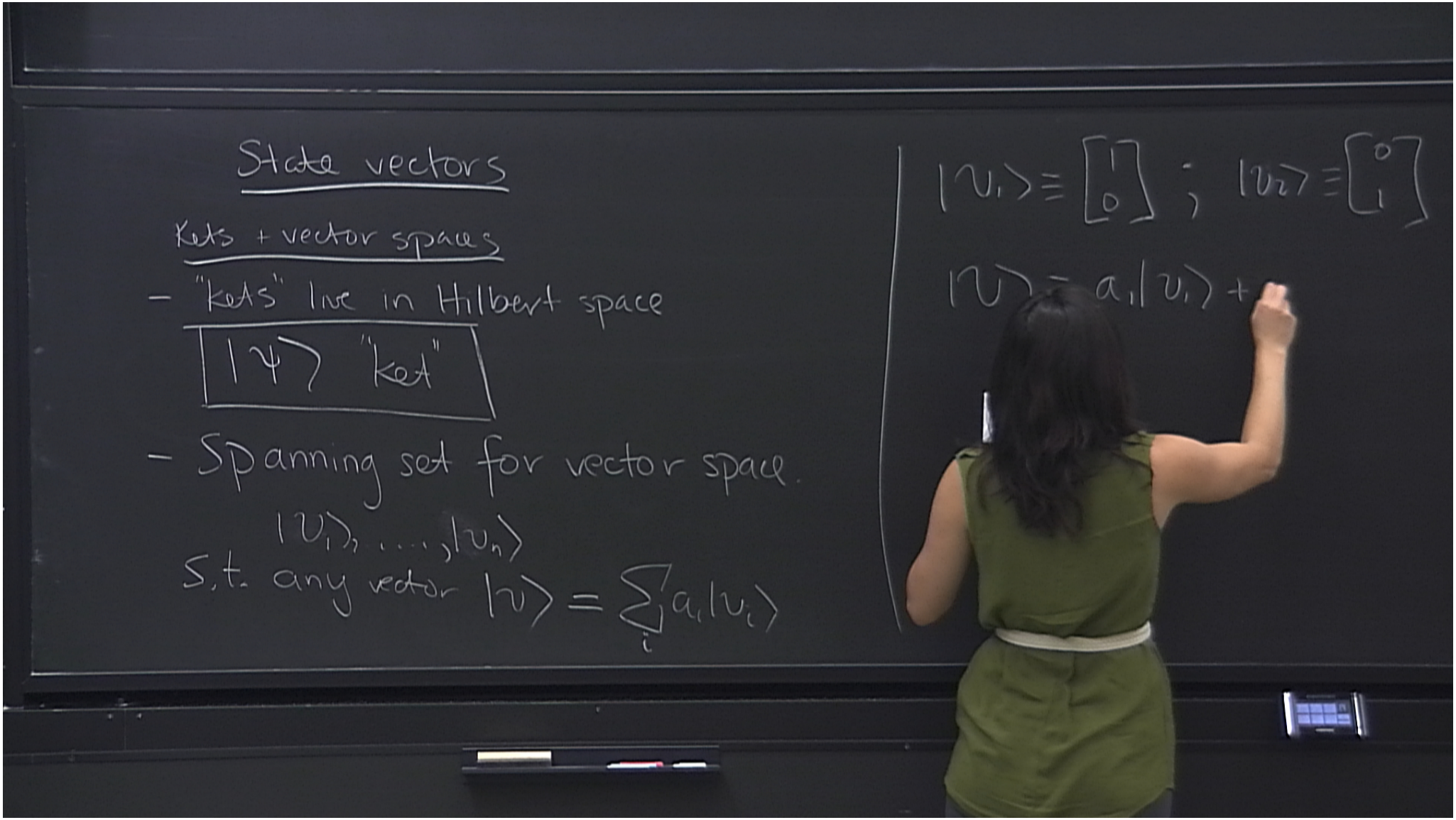
Title: 14/15 PSI - Quantum Mechanics 4

Date: Aug 22, 2014 10:30 AM

URL: http://pirsa.org/14080027

Abstract:


State vectors
Kets + vector spaces
- "kets" live in Hilbert space
|ψ⟩ "ket"
- Spanning set for vector space
|v1⟩, ..., |vn⟩
s.t. any vector |v⟩ = Σ_i a_i |v_i⟩
|v1⟩ ≡ [1 0] ; |v2⟩ ≡ [0 1]
|v⟩ = a1|v1⟩ +

vectors

r spaces

in Hilbert space

"ket"

g set for vector space.

$\ldots, |u_n\rangle$

vector $|v\rangle = \sum_i a_i |u_i\rangle$

e.g. for $\mathbb{C}^2$

$|u_1\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $|u_2\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$|v\rangle = a_1|u_1\rangle + a_2|u_2\rangle$

alternatively:

$|u_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $|u_2\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$|v\rangle = \frac{a_1+a_2}{\sqrt{2}}|u_1\rangle + \frac{a_1-a_2}{\sqrt{2}}|u_2\rangle$

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Bras and inner products

Dual to $|\phi\rangle$:

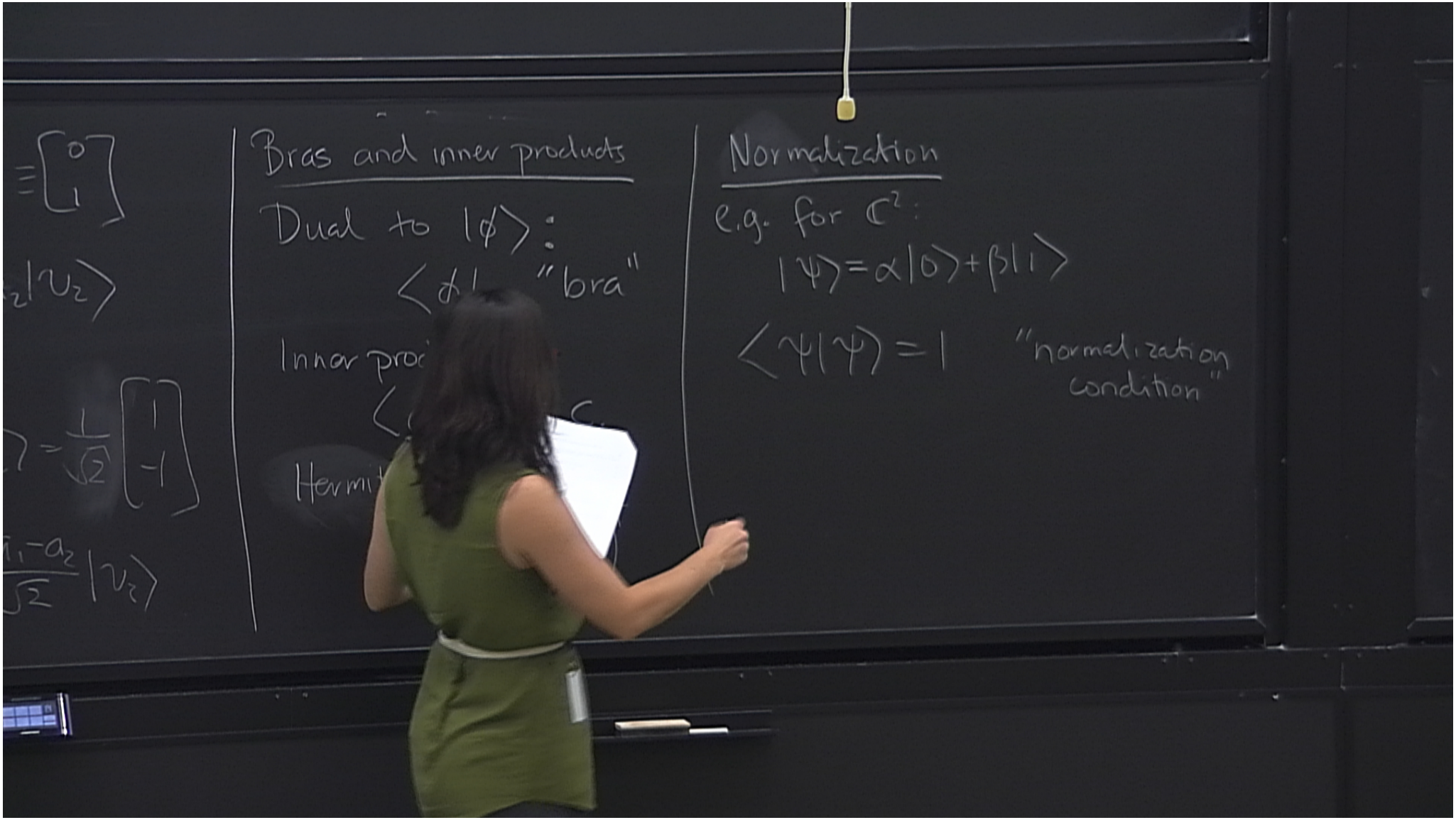
$\langle\phi|$ "bra"

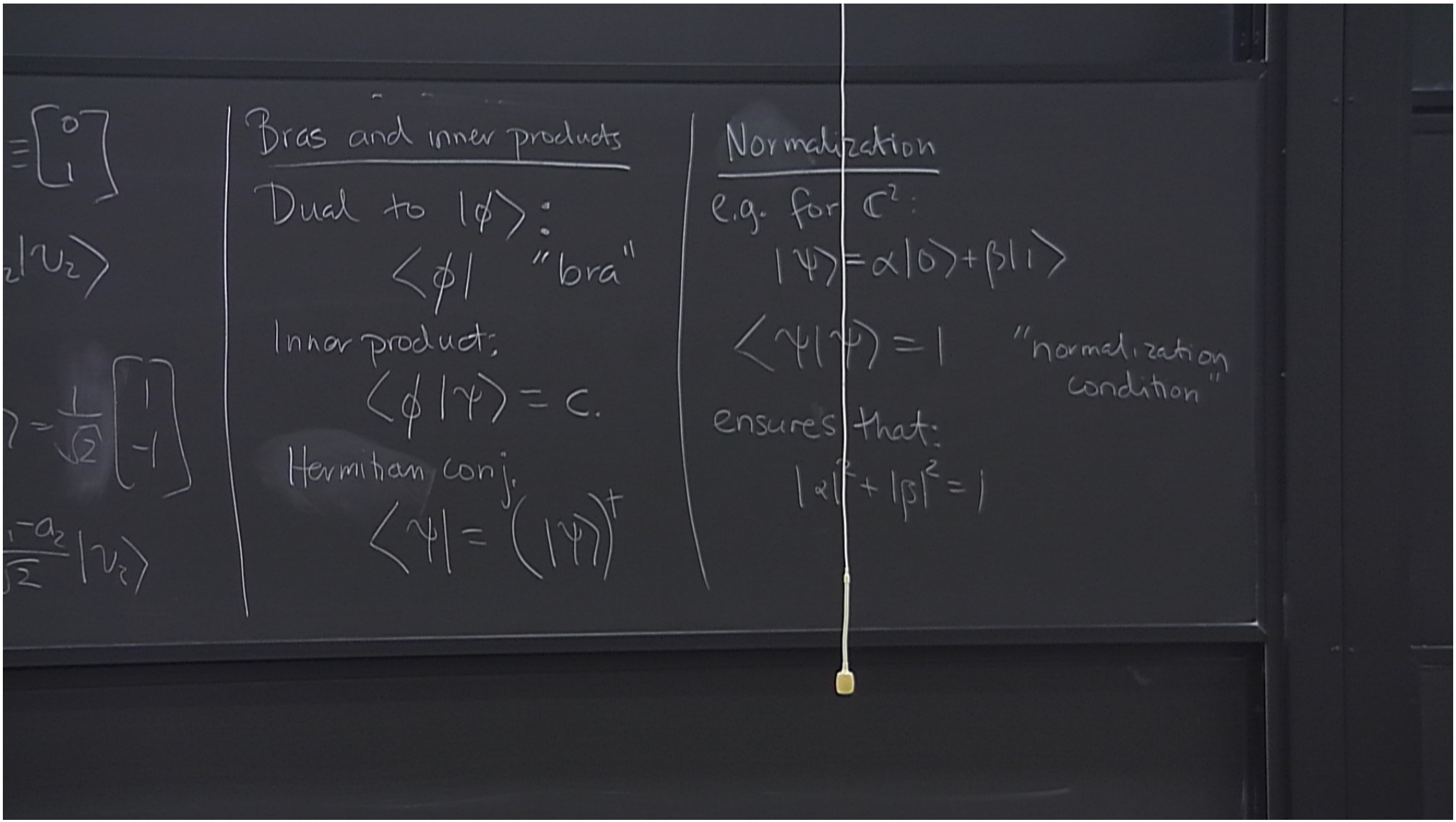
Inner product:

