Title: 14/15 PSI - Quantum Mechanics 3

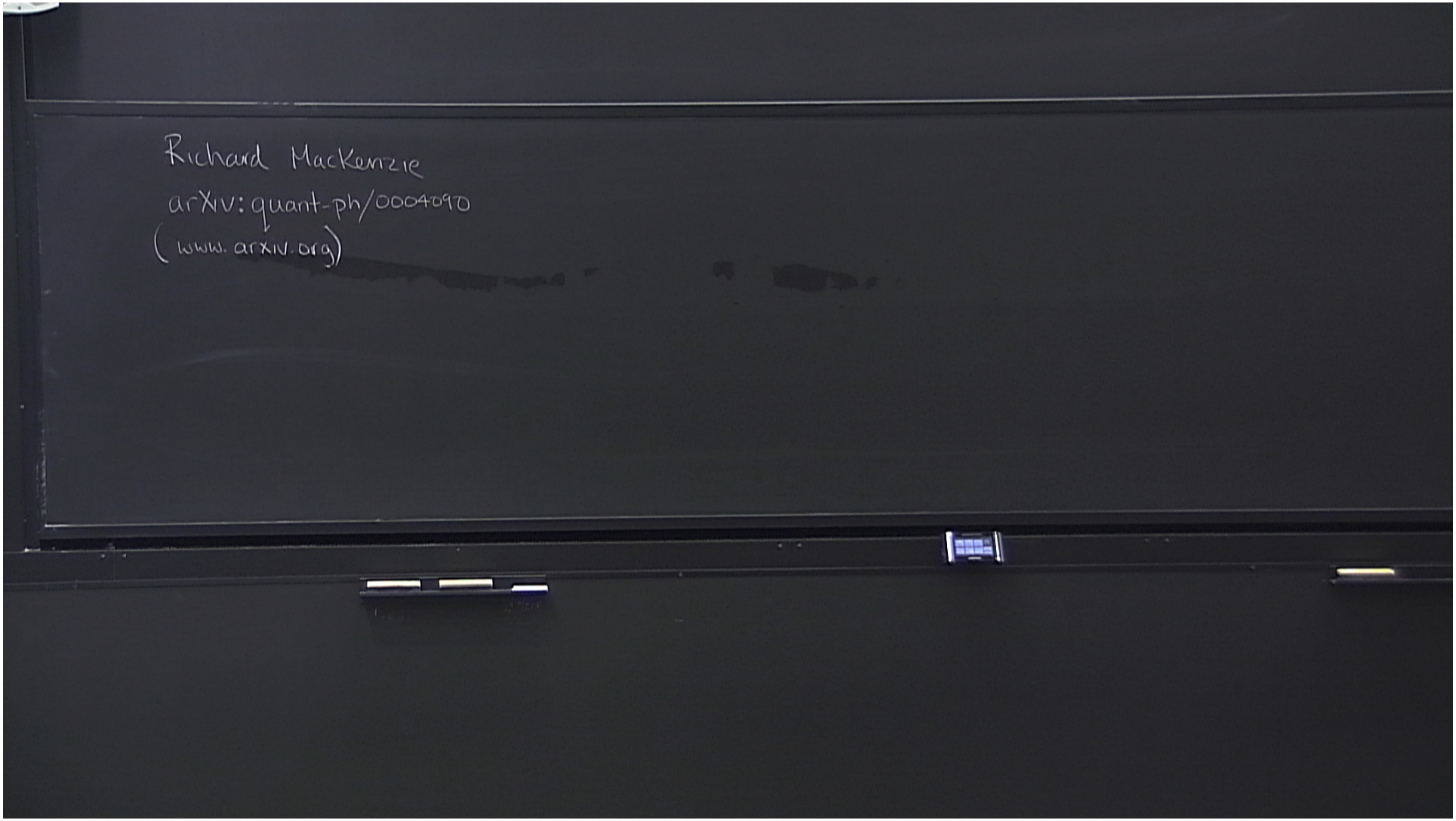
Date: Aug 21, 2014 10:30 AM

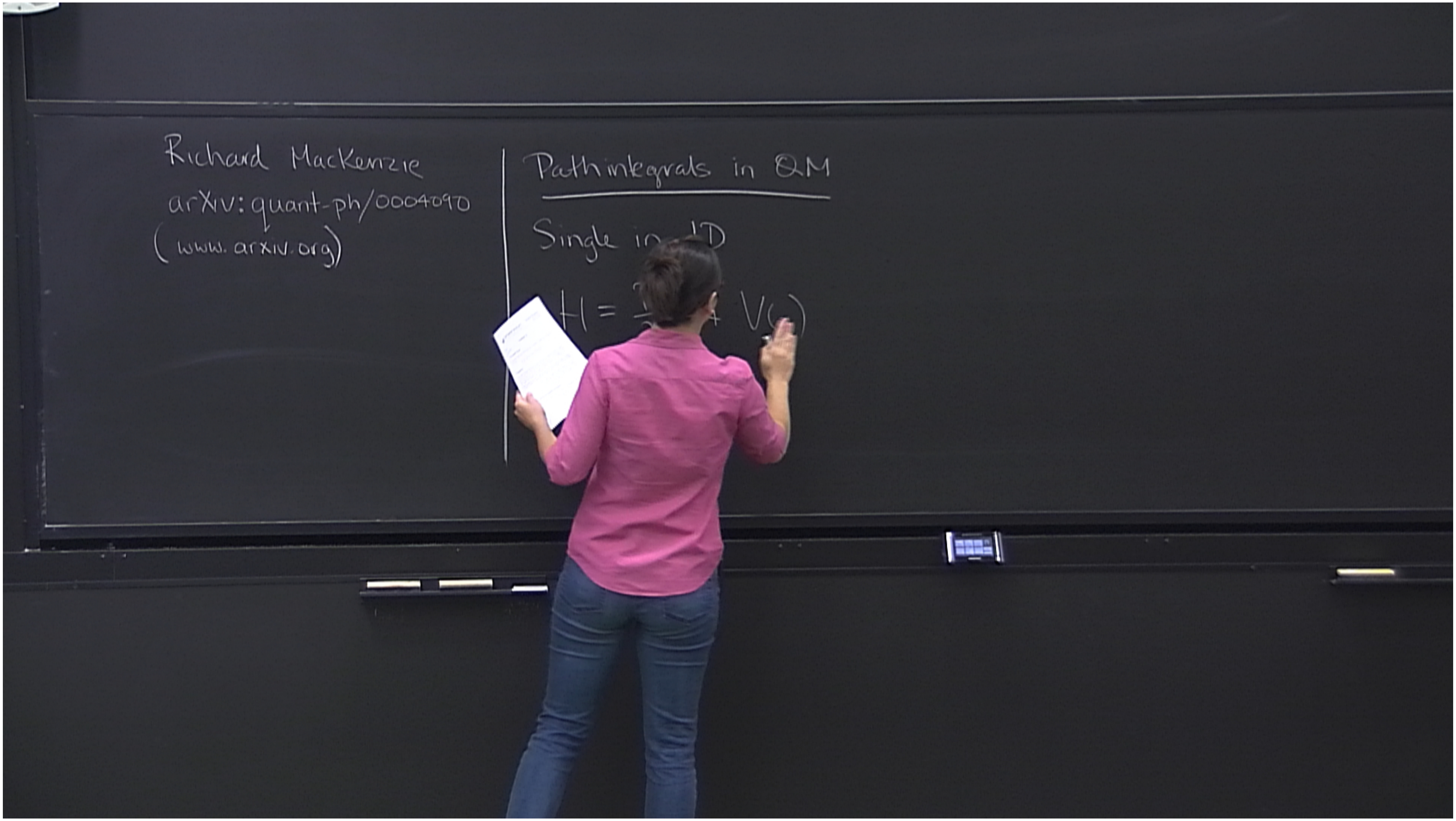
URL: http://pirsa.org/14080026

Abstract:

Gaussian Integral

$$\int_{-\infty}^{\infty} \exp\left\{-ax^2 + bx + c\right\} dx = \sqrt{\frac{\pi}{a}} \exp\left\{\frac{b^2}{4a} + c\right\}$$


Richard MacKenzie
arXiv:quant-ph/0004090
(www.arxiv.org)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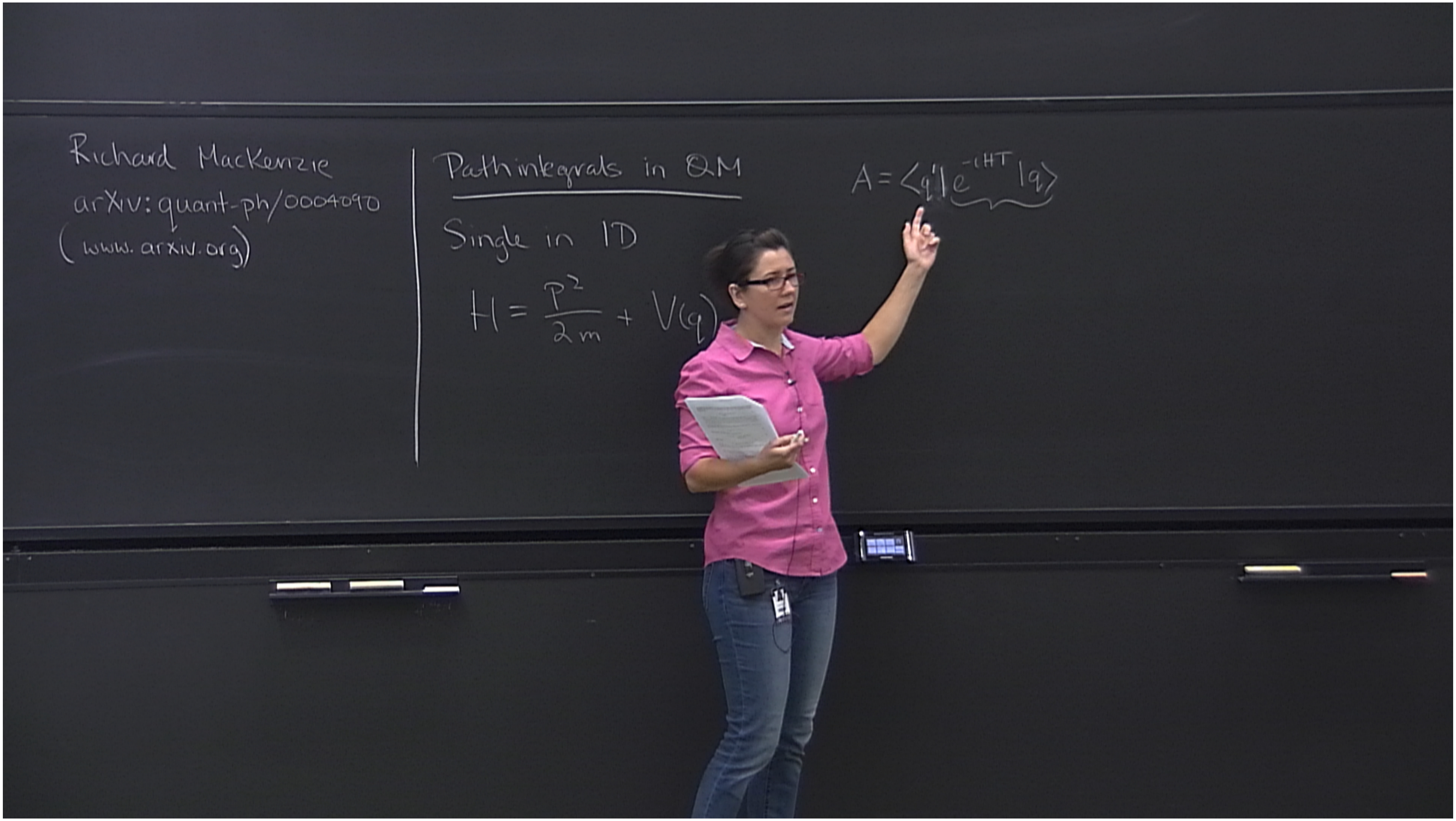
Path integrals in QM
Single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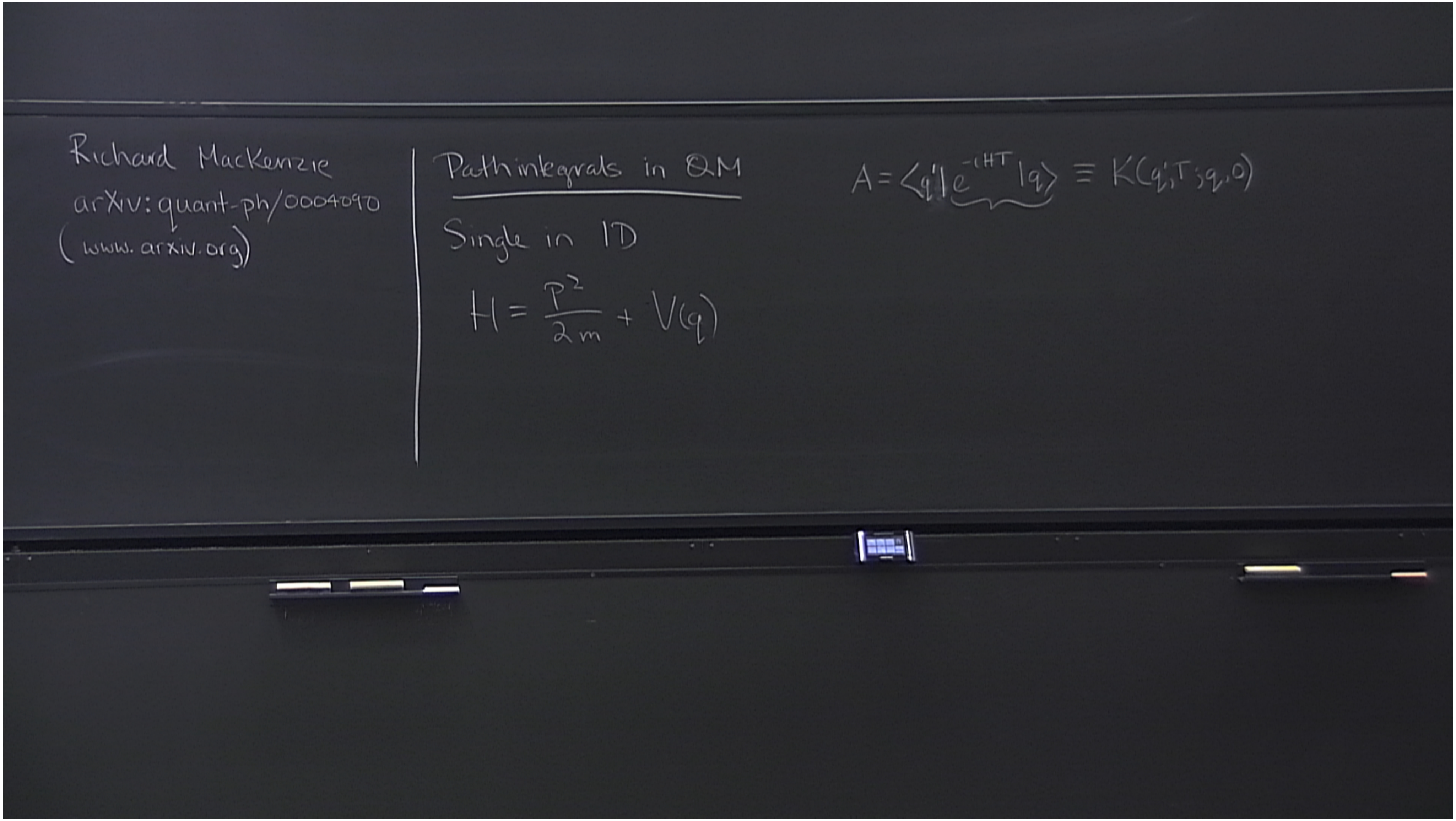
Path integrals in QM
Single in 1D
H = p²/2m + V(q)
A = ⟨q'|e^{-iHT}|q⟩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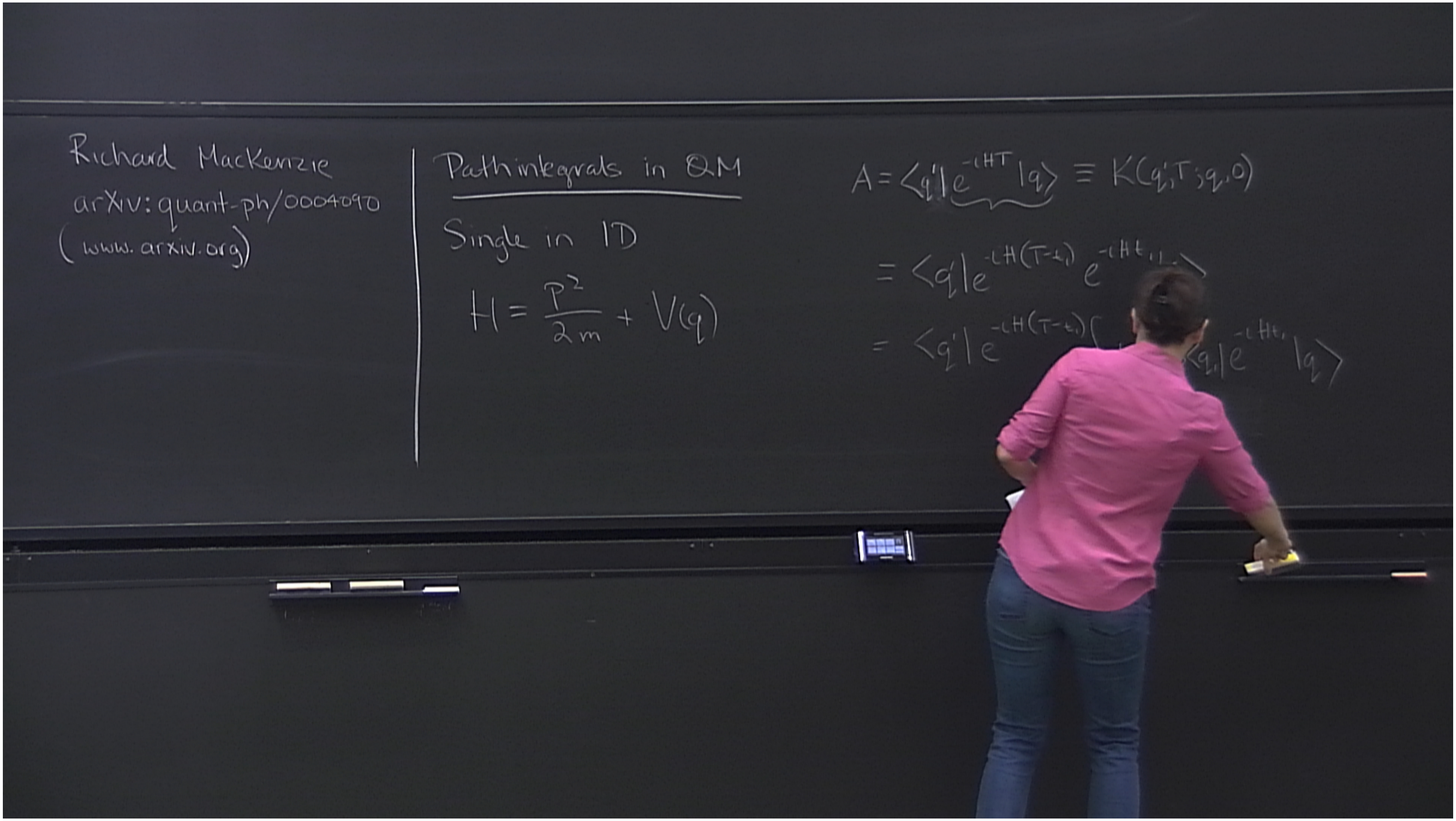
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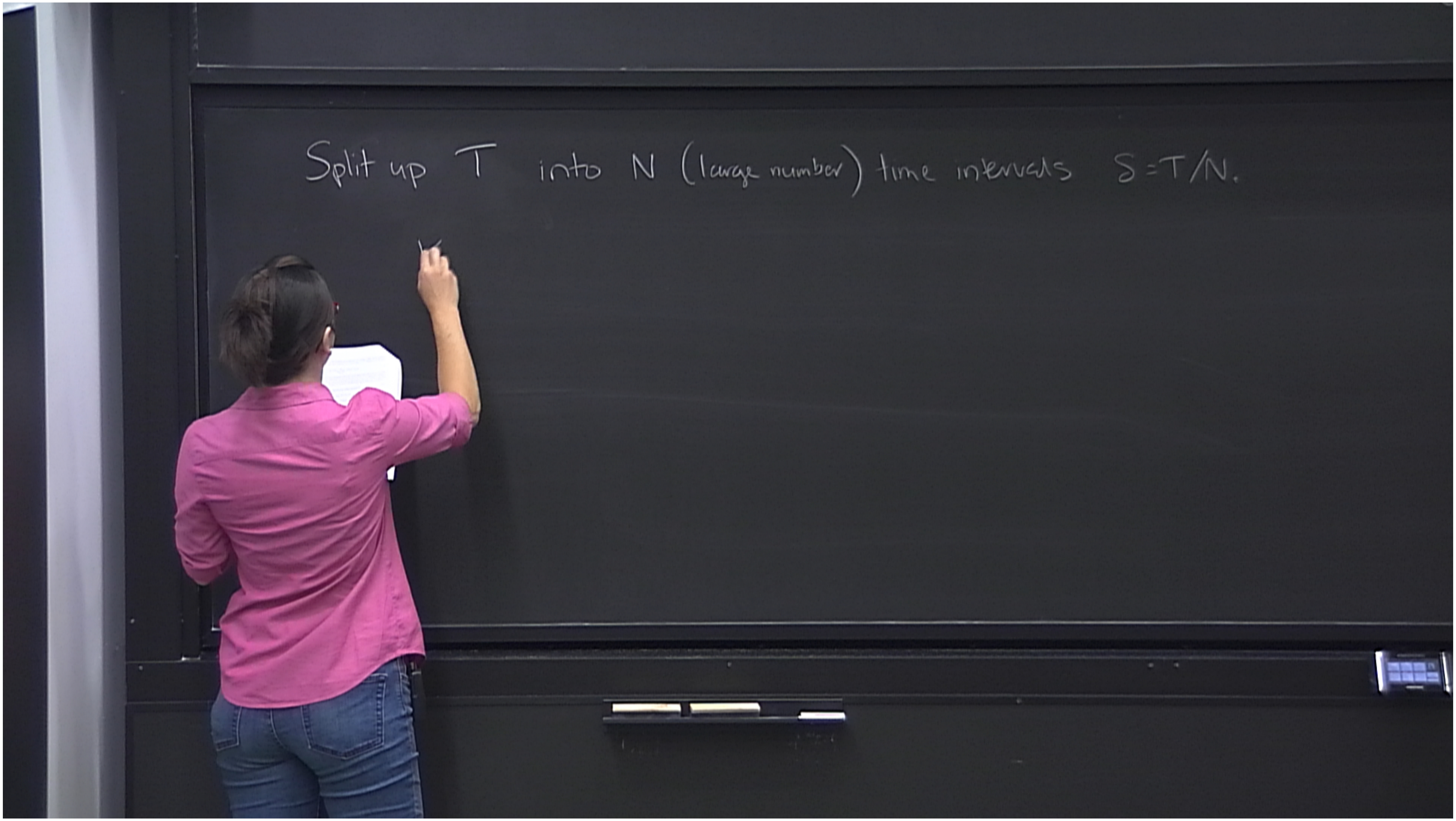
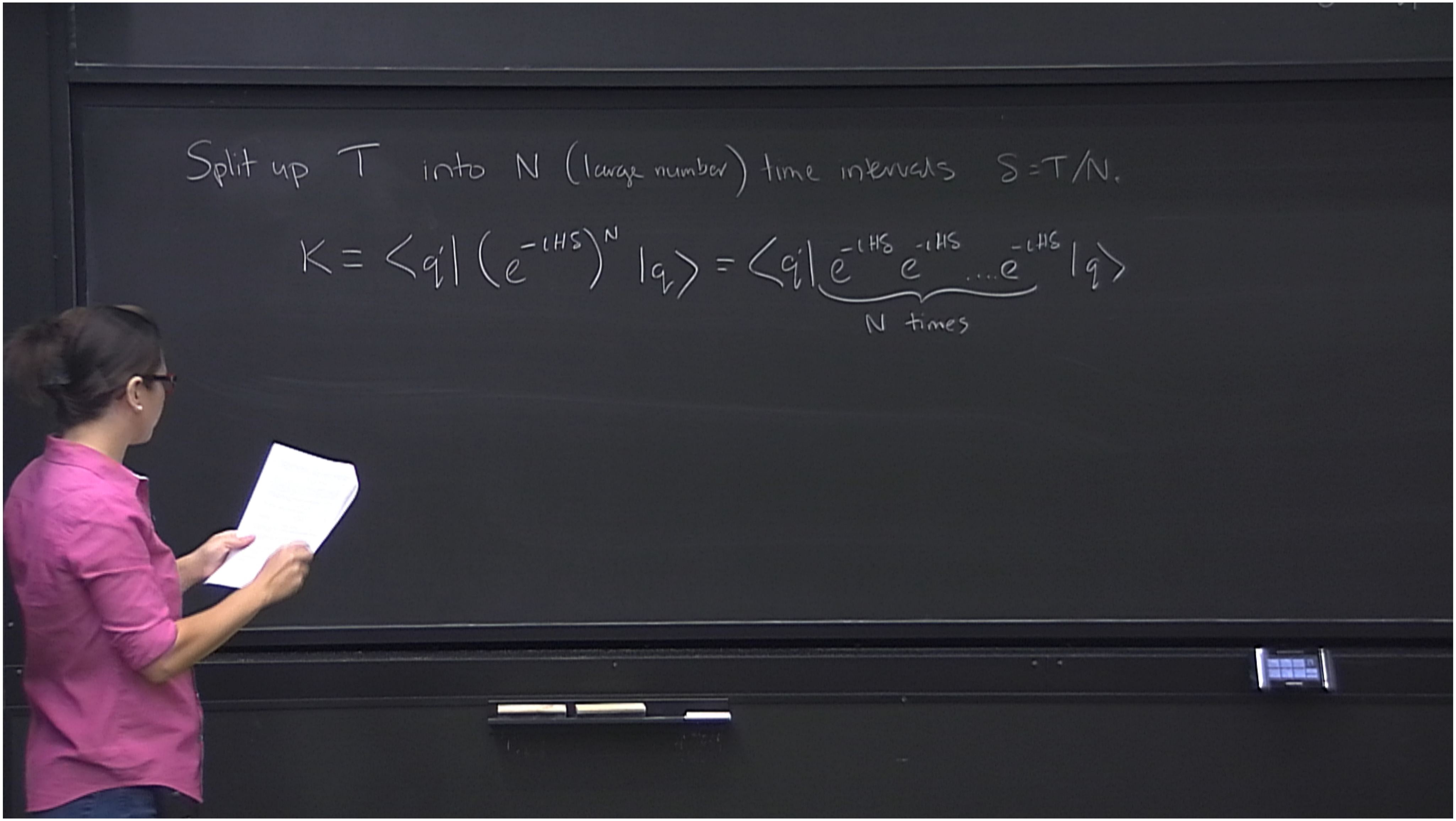
$$= \langle q' | e^{-iH(T-t_1)} e^{-iHt_1} | q \rangle$$

