

Title: 14/15 PSI - Relativity- Lecture 2

Date: Sep 18, 2014 09:00 AM

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Abstract:

last time : $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$

last time : $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$

Chose this as the combination satisfying $\nabla_\mu G^{\mu\nu} = 0$,

Proof of $\nabla_\mu (R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R) = 0$

Bianchi Identity

recall: $R_{\sigma\rho\mu\nu} = \frac{1}{2}(g_{\nu\sigma,\rho\mu} - g_{\rho\nu,\sigma\mu} - g_{\mu\sigma,\rho\nu} + g_{\rho\mu,\sigma\nu}) + \Gamma$

consider $R_{\sigma\rho\mu\nu;\eta}$

$$\left(\begin{array}{l} A_{\mu,\rho} \equiv \frac{\partial}{\partial x^\rho} A_\mu \\ \text{sim?} \\ A_{\mu;\rho} \equiv \nabla_\rho A_\mu \end{array} \right)$$

Bianchi Identity

recall :

$$R_{\sigma\rho\mu\nu} = \frac{1}{2}(g_{\nu\sigma,\rho\mu} - g_{\rho\nu,\sigma\mu} - g_{\mu\sigma,\rho\nu} + g_{\rho\mu,\sigma\nu}) + \Gamma\Gamma \text{ terms.}$$

in locally flat coords

consider

$$\left(\begin{array}{l} A_{\mu,\rho} \equiv \frac{\partial}{\partial x^\rho} A_\mu \\ \text{sim?} \\ A_{\mu,\rho} \equiv \nabla_\rho A_\mu \end{array} \right)$$

$$R_{\sigma\rho\mu\nu;\eta} = \frac{1}{2}(g_{\nu\sigma,\rho\mu\eta} - g_{\rho\nu,\sigma\mu\eta} - g_{\mu\sigma,\rho\nu\eta} + g_{\rho\mu,\sigma\nu\eta})$$

cycle

$$+ R_{\sigma\rho\eta\mu;\nu} + \frac{1}{2}(\mu_{\sigma,\rho\eta\nu} - \rho_{\mu\sigma\eta\nu} - \eta_{\sigma,\rho\mu\nu} + \rho_{\eta,\sigma\mu\nu})$$

$$+ R_{\sigma\rho\nu\eta;\mu} + \frac{1}{2}(\eta_{\sigma,\rho\nu\mu} - \rho_{\eta,\sigma\nu\mu} - \nu_{\sigma,\rho\eta\mu} + \rho_{\nu,\sigma\eta\mu})$$

Bianchi Identity

recall :

$$R_{\sigma\rho\mu\nu} = \frac{1}{2}(g_{\nu\sigma,\rho\mu} - g_{\rho\nu,\sigma\mu} - g_{\mu\sigma,\rho\nu} + g_{\rho\mu,\sigma\nu}) + \Gamma\Gamma \text{ terms.}$$

in locally flat coords

consider

$$R_{\sigma\rho\mu\nu;\eta} \underset{\text{cycle}}{=} \frac{1}{2}(g_{\nu\sigma,\rho\mu\eta} - g_{\rho\nu,\sigma\mu\eta} - g_{\mu\sigma,\rho\nu\eta} + g_{\rho\mu,\sigma\nu\eta})$$

$$+ R_{\sigma\rho\eta\mu;\nu} + \frac{1}{2}(\mu\sigma,\rho\eta\nu - \rho\mu,\sigma\eta\nu - \eta\sigma,\rho\mu\nu + \rho\eta,\sigma\mu\nu)$$

$$+ R_{\sigma\rho\nu\eta;\mu} + \frac{1}{2}(\eta\sigma,\rho\nu\mu - \rho\eta,\sigma\nu\mu - \nu\sigma,\rho\eta\mu + \rho\nu,\sigma\eta\mu) = 0.$$

$$A_{\mu,\rho} \equiv \frac{\partial}{\partial x^\rho} A_\mu$$

sim?

$$A_{\mu;\rho} \equiv \nabla_\rho A_\mu$$

... satisfying $\nabla_\mu G^{\mu\nu} = 0$, because we argued rhs should obey $\nabla_\mu T^{\mu\nu} = 0$.