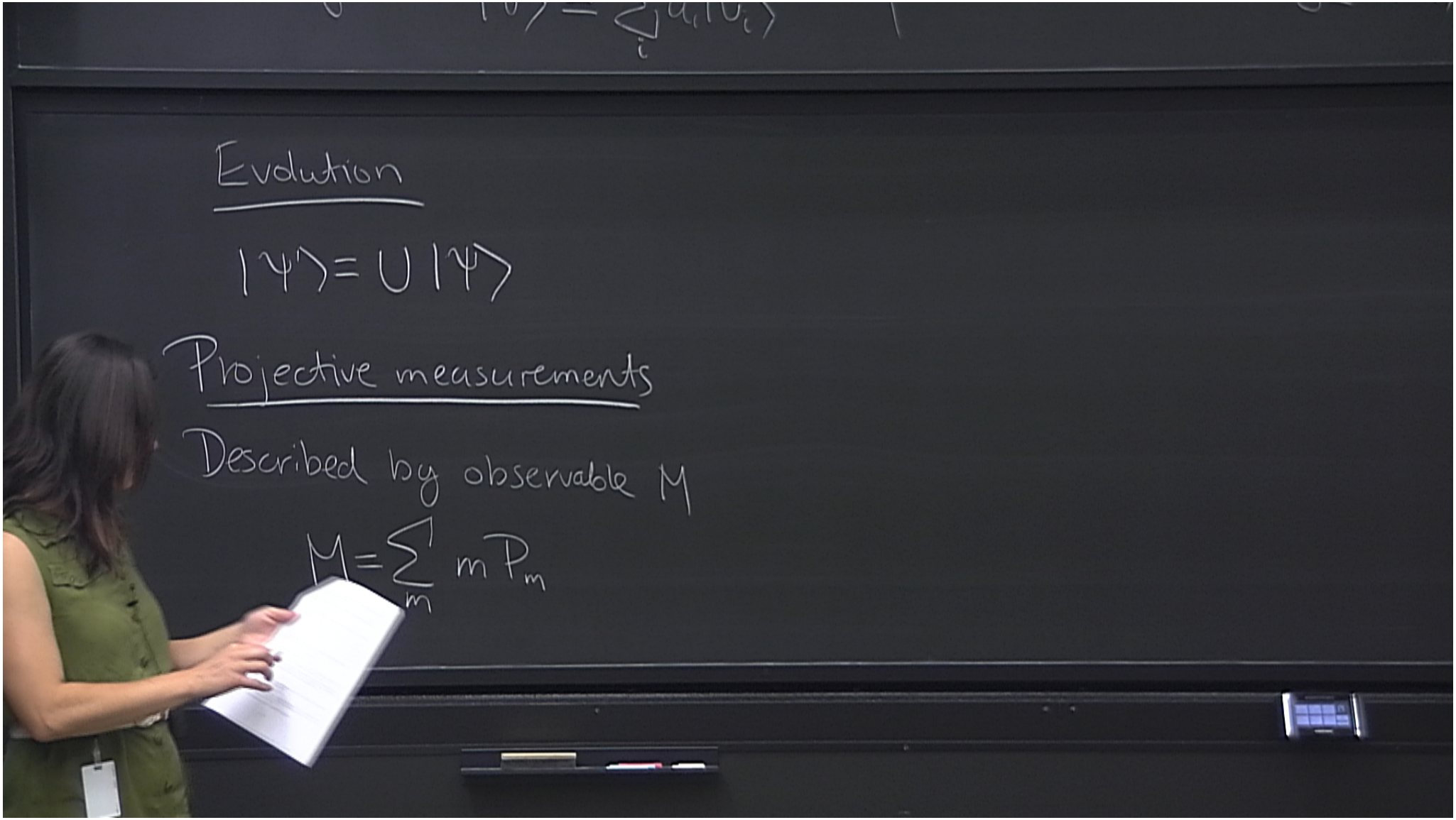
$\langle\phi|\psi\rangle = c.$

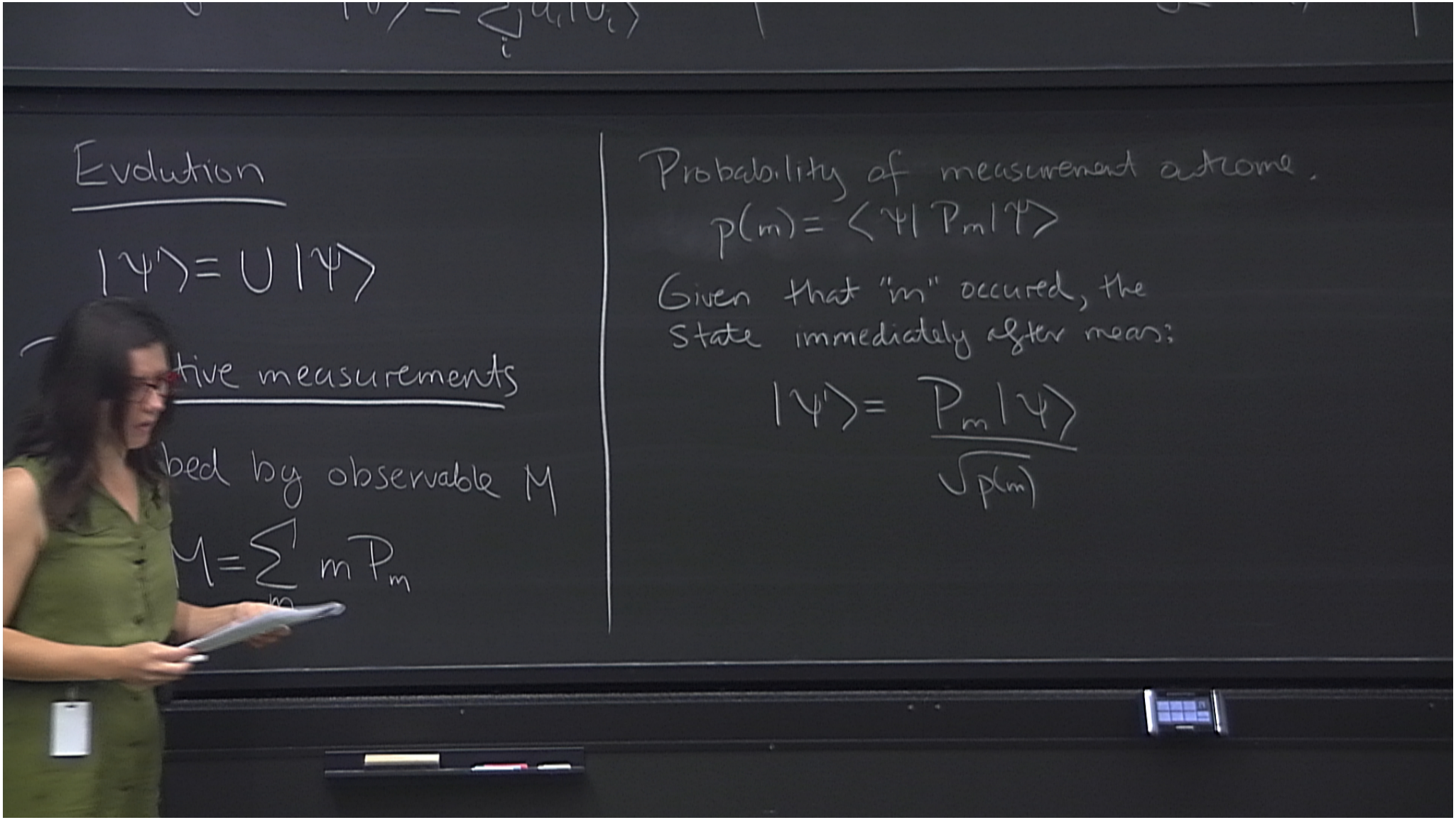
Hermitian conj.

$\langle\psi| = (|\psi\rangle)^\dagger$


Bras and inner products
Dual to $|\phi\rangle$:
$\langle\phi|$ "bra"
Normalization
e.g. for $\mathbb{C}^2$:
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$\langle\psi|\psi\rangle = 1$ "normalization condition"


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e.g. for $\mathbb{C}^2$:
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$\langle\psi|\psi\rangle = 1$ "normalization condition"
ensures that:
$|\alpha|^2 + |\beta|^2 = 1$


Evolution
$|\psi'\rangle = U|\psi\rangle$
Projective measurements
Described by observable M
$M = \sum_m m P_m$


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Probability of measurement outcome.
$p(m) = \langle\psi| P_m |\psi\rangle$
Given that "m" occured, the state immediately after mean:
$|\psi'\rangle = \frac{P_m|\psi\rangle}{\sqrt{p(m)}}$

Evolution

$$|\psi'\rangle = U|\psi\rangle$$

Projective measurements

Described by observable M

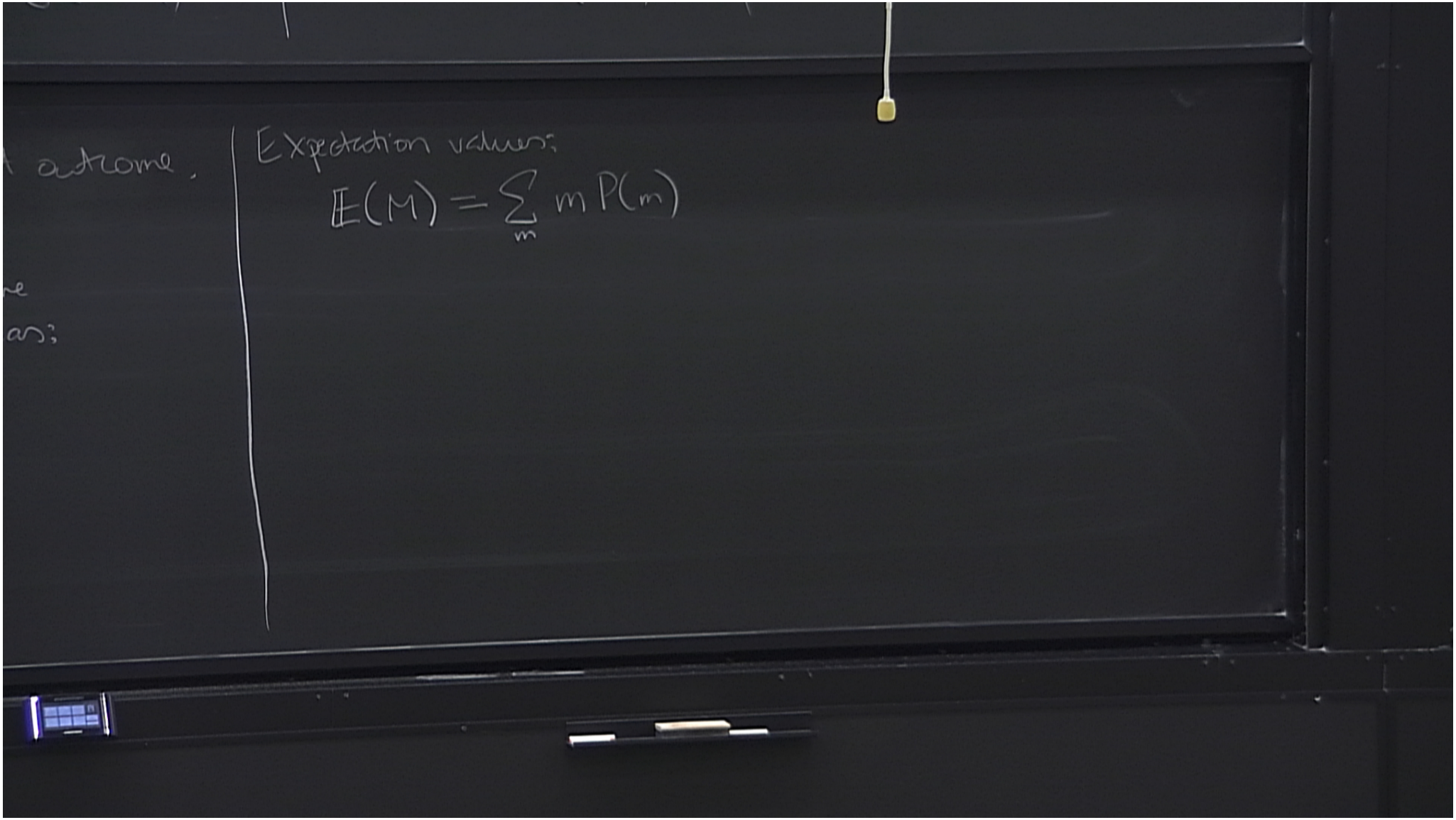
$$M = \sum_m m P_m$$

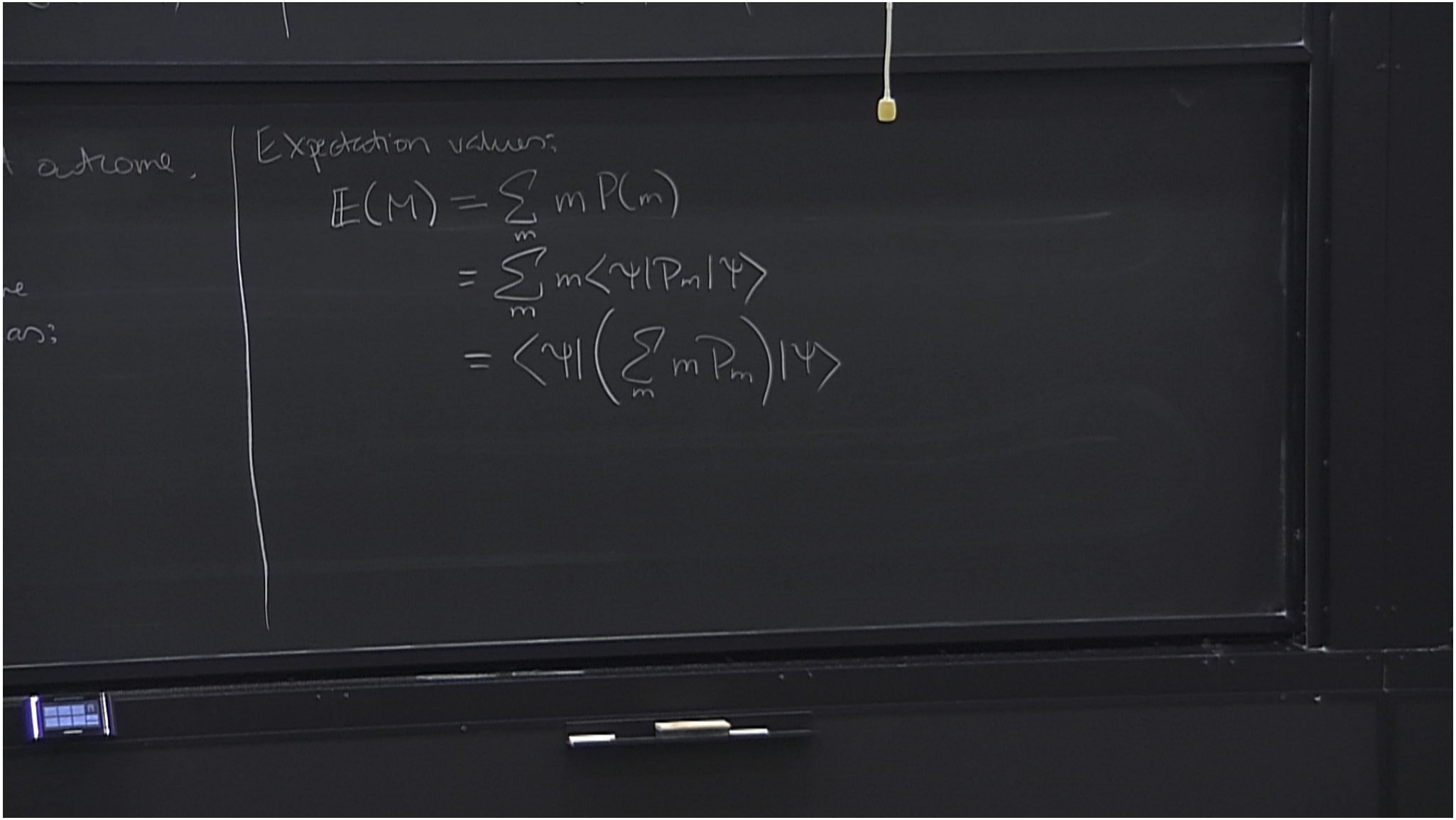
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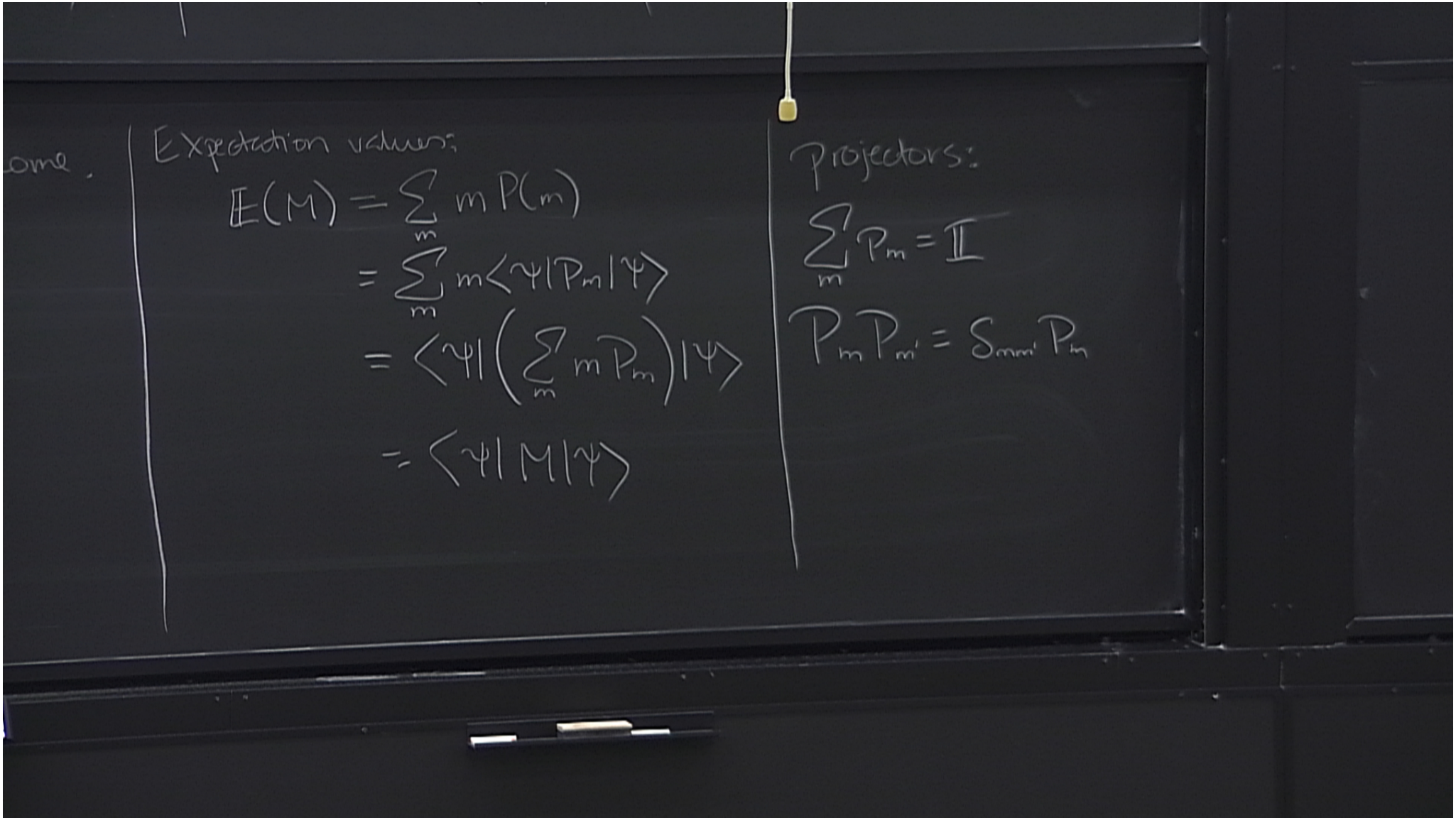
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Given that "m" occured, the state immediately after mean:

