Title: TBA

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Abstract:

Spin and Pair Density Wave Glasses

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Collaborator: D. Mross (Caltech)

Thanks: Leon Balents, T. Giamarchi, D. Huse

An old and important question

Fate of incommensurate density wave order in the presence of impurities?

This talk: focus mainly on Spin Density Waves (SDW).

Weak non-magnetic disorder → ``Spin Density Wave Glass''

Many interesting properties distinct from conventional spin glass.

Related questions: impurity effects on ``pair density wave'' superconductors (Berg, Fradkin, Kivelson, 08)

Spin Density Wave (and Pair Density Wave) order:
Order parameters, topological defects.

Some simplifying assumptions

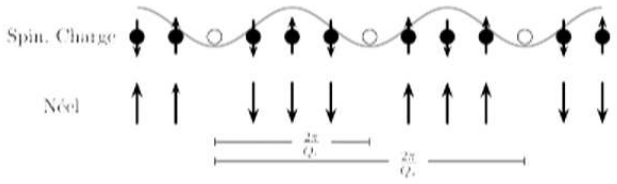
For concreteness consider

1. Uniaxial SDW (not spiral)

2. Unidirectional in real space with direction chosen by symmetry of underlying crystal.

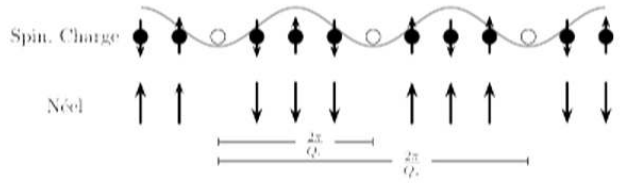
3. Incommensurate order

``Spin stripes''



``Anti-phase'' stripes, common in La-based cuprates.

Stripes and charge order



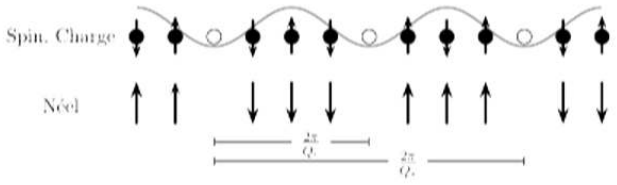
``Anti-phase'' stripes, common in La-based cuprates.

Typically spin stripe implies charge stripe (Landau argument) but charge stripe does not imply spin order.

Term in free energy $\propto \vec{S}^2 \rho$

$=>$ spin order at wave vector $\mathbf{Q}$ accompanied by charge order at wave vector $2\mathbf{Q}$.

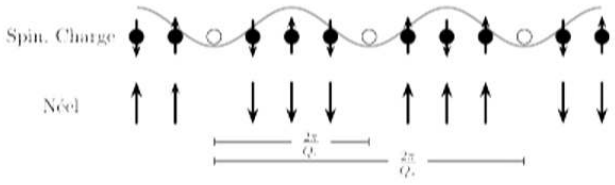
SDW order parameter



$$\vec{S}_r \sim e^{i\mathbf{Q}.\mathbf{r}} e^{i\theta_s} \vec{N} + c.c$$

SDW order parameter $= e^{i\theta_s}\vec{N}$.
θ_s: stripe displacement
$\vec{N}$: local Neel vector.
CDW order parameter $= e^{2i\theta_s}$.