$$F_{[\mu\nu]} = \frac{1}{2}(F_{\mu\nu} - F_{\nu\mu})$$

$$F_{(\mu\nu)} = \frac{1}{2}(F_{\mu\nu} + F_{\nu\mu})$$

$$F_{\mu\nu;\rho} + \text{cycles} = 0$$

$$\Leftrightarrow \epsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma} = 0$$

$$\boxed{g_{\mu\nu;\rho} = 0}$$

$$R_{\sigma\rho\mu\nu;\eta} + R_{\sigma\rho\eta\mu;\nu} + R_{\sigma\rho\nu\eta;\mu} = 0$$

$$\times g^{\sigma\mu} \quad R_{\rho\nu;\eta} - R_{\rho\eta;\nu} + R_{\rho\nu\eta;\sigma} = 0$$

$$\times g^{\rho\nu} \quad R_{;\eta} - R^{\rho}{}_{\eta;\rho} - R^{\sigma}{}_{\eta;\sigma} = 0$$

$$\Rightarrow R^{p\eta}{}_{;p} - \frac{1}{2}(Rg^{p\eta})_{;p} = 0 \Rightarrow$$

$$G^{p\eta}{}_{;p} = \left(R^{p\eta} - \frac{1}{2}g^{p\eta}R \right)_{;p} = 0.$$

Riemann:

nation

$$\frac{d^2}{d\tau^2} (x^M + \delta x^M) + \Gamma_{\nu\lambda}^M (x + \delta x) \frac{d}{d\tau} (x^\nu + \delta x^\nu) \frac{d}{d\tau} (x^\lambda + \delta x^\lambda) = 0$$

$$\frac{d^2 x^M}{d\tau^2} + \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

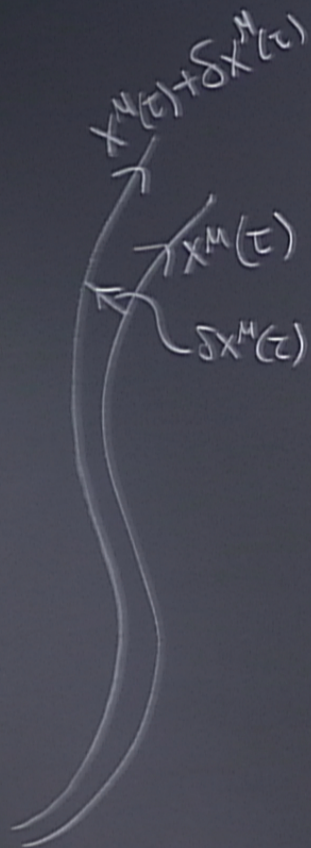
If δx^M small, difference is

$$\frac{d^2 \delta x^M}{d\tau^2} + \delta x^P \Gamma_{\nu\lambda, P}^M \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + 2 \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} = 0$$

Another derivation of Riemann:

Equation of geodesic deviation

$$\frac{d^2}{dt^2} (x^M)$$



If δx^M small, differ

Let's try to make this into a tensor equation

$$\left(\frac{D}{D\tau}\right)^2 \delta x^M = \frac{D}{D\tau} \left(\frac{d\delta x^M}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right)$$

$$= \frac{d}{d\tau} \left(\right) + \Gamma_{\alpha\beta}^M \left(\right) \frac{dx^\beta}{d\tau}$$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \kappa}^M \frac{dx^\kappa}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{d^2 x^\lambda}{d\tau^2} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \frac{d^2 x^\lambda}{d\tau^2}$$

this into a tensor equation:

$$\frac{D}{D\tau} \delta x^M = \frac{D}{D\tau} \left(\frac{d\delta x^M}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{dx^\lambda}{d\tau} \right)$$

$$= \frac{d}{d\tau} \left(\right) + \Gamma_{\alpha\beta}^M \left(\right) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$= \frac{d^2 \delta x^M}{d\tau^2} + \Gamma_{\nu\lambda, \kappa}^M \frac{dx^\kappa}{d\tau} \delta x^\nu \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^M \delta x^\nu \frac{d^2 x^\lambda}{d\tau^2} + \Gamma_{\alpha\beta}^M \frac{d\delta x^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$\downarrow$ using (*)
 $-\Gamma_{\nu\lambda, \rho}^M \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2\Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} - \Gamma_{\alpha\beta}^\lambda \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$

using (*)

$$- \Gamma_{\nu\lambda,\rho}^M \delta x^p \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^M \frac{dx^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau}$$

$$\dot{x}^\lambda = \frac{dx^\lambda}{d\tau}$$

$$= - \Gamma_{\nu\lambda,\rho}^M \delta x^p \dot{x}^\nu \dot{x}^\lambda + \Gamma_{\nu\lambda,\alpha}^M \dot{x}^\alpha \delta x^\nu \dot{x}^\lambda - \Gamma_{\nu\lambda}^M \Gamma_{\alpha\beta}^\lambda \dot{x}^\alpha \delta x^\beta$$

