Title: 3d Gravity, Universality and Poincare Series

Date: Sep 30, 2014 02:00 PM

URL: http://pirsa.org/14090005

Abstract: Modular invariance plays an important role in the AdS3/CFT2 correspondence. Using modular invariance, I discuss under what conditions a 2d CFT shows a Hawking-Page phase transition in the large c limit, and what this implies for the range of validity of the Cardy formula and the universality of its spectrum. I will also discuss partition functions obtained by summing over the modular group, how their properties are compatible with their gravity interpretation, and briefly touch on implications for the existence of pure gravity.

3D gravity, Universality + Poincaré Series

1405.5137 w. T. Hartman, B Stoica
1407.6008 w. A Maloney

3D gravity, Universality + Poincaré Series

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"modular invariance and 3D gravity"

$\gamma \in SL(2,\mathbb{Z})$: $\boxed{S : \tau \mapsto -\frac{1}{\tau}}$

$T \quad \tau \mapsto \tau + 1$

$Z(\gamma\tau) = Z(\tau)$

- "light spectrum determines heavy spectrum"

famous: vacuum

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famous: vacuum $\longrightarrow$ Cardy behavior $\rho \sim \exp 2\pi \sqrt{cE/3}$ $\leftarrow$ asymptotically

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Goal: make this slogan more precise

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: microscopic explanation BH entropy

regime" : c fixed (small!) $\Rightarrow$ $E \ggg c$ ← completely universal

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$$\boxed{AdS_3/CFT_2}$$

...e this slogan more precise

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$\Rightarrow$ necessary condition on a CFT s.t.
it can serve as a holographic dual

- no angular momentum : $\tau = \frac{i\beta}{2\pi}$ $\left(S : \beta \mapsto \beta' = \frac{4\pi^2}{\beta} \right)$

$Z(\beta) = \sum e^{-\beta E}$

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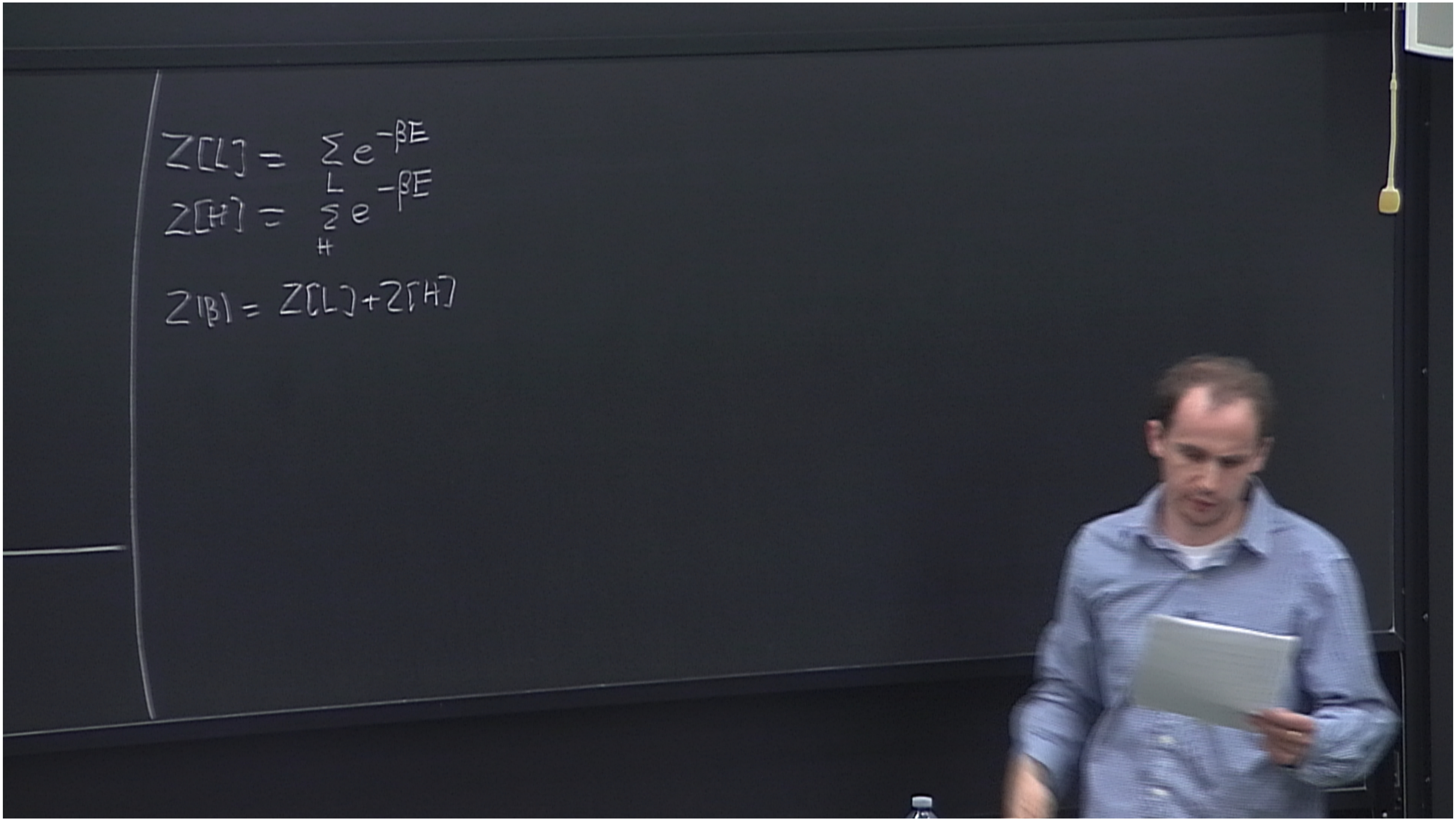
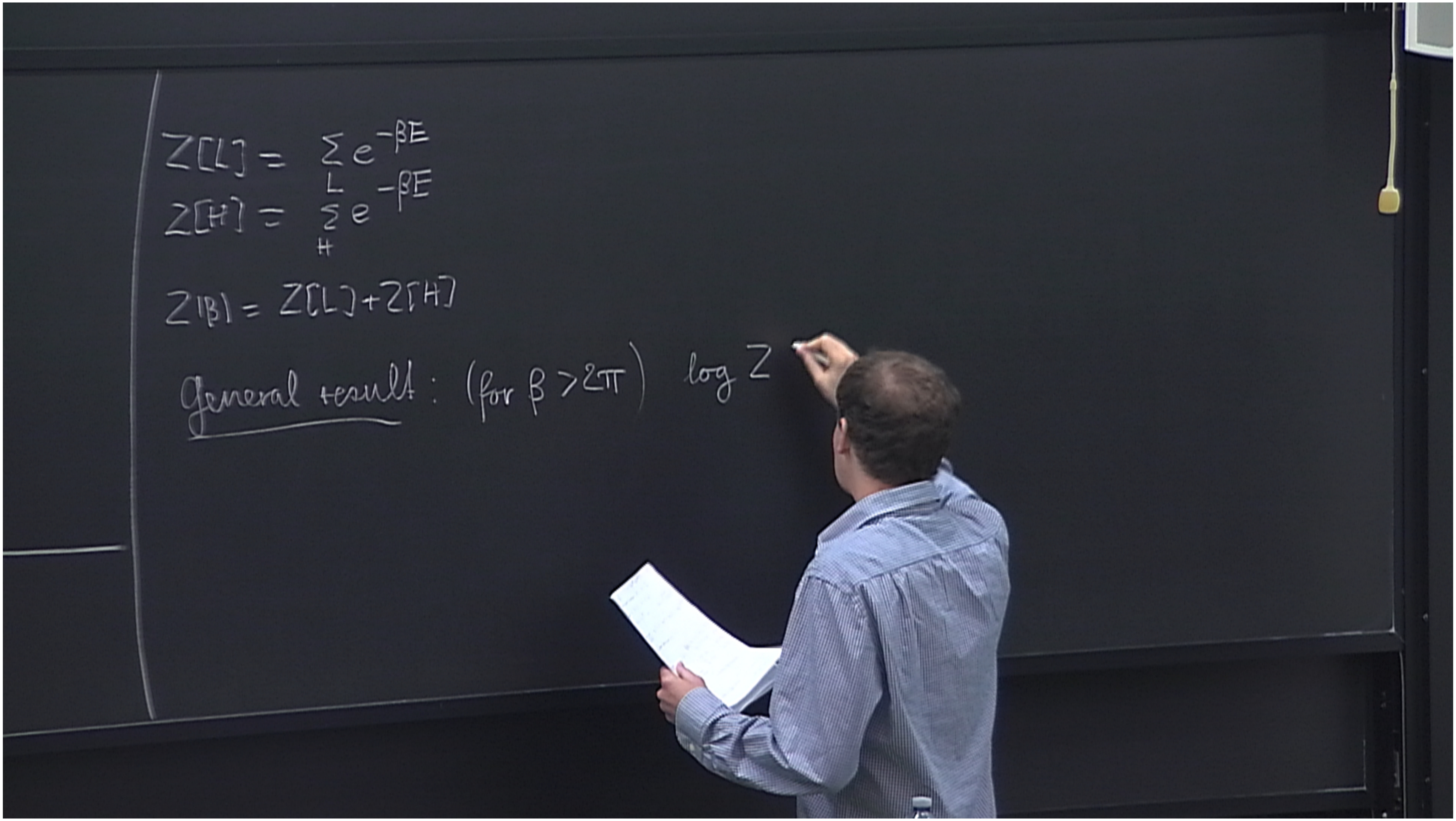
light $L = \{ E \leq \varepsilon \}$ heavy $H = \{ E > \varepsilon \}$



$Z[L] = \sum_{L} e^{-\beta E}$
$Z[H] = \sum_{H} e^{-\beta E}$
$Z(\beta) = Z[L] + Z[H]$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General result: (for $\beta > 2\pi$) $\log Z$

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General result: (for $\beta > 2\pi$) $\log Z[L] \leq \log Z \leq \log Z[L] - \log\left(1 - e^{(\beta' - \beta)\underline{\varepsilon}}\right)$

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large c limit:

$$\log Z(\beta) = \begin{cases} \log Z[L] + O(1) & \beta > 2\pi \\ \log Z'[L] + O(1) & \beta < 2\pi \end{cases}$$