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Expectation values:
$\mathbb{E}(M) = \sum_m m\,P(m)$


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$= \sum_m m \langle \psi | P_m | \psi \rangle$
$= \langle \psi | \left(\sum_m m P_m \right) | \psi \rangle$


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Expectation values:
$\mathbb{E}(M) = \sum_m m P(m)$
$= \sum_m m \langle\psi|P_m|\psi\rangle$
$= \langle\psi|\left(\sum_m m P_m\right)|\psi\rangle$
$= \langle\psi|M|\psi\rangle$
Projectors:
$\sum_m P_m = \mathbb{I}$
$P_m P_{m'} = \delta_{mm'} P_m$

Phase

$|\psi'\rangle = e^{i\theta}|\psi\rangle$; "θ" global/absolute phase

$p(m) = \langle\psi'|P_m|\psi'\rangle$

$= \langle\psi|e^{-i\theta} P_m e^{i\theta}|\psi\rangle$

$= \langle\psi|P_m|\psi\rangle$

$\Rightarrow$ θ not experimentally detectable

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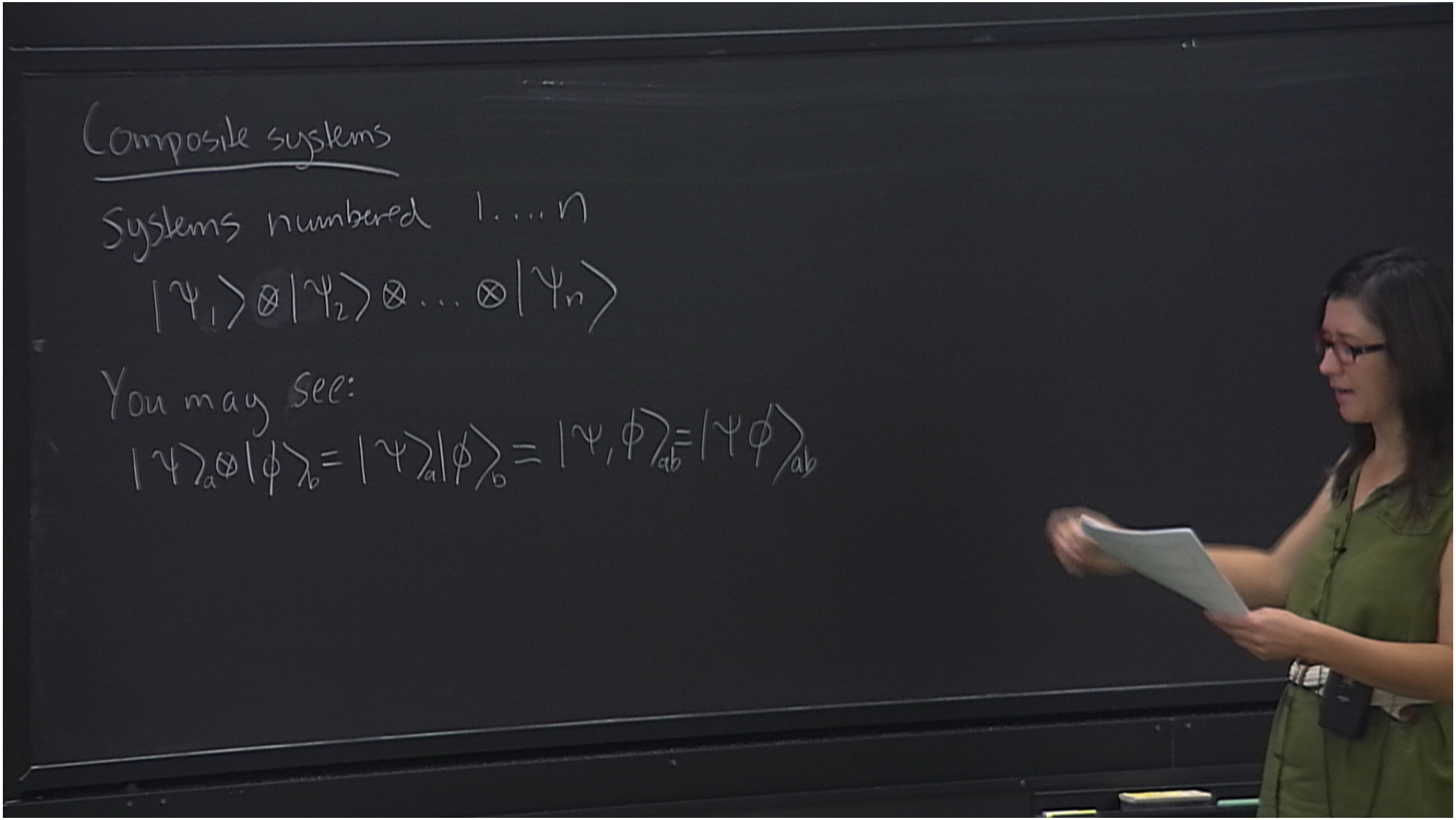
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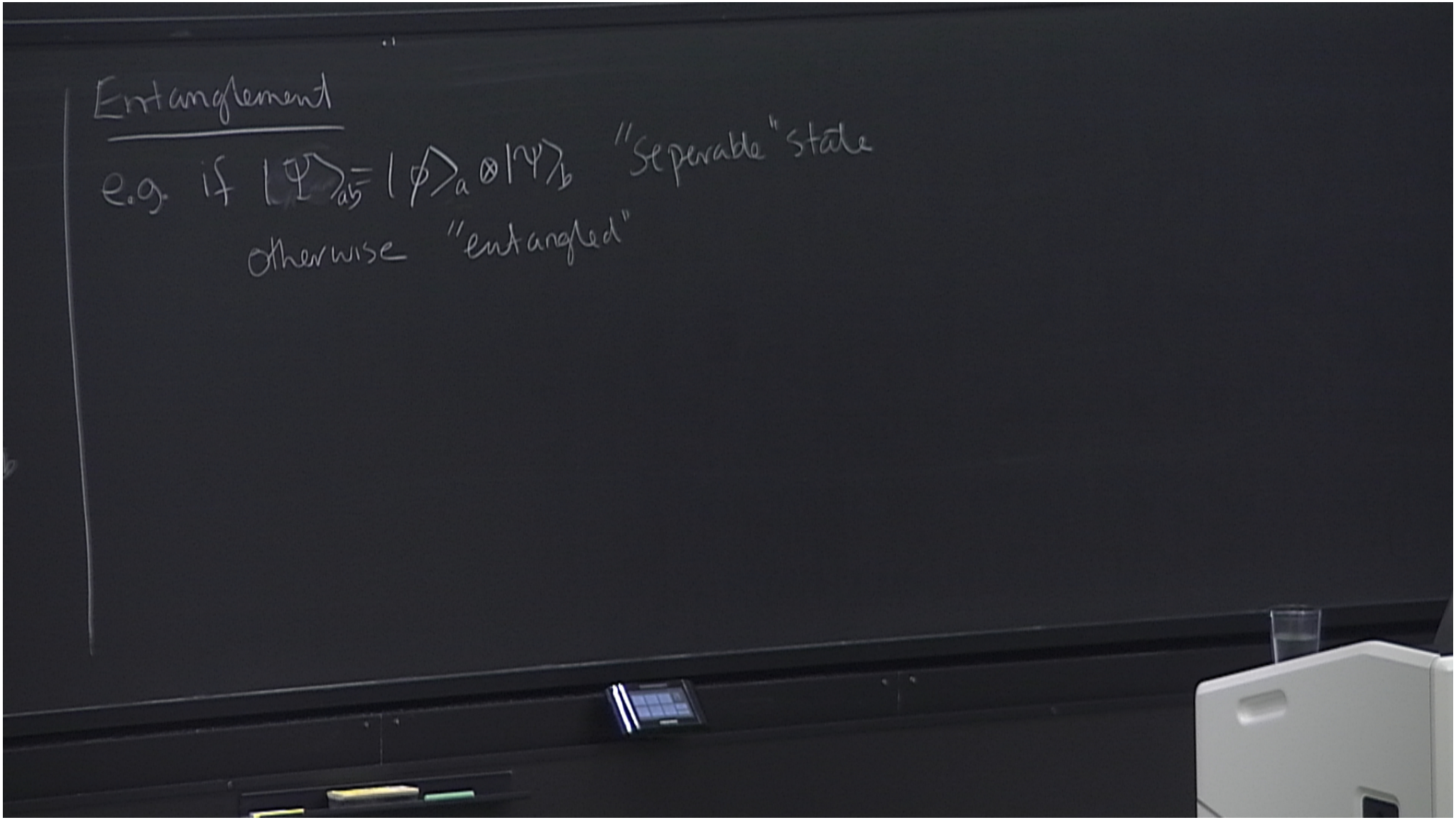
$|\psi\rangle = \alpha|0\rangle + e^{i\phi}\beta|1\rangle$

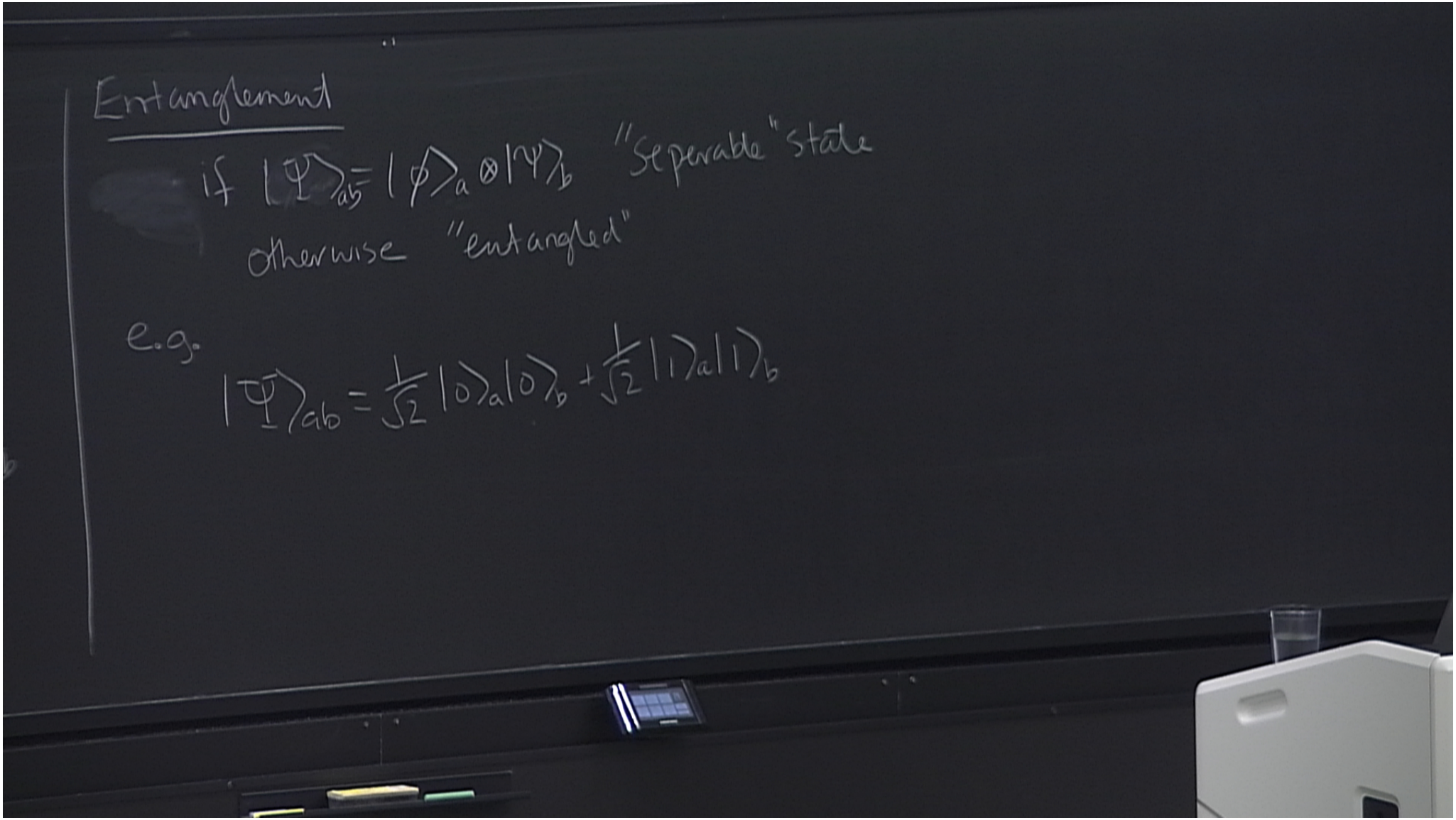
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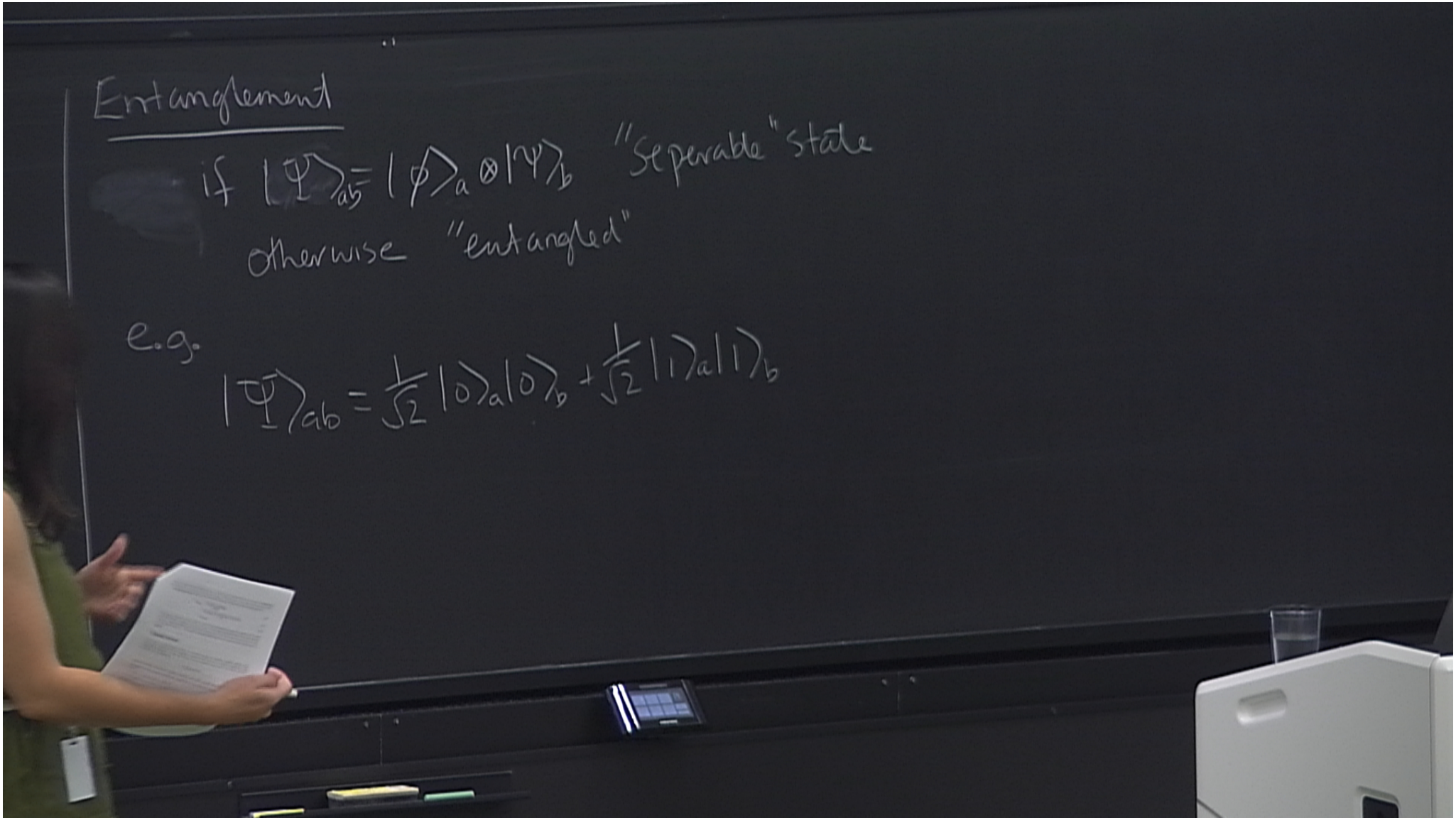
"ϕ" relative phase