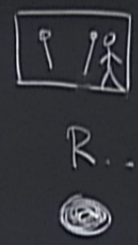
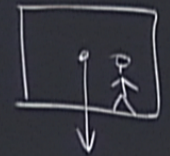
$$= \underline{R_{\nu\lambda\rho}^M \dot{x}^\nu \dot{x}^\lambda \delta x^\rho}$$

this is the test of whether there is really a gravitational field present.

$$[Th^m:] \quad R_{\mu\rho}^M = 0 \quad \forall x \Leftrightarrow \exists \text{ coordinates in which } g_{\mu\nu} = \eta_{\mu\nu} \quad \forall x$$

↓ using (*)

$$- \Gamma^M_{\nu\lambda,\rho} \delta x^\rho \frac{dx^\nu}{d\lambda} \frac{dx^\lambda}{d\tau} - 2 \Gamma^M_{\nu\lambda,\alpha} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} \frac{dx^\alpha}{d\lambda}$$



$$\dot{x}^\lambda = \frac{dx^\lambda}{d\tau}$$

$$= - \Gamma^M_{\nu\lambda,\rho} \delta x^\rho \dot{x}^\nu \dot{x}^\lambda + \Gamma^M_{\nu\lambda,\alpha} \dot{x}^\alpha \delta x^\nu \dot{x}^\lambda$$

$x^0 = ct$

$$= \underline{R^M_{\nu\lambda\rho} \dot{x}^\nu \dot{x}^\lambda \delta x^\rho}$$

$$- \left(1 - \frac{2GM}{r^2}\right) dt^2 \dots$$

(1)

$$c^2 \delta x^0 \sim \frac{GM \delta x^0}{r^3} \frac{d^2 \delta x^0}{dt^2}$$

this is the test of whether there is really a gravitational

$$[Th^M] \quad R^M_{\mu\nu\rho} = 0 \quad \forall x \Leftrightarrow] \text{ coordinates in which } g_{\mu\nu} = \eta_{\mu\nu}$$

Simplest solution of Einstein (apart from flats)

Schwarzschild : $G_{\mu\nu} = 0$ $\underbrace{g^{\mu\nu}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)} = 0$

$$g^{\mu\nu}g_{\mu\nu} = \delta^\nu_\nu$$

$$R - 2R = 0 \Rightarrow \underline{R=0}$$

- from flat space):

For $T_{\mu\nu} = 0$ i.e. outside any matter

Einstein's equations read $R_{\mu\nu} = 0$

Assume spherical symmetry: (e.g. Sun)

line element can depend only on rotational invariant quantities:

$$t, dt, r^2 = \underline{x} \cdot \underline{x}, r dr = \underline{x} \cdot d\underline{x}, d\underline{x}^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$\underline{x}' = 0 \underline{x}$$

$$t, \underline{x}$$

Assume spherical symmetry: (e.g. Sun)

line element can depend only on rotational invariant quantities:

$$t, dt, r^2 = \underline{x} \cdot \underline{x}, r dr = \underline{x} \cdot d\underline{x}, d\underline{x}^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$\underline{x}' = 0 \underline{x}$$

$$t, \underline{x}$$

$$\Rightarrow ds^2 = -C(r,t)dt^2 + E(r,t)dr^2 - 2D(r,t)dt dr + F(r,t)r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Sun)

quantities:

$$-r^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$+ F(r,t) r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

most general spherically symmetric
Metric.

, θ, φ coords, $F = 1$.

$$r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

↙ area of a sphere = $4\pi r^2$.