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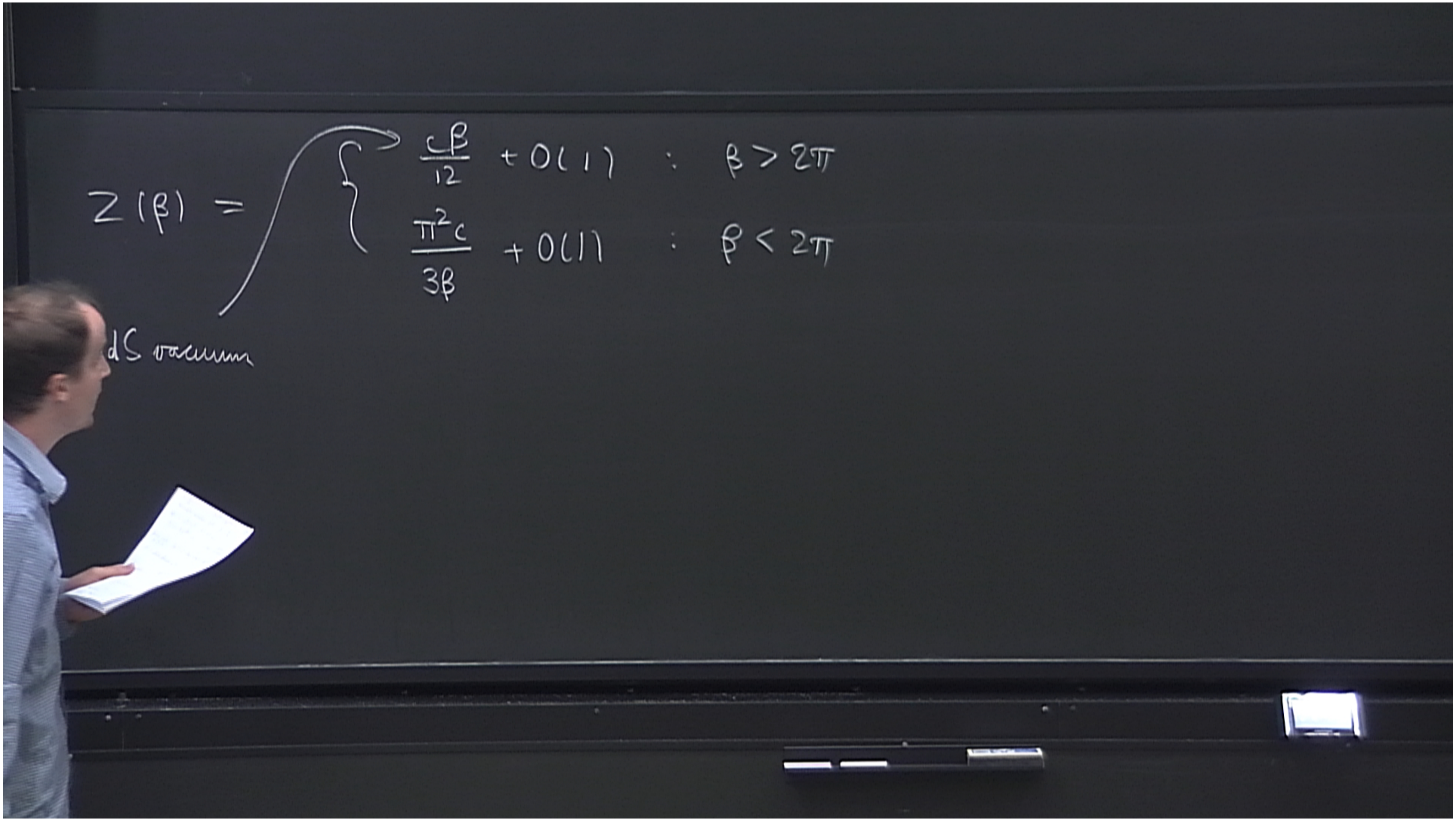
$Z(\beta) = \begin{cases} \frac{c\beta}{12} + O(1) & : \beta > 2\pi \\ \frac{\pi^2 c}{3\beta} + O(1) & : \beta < 2\pi \end{cases}$
dS vacuum

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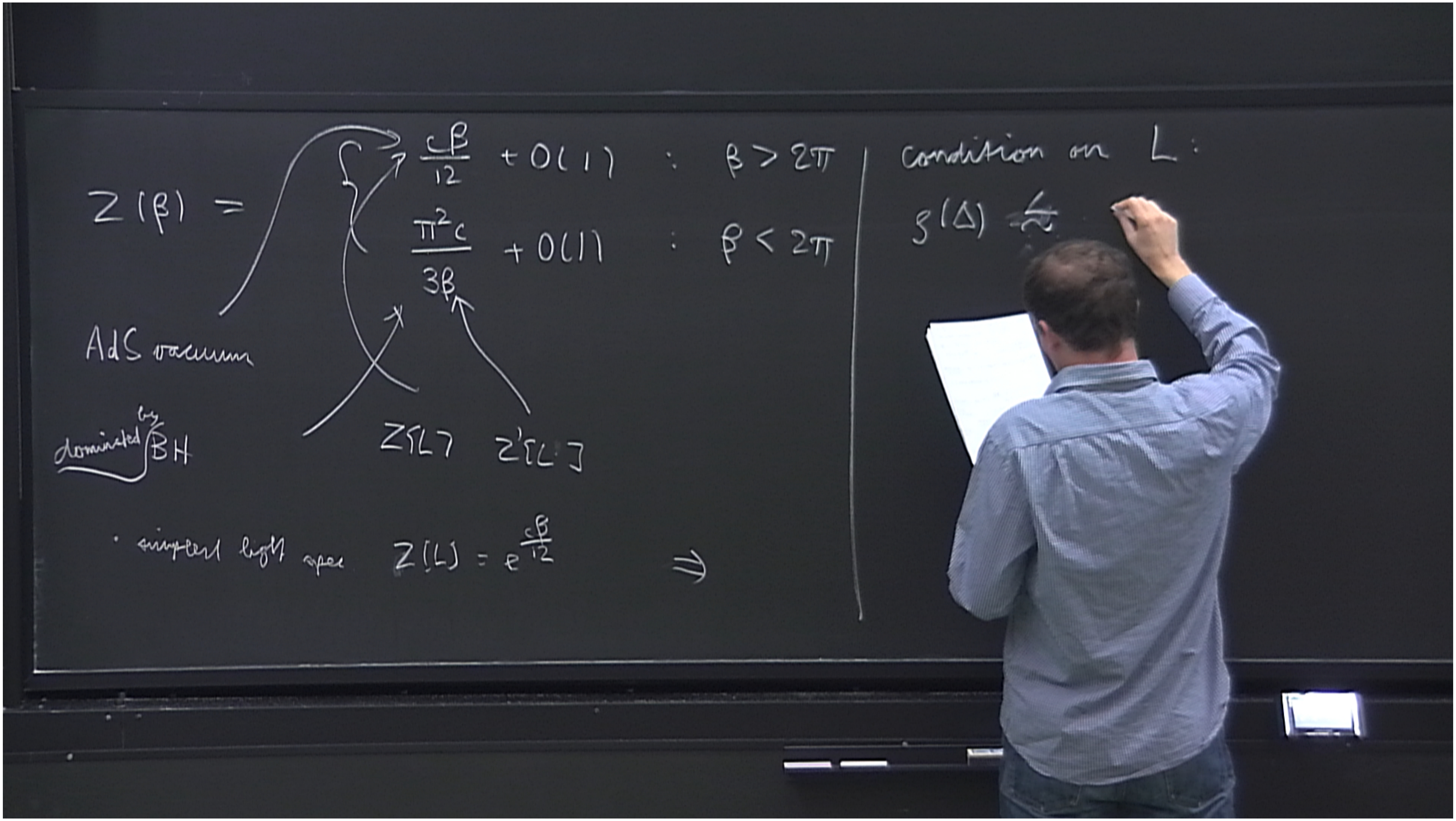
AdS vacuum

dominated by BH


Z(β) =
cβ/12 + O(1) : β > 2π
π²c/3β + O(1) : β < 2π
AdS vacuum
dominated by BH
Z[L] Z'[L']
simplest light spec Z[L] = e^{cβ/12} ⇒
condition on L:
ρ(Δ)

condition on L:

$$\boxed{\rho(\Delta) \lesssim \exp(2\pi\Delta) \quad : \begin{array}{l} \Delta \le \frac{c}{12} + \varepsilon \\ E \le \varepsilon \end{array}}$$

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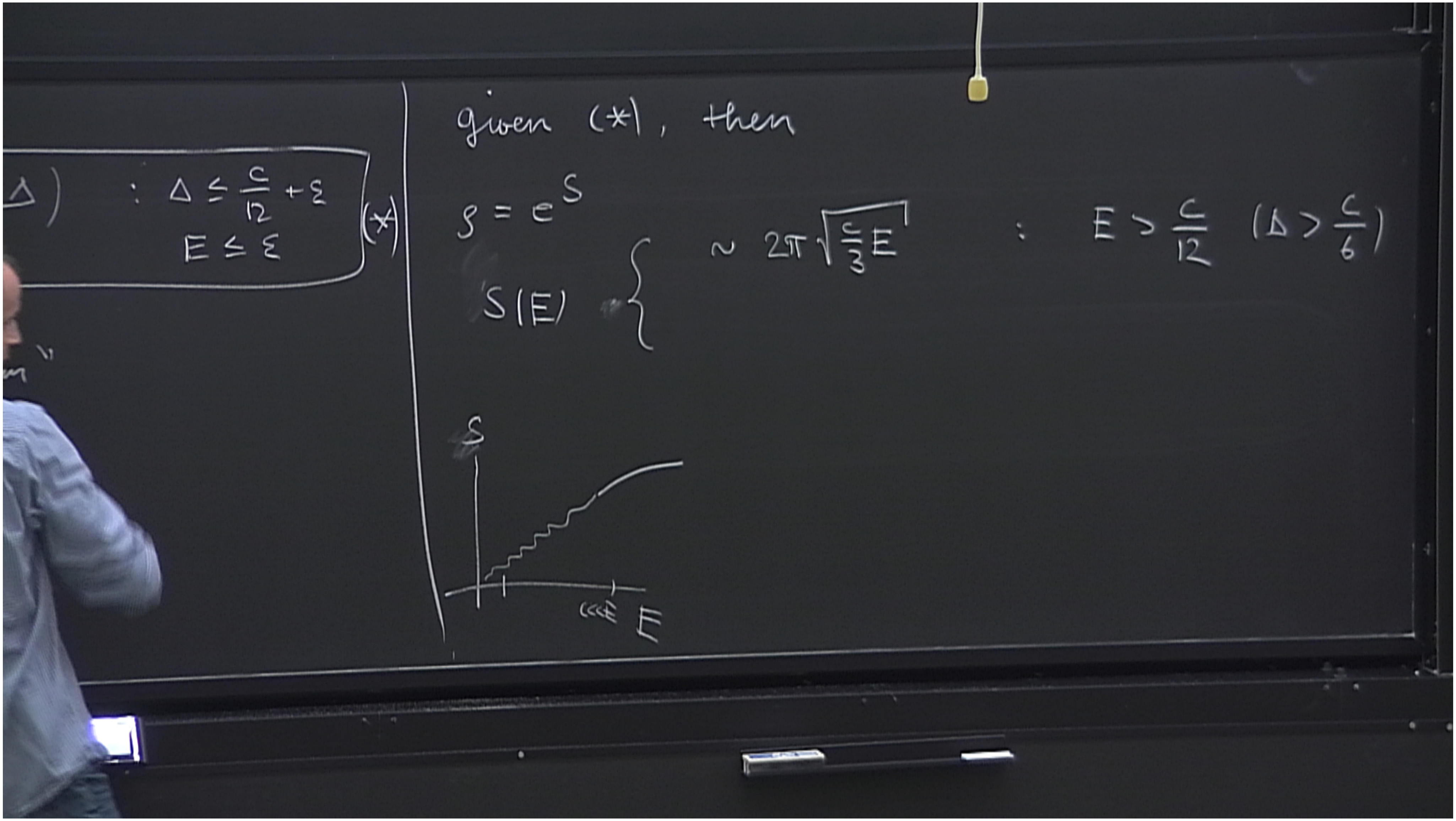
"sparse light spectrum"


$\Delta)$: $\Delta \leq \frac{c}{12} + \varepsilon$
$E \leq \varepsilon$ $(*)$
given $(*)$, then
$\rho = e^S$
$S(E) \begin{cases} \sim 2\pi\sqrt{\frac{c}{3}E} & : E > \frac{c}{12} \quad (\Delta > \frac{c}{6}) \end{cases}$
S
E

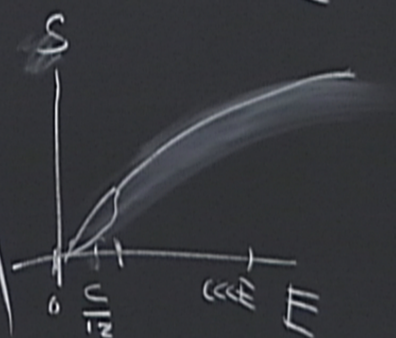
$$\Delta) \quad : \Delta \le \frac{c}{12} + \varepsilon$$
$$E \le \varepsilon \qquad (*)$$

m"

given $(*)$, then

$$\rho = e^{S}$$

$$S(E) \begin{cases} \sim 2\pi\sqrt{\frac{c}{3}E} & : \; E > \frac{c}{12} \quad \left(\Delta > \frac{c}{6}\right) \\ \lesssim 2\pi E + \frac{\pi c}{6} & : \; 0 < E < \frac{c}{12} \end{cases}$$