SDW order parameter



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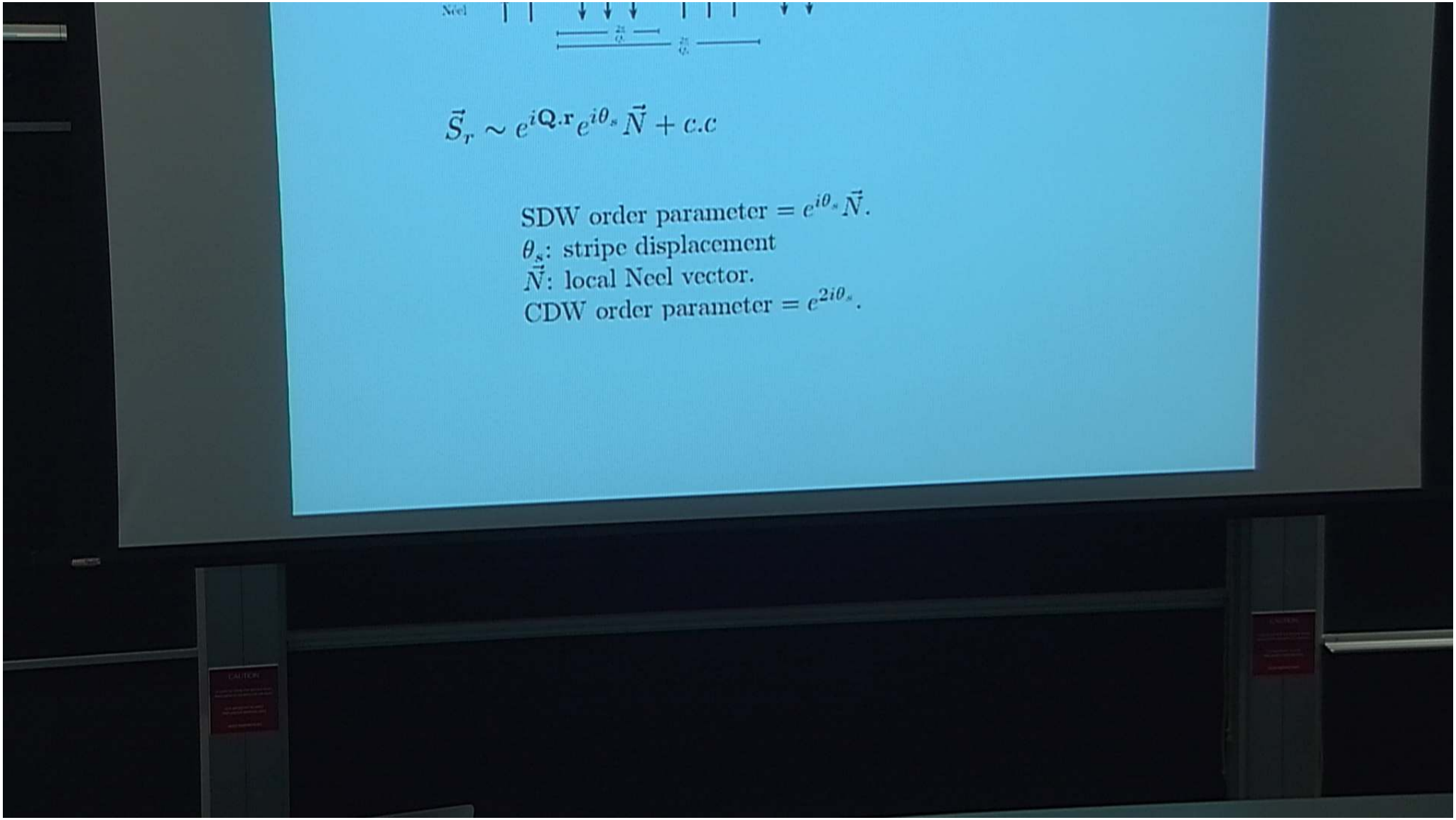
Neel
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A convenient formal description

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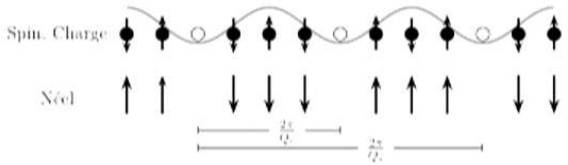

$$\vec{S}_r \sim e^{i\mathbf{Q}.\mathbf{r}} e^{i\theta_s}\vec{N} + c.c$$

Both order parameters are composites of fields $b = e^{i\theta_s}, \vec{N}$.

SDW order parameter $\vec{S}_{\mathbf{Q}} = b\vec{N}$.

CDW order parameter $\rho_{\mathbf{2Q}} \sim b^2$.

Formulation in terms of $b, \vec{N}$ has Z_2 "gauge" redundancy $(b, \vec{N} \to -b, -\vec{N})$.

Zaanen, Nussinov, 2000
Sachdev,.....2001

No fractionalization, etc in this talk. `Slave' formulation useful nevertheless.

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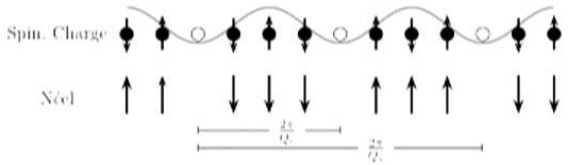

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Closely related order: Pair Density Wave (PDW) superconductor

Cooper pairing at non-zero momentum (eg, FFLO).

$$\Delta(x) \sim \Delta_0 \cos(\mathbf{Q} \cdot \mathbf{x})$$

Many names:
Larkin-Ovchinnikov SC, PDW SC,
Striped SC, Amperean paired SC

Proposed for $La_{2-x}Ba_xCu2O4$ (Berg, Fradkin, Kim, Kivelson, ..., 2007)

Very similar to SDW with XY spin anisotropy.