1) can remove $F(r,t)$ by defining $\begin{matrix} t' = t \\ r' = \sqrt{F(r,t)} r \end{matrix}$ } , in t', r', θ, φ coords, $F = 1$.
 (and then drop primes)

2) remove $D(r,t)$ by setting:

$$dt' = f(t,r)$$

$$r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

└ area

1) can remove $F(r,t)$ by defining $\begin{matrix} t' = t \\ r' = \sqrt{F(r,t)} r \end{matrix}$ } , in t', r', θ, φ coords
(and then drop primes)

2) remove $D(r,t)$ by setting:

$$dt' = f(t,r) (C dt + D dr)$$

If there is a
function
 $t'(t,r)$

$$= \frac{\partial t'}{\partial t} dt + \frac{\partial t'}{\partial r} dr$$

$$\Rightarrow \frac{\partial t'}{\partial t} = fC \quad \frac{\partial t'}{\partial r} = fD$$

$$\Rightarrow \frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial^2 t'}{\partial t \partial r} \Rightarrow \frac{\partial}{\partial r}(fC) = \frac{\partial}{\partial t}(fD)$$

$\{t, r, \theta, \varphi\}$ coords, $F=1$.

$$r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

area of a sphere = $4\pi r^2$.

given $f(\theta, r)$ $\rightarrow \frac{\partial}{\partial t} f(\theta, r)$

$$\frac{\partial t'}{\partial r} = fD$$

$$\Rightarrow \frac{\partial}{\partial r}(fC) = \frac{\partial}{\partial t}(fD)$$

solution does exist.

1) can remove $F(r, t)$ by defining $\left. \begin{array}{l} t' = t \\ r' = \sqrt{F(r, t)} r \end{array} \right\}$
(and then drop primes)

2) remove $D(r, t)$ by setting:

$$dt' = f(t, r) (C dt + D dr)$$

If there is a
function

$$t'(t, r)$$

$$-\frac{(C dt + D dr)^2}{C}$$

$$= \frac{\partial t'}{\partial t} dt + \frac{\partial t'}{\partial r} dr$$

$$\Rightarrow \frac{\partial t'}{\partial t} = fC \quad \frac{\partial t'}{\partial r} = fD$$

$$\Rightarrow \frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial^2 t'}{\partial t \partial r} \Rightarrow \frac{\partial}{\partial r} (fC) = \frac{\partial}{\partial t} (fD)$$

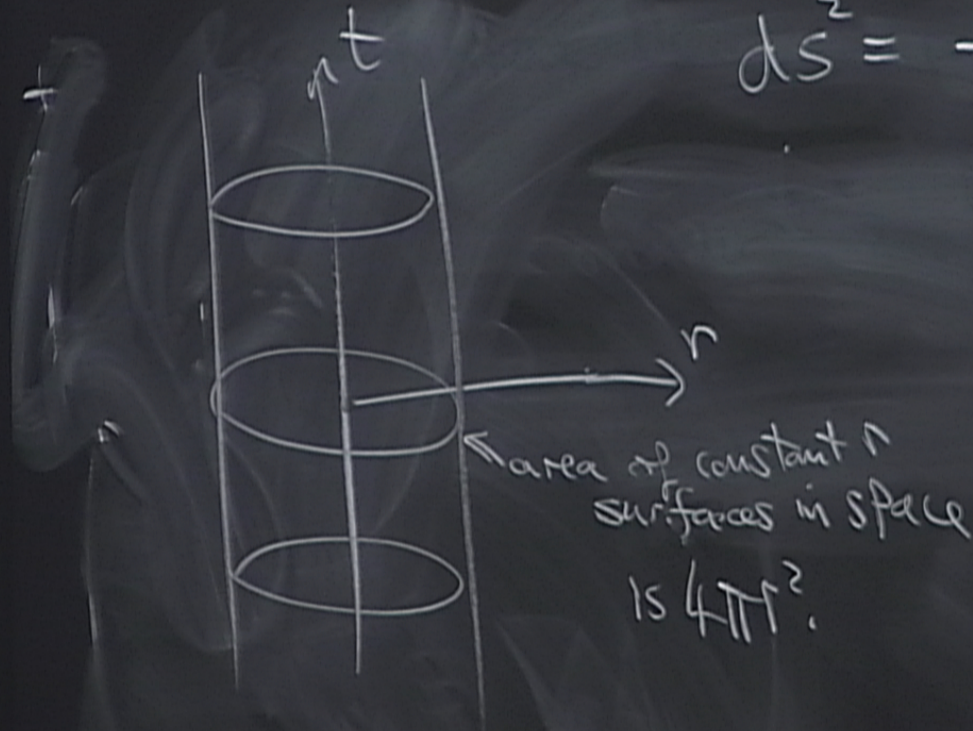
$$-C dt^2 - 2D dr dt - \frac{D^2}{C} dr^2 \Rightarrow ds^2 = -\frac{1}{f^2} \frac{dt'^2}{C} + \frac{D^2}{C} dr^2 + E dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