$$= \langle q' | e^{-iH(T-t_1)} \int dq_1 | q_1 \rangle \langle q_1 | e^{-iHt_1} | q \rangle$$

$$= \int dq_1 \, K(q', T; q_1, t_1) K(q_1, t_1; q, 0)$$ $\Rightarrow$ "Sum" over intermediate positions "q_1"


Split up T into N (large number) time intervals δ = T/N.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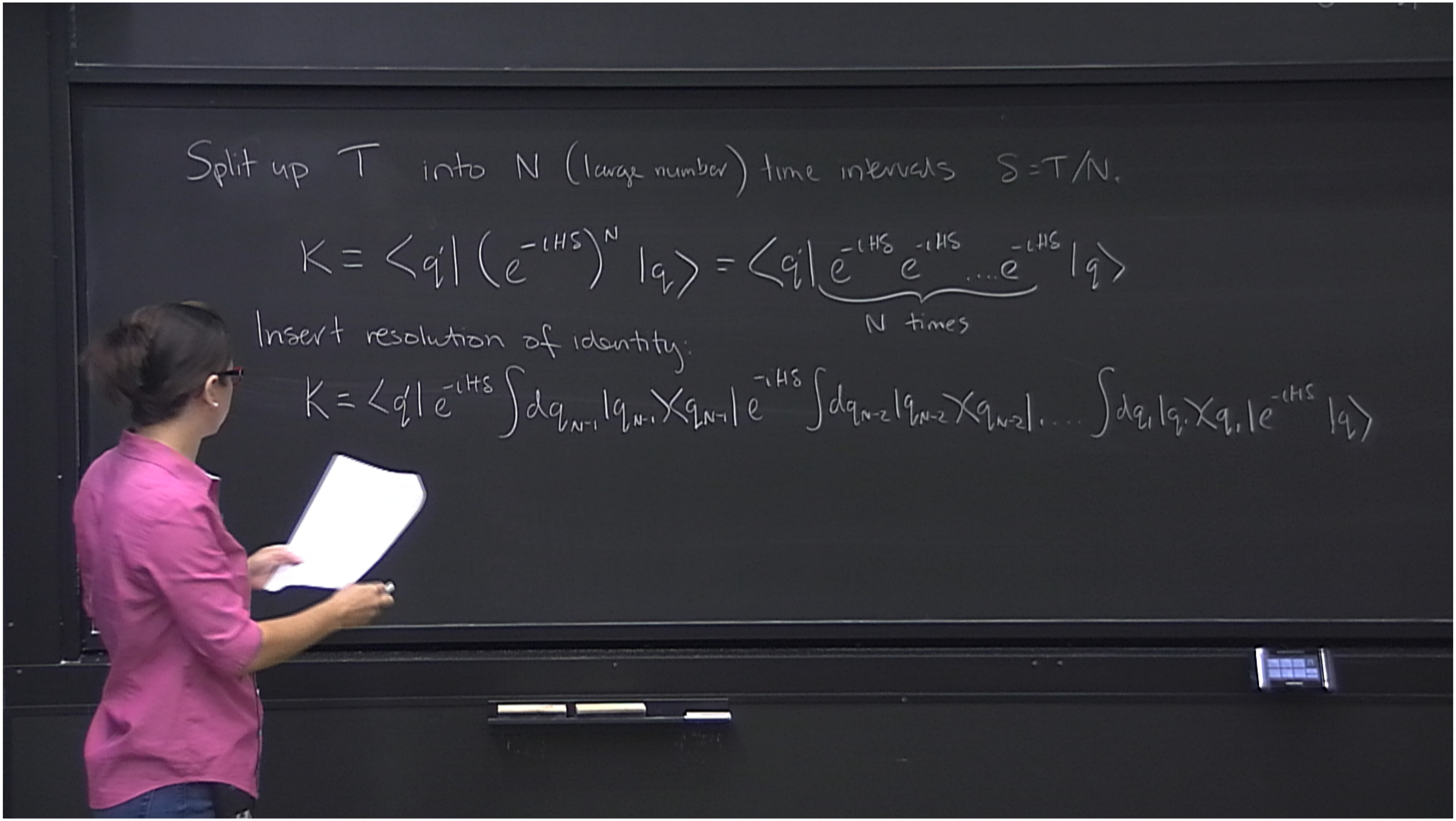
$K = \langle q' | (e^{-iH\delta})^N | q \rangle = \langle q' | \underbrace{e^{-iH\delta} e^{-iH\delta} \ldots e^{-iH\delta}}_{N \text{ times}} | q \rangle$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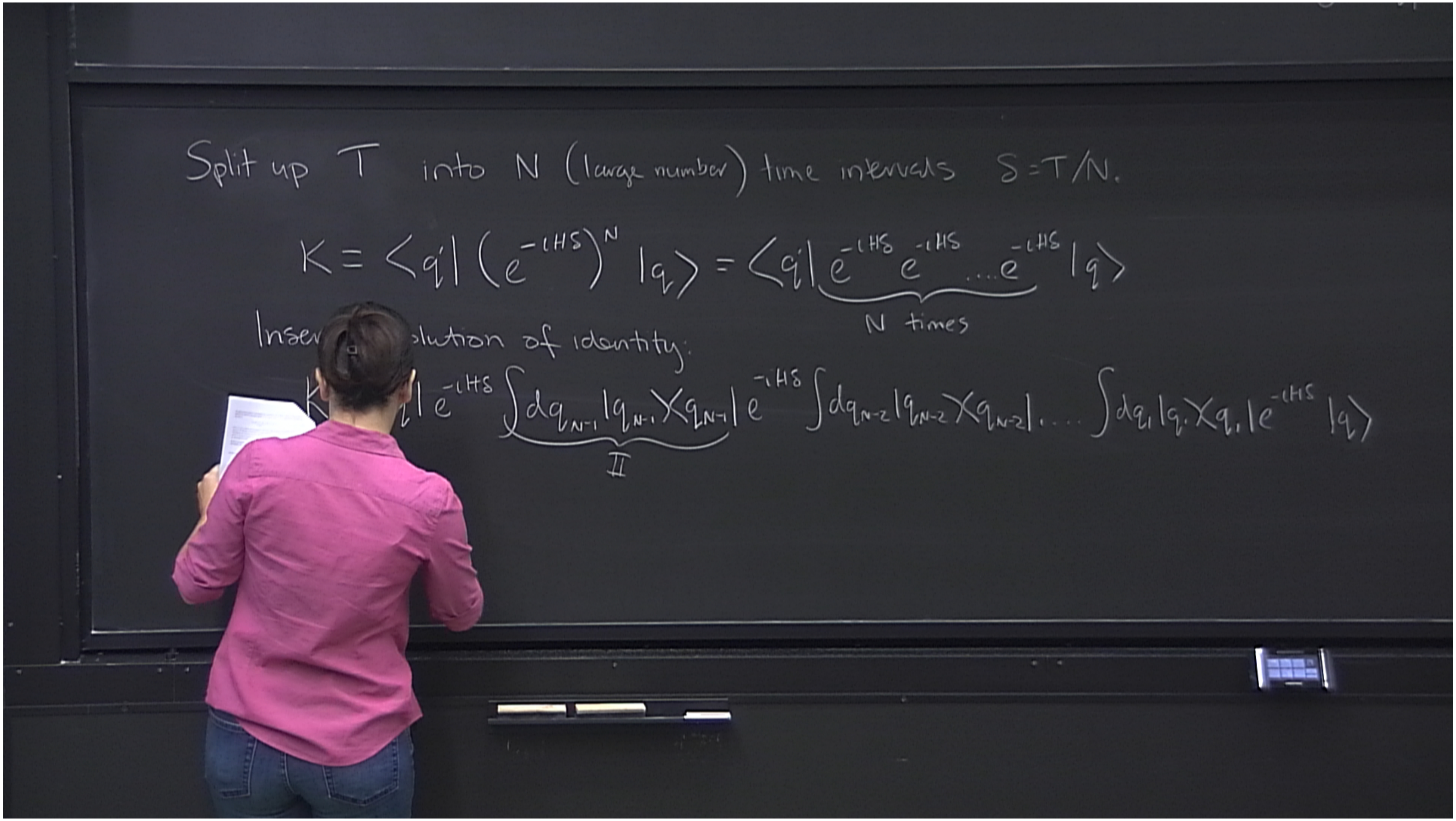
Insert resolution of identity:
$K = \langle q'| e^{-iH\delta} \int dq_{N-1} |q_{N-1}\rangle\langle q_{N-1}| e^{-iH\delta} \int dq_{N-2} |q_{N-2}\rangle\langle q_{N-2}| \ldots \int dq_1 |q_1\rangle\langle q_1| e^{-iH\delta} |q\rangle$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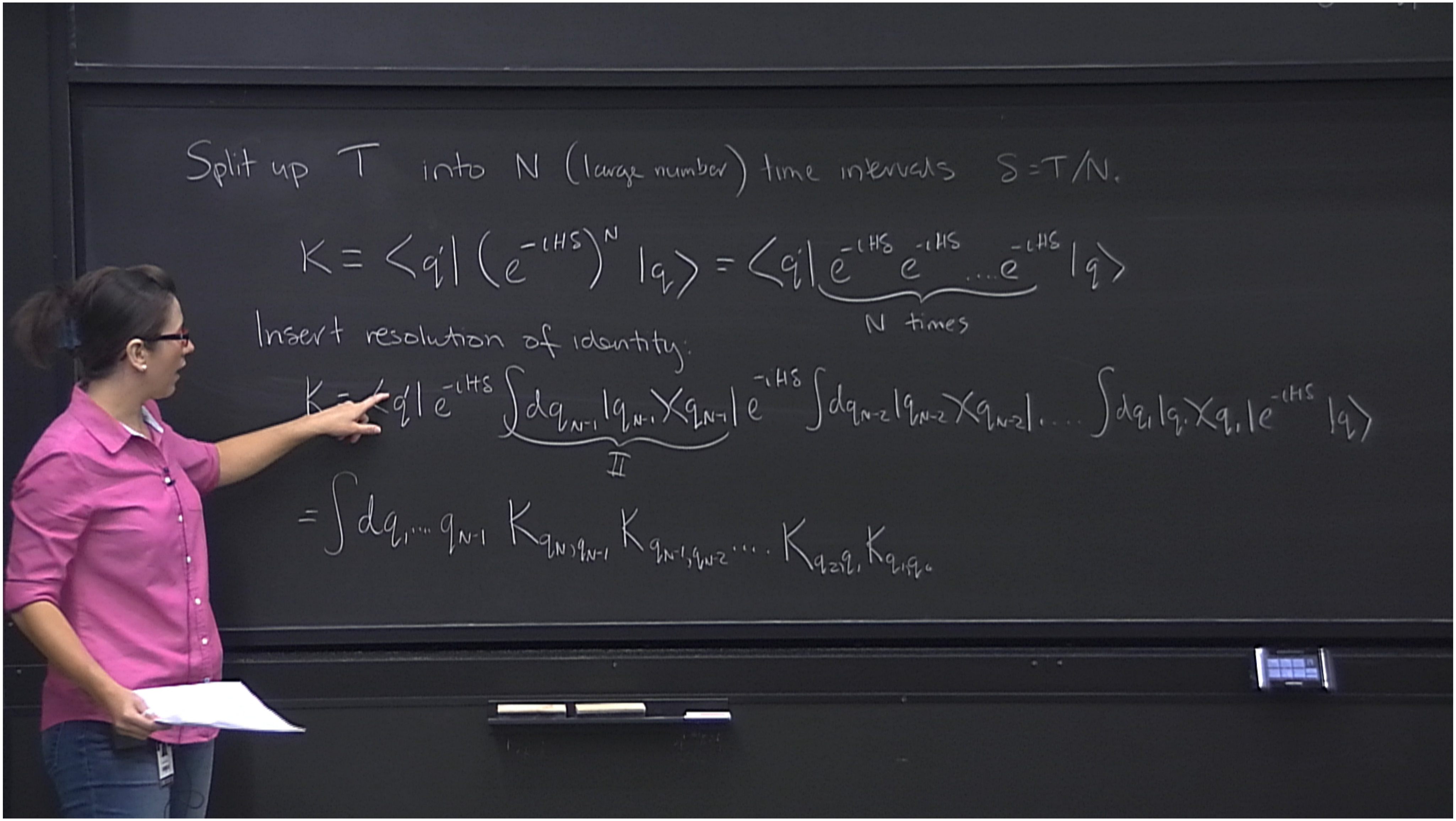
Inser solution of identity:
$q'|e^{-iH\delta}\underbrace{\int dq_{N-1}|q_{N-1}\rangle\langle q_{N-1}|}_{\mathbb{I}}e^{-iH\delta}\int dq_{N-2}|q_{N-2}\rangle\langle q_{N-2}|\dots\int dq_1|q_1\rangle\langle q_1|e^{-iH\delta}|q\rangle$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$= \int dq_1 \cdots q_{N-1} K_{q_N, q_{N-1}} K_{q_{N-1}, q_{N-2}} \cdots K_{q_2 q_1} K_{q_1 q_0}$

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$$= \int dq_1 \ldots q_{N-1} \, K_{q_N, q_{N-1}} K_{q_{N-1}, q_{N-2}} \ldots K_{q_2 q_1} K_{q_1 q_0}$$

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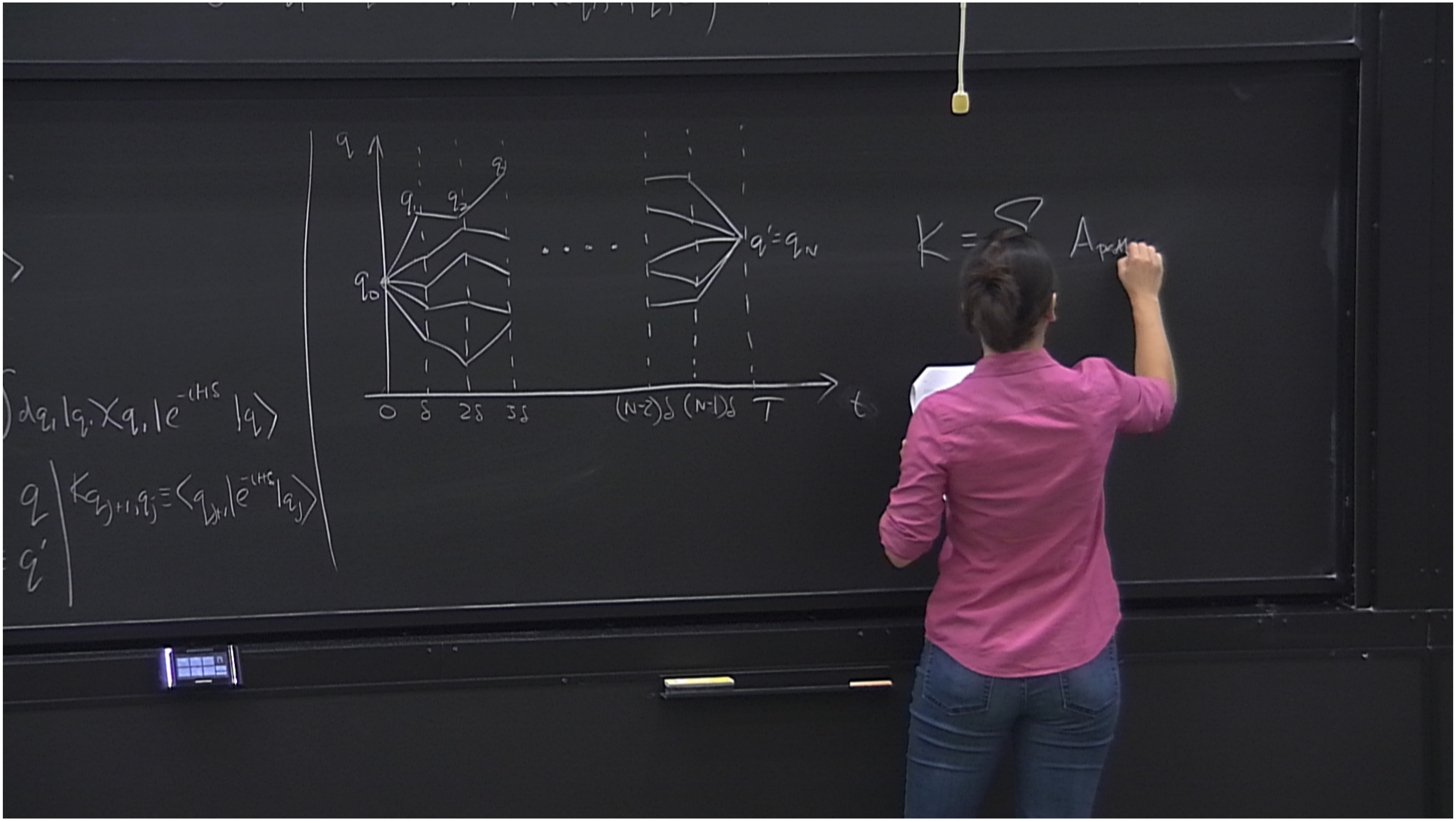
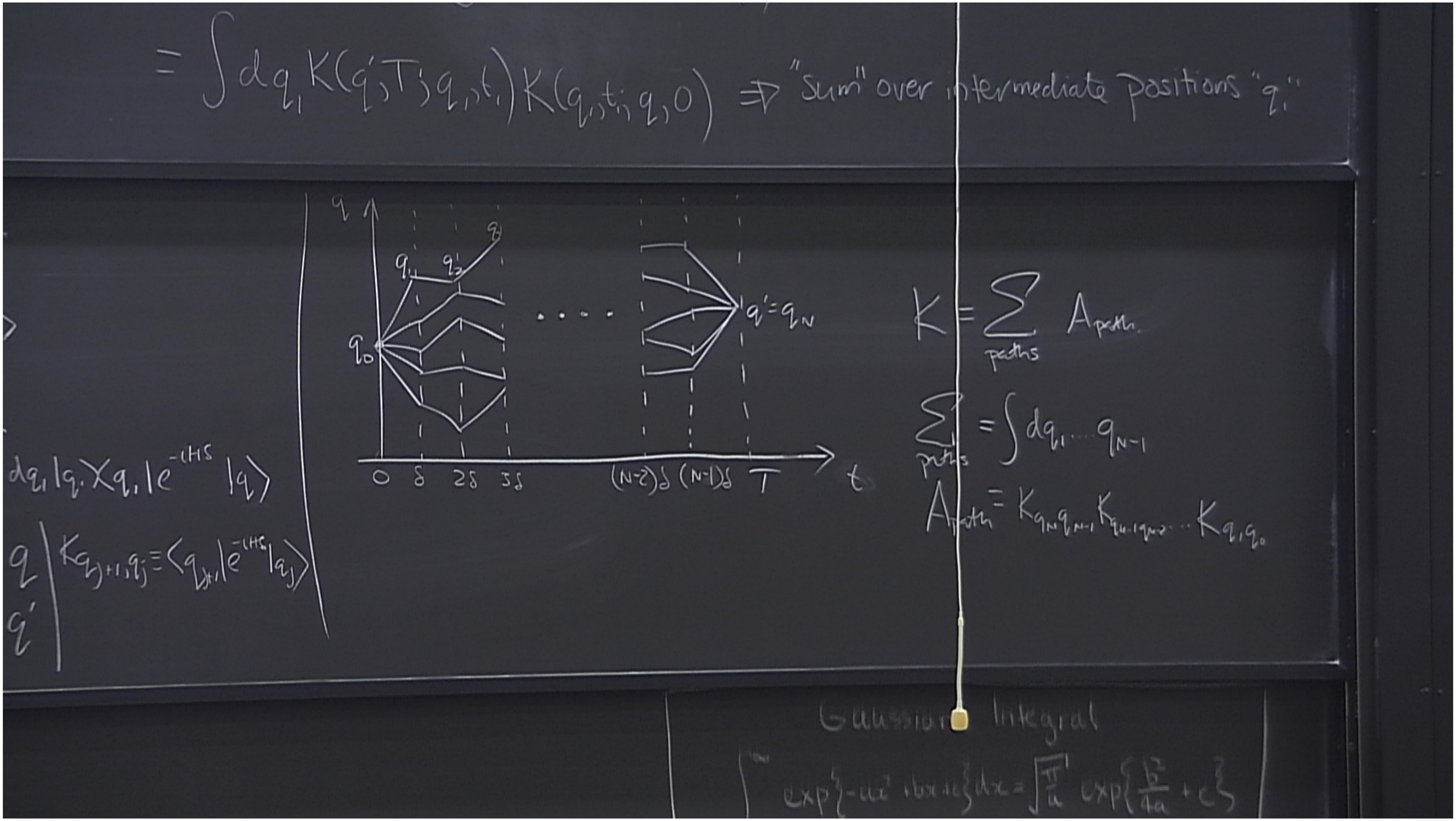
$$= \int dq_1 \cdots q_{N-1}\; K_{q_N,q_{N-1}}\,K_{q_{N-1},q_{N-2}}\cdots K_{q_2,q_1}\,K_{q_1,q_0}$$

