Title: 14/15 PSI - Relativity- Lecture 2

Date: Sep 19, 2014 09:00 AM

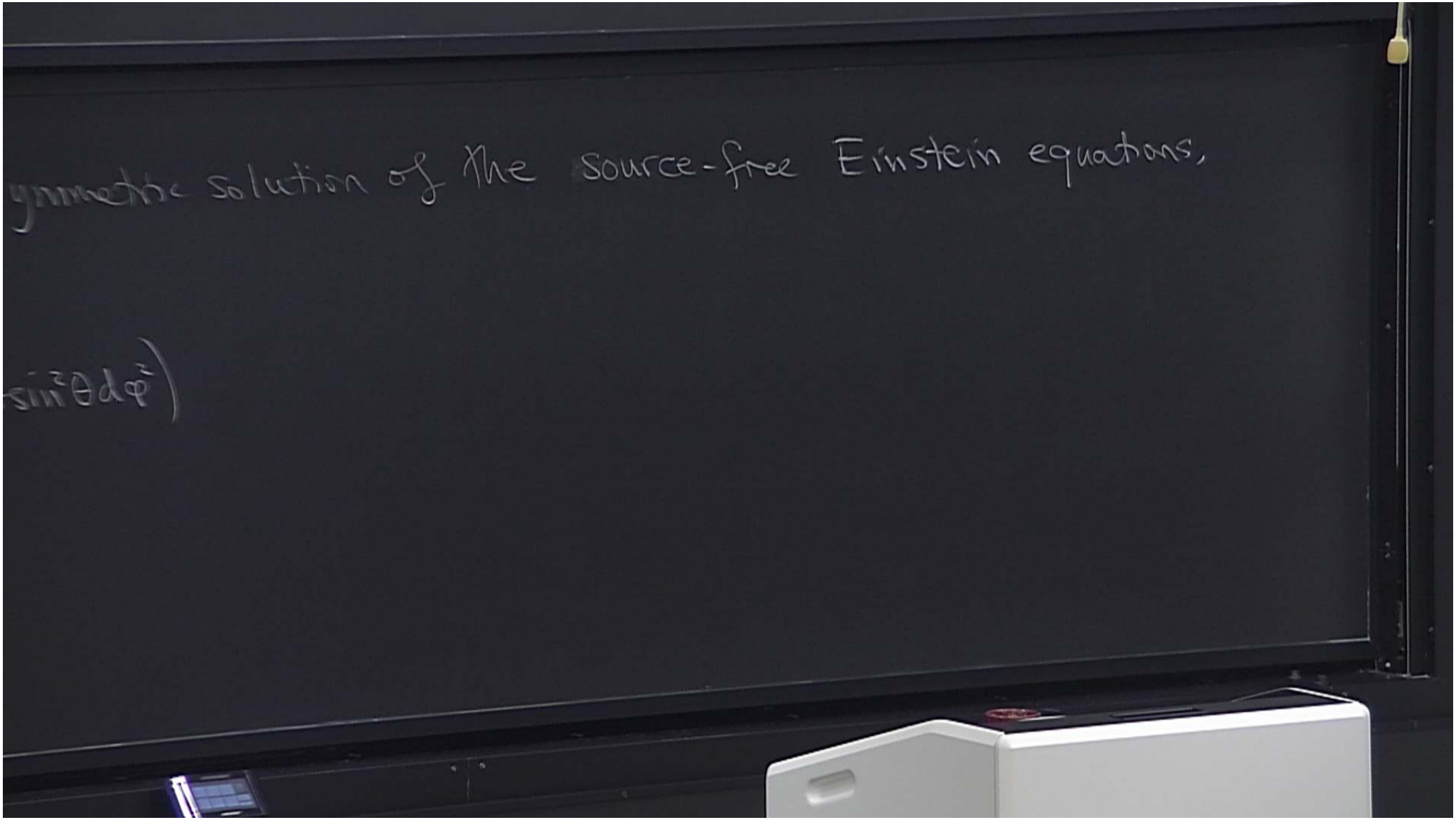
URL: http://pirsa.org/14090022

Abstract:

last time: showed that most general spherically symmetric solution

$G_{\mu\nu} = 0$, is

$$-dt^2\left(1+\frac{k}{r}\right) + \frac{dr^2}{\left(1+\frac{k}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$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$\sin^2\theta d\varphi^2$)

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look at large $r \Rightarrow g_{\mu\nu} \approx \eta_{\mu\nu} + \underbrace{h_{\mu\nu}}_{\sim \frac{k}{r}}$ recall that $h_{00} = -\frac{2\phi}{c^2}$

ymmetric solution of the source-free Einstein equations,

$\sin^2\theta d\varphi^2)$

$= -\frac{2\phi}{c^2}$, recall that $\phi = -\frac{GM}{r}$ is the Newtonian potential outside a spherical mass M.

