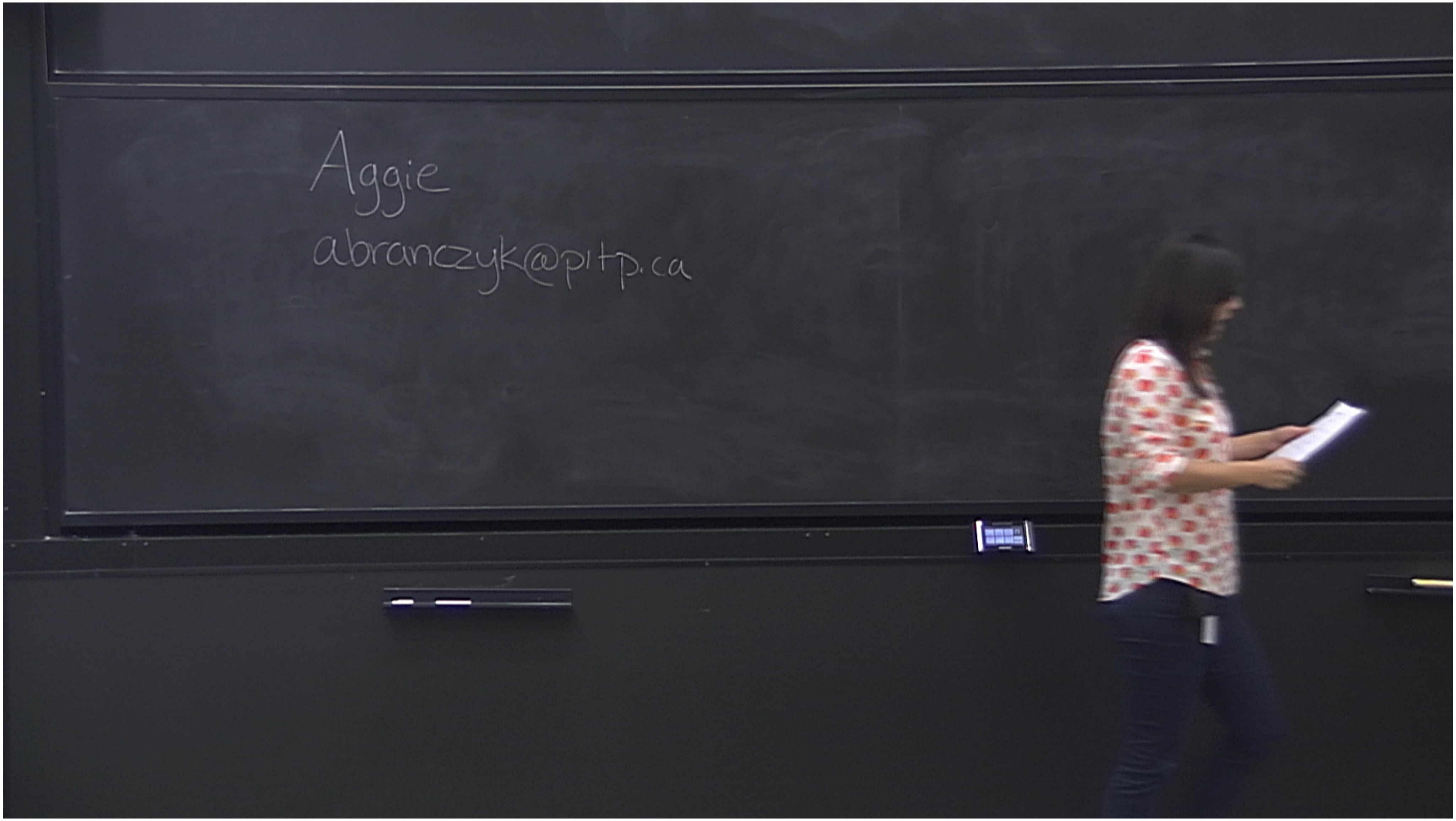
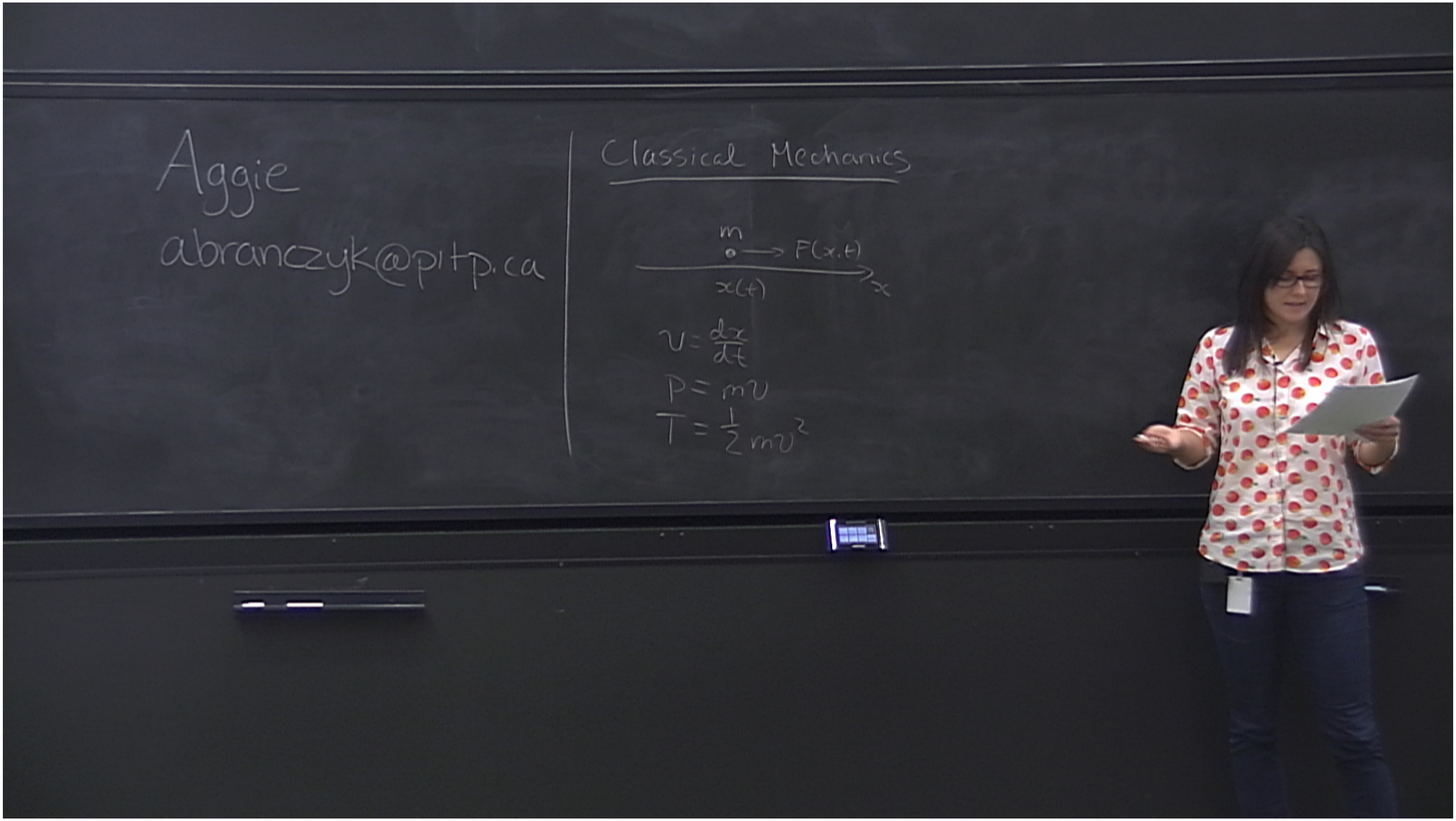
Title: 14/15 PSI - Quantum Mechanics 1

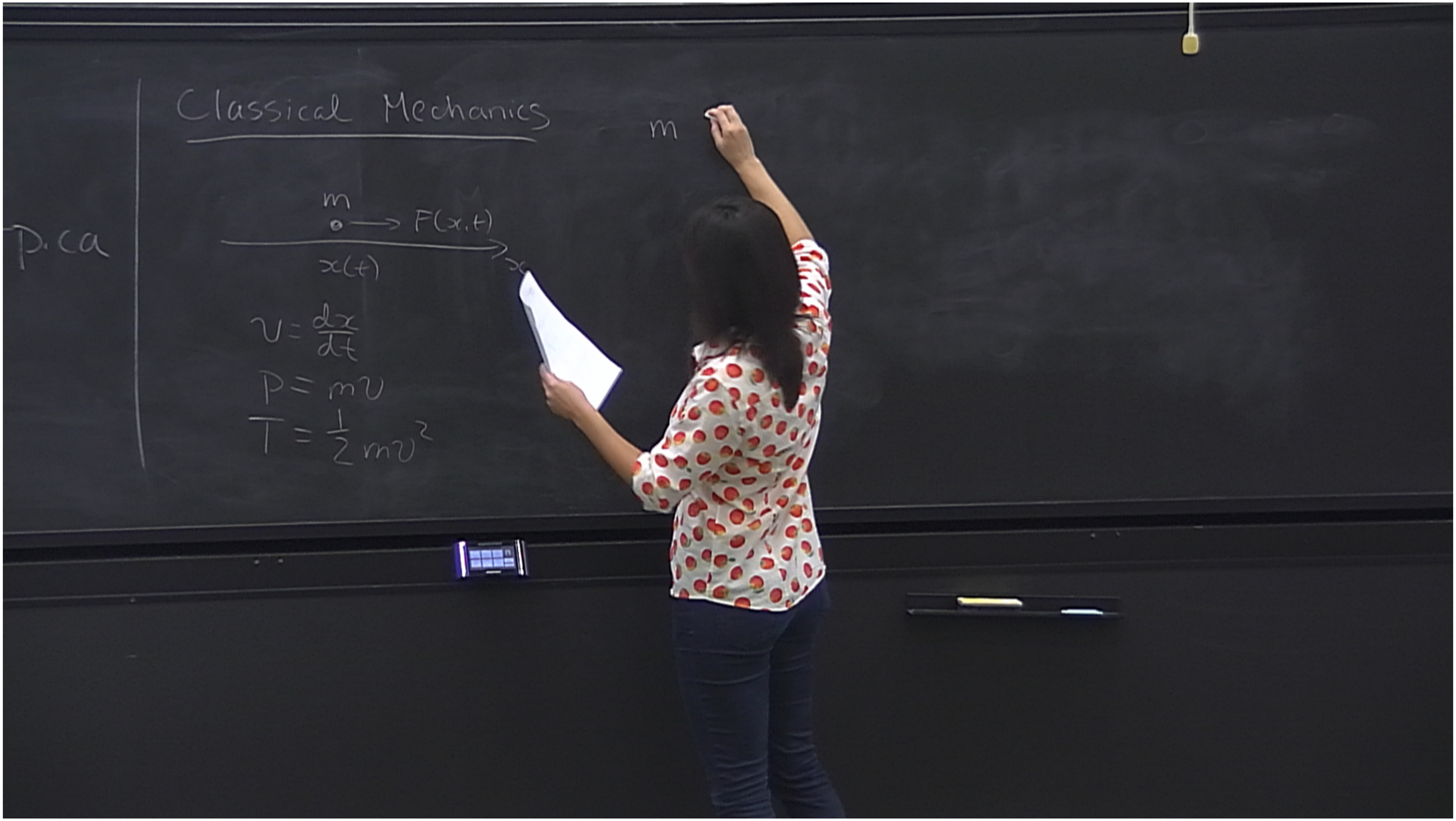
Date: Aug 19, 2014 10:30 AM

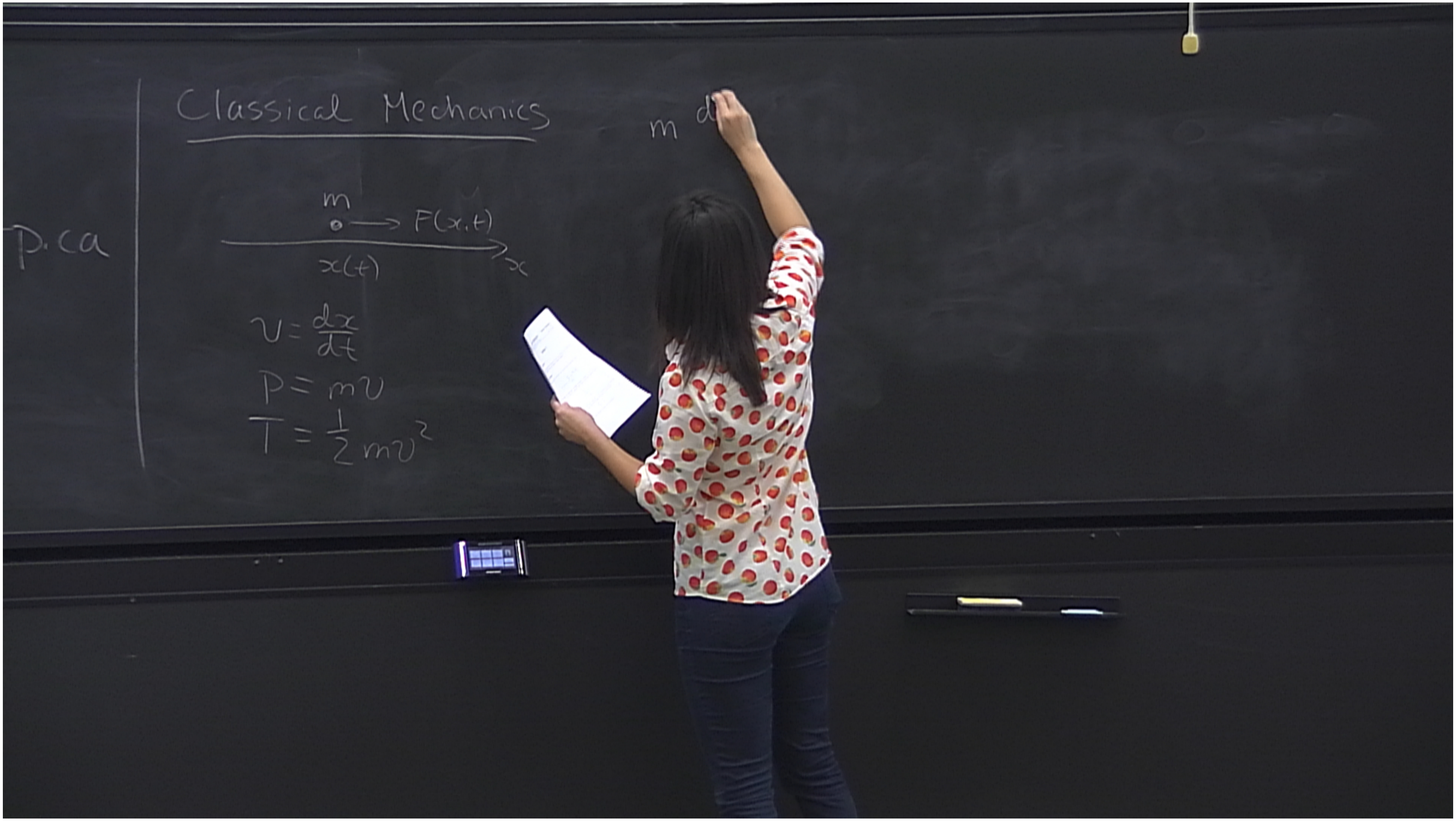
URL: http://pirsa.org/14080024

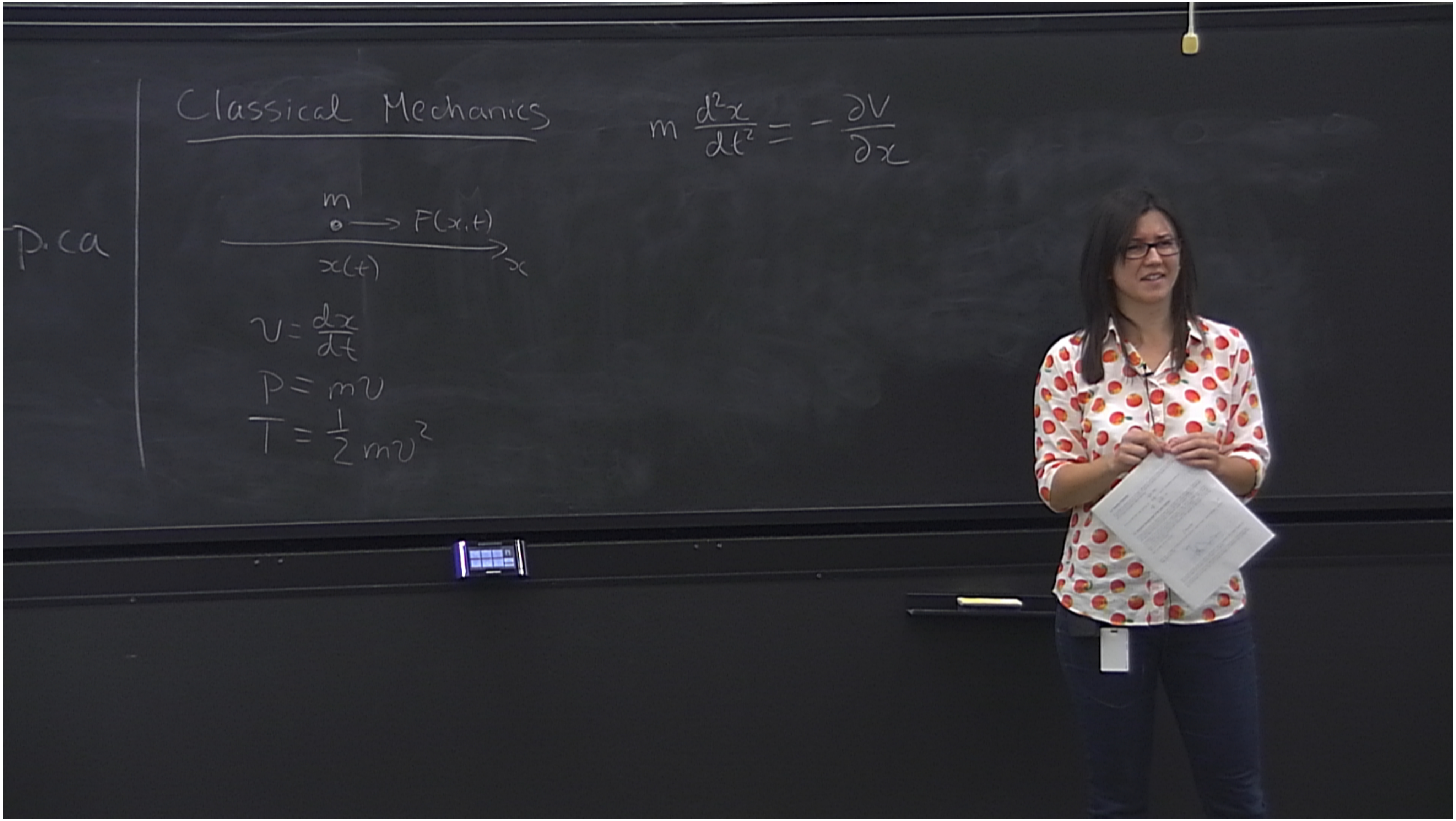
Abstract:

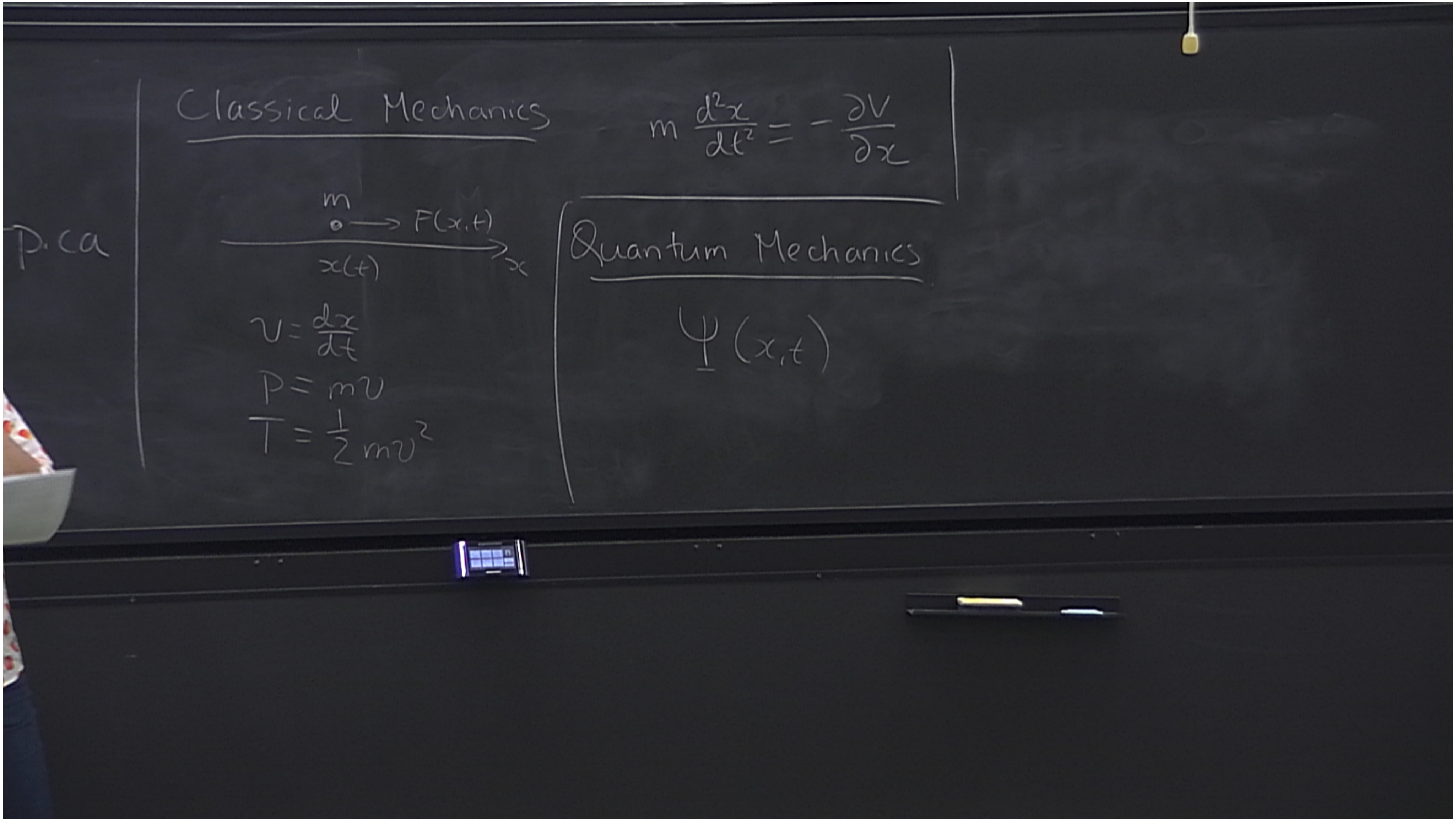

Aggie
abranczyk@pitp.ca

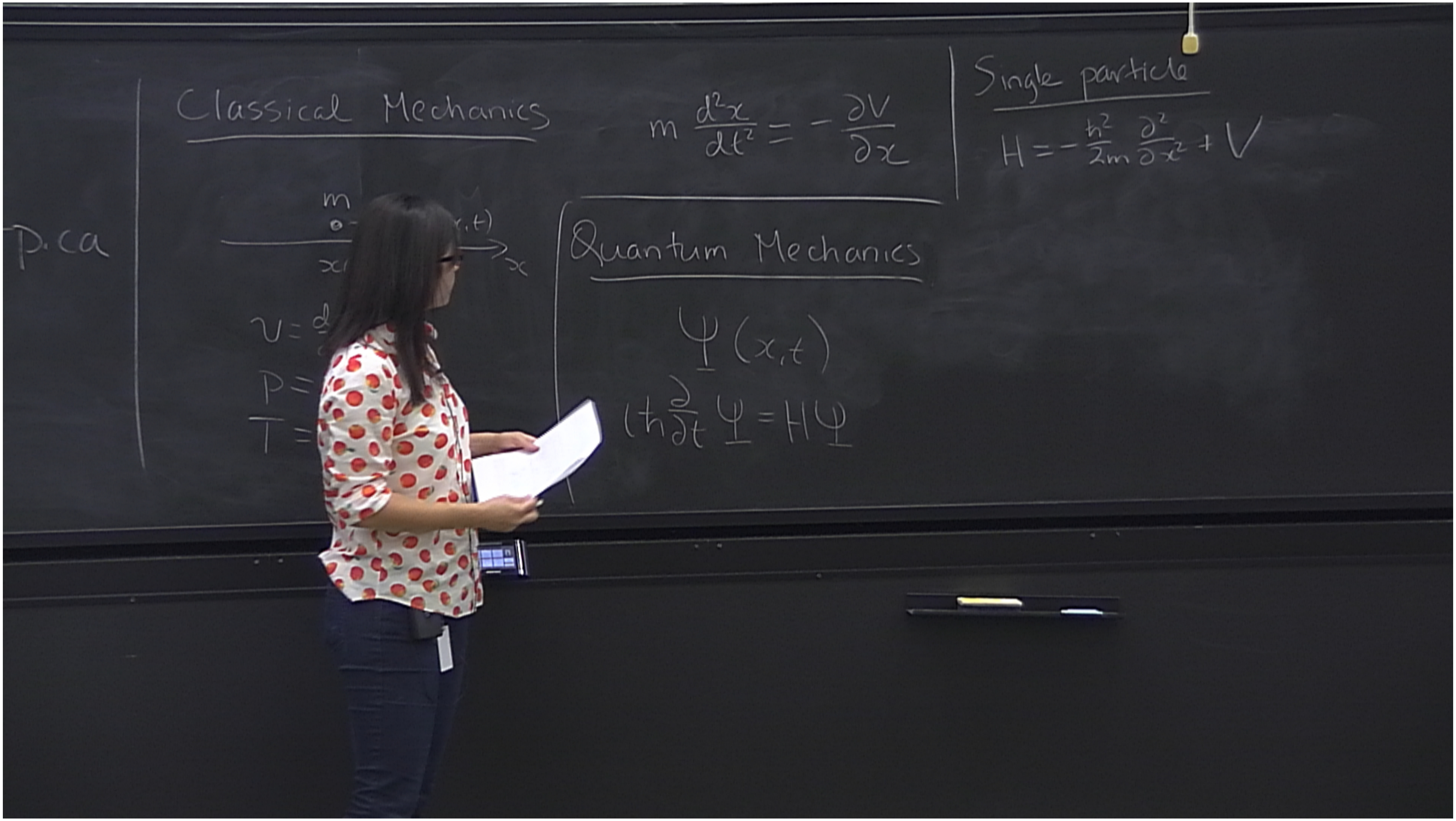

Aggie
abranczyk@pitp.ca
Classical Mechanics
m
F(x,t)
x(t)
x
v = dx/dt
p = mv
T = 1/2 mv^2