However: different action of time reversal.

SDW breaks time reversal; PDW preserves it.

(Formal: Spin U(1) commutes with T-reversal while charge U(1) does not.)

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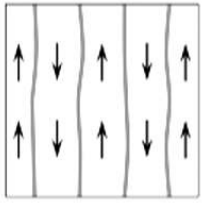
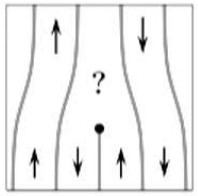
Topological defects in SDW phases

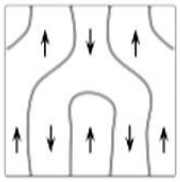
Important class of defects: dislocations in CDW order.



Point defect in 2d, line defect in 3d.



Spin order is frustrated at a strength-1 dislocation.



Spin order not frustrated at a double dislocation.

Strength-1 dislocation necessarily accompanied by 'disclination' where $\vec{N}$ twists by π.

Alternate: Single dislocation bound to Z_2 gauge vortex in $b, \vec{N}$ description.

Fate of incommensurate SDW to non-magnetic impurities?

Impurity: linear coupling to CDW order parameter

$$\int d^d\mathbf{x} v(\mathbf{x}) \rho_{\mathbf{2Q}}(\mathbf{x}) + c.c$$

$$v(\mathbf{x}) \text{ random.}$$

But no linear coupling to SDW order parameter

Larkin/Imry-Ma: destruction of CDW order

Balance of random `field' versus elastic energies:

Large region of size L:
Energy gain in following random potential ~ $v_0 L^{d/2}$

Elastic energy cost ~ $K\, L^{d-2}$

Elastic energy lost by following random potential overwhelmed by potential energy gain for $d < 4$.

CDW long range order lost beyond ``Larkin length'' $\xi_L \sim (K/v_0)^{2/(4-d)}$

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SDW => CDW;

Therefore no CDW => no SDW long range order beyond ξ_L

Long length scales: ``Spin Density Wave Glass''

Physics?

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Physics?

SDW glasses in d = 3

Pinned Charge Density Waves: Elastic/Bragg glass

Weak disorder in d = 3: CDW enters an `elastic glass' phase where long dislocation loops do not occur.

Giamarchi, Le Doussal, 94
Gingras, Huse, 95
D. S. Fisher, 97

Describe by random field XY model without vortices.

Many approximate treatments (eg, `Functional' RG near d = 4):

Power law order for CDW order parameter:

$$\overline{\rho^*_{\mathbf{2Q}}(\mathbf{x})\rho_{\mathbf{2Q}}(\mathbf{x}')} \sim \frac{1}{|\mathbf{x}-\mathbf{x}'|^{\eta_c}}$$

Delta function Bragg peaks replaced by power law peaks.

Estimate: $\eta_c \approx 1.1$ (extrapolate from leading order Functional RG

Fate of spin order?

Mross, TS, 14

No long range SDW order.

But no long dislocations => no frozen Z_2 gauge vortices (i.e no π-disclinations of N-vector).

$\vec{N}$ has true long range order.
=> SDW order has power law Bragg peaks.

$$\begin{aligned}\overline{\vec{S}^*_{\mathbf{Q}}(\mathbf{x}) \cdot \vec{S}_{\mathbf{Q}}(\mathbf{x}')} &= \overline{e^{i\theta_s(\mathbf{x})} e^{-i\theta_s(\mathbf{x}')} \vec{N}(\mathbf{x}) \cdot \vec{N}(\mathbf{x}')} \\ &\sim \overline{e^{i\theta_s(\mathbf{x})} e^{-i\theta_s(\mathbf{x}')}} \\ &\sim \frac{1}{|\mathbf{x}-\mathbf{x}'|^{\eta_S}}\end{aligned}$$

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Long range spin nematic order

Mross, TS, 14

Meaning of ordering of $\vec{N}$?

$\vec{N}$ not gauge invariant and hence not observable.
$\vec{N}$ order => Long range spin quadrupole order (= spin nematic).