$$\rightarrow \quad h_{00} \approx -\frac{k}{r} = \frac{2GM}{rc^2}$$

$\Rightarrow$ Schwarzchild metric represents the exterior of a spherical mass,

$$-dt^2\left(1-\frac{2GM}{rc^2}\right) + \frac{dr^2}{\left(1-\frac{2GM}{rc^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

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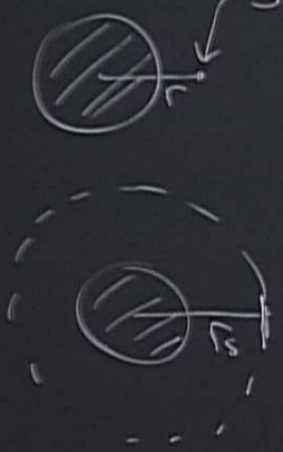
(in fact this leads to the definition of the mass of a body in G.R.)

always inside r; it looks like metric becomes singular at $r = \frac{2GM}{c^2}$

$= r_s$

Schwarzchild radius



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we shall see that this is a coordinate singularity, i

is not singular at $r = r_s$.

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coordinate singularity, ∃ coordinates in which $g_{\mu\nu}$

Schwarzschild valid here

: provided the mass M is always inside

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There is a huge physical effect at $r = r_s$:

always inside r; it looks like metric becomes singular at $r = \frac{2GM}{c^2}$

$= r_s$ Schwarzchild radius

coordinate singularity, ∃ coordinates in which $g_{\mu\nu}$

gravitational time dilation

Recall. $\Delta\tau = (-g_{00})^{\frac{1}{2}} \Delta t \Rightarrow \Delta t = \frac{\Delta\tau}{(-g_{00})^{\frac{1}{2}}} = \frac{\Delta\tau}{\sqrt{1 - \frac{2GM}{rc^2}}} \Rightarrow \infty$ as r decreases to r_s.

$(-g_{\mu\nu} dx^\mu dx^\nu)^{\frac{1}{2}}$

$r_s = \frac{2GM}{rc^2}$; for Sun, $\frac{2 \times 6.7 \times 10^{-11} \cdot 2 \times 10^{30}}{(3 \times 10^8)^2} = 3\,km$.

black hole Milky way S2

obviously, $r_{sun} \gg r_{schw}$. Sun is not a black hole.

However, there is a black hole in the centre of our galaxy.

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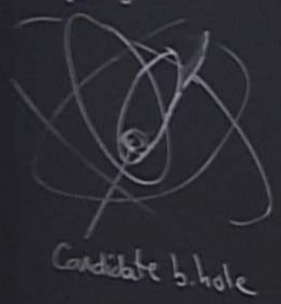

However, there is a black hole in the centre of our galaxy. $\therefore M \approx 4 \times 10^6 M_\odot$

$\Rightarrow r_{schw} \approx 10^0\,\text{m}$

distance $\approx 2 \times 10^{20}\,\text{m}$.

$\left.\begin{array}{l} \\ \\ \end{array}\right\} \theta \sim 10^{-10}$ radians.

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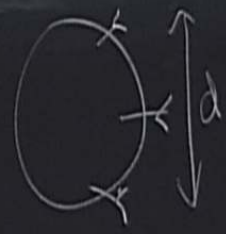
$\theta \sim 10^{-10}$ radians.

VLBI $\theta_{res} \sim \frac{\lambda}{d}$



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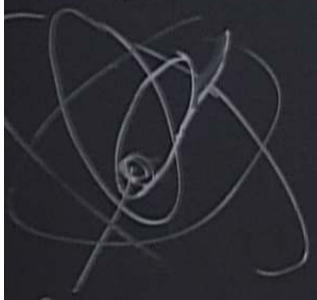

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$r_s \sim 695{,}000\,km$
$10^9\,m$

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distance $\approx 2 \times 10^{20}\,m.$