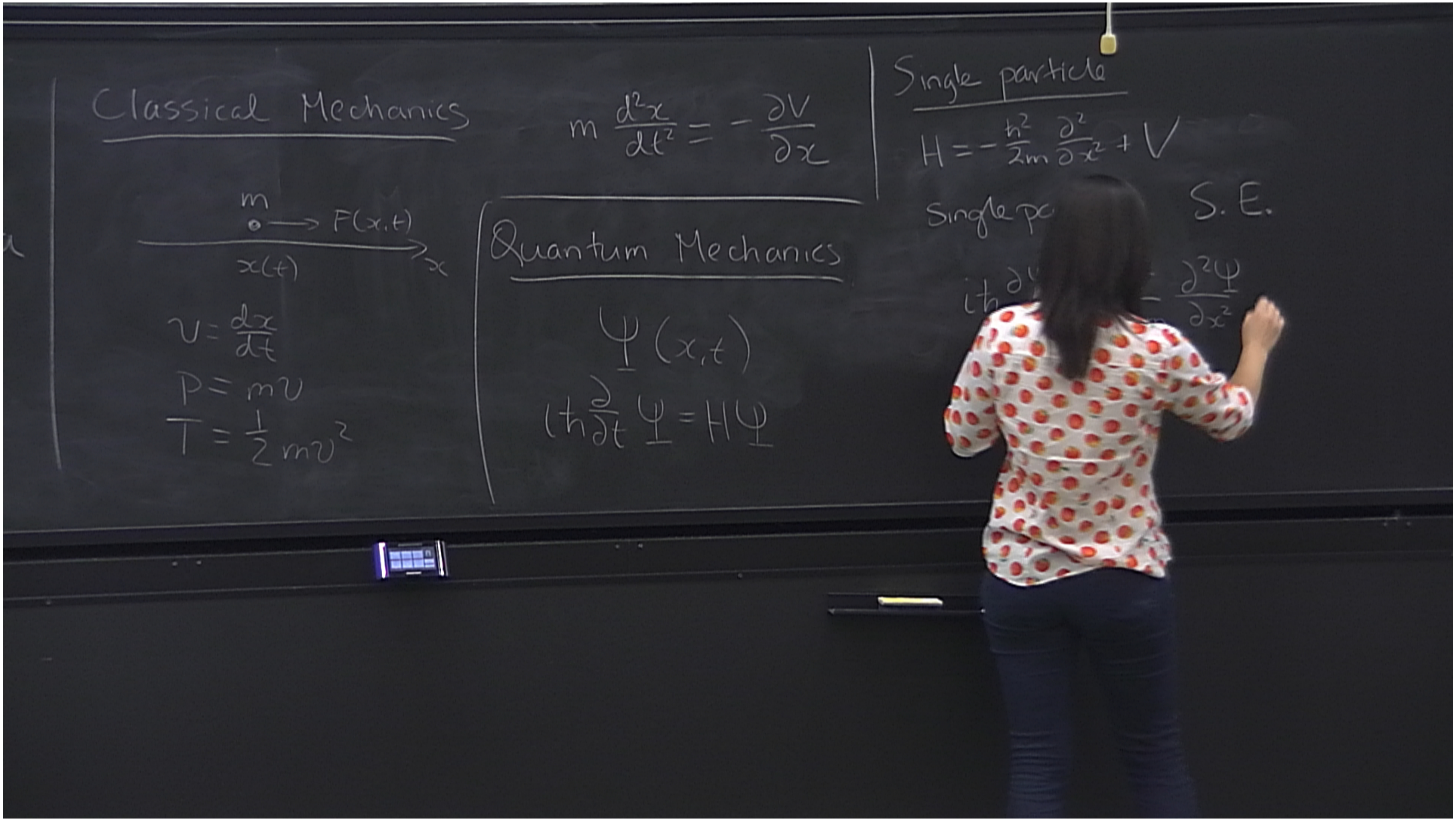

Classical Mechanics
m
m
F(x,t)
x(t)
v = dx/dt
p = mv
T = 1/2 mv^2

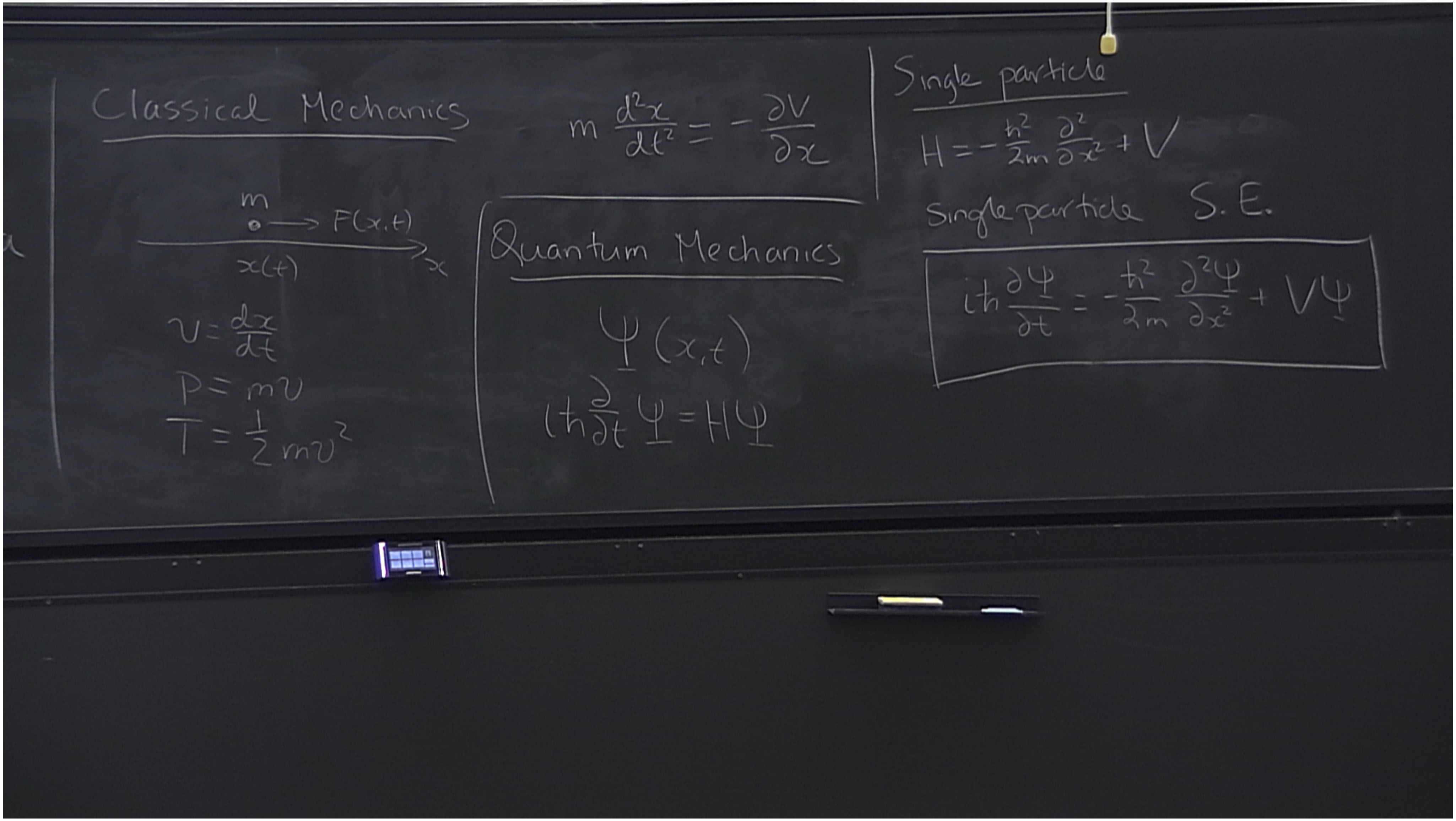

Classical Mechanics
m
F(x,t)
x(t)
x
v = dx/dt
p = mv
T = 1/2 mv^2
m d

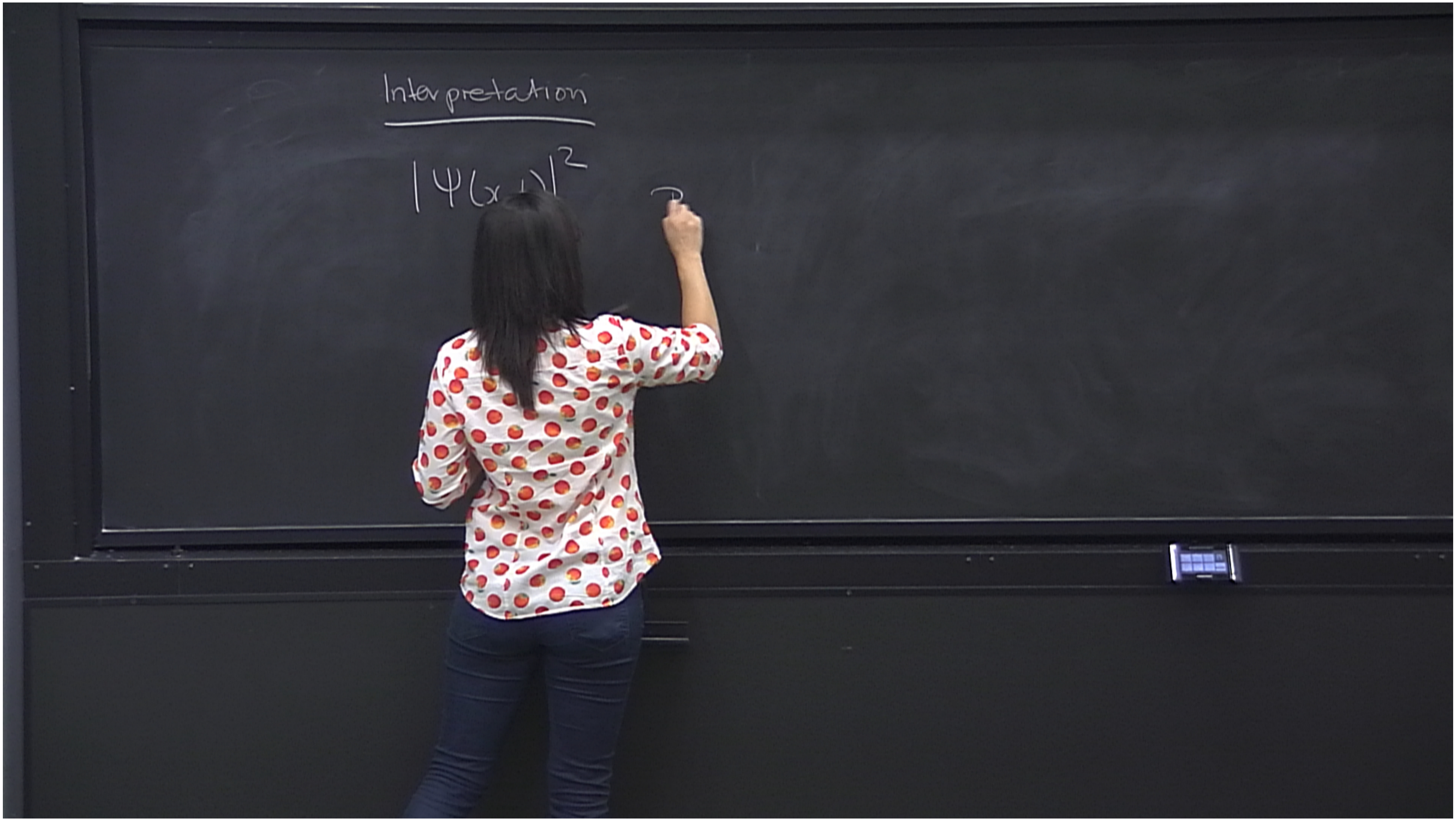

Classical Mechanics
m d²x/dt² = − ∂V/∂x
m
F(x,t)
x(t)
x
v = dx/dt
p = mv
T = 1/2 mv²

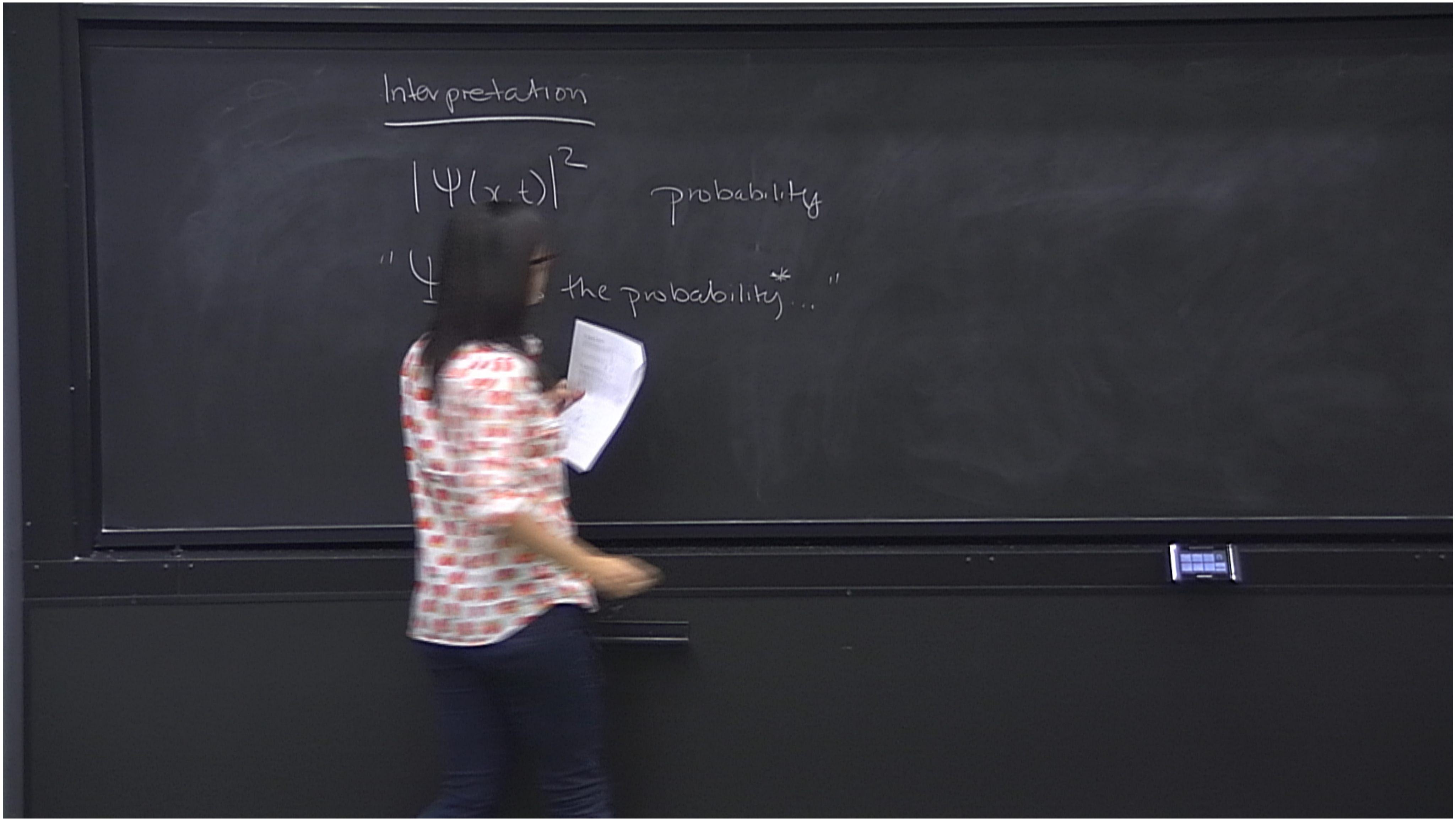

p.ca
Classical Mechanics
$m \frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x}$
m
F(x,t)
x(t)
x
$v = \frac{dx}{dt}$
$p = mv$
$T = \frac{1}{2}mv^2$
Quantum Mechanics
$\Psi(x,t)$

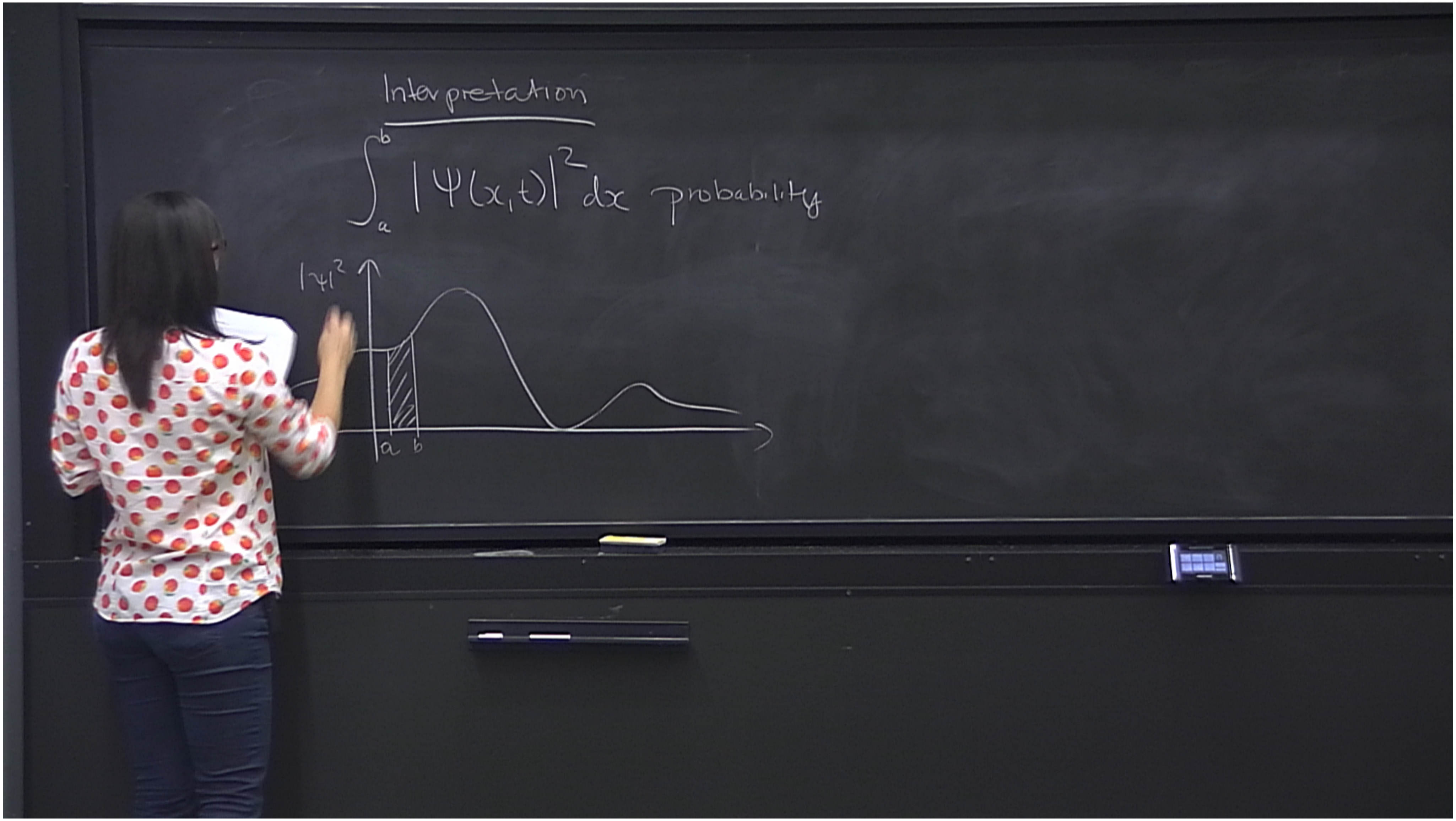

Classical Mechanics
m d²x/dt² = − ∂V/∂x
Quantum Mechanics
Ψ(x,t)
iħ ∂/∂t Ψ = HΨ
Single particle
H = − ħ²/2m ∂²/∂x² + V

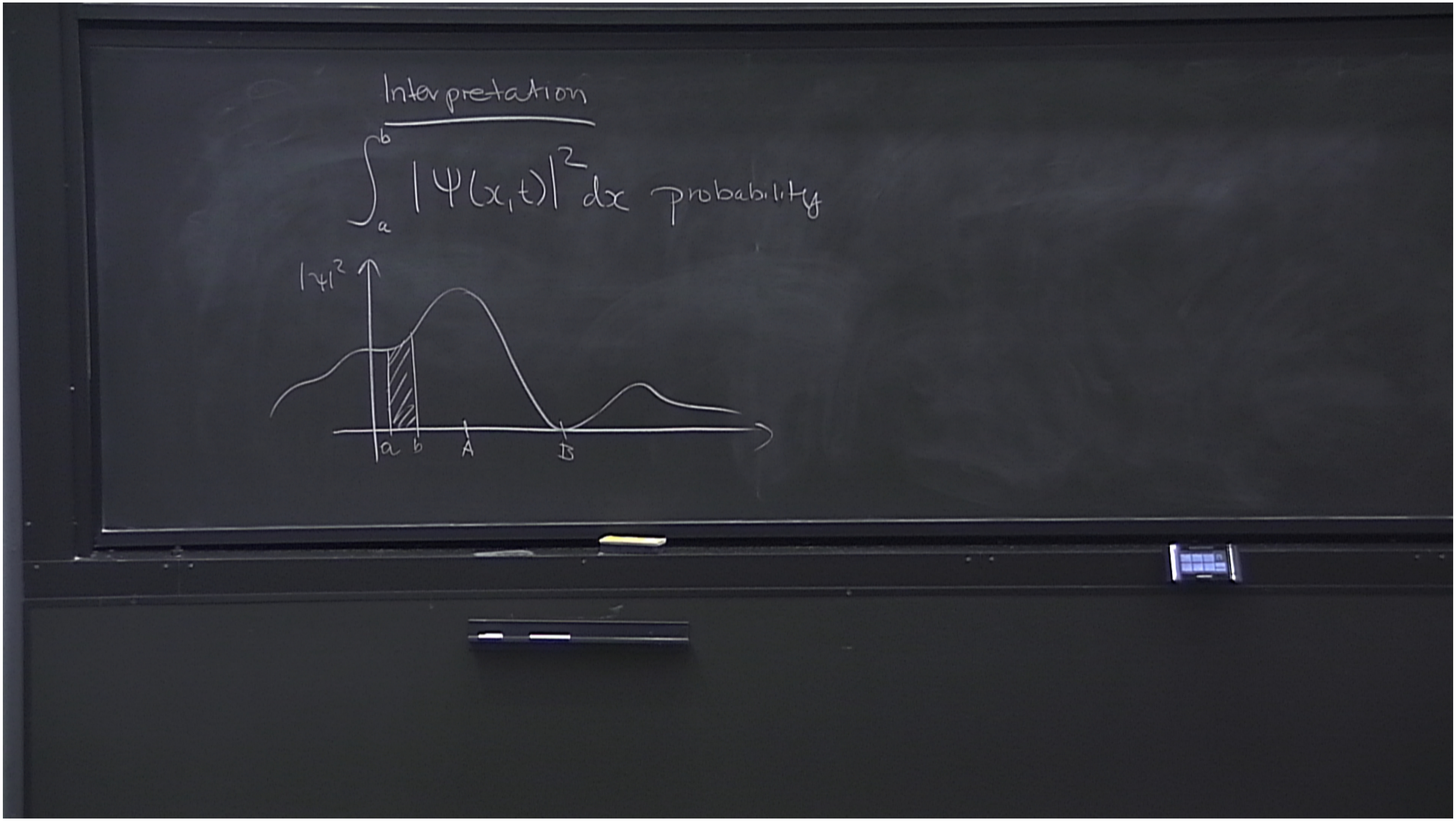

Classical Mechanics
m d²x/dt² = − ∂V/∂x
m
F(x,t)
x(t)
x
v = dx/dt
p = mv
T = 1/2 mv²
Quantum Mechanics
Ψ(x,t)
iħ ∂/∂t Ψ = HΨ
Single particle
H = − ħ²/2m ∂²/∂x² + V
S. E.