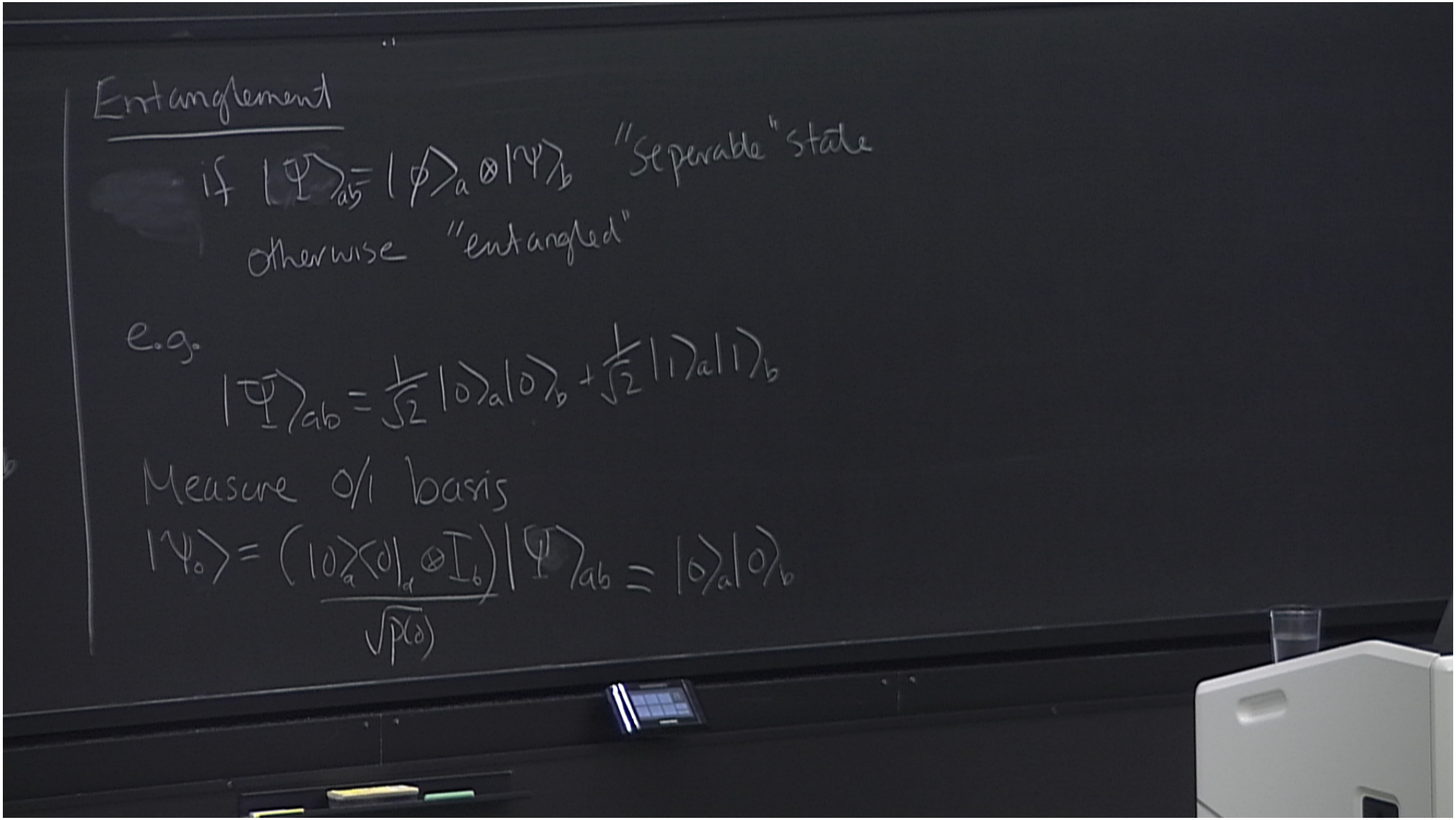
$$p(m) = \langle\psi|P_m|\psi\rangle$$
$$= \left(\alpha^*\langle 0| + e^{-i\phi}\langle 1|\right) P_m \left(\alpha|0\rangle + e^{i\phi}|1\rangle\right)$$
$$= |\alpha|^2\langle 0|P_m|0\rangle + |\beta|^2\langle 1|P_m|1\rangle + 2\,\mathrm{Re}\left[\alpha^*\beta e^{-i\phi}\langle 0|P_m|1\rangle\right]$$

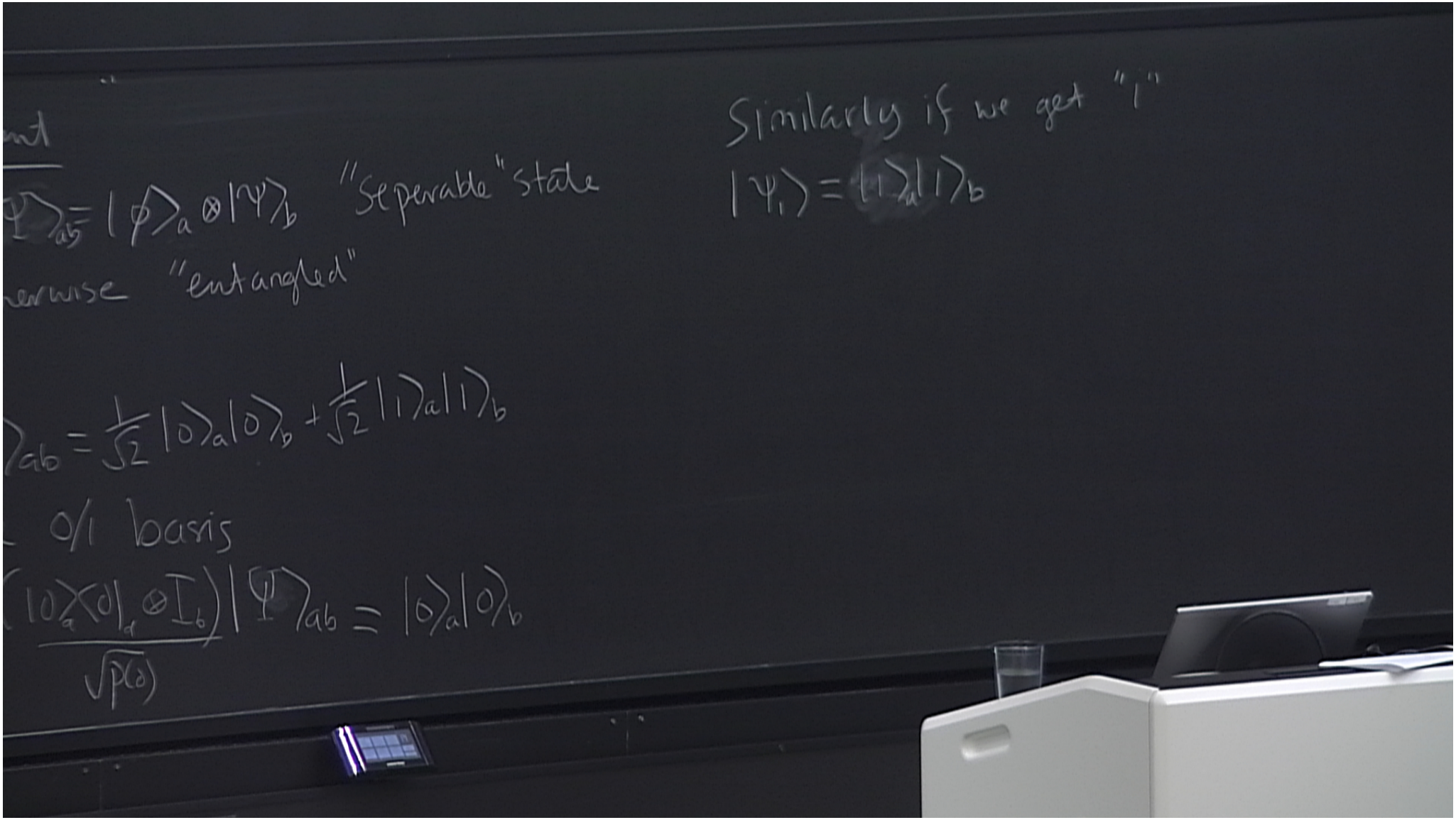

Composite systems
Systems numbered 1....n
$|\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$
You may see:
$|\psi\rangle_a \otimes |\phi\rangle_b = |\psi\rangle_a |\phi\rangle_b = |\psi,\phi\rangle_{ab} = |\psi\phi\rangle_{ab}$

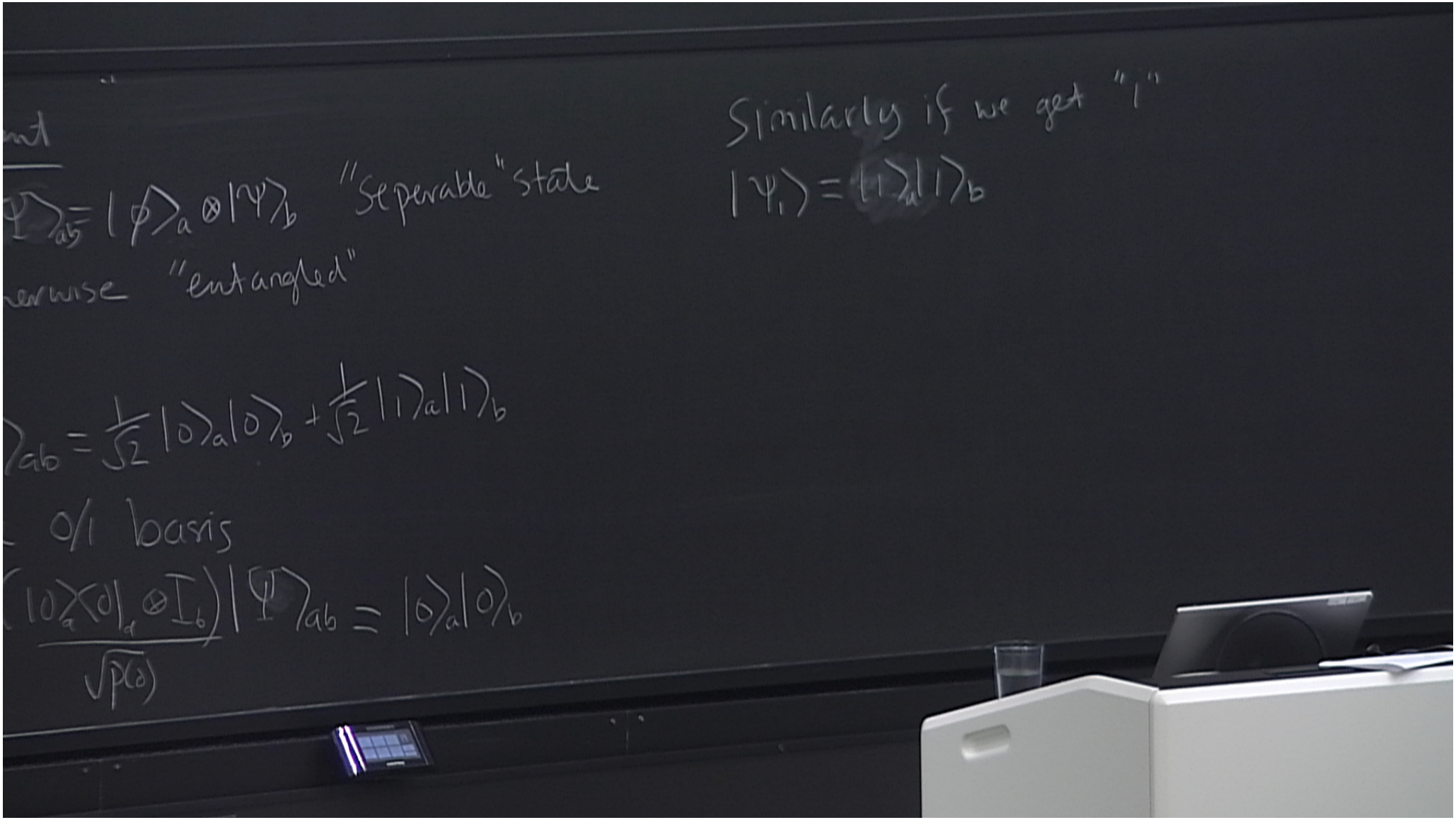

Entanglement
e.g. if $|\Psi\rangle_{ab} = |\phi\rangle_a \otimes |\psi\rangle_b$ "seperable" state
otherwise "entangled"


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Measure 0/1 basis
$|\psi_0\rangle = \frac{(|0\rangle_a\langle 0|_a \otimes I_b)}{\sqrt{p(0)}}|\Psi^-\rangle_{ab} = |0\rangle_a|0\rangle_b$


Similarly if we get "1"
"Seperable" state
"entangled"
0/1 basis


Similarly if we get "1"
$|\psi_1\rangle = |1\rangle_a|1\rangle_b$
"Seperable" state
$|0\rangle_a|0\rangle_b + \frac{1}{\sqrt{2}}|1\rangle_a|1\rangle_b$
0/1 basis
$|0\rangle_a\langle 0|_a \otimes I_b)|\Psi\rangle_{ab} = |0\rangle_a|0\rangle_b$
$\sqrt{p(0)}$

nt

$|\psi\rangle_{ab} = |\phi\rangle_a \otimes |\psi\rangle_b$ "Seperable" state

erwise "entangled"

$\rangle_{ab} = \frac{1}{\sqrt{2}}|0\rangle_a|0\rangle_b + \frac{1}{\sqrt{2}}|1\rangle_a|1\rangle_b$