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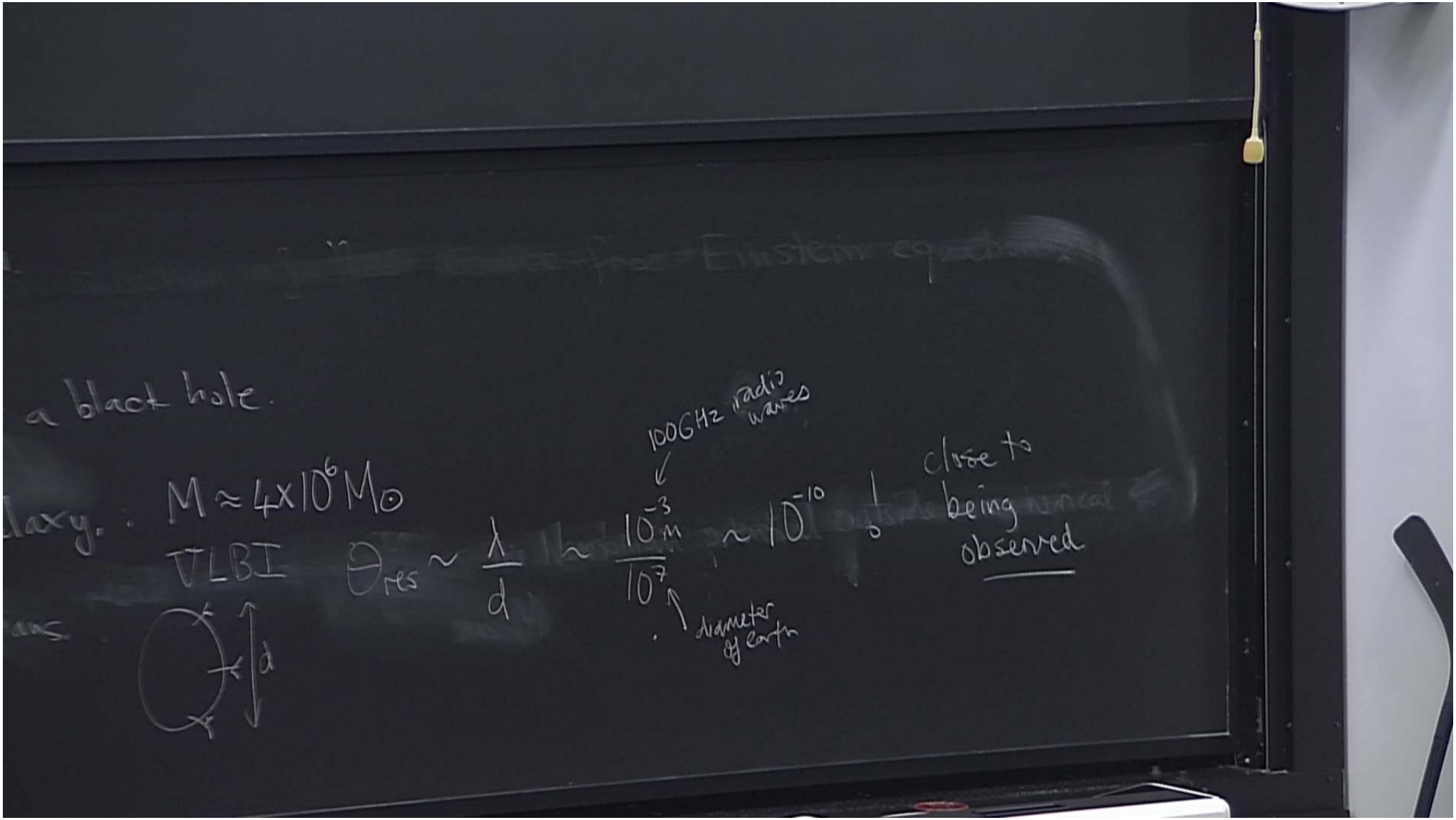
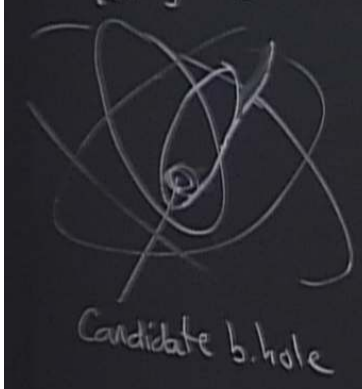

a black hole.
$M \approx 4 \times 10^6 M_\odot$
VLBI
$\theta_{res} \sim \frac{\lambda}{d}$
$\frac{10^{-3}\,m}{10^7} \sim 10^{-10}$
100GHz radio waves
diameter of earth
close to being observed

$r_s = \frac{2GM}{rc^2}$; for Sun, $\frac{2 \times 6.7 \times 10^{-11} \cdot 2 \times 10^{30}}{(3 \times 10^8)^2} = 3\,km.$



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$10^9\,m$ distance $\approx 2 \times 10^{20}\,m.$ } $\theta \sim 10^{-10}$ radians.

$\frac{10^{10}}{3 \times 10^8} = 30$

Proper surface area of black hole

$$= 4\pi \cdot r_s^2 = 4\pi \cdot \left(\frac{2GM}{c^2}\right)^2$$

$$= 16\pi \frac{(GM)^2}{c^4}$$

– completely coordinate independent.