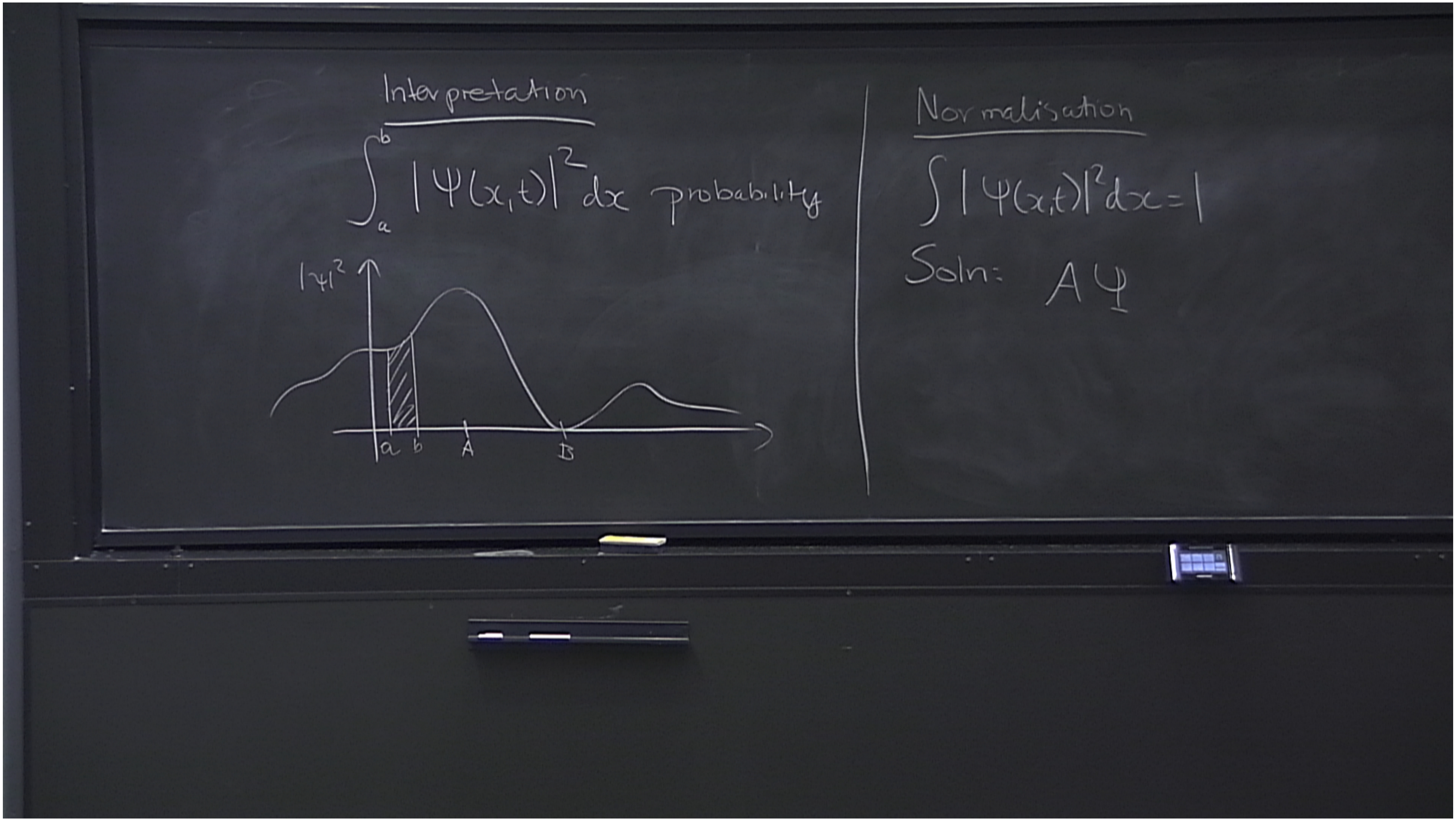

Classical Mechanics
$m \frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x}$
m
$F(x,t)$
$x(t)$
x
$v = \frac{dx}{dt}$
$p = mv$
$T = \frac{1}{2} m v^2$
Quantum Mechanics
$\Psi(x,t)$
$i\hbar \frac{\partial}{\partial t} \Psi = H \Psi$
Single particle
$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$
Single particle S. E.
$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$

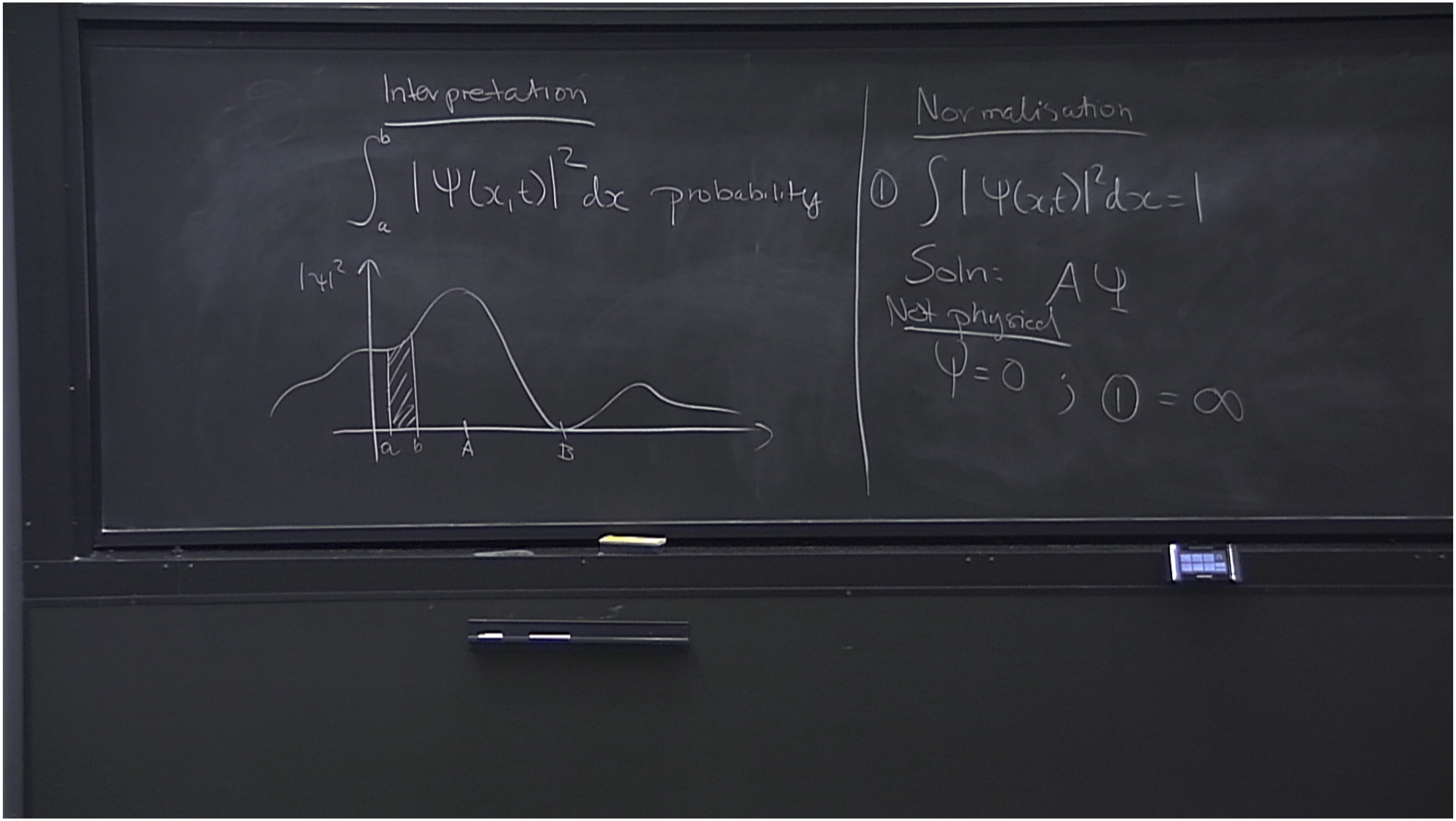

Interpretation

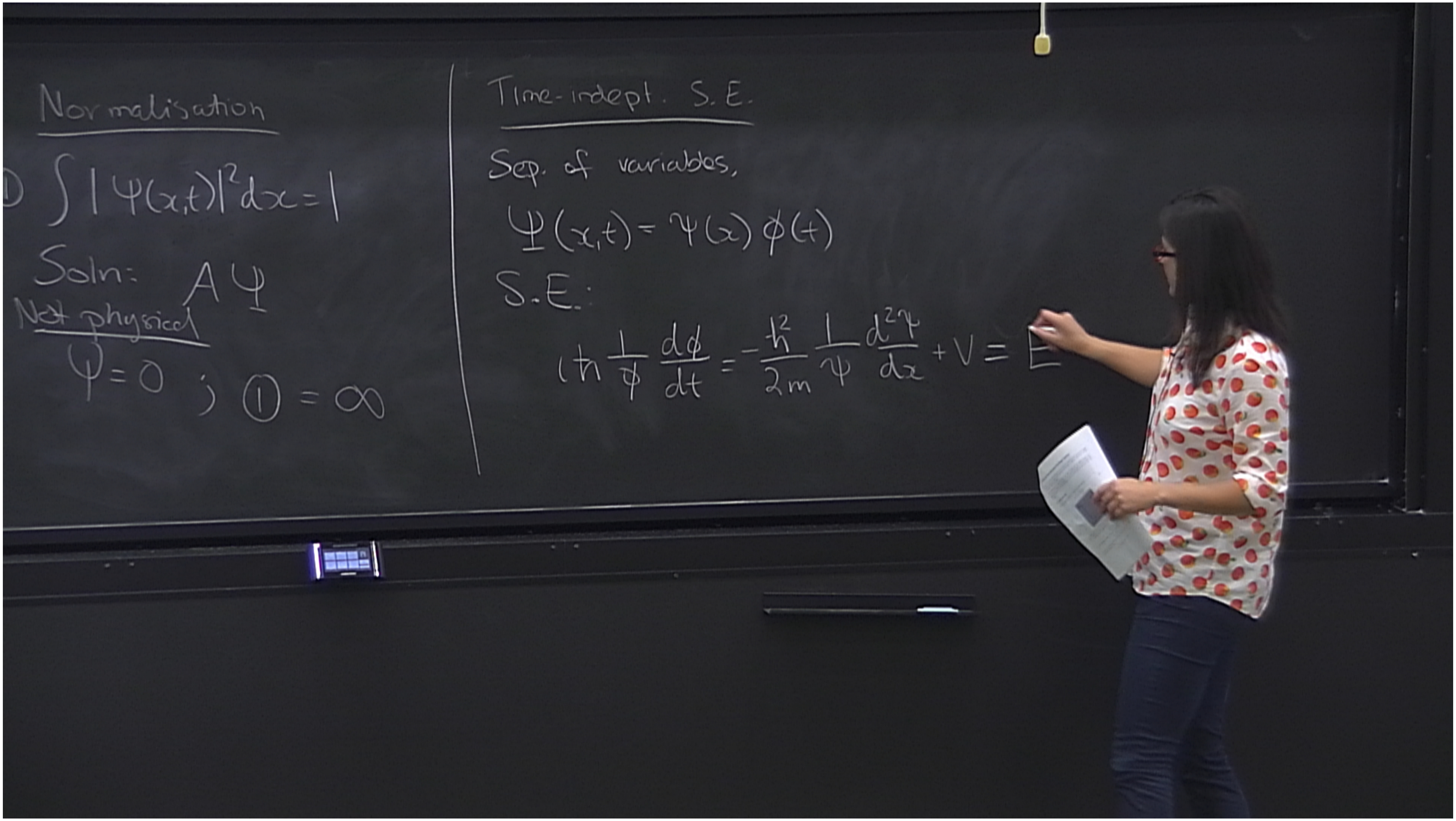

Interpretation
|Ψ(x,t)|²
probability
" Ψ
the probability* ... "

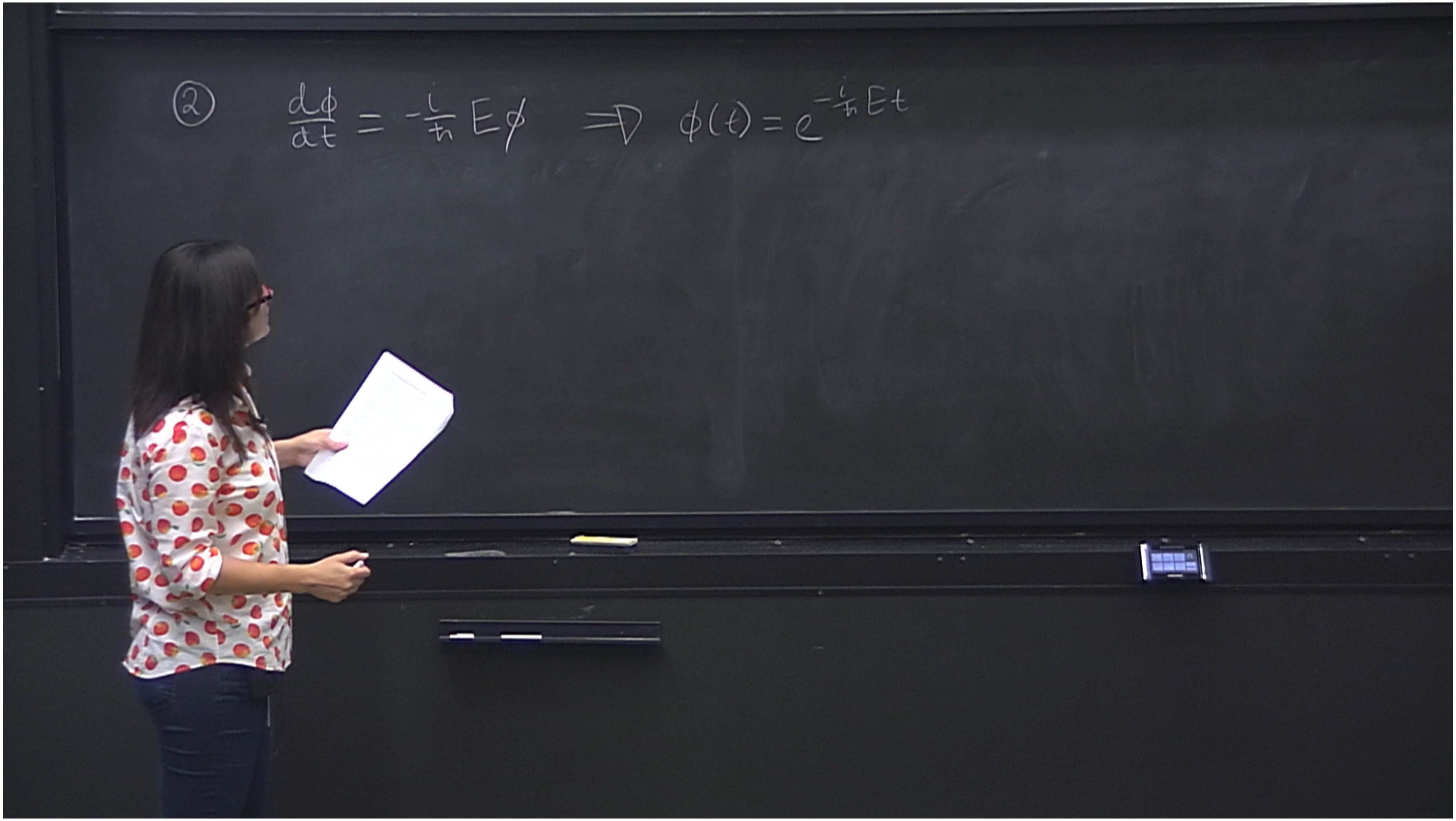

Interpretation
$\int_a^b |\Psi(x,t)|^2 dx$ probability
$|\Psi|^2$
a b

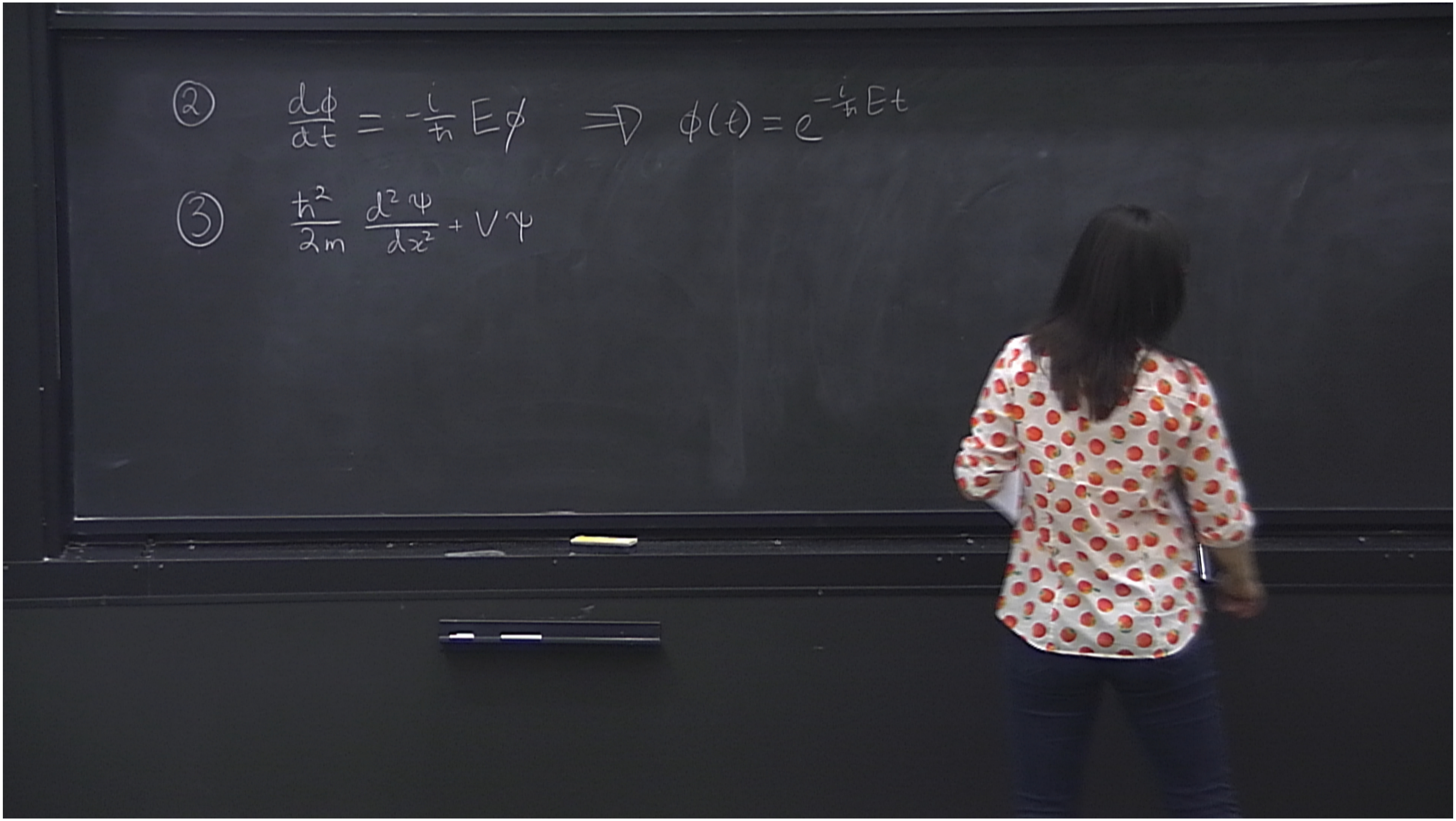

Interpretation
$\int_a^b |\Psi(x,t)|^2 dx$ probability
$|\psi|^2$
a
b
A
B

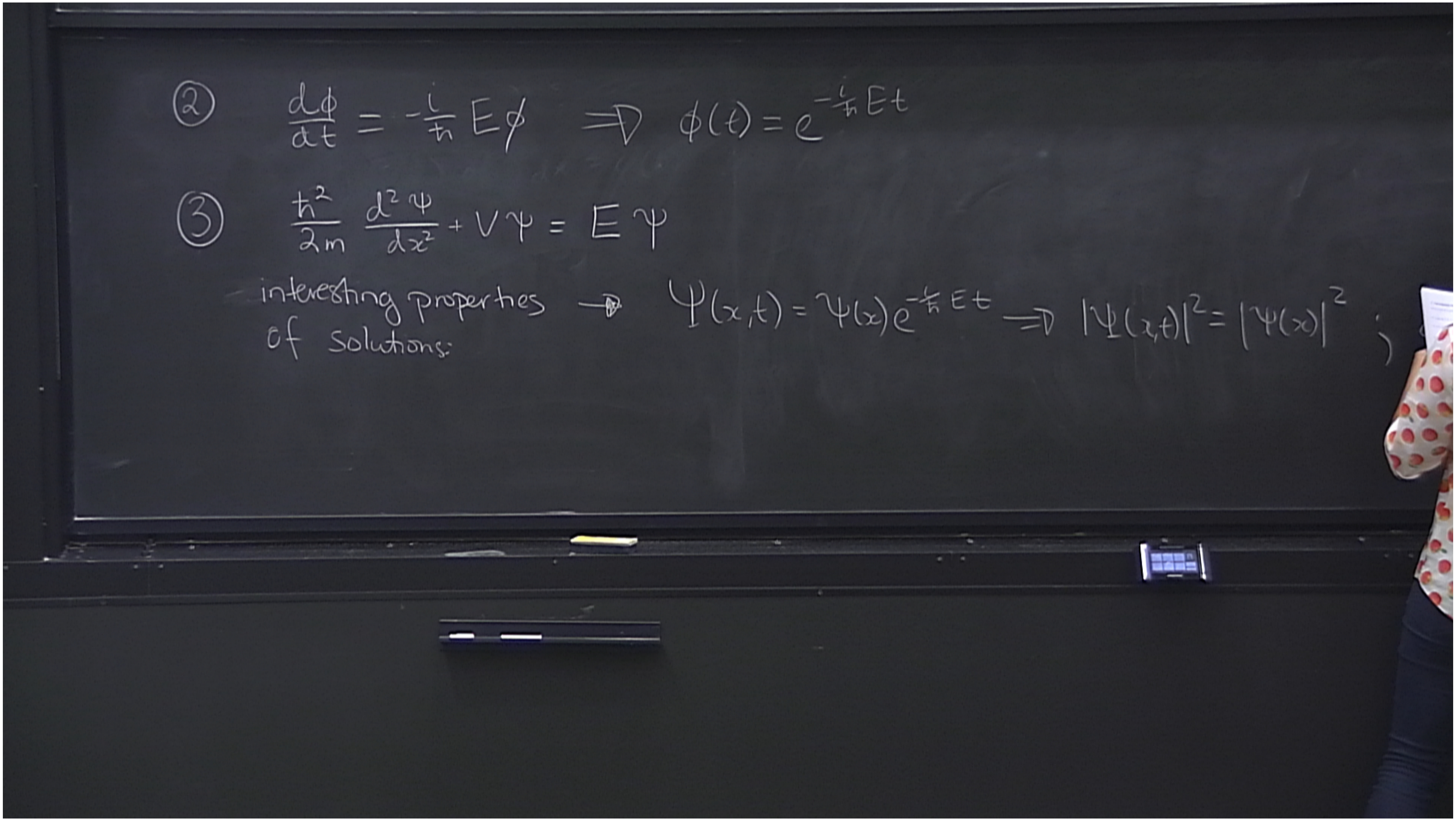

Interpretation
$\int_a^b |\Psi(x,t)|^2 dx$ probability
$|\Psi|^2$
a b A B
Normalisation
$\int |\Psi(x,t)|^2 dx = 1$
Soln: $A\Psi$

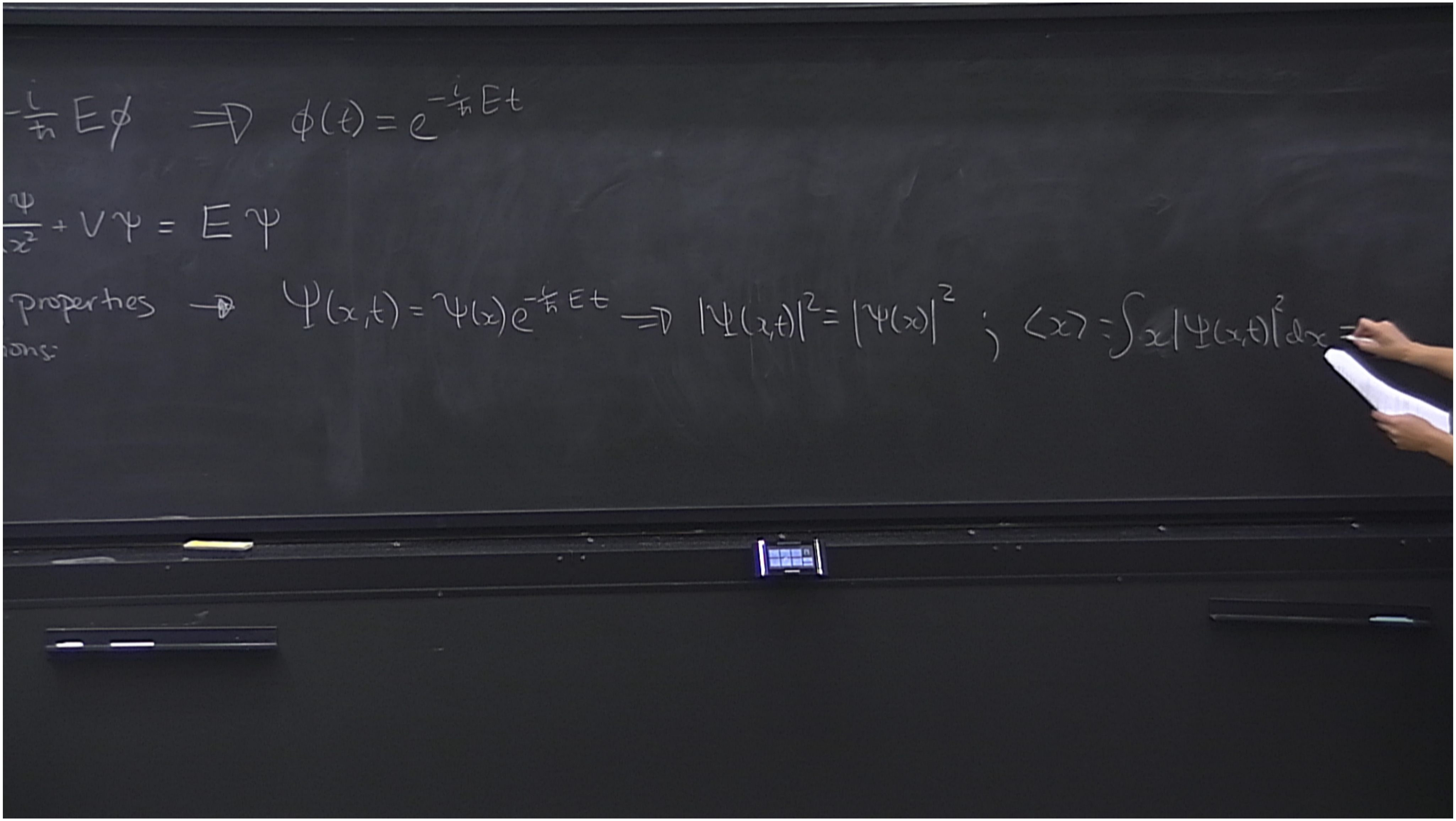

Interpretation
$\int_a^b |\Psi(x,t)|^2 dx$ probability
$|\psi|^2$
a b A B
Normalisation
① $\int |\Psi(x,t)|^2 dx = 1$
Soln: $A\Psi$
Not physical
$\Psi = 0$; ① $= \infty$

