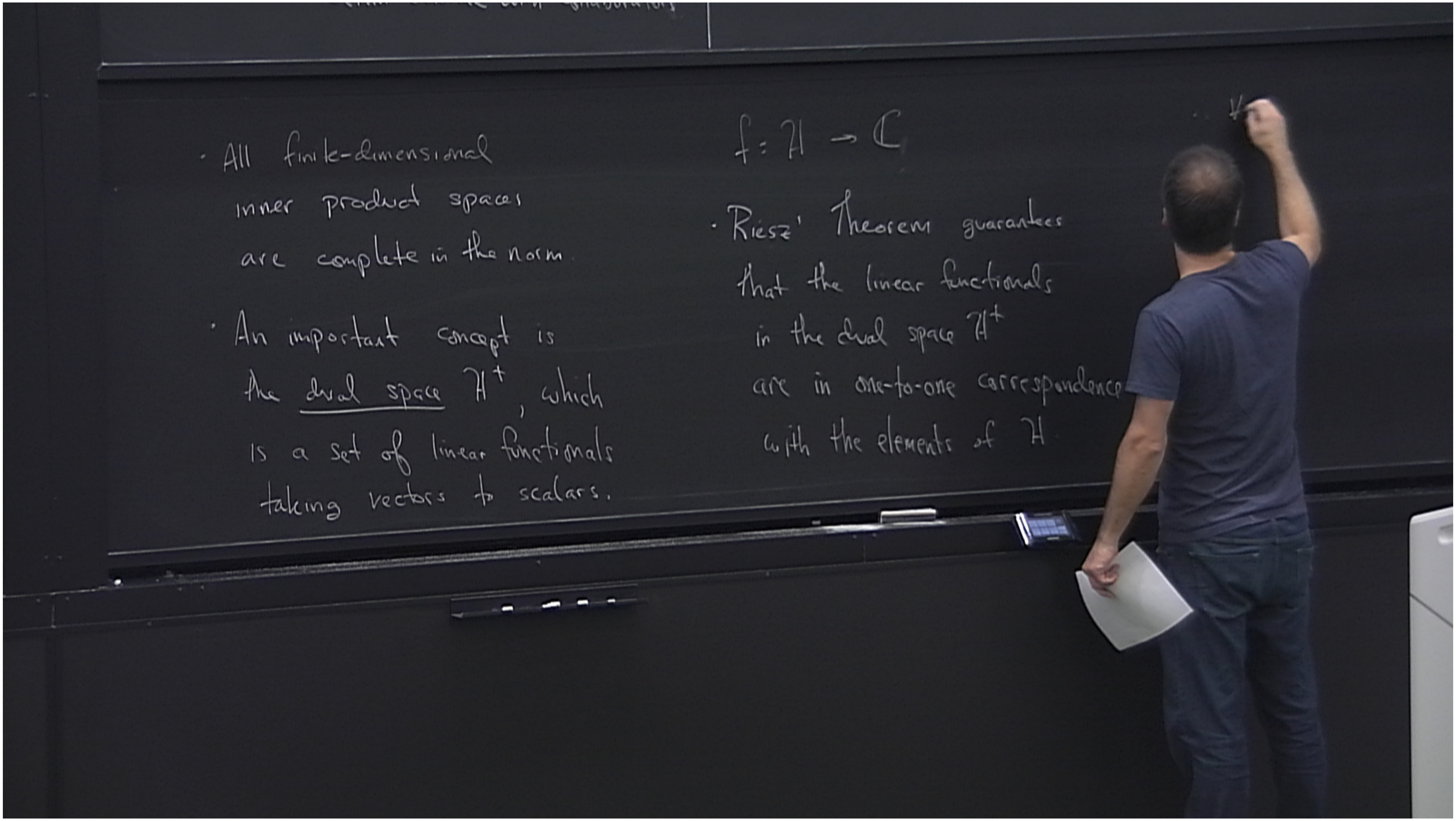

Postulate 1.
A system, or more precisely
a preparation procedure,
is described by a Hilbert
space vector.