Tests of GR

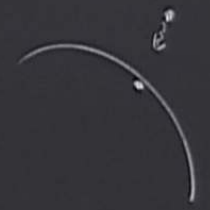
2 different objects falling in the Earth's gravitational field experience the same acceleration a, up to

① Equivalence principle: $\frac{\Delta a}{a} \lesssim 10^{-13}$ (Adelberger, U. Washington)

proposed STEP satellite 1401.4784 Overduin et al

$\Rightarrow \frac{\Delta a}{a} \lesssim 10^{-18}$

② Gravitational redshift: 2 Masers on earth and in orbit



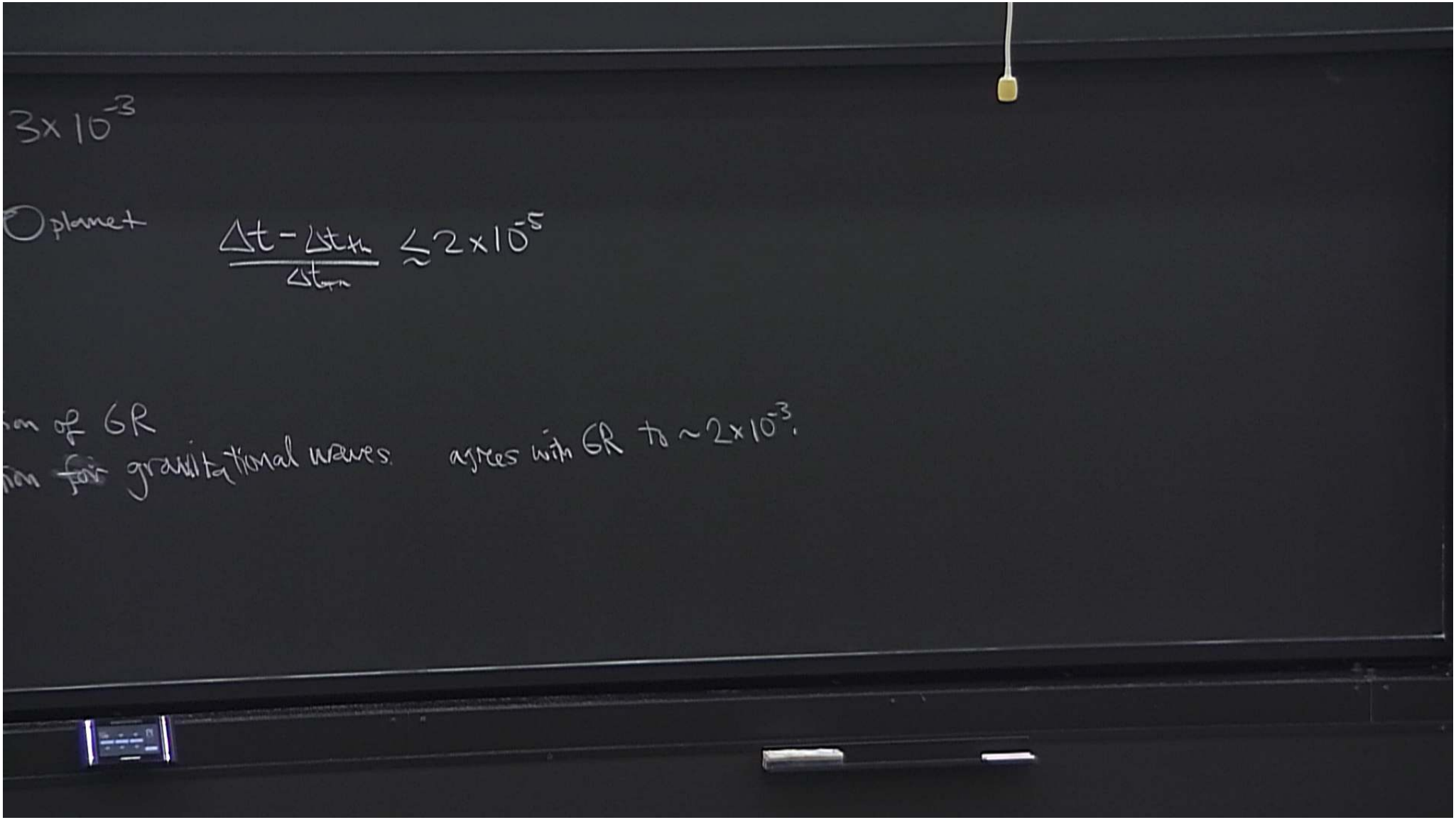
$= \frac{\Delta\nu - \Delta\nu_{th}}{\Delta\nu_{th}} < 2 \times 10^{-4}$

③ Deflection of light $\quad \frac{\Delta\phi - \Delta\phi_{th}}{\Delta\phi_{th}} \lesssim 3\times10^{-3}$

④ Shapiro time delay

$\frac{\Delta t - \Delta t_{th}}{\Delta t_{th}} \lesssim 2\times10^{-5}$

⑤ Pulsars $\quad$ binary pulsar (Hulse + Taylor) $\Rightarrow$ confirmation of GR prediction ~~for~~ gravitational waves.


3×10^{-3}
planet
$\frac{\Delta t - \Delta t_{th}}{\Delta t_{th}} \lesssim 2\times10^{-5}$
of GR
gravitational waves.
agrees with GR to $\sim 2\times10^{-3}$.