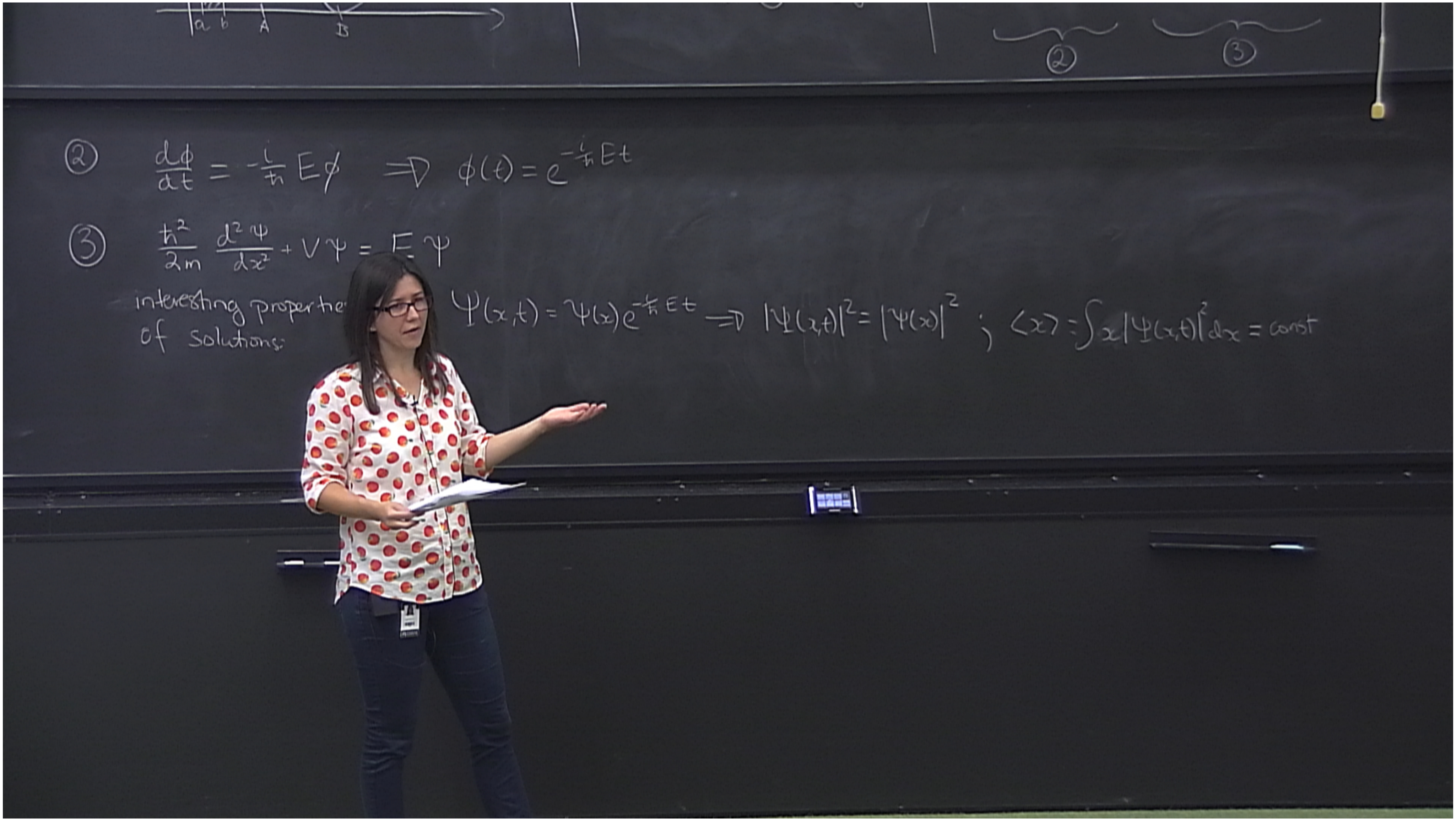

Normalisation
① $\int |\Psi(x,t)|^2 dx = 1$
Soln: $A\Psi$
Not physical
$\psi = 0$; ① $= \infty$
Time-indept. S.E.
Sep. of variables,
$\Psi(x,t) = \psi(x)\,\phi(t)$
S.E.:
$i\hbar \frac{1}{\phi}\frac{d\phi}{dt} = -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx} + V = E$

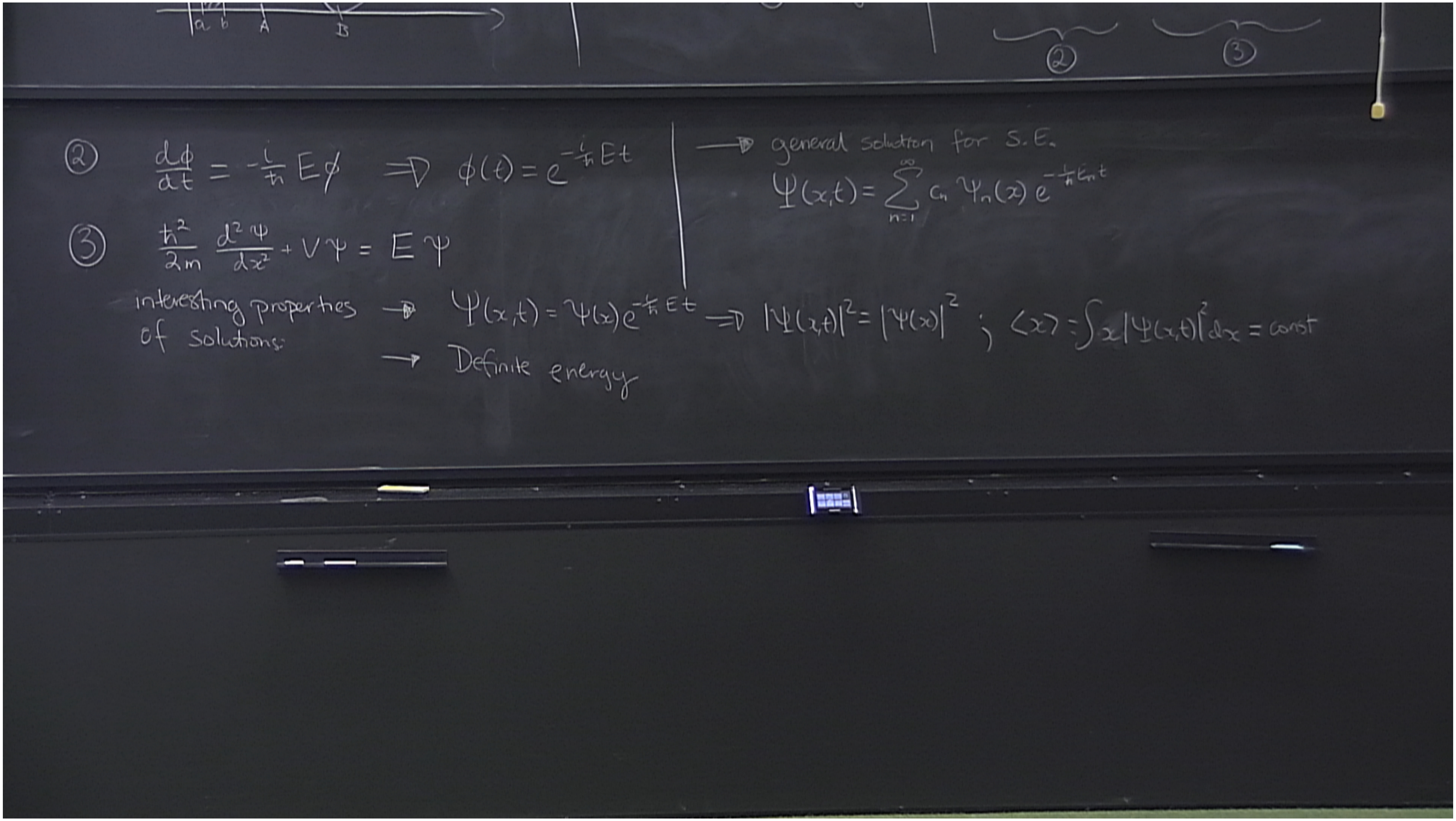

② $\frac{d\phi}{dt} = -\frac{i}{\hbar} E\phi \Rightarrow \phi(t) = e^{-\frac{i}{\hbar}Et}$


② $\frac{d\phi}{dt} = -\frac{i}{\hbar} E\phi \Rightarrow \phi(t) = e^{-\frac{i}{\hbar}Et}$
③ $\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi$

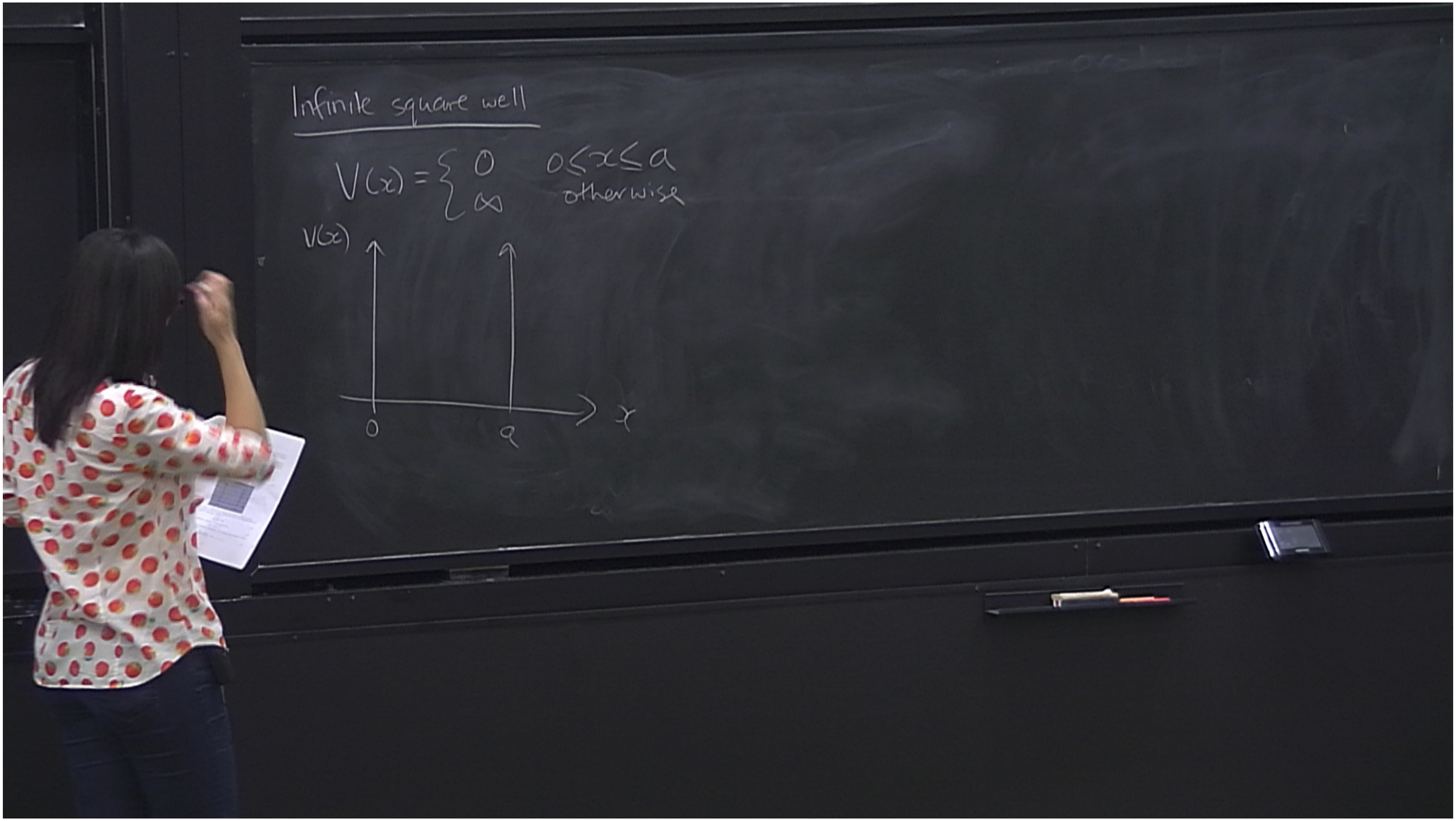

② $\frac{d\phi}{dt} = -\frac{i}{\hbar} E\phi \Rightarrow \phi(t) = e^{-\frac{i}{\hbar}Et}$
③ $\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$
interesting properties of solutions:
$\rightarrow \Psi(x,t) = \psi(x)e^{-\frac{i}{\hbar}Et} \Rightarrow |\Psi(x,t)|^2 = |\psi(x)|^2$;

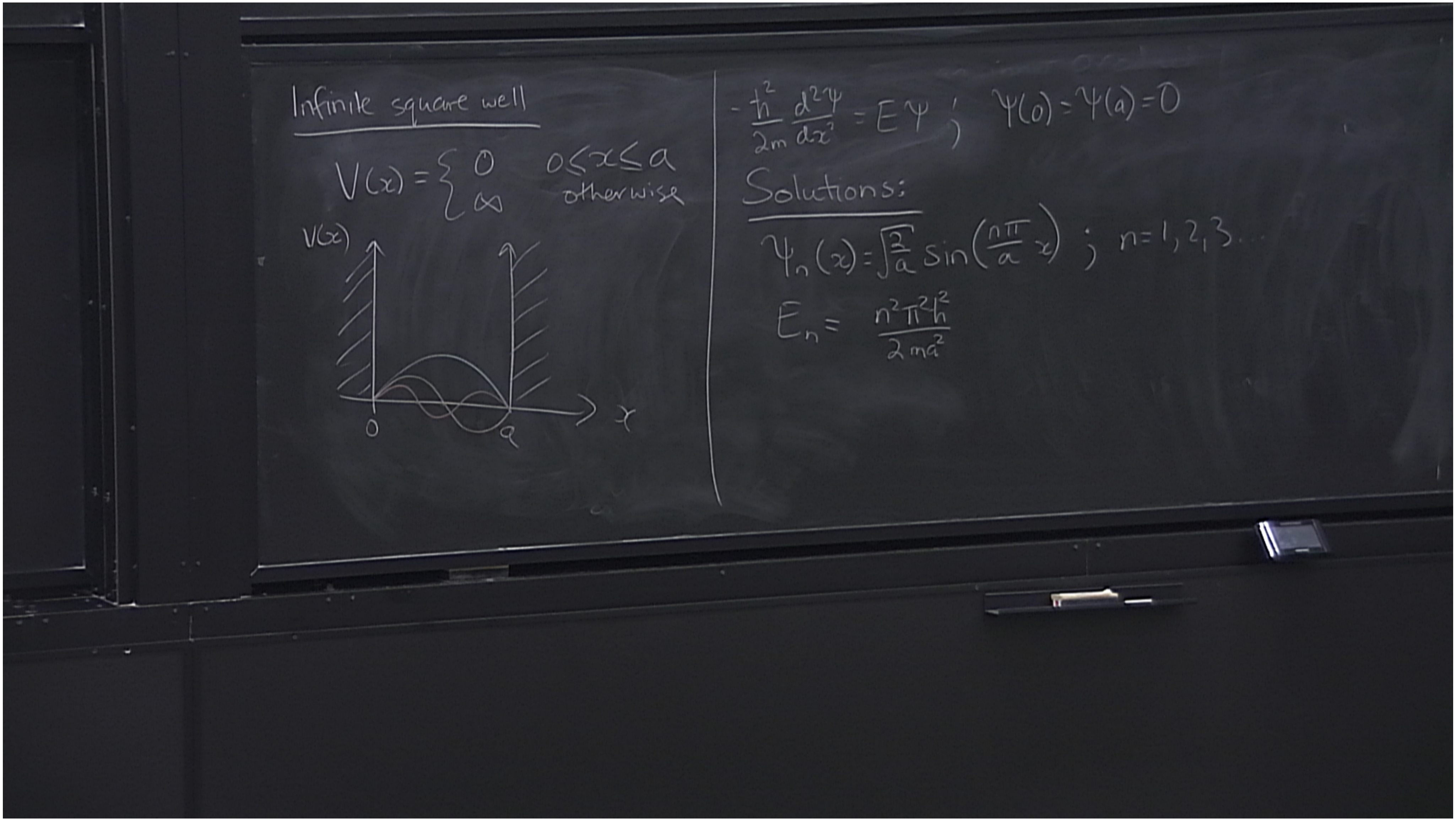

-i/ħ Eφ ⇒ φ(t) = e^{-i/ħ Et}
ψ/x² + Vψ = Eψ
properties → Ψ(x,t) = ψ(x)e^{-i/ħ Et} ⇒ |Ψ(x,t)|² = |ψ(x)|² ; ⟨x⟩ = ∫x|Ψ(x,t)|²dx =
ons:

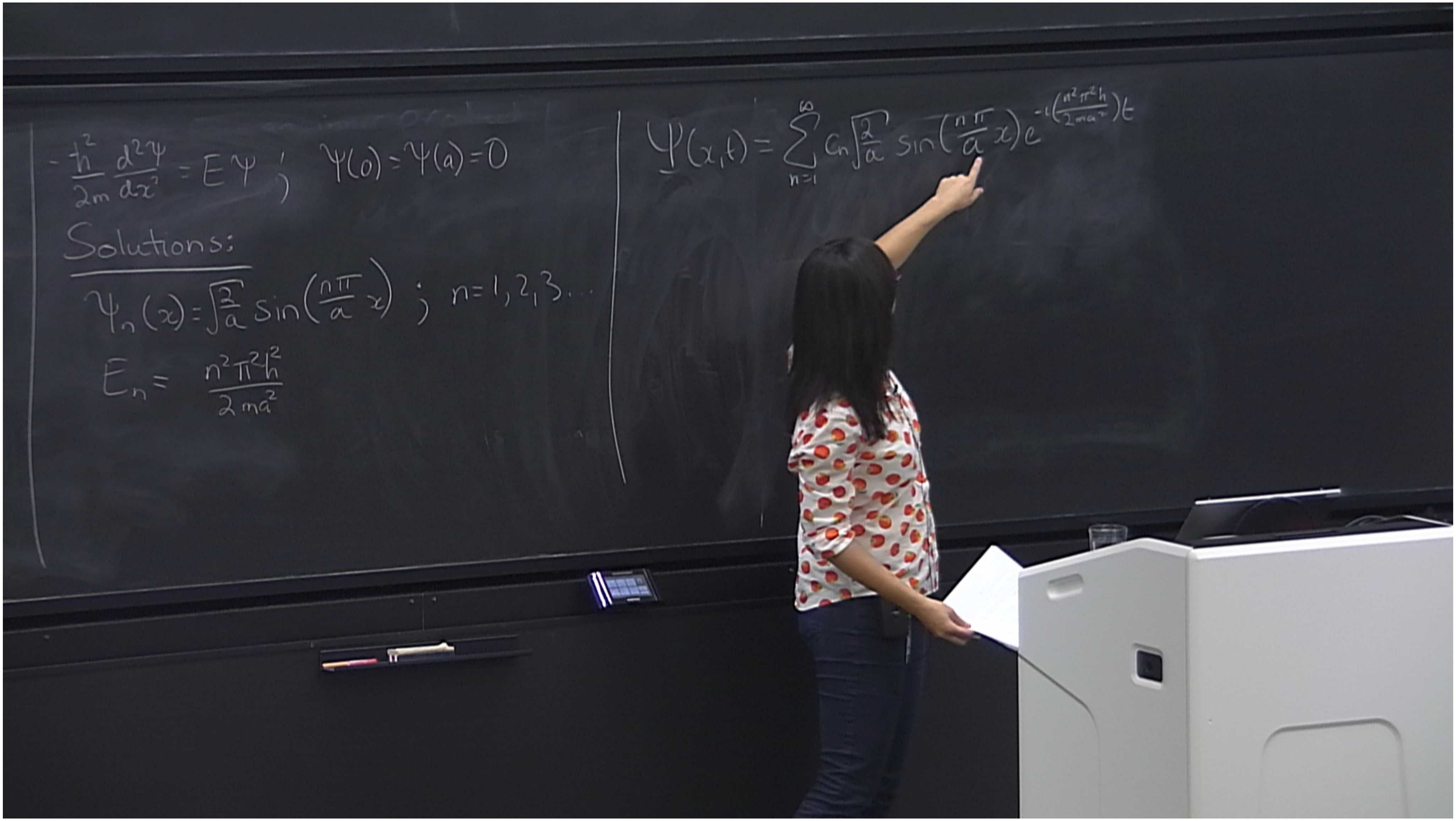

a b A B
② ③
② dφ/dt = -i/ħ Eφ ⇒ φ(t) = e^(-i/ħ Et)
③ ħ²/2m d²ψ/dx² + Vψ = Eψ
interesting properties of solutions:
Ψ(x,t) = ψ(x)e^(-i/ħ Et) ⇒ |Ψ(x,t)|² = |ψ(x)|² ; <x> = ∫x|Ψ(x,t)|² dx = const

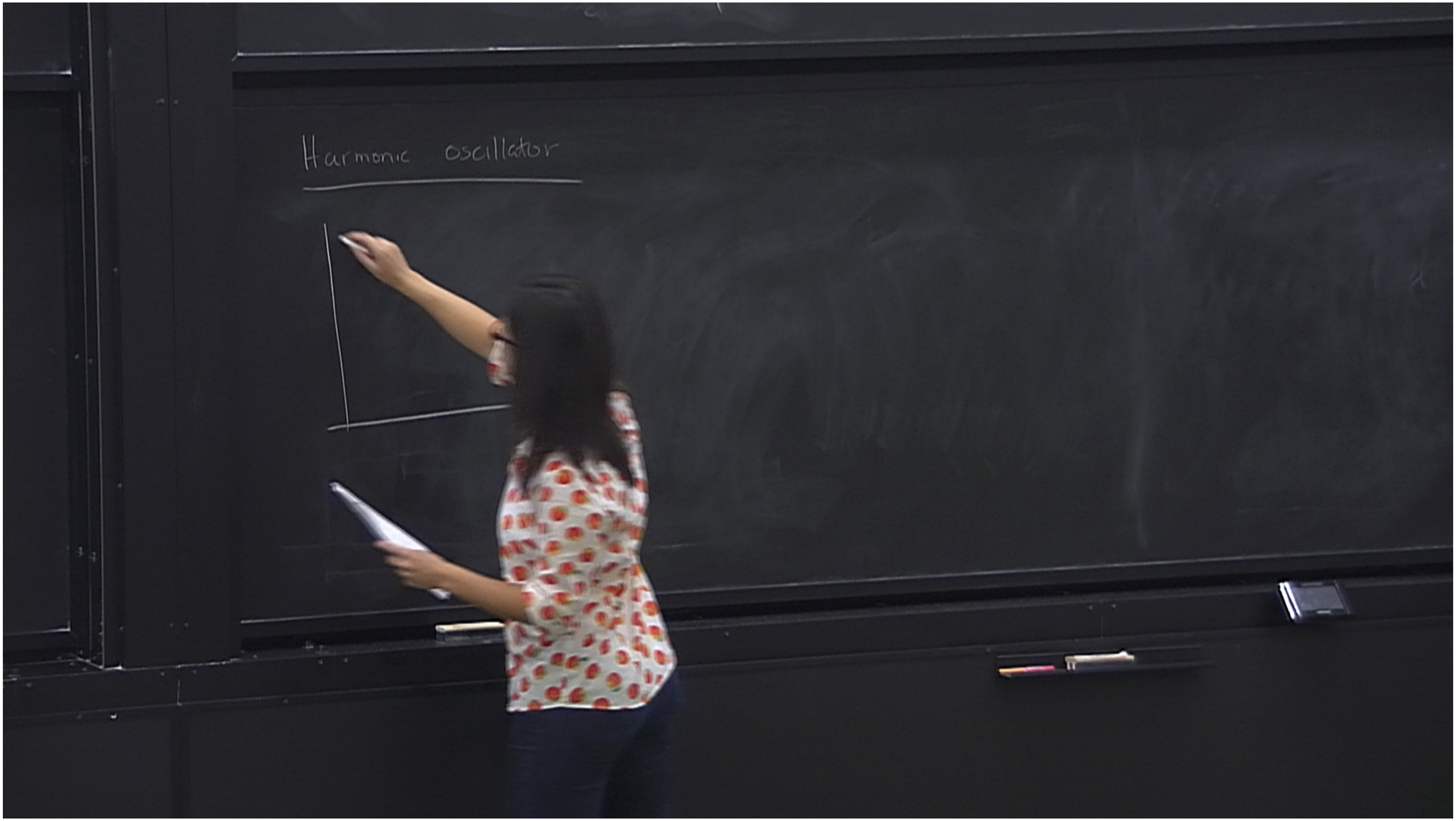


② $\frac{d\phi}{dt} = -\frac{i}{\hbar} E \phi \Rightarrow \phi(t) = e^{-\frac{i}{\hbar}Et}$
③ $\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = E\psi$
→ general solution for S.E.
$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-\frac{i}{\hbar}E_n t}$
interesting properties of solutions:
→ $\Psi(x,t) = \psi(x) e^{-\frac{i}{\hbar}Et} \Rightarrow |\Psi(x,t)|^2 = |\psi(x)|^2$; $\langle x \rangle = \int x |\Psi(x,t)|^2 dx = \text{const}$
→ Definite energy