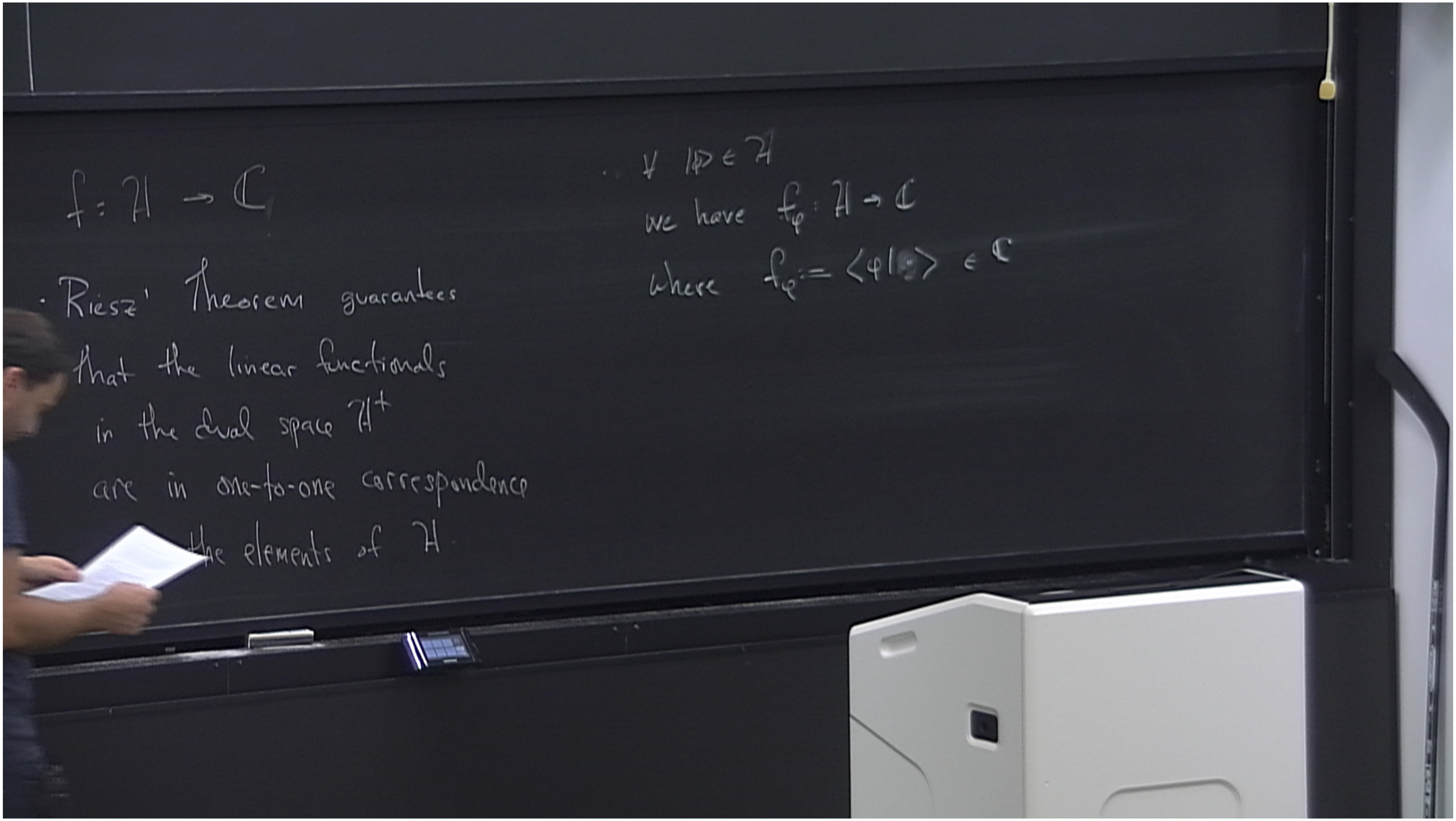

A Hilbert space is
an inner product space
that is complete in the norm

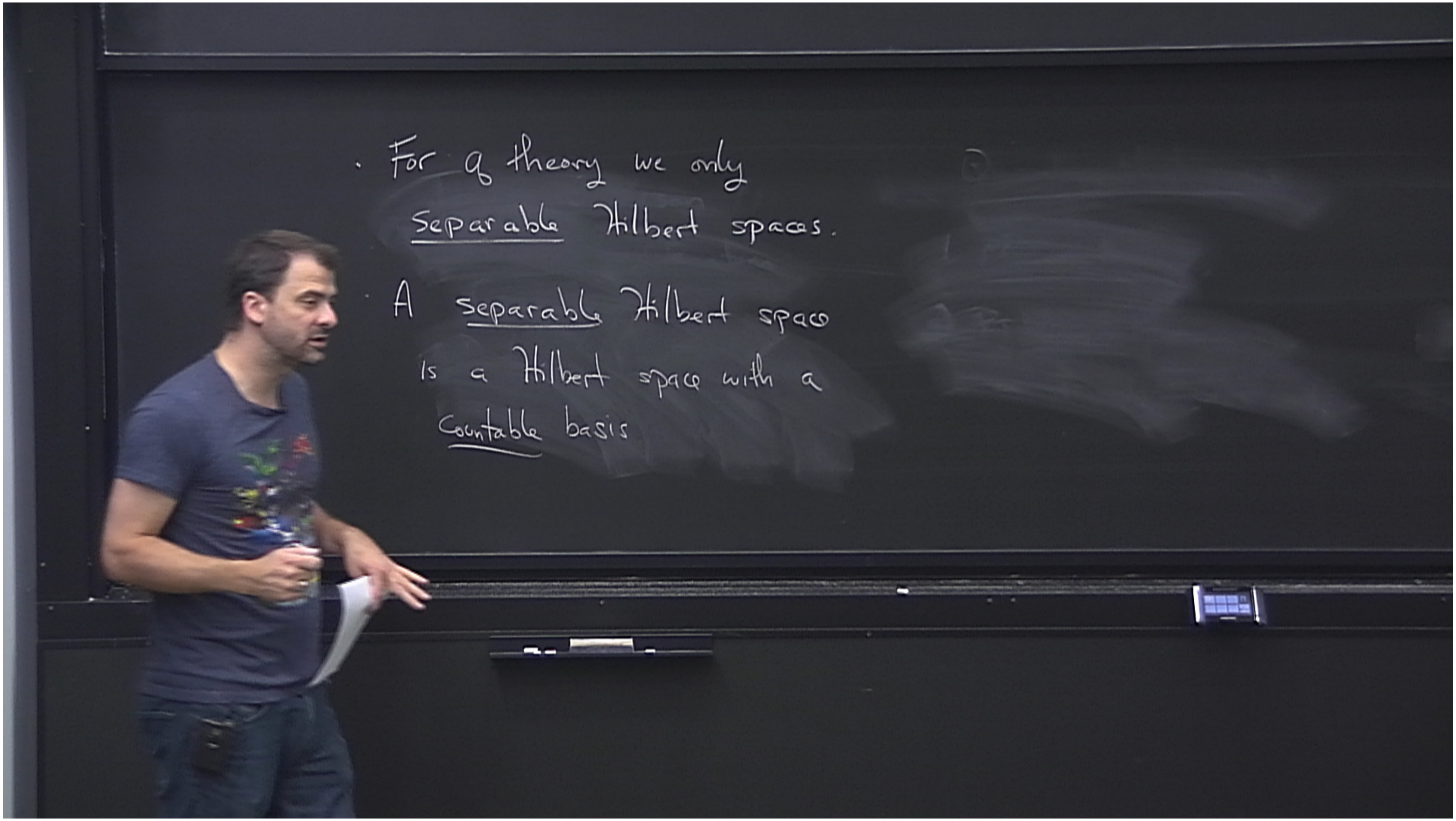