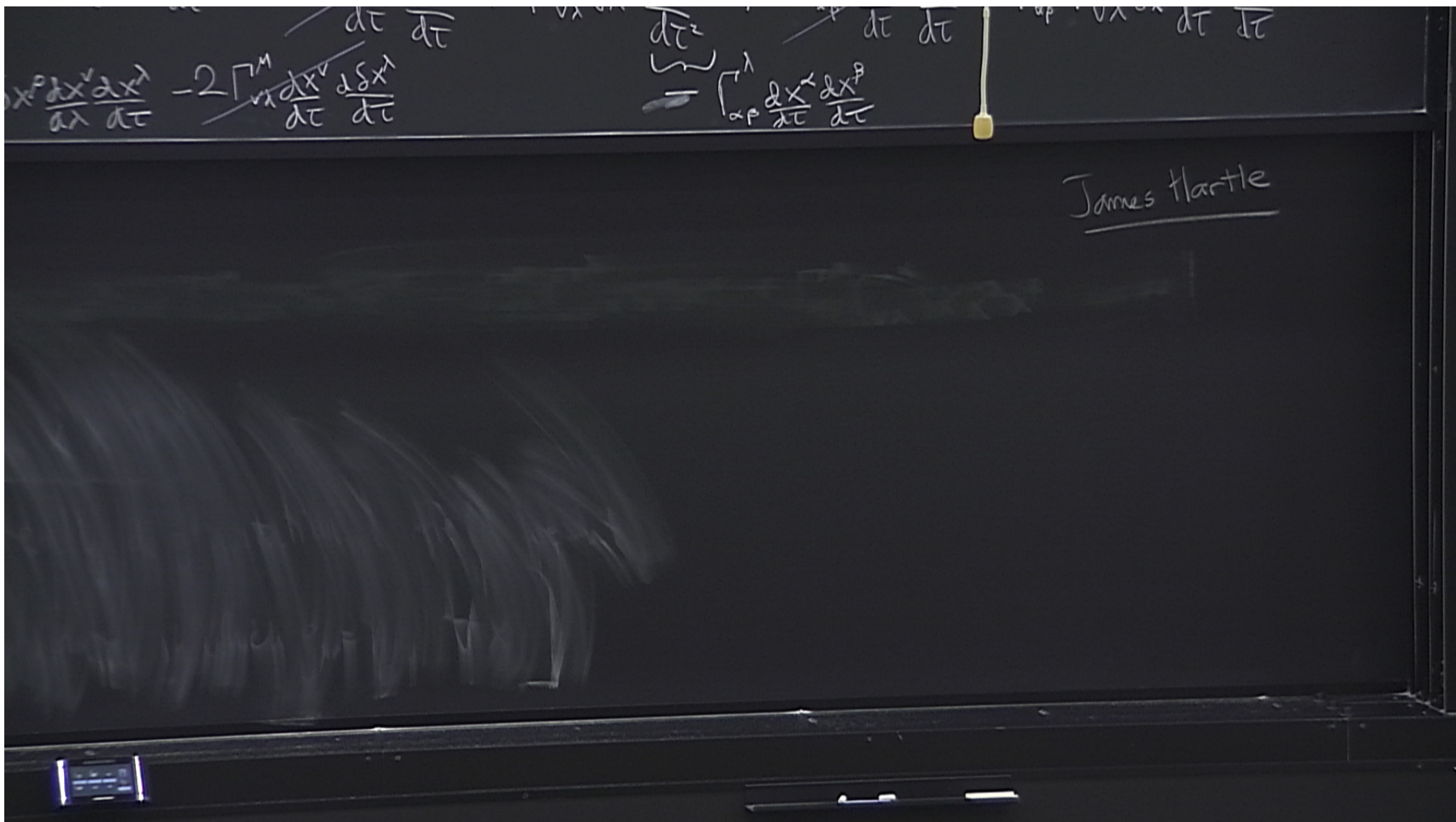
$\Rightarrow$ in the new coordinates (dropping primes)