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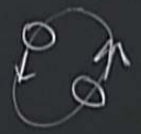
⑤ Pulsars  binary pulsar (Hulse + Taylor) ⇒ confirmation of GR prediction for gravitational waves agrees w

⑥ Direct Detection of Gravitational Waves LIGO

(7) Perihelion shift of Mercury (Einstein, ~1917)

Orbits of a massive particle in Schwarzschild metric

$r_s = \frac{2GM}{c^2}$

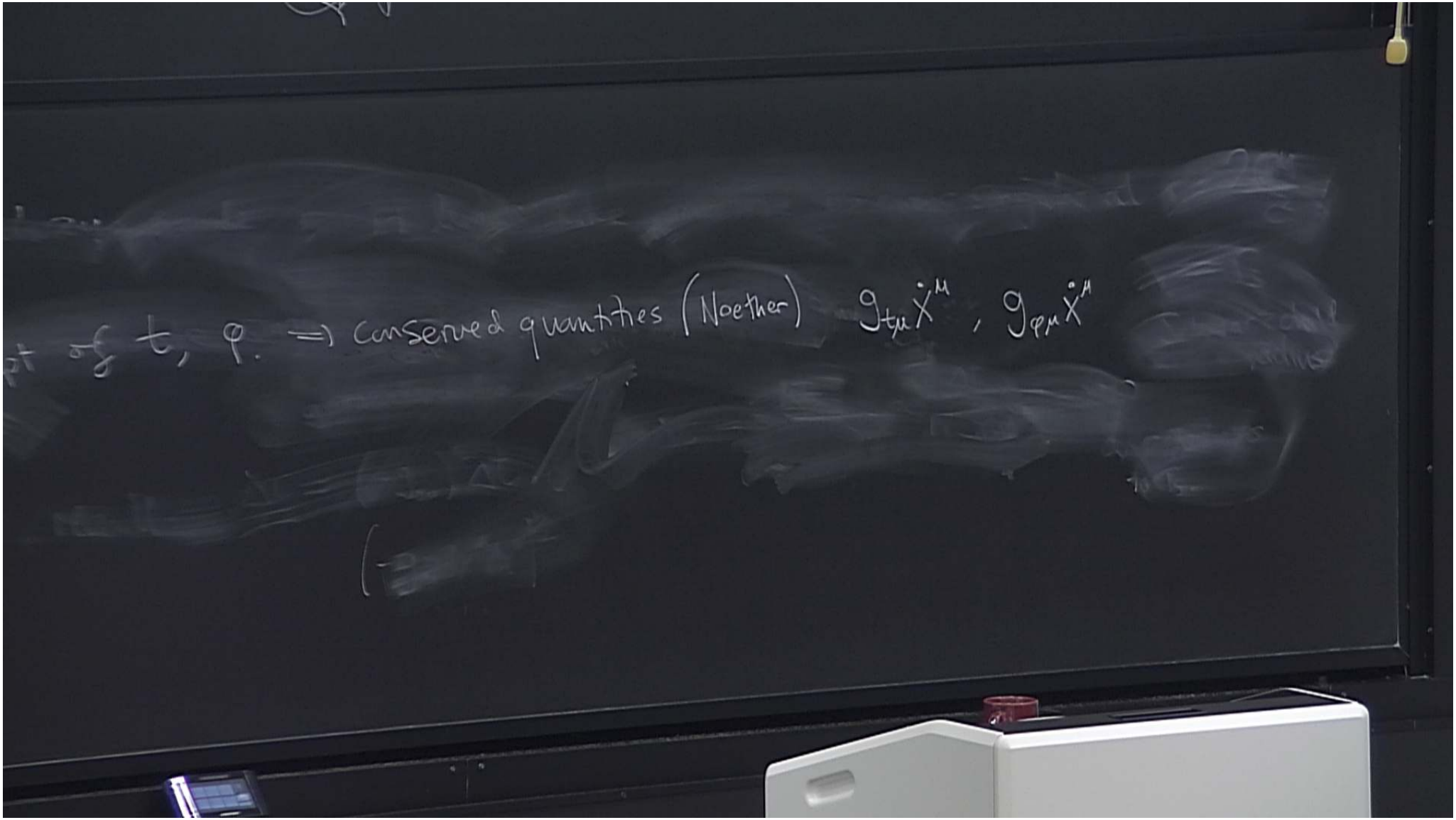
$c = 1$

$$-\left(1-\frac{r_s}{r}\right)dt^2 + \left(1-\frac{r_s}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