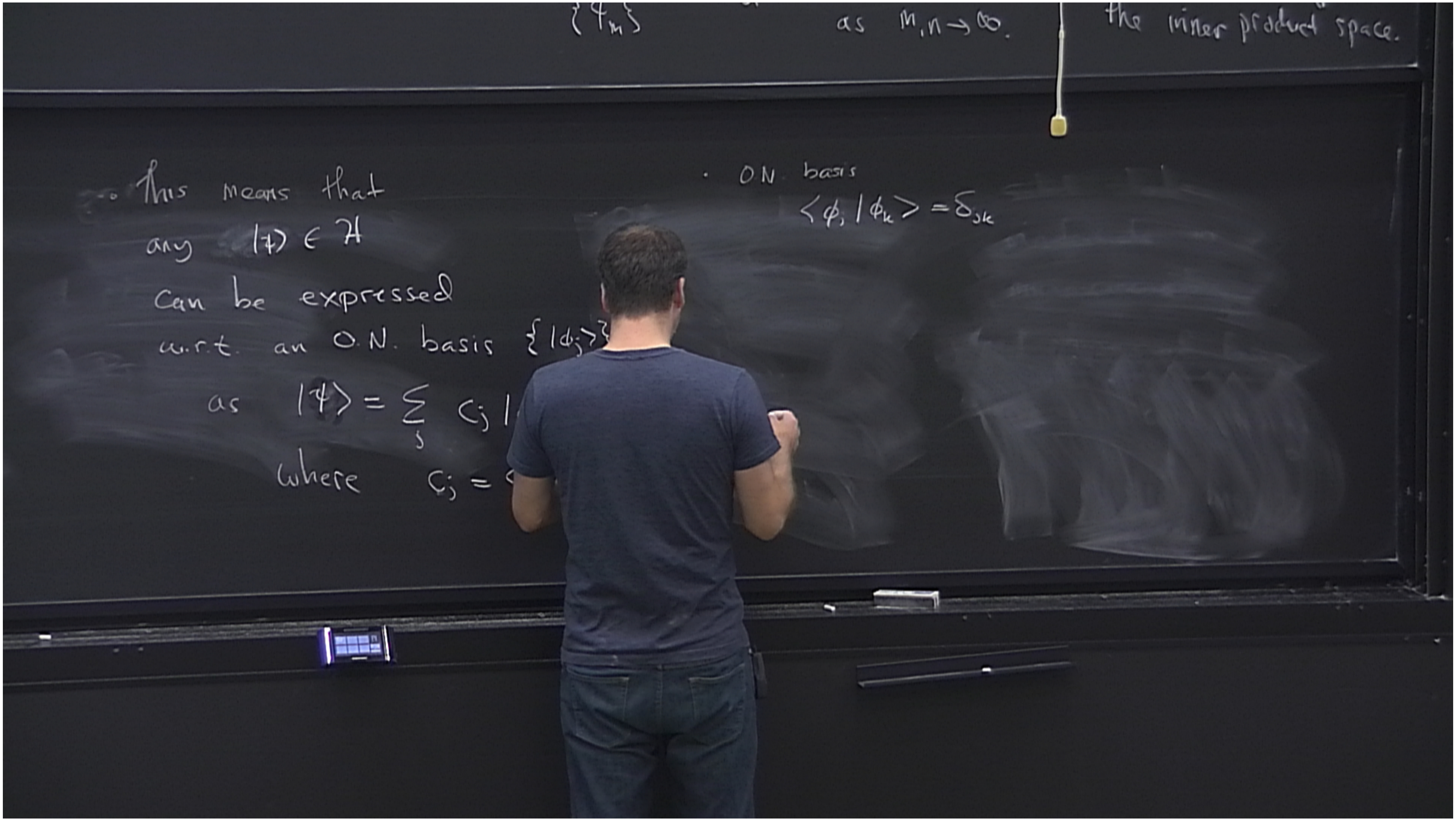
A Hilbert space is an inner product space that is complete in the norm
Defn: A Cauchy sequence
a sequence satisfying
$\{\psi_n\}$
Complete in the norm means that the limit of every Cauchy sequence
$\psi := \lim_{n\to\infty} \psi_n$