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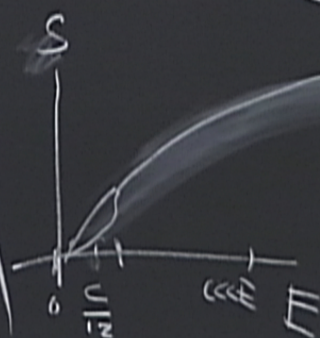
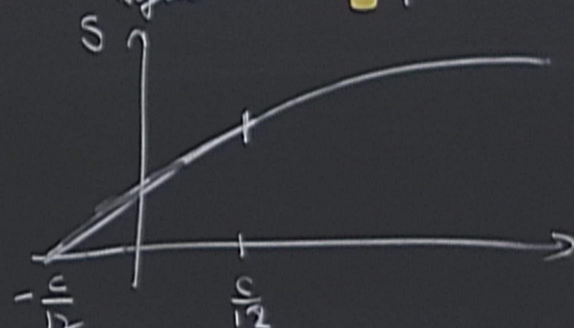


symmetric orbifold



$$Z(\beta_L, \beta_R) = \sum e^{-E_L \beta_L} e^{-E_R \beta_R}$$

$$\hat{R}$$

$$E = E_L + E_R$$

$$Z(\beta_L, \beta_R) = \sum e^{-E_L \beta_L} e^{-E_R \beta_R}$$

$$\hat{\mathbb{R}}$$

$$S : \begin{cases} \beta_L \mapsto \dfrac{4\pi^2}{\beta_L} \\ \beta_R \mapsto \dfrac{4\pi^2}{\beta_R} \end{cases}$$

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$\uparrow$ $\mathbb{R}$ $\qquad$ $\uparrow$ invariant under $\to S : \begin{cases} \beta_L \mapsto \dfrac{4\pi^2}{\beta_L} \\ \beta_R \mapsto \dfrac{4\pi^2}{\beta_R} \end{cases}$

$$E = E_L + E_R$$

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$\uparrow$ IR $\quad \uparrow$ invariant under

$$S : \begin{cases} \beta_L \mapsto \dfrac{4\pi^2}{\beta_L} \\ \beta_R \mapsto \dfrac{4\pi^2}{\beta_R} \end{cases}$$

$E = E_L + E_R$

$\Delta_L = E_L - \frac{c}{24} \ldots$

$$\boxed{E_L < 0 \text{ or } E_R < 0}$$

"censored spectrum"

condition : (analog $(*)$)

$$\rho(E_L, E_R) \approx \exp\sqrt{4\pi \Delta_L \Delta_R}$$

- $Z(\beta_L, \beta_R) = \sum e^{-E_L \beta_L} e^{-E_R \beta_R}$

$\uparrow$ $\mathbb{R}$ $\quad\uparrow$ invariant under $\to S : \begin{cases} \beta_L \mapsto \frac{4\pi^2}{\beta_L} \\ \beta_R \mapsto \frac{4\pi^2}{\beta_R} \end{cases}$

$E = E_L + E_R$

---

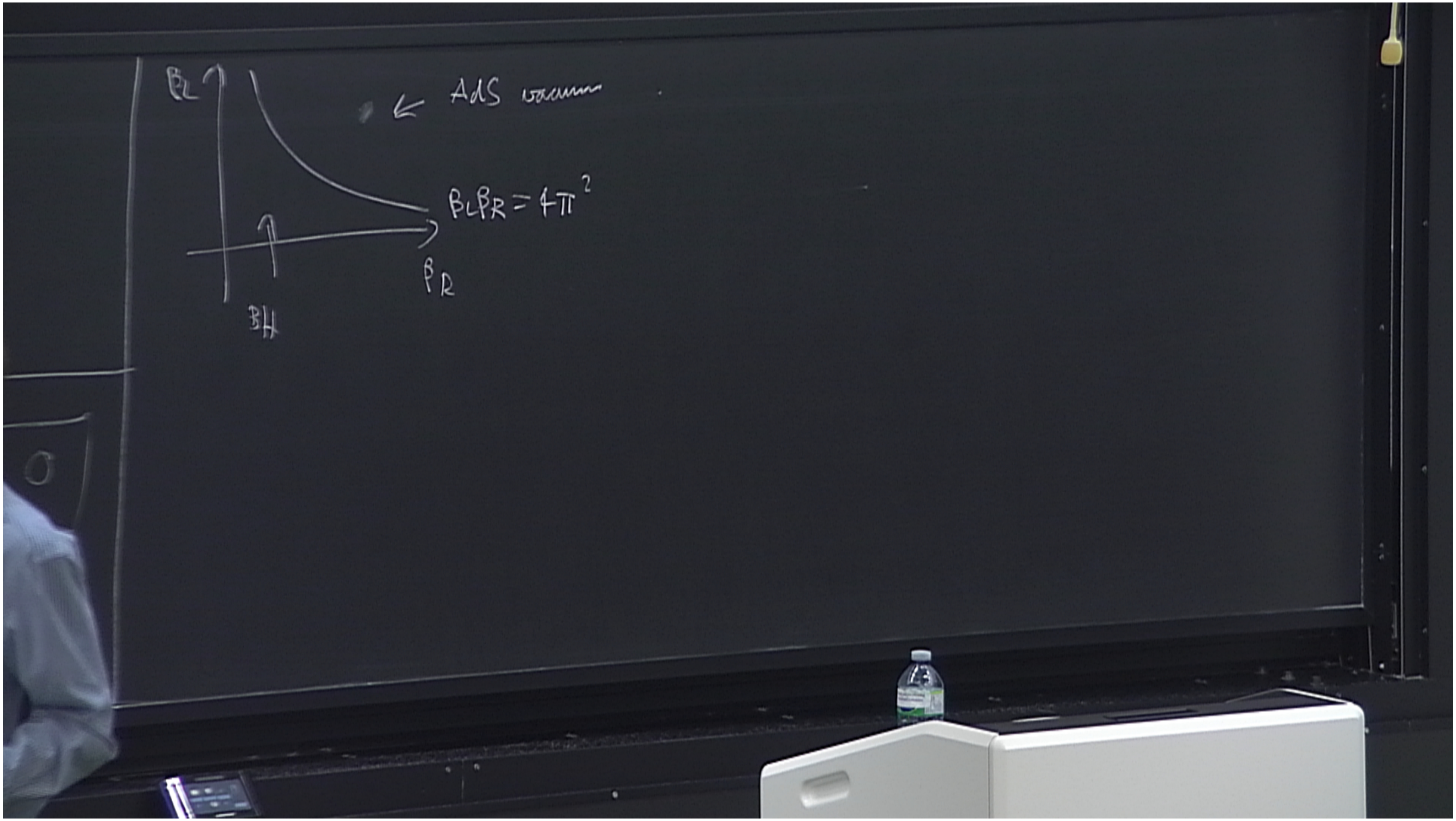
condition (analog $(*)$) : $\quad g(E_L, E_R) \simeq \exp\sqrt{4\pi \Delta_L \Delta_R}$