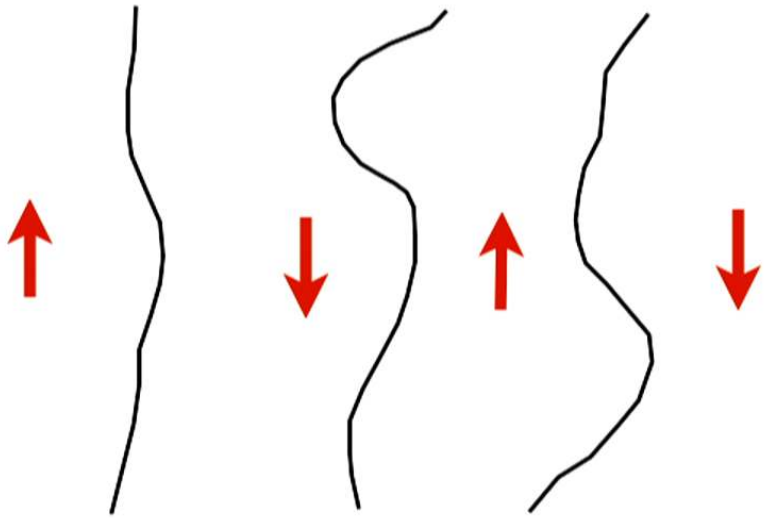
Order parameter

$$Q_{\alpha\beta} = N_\alpha N_\beta - \delta_{\alpha\beta}\frac{\vec{N}^2}{3}$$

Spontaneous spin anisotropy without long range SDW order.

Similar: thermal/quantum melting of spin stripes → spin nematic (Zaanen 2000).

Physical picture



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Spins retain common axis along which they point up or down (spin nematic LRO).

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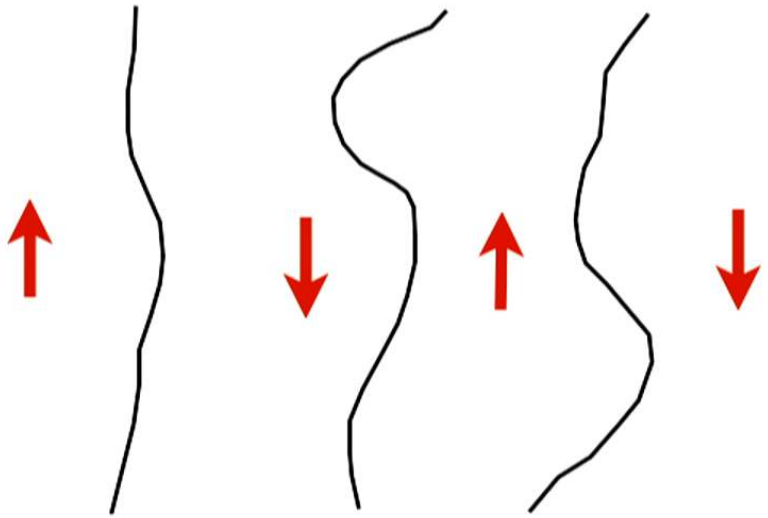

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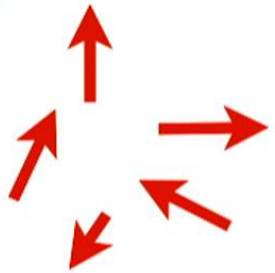
Uniaxial spin glass

Spin freezing => Edwards-Anderson spin glass order parameter.

$$lt_{t\to\infty}\overline{\vec{S}(\mathbf{x},t)\cdot\vec{S}(\mathbf{x},0)} = q_{EA} \neq 0.$$

Uniaxial spin glass in a Heisenberg system with easy axis determined spontaneously.

Clearly distinct from the `conventional' Heisenberg spin glass.



Heisenberg spin glass

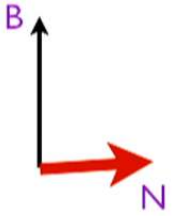
Contrast with the `standard' Heisenberg spin glass

Spin Density Wave Glass

1. Power law correlations of SDW order coexisting with spin glass order parameter.

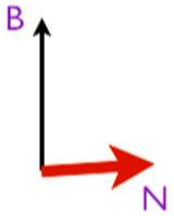
2. Spontaneous spin anisotropy, propagating gapless nematic director waves (Goldstone modes of spin nematic order).