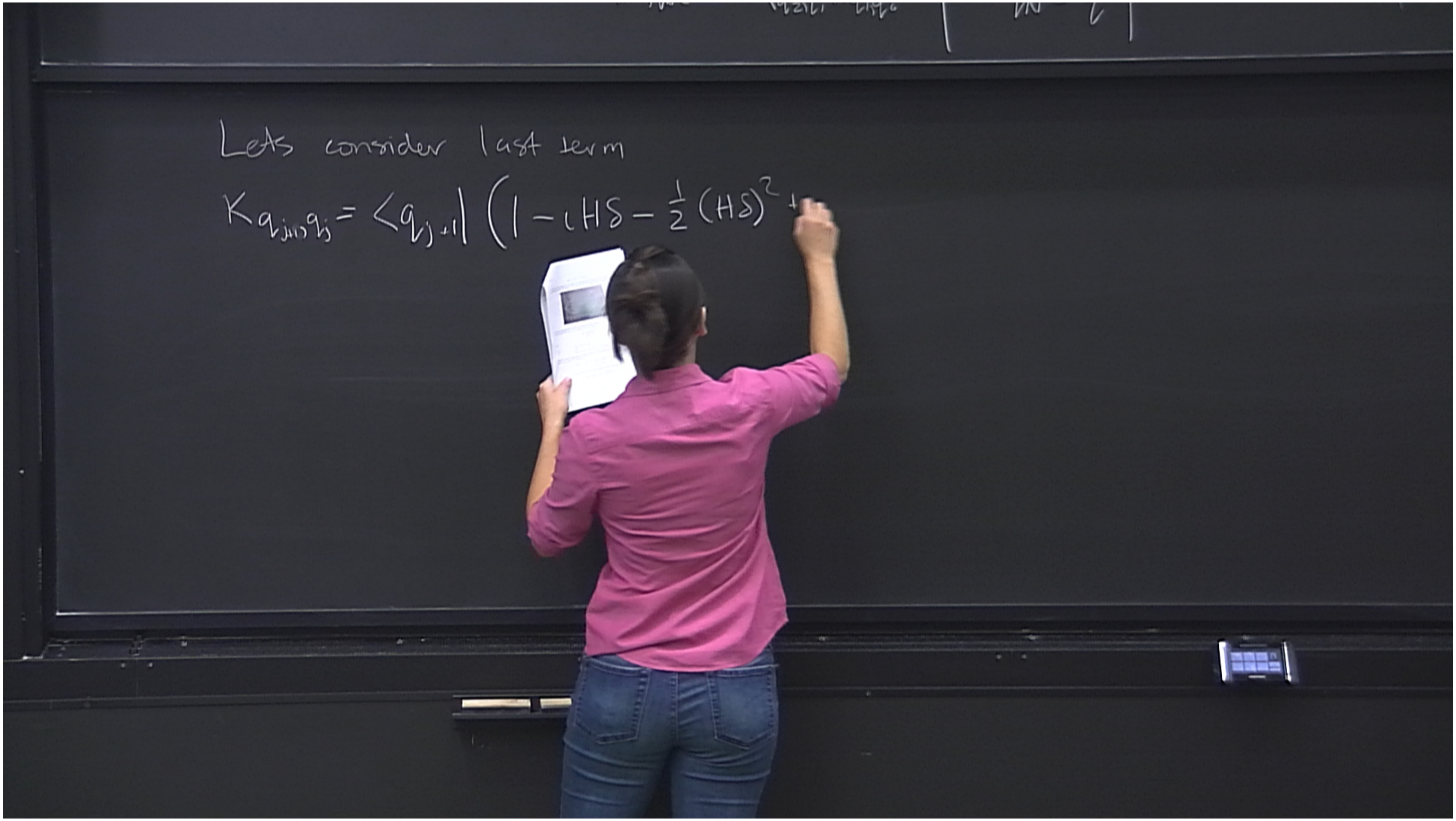
$q_0 \equiv q$

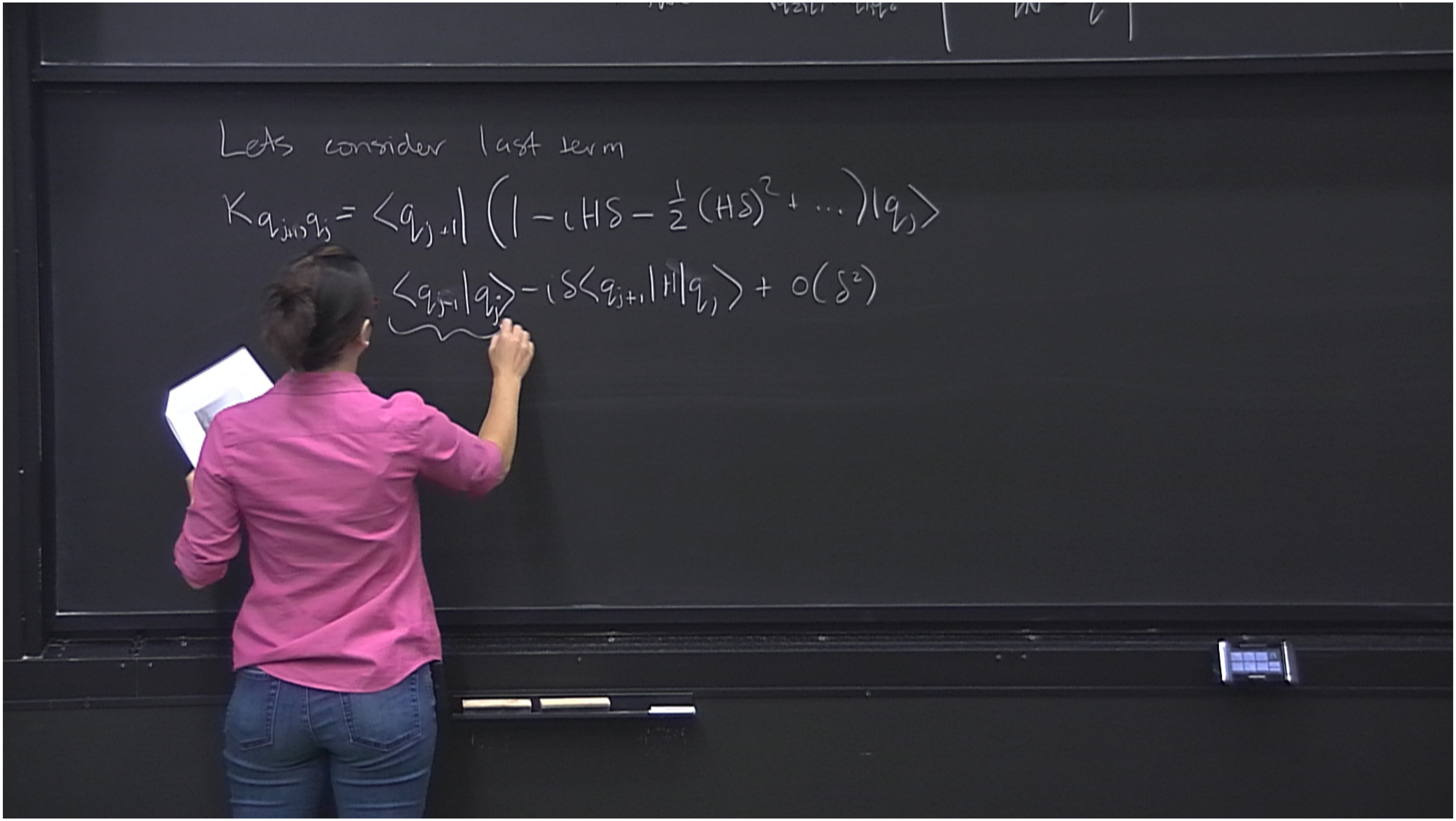
$q_N \equiv q'$

$K_{q_{j+1},q_j} = \langle q_{j+1}|e^{-iH\delta}|q_j\rangle$


q
q_0
q' = q_N
0 δ 2δ 3δ (N-2)δ (N-1)δ T
t
K_{q_{j+1}, q_j} = ⟨q_{j+1}| e^{-iHδ} |q_j⟩
K = Σ A


= ∫dq₁ K(q′, T; q₁, t₁) K(q₁, t₁; q, 0) ⇒ "sum" over intermediate positions "q₁"
K = Σ_paths A_path
Σ_paths = ∫dq₁ … q_{N−1}
A_path = K_{q_N q_{N−1}} K_{q_{N−1} q_{N−2}} … K_{q₁ q₀}
K_{q_{j+1}, q_j} = ⟨q_{j+1}| e^{−iHδ} |q_j⟩
0 δ 2δ 3δ (N−2)δ (N−1)δ T
q′ = q_N
Gaussian Integral


Lets consider last term
$K_{q_{j+1} q_j} = \langle q_{j+1} | \left(\left(1 - iH\delta - \frac{1}{2}(H\delta)^2 + \right.\right.$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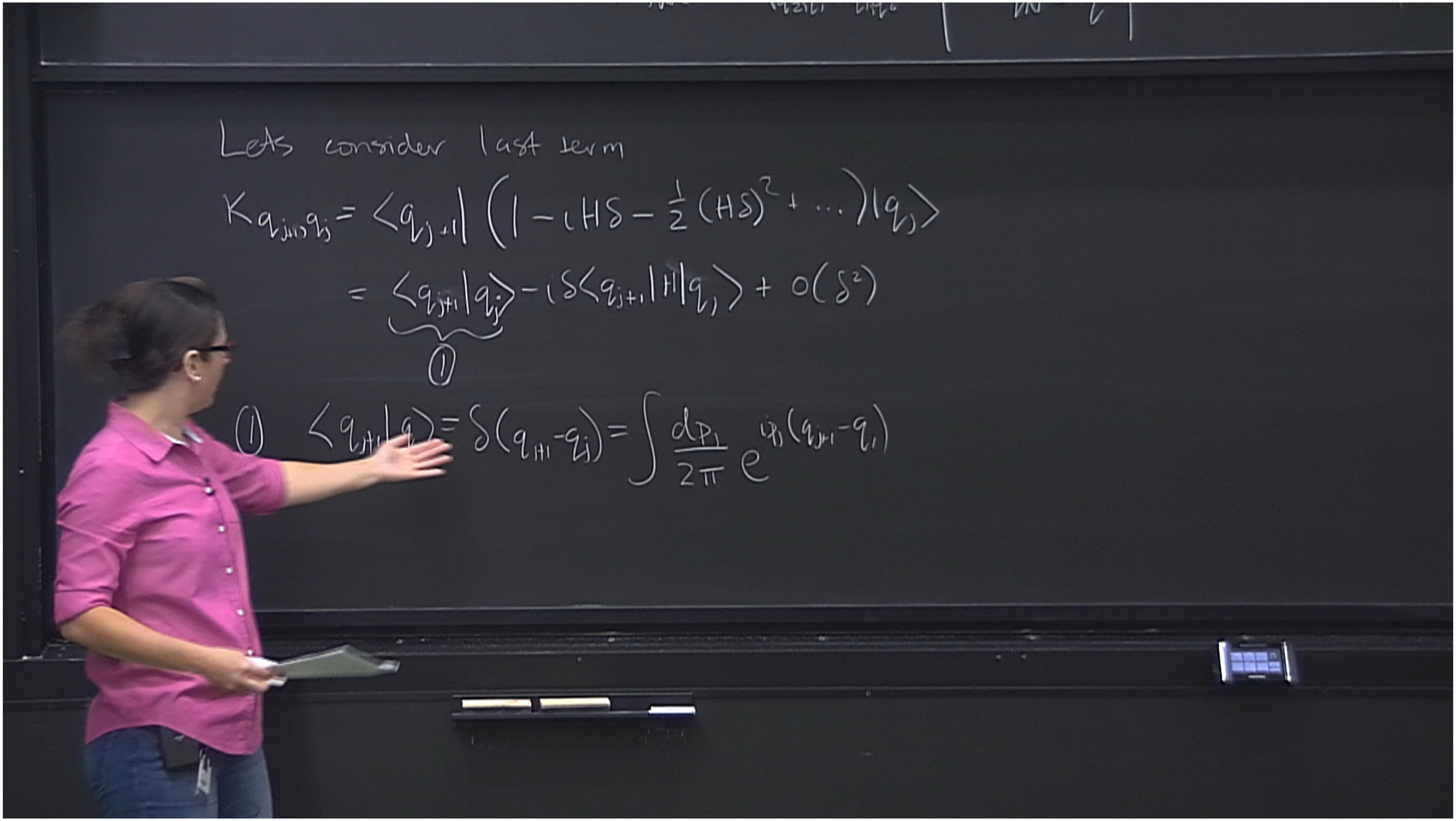
$K_{q_{j+1} q_j} = \langle q_{j+1} | \left(1 - iH\delta - \frac{1}{2}(H\delta)^2 + \dots \right) | q_j \rangle$
$\langle q_{j+1} | q_j \rangle - i\delta \langle q_{j+1} | H | q_j \rangle + O(\delta^2)$


Lets consider last term
$K_{q_{j+1} q_j} = \langle q_{j+1}| \left(1 - iH\delta - \frac{1}{2}(H\delta)^2 + \dots\right)|q_j\rangle$
$= \underbrace{\langle q_{j+1}|q_j\rangle}_{1} - i\delta\langle q_{j+1}|H|q_j\rangle + O(\delta^2)$
1 $\langle q_{j+1}|q_j\rangle = \delta(q_{j+1}-q_j) = \int \frac{dp_j}{2\pi} e^{ip_j(q_{j+1}-q_j)}$

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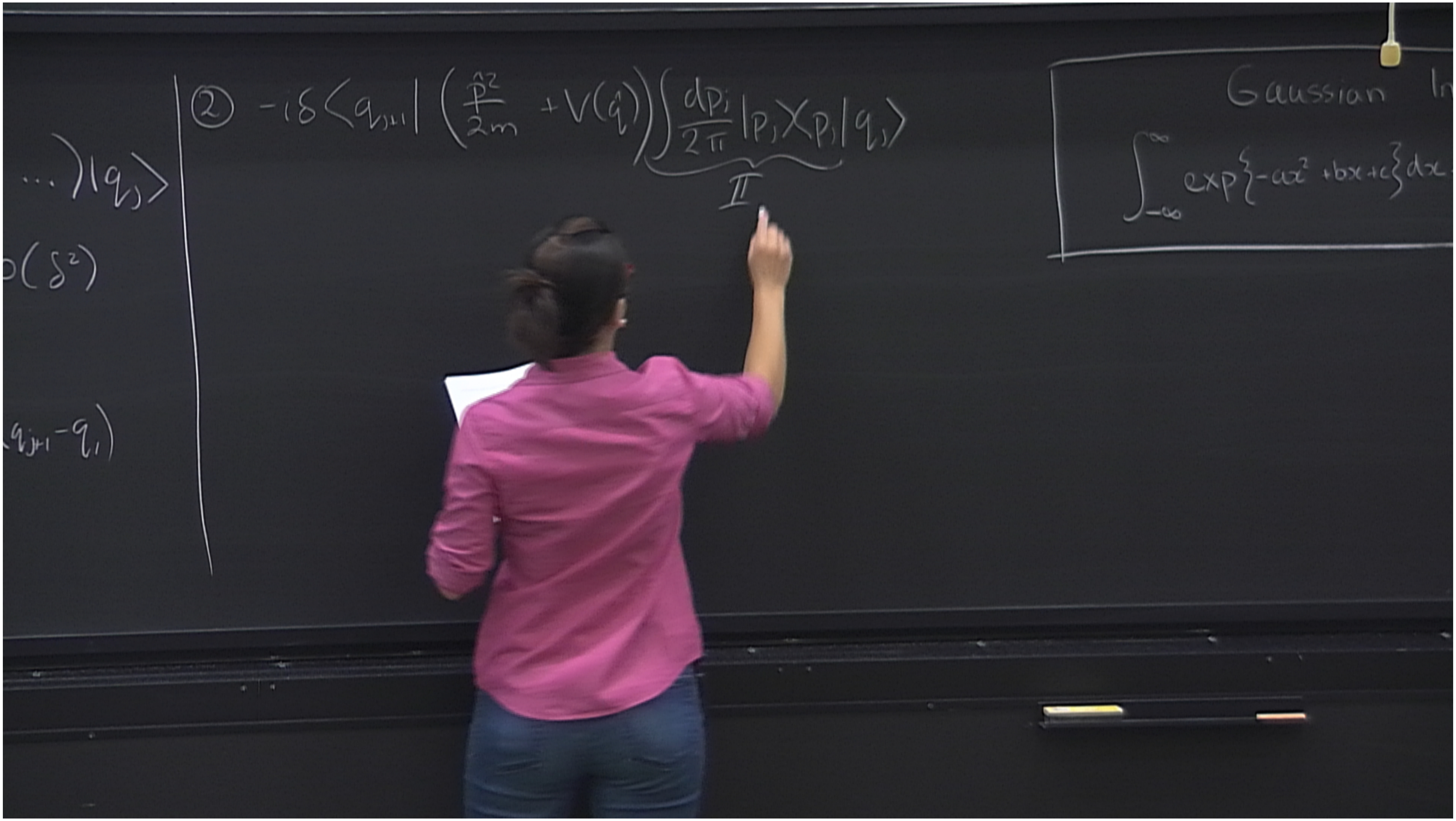
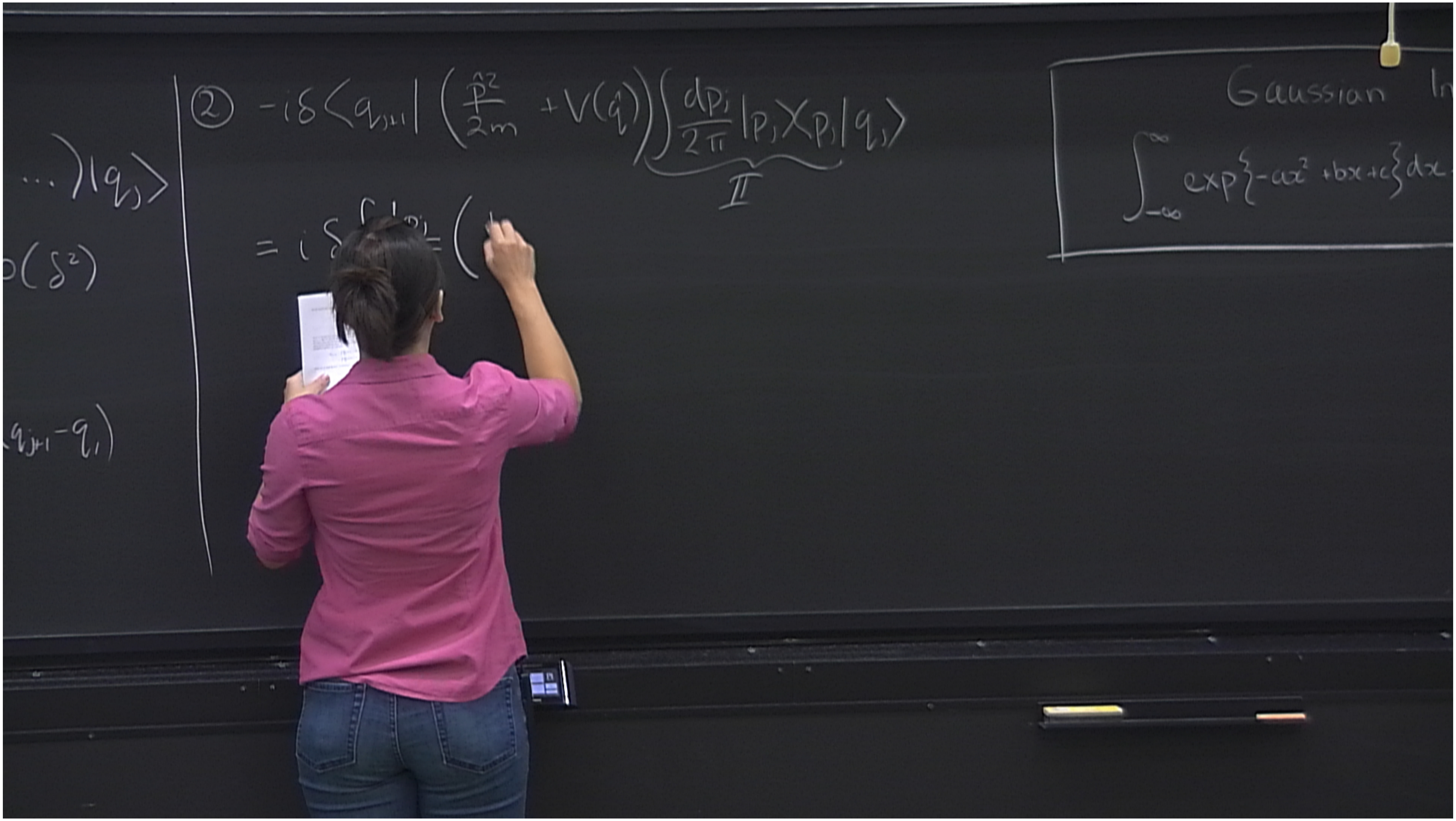
2