$$ds^2 = -\underline{A(r,t)} dt^2 + \underline{B(r,t)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$\Rightarrow$ in the new coordinates (comp)

$$ds^2 = -A(r,t)dt^2 + B(r,t)$$





new coordinates (dropping primes)

$$= -A(r,t)dt^2 + B(r,t)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$g_{\mu\nu} = \begin{pmatrix} -A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$= \frac{d}{dt^2} + \Gamma_{\lambda\kappa}^{\mu} \frac{dx^{\lambda}}{dt} \frac{dx^{\kappa}}{dt} + \Gamma_{\lambda\kappa}^{\mu} \frac{dx^{\lambda}}{dt} \frac{dx^{\kappa}}{dt} + \Gamma_{\lambda\kappa}^{\mu} \frac{dx^{\lambda}}{dt} \frac{dx^{\kappa}}{dt} + \Gamma_{\lambda\kappa}^{\mu} \frac{dx^{\lambda}}{dt} \frac{dx^{\kappa}}{dt}$$

↓ using (*)

$$- \Gamma_{\lambda\kappa}^{\mu} \delta x^{\lambda} \frac{dx^{\kappa}}{dt} \frac{dx^{\lambda}}{dt} - 2 \Gamma_{\lambda\kappa}^{\mu} \frac{dx^{\lambda}}{dt} \frac{dx^{\kappa}}{dt}$$

$$\dot{\cdot} = \frac{\partial}{\partial t}$$

$$\dot{\cdot}' = \frac{\partial}{\partial r}$$

$$\Rightarrow \Gamma_{tt}^t = \frac{\dot{A}}{2A}, \quad \Gamma_{rr}^t = \frac{\dot{B}}{2A}, \quad \Gamma_{tr}^t = \frac{A'}{2A}$$

$$\Gamma_{tt}^r = \frac{A'}{2B}, \quad \Gamma_{tr}^r = \frac{\dot{B}}{2B}, \quad \Gamma_{rr}^r = \frac{B'}{2B}, \quad \Gamma_{\theta\theta}^r = -\frac{r}{B}, \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

$$\Gamma = \frac{1}{2} g(g, g, g)$$

$$\Gamma_{r\theta}^{\theta} = \frac{1}{r}, \quad \Gamma_{\varphi\varphi}^{\theta} = -\sin \theta \cos \theta, \quad \Gamma_{r\varphi}^{\varphi} = \frac{1}{r}, \quad \Gamma_{\varphi\theta}^{\varphi} = \tan \theta$$

$$\delta x^\rho \frac{dx^\nu}{dx^\lambda} \frac{dx^\lambda}{d\tau} - 2 \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = \Gamma^\lambda_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\frac{A'}{2A}$$

$$\frac{B'}{2B}, \quad \Gamma^r_{\theta\theta} = -\frac{r}{B}, \quad \Gamma^r_{\varphi\varphi} = -\frac{r \sin^2 \theta}{B}$$

$$, \quad \Gamma^\varphi_{\varphi\theta} = \cot \theta$$

with other components
given by symmetry $\Gamma^r_{\mu\nu} = \Gamma^r_{\nu\mu}$
or equal to zero.

James Hartle

$$= \frac{d^2 \delta x^\lambda}{d\tau^2} + \Gamma_{\nu\lambda}^{\mu} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} + \Gamma_{\nu\lambda}^{\mu} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} + \Gamma_{\nu\lambda}^{\mu} \delta x^\nu \frac{d^2 \delta x^\lambda}{d\tau^2} + \Gamma_{\nu\lambda}^{\mu} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} + \Gamma_{\nu\lambda}^{\mu} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} - \Gamma_{\nu\lambda}^{\mu} \delta x^\nu \frac{d\delta x^\lambda}{d\tau} - 2\Gamma_{\nu\lambda}^{\mu} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} = \Gamma_{\nu\lambda}^{\mu} \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau}$$

using (*)