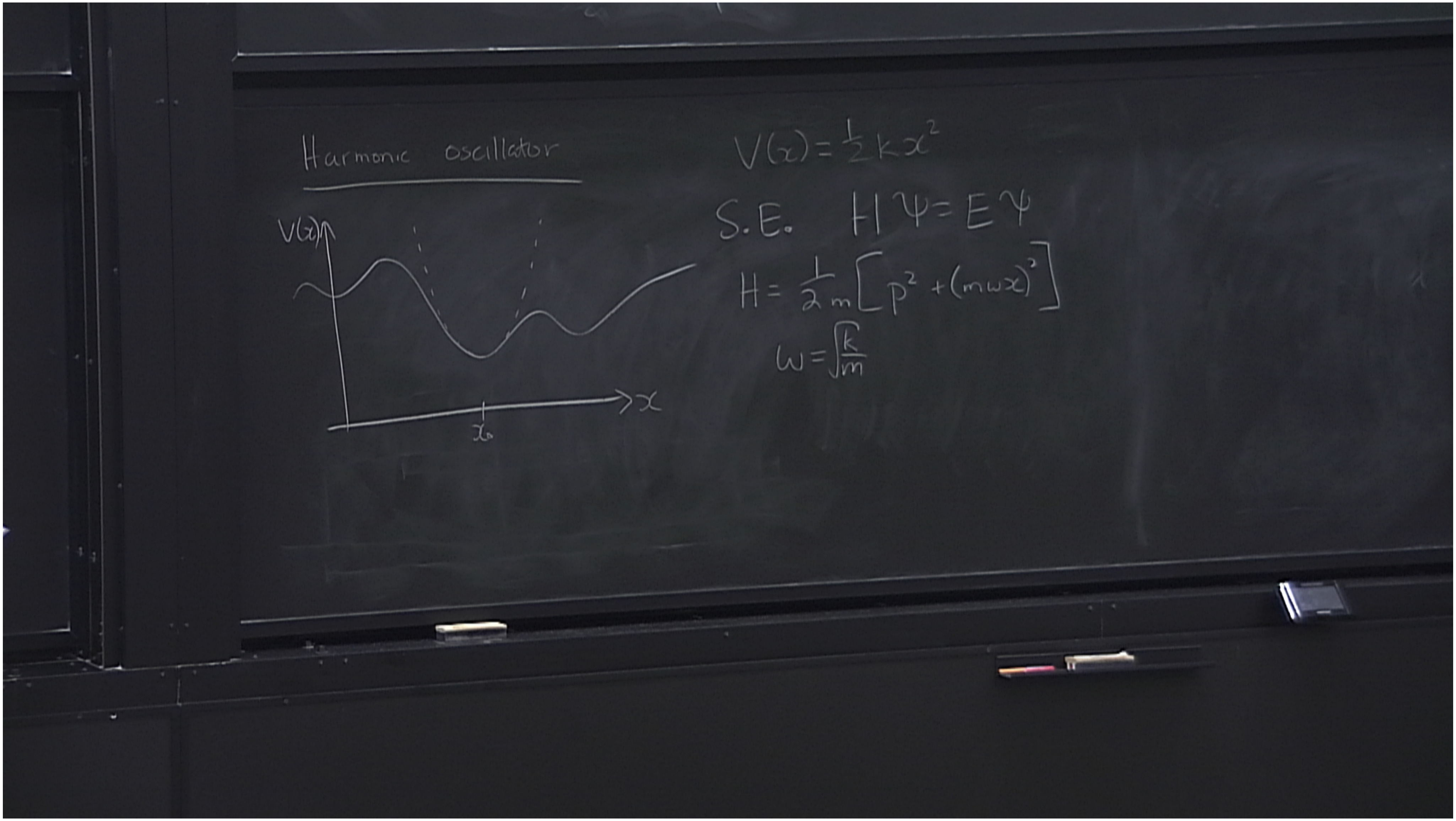

Infinite square well
V(x) = { 0 0≤x≤a
∞ otherwise
V(x)
0
a
x

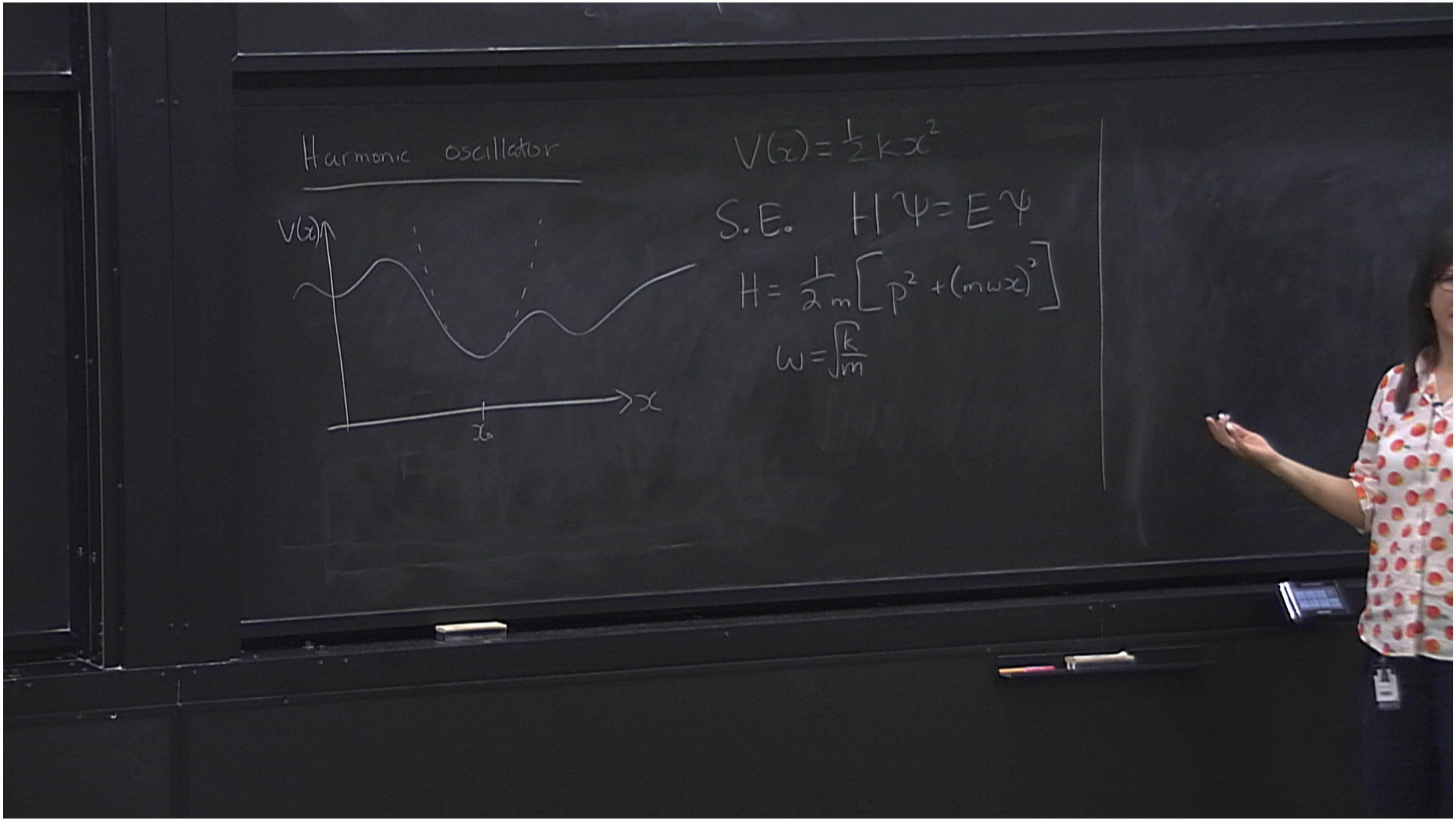

Infinite square well
V(x) = { 0   0 ≤ x ≤ a
∞   otherwise
V(x)
0
a
x
- ħ²/2m d²ψ/dx² = Eψ ; ψ(0) = ψ(a) = 0
Solutions:
ψn(x) = √(2/a) sin(nπ/a x) ; n = 1, 2, 3 ...
En = n²π²ħ²/2ma²

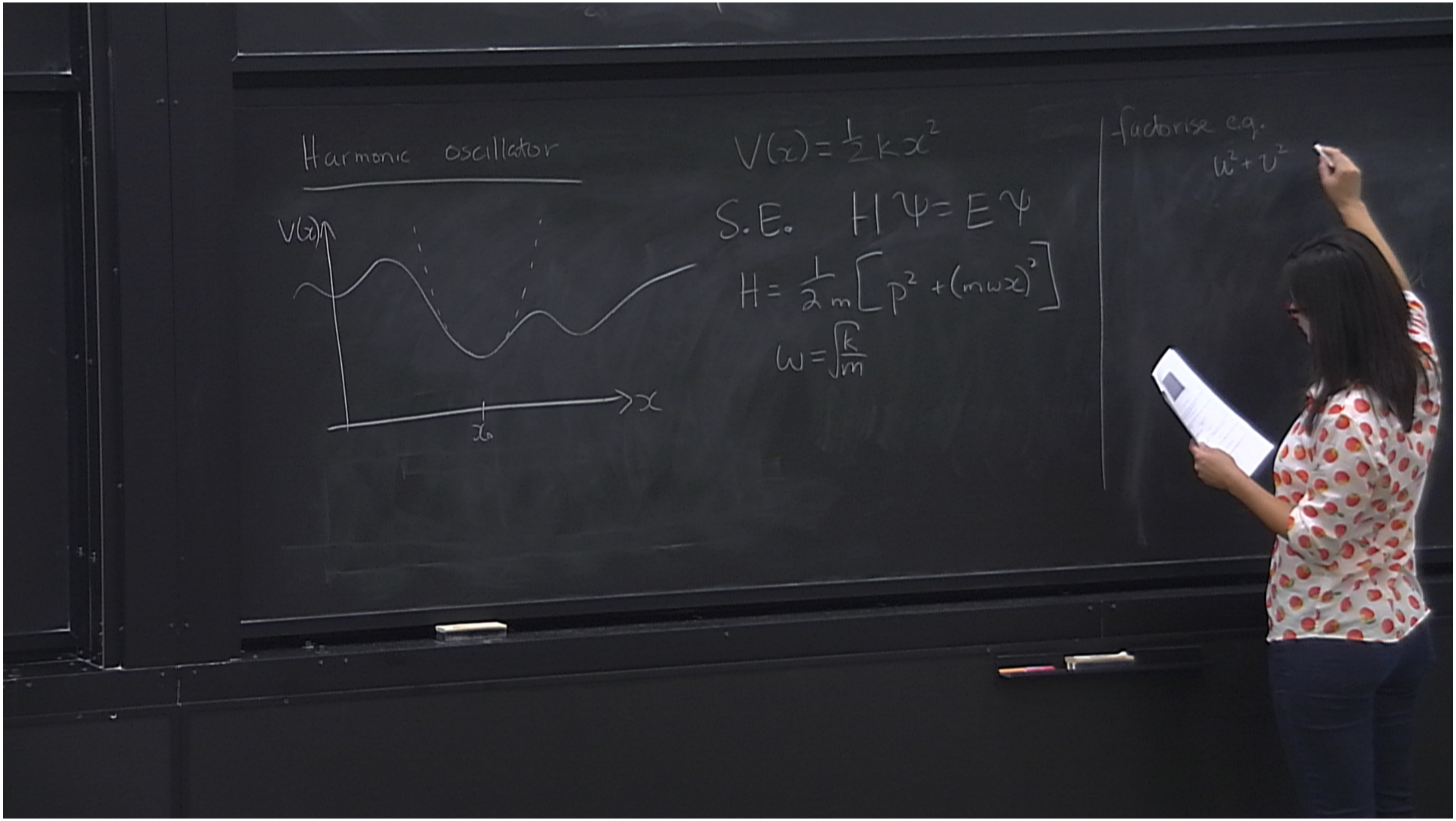

$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$; $\psi(0) = \psi(a) = 0$
Solutions:
$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$; $n = 1, 2, 3 \ldots$
$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$
$\Psi(x,t) = \sum_{n=1}^{\infty} c_n\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)e^{-i\left(\frac{n^2\pi^2\hbar}{2ma^2}\right)t}$

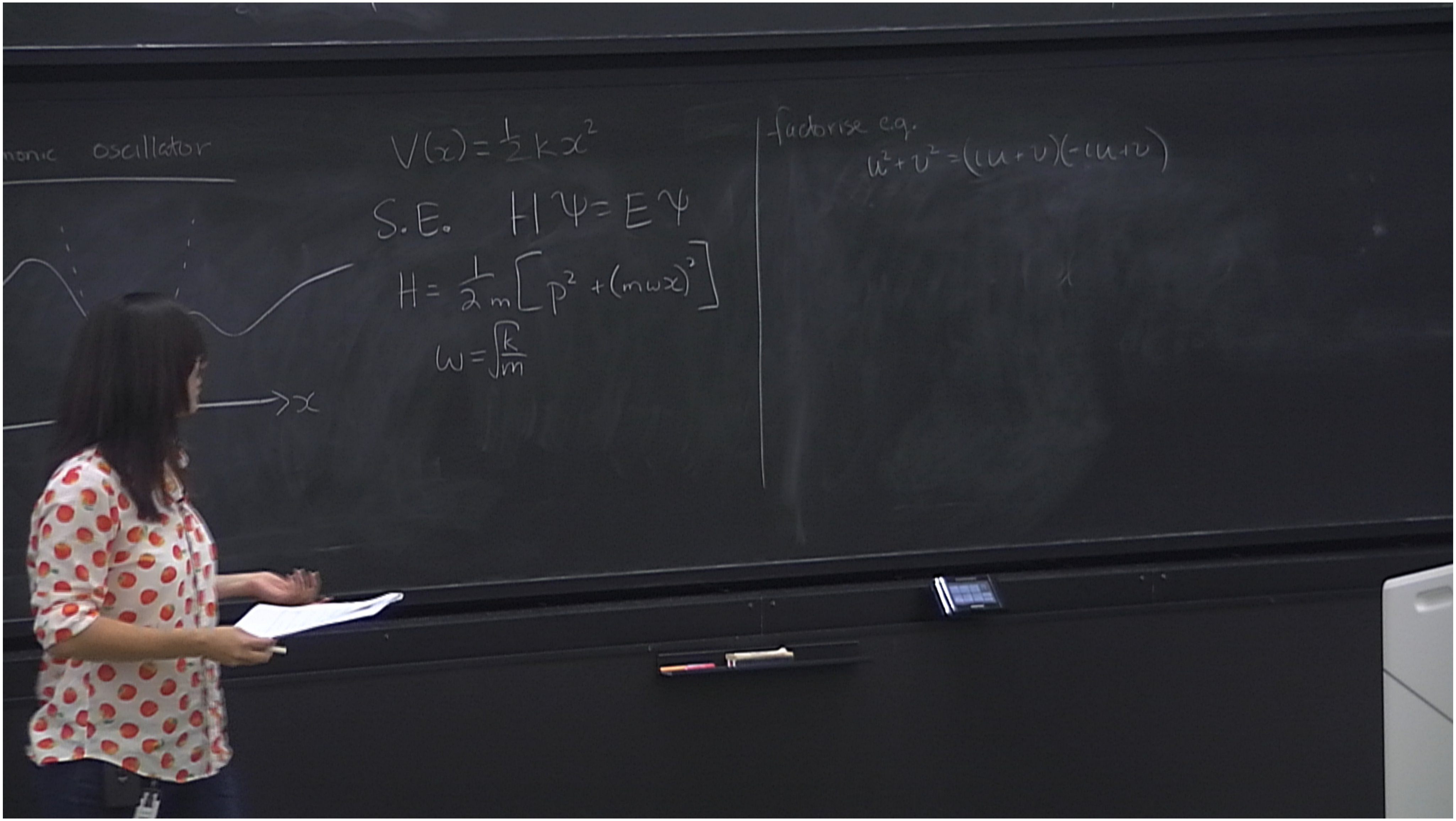

Harmonic oscillator

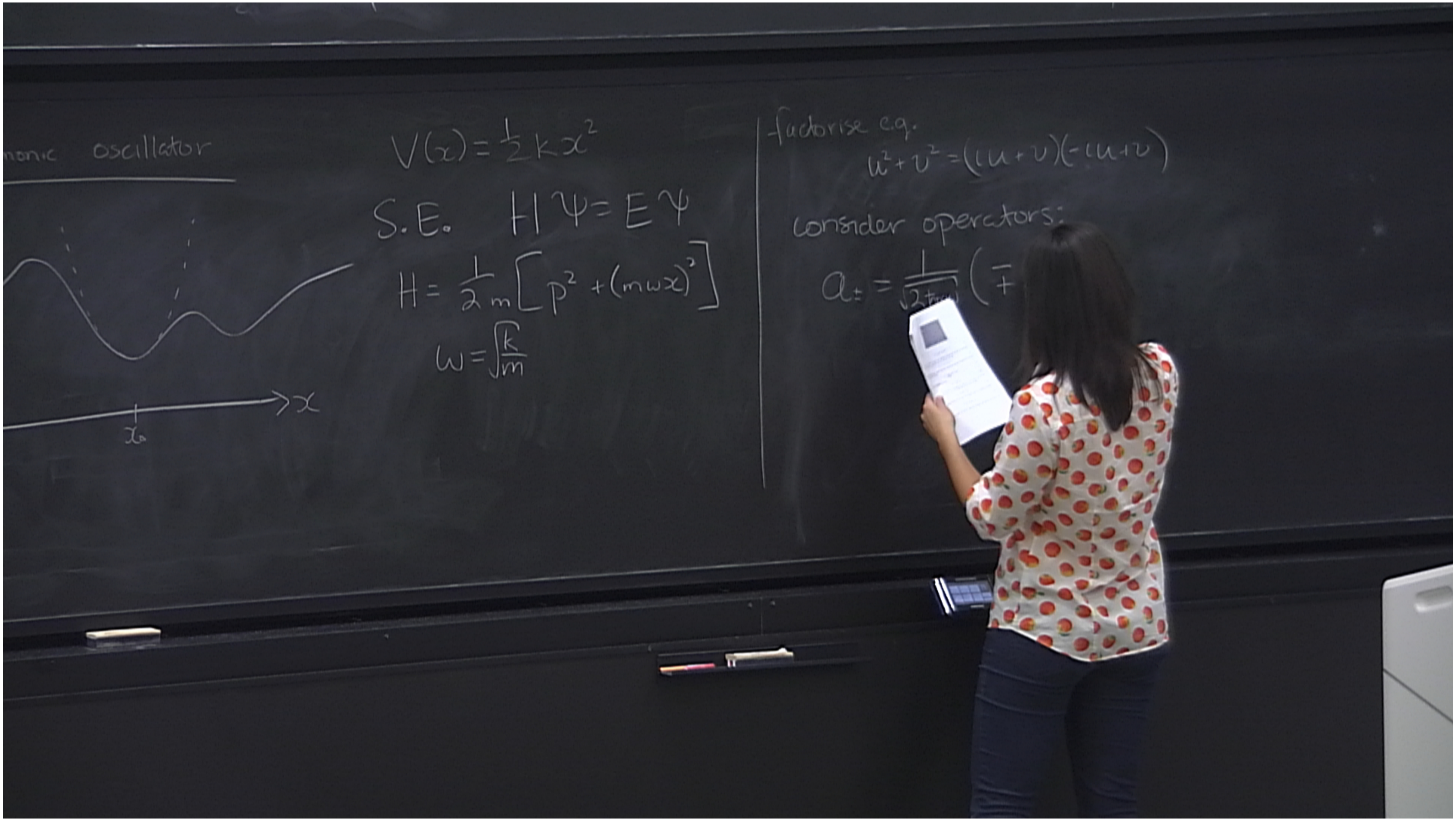

Harmonic oscillator
V(x)
x
x₀

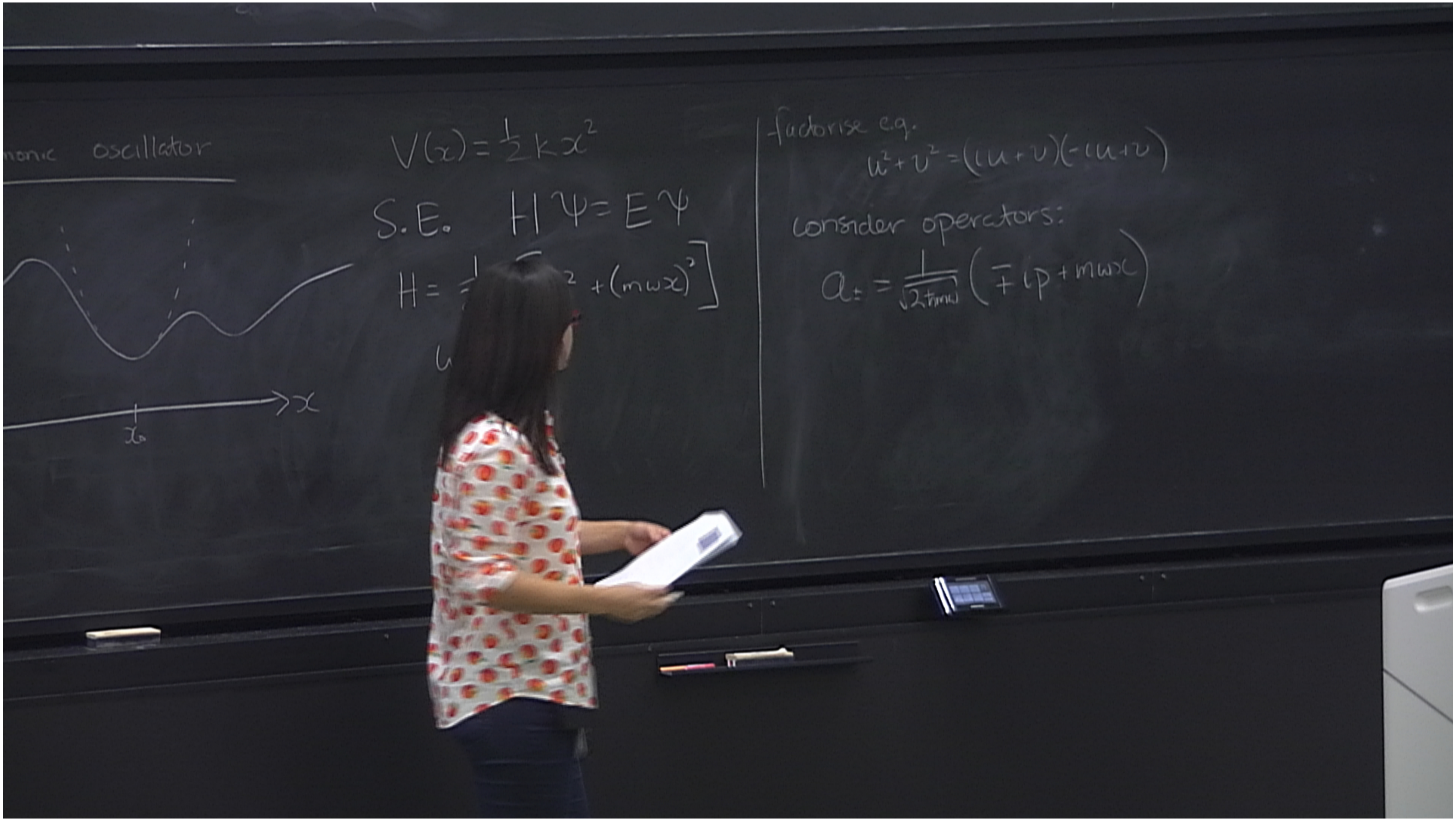

Harmonic oscillator
V(x)
x
x₀
$V(x) = \frac{1}{2} k x^2$
S.E. $H\Psi = E\Psi$
$H = \frac{1}{2m}\left[p^2 + (m\omega x)^2\right]$
$\omega = \sqrt{\frac{k}{m}}$

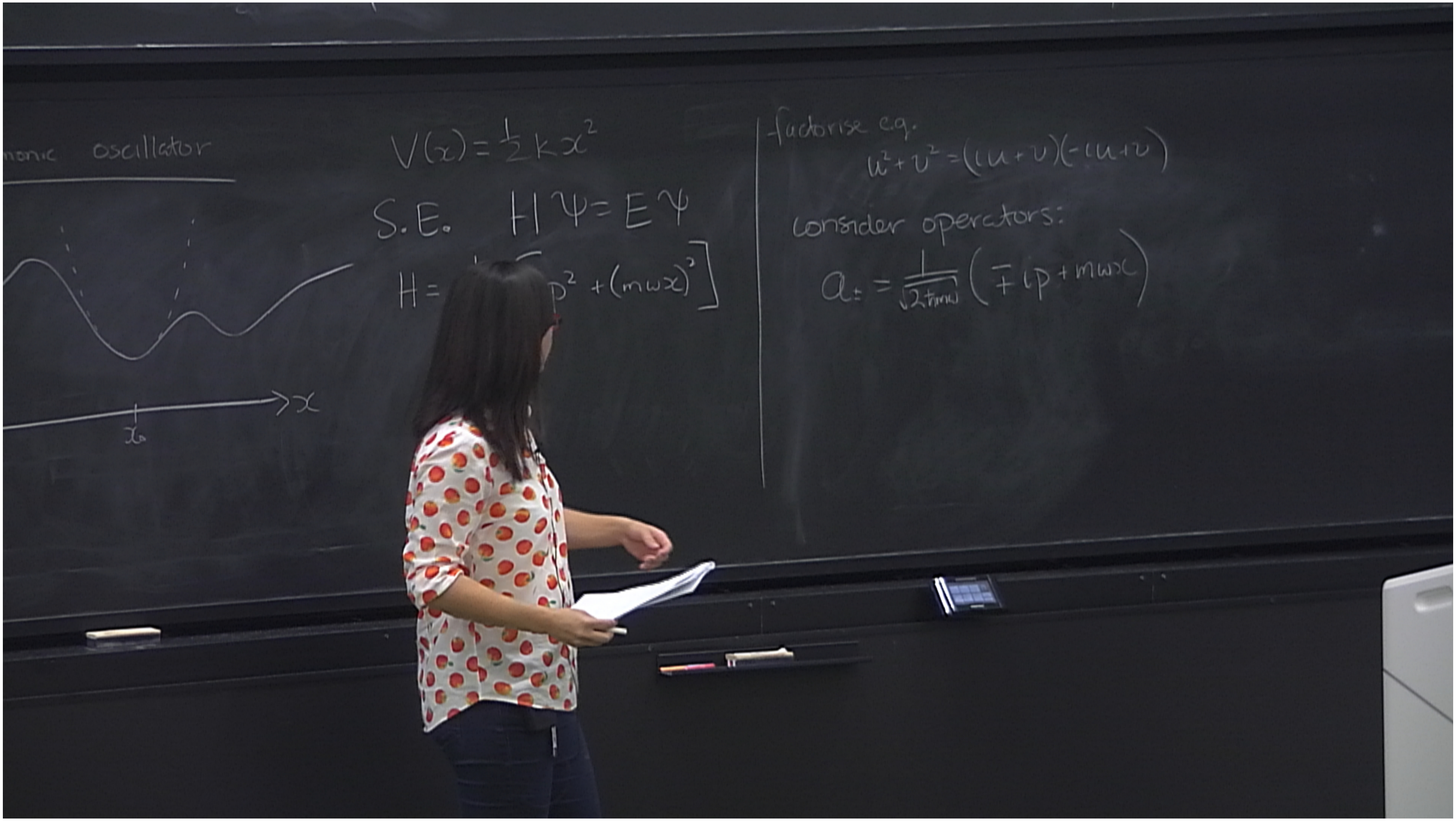

Harmonic oscillator
$V(x) = \frac{1}{2} k x^2$
S.E. $H\Psi = E\Psi$
$H = \frac{1}{2m}\left[p^2 + (m\omega x)^2\right]$
$\omega = \sqrt{\frac{k}{m}}$
$V(x)$
x
x_0