$② \ -i\delta \langle q_{j+1}| \left(\frac{\hat{p}^2}{2m} + V(\hat{q})\right) \underbrace{\int \frac{dp_j}{2\pi} |p_j\rangle\langle p_j|}_{I} q_j\rangle$
$\ldots)|q_j\rangle$
$O(\delta^2)$
$(q_{j+1} - q_j)$
Gaussian In
$\int_{-\infty}^{\infty} \exp\{-ax^2 + bx + c\} dx$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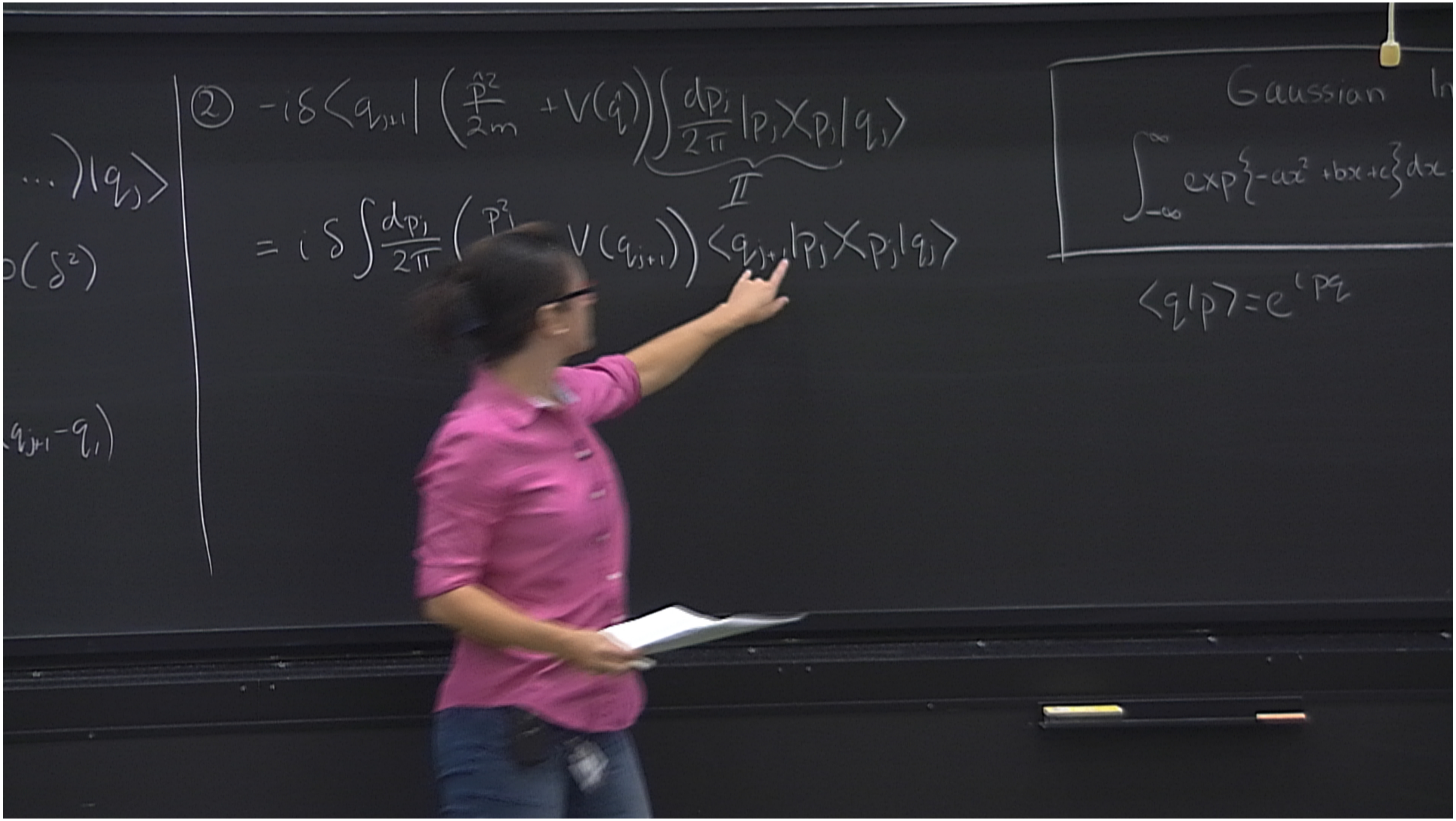
Gaussian
$\int_{-\infty}^{\infty}\exp\{-ax^2+bx+c\}dx$


② $-i\delta\langle q_{j+1}|\left(\frac{\hat{p}^2}{2m}+V(\hat{q})\right)\underbrace{\int\frac{dp_j}{2\pi}|p_j\rangle\langle p_j|q_j\rangle}_{I}$
$= i\delta\int\frac{dp_j}{2\pi}\left(\frac{p_j^2}{2m}+V(q_{j+1})\right)\langle q_{j+1}|p_j\rangle\langle p_j|q_j\rangle$
Gaussian In
$\int_{-\infty}^{\infty}\exp\{-ax^2+bx+c\}dx$
$\langle q|p\rangle = e^{ipq}$

$\ldots)|q_j\rangle$

$o(\delta^2)$

$(q_{j+1}-q_1)$

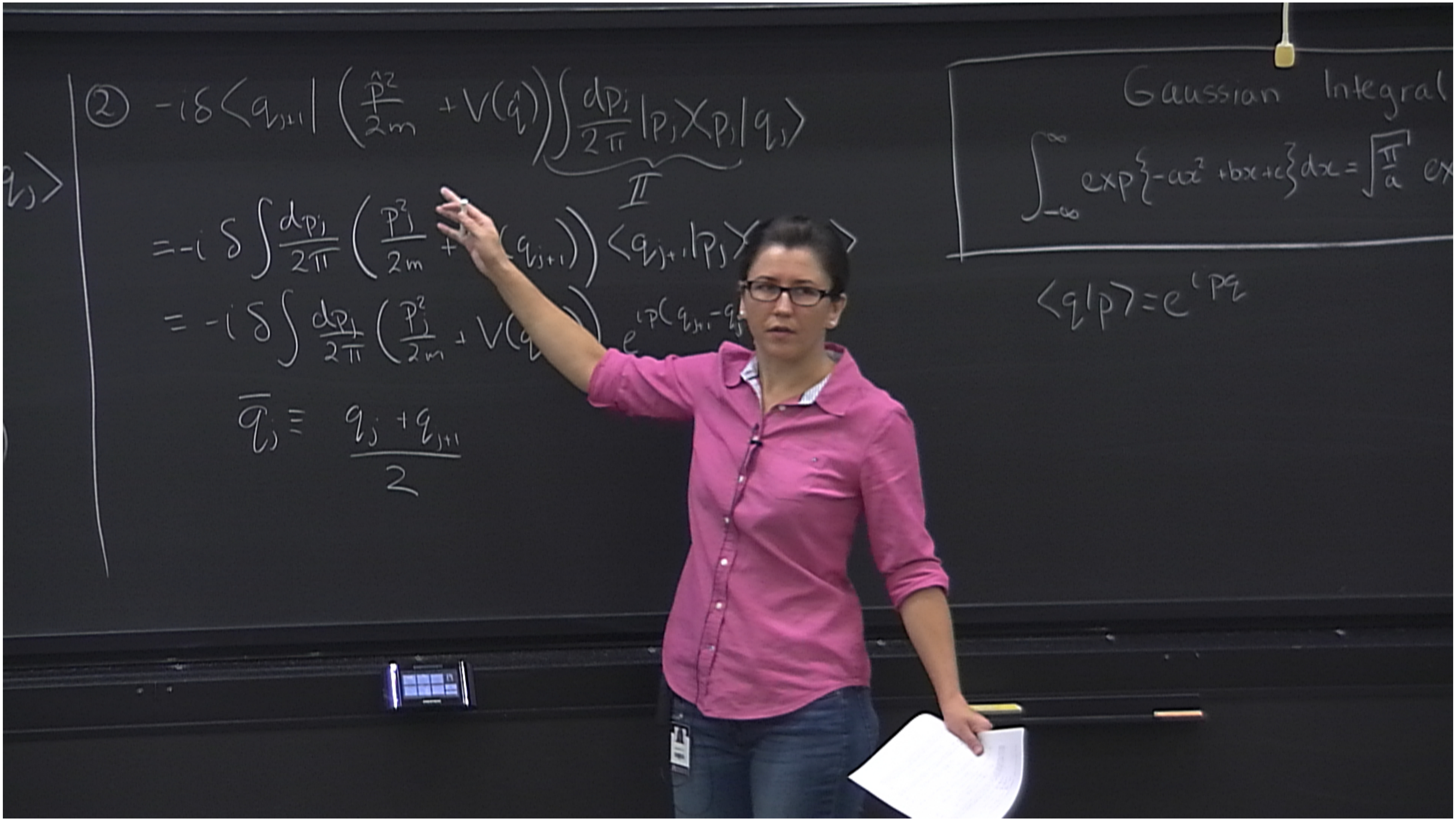
② $-i\delta\langle q_{j+1}|\left(\frac{\hat{p}^2}{2m}+V(\hat{q})\right)\underbrace{\int\frac{dp_j}{2\pi}|p_j\rangle\langle p_j|}_{\mathbb{I}}q_j\rangle$

$$=-i\delta\int\frac{dp_j}{2\pi}\left(\frac{p_j^2}{2m}+V(q_{j+1})\right)\langle q_{j+1}|p_j\rangle\langle p_j|q_j\rangle$$

$$=-i\delta\int\frac{dp_j}{2\pi}\left(\frac{p_j^2}{2m}+V(q_{j+1})\right)e^{ip(q_{j+1}-q_j)}$$

$$\boxed{\text{Gaussian In}\ldots \quad \int_{-\infty}^{\infty}\exp\{-ax^2+bx+c\}dx\ldots}$$

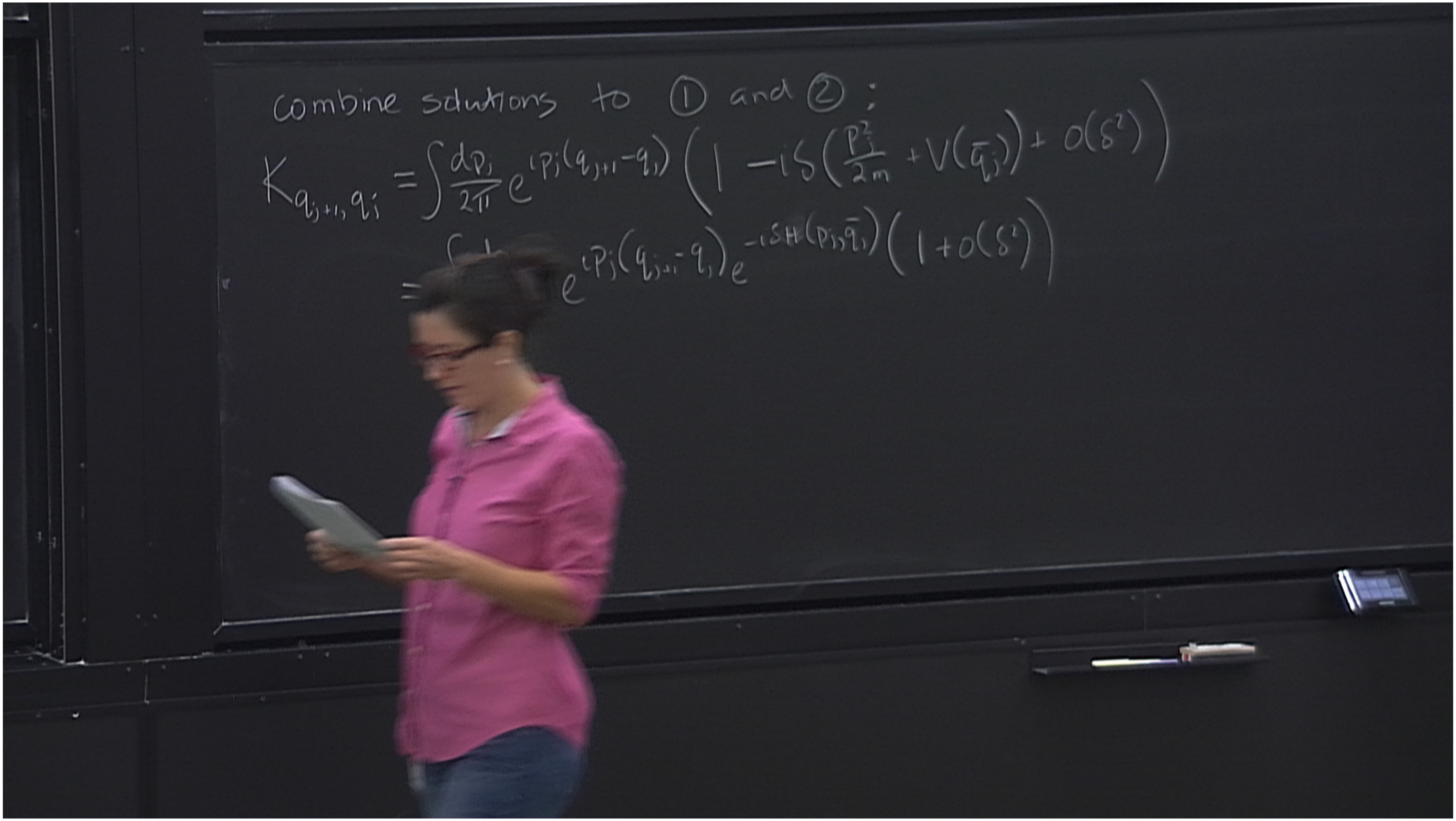
$\langle q|p\rangle=e^{ipq}$

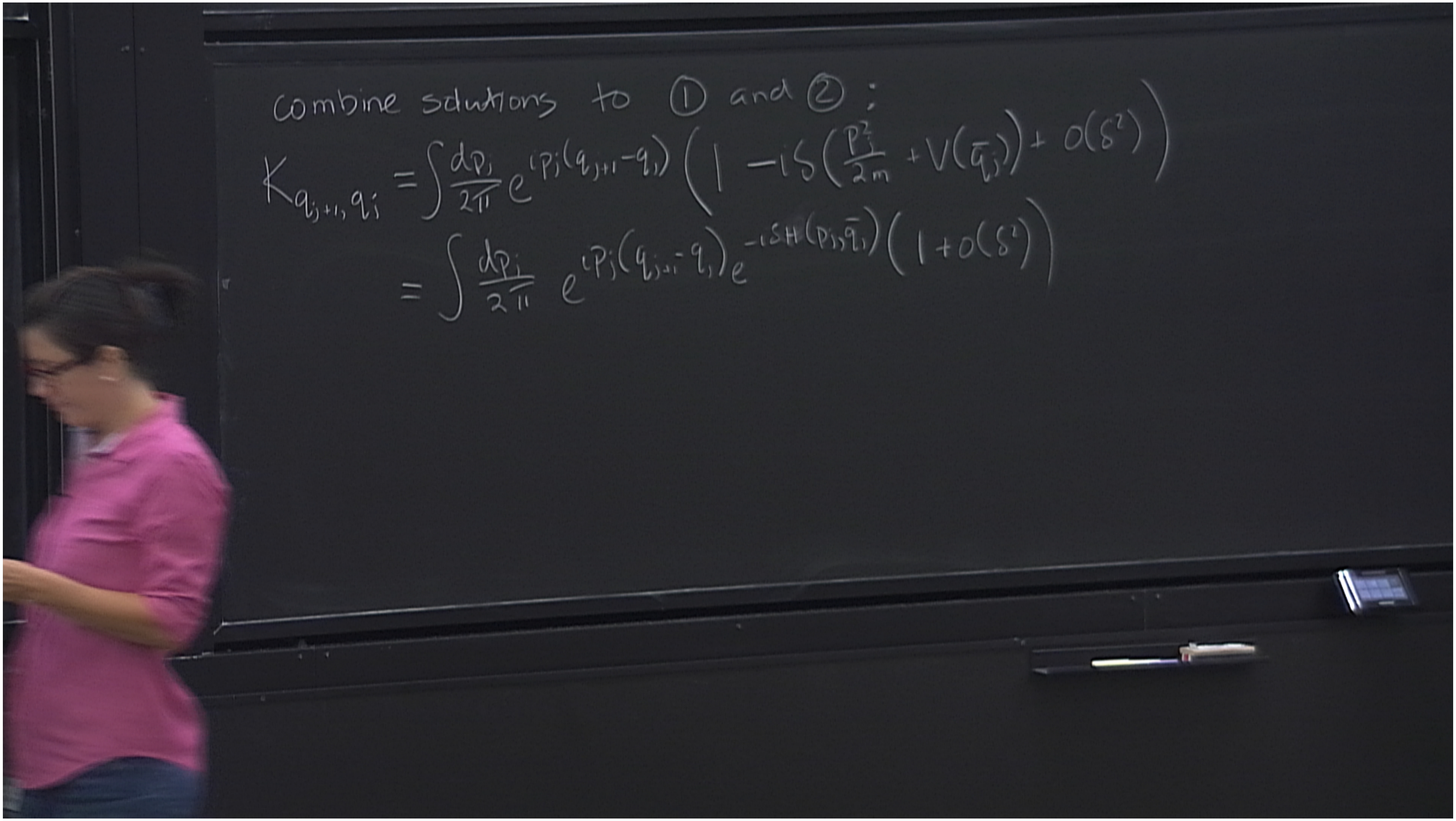


② $-i\delta\langle q_{j+1}|\left(\frac{\hat{p}^2}{2m}+V(\hat{q})\right)\int\frac{dp_j}{2\pi}|p_j\rangle\langle p_j|q_j\rangle$
Gaussian Integral
$\int_{-\infty}^{\infty}\exp\{-ax^2+bx+c\}dx=\sqrt{\frac{\pi}{a}}$
$\langle q|p\rangle=e^{ipq}$
$\bar{q}_j\equiv\frac{q_j+q_{j+1}}{2}$

combine solutions to 1 and 2 :