3. Different magnetic field behavior.
Director orients perpendicular to field.

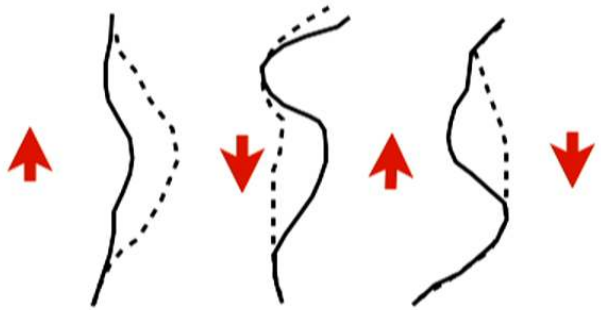


Contrast with the `standard' Heisenberg spin glass: effects of weak spin anisotropy

Weak intrinsic spin anisotropy can pin director, for example along an easy axis.



Example: B along an easy axis
$B \ll B_p$: glassy as stripes reorganize to accommodate magnetization
$B \gg B_p$: glass effects reduced in magnetization as spins reorient.
B_p: field required to 'depin' director.



Scale B_p for glassy effects to disappear is set by the weak anisotropy and not by J. (unlike Heisenberg spin glass).

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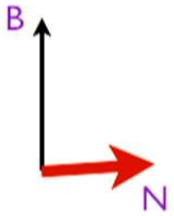

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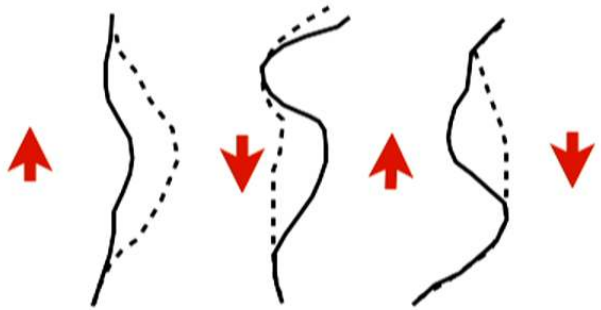

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Comments
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2. SDW systems – opportunity to study Bragg glass physics in experiments?
3. Even some classic spin glasses (eg CuMn) show strong short range SDW order. SDW glass physics relevant starting point?

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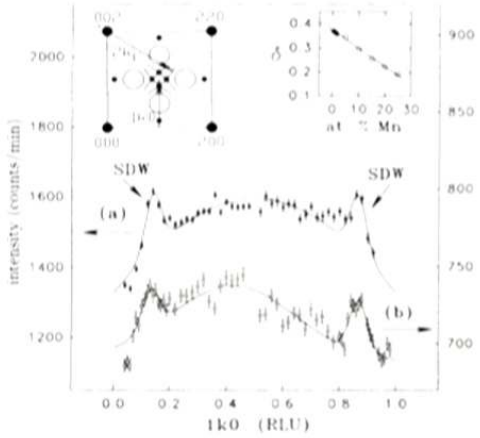
Lamelas, Werner, Shapiro, Mydosh, PR B 1995



SDW and PDW glasses in d = 2

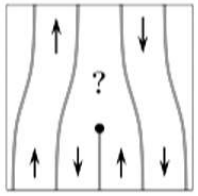
Pinned SDW (or PDW) in 2d

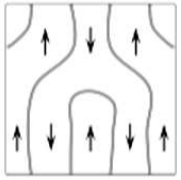
Weak disorder:

Larkin-Imry-Ma: Accompanying CDW disordered beyond finite length scale ξ_L

Fate of dislocations?

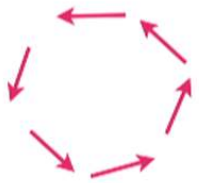
Single and double dislocations possibly different.





Competing effects

Dislocations cost elastic energy, and see a random correlated potential; can nucleate near potential minima.



Elastic energy cost versus optimal potential energy gain:

Single dislocation:

Elastic energy of both CDW distortion and spin disclination:
$\pi(K_c + K_s) \ln L$

Doubled dislocation: No spin disclination => different elastic energy cost.
$4\pi K_c \ln L$

Kc, Ks: elastic constants for CDW and spin distortions.

Doubled dislocations

Same analysis as for ordinary pinned CDW

Zeng, Leath, D. Fisher, 1999
Le Doussal, Giamarchi, 2000

Length scales $L \gg \xi_L$:
Average elastic energy cost $\overline{E_{elast}} \sim K \ln(L)$.
Optimal potential energy gain for isolated dislocations $E_V \sim (\ln(L))^{\frac{3}{2}}$ (See later.)
Always favor isolated strength-2 dislocations at long scales.