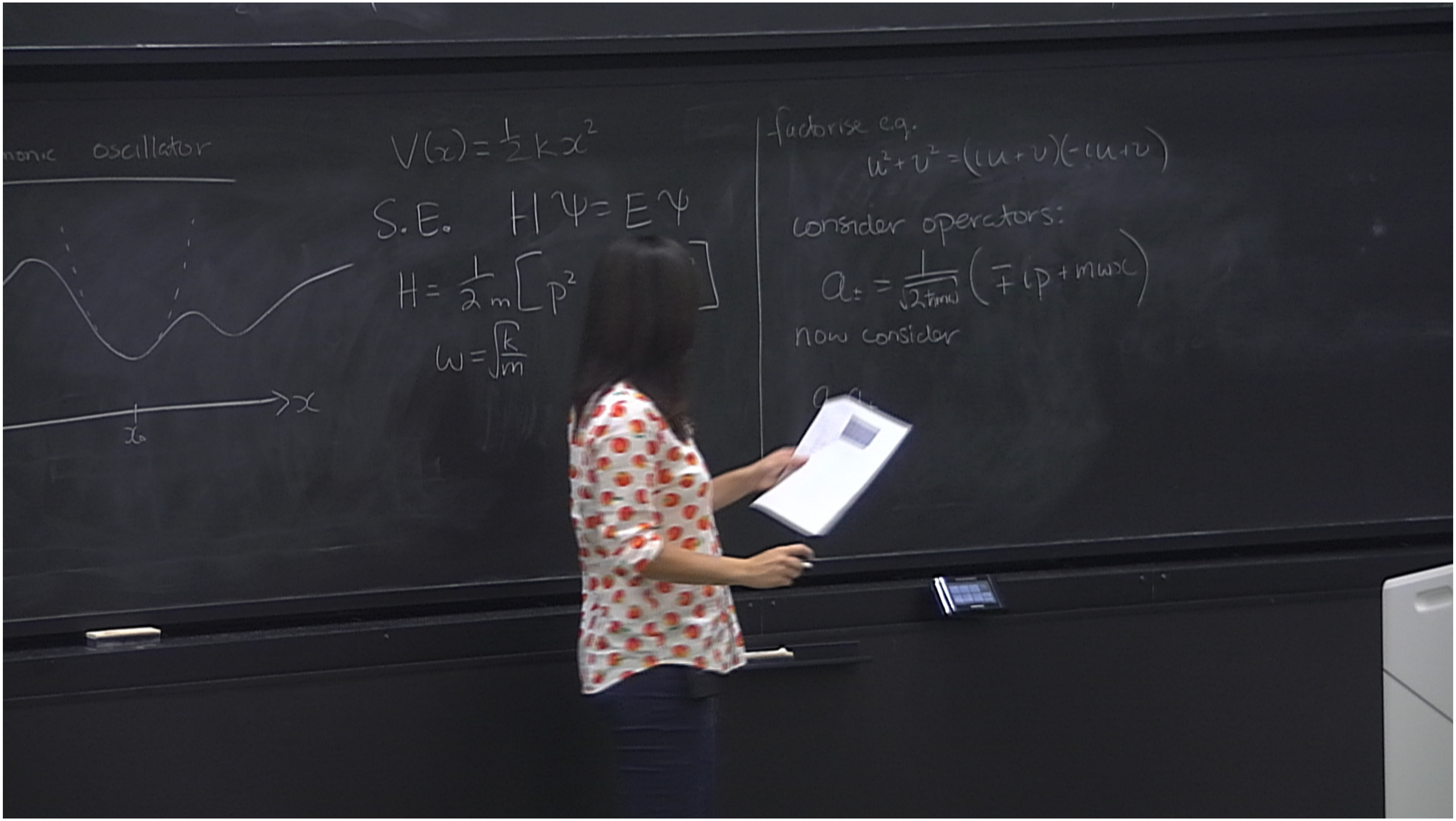

Harmonic oscillator
V(x)
x
x₀
V(x) = ½ k x²
S.E. HΨ = EΨ
H = 1/2m [p² + (mωx)²]
ω = √(k/m)
factorise e.g.
u² + v²


monic oscillator
V(x) = ½ k x²
S.E. HΨ = EΨ
H = 1/2m [p² + (mωx)²]
ω = √(k/m)
x
factorise e.g.
u² + v² = (iu + v)(-iu + v)

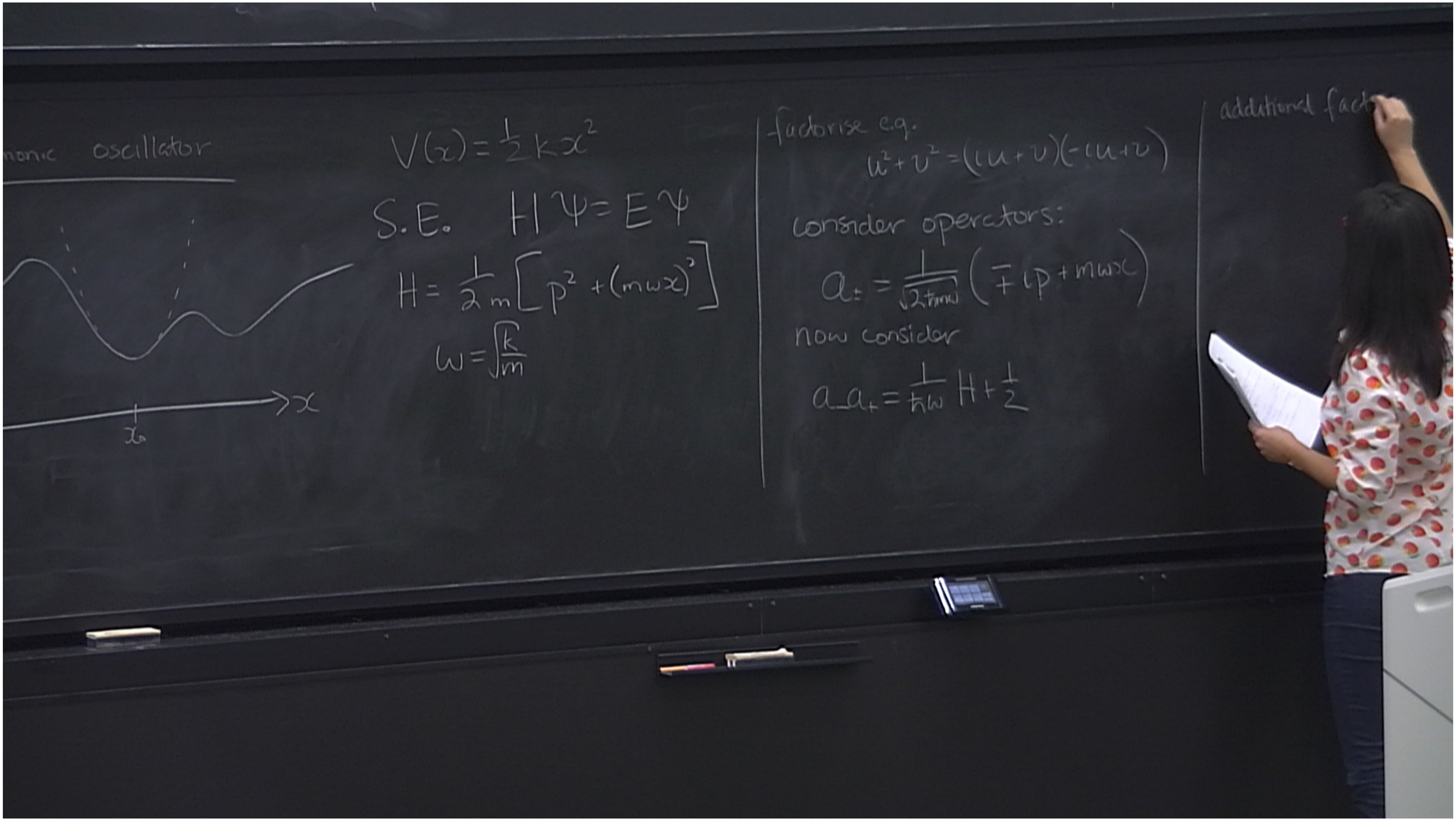

oscillator
V(x) = ½ k x²
S.E. HΨ = EΨ
H = 1/2m [p² + (mωx)²]
ω = √(k/m)
x₀
x
factorise e.g.
u² + v² = (iu + v)(-iu + v)
consider operators:
a± =

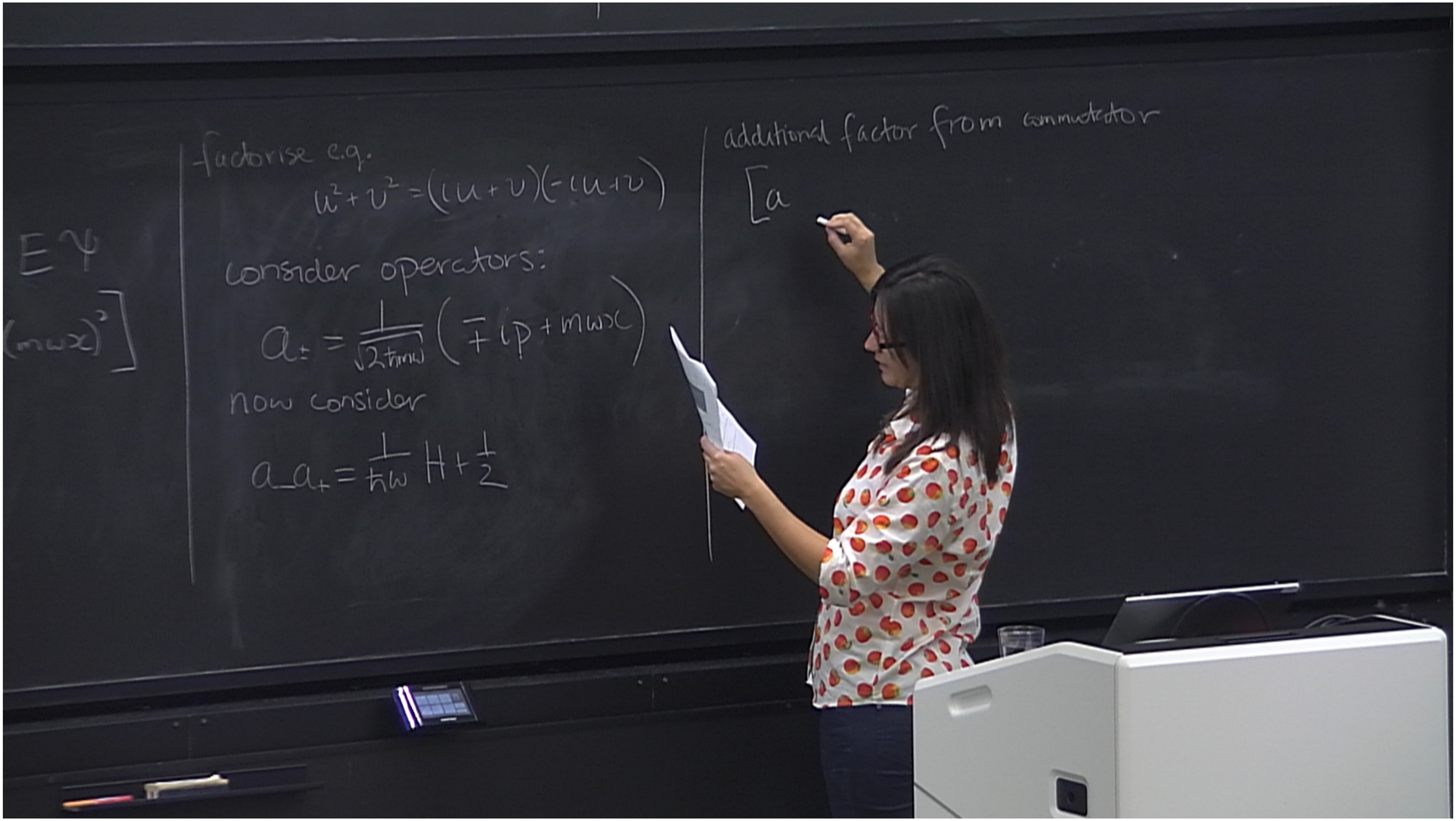

monic oscillator
V(x) = ½ k x²
S.E. HΨ = EΨ
(mωx)²]
factorise e.g.
u² + v² = (iu + v)(−iu + v)
consider operators:
a± = 1/√(2ħmω) (∓ip + mωx)

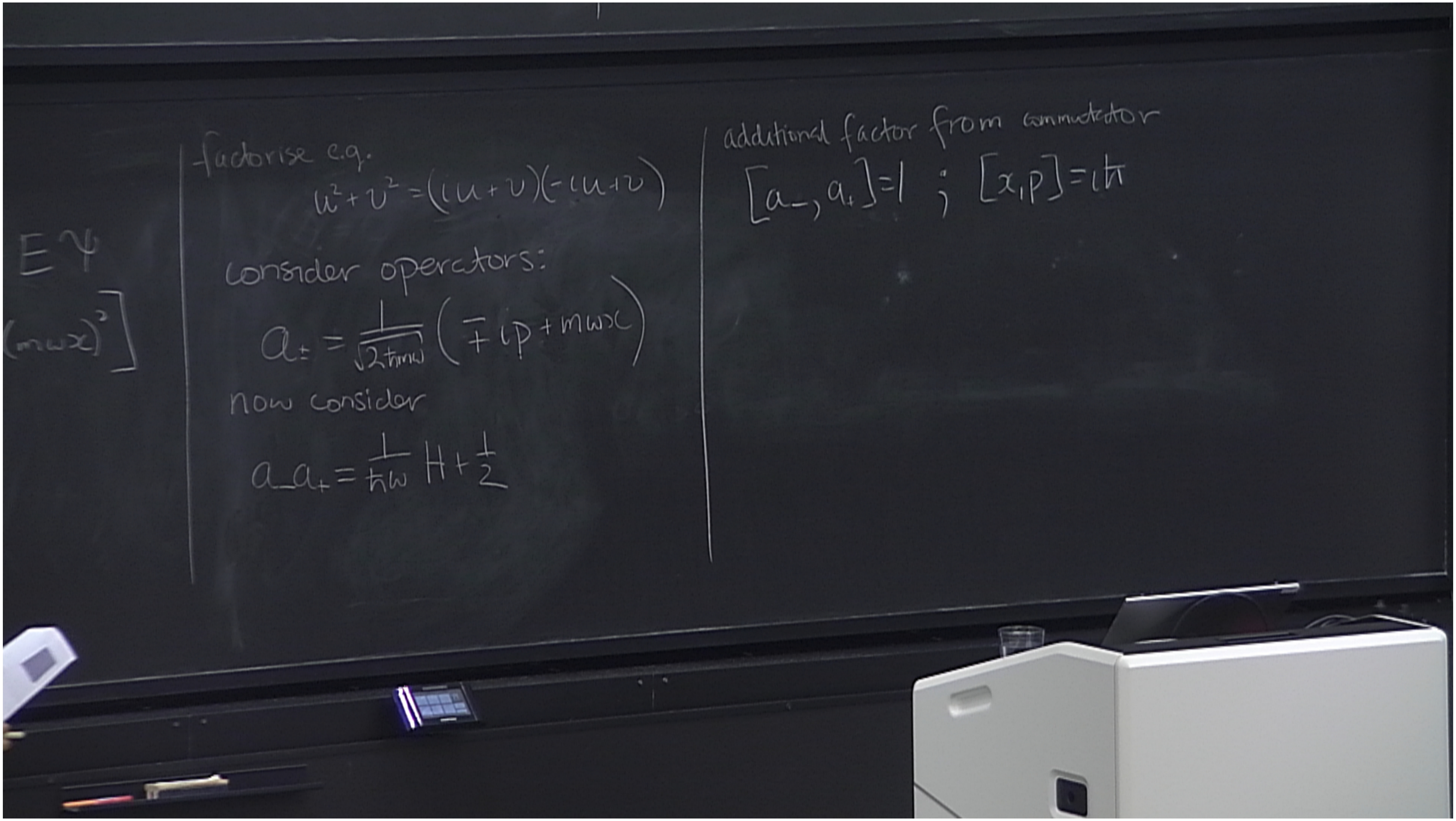

oscillator
V(x) = ½ k x²
S.E. HΨ = EΨ
x
x₀
factorise eg.
u² + v² = (iu + v)(-iu + v)
consider operators:
a± = 1/√(2ħmω) (∓ip + mωx)

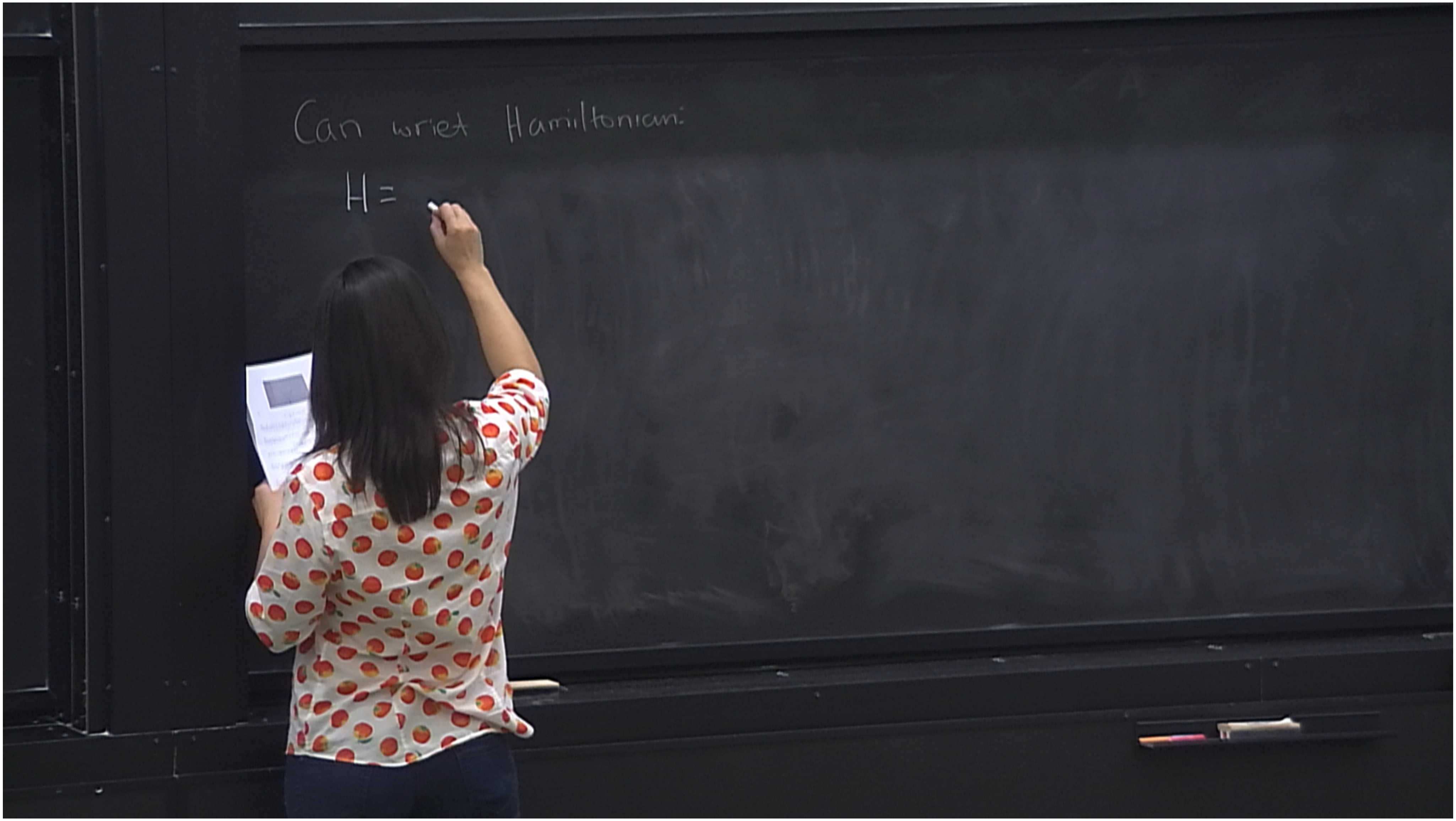


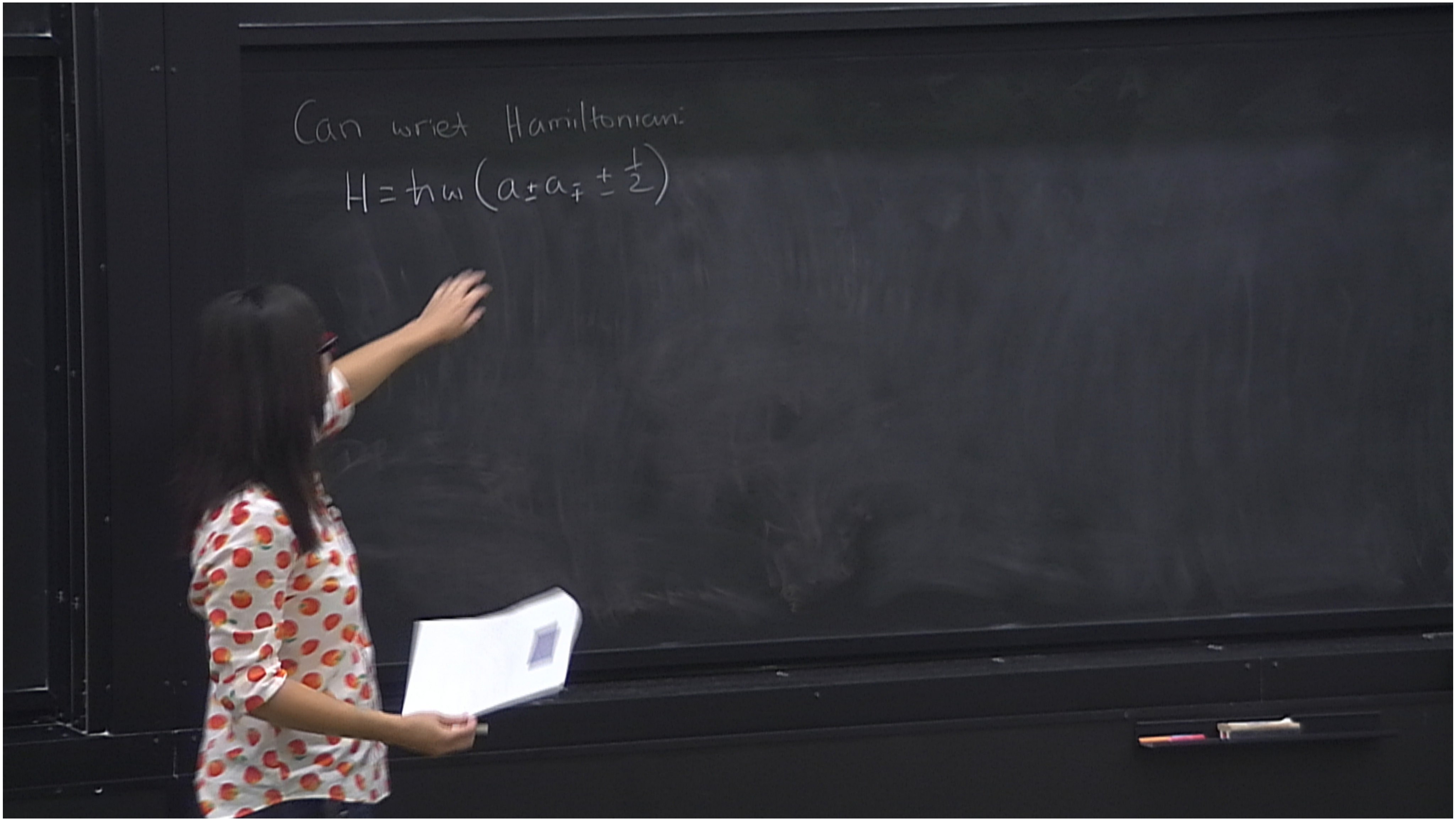
oscillator
$V(x) = \frac{1}{2}kx^2$
S.E. $H\Psi = E\Psi$
$H = \frac{1}{2m}[p^2$
$\omega = \sqrt{\frac{k}{m}}$
x
x_0
factorise e.g.
$u^2 + v^2 = (iu + v)(-iu + v)$
consider operators:
$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$
now consider

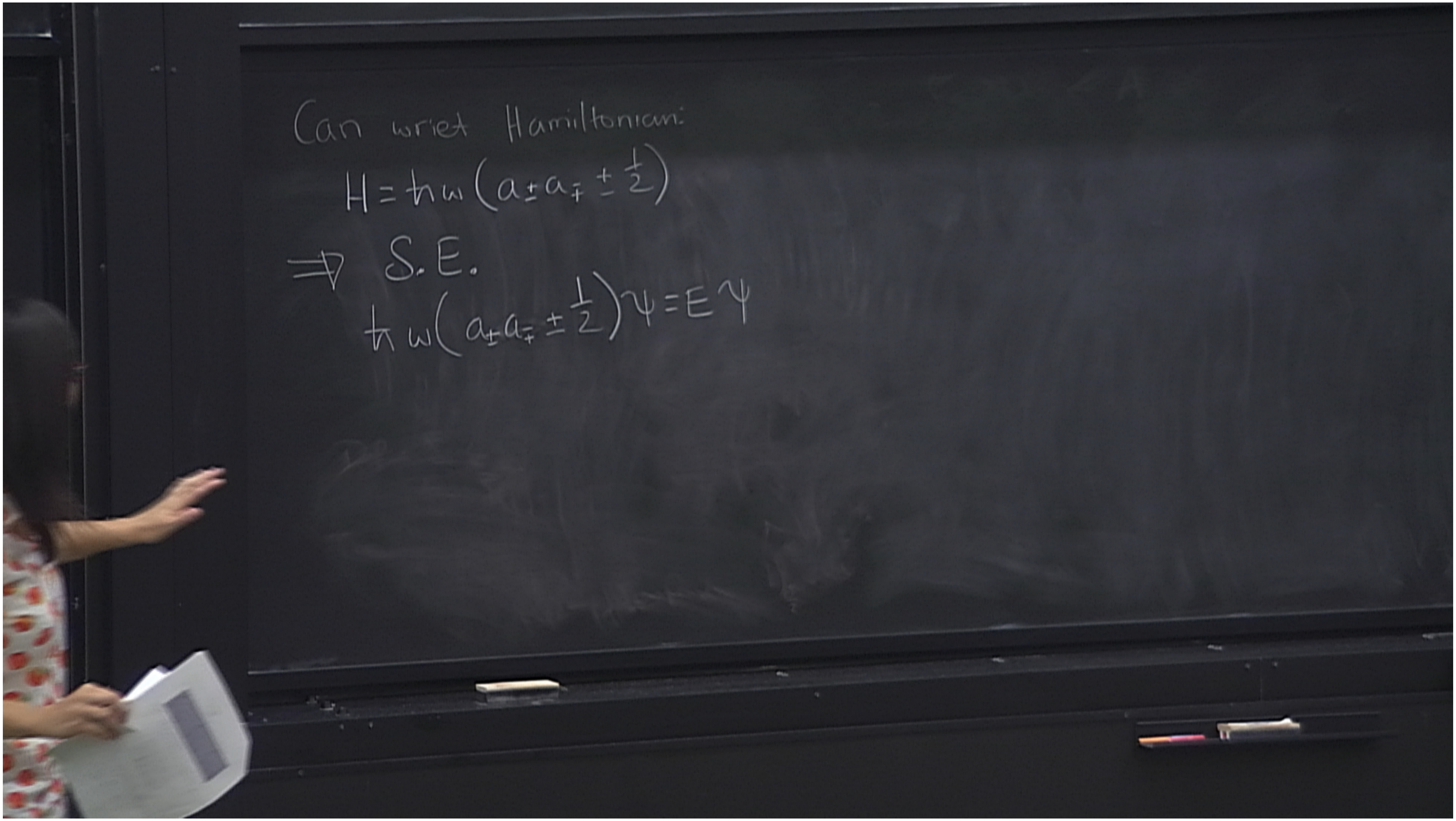

monic oscillator
V(x) = ½ k x²
S.E. HΨ = EΨ
H = 1/2m [p² + (mωx)²]
ω = √(k/m)
x
x₀
factorise e.g.
u² + v² = (iu + v)(-iu + v)
consider operators:
a± = 1/√(2ħmω) (∓ip + mωx)
now consider
a₋a₊ = 1/ħω H + ½
additional fact