$$K_{q_{j+1}, q_j} = \int \frac{dp_j}{2\pi} e^{ip_j(q_{j+1}-q_j)} \Big($$


combine solutions to 1 and 2 :
$K_{q_{j+1}, q_j} = \int \frac{dp_j}{2\pi} e^{i p_j (q_{j+1} - q_j)} \left(1 - i\delta \left(\frac{p_j^2}{2m} + V(\bar{q}_j) \right) + O(\delta^2) \right)$
$= \ldots e^{i p_j (q_{j+1} - q_j)} e^{-i\delta H(p_j, \bar{q}_j)} \left(1 + O(\delta^2) \right)$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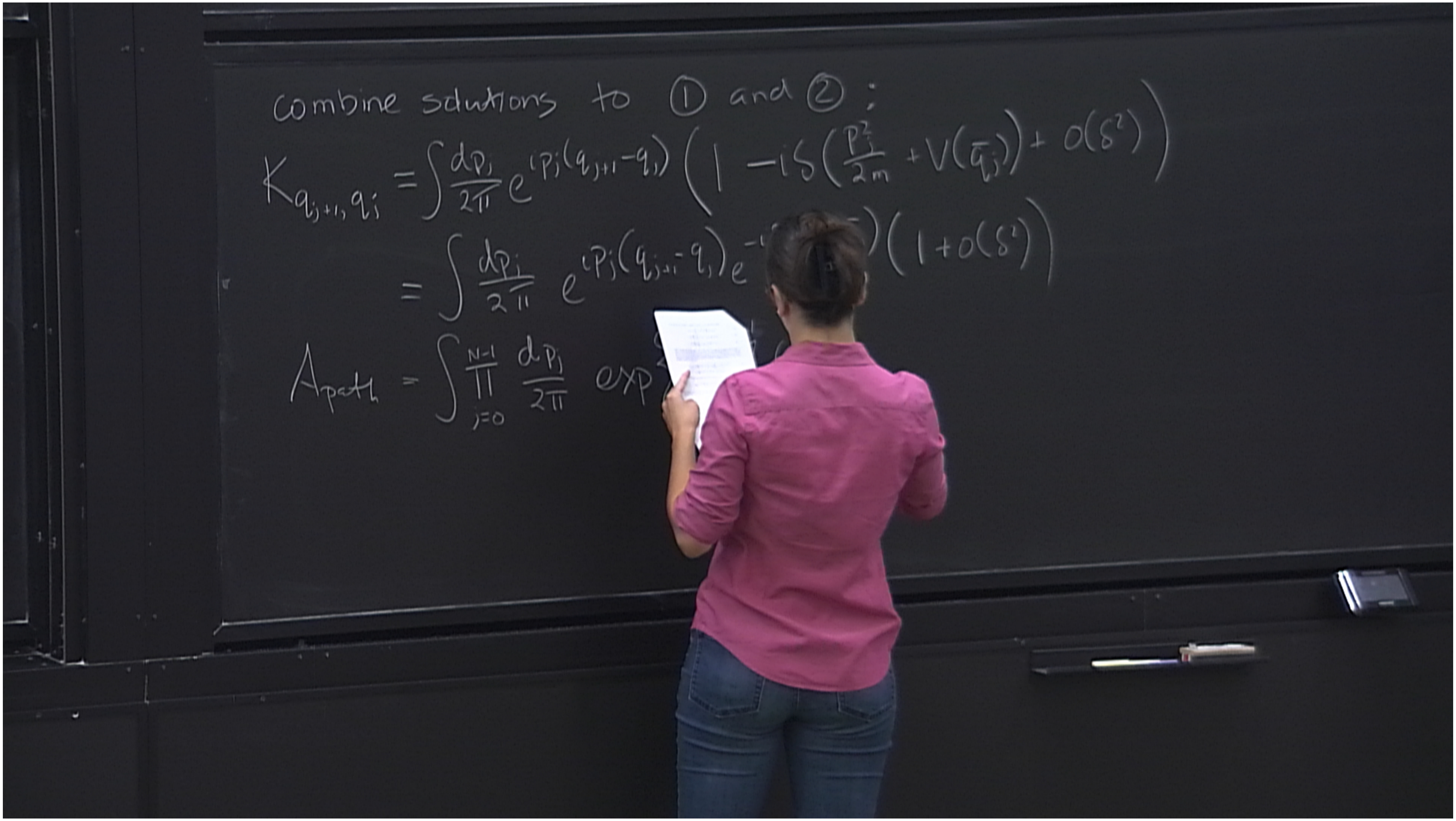
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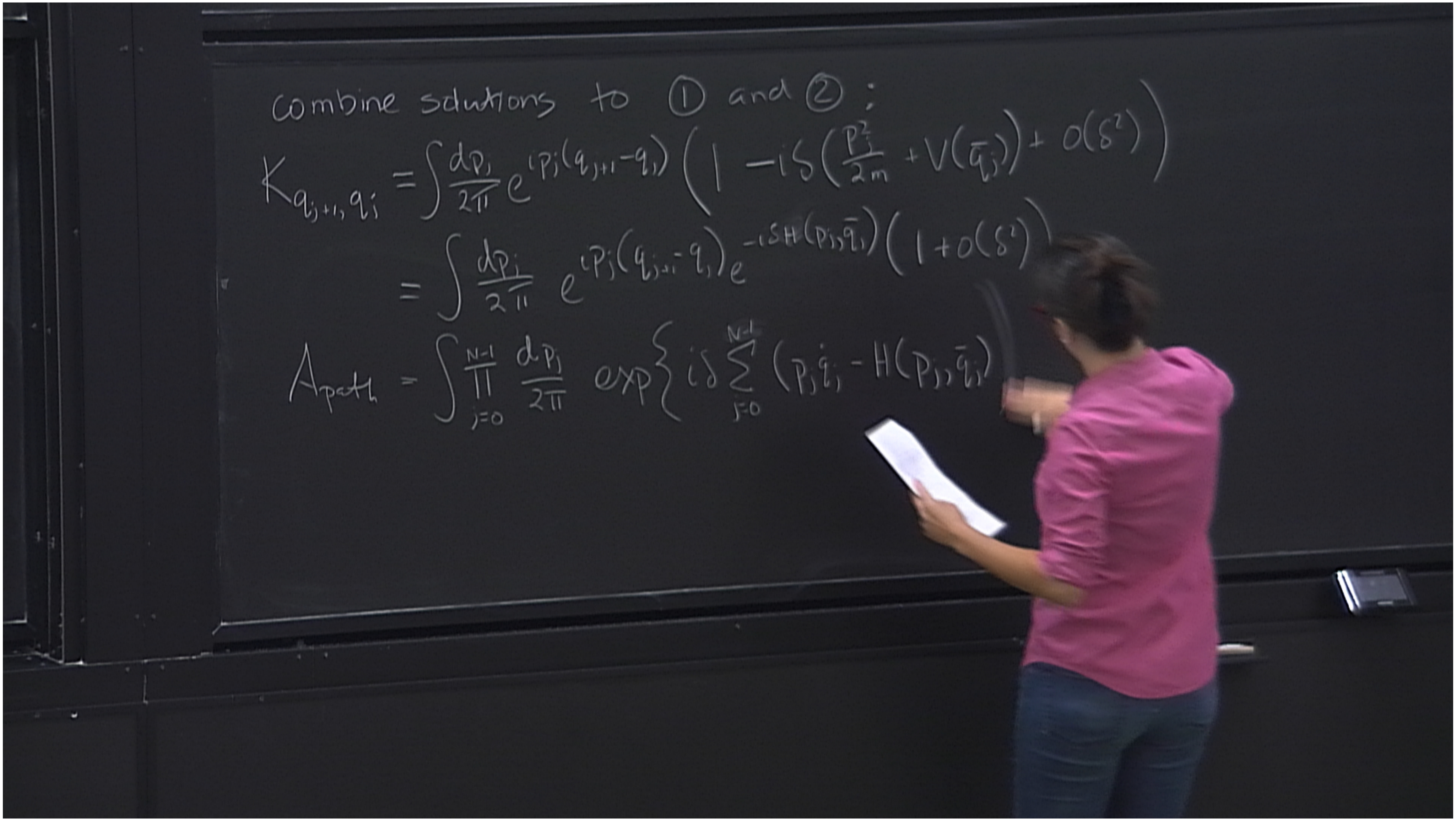
$= \int \frac{dp_j}{2\pi} e^{ip_j(q_{j+1}-q_j)} e^{-i} \left(1 + O(\delta^2)\right)$
$A_{path} = \int \prod_{j=0}^{N-1} \frac{dp_j}{2\pi} \exp$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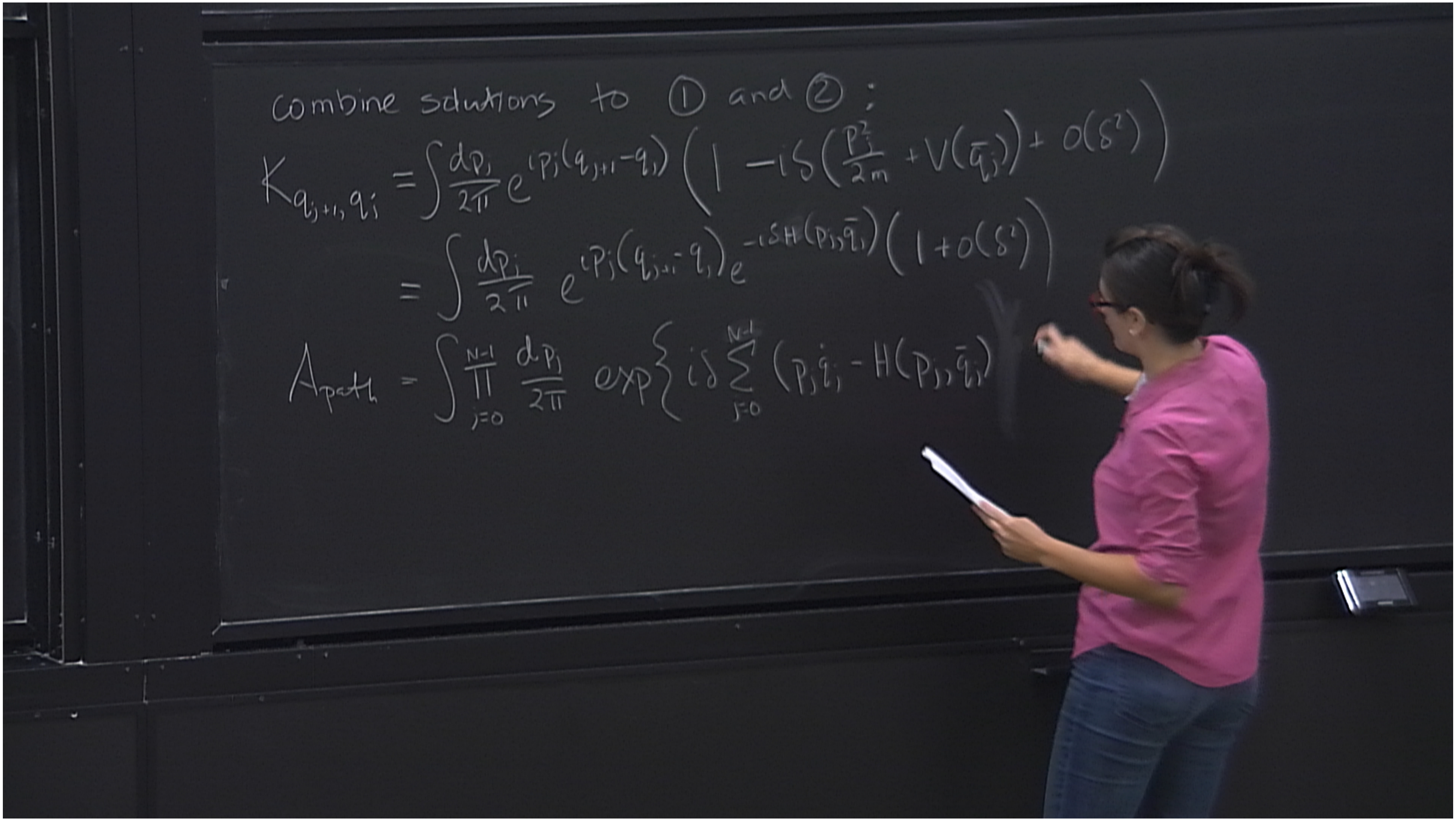
$A_{path} = \int \prod_{j=0}^{N-1} \frac{dp_j}{2\pi} \exp\left\{ i\delta \sum_{j=0}^{N-1} \left(p_j \dot{q}_j - H(p_j, \bar{q}_j)\right)\right.$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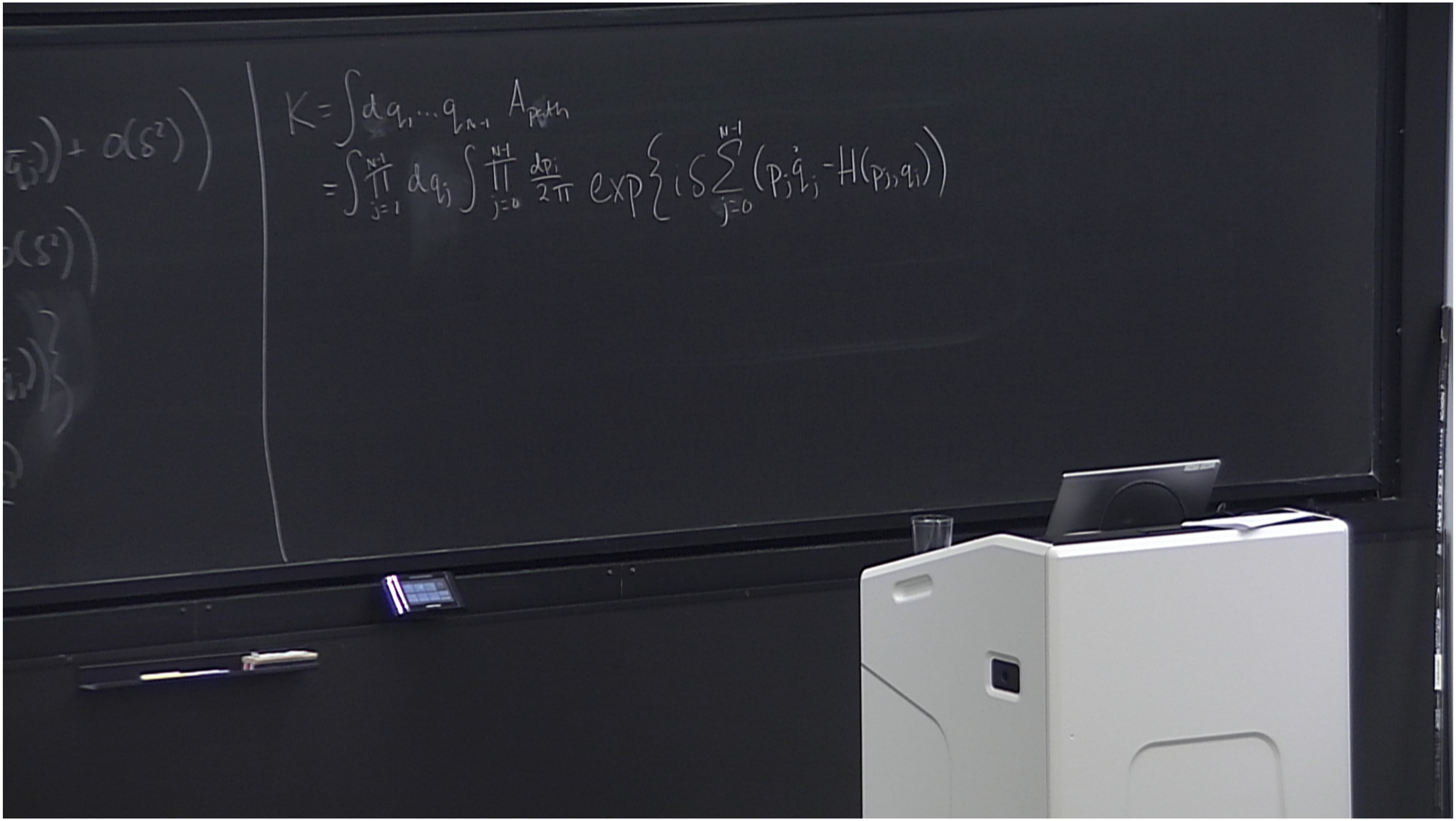
$$= \int \frac{dp_j}{2\pi} e^{ip_j(q_{j+1}-q_j)} e^{-i\delta H(p_j,\bar{q}_j)}\left(1+O(\delta^2)\right)$$

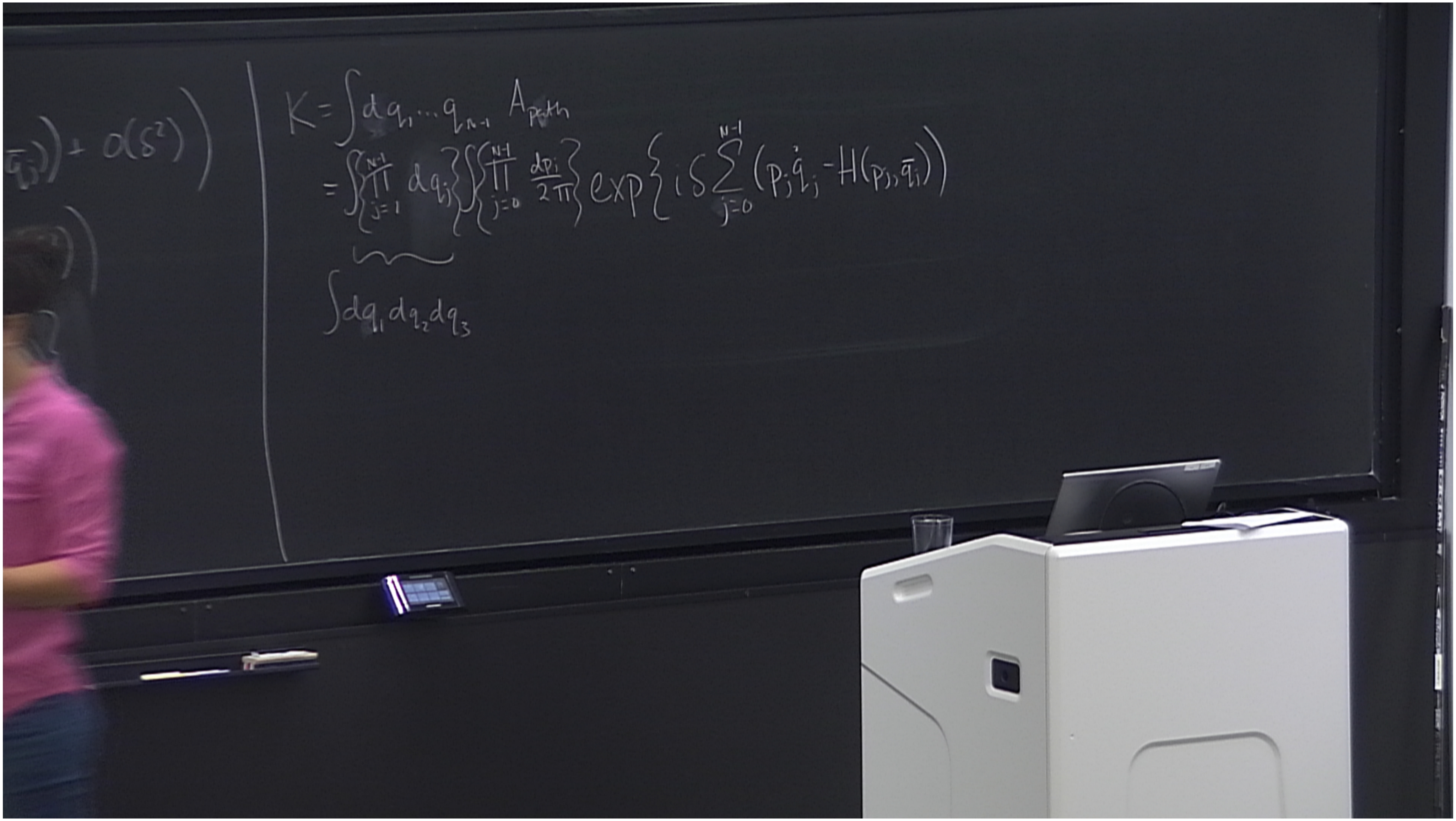
$$A_{path} = \int \prod_{j=0}^{N-1} \frac{dp_j}{2\pi} \exp\left\{ i\delta \sum_{j=0}^{N-1} \left(p_j \dot{q}_j - H(p_j,\bar{q}_j)\right)\right\}$$

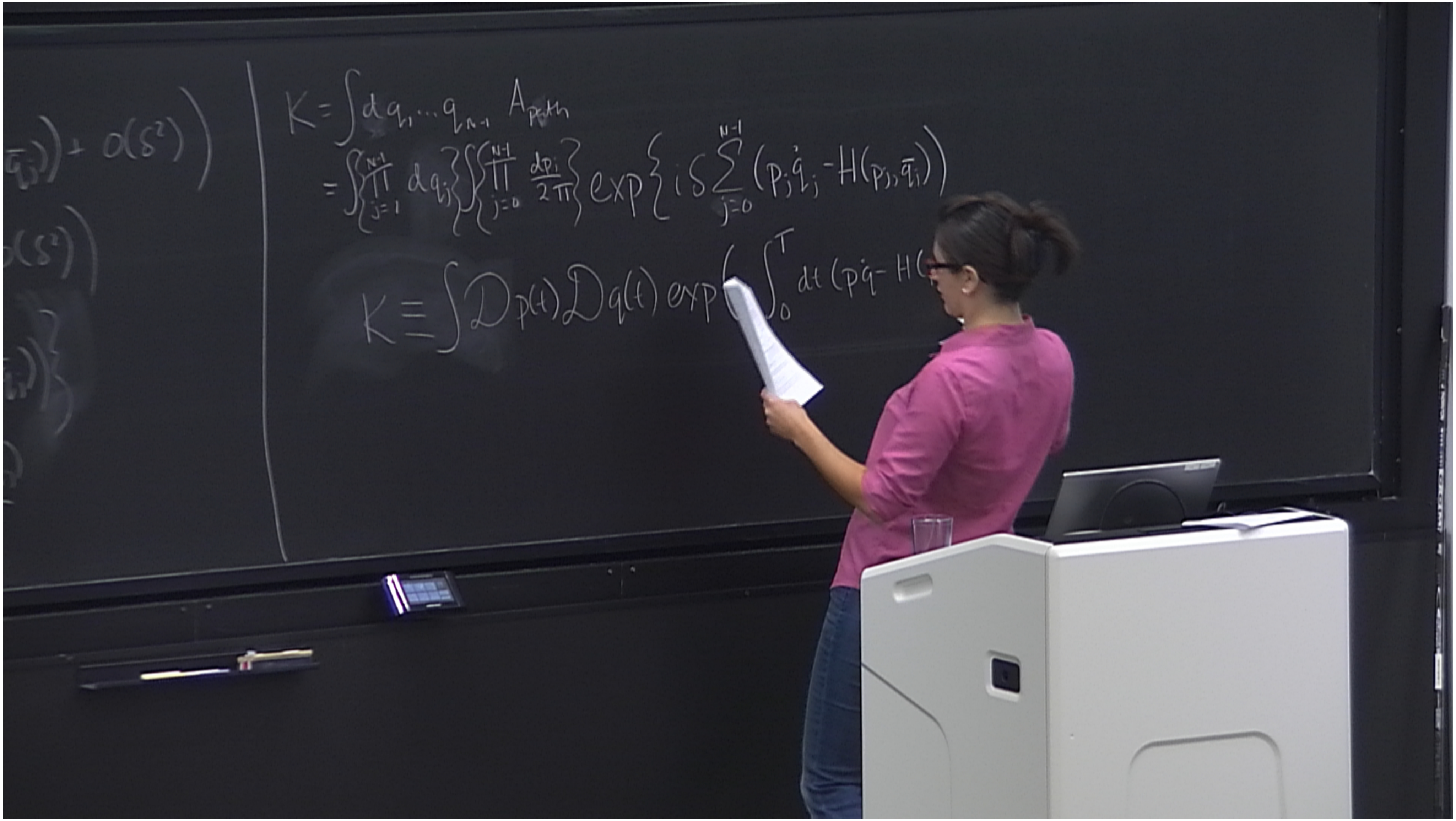
(ignoring $(1+O(\delta^2))^N$) also $\dot{q} \equiv \frac{(q_{j+1}-q_j)}{\delta}$