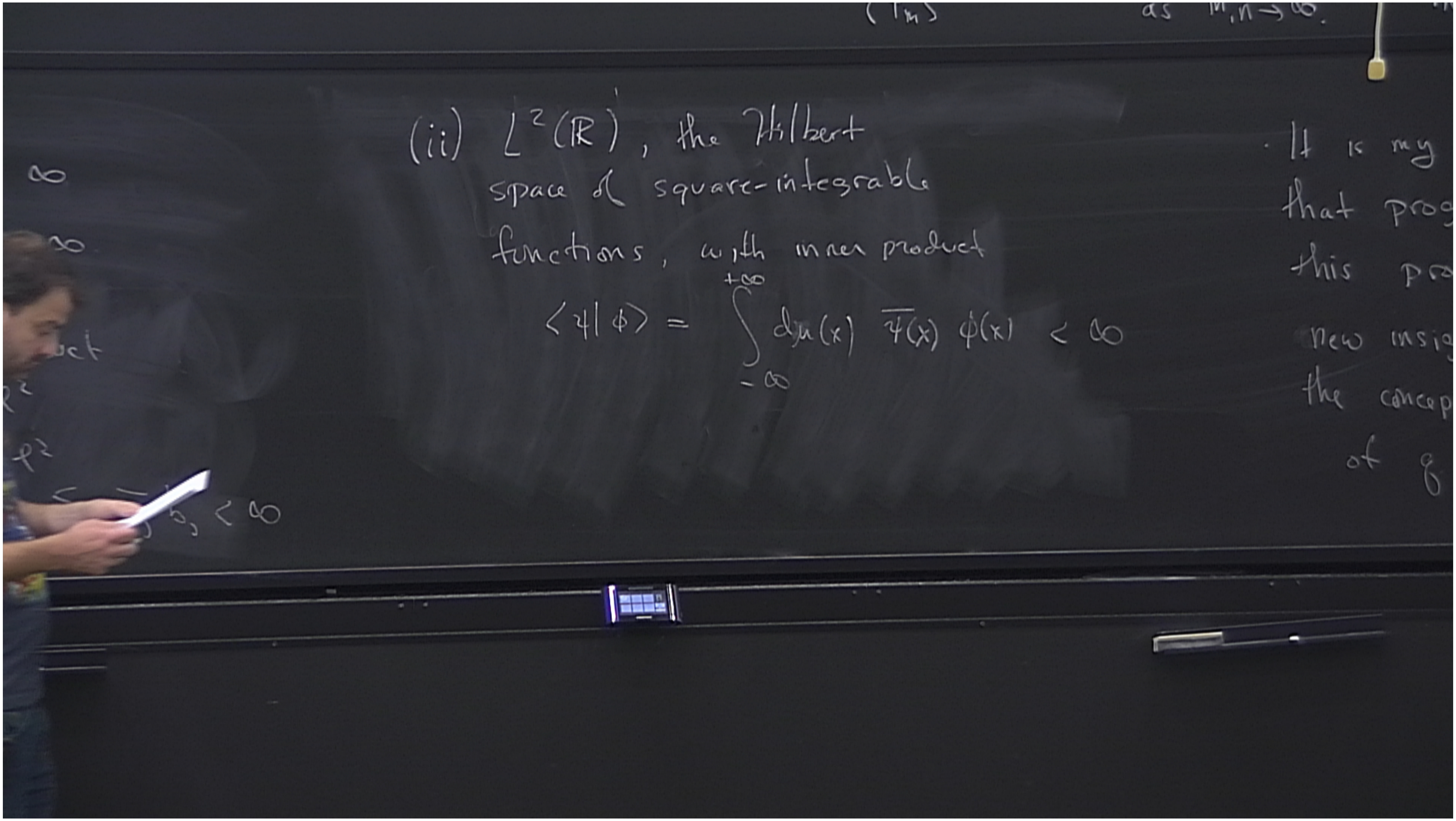

A Hilbert space is
an inner product space
that is complete in the norm
Def^n: A Cauchy sequence is
a sequence {ψ_m} satisfying ‖ψ_m − ψ_n‖ → 0
as m, n → ∞.
Complete in the norm
means that the limit
of every Cauchy
sequence
ψ := lim_{m→∞} ψ_m
is an element of
the inner product space.


· All finite-dimensional inner product spaces are complete in the norm.
· An important concept is the dual space $\mathcal{H}^+$, which is a set of linear functionals taking vectors to scalars.
$f : \mathcal{H} \to \mathbb{C}$
· Riesz' Theorem guarantees that the linear functionals in the dual space $\mathcal{H}^+$ are in one-to-one correspondence with the elements of $\mathcal{H}$.


$f : \mathcal{H} \to \mathbb{C}$
· Riesz' Theorem guarantees
that the linear functionals
in the dual space $\mathcal{H}^{\dagger}$
are in one-to-one correspondence
the elements of $\mathcal{H}$.
$\forall \; |\varphi\rangle \in \mathcal{H}$
we have $f_\varphi : \mathcal{H} \to \mathbb{C}$
where $f_\varphi := \langle \varphi | \cdot \rangle \in \mathbb{C}$


· For a theory we only
Separable Hilbert spaces.
· A separable Hilbert space
is a Hilbert space with a
countable basis


as $m,n \to \infty$.
the inner product space.
This means that
any $|\psi\rangle \in \mathcal{H}$
Can be expressed
w.r.t. an O.N. basis $\{|\phi_j\rangle\}$
as $|\psi\rangle = \sum_j c_j |$
where $c_j =$
O.N. basis
$\langle \phi_j | \phi_k \rangle = \delta_{jk}$