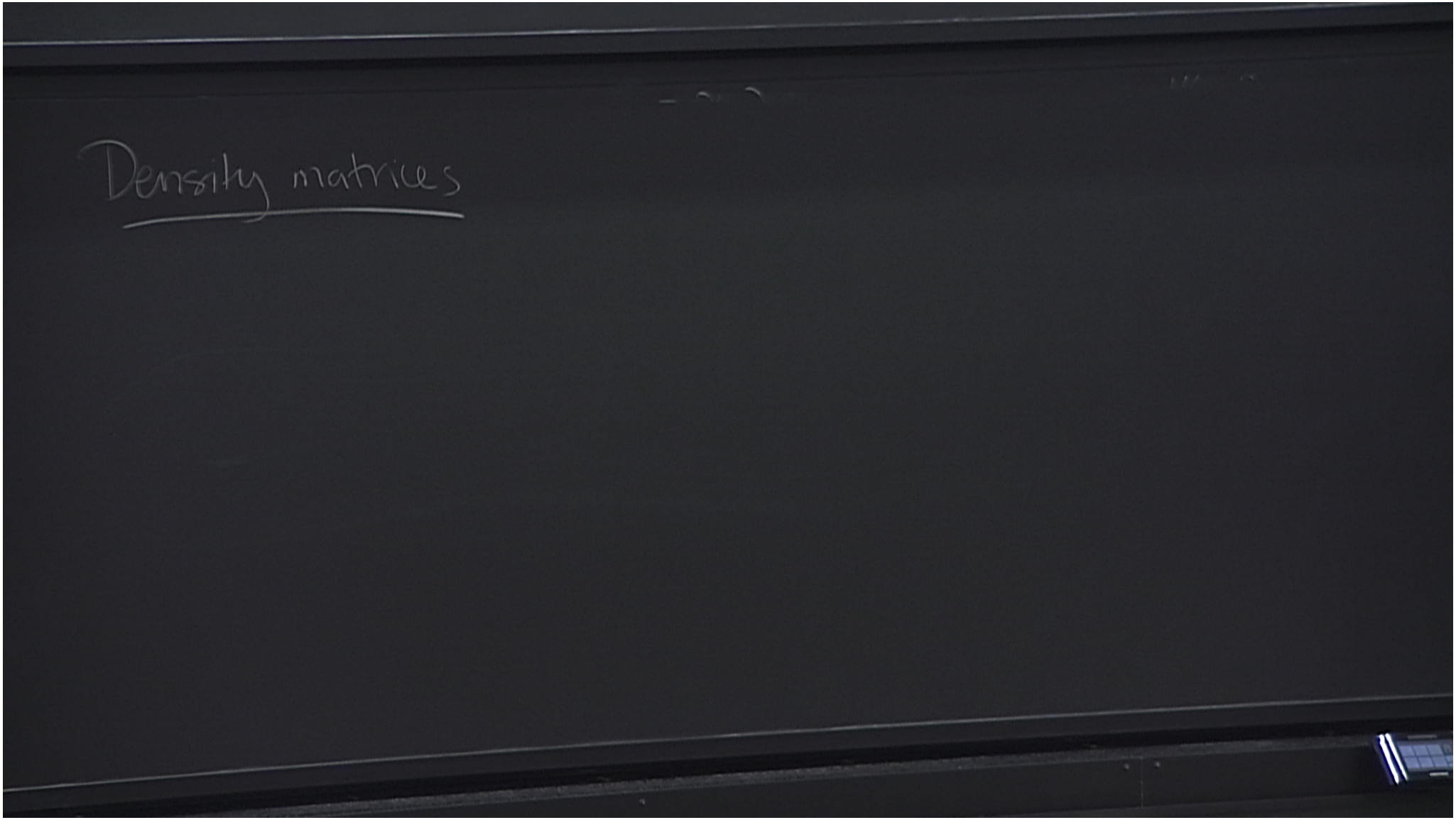
0/1 basis

$\frac{|0\rangle_a\langle 0|_a \otimes I_b}{\sqrt{P(0)}} |\psi\rangle_{ab} = |0\rangle_a|0\rangle_b$

Similarly if we get "1"

$|\psi_1\rangle = |1\rangle_a|1\rangle_b$

Can't use this to comunicate f.t.l. because we choose basis not outcome.


Density matrices

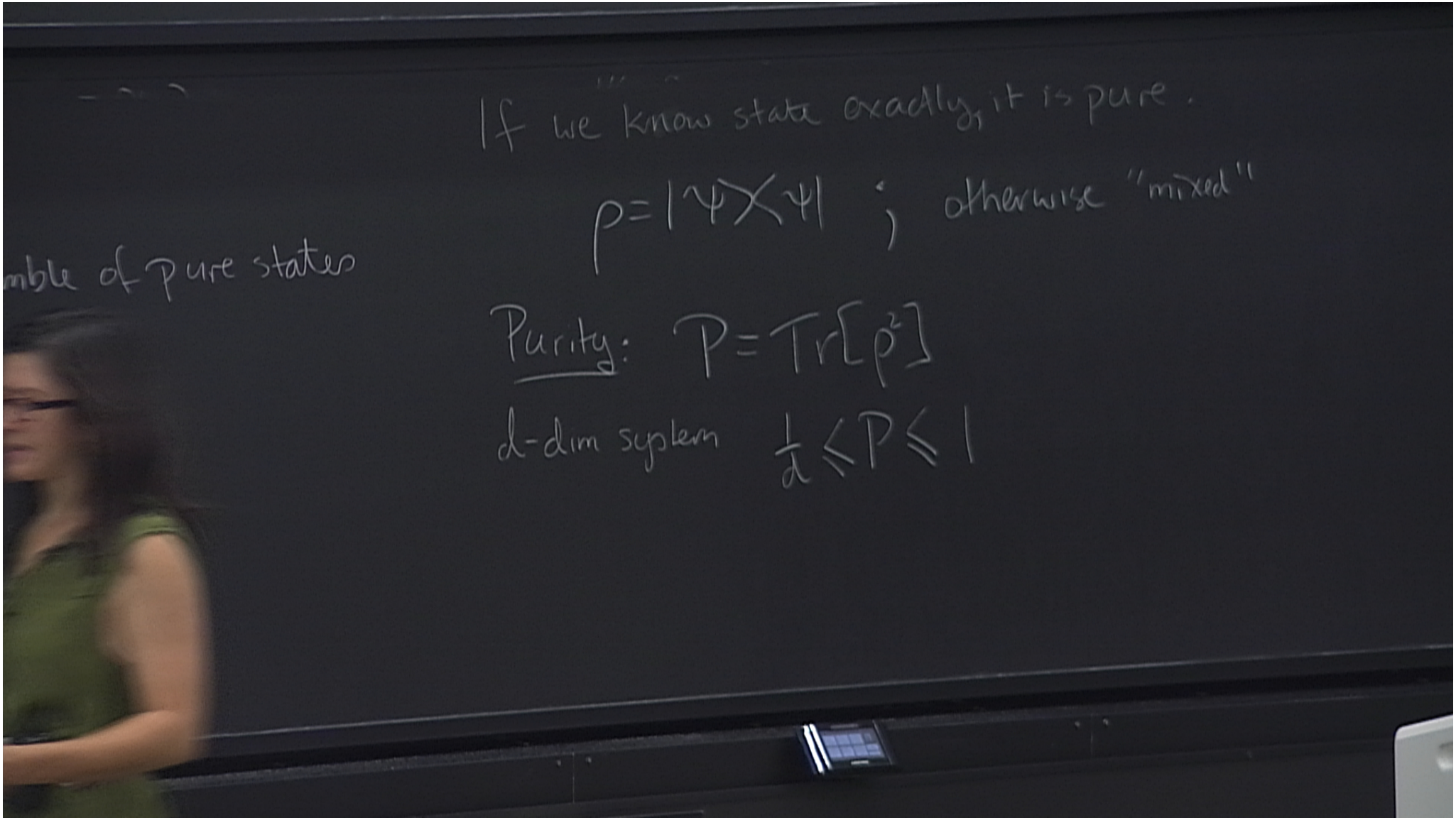
Density matrices

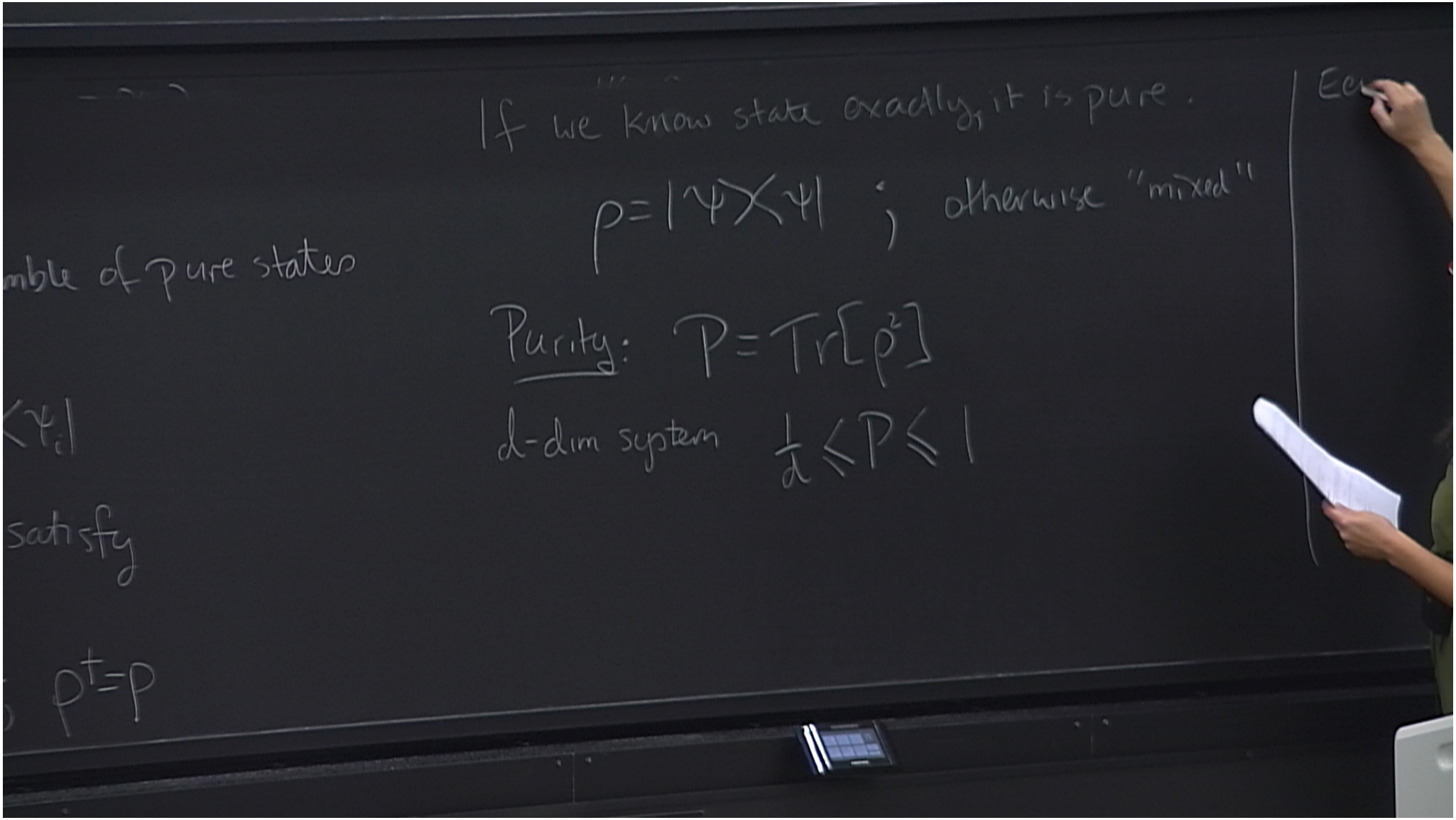
$\{p_i, |\psi_i\rangle\}$ is an ensemble of pure states

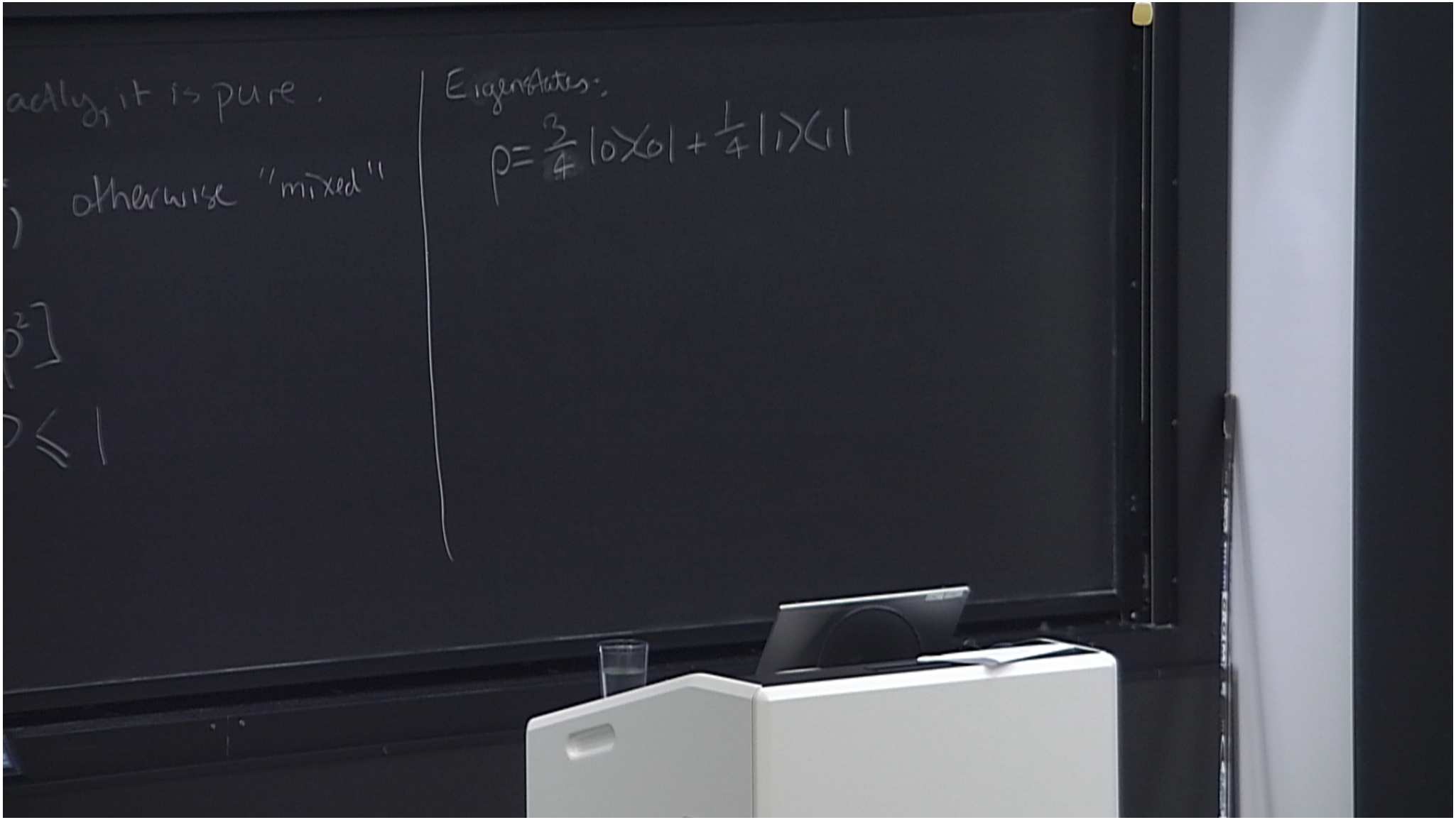
$$\rho \equiv \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

To be physical, must satisfy

- $\mathrm{Tr}[\rho] = 1$
- Positive-semi definite


If we know state exactly, it is pure.
$\rho = |\psi\rangle\langle\psi|$; otherwise "mixed"
mble of pure states
Purity: $P = Tr[\rho^2]$
d-dim system $\frac{1}{d} \leq P \leq 1$


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$\langle\psi_i|$
satisfy
$\rho^\dagger = \rho$