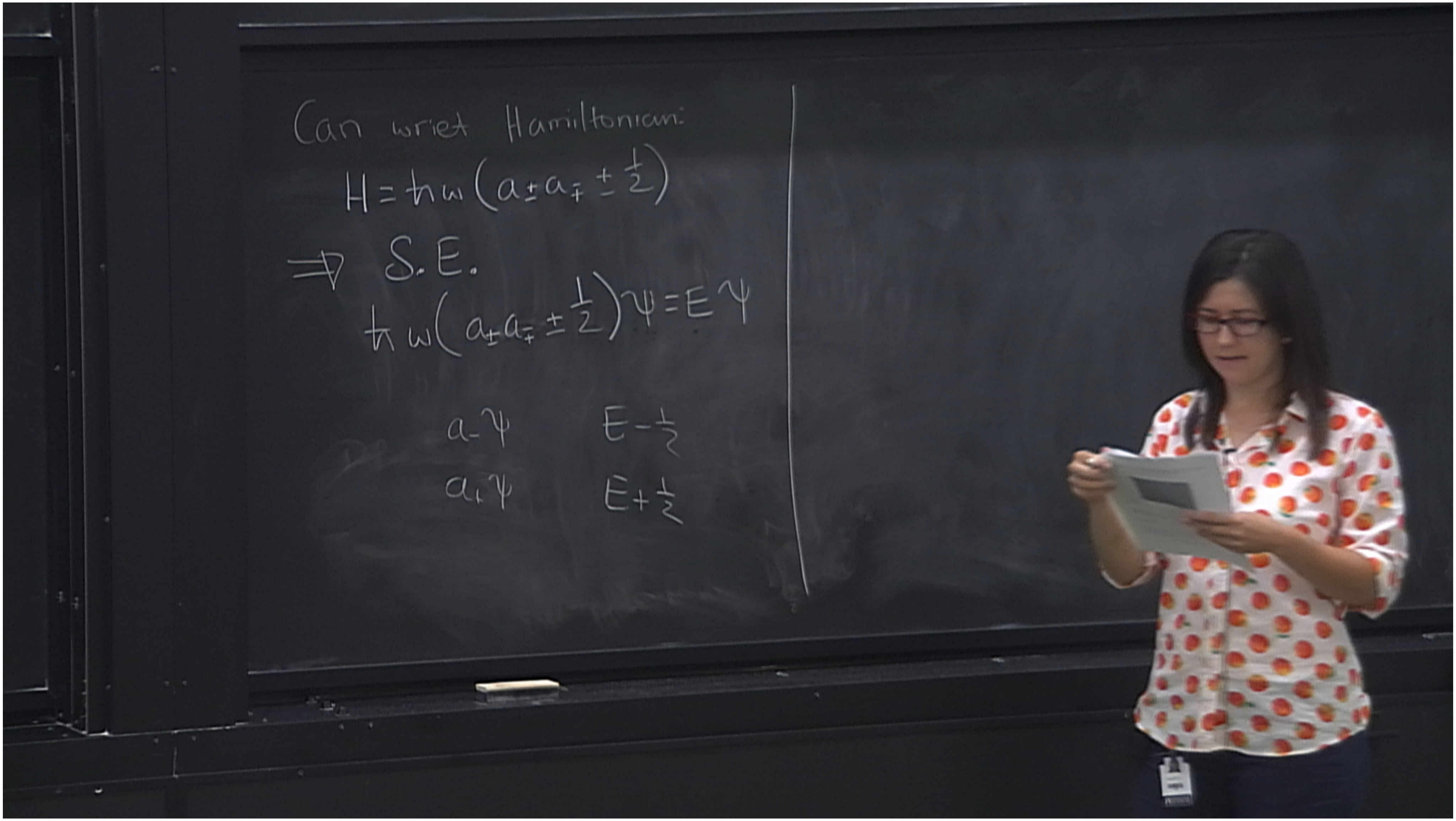

EΨ
(mωx)²]
factorise e.g.
u² + v² = (iu + v)(-iu + v)
consider operators:
a± = 1/√(2ħmω) (∓ip + mωx)
now consider
a₋a₊ = 1/(ħω) H + 1/2
additional factor from commutator
[a

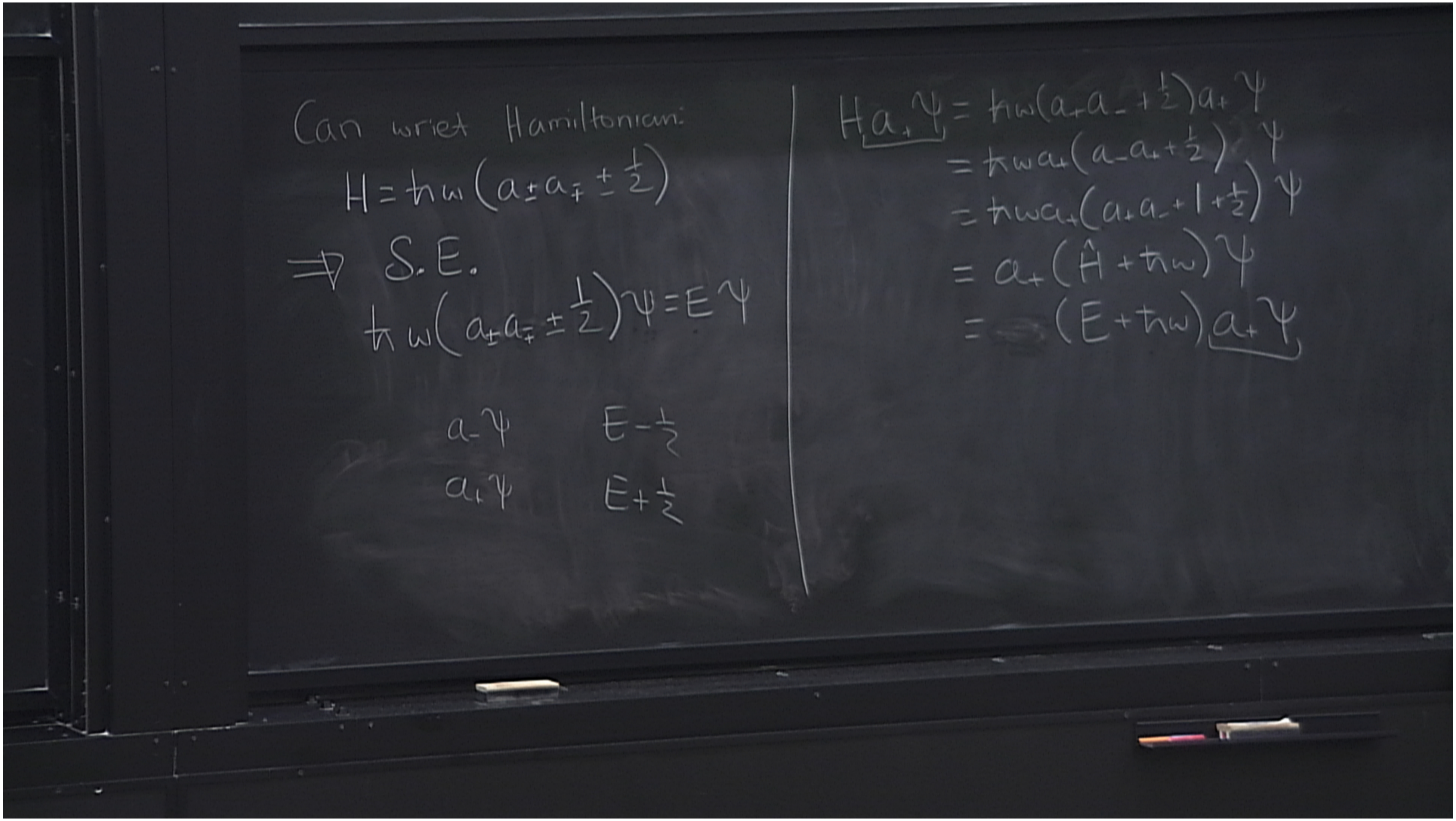

factorise e.g.
$u^2+v^2=(iu+v)(-iu+v)$
consider operators:
$a_\pm=\frac{1}{\sqrt{2\hbar m\omega}}(\mp ip+m\omega x)$
now consider
$a_-a_+=\frac{1}{\hbar\omega}H+\frac{1}{2}$
additional factor from commutator
$[a_-,a_+]=1$; $[x,p]=i\hbar$
$E\psi$
$(m\omega x)^2]$

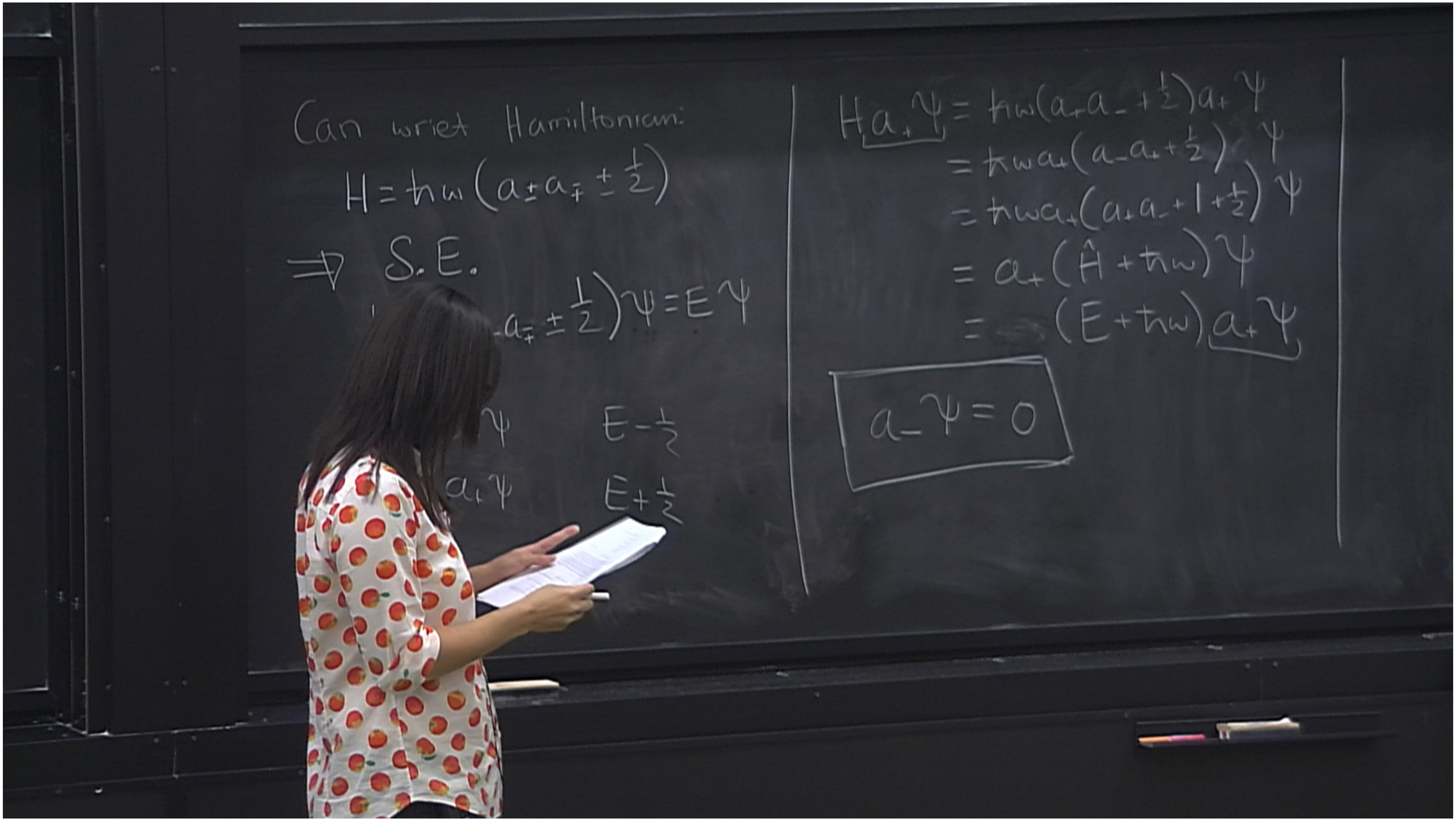

Can wriet Hamiltonian:
H =


Can wriet Hamiltonian:
$H = \hbar\omega(a_{\pm}a_{\mp} \pm \frac{1}{2})$


Can wriet Hamiltonian:
H = ħω(a±a∓ ± 1/2)
⇒ S.E.
ħω(a±a∓ ± 1/2)ψ = Eψ


Can wriet Hamiltonian:
H = ħω(a±a∓ ± 1/2)
⇒ S.E.
ħω(a±a∓ ± 1/2)ψ = Eψ
a-ψ E - 1/2
a+ψ E + 1/2


Can wriet Hamiltonian:
$H = \hbar\omega(a_\pm a_\mp \pm \frac{1}{2})$
$\Rightarrow$ S.E.
$\hbar\omega(a_\pm a_\mp \pm \frac{1}{2})\psi = E\psi$
$a_-\psi$ $E - \frac{1}{2}$
$a_+\psi$ $E + \frac{1}{2}$
$H a_+\psi = \hbar\omega(a_+a_- + \frac{1}{2})a_+\psi$
$= \hbar\omega a_+(a_-a_+ + \frac{1}{2})\psi$
$= \hbar\omega a_+(a_+a_- + 1 + \frac{1}{2})\psi$
$= a_+(\hat{H} + \hbar\omega)\psi$
$= (E + \hbar\omega)a_+\psi$


Can wriet Hamiltonian:
$H = \hbar\omega(a_\pm a_\mp \pm \frac{1}{2})$
$\Rightarrow$ S.E.
$a_\mp \pm \frac{1}{2})\psi = E\psi$
ψ $E - \frac{1}{2}$
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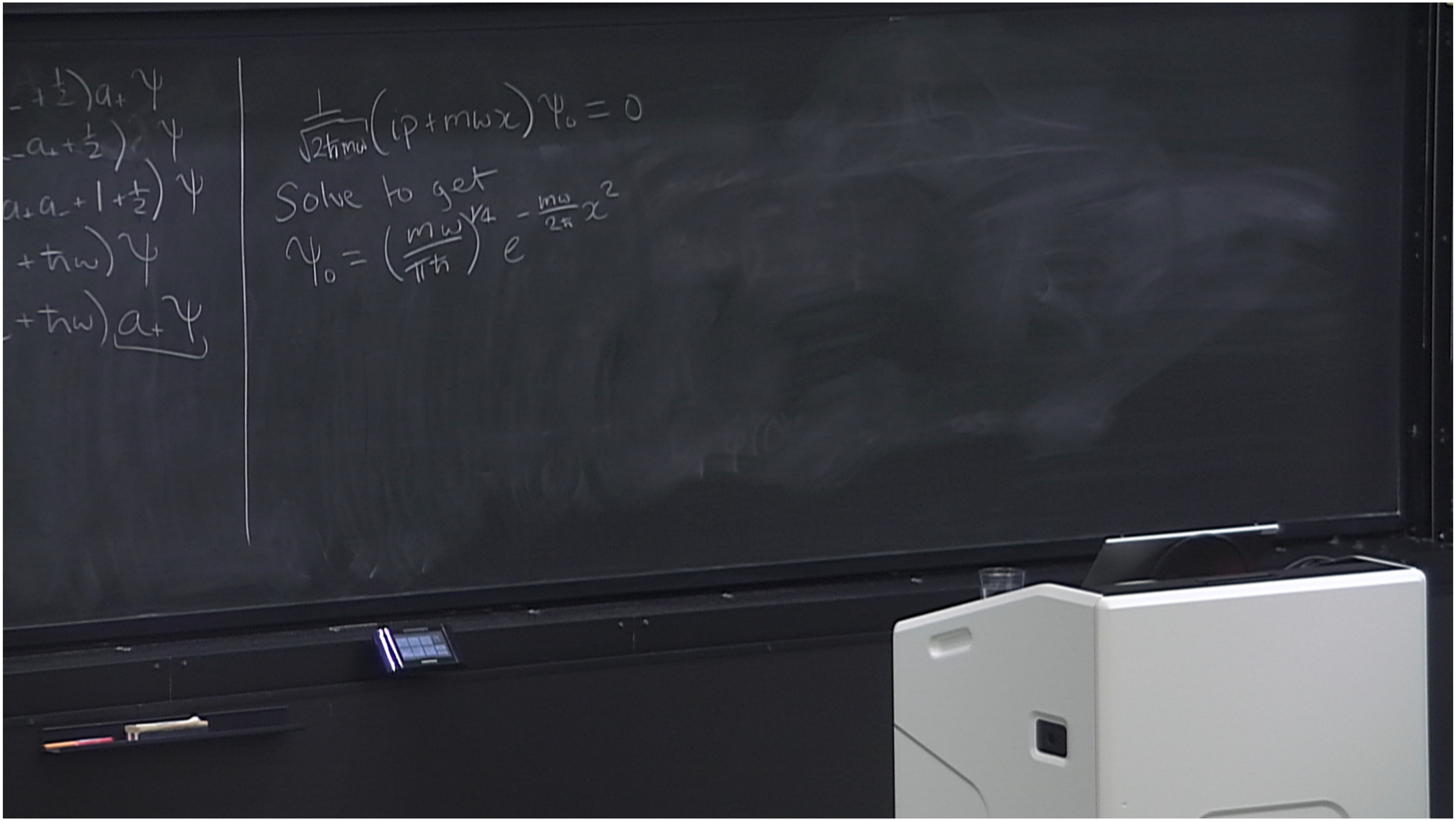
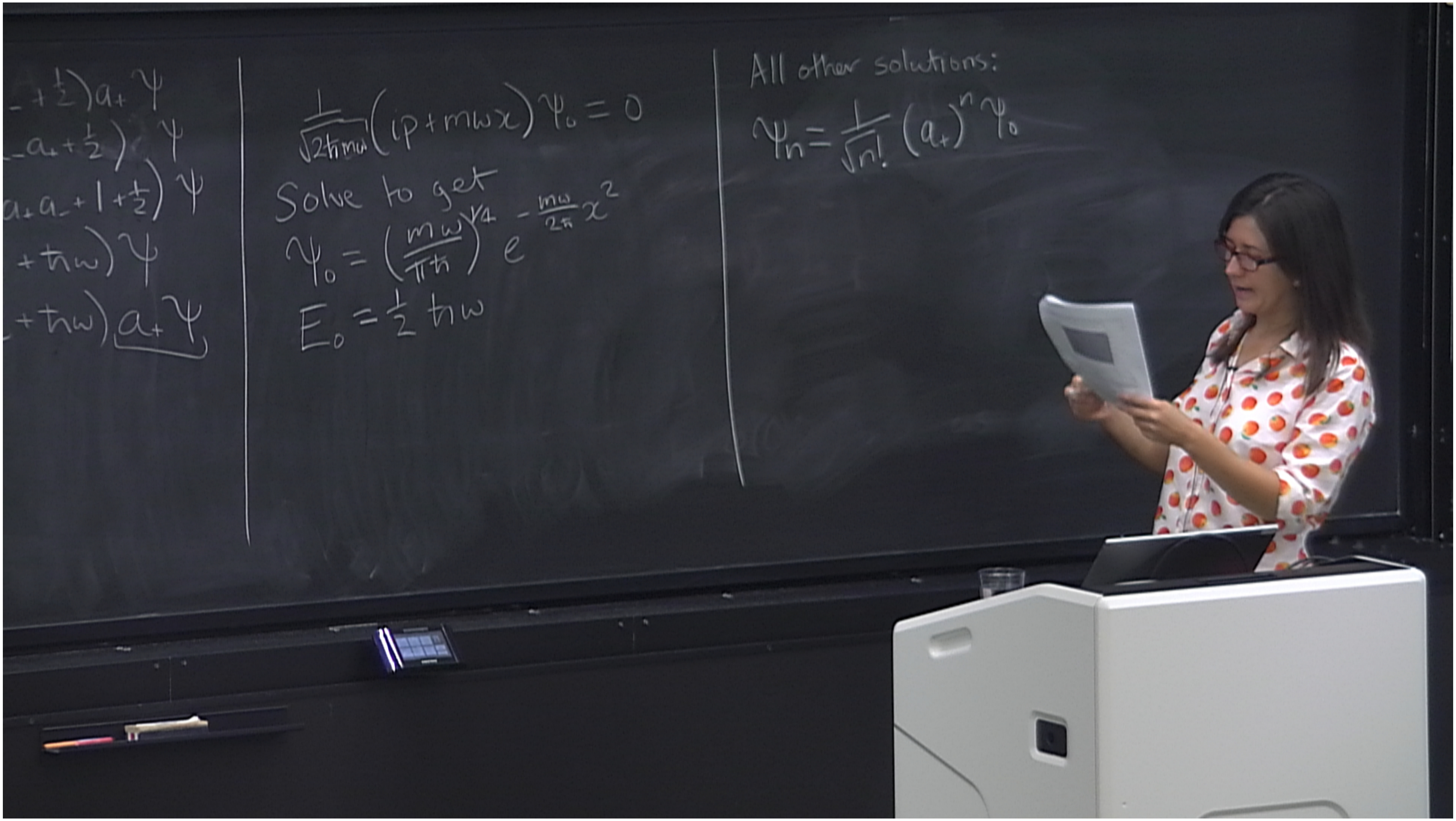
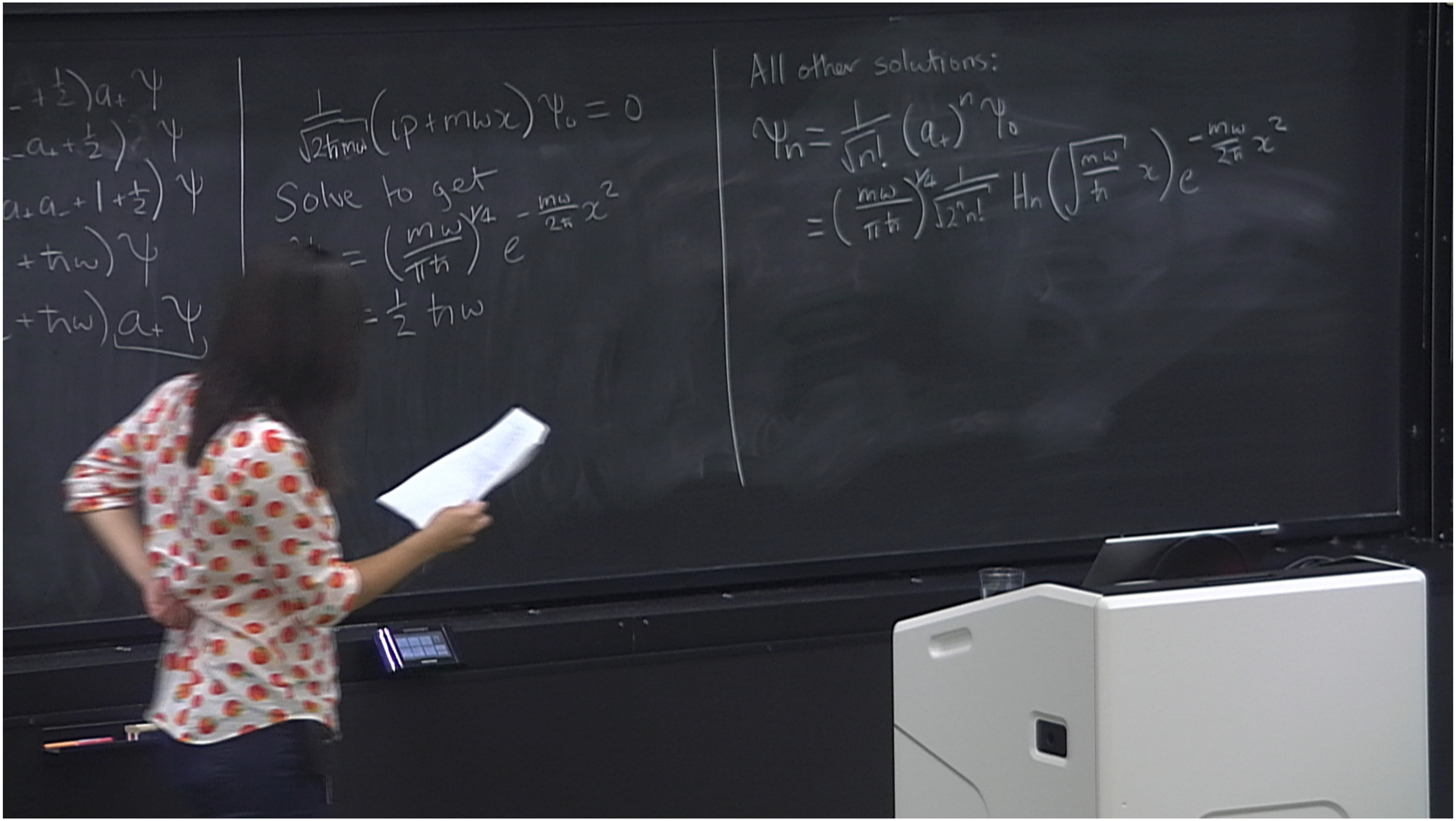
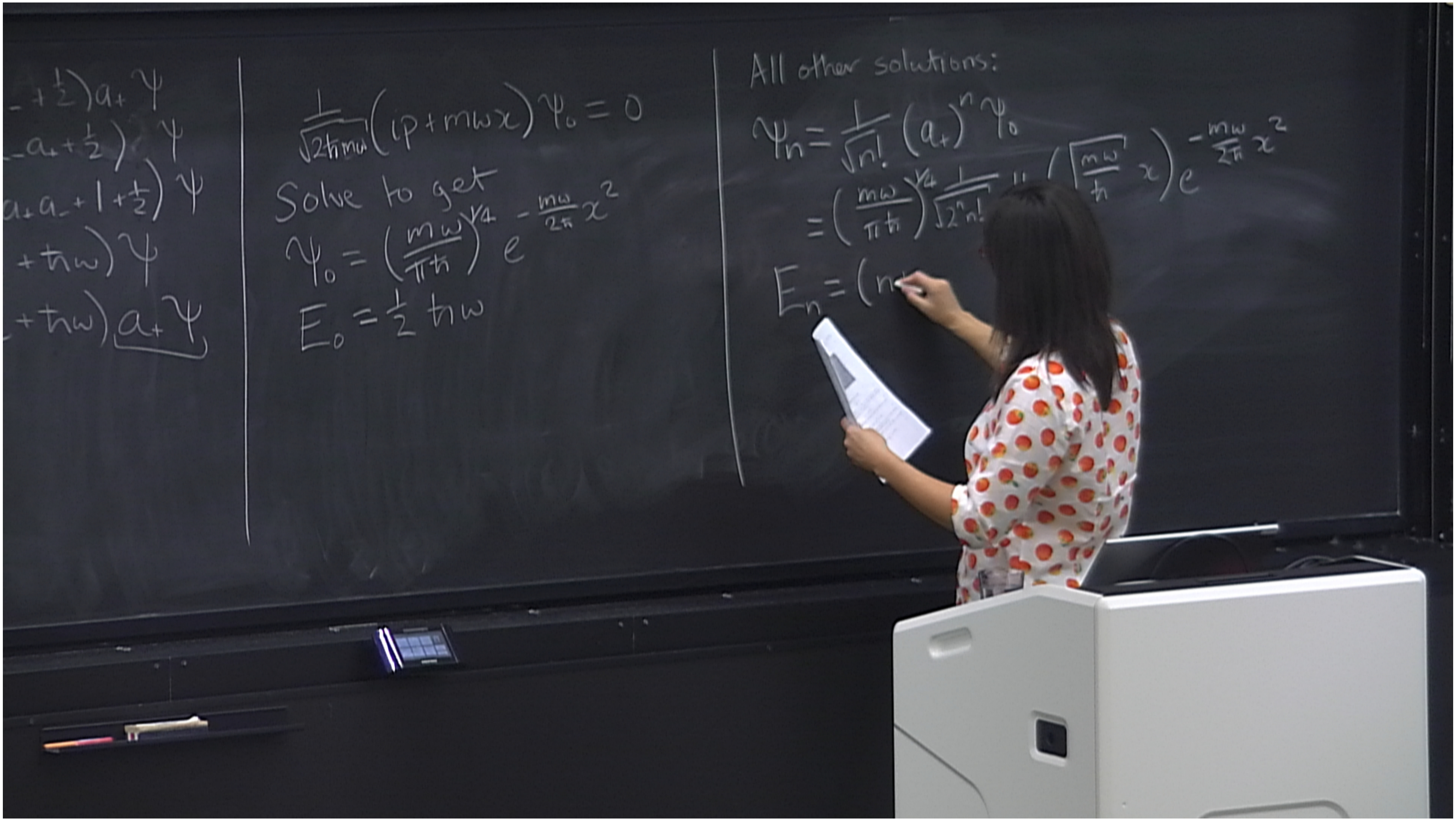
$= a_+(\hat{H} + \hbar\omega)\psi$
$= (E + \hbar\omega)a_+\psi$
$a_-\psi = 0$