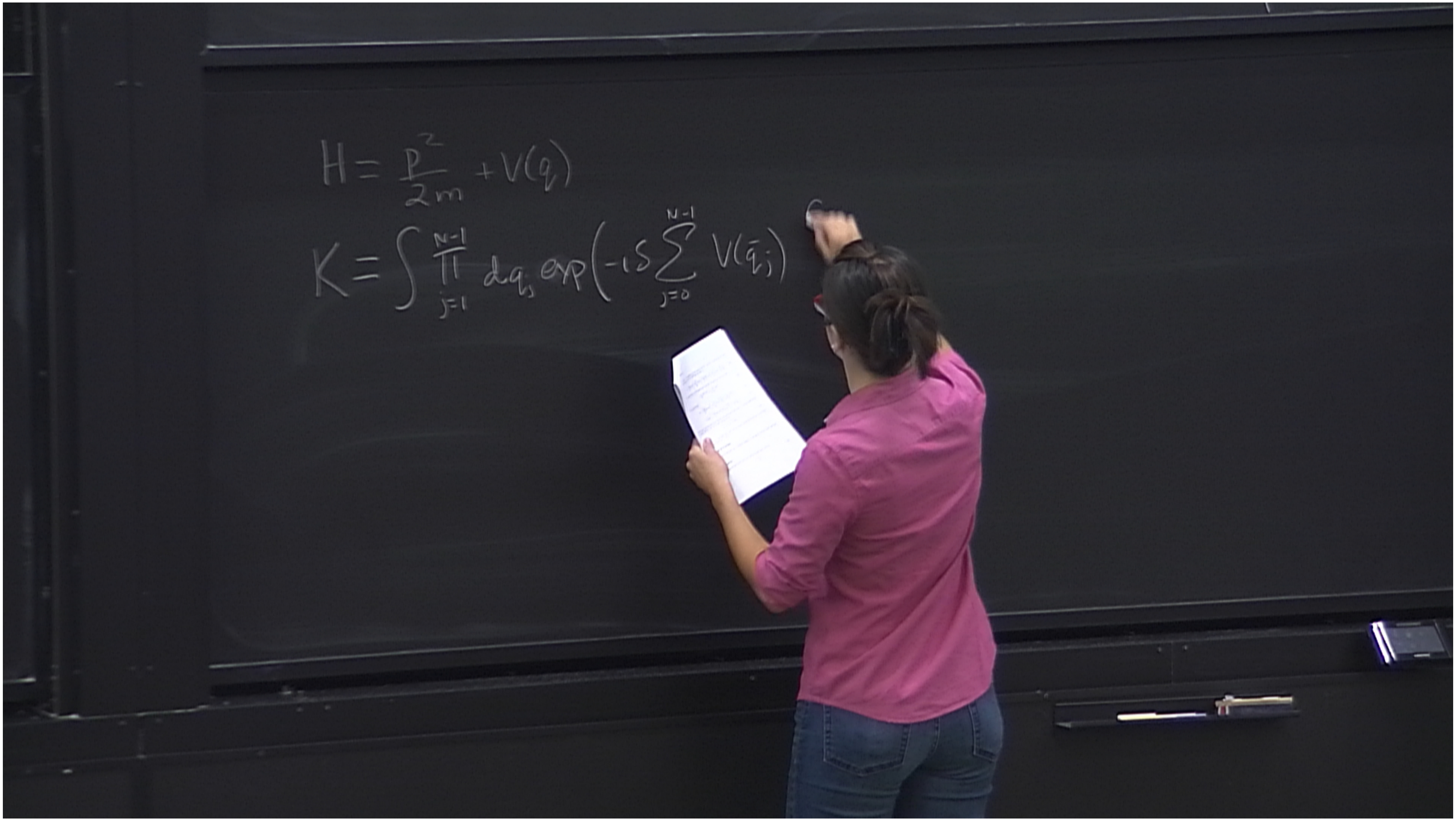
$K = \int dq_1 \cdots q_{N-1}$

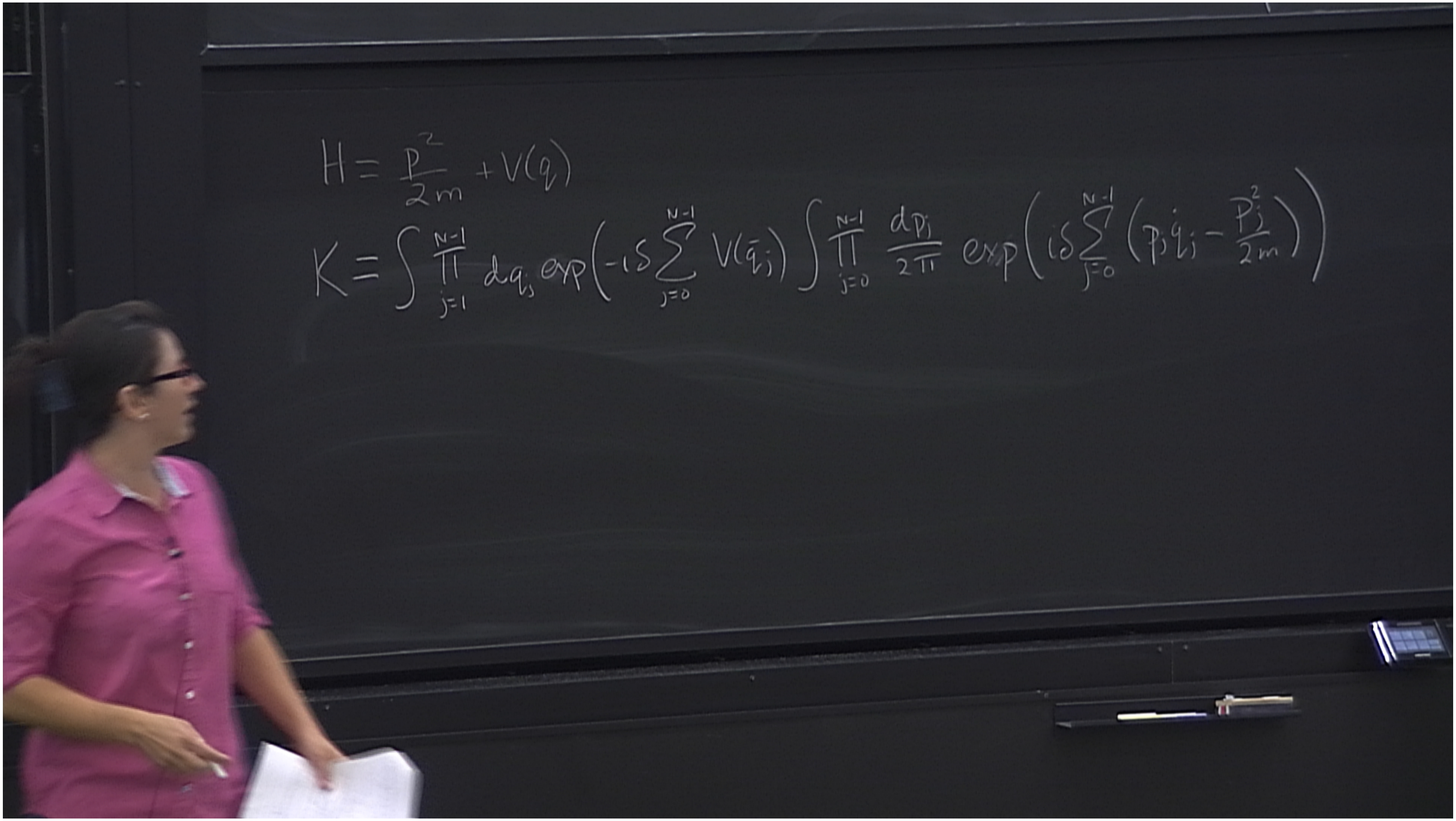
$= \int \prod_{j}$


$K = \int dq_1 \ldots q_{N-1}\, A_{path}$
$= \int \prod_{j=1}^{N-1} dq_j \int \prod_{j=0}^{N-1} \frac{dp_j}{2\pi} \exp\Big\{ i\delta \sum_{j=0}^{N-1} \big(p_j \dot{q}_j - H(p_j, q_j)\big)\Big\}$


$K = \int dq_1 \dots q_{N-1} \, A_{path}$
$= \int \left\{ \prod_{j=1}^{N-1} dq_j \right\} \int \left\{ \prod_{j=0}^{N-1} \frac{dp_j}{2\pi} \right\} \exp\left\{ i\delta \sum_{j=0}^{N-1} \left(p_j \dot{q}_j - H(p_j, \bar{q}_j) \right) \right\}$
$\int dq_1 \, dq_2 \, dq_3$
$\bar{q}_j)) + O(\delta^2))$


$K = \int dq_1 \ldots q_{N-1}\, A_{path}$
$= \int \left\{ \prod_{j=1}^{N-1} dq_j \right\} \int \left\{ \prod_{j=0}^{N-1} \frac{dp_i}{2\pi} \right\} \exp\left\{ i\delta \sum_{j=0}^{N-1} \left(p_j \dot{q}_j - H(p_j, \bar{q}_i) \right) \right\}$
$K \equiv \int \mathcal{D}p(t) \mathcal{D}q(t) \exp\left(\int_0^T dt\, (p\dot{q} - H($


$H = \frac{p^2}{2m} + V(q)$
$K = \int \prod_{j=1}^{N-1} dq_j \exp\left(-i\varepsilon \sum_{j=0}^{N-1} V(\bar{q}_j)\right)$


$H = \frac{p^2}{2m} + V(q)$
$K = \int \prod_{j=1}^{N-1} dq_j \exp\left(-i\delta \sum_{j=0}^{N-1} V(\bar{q}_j)\right) \int \prod_{j=0}^{N-1} \frac{dp_j}{2\pi} \exp\left(i\delta \sum_{j=0}^{N-1} \left(p_j \dot{q}_j - \frac{p_j^2}{2m}\right)\right)$

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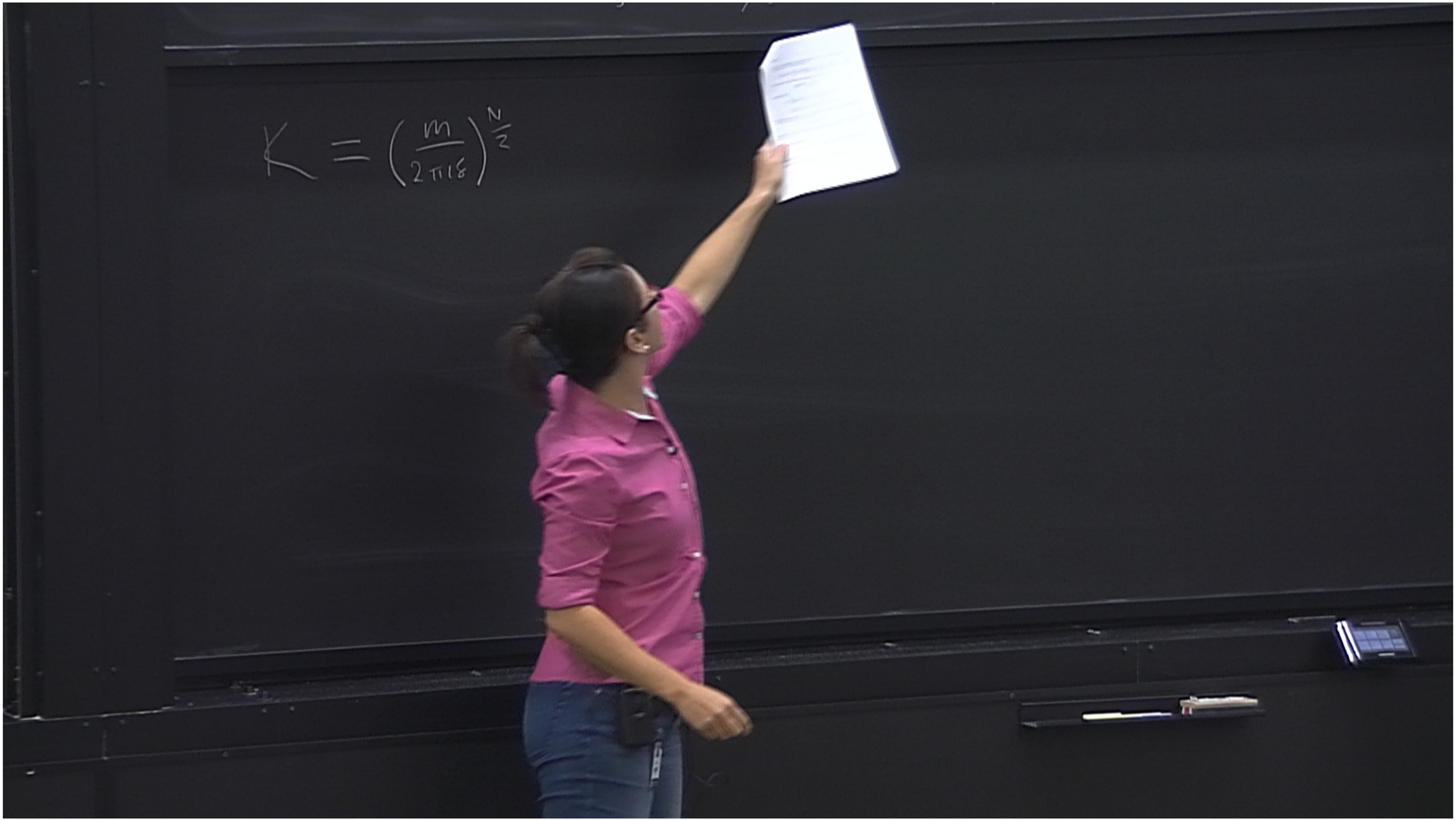
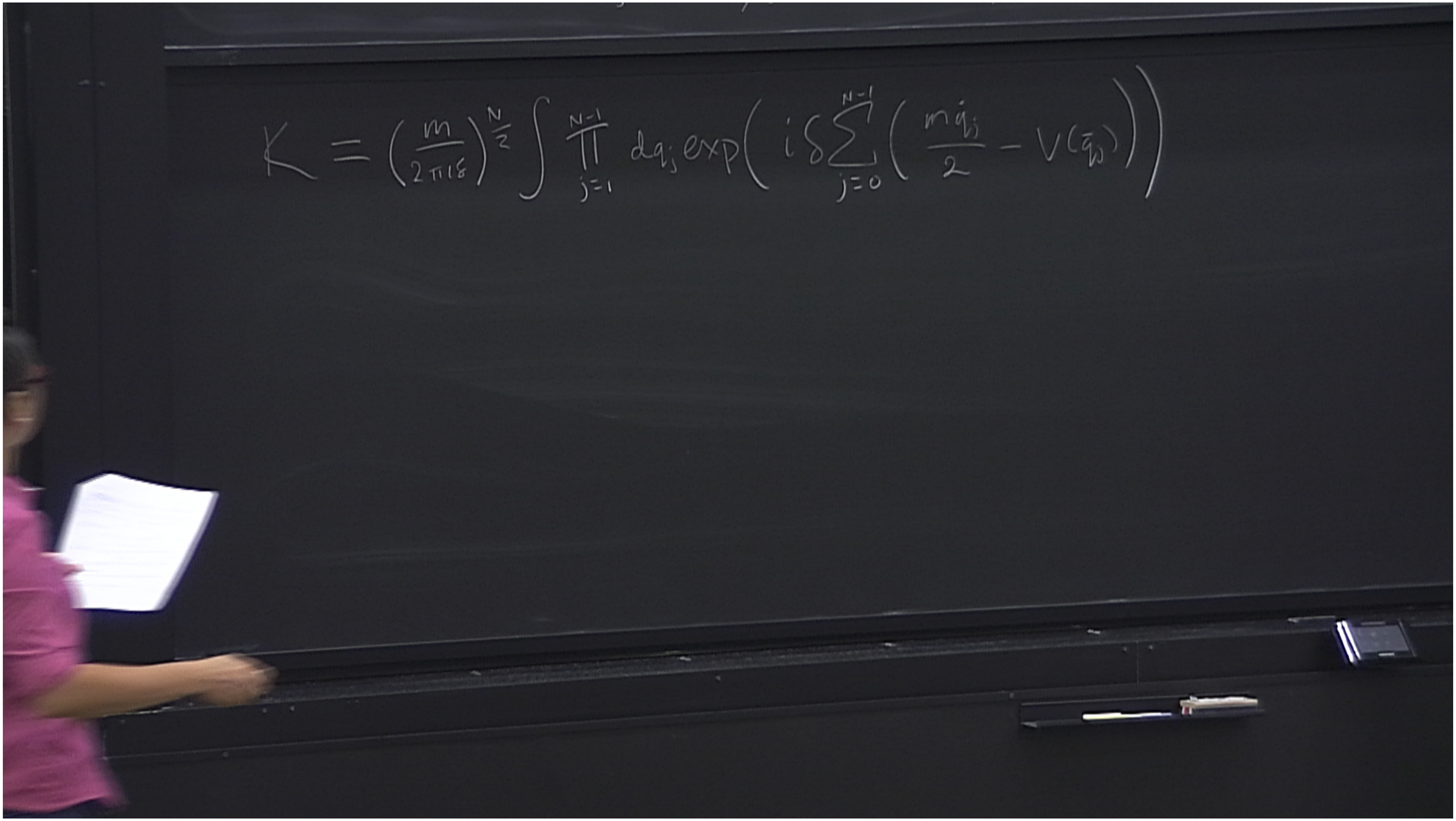
$$\boxed{\int \frac{dp}{2\pi}\, e^{i\delta\left(p\dot{q} - \frac{p^2}{2m}\right)} = \sqrt{\frac{m}{2\pi i\delta}}\; e^{i\frac{\delta m \dot{q}^2}{2}}}$$

$$K = \int \prod_{j=1}^{N-1} \delta q_j \exp\Big(-i\delta \sum_{j=0}^{N-1} V(\bar{q}_j)\Big) \prod_{j=0}^{N-1} \left(\sqrt{\frac{m}{2\pi i\delta}}\; e^{i\frac{\delta m \dot{q}_j^2}{2}}\right)$$