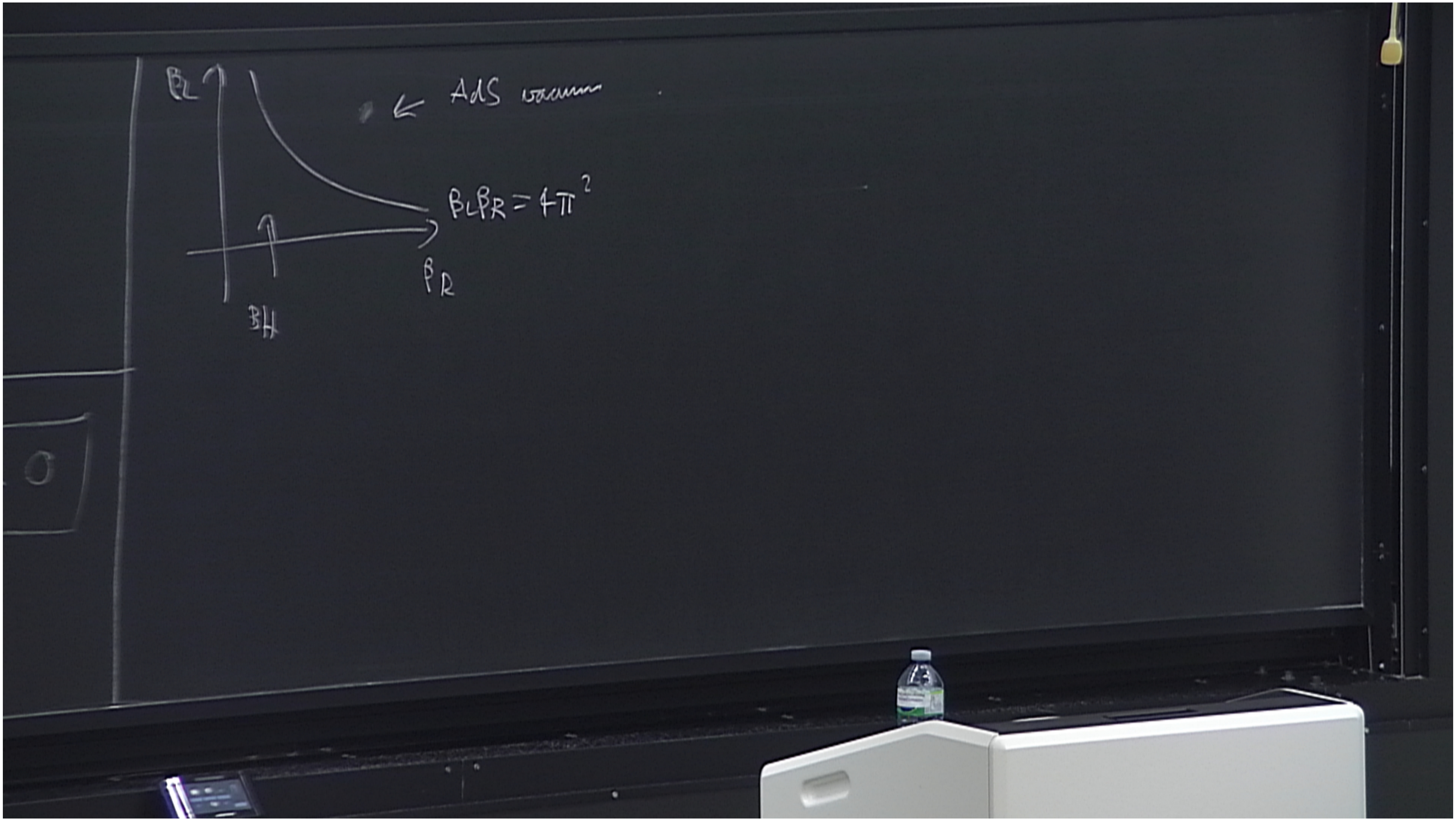
$\Delta_L = E_L - \frac{c}{24} \ldots$

$\boxed{E_L < 0 \text{ or } E_R < 0}$

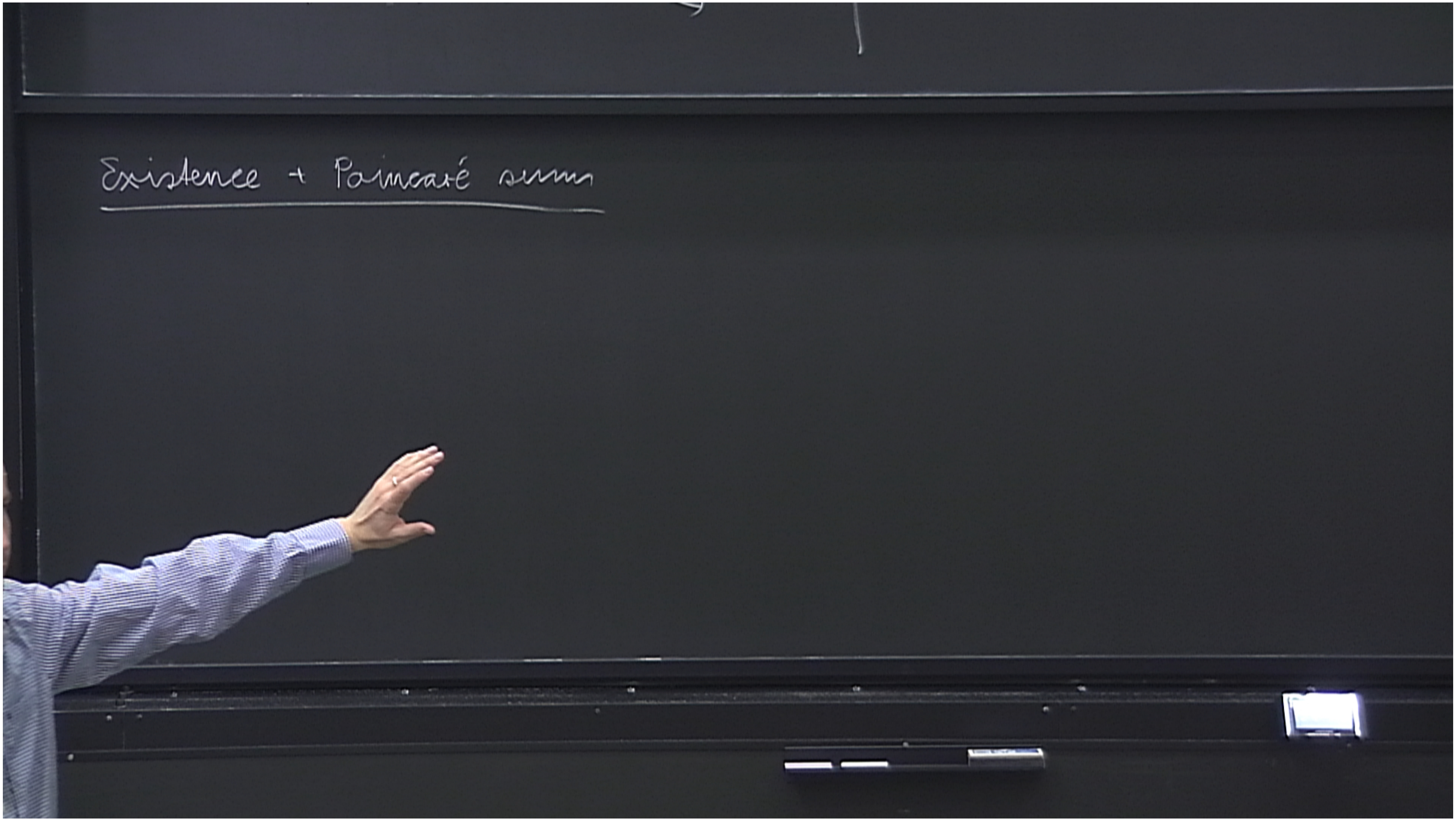
"uncensored spectrum"

$|J| > M\ell$


β_L
AdS vacuum
$\beta_L \beta_R = 4\pi^2$
β_R
BH



βL
AdS vacuum
βLβR = 4π²
βR
BH

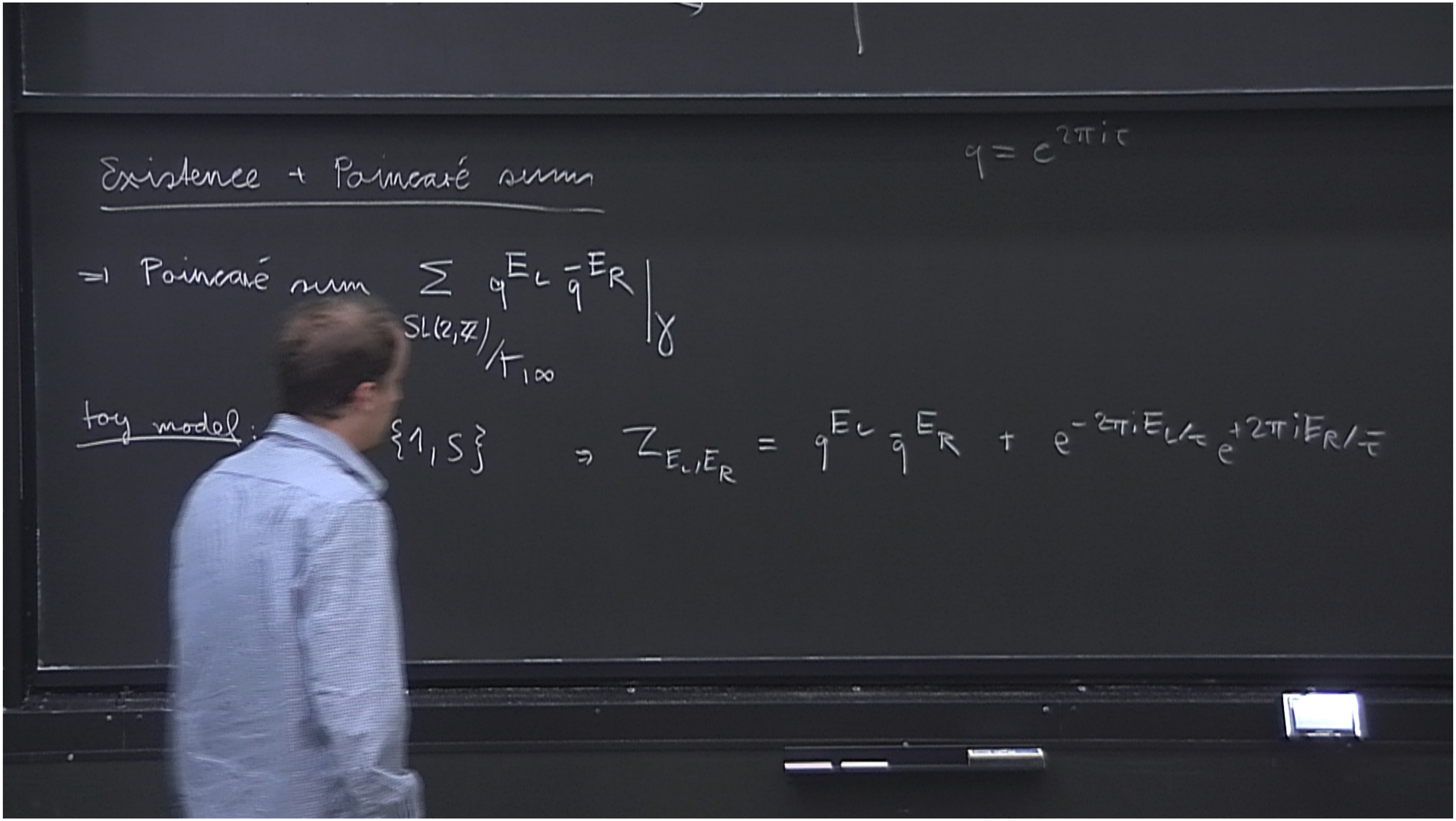

Existence + Poincaré sums

Existence + Poincaré sums

$\Rightarrow$ Poincaré sum $\sum_{SL(2,\mathbb{Z})} q^{E_L} \bar{q}^{E_R} \Big|_{\gamma}$

Existence + Poincaré sum

Poincaré sum $\displaystyle\sum_{SL(2,\mathbb{Z})/\Gamma_{\infty}} q^{E_L}\bar{q}^{E_R}\Big|_{\gamma}$


Existence + Poincaré sum
$q = e^{2\pi i \tau}$
⇒ Poincaré sum $\sum_{SL(2,\mathbb{Z})/\Gamma_{\infty}} q^{E_L} \bar{q}^{E_R} \Big|_{\gamma}$
toy model: $\{1, S\}$ ⇒ $Z_{E_L,E_R} = q^{E_L} \bar{q}^{E_R} + e^{-2\pi i E_L/\tau} e^{+2\pi i E_R/\bar{\tau}}$