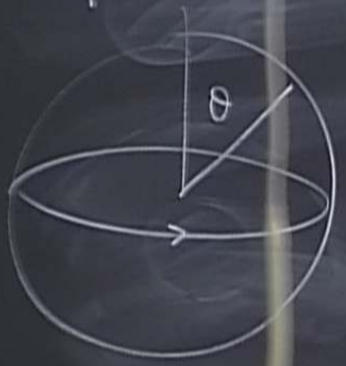
for massive particle, $-\left(1-\frac{r_s}{r}\right)\dot{t}^2 + \left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2 + r^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right) = -1$

$$\frac{dx^\mu}{d\tau} = \dot{x}^\mu$$

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$$


t, φ. ⇒ Conserved quantities (Noether) $g_{t\mu}\dot{X}^\mu$, $g_{\varphi\mu}\dot{X}^\mu$

spherical symmetry



$\Rightarrow \theta = \pi/2$ should be a solution,

Conserved quantities: metric is indept of t, φ. $\Rightarrow$ Cu

$\Rightarrow$ energy $E = \left(1 - \frac{r_s}{r}\right)\dot{t}$ } Conserved

angular momentum $J = r^2\dot{\varphi}$

$\Rightarrow$ 3 eq^{n}s for 3 unknowns, $\dot{t}, \dot{r}, \dot{\varphi}$ $\Rightarrow$ can solve.

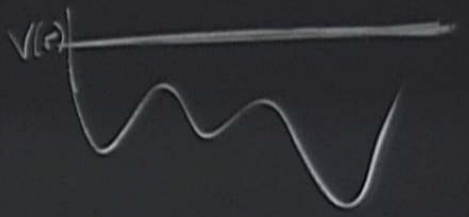
eliminate $\dot{t}, \dot{\varphi}$

$$\Rightarrow -\frac{E^2}{\left(1-\frac{r_s}{r}\right)} + \frac{\dot{r}^2}{\left(1-\frac{r_s}{r}\right)} + \frac{J^2}{r^2} = -1$$

$$\Rightarrow \quad \dot{r}^2 + V(r) = E^2 - 1 = \text{constant}$$

3×10^{-3}

) planet $\quad \frac{\Delta t - \Delta t_{th}}{\Delta t_{th}} \lesssim 2 \times 10^{-5}$



of GR

for gravitational waves. agrees with GR to $\sim 2 \times 10^{-3}$.

$\Rightarrow$ 3 eq^{n}s for 3 unknowns, $\dot{t}, \dot{r}, \dot{\varphi}$ $\Rightarrow$ can solve.

minate $\dot{t}, \dot{\varphi}$

$$\Rightarrow -\frac{E^2}{\left(1-\frac{r_s}{r}\right)} + \frac{\dot{r}^2}{\left(1-\frac{r_s}{r}\right)} + \frac{J^2}{r^2} = -1$$

$$\Rightarrow \dot{r}^2 + V(r) = E^2 - 1 = \text{constant}$$

usual Newtonian potential

usual centripetal force (ang. momentum barrier)