$K = \left(\frac{m}{2\pi i \varepsilon}\right)^{\frac{N}{2}}$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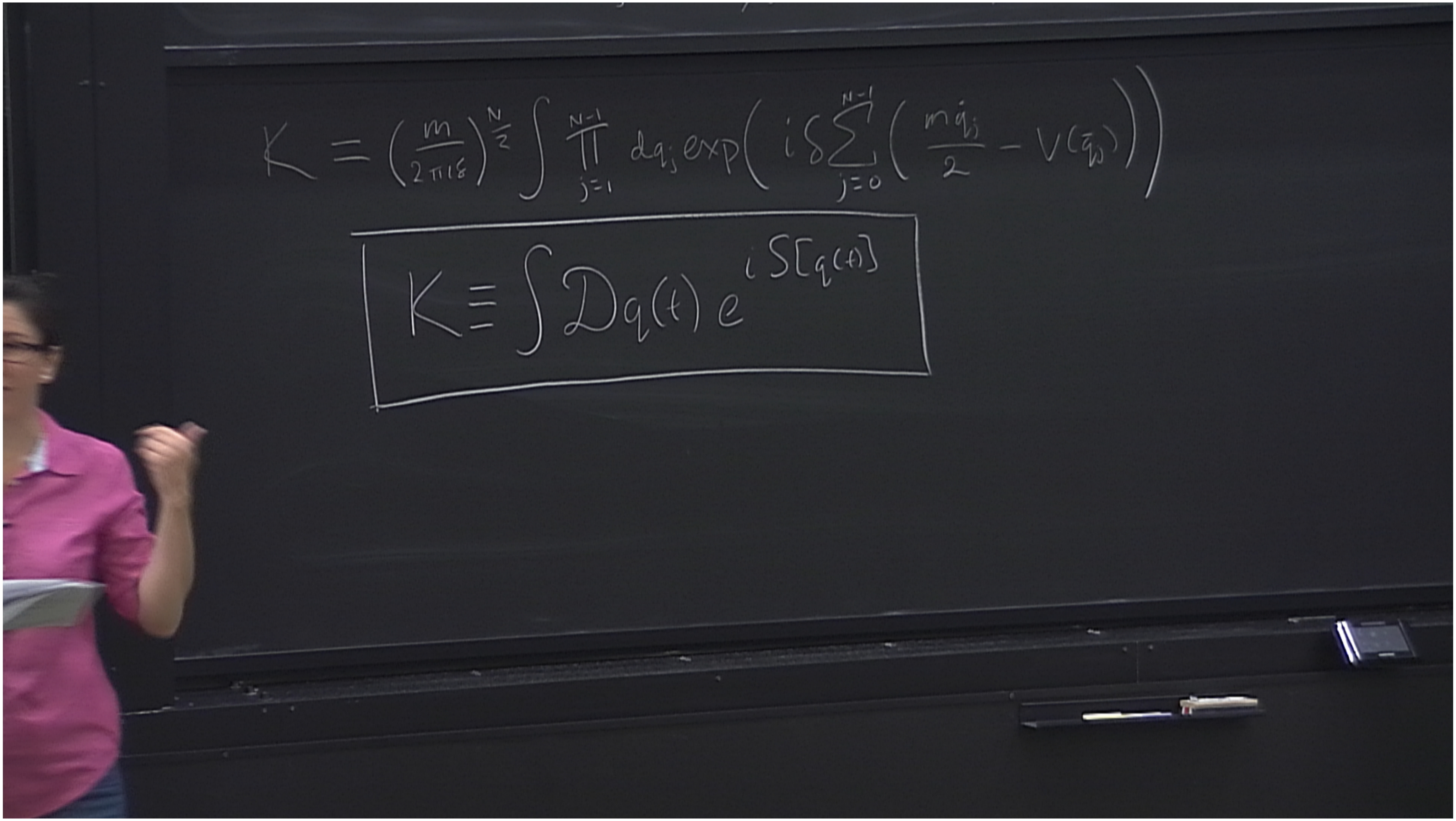
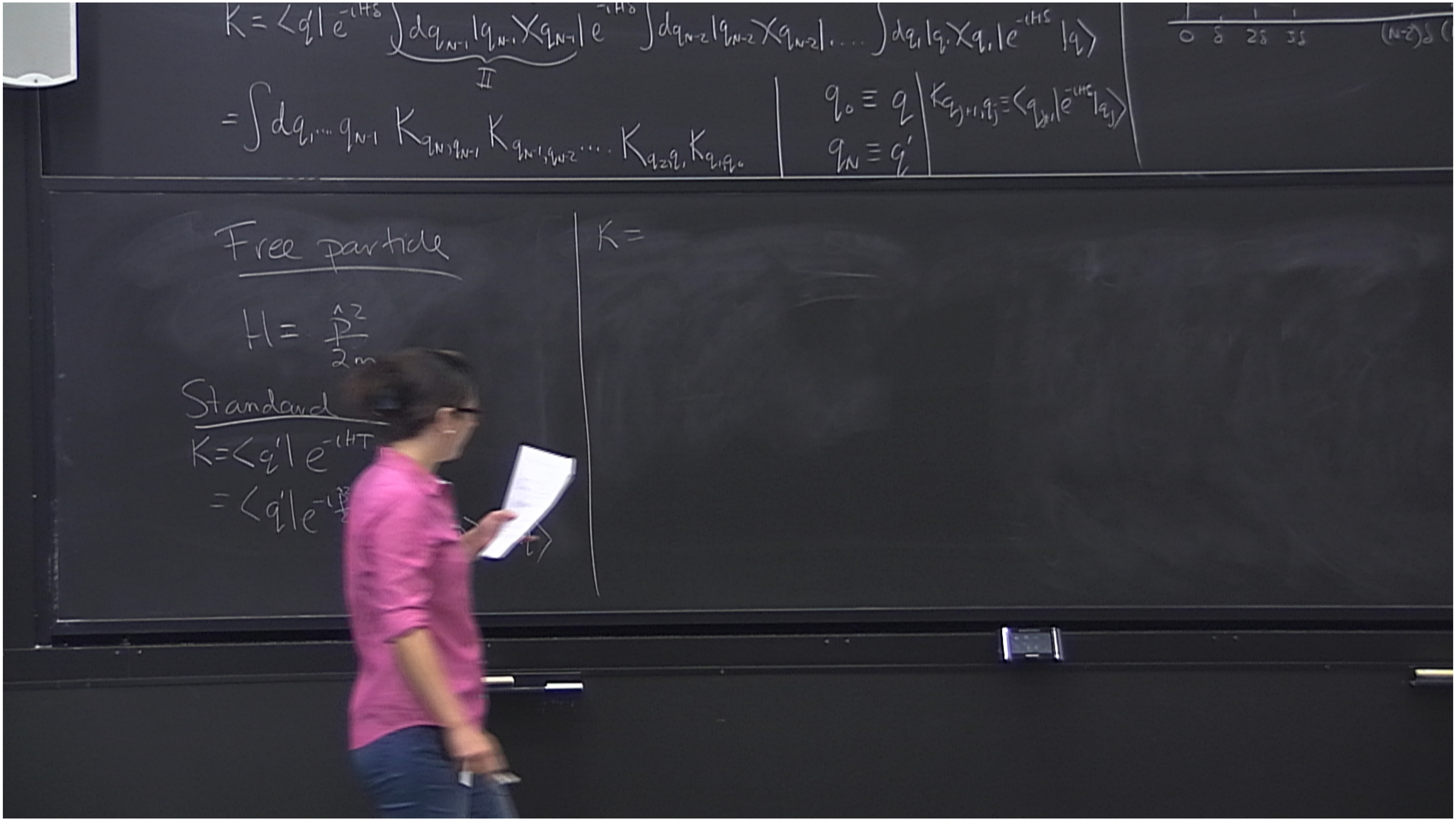
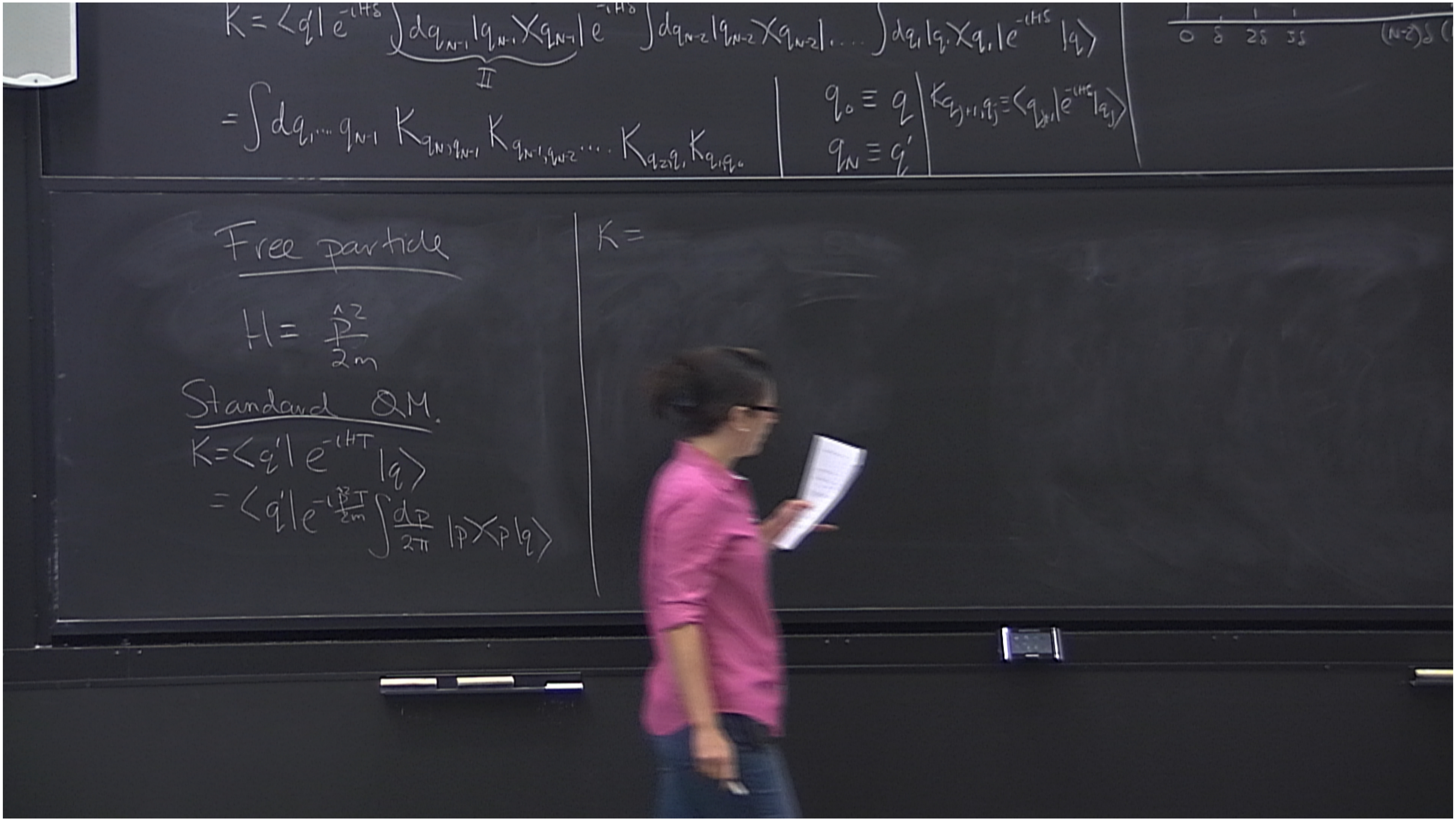
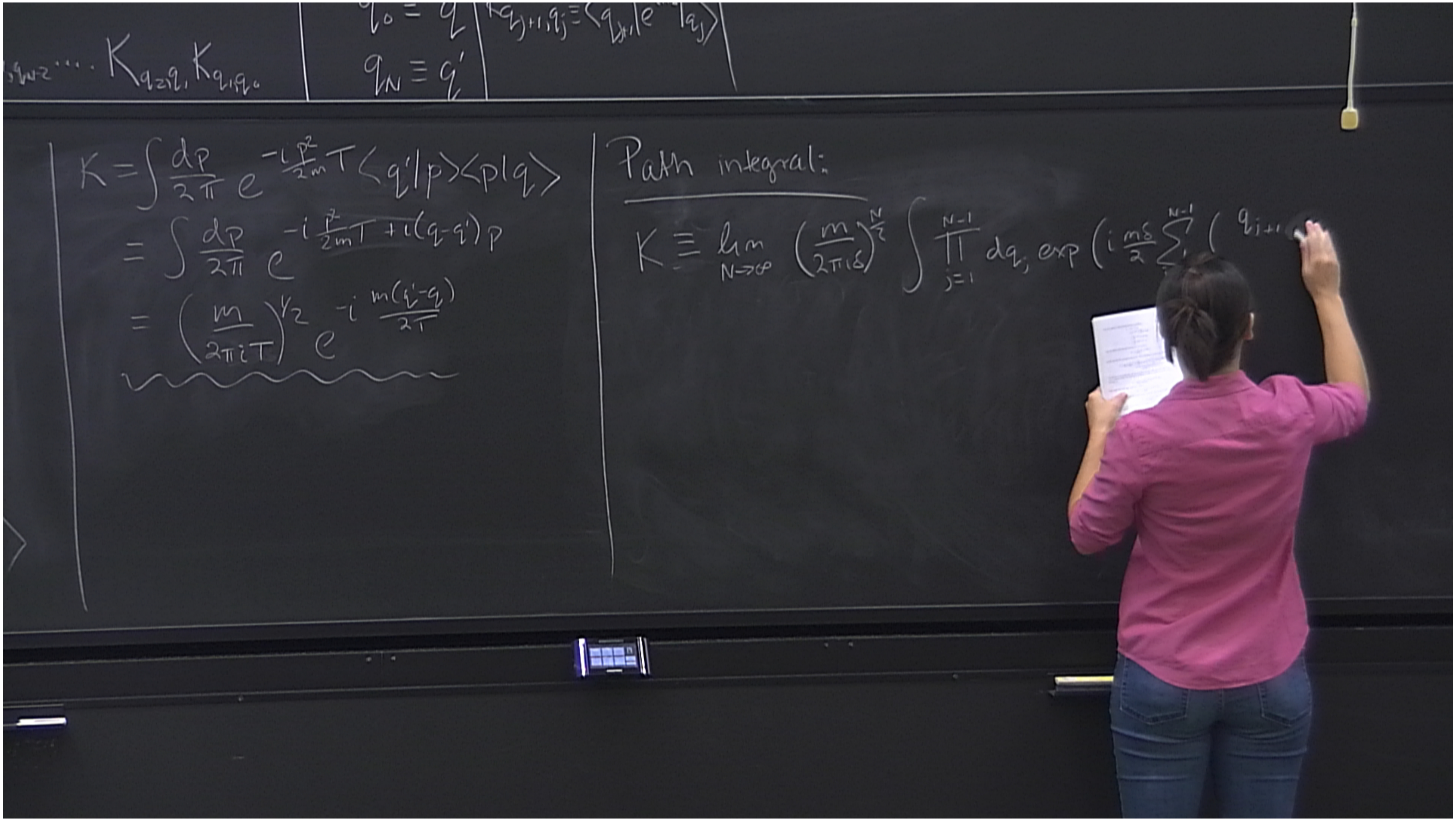

$K = \left(\frac{m}{2\pi i \varepsilon}\right)^{\frac{N}{2}} \int \prod_{j=1}^{N-1} dq_j \exp\left(i\varepsilon \sum_{j=0}^{N-1} \left(\frac{m \dot{q}_j^2}{2} - V(\bar{q}_j) \right)\right)$
$\boxed{K \equiv \int \mathcal{D}q(t)\, e^{iS[q(t)]}}$


K = ⟨q'| e^{-iHδ} ∫dq_{N-1} |q_{N-1}⟩⟨q_{N-1}| e^{-iHδ} ∫dq_{N-2} |q_{N-2}⟩⟨q_{N-2}| . . . ∫dq_1 |q_1⟩⟨q_1| e^{-iHδ} |q⟩
II
= ∫dq_1 ... q_{N-1} K_{q_N,q_{N-1}} K_{q_{N-1},q_{N-2}} . . . K_{q_2 q_1} K_{q_1 q_0}
q_0 ≡ q
q_N ≡ q'
K_{q_{j+1},q_j} ≡ ⟨q_{j+1}| e^{-iHδ} |q_j⟩
0 δ 2δ 3δ (N-2)δ
Free particle
H = p̂²/2m
Standard
K = ⟨q'| e^{-iHT}
= ⟨q'| e^{-i
K =


$K = \langle q' | e^{-iH\delta} \underbrace{\int dq_{N-1} |q_{N-1}\rangle\langle q_{N-1}|}_{I} e^{-iH\delta} \int dq_{N-2} |q_{N-2}\rangle\langle q_{N-2}| \dots \int dq_1 |q_1\rangle\langle q_1| e^{-iH\delta} |q\rangle$
$= \int dq_1 \dots q_{N-1} K_{q_N, q_{N-1}} K_{q_{N-1}, q_{N-2}} \dots K_{q_2 q_1} K_{q_1 q_0}$
$q_0 \equiv q$
$q_N \equiv q'$
$K_{q_{j+1}, q_j} \equiv \langle q_{j+1} | e^{-iH\delta} | q_j \rangle$
$0 \quad \delta \quad 2\delta \quad 3\delta \quad (N-2)\delta$
Free particle
$H = \frac{\hat{p}^2}{2m}$
Standard QM.
$K = \langle q' | e^{-iHT} | q \rangle$
$= \langle q' | e^{-i\frac{\hat{p}^2 T}{2m}} \int \frac{dp}{2\pi} |p\rangle\langle p | q \rangle$
$K =$


$K = \int \frac{dp}{2\pi} e^{-i\frac{p^2}{2m}T} \langle q'|p\rangle\langle p|q\rangle$
$= \int \frac{dp}{2\pi} e^{-i\frac{p^2}{2m}T + i(q-q')p}$
$= \left(\frac{m}{2\pi i T}\right)^{1/2} e^{-i\frac{m(q'-q)}{2T}}$
Path integral:
$K \equiv \lim_{N\to\infty} \left(\frac{m}{2\pi i\delta}\right)^{\frac{N}{2}} \int \prod_{j=1}^{N-1} dq_j \exp\left(i\frac{m\delta}{2}\sum^{N-1}\left(q_{j+1}\right.\right.$
$q_0 \equiv q$
$q_N \equiv q'$

$\ldots q_{N-2} \cdots K_{q_2 q_1} K_{q_1 q_0}$

$q_0 = q$

$q_N \equiv q'$

$K_{q_{j+1}, q_j} \equiv \langle q_{j+1} | e^{\ldots} | q_j \rangle$

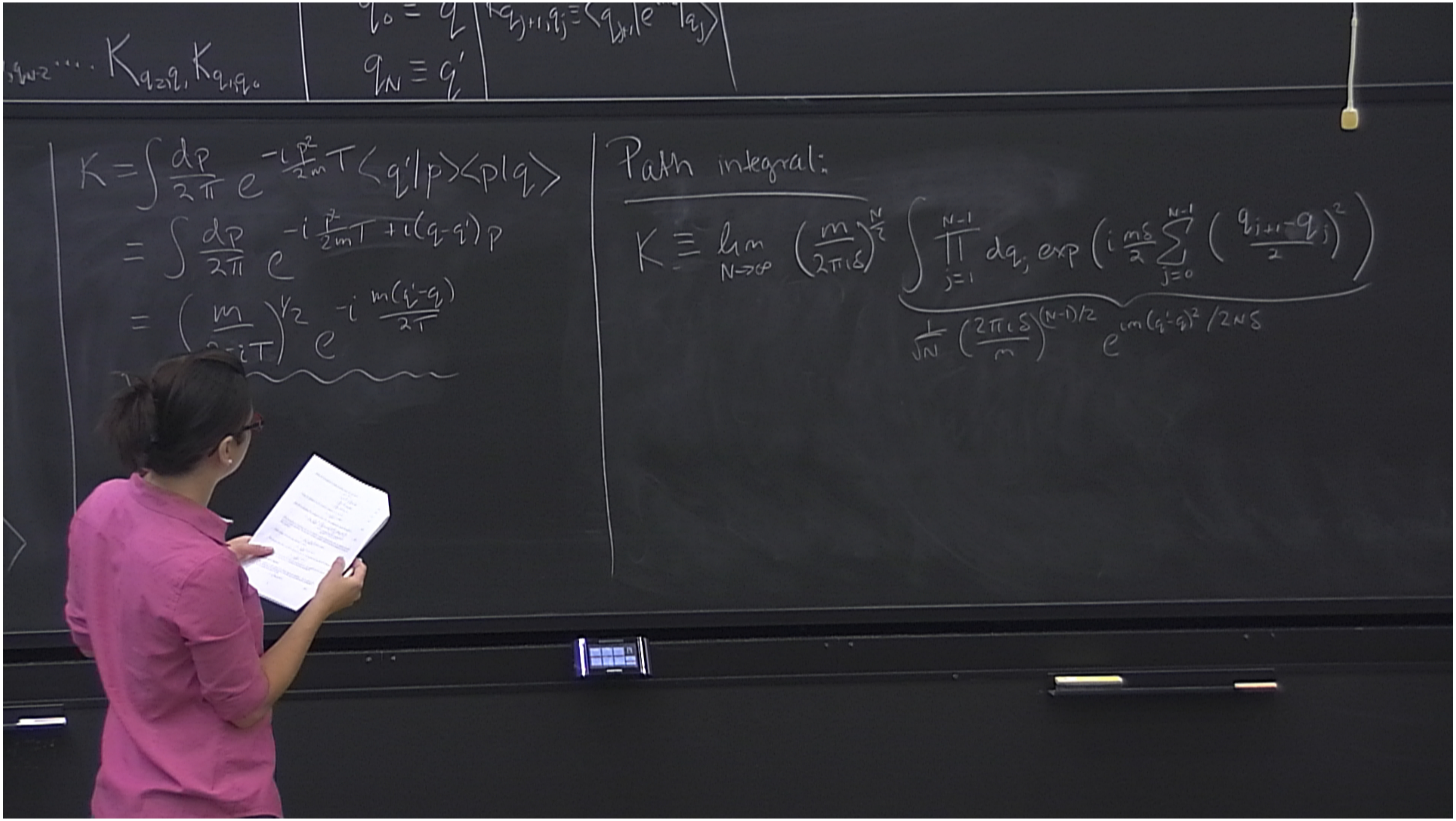
$$K = \int \frac{dp}{2\pi} e^{-i \frac{p^2}{2m} T} \langle q' | p \rangle \langle p | q \rangle$$

$$= \int \frac{dp}{2\pi} e^{-i \frac{p^2}{2m} T + i(q - q')p}$$

$$= \left(\frac{m}{2\pi i T} \right)^{1/2} e^{-i \frac{m(q'-q)}{2T}}$$

Path integral:

$$K \equiv \lim_{N \to \infty} \left(\frac{m}{2\pi i \delta} \right)^{\frac{N}{2}} \int \prod_{j=1}^{N-1} dq_j \exp\left(i \frac{m\delta}{2} \sum_{j=0}^{N-1} \left(\frac{q_{j+1} - q_j}{2} \right)^2 \right)$$


$\ldots K_{q_2 q_1} K_{q_1 q_0}$
$q_0 \equiv q$
$q_N \equiv q'$
$K_{q_{j+1}, q_j} \equiv \langle q_{j+1} | e^{\ldots} | q_j \rangle$
$K = \int \frac{dp}{2\pi} e^{-i \frac{p^2}{2m} T} \langle q' | p \rangle \langle p | q \rangle$
$= \int \frac{dp}{2\pi} e^{-i \frac{p^2}{2m} T + i(q-q')p}$
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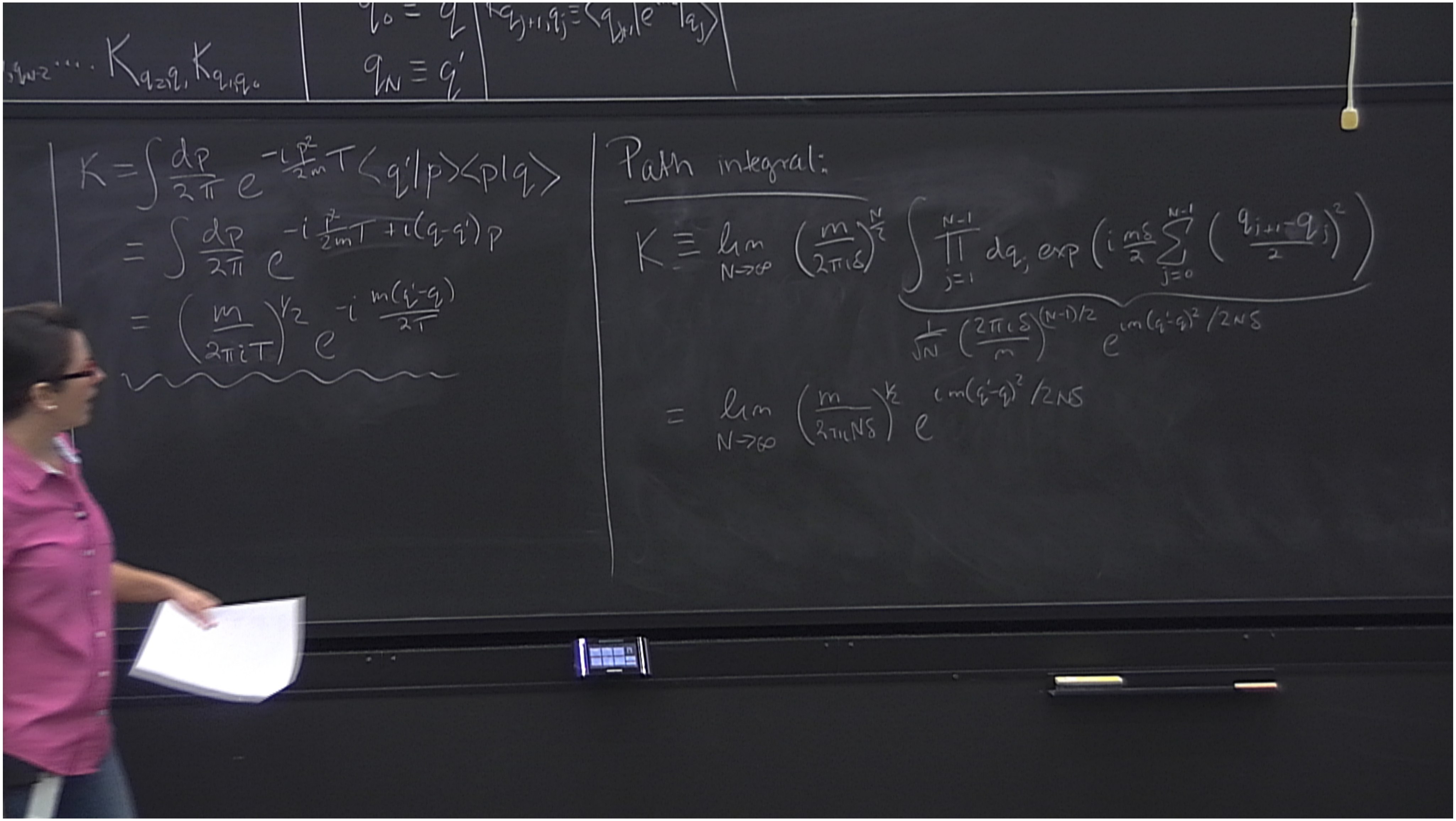
Path integral:
$K \equiv \lim_{N \to \infty} \left(\frac{m}{2\pi i \delta} \right)^{\frac{N}{2}} \underbrace{\int \prod_{j=1}^{N-1} dq_j \exp\left(i \frac{m\delta}{2} \sum_{j=0}^{N-1} \left(\frac{q_{j+1} - q_j}{\delta} \right)^2 \right)}_{\frac{1}{\sqrt{N}} \left(\frac{2\pi i \delta}{m} \right)^{(N-1)/2} e^{im(q'-q)^2/2N\delta}}$