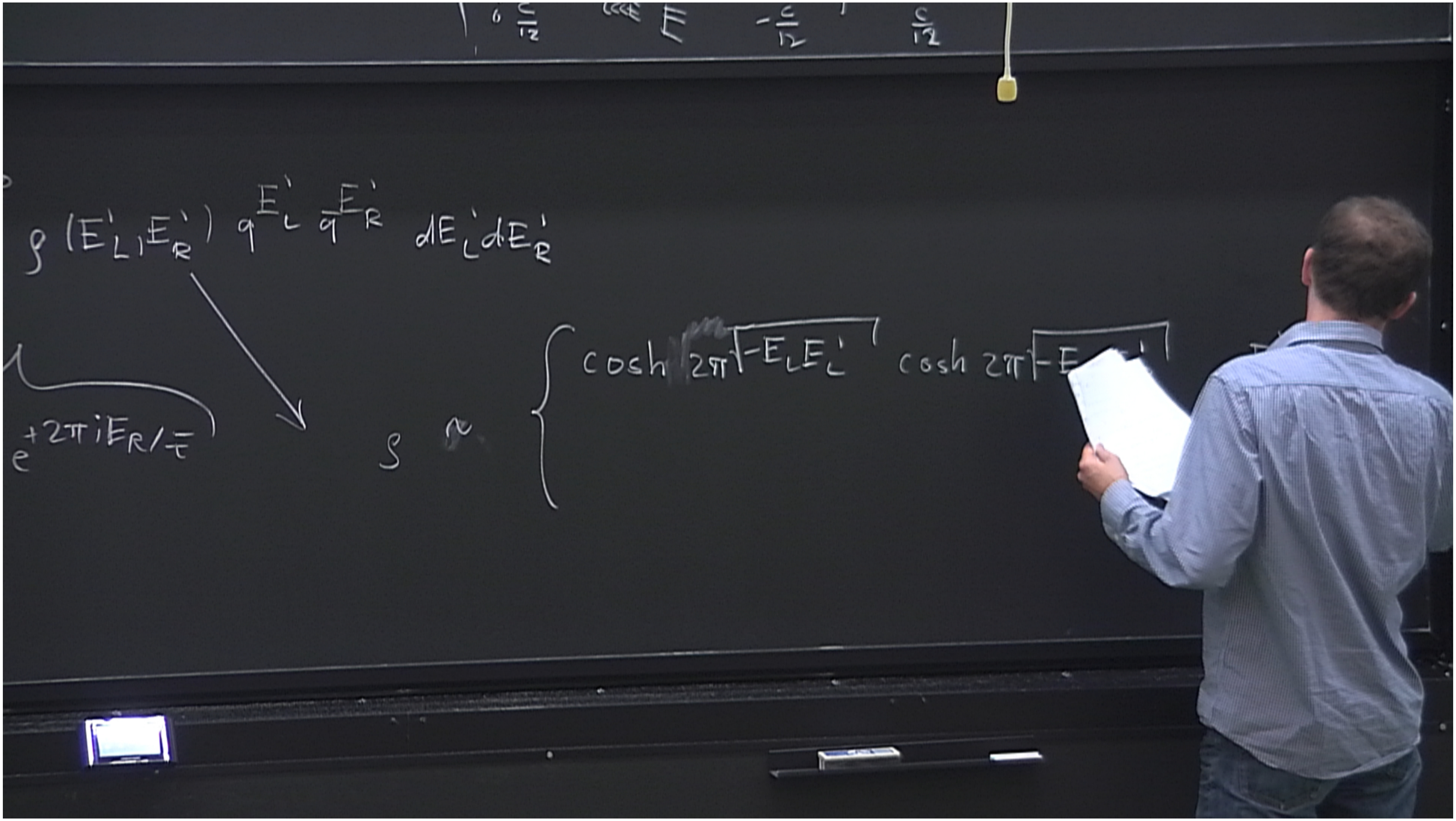
Existence + Poincaré sum

$q = e^{2\pi i \tau}$

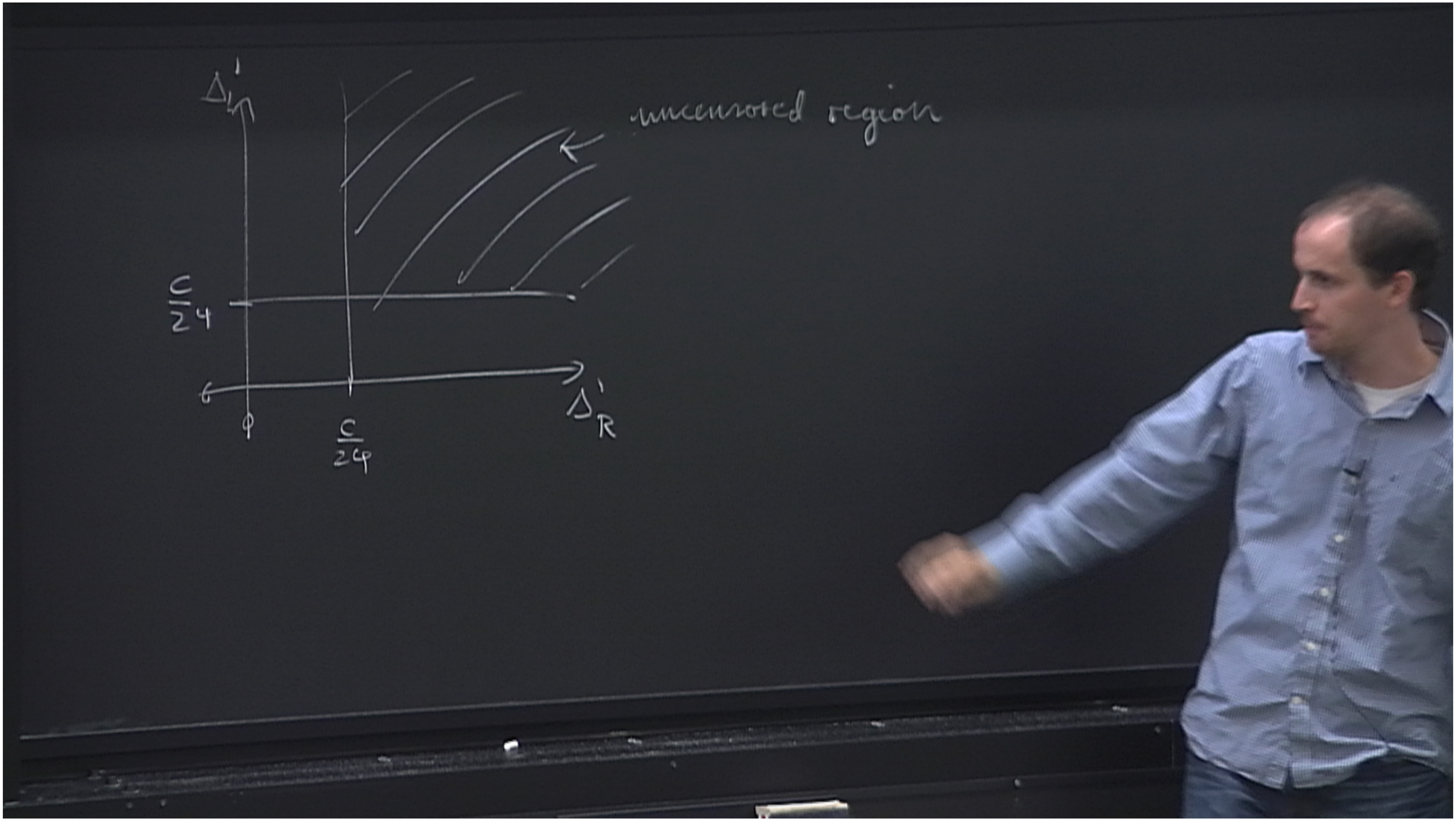
$\int_0^\infty \int_0^\infty \rho(E_L', E_R') \, q$

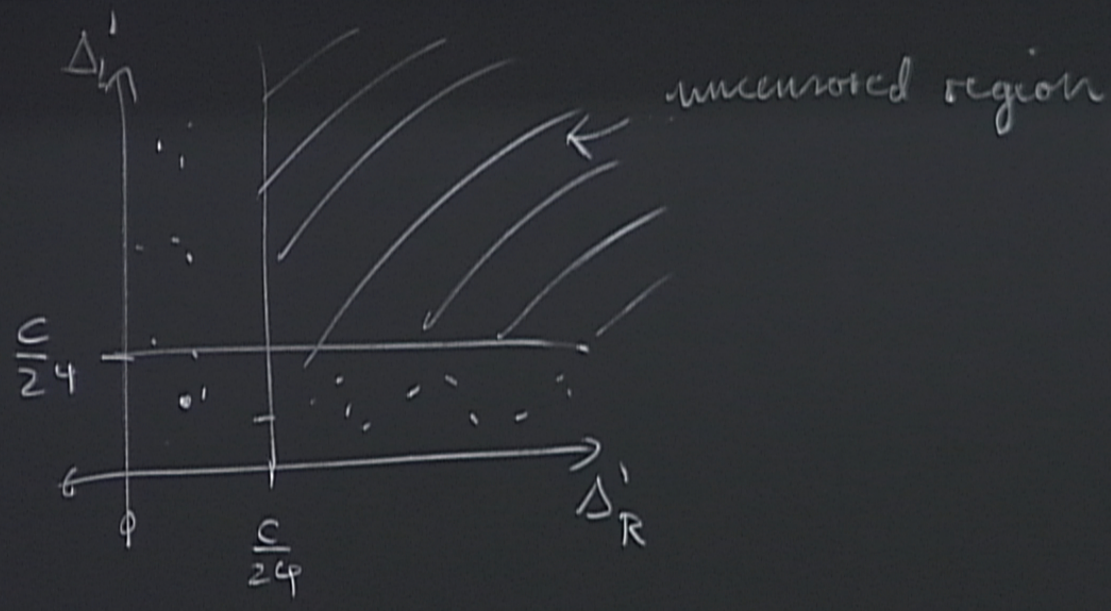
$\Rightarrow$ Poincaré sum $\sum_{SL(2,\mathbb{Z})/\Gamma_\infty} q^{E_L} \bar{q}^{E_R} \Big|_\gamma$

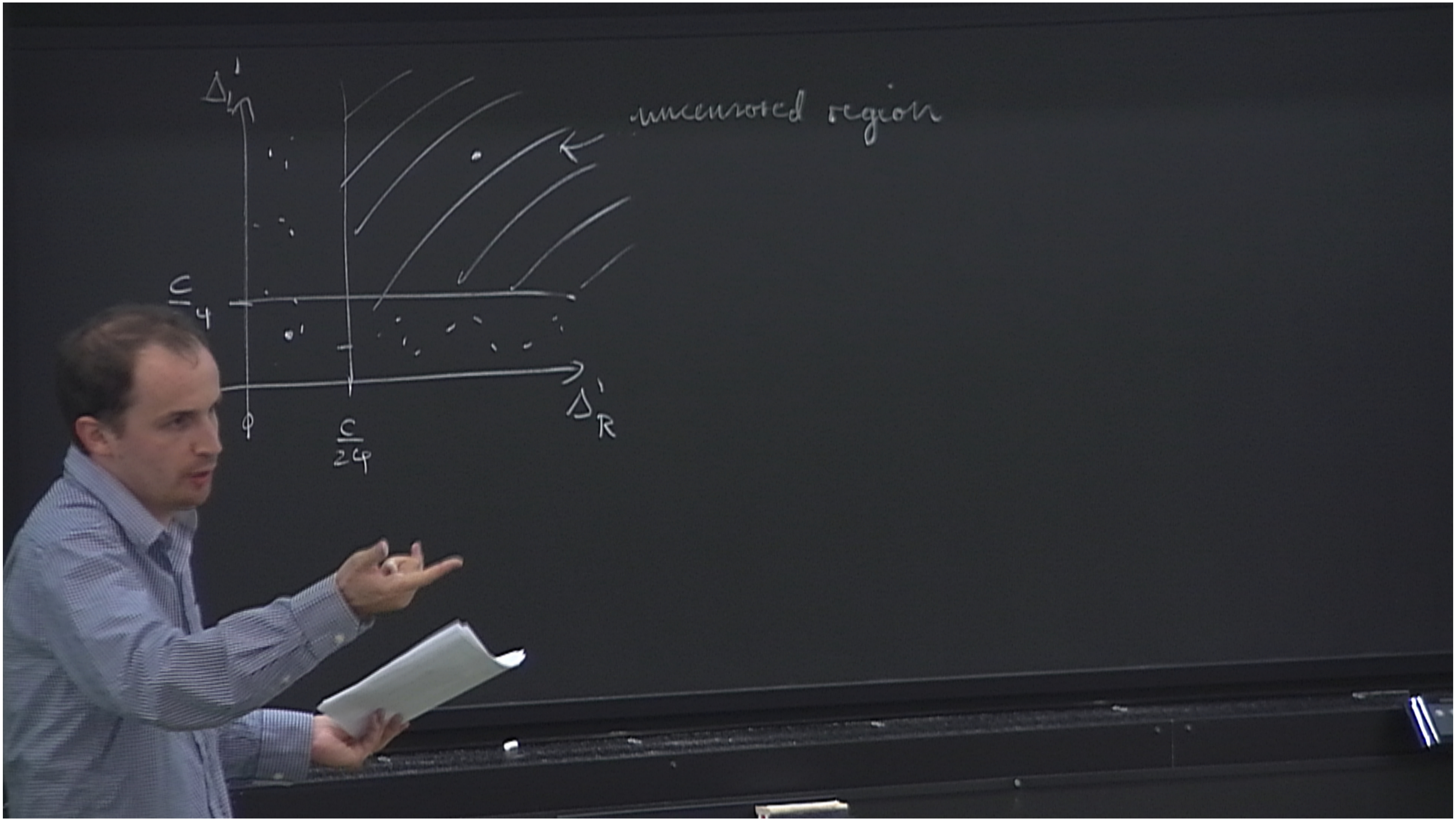
toy model: $\Gamma = \{1, S\}$ $\Rightarrow$ $Z_{E_L, E_R} = q^{E_L} \bar{q}^{E_R} + e^{-2\pi i E_L/\tau} e^{+2\pi i E_R/\bar{\tau}}$

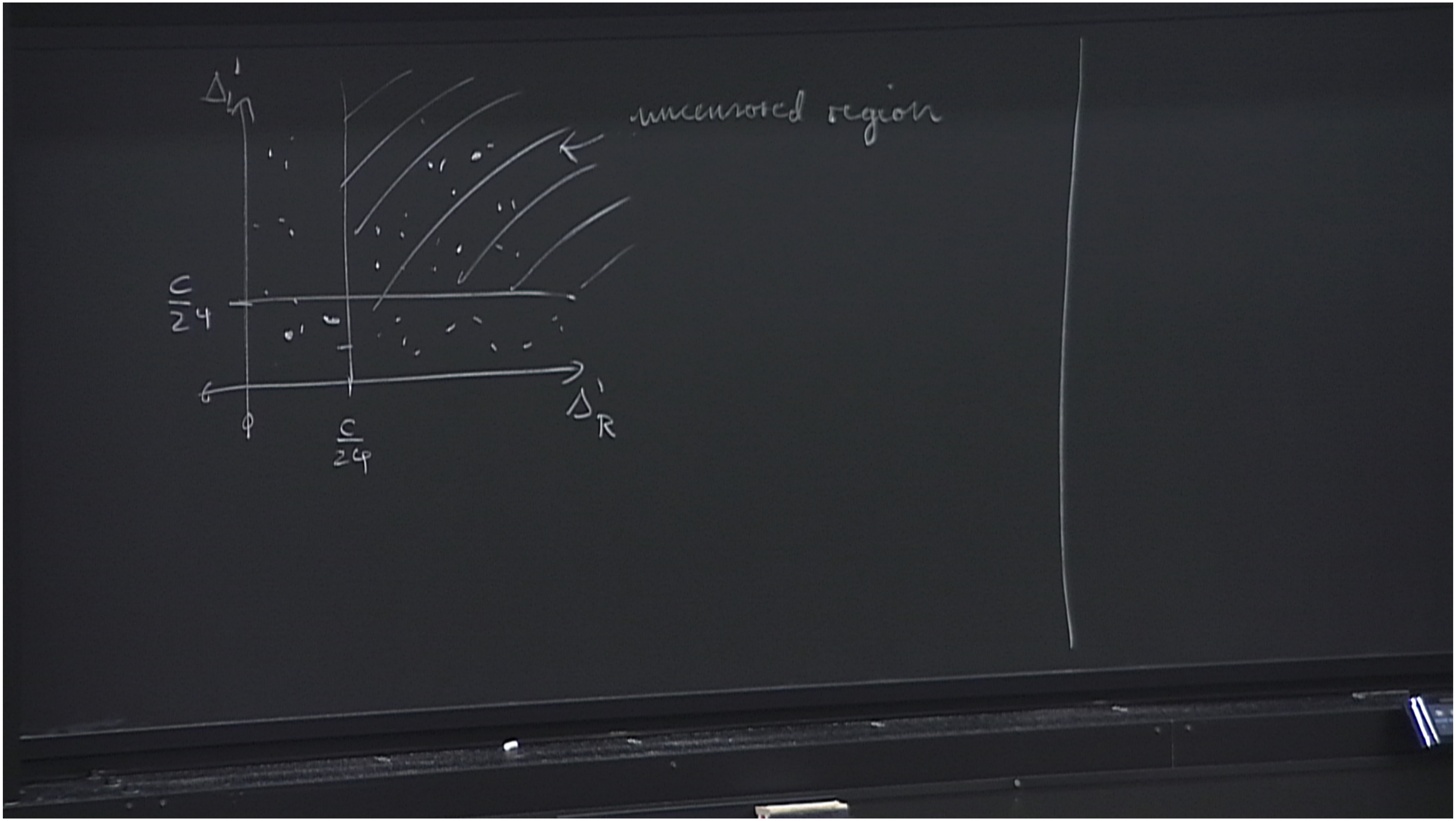


ρ(E'_L, E'_R) q^{E'_L} q̄^{E'_R} dE'_L dE'_R
e^{+2πiE_R/τ}
cosh 2π√(-E_L E'_L) cosh 2π√(-E