$$V(r) = -\frac{r_s}{r} + \frac{J^2}{r^2} - \frac{r_s J^2}{r^3}$$

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GR correction

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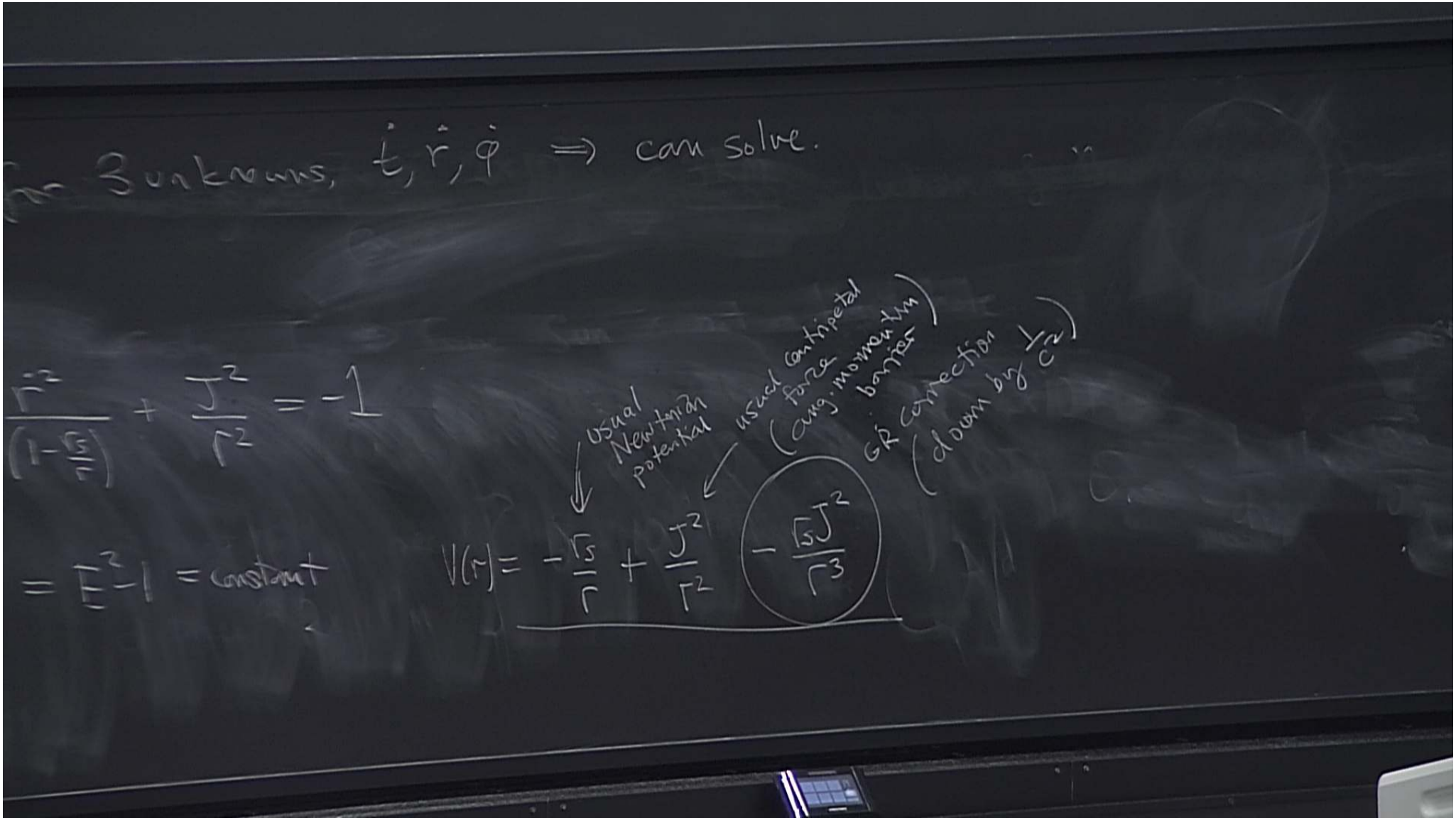
3 unknowns, $\dot{t}, \dot{r}, \dot{\varphi}$ $\Rightarrow$ can solve.
$\frac{\dot{r}^2}{(1-\frac{r_s}{r})} + \frac{J^2}{r^2} = -1$
$= E^2 - 1 = \text{constant}$
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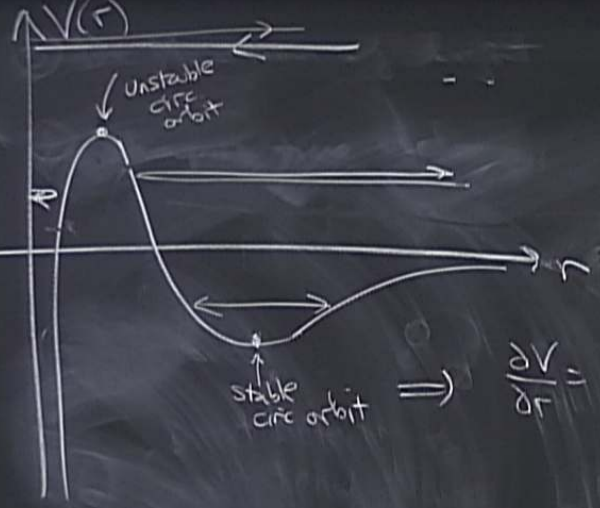
usual Newtonian potential
usual centripetal force (ang. momentum barrier)
GR correction (down by $\frac{1}{c^2}$)

Gravitational redshift: 2 Masers on earth and in orbit

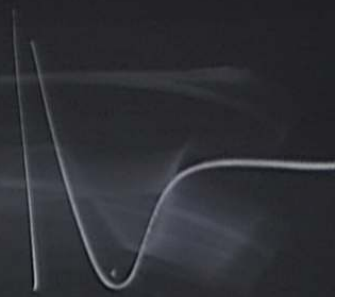
$= \frac{\Delta f - \Delta f_0}{\Delta f_0} < 2 \times 10^{-16}$ Vessot et al.



$\Rightarrow \frac{dV}{dr} = 0 = \frac{r_s}{r^2} - \frac{2J^2}{r^3} + \frac{3r_s J^2}{r^4} \Rightarrow r = \frac{J^2}{r_s}\left(1 \pm \sqrt{1 - 3r_s^2/J^2}\right)$; $r_{stable} = \frac{J^2}{r_s}\left(1 + \sqrt{1 - 3r_s^2/J^2}\right)$

$$+\frac{3[J]^2}{r^4} \Rightarrow r = \frac{J^2}{r_s}\left(1 \pm \sqrt{1 - 3r_s^2/J^2}\right) \quad ; \quad r_{stable} = \frac{J^2}{r_s}\left(1 + \sqrt{1 - 3r_s^2/J^2}\right)$$

as you decrease J, there is an innermost stable circular orbit : ISCO.