(ii) $L^2(\mathbb{R})$, the Hilbert
space of square-integrable
functions, with inner product
$\langle \psi | \phi \rangle = \int_{-\infty}^{+\infty} d\mu(x)\, \overline{\psi}(x)\, \phi(x) < \infty$
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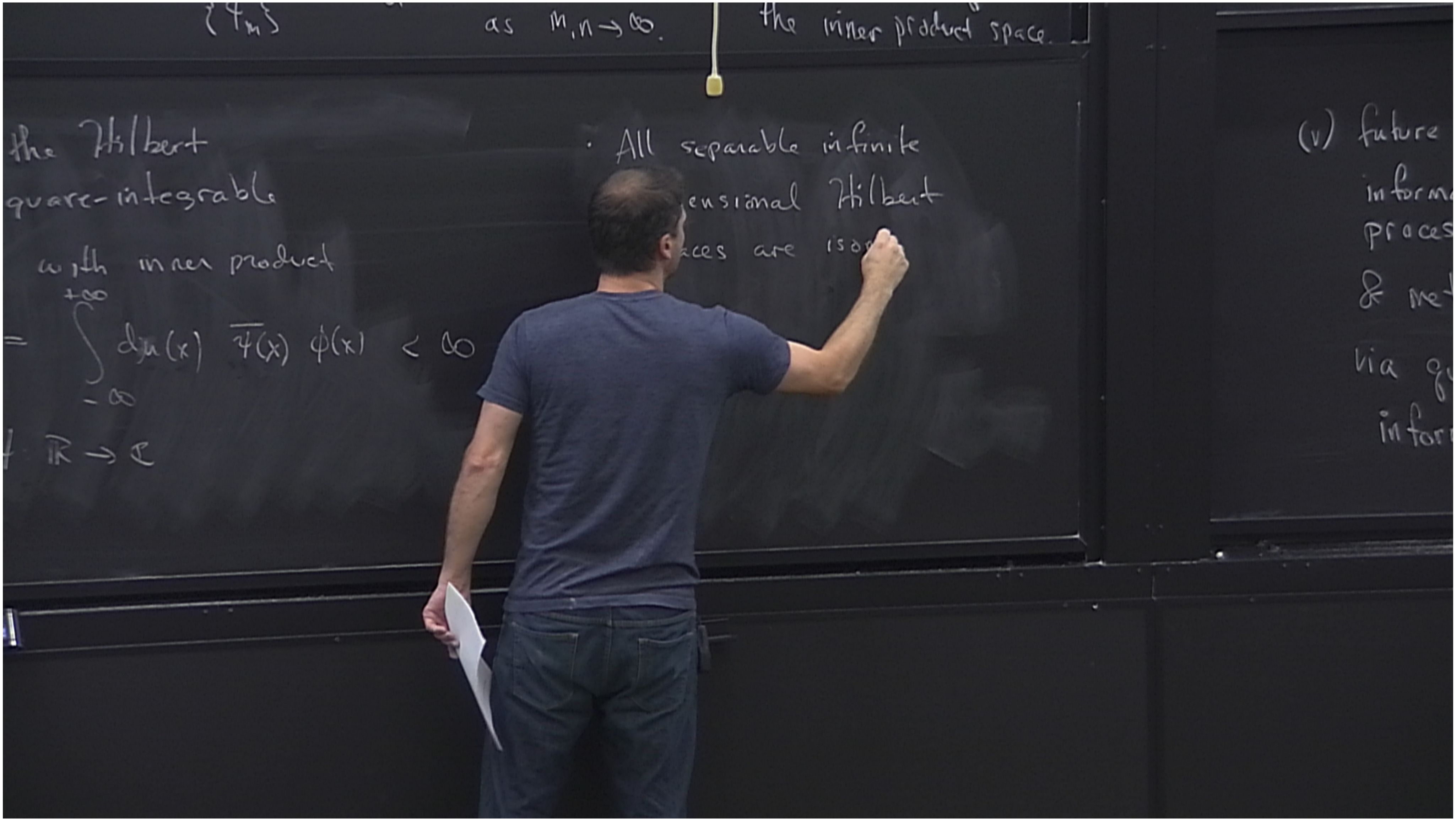
as m,n → ∞.
the inner product space.
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All separable infinite
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$\{\psi_m\}$ as $m, n \to \infty$. the inner product space.

the Hilbert
quare-integrable
with inner product

$$= \int_{-\infty}^{+\infty} d\mu(x)\, \overline{\psi}(x)\, \phi(x) < \infty$$

$\psi : \mathbb{R} \to \mathbb{C}$

· All separable infinite dimensional Hilbert spaces are <u>isomorphic</u>,

that is, there is an invertible, structure preserving map between any two separable Hilbert spaces.

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