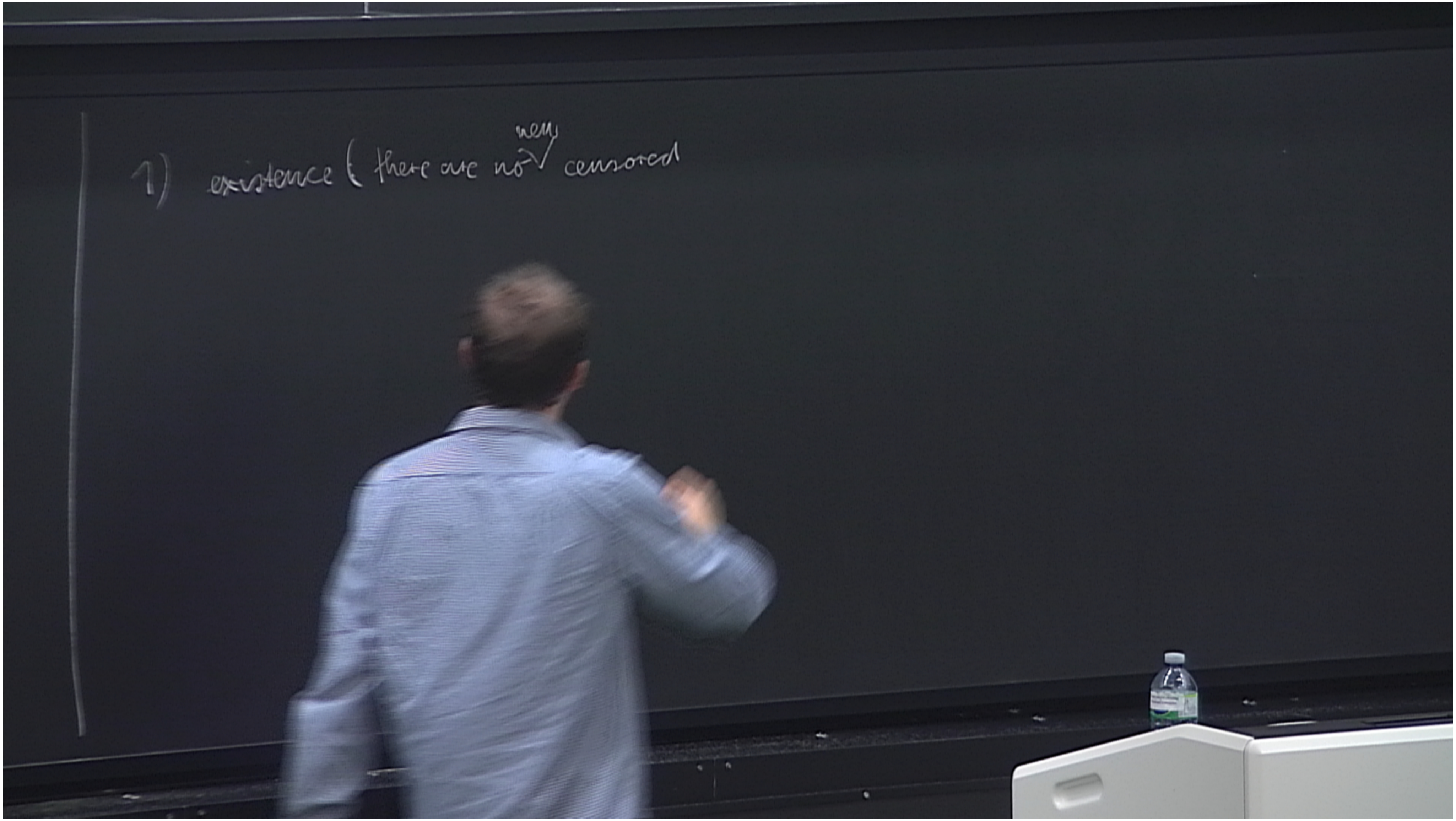

Δ_L'
uncensored region
$\frac{c}{24}$
Δ_R'
0
$\frac{c}{24}$

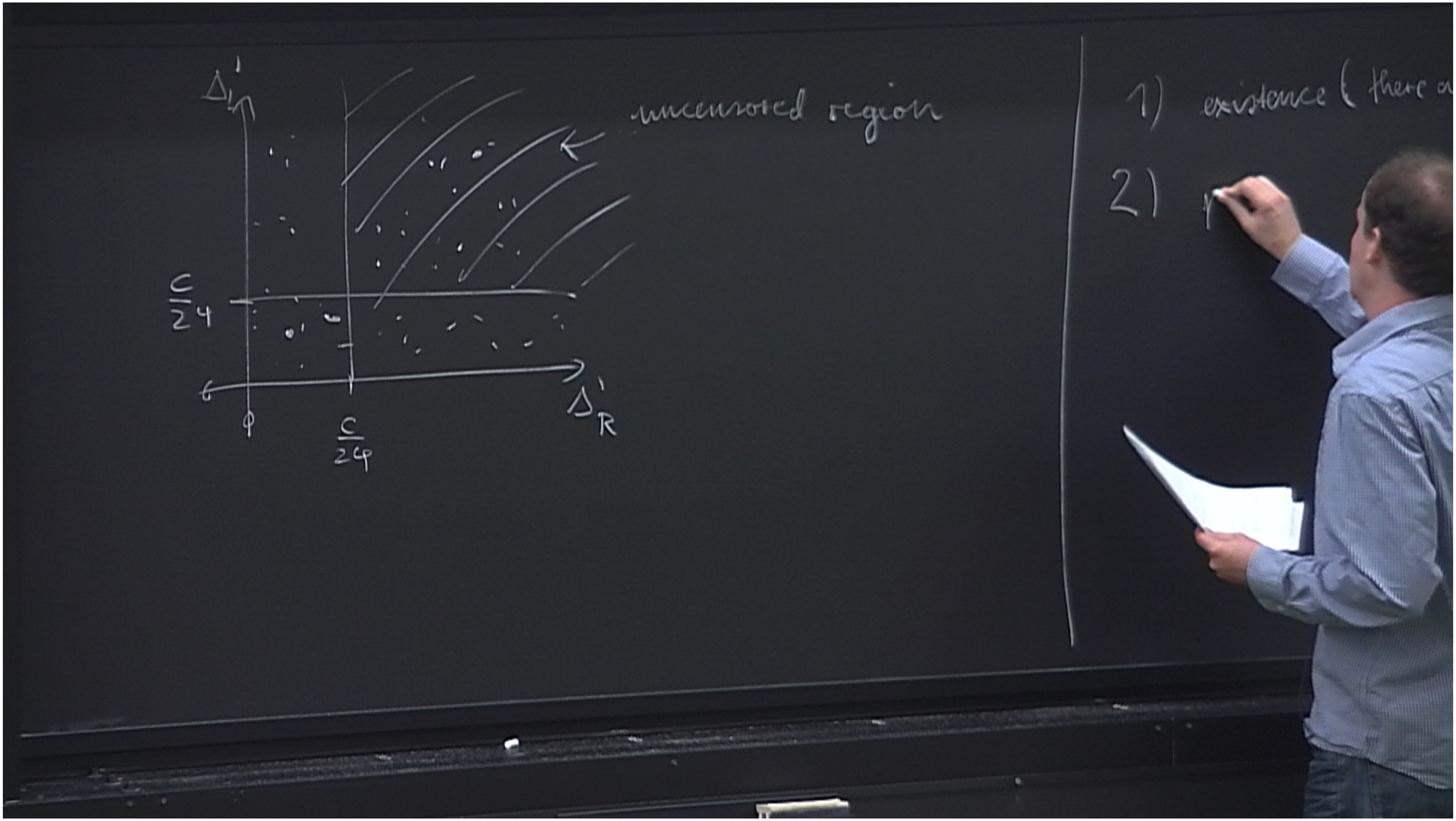

uncensored region
Δ_L'
Δ_R'
$\frac{c}{24}$
$\frac{c}{24}$
0


uncensored region
Δ'_L
Δ'_R
$\frac{c}{4}$
ϕ
$\frac{c}{24}$



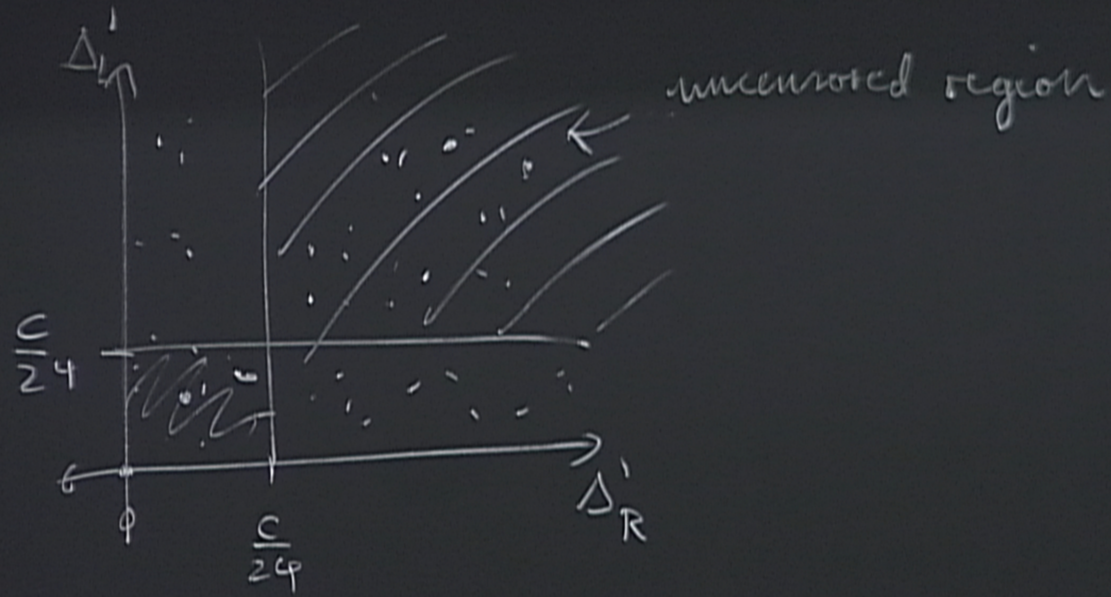
Δ_L
$\frac{c}{24}$
Δ_R
0
$\frac{c}{24}$
uncensored region


1) existence (there are no new censored


uncensored region
1) existence (there a
2)
c
24
c
24
Δ
R
Δ

1) existence (there are no new censored states)