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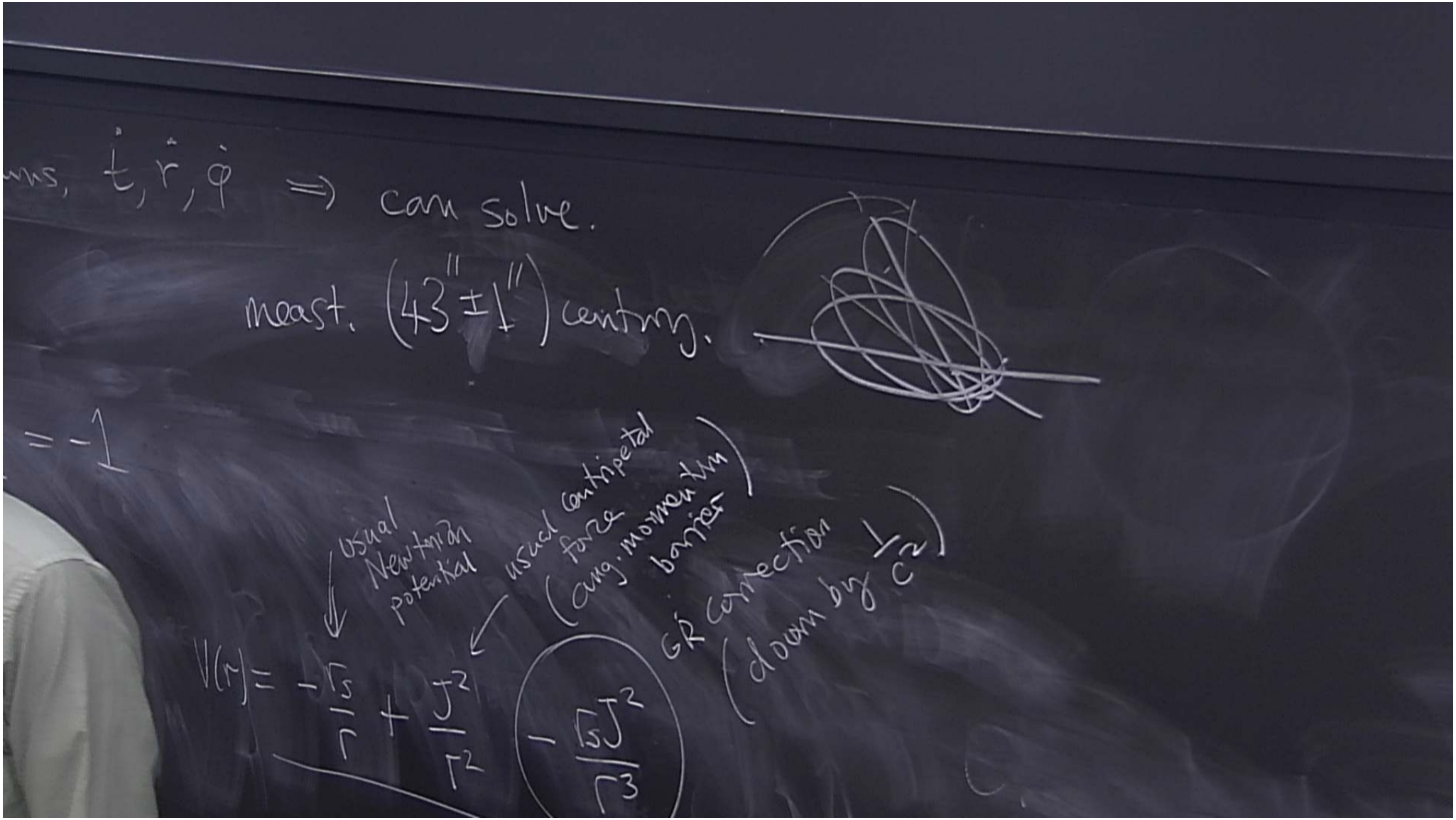
at $J^2 = 3r_s^2$; $\underline{r_{stable} = 3r_s}$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meast. $(43'' \pm 1'')$ centry.
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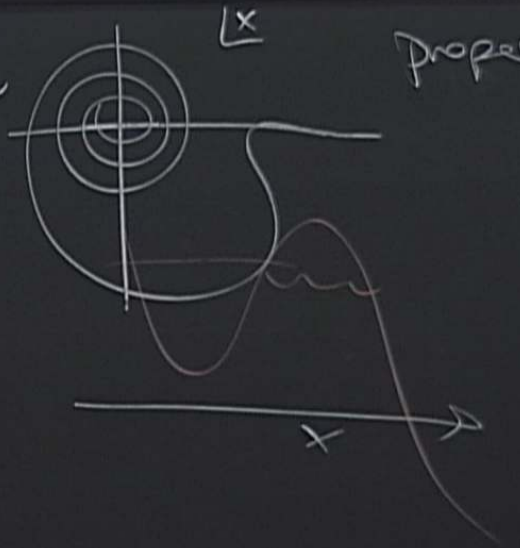
Stable - $3r_s$

g in the Earth's gravitational
same acceleration a, up to

Adelberger, U. Washington)

4784 Overduin et al



Proper surface area of black hole

$= 4\pi \cdot r_s^2 = 4\pi \left(\frac{2GM}{c^2}\right)^2$

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