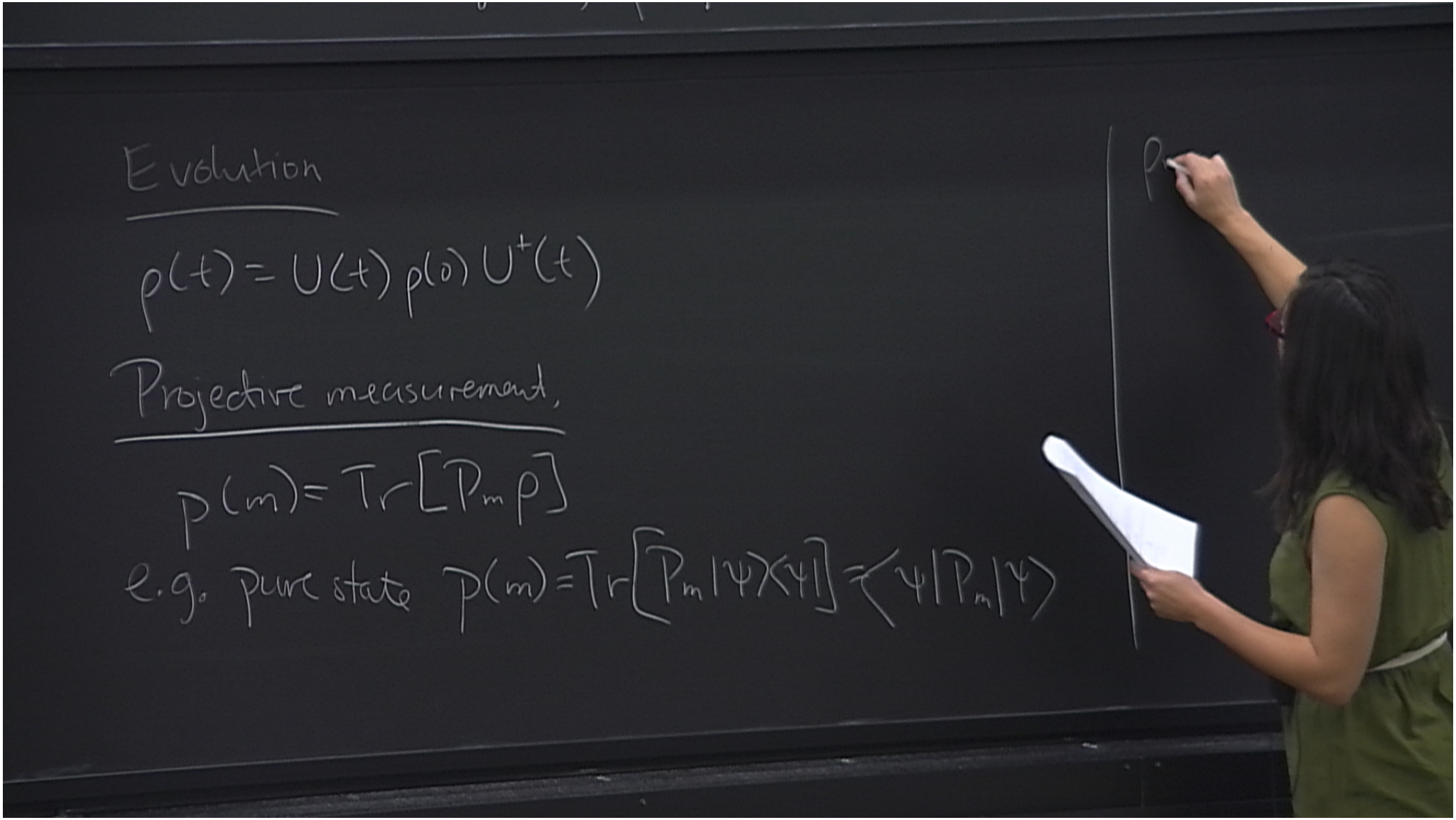

actly, it is pure.
otherwise "mixed"
Eigenstates:
$\rho = \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|$

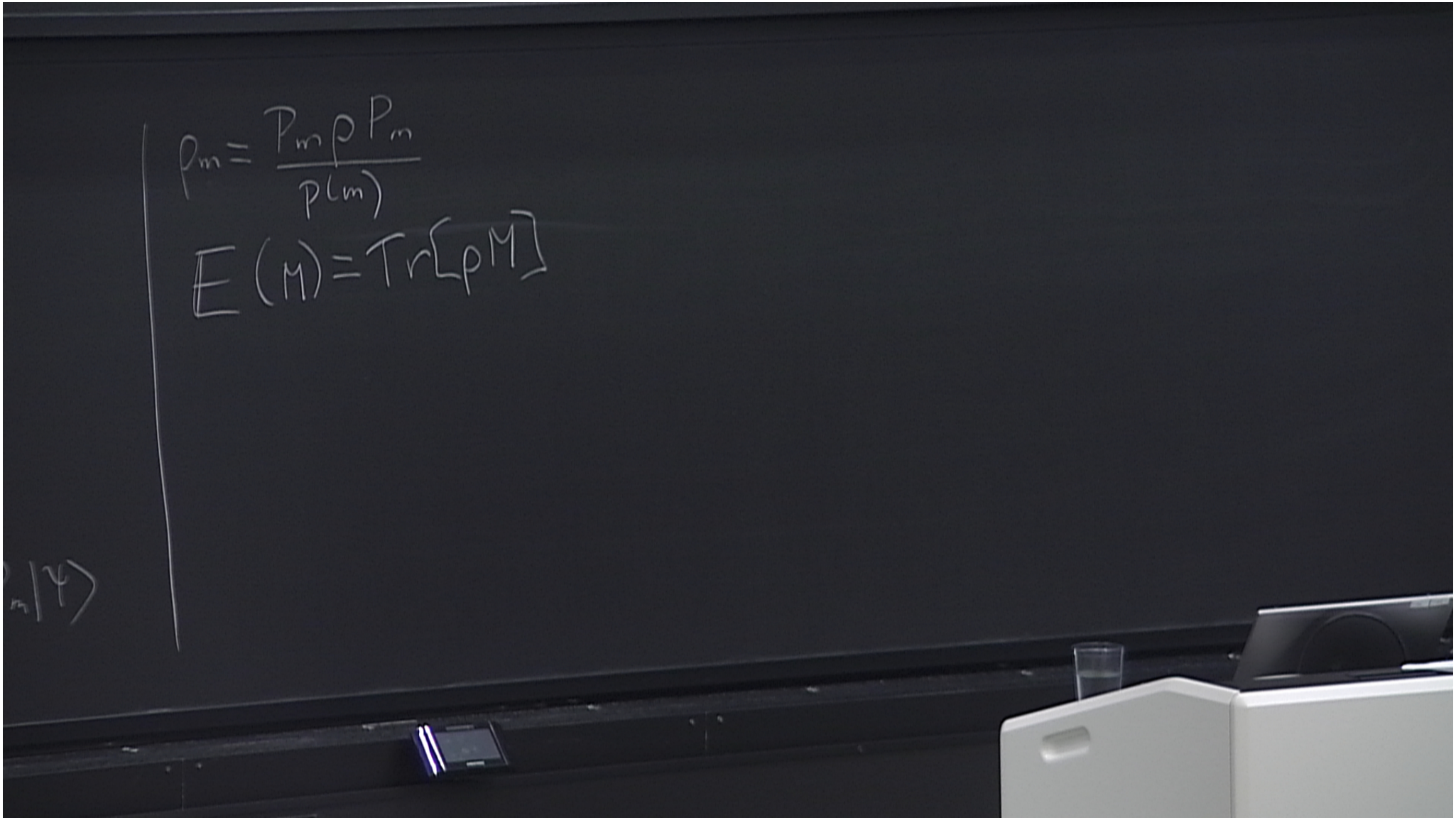
Evolution

$$\rho(t) = U(t)\,\rho(0)\,U^{+}(t)$$

Projective measurement.

$$p(m) = Tr[P_m \rho]$$


Evolution
$\rho(t) = U(t)\,\rho(0)\,U^{\dagger}(t)$
Projective measurement.
$p(m) = \mathrm{Tr}[P_m \rho]$
e.g. pure state $p(m) = \mathrm{Tr}[P_m |\psi\rangle\langle\psi|] = \langle\psi| P_m |\psi\rangle$


$\rho_m = \frac{P_m \rho P_m}{p(m)}$
$E(M) = Tr[\rho M]$

systems a & b. $|\Psi\rangle_{ab}$

operator for 'a'

Define:

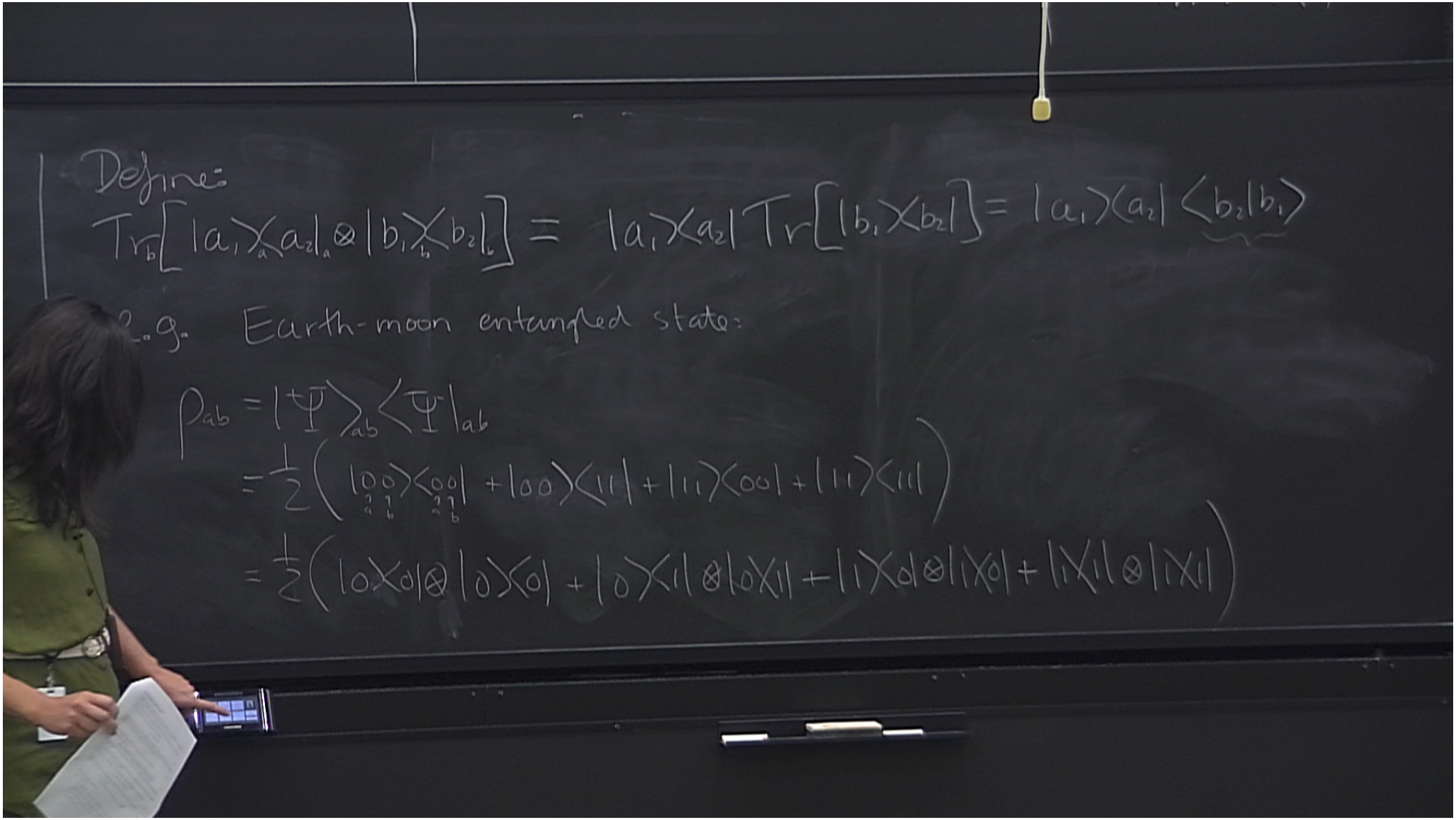
$$\mathrm{Tr}_b\left[|a_1\rangle\langle a_2|_a \otimes |b_1\rangle\langle b_2|_b\right] = |a_1\rangle\langle a_2| \mathrm{Tr}\left[|b_1\rangle\langle b_2|\right.$$

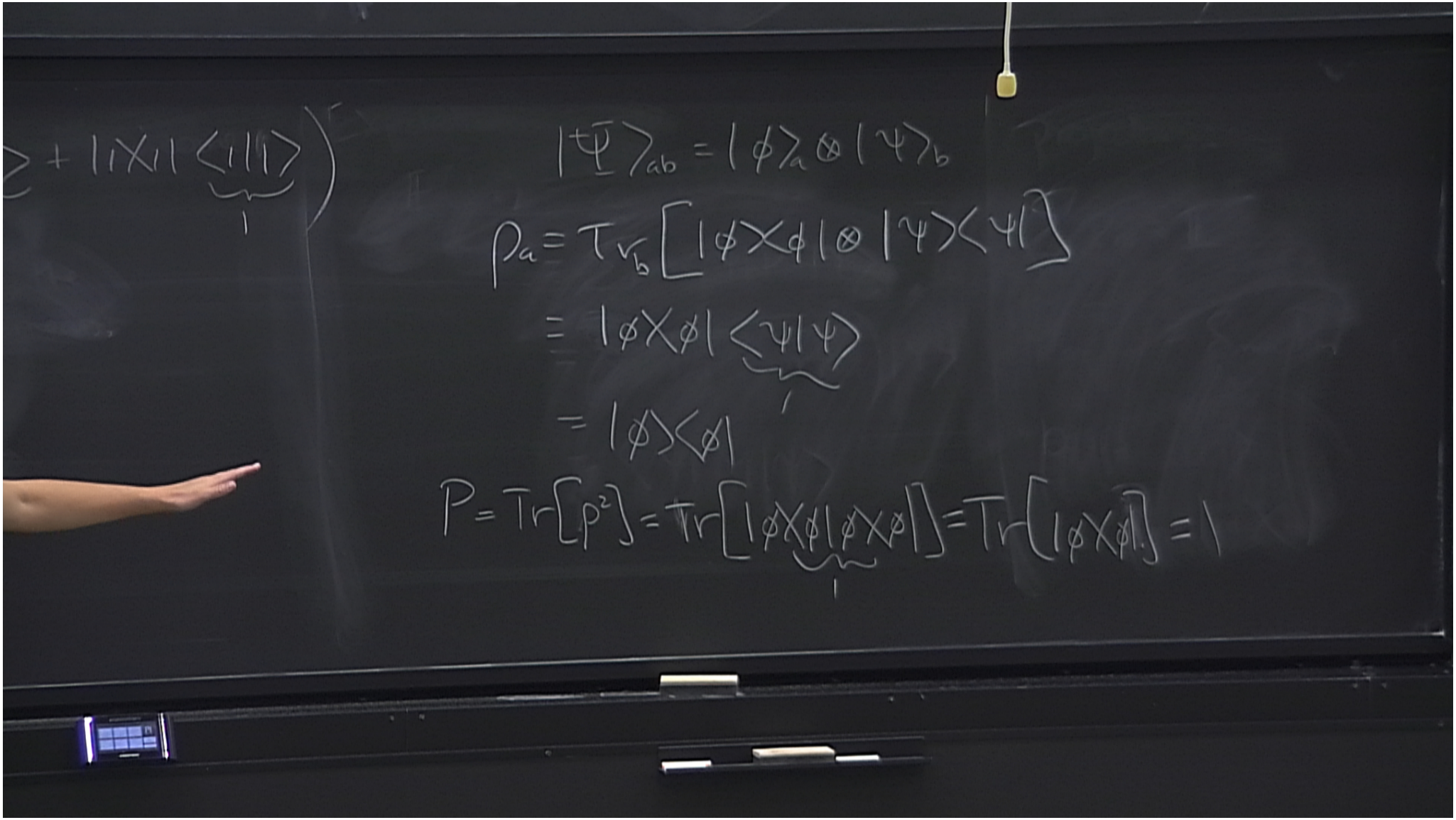
Define:

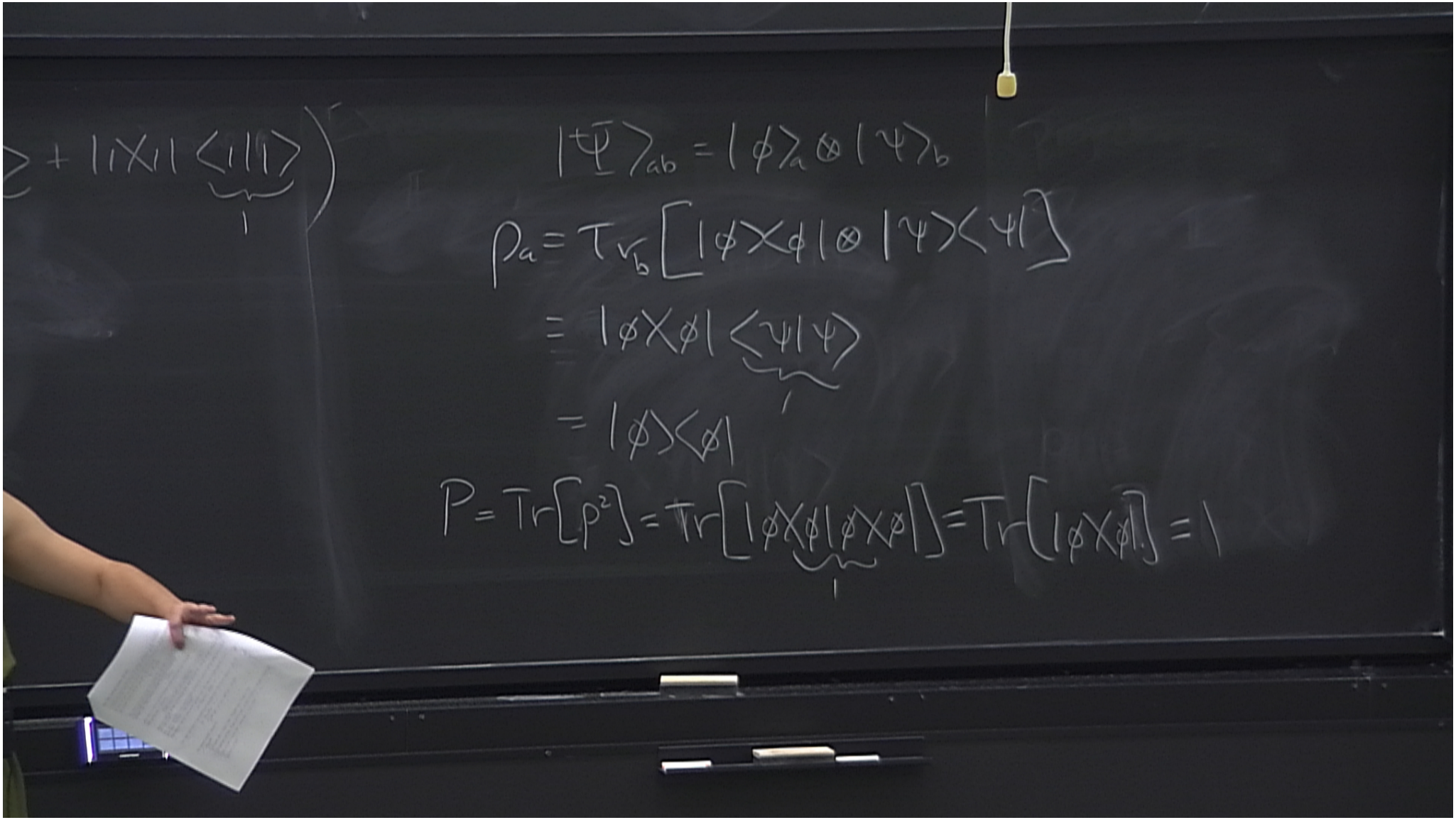
$$\mathrm{Tr}_b\left[|a_1\rangle\langle a_2|_a \otimes |b_1\rangle\langle b_2|_b\right] \equiv |a_1\rangle\langle a_2| \, \mathrm{Tr}\left[|b_1\rangle\langle b_2|\right] = |a_1\rangle\langle a_2| \langle b_2|b_1\rangle$$

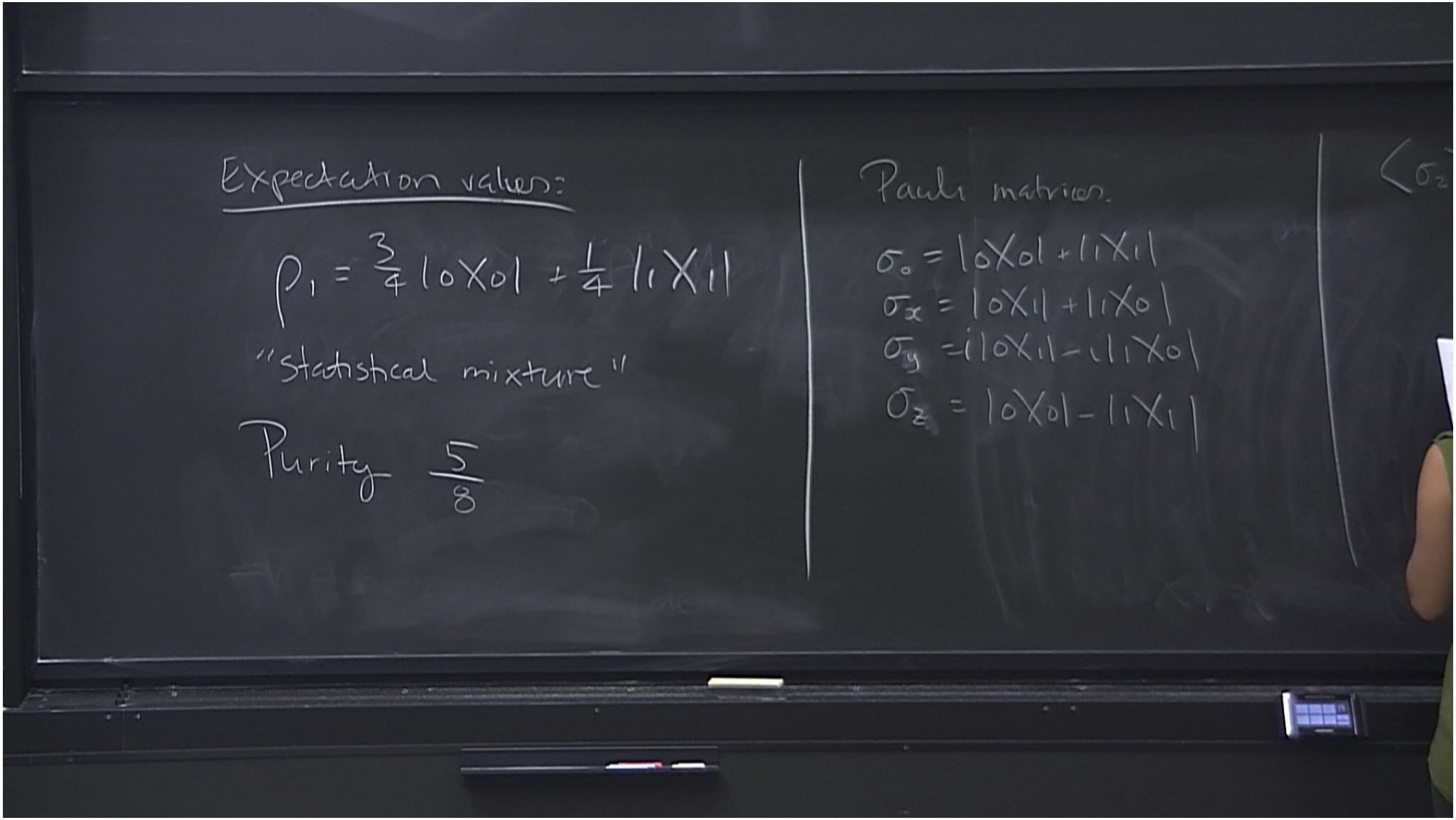
e.g. Earth-moon entangled state:

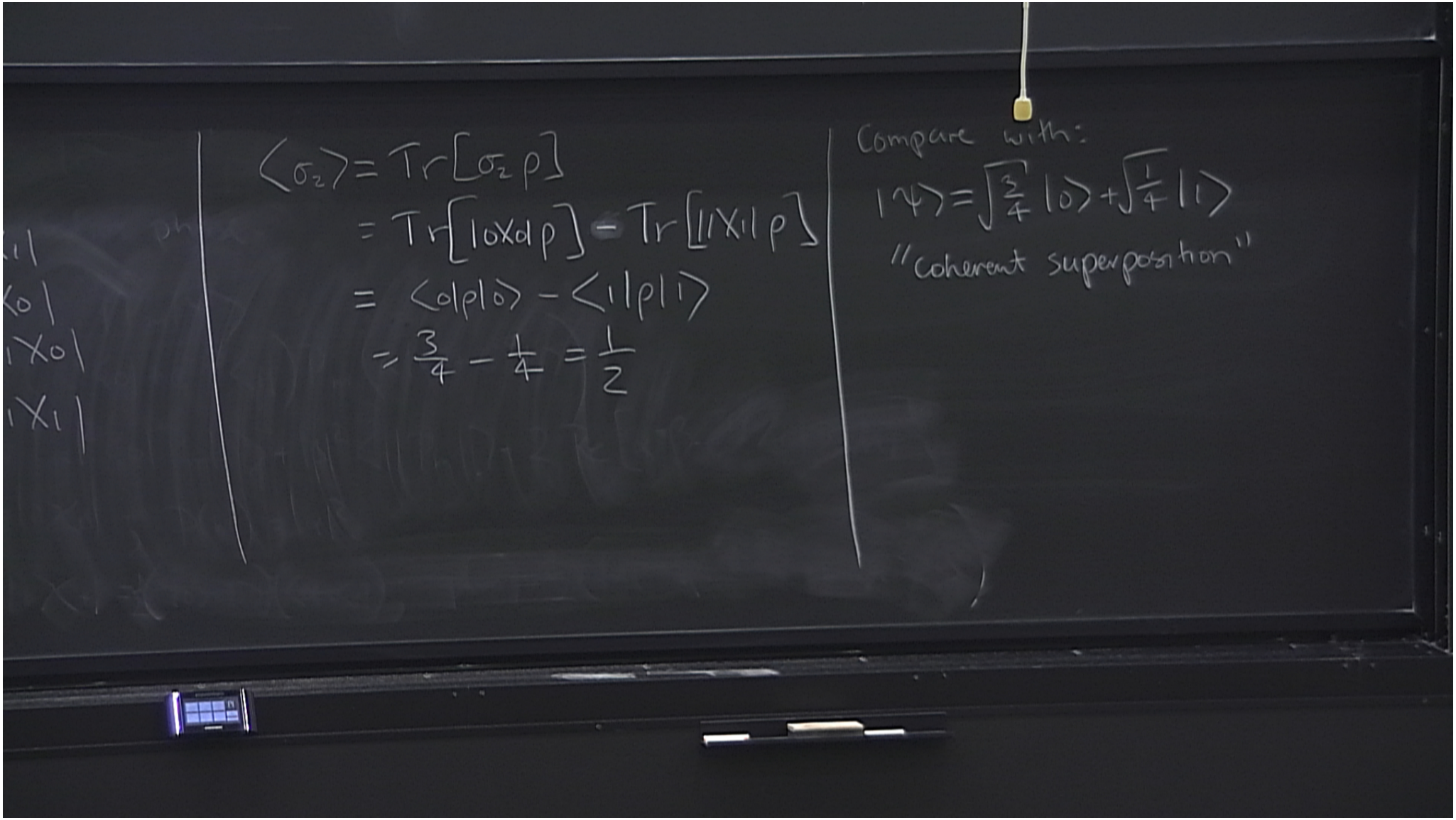
$$\rho_{ab} = |\Psi^+\rangle_{ab}\langle\Psi^-|_{ab}$$
$$= \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$$

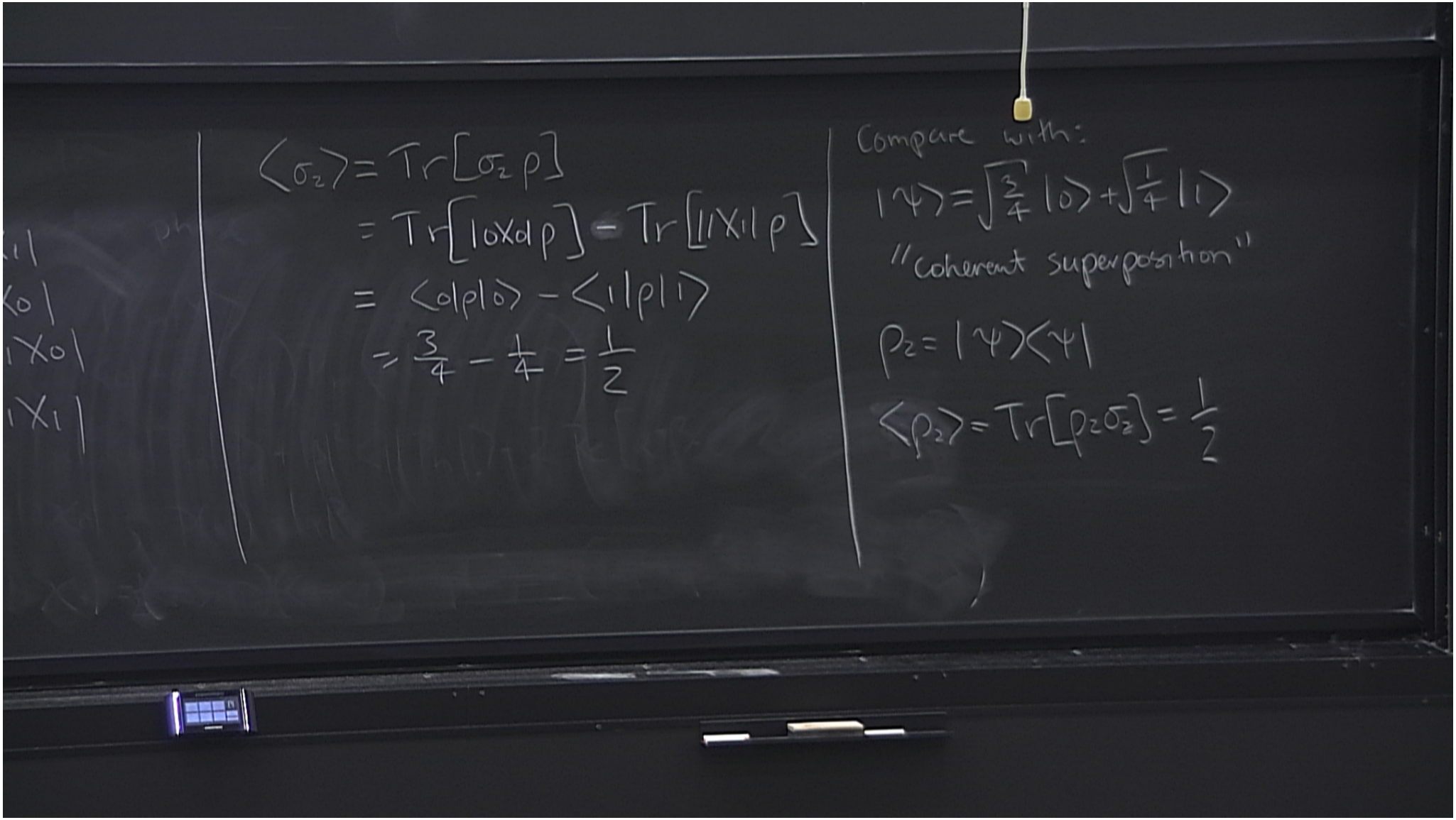

Define:
$\mathrm{Tr}_b[|a_1\rangle\langle a_2|_a \otimes |b_1\rangle\langle b_2|_b] = |a_1\rangle\langle a_2| \mathrm{Tr}[|b_1\rangle\langle b_2|] = |a_1\rangle\langle a_2| \langle b_2|b_1\rangle$
e.g. Earth-moon entangled state:
$\rho_{ab} = |\Psi^+\rangle_{ab}\langle\Psi^-|_{ab}$
$= \frac{1}{2}(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|)$
$= \frac{1}{2}(|0\rangle\langle 0| \otimes |0\rangle\langle 0| + |0\rangle\langle 1| \otimes |0\rangle\langle 1| + |1\rangle\langle 0| \otimes |1\rangle\langle 0| + |1\rangle\langle 1| \otimes |1\rangle\langle 1|)$