2) positivity, if $E_L, E_R < 0$



1) existence (there are

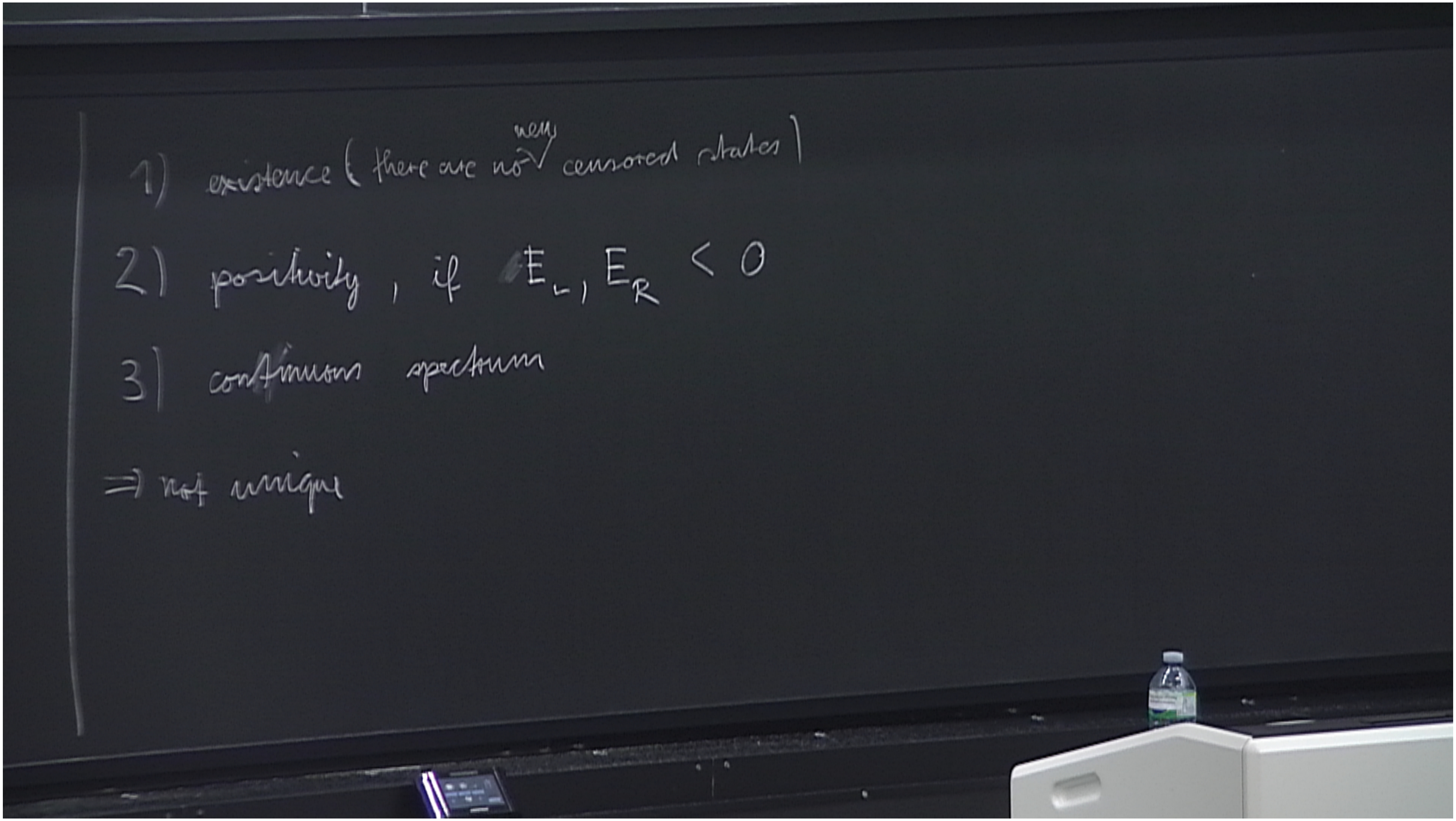
2) positivity , if

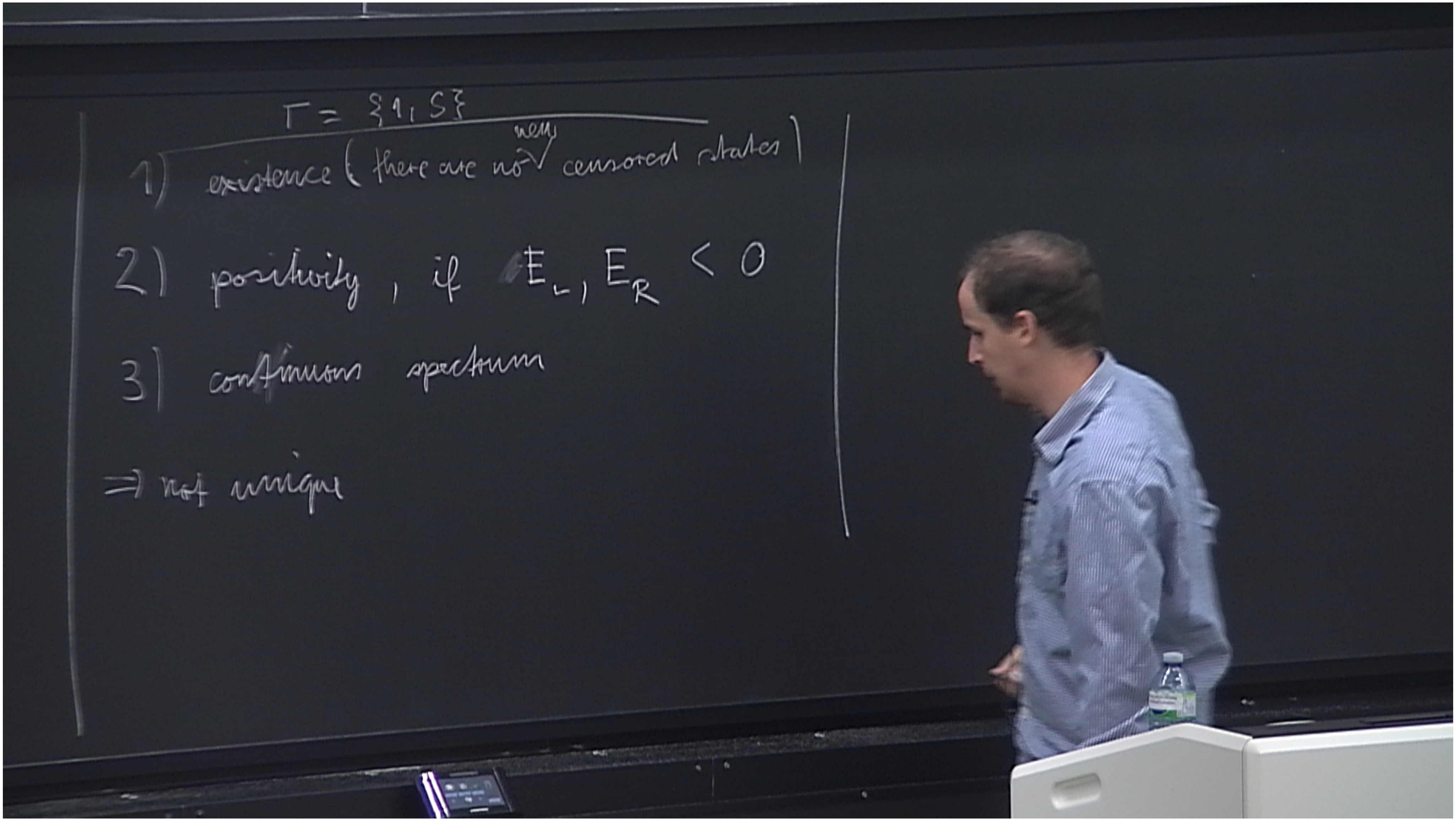
3)

1) existence (there are no new censored states)

2) positivity , if $E_L, E_R < 0$

3) continuous spectrum


1) existence (there are no new censored states)
2) positivity , if E_L , E_R < 0
3) continuous spectrum
⇒ not unique


Γ = {1, S}
1) existence (there are no new censored states)
2) positivity , if E_L , E_R < 0
3) continuous spectrum
⇒ not unique

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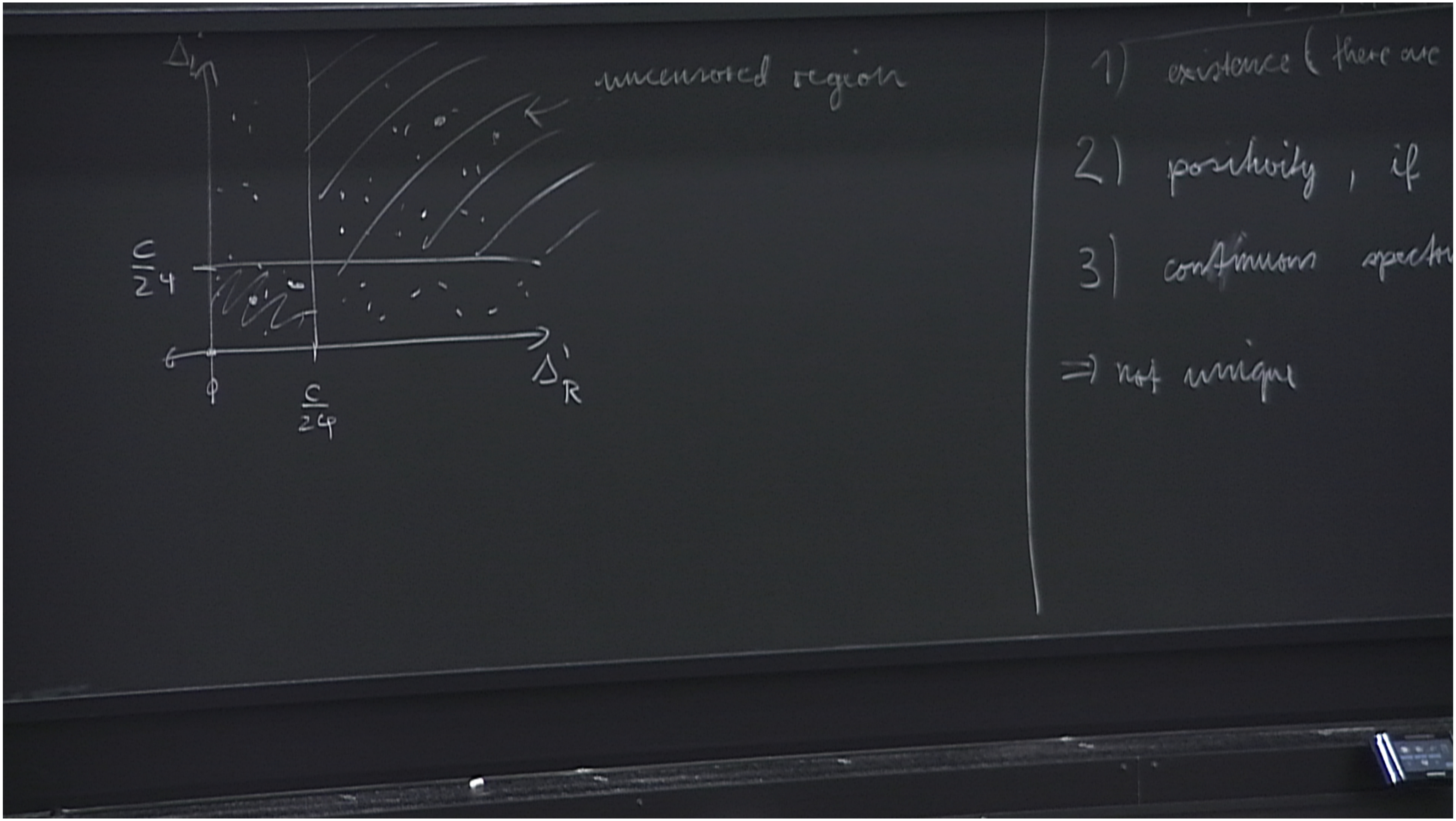
itivity , if $E_L, E_R < 0$