Doubled dislocations

Same analysis as for ordinary pinned CDW

Zeng, Leath, D. Fisher, 1999
Le Doussal, Giamarchi, 2000

Length scales $L \gg \xi_L$:
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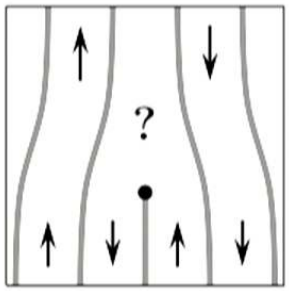
Meaning in Pair Density Wave context

Spin Density Wave	Pair Density Wave
Spin Nematic	Charge-4 superconductivity
Strength-1 CDW dislocation bound to π-disclination of spin	Strength-1 CDW dislocation bound to **hc/4e SC vortex**

Agterberg, Tsunetsugu, 2008. Berg, Fradkin, Kivelson 2009



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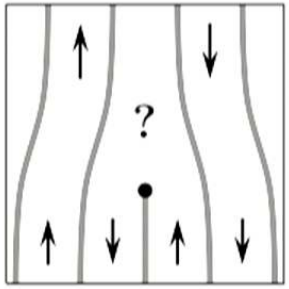
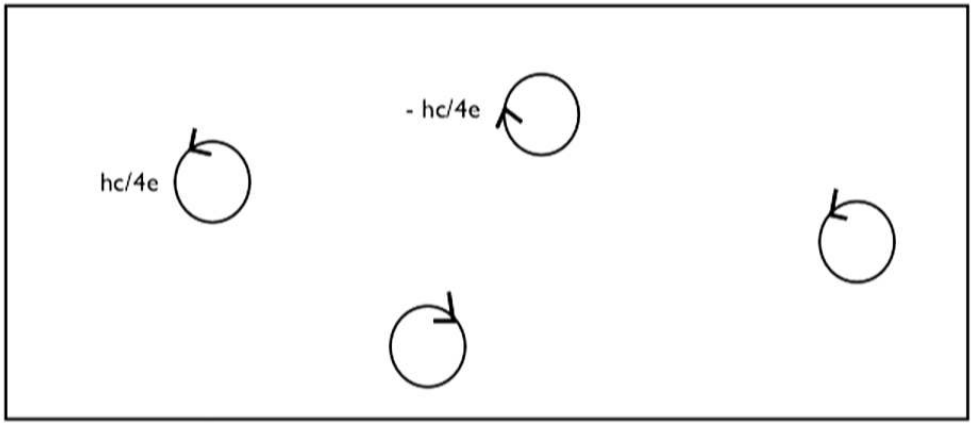


Spontaneous vortices in weakly disordered PDW?

Does weakly disordered 2d PDW necessarily generate strength-1 dislocations
=> ***random hc/4e SC vortices of either sign?***

For PDW this breaks T-reversal symmetry locally.

Detect through local vortex probes in LBCO - key experimental test?



Claim

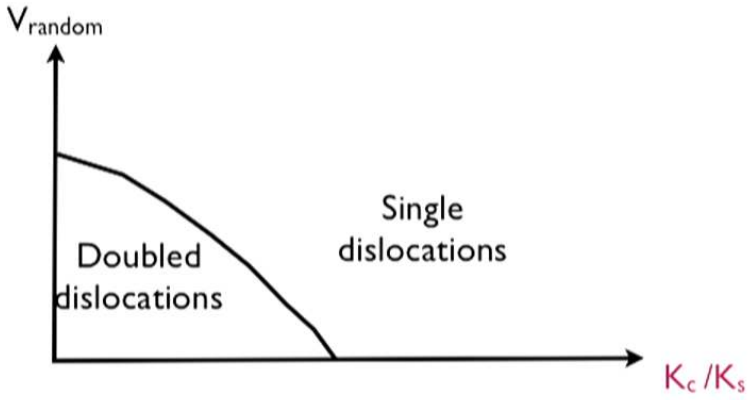
Weak non-magnetic disorder: Strength-1 dislocations are not ***necessarily*** generated at long scales (depends on bare elastic constants). Mross, TS, 14