$$\Rightarrow \Gamma_{tt}^t = \frac{\dot{A}}{2A}, \quad \Gamma_{rr}^t = \frac{\dot{B}}{2A}, \quad \Gamma_{tr}^t = \frac{A'}{2A}$$

$$\Gamma_{tt}^r = \frac{A'}{2B}, \quad \Gamma_{tr}^r = \frac{\dot{B}}{2B}, \quad \Gamma_{rr}^r = \frac{B'}{2B}, \quad \Gamma_{\theta\theta}^r = -\frac{r}{B}, \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

$$\Gamma_{r\theta}^\theta = \frac{1}{r}, \quad \Gamma_{\varphi\varphi}^\theta = -\sin \theta \cos \theta, \quad \Gamma_{r\varphi}^\varphi = \frac{1}{r}, \quad \Gamma_{\varphi\theta}^\varphi = \cot \theta$$

with other components
given by symmetry or
equal to zero

$$\Rightarrow R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{B^2} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}; \quad R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A\dot{B}'}{4AB} + \frac{B'}{B^2} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB}$$

$$R_{tr} = \frac{\dot{B}}{B^2}, \quad R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}, \quad R_{\varphi\varphi} = \sin^2 \theta R_{\theta\theta}$$

James Hartle

$$\Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

with other components
given by Symmetry $\Gamma_{\mu\nu}^\alpha = \Gamma_{\nu\mu}^\alpha$
or equal to zero.

$$= -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{B^2} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB}$$

$$= 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}, \quad R_{\varphi\varphi} = \sin^2 \theta R_{\theta\theta},$$

with
other components
zero

$$= \frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau}$$

↓ using (*)

$$- \Gamma_{\nu\lambda\rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - \Gamma_{\alpha\beta}^\lambda \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\Rightarrow \Gamma_{tt}^t = \frac{\dot{A}}{2A}, \quad \Gamma_{rr}^t = \frac{\dot{B}}{2A}, \quad \Gamma_{tr}^t = \frac{A'}{2A}$$

$$\Gamma_{tt}^r = \frac{A'}{2B}, \quad \Gamma_{tr}^r = \frac{\dot{B}}{2B}, \quad \Gamma_{rr}^r = \frac{B'}{2B}, \quad \Gamma_{\theta\theta}^r = -\frac{r}{B}, \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

$$\Gamma_{r\theta}^\theta = \frac{1}{r}, \quad \Gamma_{\varphi\varphi}^\theta = -\sin \theta \cos \theta, \quad \Gamma_{r\varphi}^\varphi = \frac{1}{r}, \quad \Gamma_{\varphi\theta}^\varphi = \tan \theta$$

with other components
given by symmetry $\Gamma_{\mu\nu}^\mu =$
or equal to zero

$$\Rightarrow R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}; \quad R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB}$$

$$R_{tr} = \frac{\dot{B}}{Br}, \quad R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}, \quad R_{\varphi\varphi} = \sin^2 \theta R_{\theta\theta}$$

$$= \frac{d^2 \delta x^\mu}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \frac{d\delta x^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{d^2 x^\lambda}{d\tau^2} + \Gamma_{\nu\lambda}^\mu \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \frac{d\delta x^\lambda}{d\tau} + \Gamma_{\nu\lambda}^\mu \delta x^\nu \delta x^\lambda \frac{d^2}{d\tau^2}$$

↓ using (*)

$$- \Gamma_{\nu\lambda\rho}^\mu \delta x^\rho \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} - 2 \Gamma_{\nu\lambda}^\mu \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau} - \Gamma_{\nu\lambda}^\mu \frac{d\delta x^\nu}{d\tau} \frac{d\delta x^\lambda}{d\tau}$$

$$\Rightarrow \Gamma_{tt}^t = \frac{\dot{A}}{2A}, \quad \Gamma_{rr}^t = \frac{\dot{B}}{2A}, \quad \Gamma_{tr}^t = \frac{A'}{2A}$$

$$\Gamma_{tt}^r = \frac{A'}{2B}, \quad \Gamma_{tr}^r = \frac{\dot{B}}{2B}, \quad \Gamma_{rr}^r = \frac{B'}{2B}, \quad \Gamma_{\theta\theta}^r = -\frac{r}{B}, \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