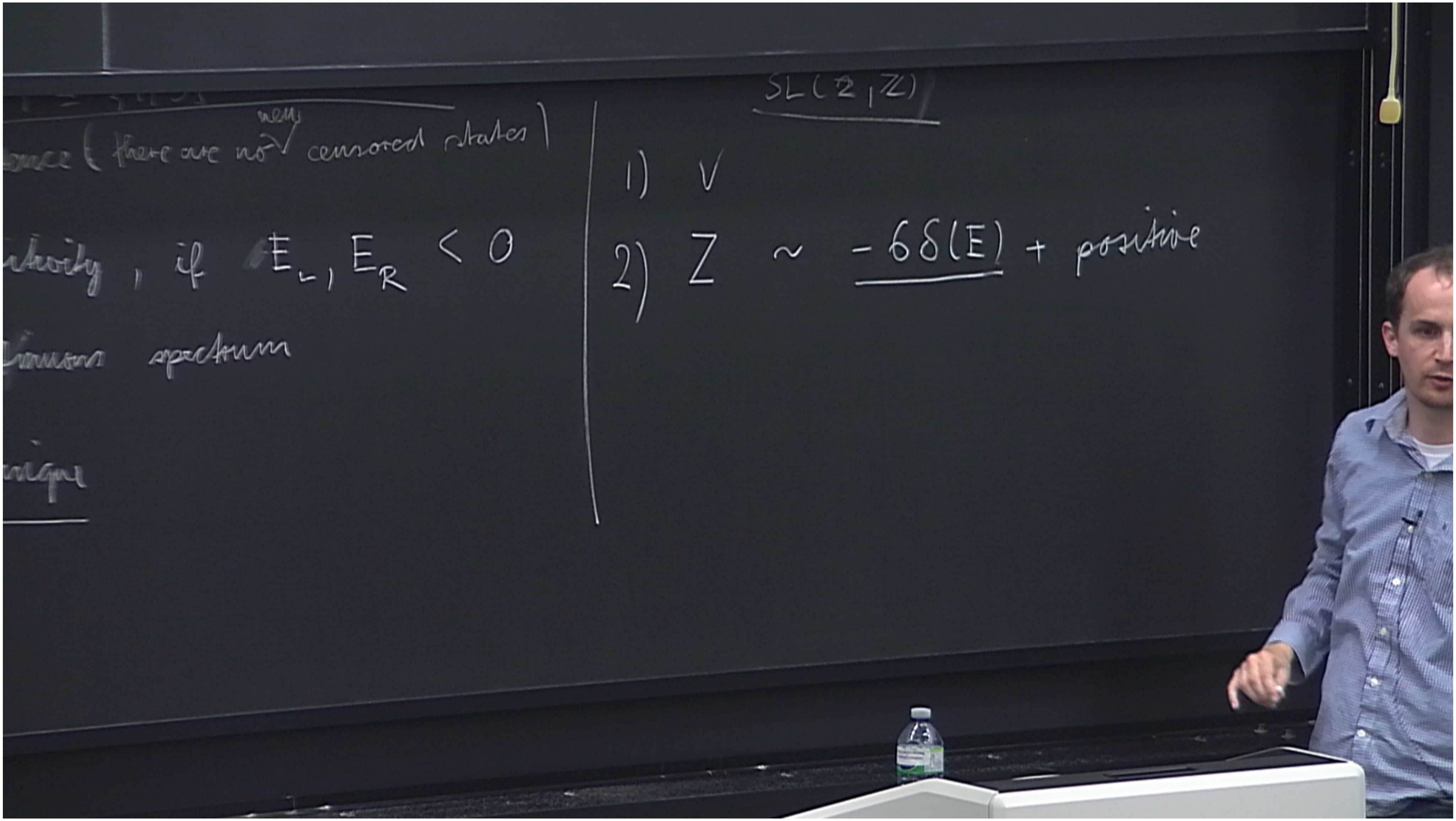
tinuous spectrum

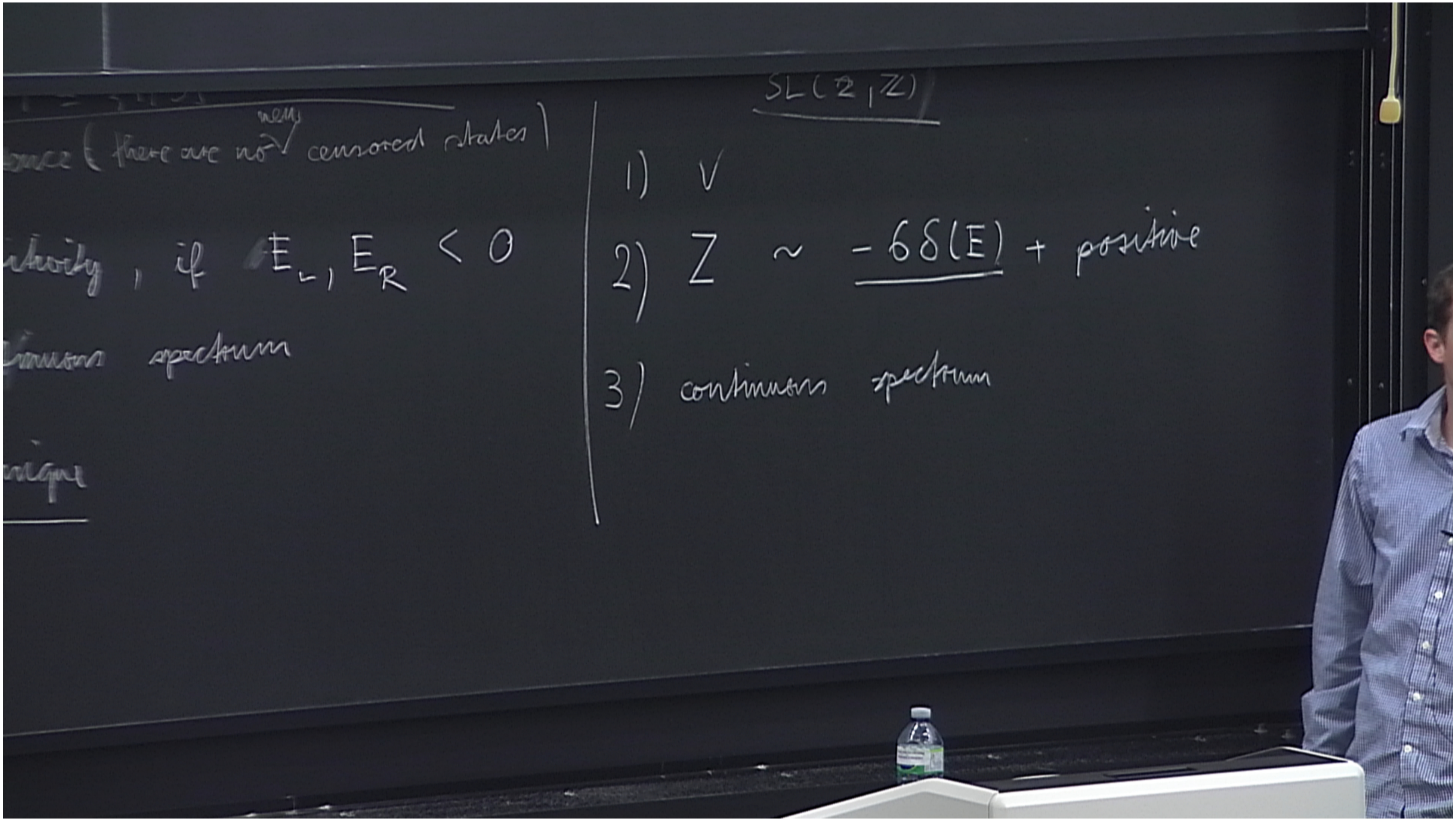
nique

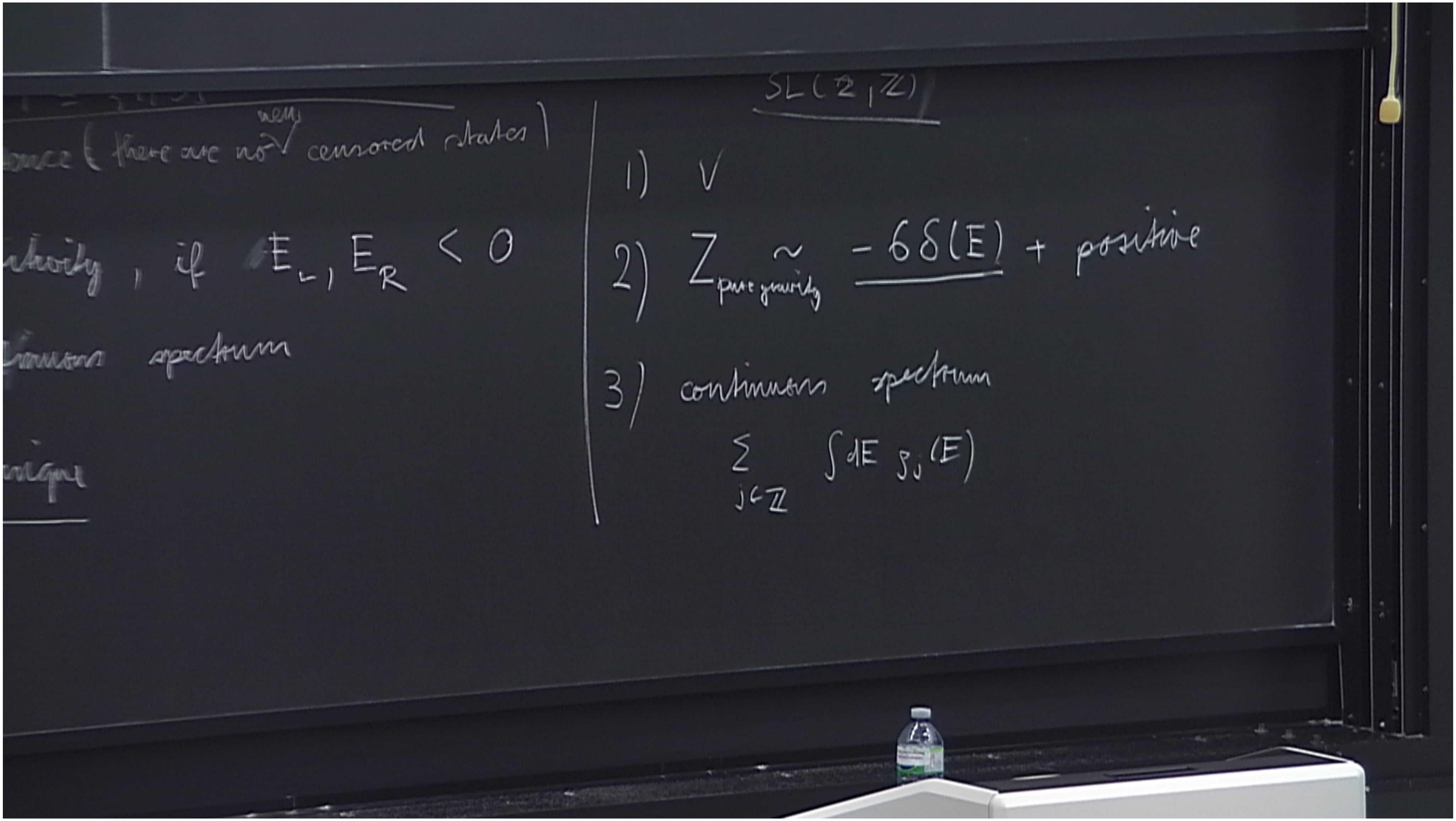
$\underline{SL(2,\mathbb{Z})}$

1) $\surd$


uncensored region
Δ_L
c/24
0
Δ_R
1) existence (there are
2) positivity , if
3) continuous spectr
⇒ not unique


SL(2,Z)
(there are no very censored states)
1) ✓
, if E_L, E_R < 0
2) Z ~ -6δ(E) + positive
spectrum


SL(2,Z)
(there are no new censored states)
if E_L, E_R < 0
spectrum
1) ✓
2) Z ~ −6δ(E) + positive
3) continuous spectrum


SL(2, Z)
(there are no new censored states)
, if E_L, E_R < 0
spectrum
1) ✓
2) Z_pure gravity ~ −6δ(E) + positive
3) continuous spectrum
Σ_{j∈Z} ∫dE ρ_j(E)