For $K_s >> K_c$

1. Spin nematic LRO persists in 2d in SDW glass

2. Weakly disordered PDW preserves time reversal symmetry (no random half-vortices of SC order).

Schematic phase diagram



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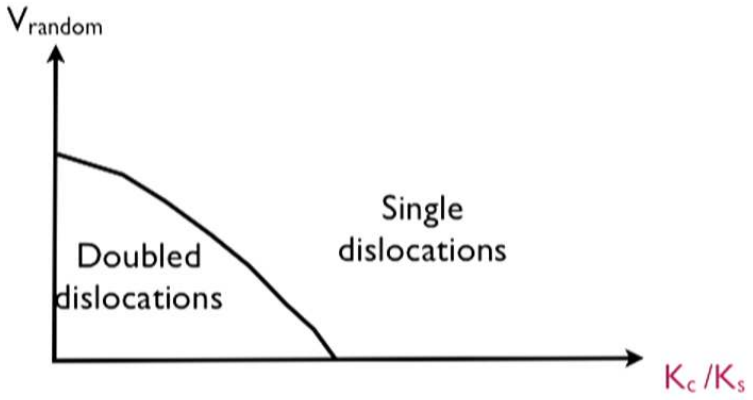


Dislocation energetics: elastic model

PDW example:

$$H = \int d^2x \frac{K_s}{2}(\vec{\nabla}\theta_s)^2 + \frac{K_c}{2}(\vec{\nabla}\theta_c + \vec{\eta})^2 - V\cos(2\theta_c - \alpha(x))$$

$\alpha(x)$ uniform in $[0, 2\pi)$

$\overline{\eta_i(x)\eta_j(x')} = \sigma\delta_{ij}\delta^2(x - x')$

Dislocation energetics in random phase shift model

Nattermann et al, 1995

$$H_o = \int d^2x \frac{K_c}{2} (\vec{\nabla}\theta_c + \vec{\eta})^2$$

Region of size L: Strength-m dislocations see a random potential $U(x)$ with variance $\sim K_c^2 \sigma m^2 \ln L$.

Simple model: Assume $U(x)$ spatially uncorrelated.

Minimum value $U(x) \sim -K_c m \sqrt{\sigma} \ln L$ = optimal potential energy gain of isolated dislocation.

Are single dislocations necessarily introduced?

Suppress single dislocations by hand in the SDW glass, and examine stability of resulting state to introducing them.

Long range spin nematic order: elastic energy $\approx$ Ks ln(L).

Random energy gain in ***fully disordered*** CDW state just a constant

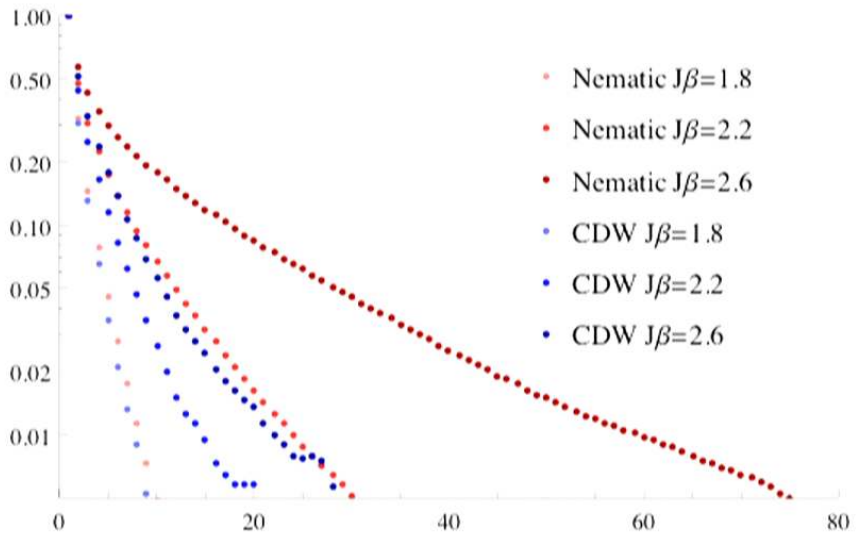
=> Single dislocations not favored

SDW glass coexisting with spin nematic LRO stable.