$$\Gamma_{r\theta}^\theta = \frac{1}{r}, \quad \Gamma_{\varphi\varphi}^\theta = -\sin \theta \cos \theta, \quad \Gamma_{r\varphi}^\varphi = \frac{1}{r}, \quad \Gamma_{\varphi\theta}^\varphi = \tan \theta$$

with other components
given by symmetry $\Gamma_{\mu\nu}^\mu =$
or equal to zero

$$\Rightarrow R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}; \quad R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB}$$

$$R_{tr} = \frac{\dot{B}}{Br}, \quad R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}, \quad R_{\varphi\varphi} = \sin^2 \theta R_{\theta\theta}$$

$$- \Gamma_{\nu\lambda\rho}^{\mu} \delta x^{\rho} \frac{dx^{\nu}}{d\lambda} \frac{dx^{\lambda}}{d\tau} - 2 \delta x^{\mu} \frac{dx^{\nu}}{d\tau} \frac{d\delta x^{\nu}}{d\tau}$$

$$\Rightarrow \Gamma_{tt}^t = \frac{\dot{A}}{2A}, \quad \Gamma_{rr}^t = \frac{\dot{B}}{2A}, \quad \Gamma_{tr}^t = \frac{A'}{2A}$$

$$\Gamma_{tt}^r = \frac{A'}{2B}, \quad \Gamma_{tr}^r = \frac{\dot{B}}{2B}, \quad \Gamma_{rr}^r = \frac{B'}{2B}, \quad \Gamma_{\theta\theta}^r = -\frac{r}{B}, \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

$$\Gamma_{r\theta}^{\theta} = \frac{1}{r}, \quad \Gamma_{\varphi\varphi}^{\theta} = -\sin \theta \cos \theta, \quad \Gamma_{r\varphi}^{\varphi} = \frac{1}{r}, \quad \Gamma_{\varphi\theta}^{\varphi} = \tan \theta$$

$$\Rightarrow R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A^{12}}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}; \quad R_{rr} = -\frac{A''}{2A} + \frac{A^{12}}{4A^2}$$

$$R_{tr} = \frac{\dot{B}}{Br} \Leftarrow \quad R_{\theta\theta} = 1 - \frac{1}{B} +$$

$$\frac{A'}{2A}, \quad \Gamma_{\theta\theta}^r = -\frac{r}{B}, \quad \Gamma_{\varphi\varphi}^r = -\frac{r \sin^2 \theta}{B}$$

$$, \quad \Gamma_{\varphi\theta}^\varphi = \cot \theta$$

with other components
given by symmetry $\Gamma_{\mu\nu}^\alpha =$
or equal to zero.

$$+ \frac{\dot{A}\dot{B}}{4AB}; \quad R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{B^2} - \frac{B''}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{B'^2}{4AB}$$

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}, \quad R_{\varphi\varphi} = \sin^2 \theta R_{\theta\theta}.$$

$R_{tr}: \dot{B}=0 \Rightarrow B(r)$, all time derivatives disappear!

$$R_{tt} + \frac{A}{B} R_{rr} \quad \frac{A'}{B} + \frac{B'A}{B^2} = 0 \quad \Rightarrow (AB)' = 0 \quad \Rightarrow A = \frac{f(t)}{B}$$

$$R_{\theta\theta}: \quad 1 - \frac{1}{B} + r \frac{B'}{B^2} = 0 \quad \Rightarrow \left(\frac{r}{B} \right)' = 1 \quad \Rightarrow \frac{r}{B} = r + K$$

appear!

$$\Rightarrow A = \frac{f(t)}{B}$$

$$\Rightarrow \frac{r}{B} = r + \overset{\substack{\text{constant} \\ \downarrow}}{K} \Rightarrow B = \frac{1}{1 + \frac{K}{r}} \Rightarrow A = f(t) \left(1 + \frac{K}{r}\right)$$

$\Rightarrow$ redefine t to set $f(t)$ to unity $\left(f(t)dt^2 = dt'^2 \right)$

$$\Rightarrow ds^2 = -\left(1 + \frac{k}{r}\right)dt^2 + \frac{1}{\left(1 + \frac{k}{r}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

nity

$$\left(f(t) dt^2 = dt'^2 \right)$$

$$- A dt^2 + B dr^2 + \dots$$

$$(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$\int_0^t f(t) dt = t^2$$