$\ldots K_{q_2 q_1} K_{q_1 q_0}$
$q_0 \equiv q$
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$K_{q_{j+1},q_j} \equiv \langle q_{j+1}|e^{\ldots}|q_j\rangle$
$K = \int \frac{dp}{2\pi} e^{-i\frac{p^2}{2m}T} \langle q'|p\rangle\langle p|q\rangle$
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$= \lim_{N\to\infty} \left(\frac{m}{2\pi i N\delta}\right)^{1/2} e^{im(q'-q)^2/2N\delta}$

$$\ldots K_{q_2 q_1} K_{q_1 q_0} \qquad q_0 \equiv q, \quad q_N \equiv q' \qquad K_{q_{j+1}, q_j} \equiv \langle q_{j+1} | e^{\cdots} | q_j \rangle$$

$$K = \int \frac{dp}{2\pi} e^{-i \frac{p^2}{2m} T} \langle q' | p \rangle \langle p | q \rangle$$

$$= \int \frac{dp}{2\pi} e^{-i \frac{p^2}{2m} T + i(q-q')p}$$

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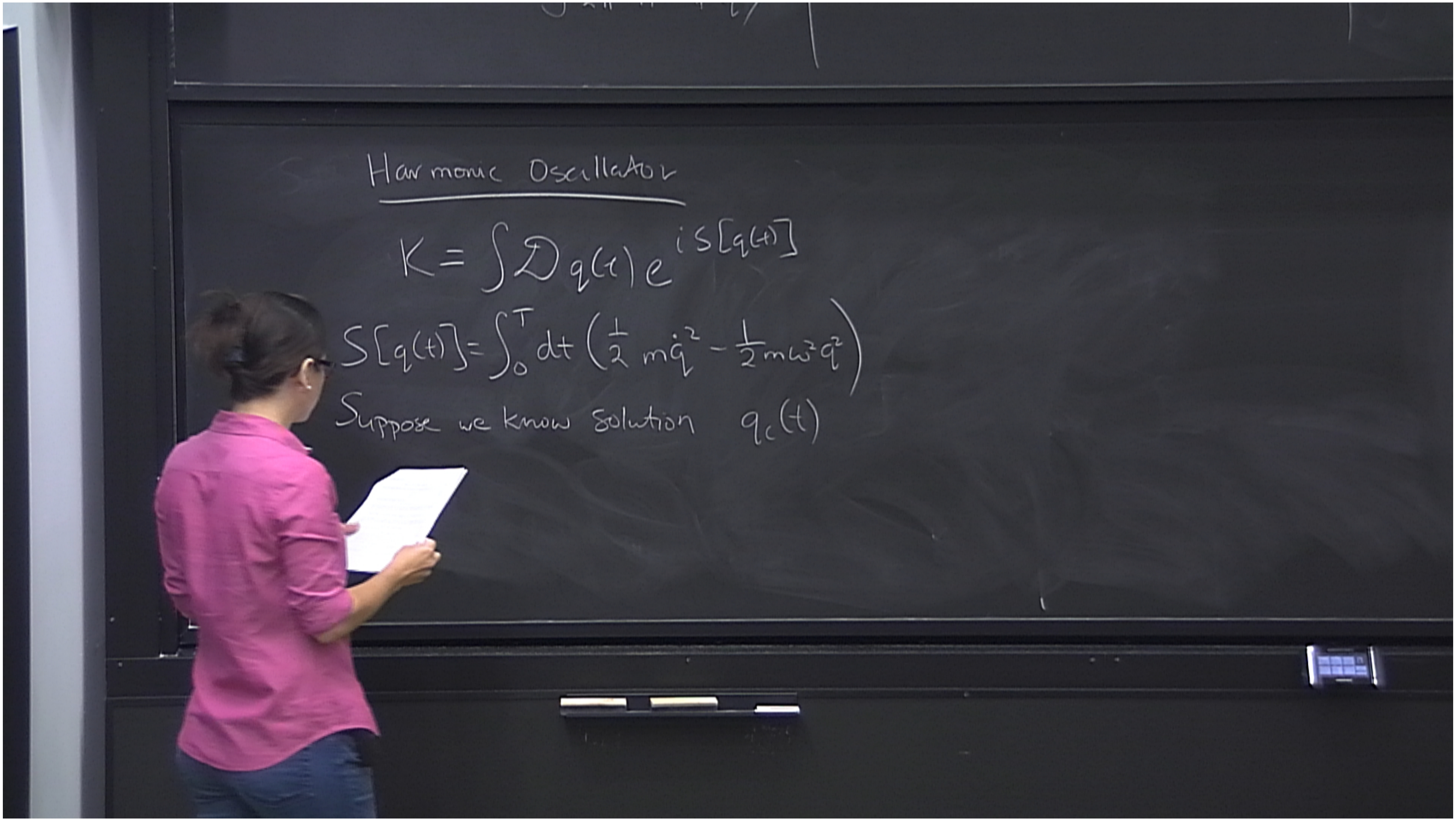
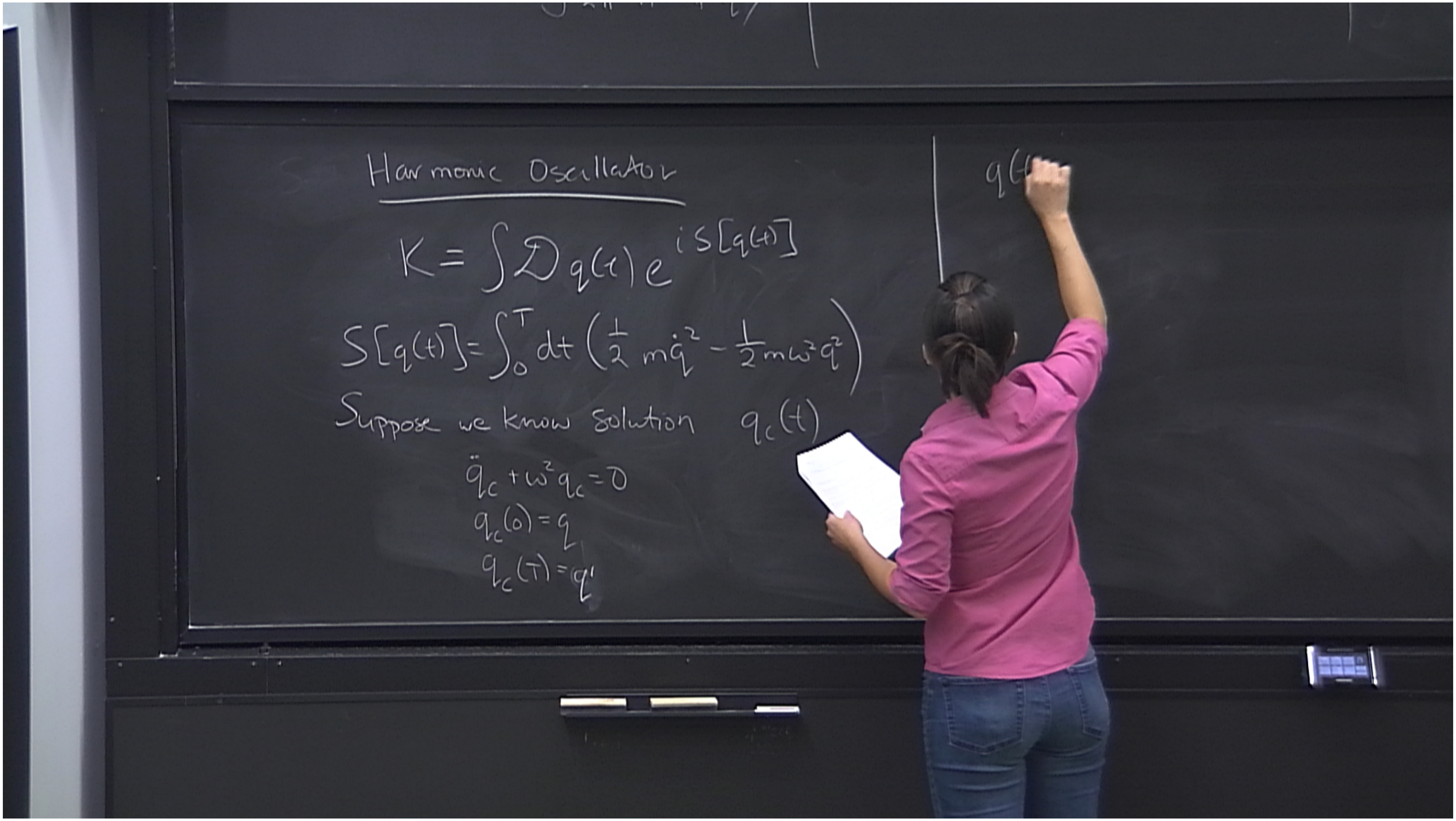
Path integral:

$$K \equiv \lim_{N \to \infty} \left(\frac{m}{2\pi i \delta} \right)^{\frac{N}{2}} \underbrace{\int \prod_{j=1}^{N-1} dq_j \exp\left(i \frac{m\delta}{2} \sum_{j=0}^{N-1} \left(\frac{q_{j+1} - q_j}{2} \right)^2 \right)}_{\frac{1}{\sqrt{N}} \left(\frac{2\pi i \delta}{m} \right)^{(N-1)/2} e^{im(q'-q)^2/2N\delta}}$$

$$= \lim_{N \to \infty} \left(\frac{m}{2\pi i N\delta} \right)^{\frac{1}{2}} e^{im(q'-q)^2/2N\delta} \qquad \boxed{N\delta = T}$$

$$= \left(\frac{m}{2\pi i T} \right)^{\frac{1}{2}} e^{im(q'-q)^2/2T}$$


Harmonic Oscillator
$K = \int \mathcal{D}q(t)\, e^{iS[q(t)]}$
$S[q(t)] = \int_0^T dt \left(\frac{1}{2} m\dot{q}^2 - \frac{1}{2} m\omega^2 q^2 \right)$
Suppose we know solution $q_c(t)$


Harmonic Oscillator
$K = \int \mathcal{D}q(t)\, e^{iS[q(t)]}$
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Suppose we know solution $q_c(t)$
$\ddot{q}_c + \omega^2 q_c = 0$
$q_c(0) = q$
$q_c(T) = q'$

$$q(t) = q_c(t) + y(t)$$

$$S[q(t)] = S[q_c(t) + y(t)]$$

$$= \int_0^T dt \left(\frac{1}{2} m \dot{q}_c^2 - \frac{1}{2} m\omega^2 q_c^2\right) + (\text{linear in } y) + \int_0^T dt \left(\frac{1}{2} m \dot{y}^2 - \frac{1}{2} m\omega^2 y^2\right)$$

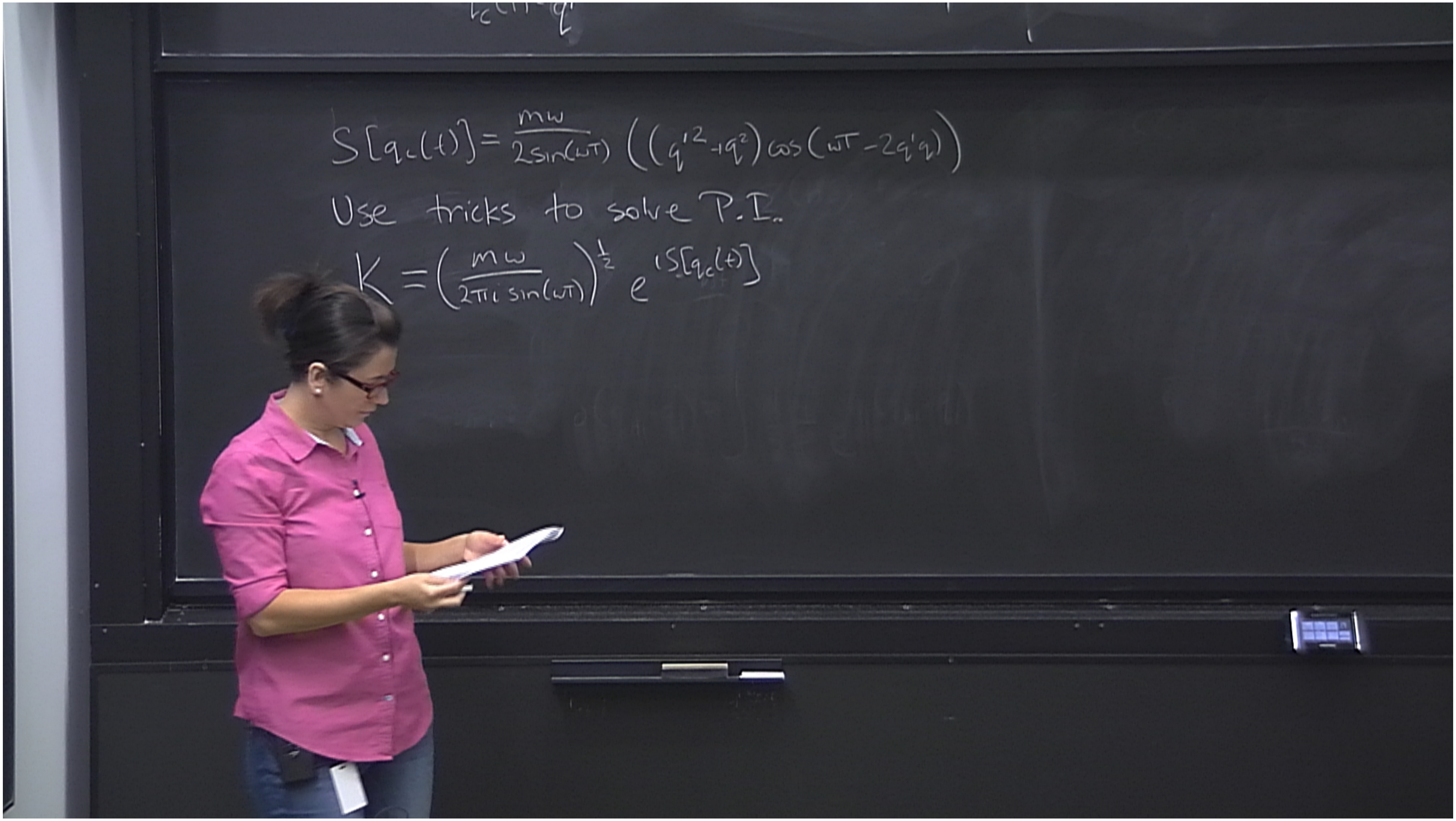
$$q(t) = q_c(t) + y(t)$$

$$S[q(t)] = S[q_c(t) + y(t)]$$

$$= \int_0^T dt \left(\frac{1}{2} m \dot{q}_c^2 - \frac{1}{2} m \omega^2 q_c^2\right) + \underbrace{\left(\text{linear in } y\right)}_{=0} + \int_0^T dt \left(\frac{1}{2} m \dot{y}^2 - \frac{1}{2} m \omega^2 y^2\right)$$

$$= S[q_c(t)] + S[y(t)]$$

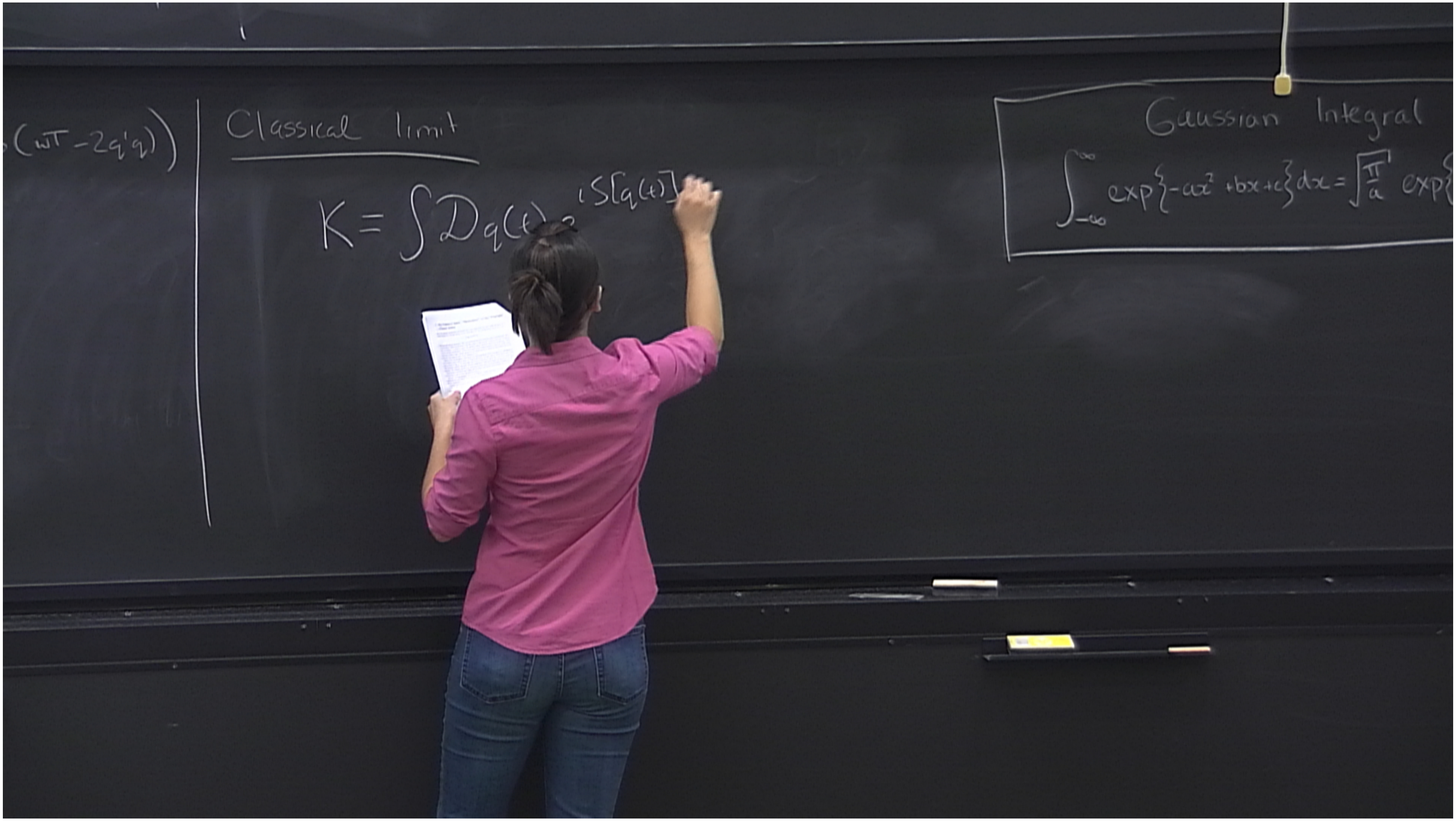
$$K = e^{iS[q_c(t)]} \int \mathcal{D} y(t)\, e^{iS[y(t)]}$$


$S[q_c(t)] = \frac{m\omega}{2\sin(\omega T)}\left(\left(q'^2+q^2\right)\cos(\omega T - 2q'q)\right)$
Use tricks to solve P.I.
$K = \left(\frac{m\omega}{2\pi i \sin(\omega T)}\right)^{\frac{1}{2}} e^{iS[q_c(t)]}$

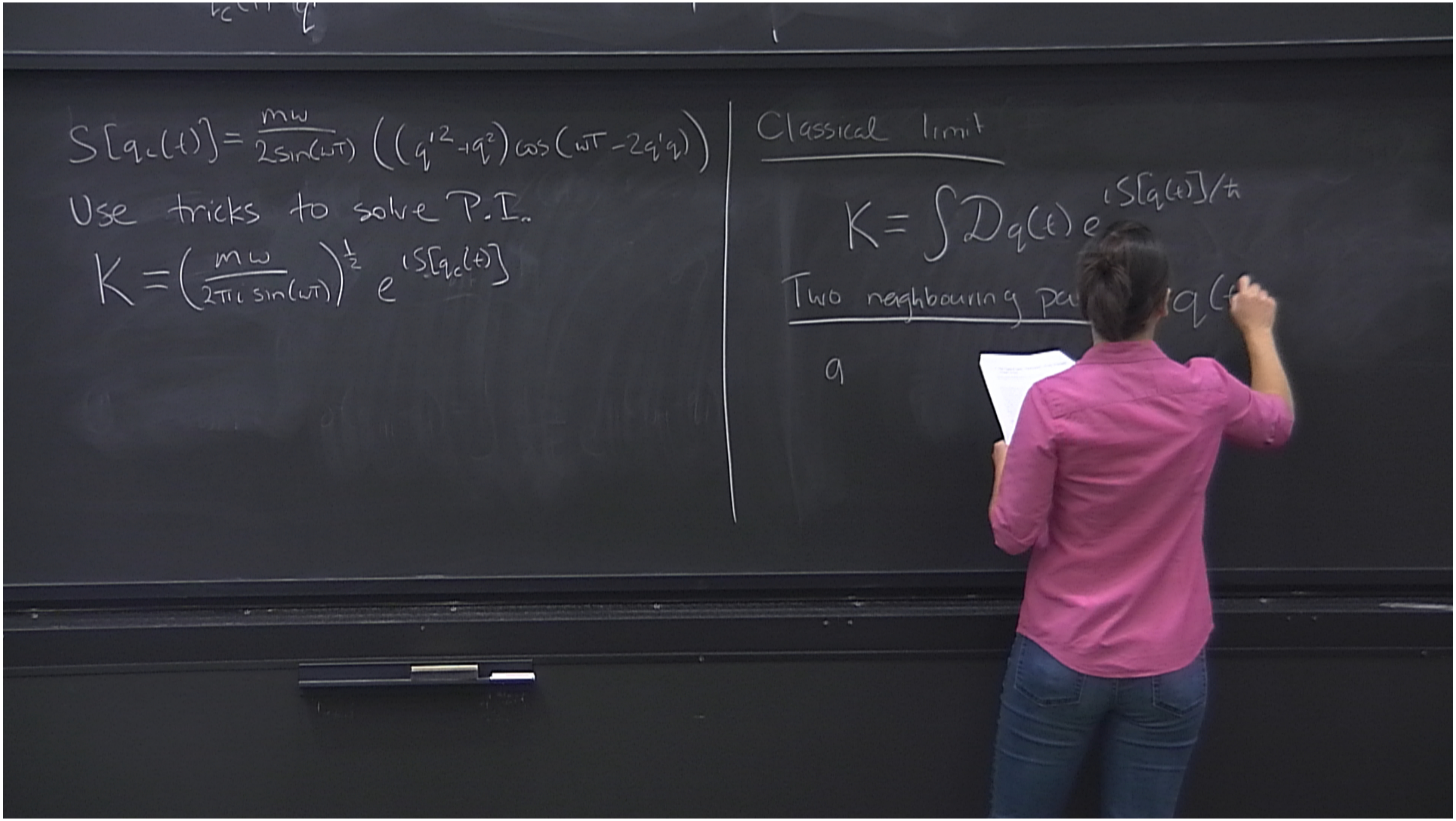
$$S[q_c(t)] = \frac{m\omega}{2\sin(\omega T)} \left((q'^2 + q^2) \cos(\omega T - 2q'q) \right)$$

Use tricks to solve P.I.

$$K = \left(\frac{m\omega}{2\pi i \sin(\omega T)} \right)^{\frac{1}{2}} e^{iS[q_c(t)]}$$



$(\omega T - 2q'q))$
Classical limit
$K = \int \mathcal{D}q(t)\, e^{iS[q(t)]}$
Gaussian Integral
$\int_{-\infty}^{\infty} \exp\{-ax^2 + bx + c\}\, dx = \sqrt{\frac{\pi}{a}} \exp\{$


$S[q_c(t)] = \frac{m\omega}{2\sin(\omega T)}\left(\left(q'^2+q^2\right)\cos(\omega T - 2q'q)\right)$
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$K = \left(\frac{m\omega}{2\pi i \sin(\omega T)}\right)^{\frac{1}{2}} e^{iS[q_c(t)]}$
Classical limit
$K = \int \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}$
Two neighbouring

$+q^2)\cos(\omega T - 2q'q))$

P. I.

$_c(t)]$

Classical limit

$$K = \int \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}$$

Two neighbouring paths $\quad q(t)$ and $q'(t) = q(t) + \eta(t)$

$$S[q'] = S[q+\eta] = S[q] + \int dt\, \eta(t) \frac{\delta S[q]}{\delta q(t)} + O(\eta^2)$$

Gaussian Int

$$\int_{-\infty}^{\infty} \exp\{-ax^2 + bx + c\}\, dx =$$

$+q^2) \cos(\omega T - 2q'q))$

P.I.

$_c(t)]$

Classical limit

$$K = \int \mathcal{D}q(t)\, e^{iS[q(t)]/\hbar}$$

Two neighbouring paths $\quad q(t)$ and $q'(t) = q(t) + \eta(t)$

$$S[q'] = S[q+\eta] = S[q] + \int dt\, \eta(t) \frac{\delta S[q]}{\delta q(t)} + O(\eta^2)$$

Sum two paths: $\quad A \simeq e^{iS[q]/\hbar}\left(1 + \exp \frac{i}{\hbar}\int dt\, \eta(t) \frac{\delta S[q]}{\delta q(t)}\right)$

Gaussian Int

$$\int_{-\infty}^{\infty} \exp\{-ax^2 + bx + c\}\, dx =$$