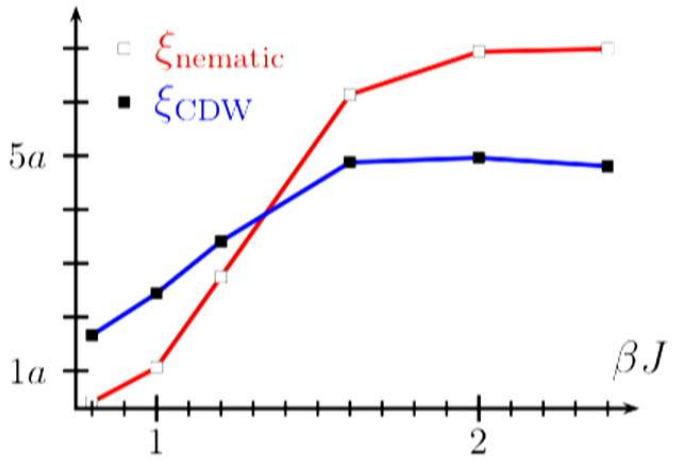
Numerics
Simple model of 2 coupled XY orders
(one for CDW and the other for SDW
or PDW).
$$H = -J \sum_{<ij>} \cos(\nabla\theta_N)\cos(\nabla\theta_s) - h\sum_i \cos(2\theta_{si} - \alpha_i)$$
CDW order parameter $= e^{2i\theta_c}$
(XY) SDW order parameter $= e^{i(\theta_N+\theta_s)}$
$\alpha_i \in [0, 2\pi]$, random
Study through Monte Carlo simulations at not too low temperature

Persistence of nematic correlations.

Mross, TS, 14



Correlation lengths: CDW versus spin nematic



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Study through Monte Carlo simulations at not too low temperature.

Some experimental applications

1. Proposed PDW in LBCO: Possibility of spontaneous half-vortex formation by non-magnetic disorder.

2. ``Spin glass'' phase in underdoped cuprates.

- known to have substantive incommensurate SDW order

- 2d SDW glass with long range spin nematic order?

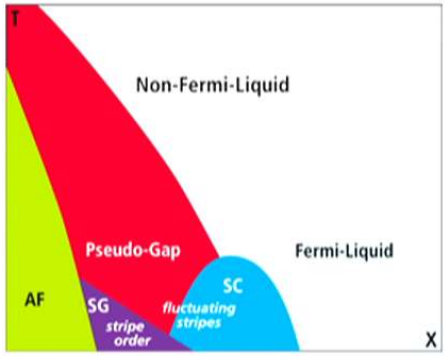

Image credit:
http://www.psi.ch/swissfel/correlated-electron-phases

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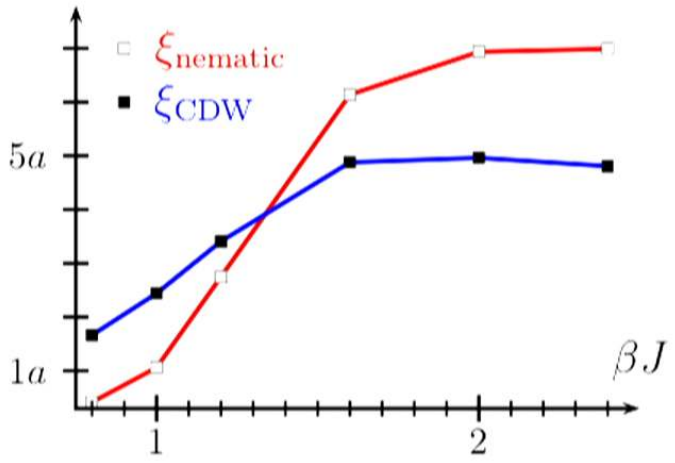
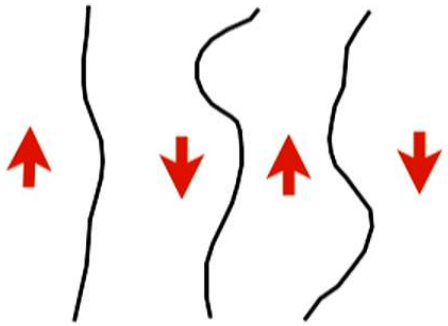


Summary

Weakly disordered uniaxial incommensurate SDW → `Spin Density Wave Glass' without long range SDW order.

In 3d (and may be also 2d) spin nematic order survives: uniaxial spin glass in a Heisenberg spin system.

In 3d, but not 2d, SDW order has power law correlations (SDW Bragg glass)

Interesting experimental opportunity to probe Bragg glass physics?

Similar phenomena in PDWs: Spontaneous T-breaking induced by disorder?