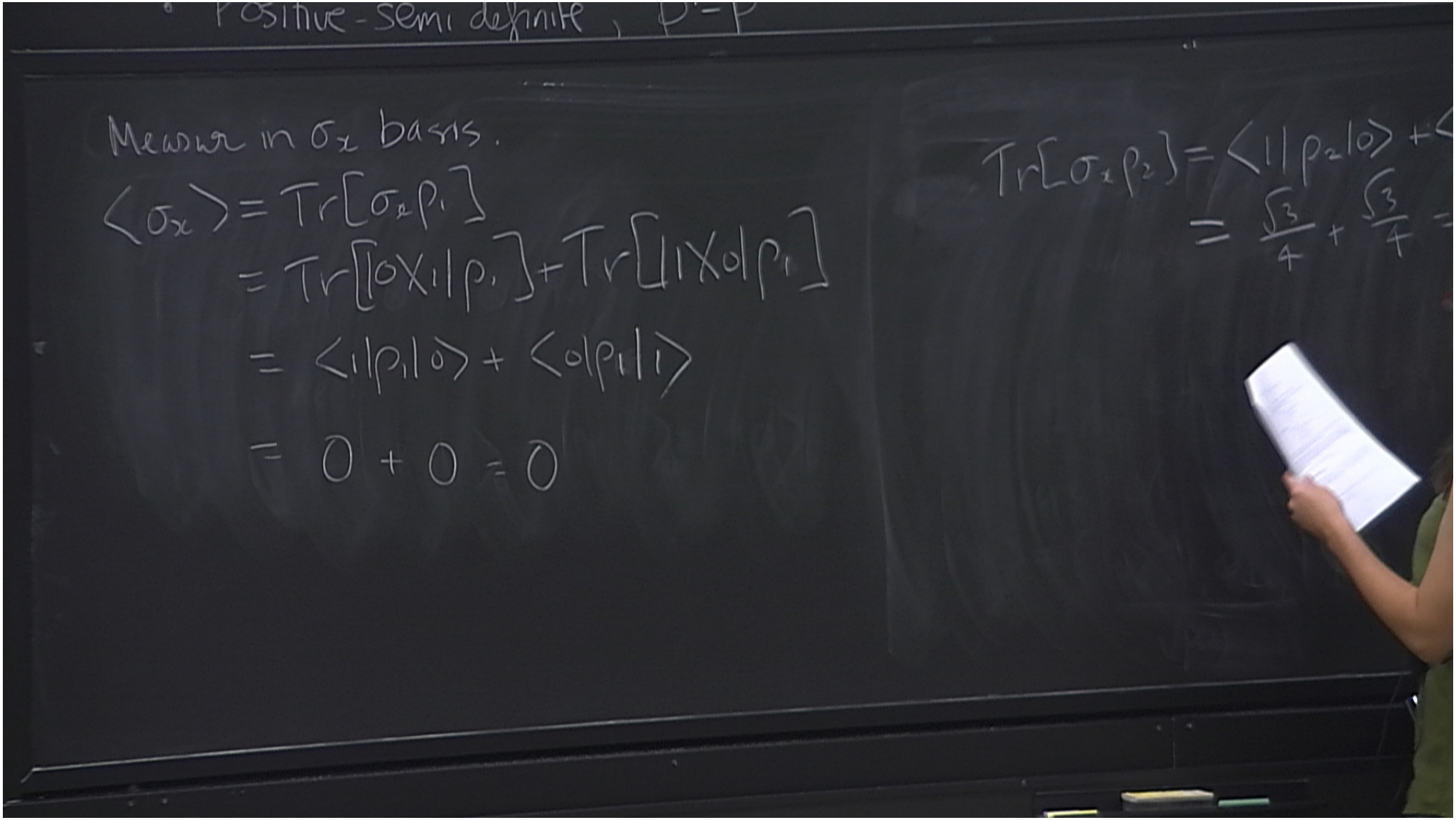

+ |1><1| <1|1|> = 1
|Ψ⁻>ab = |φ>a ⊗ |ψ>b
ρa = Trb[|φ><φ| ⊗ |ψ><ψ|]
= |φ><φ| <ψ|ψ> = 1
= |φ><φ|
P = Tr[ρ²] = Tr[|φ><φ| |φ><φ|] = Tr[|φ><φ|] = 1

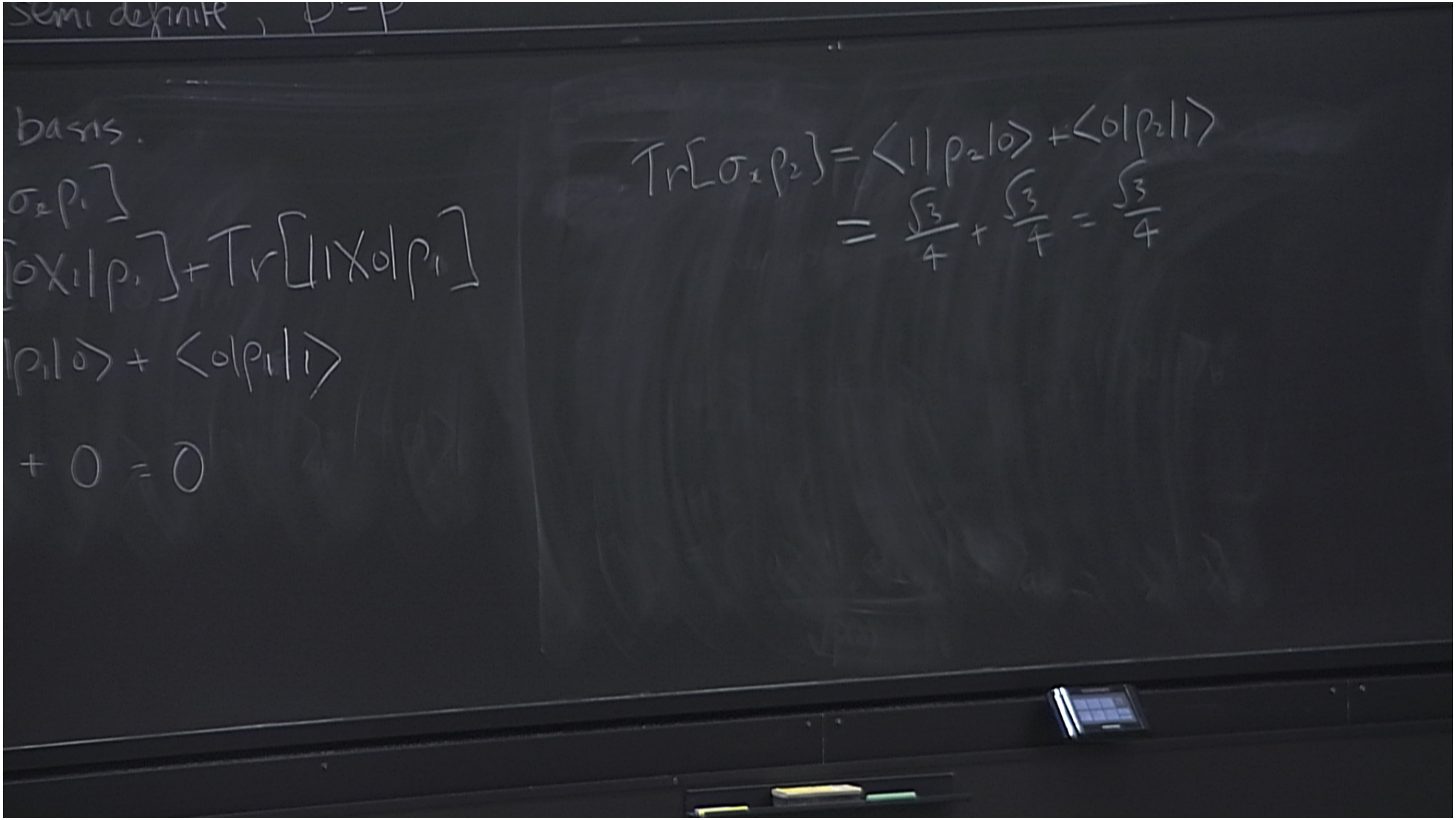

$+|1\rangle\langle 1|\otimes|1\rangle\langle 1|)$
$|\Psi^-\rangle_{ab} = |\phi\rangle_a \otimes |\psi\rangle_b$
$\rho_a = Tr_b[|\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|]$
$= |\phi\rangle\langle\phi| \langle\psi|\psi\rangle$
$= |\phi\rangle\langle\phi|$
$P = Tr[\rho^2] = Tr[|\phi\rangle\langle\phi|\phi\rangle\langle\phi|] = Tr[|\phi\rangle\langle\phi|] = 1$

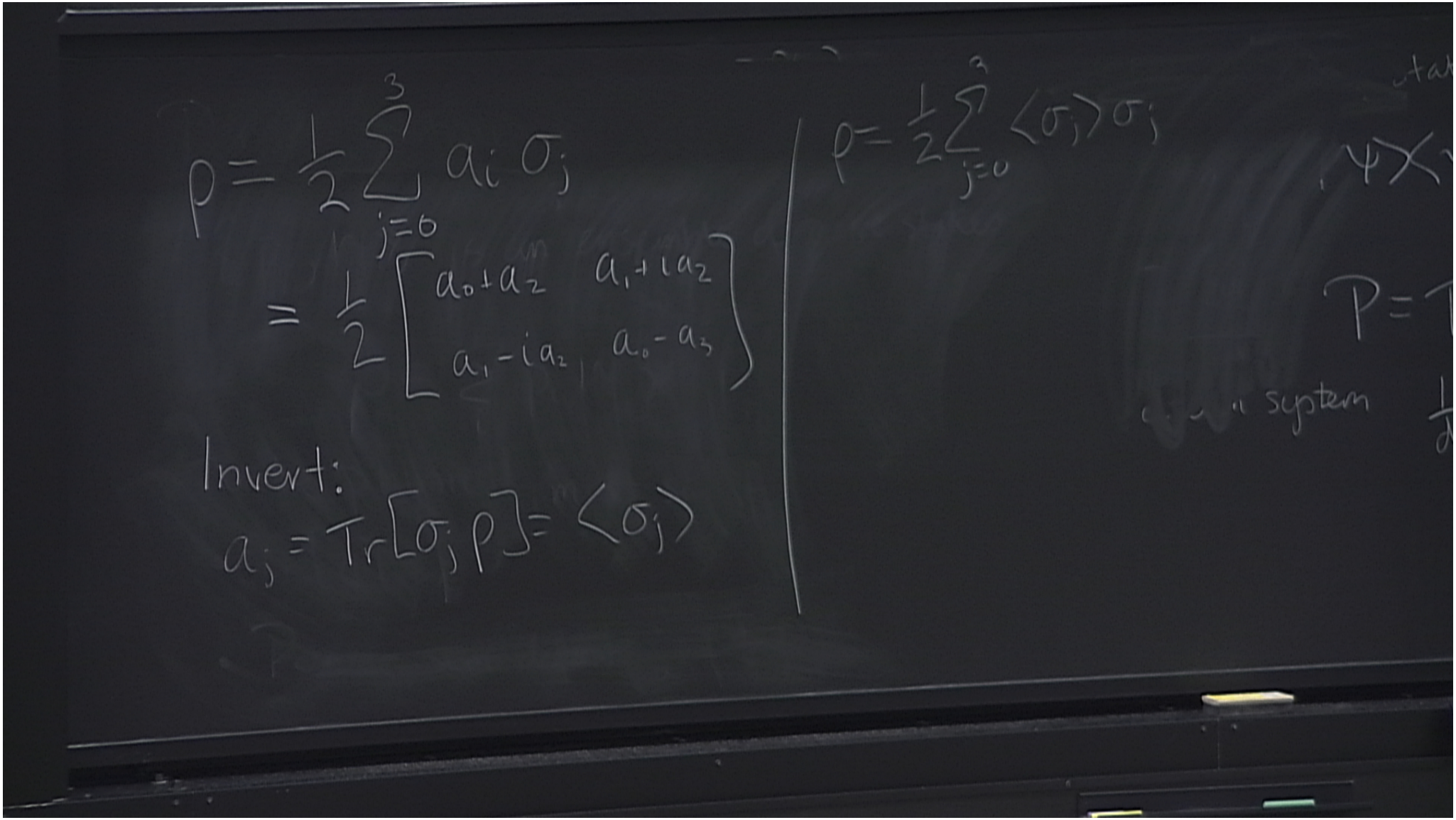

Expectation values:
$\rho_1 = \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|$
"statistical mixture"
Purity $\frac{5}{8}$
Pauli matrices.
$\sigma_0 = |0\rangle\langle 0| + |1\rangle\langle 1|$
$\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$
$\sigma_y = i|0\rangle\langle 1| - i|1\rangle\langle 0|$
$\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$
$\langle \sigma_z$


$\langle \sigma_z \rangle = Tr[\sigma_z \rho]$
$= Tr[|0\rangle\langle 0|\rho] - Tr[|1\rangle\langle 1|\rho]$
$= \langle 0|\rho|0\rangle - \langle 1|\rho|1\rangle$
$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$
Compare with:
$|\psi\rangle = \sqrt{\frac{3}{4}}|0\rangle + \sqrt{\frac{1}{4}}|1\rangle$
"coherent superposition"


$\langle\sigma_z\rangle = Tr[\sigma_z \rho]$
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Compare with:
$|\psi\rangle = \sqrt{\frac{3}{4}}|0\rangle + \sqrt{\frac{1}{4}}|1\rangle$
"coherent superposition"
$\rho_2 = |\psi\rangle\langle\psi|$
$\langle\rho_2\rangle = Tr[\rho_2\sigma_z] = \frac{1}{2}$


Positive-semi definite, ρ†=ρ
Measure in σx basis.
⟨σx⟩ = Tr[σxρ1]
= Tr[|0⟩⟨1|ρ1] + Tr[|1⟩⟨0|ρ1]
= ⟨1|ρ1|0⟩ + ⟨0|ρ1|1⟩
= 0 + 0 = 0
Tr[σxρ2] = ⟨1|ρ2|0⟩ +
= √3/4 + √3/4 =


semi definite ; $\rho^2 = \rho$
basis.
$\sigma_2 \rho_1]$
$[\sigma_x | \rho_1] + Tr[|1\rangle\langle 0| \rho_1]$
$|\rho_1|0\rangle + \langle 0|\rho_1|1\rangle$
$+ 0 = 0$
$Tr[\sigma_x \rho_2] = \langle 1|\rho_2|0\rangle + \langle 0|\rho_2|1\rangle$
$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$


$\rho = \frac{1}{2}\sum_{j=0}^{3} a_j \sigma_j$
$= \frac{1}{2}\begin{bmatrix} a_0 + a_z & a_1 + i a_2 \\ a_1 - i a_2 & a_0 - a_3 \end{bmatrix}$
Invert:
$a_j = Tr[\sigma_j \rho] = \langle \sigma_j \rangle$
$\rho = \frac{1}{2}\sum_{j=0}^{3} \langle \sigma_j \rangle \sigma_j$