1/√(2ħmω) (ip + mωx) ψ₀ = 0
Solve to get
ψ₀ = (mω/πħ)^(1/4) e^(−(mω/2ħ) x²)


$\frac{1}{\sqrt{2\hbar m\omega}}(ip + m\omega x)\psi_0 = 0$
Solve to get
$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$
$E_0 = \frac{1}{2}\hbar\omega$
All other solutions:
$\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0$


Solve to get
All other solutions: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$\frac{1}{\sqrt{2\hbar m\omega}}(ip + m\omega x)\psi_0 = 0$
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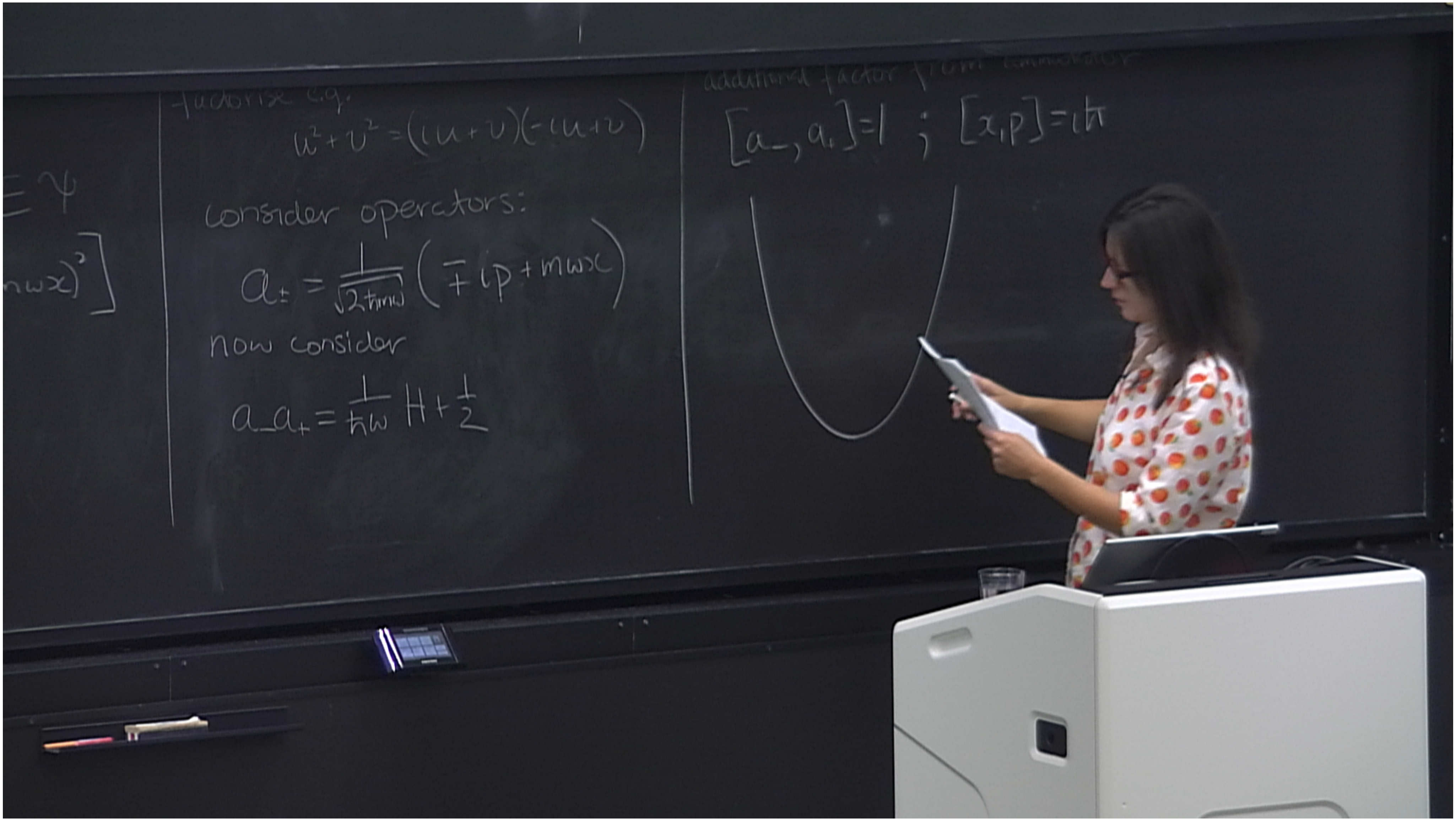
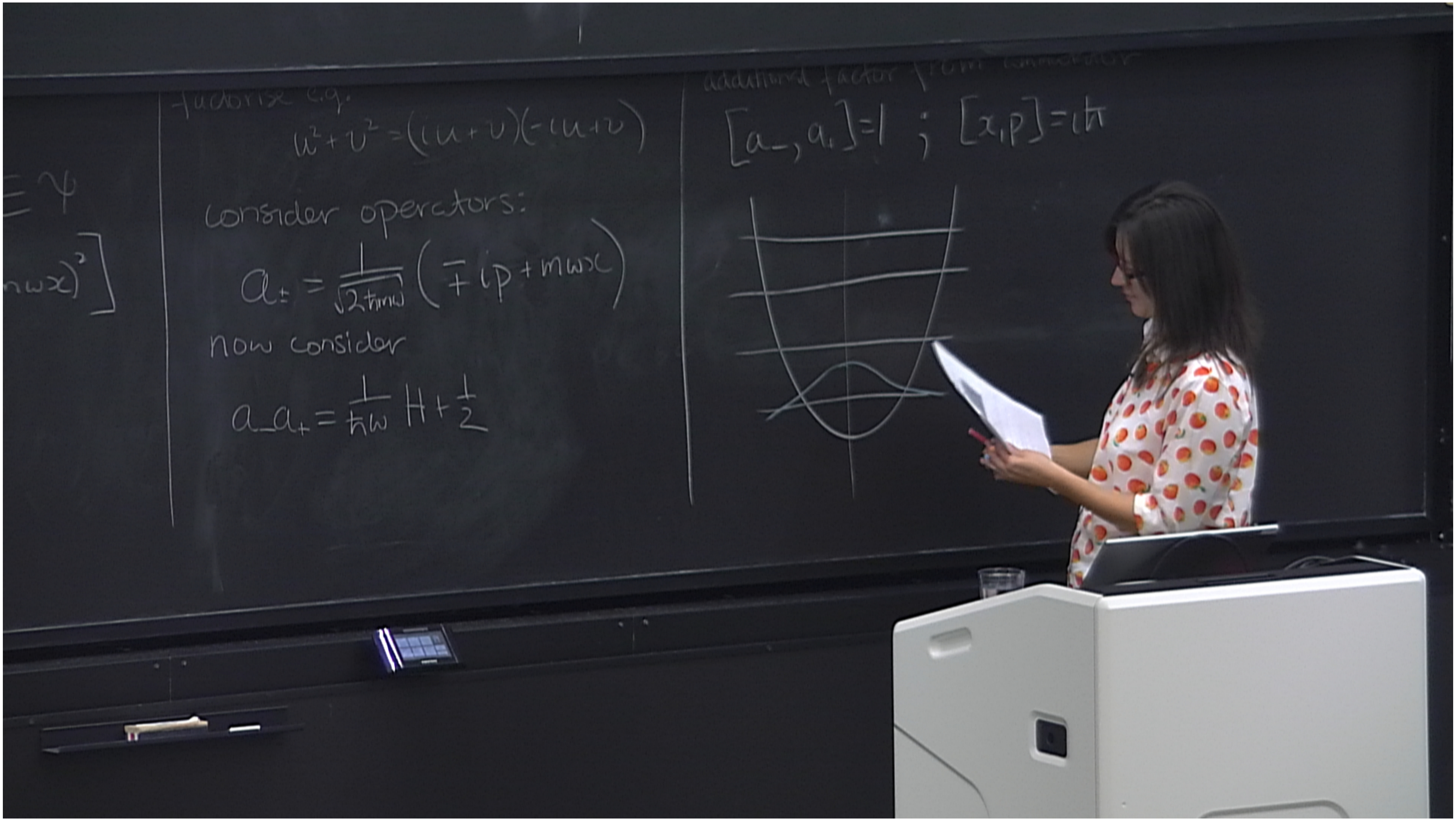
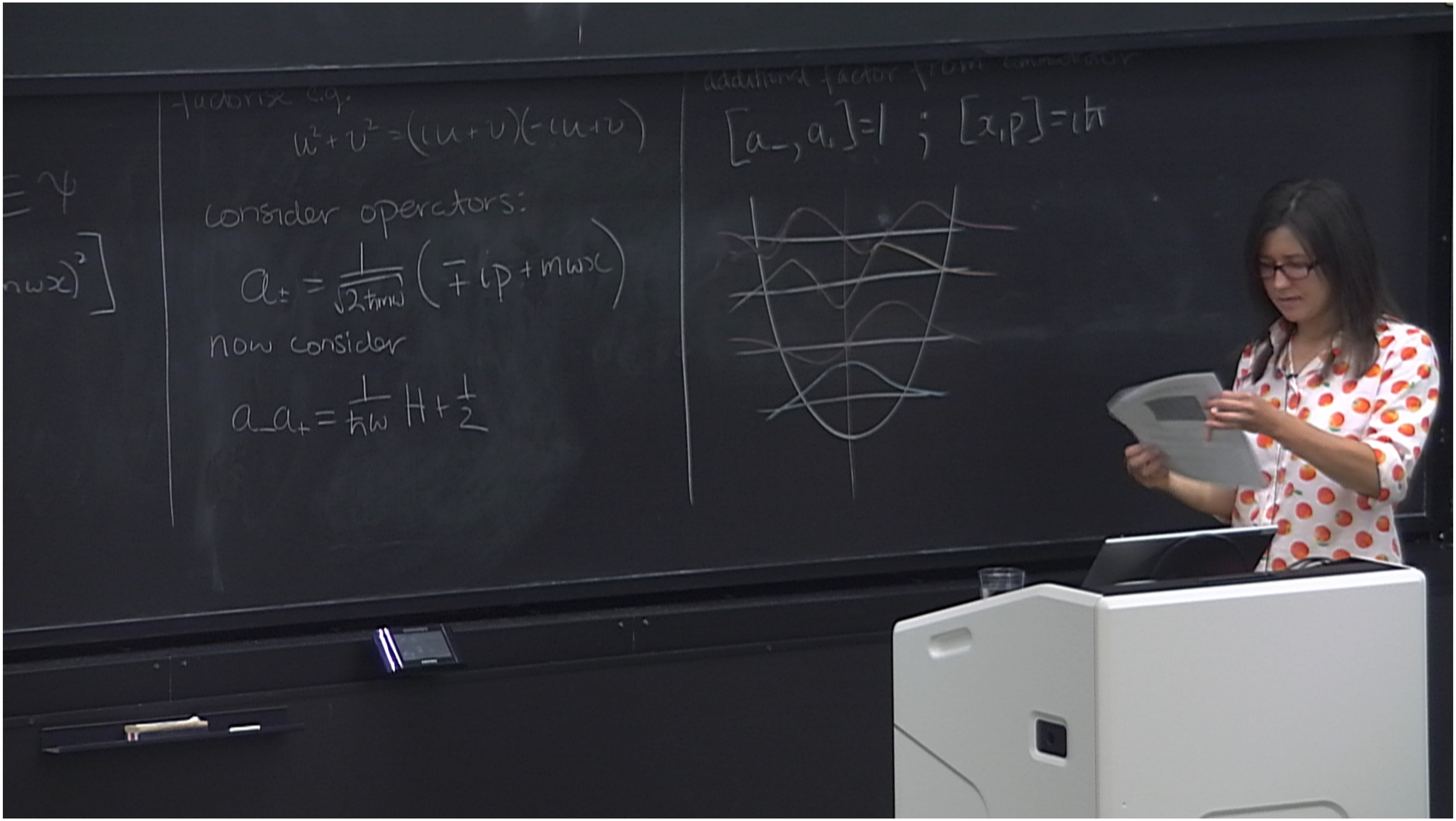
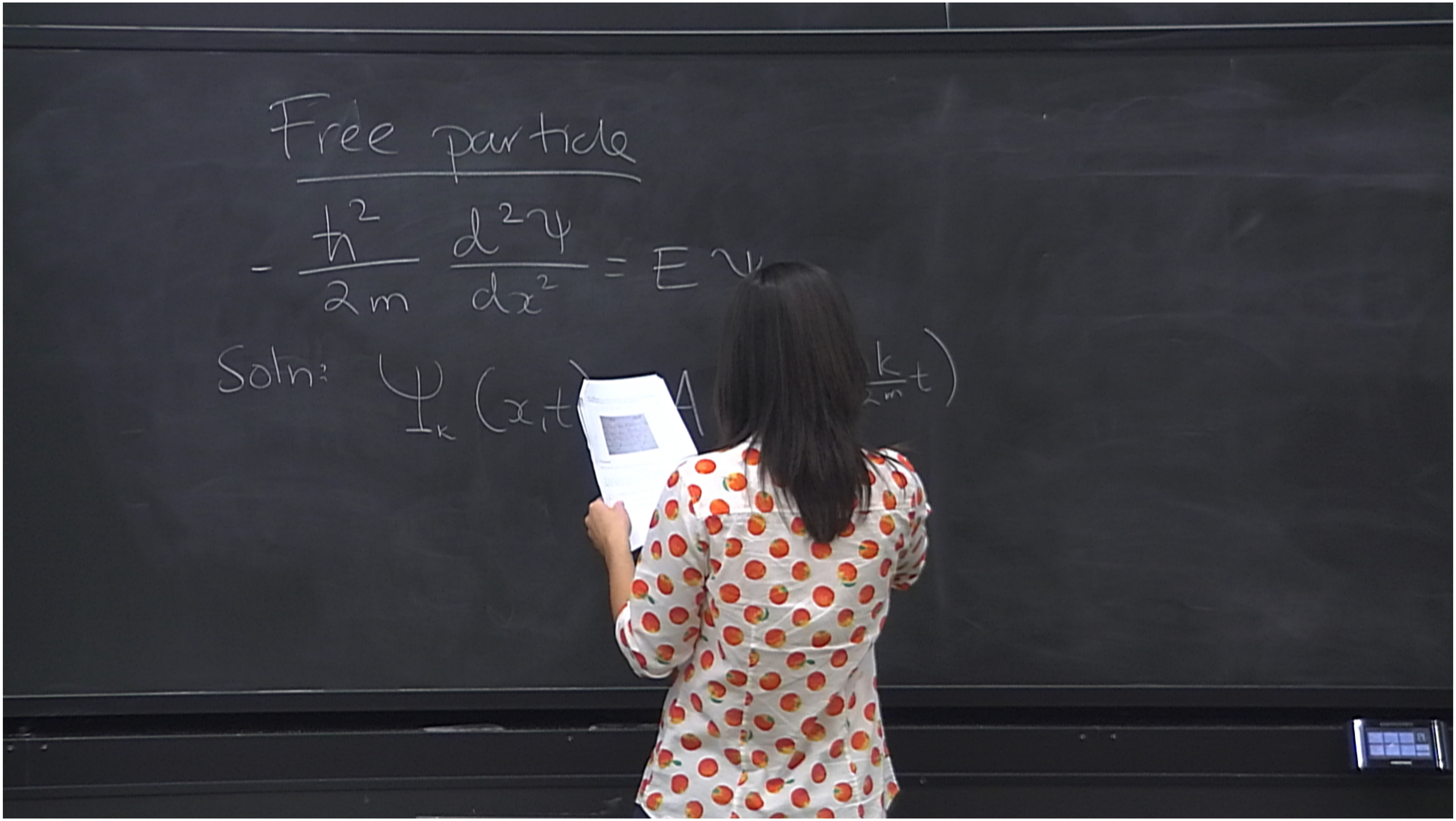
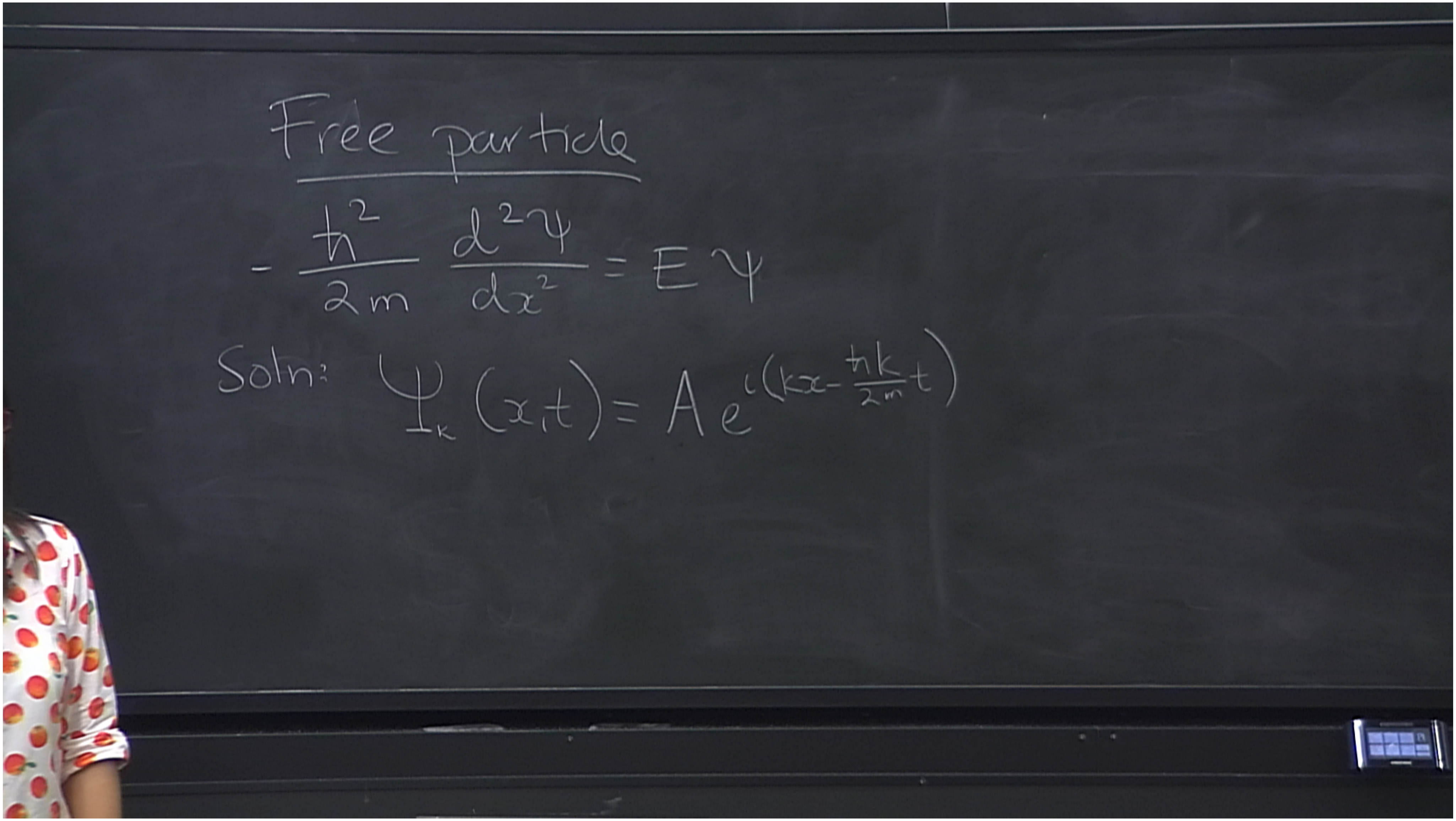

factorise e.g.
u^2 + v^2 = (iu+v)(-iu+v)
consider operators:
a± = 1/√(2ħmω) (∓ip + mωx)
now consider
a_a+ = 1/ħω H + 1/2
[a_, a+] = 1 ; [x, p] = iħ

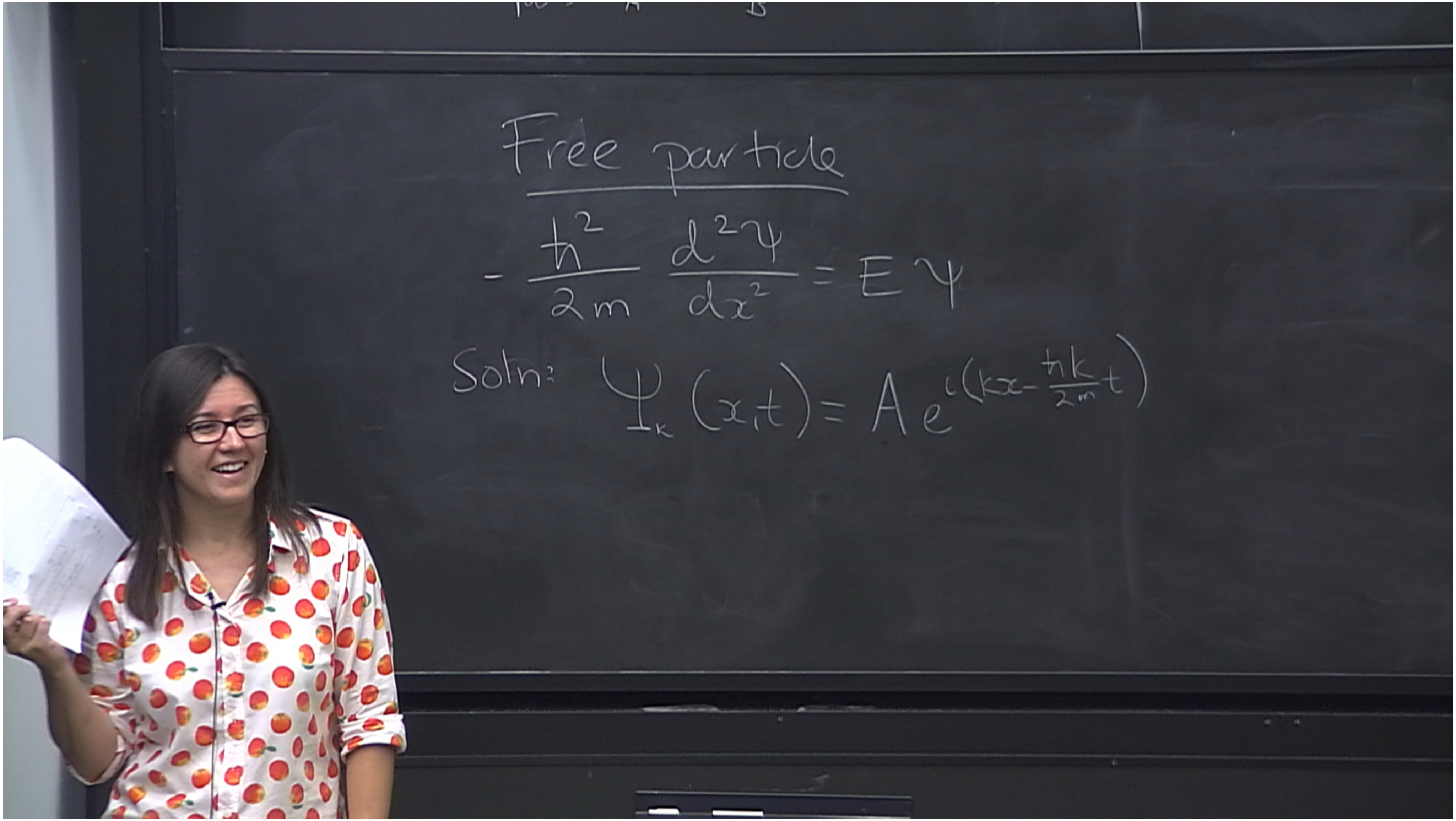

factorise e.g.
u² + v² = (iu + v)(−iu + v)
consider operators:
a± = 1/√(2ħmω) (∓ip + mωx)
now consider
a₋a₊ = 1/ħω H + 1/2
[a₋, a₊] = 1 ; [x, p] = iħ

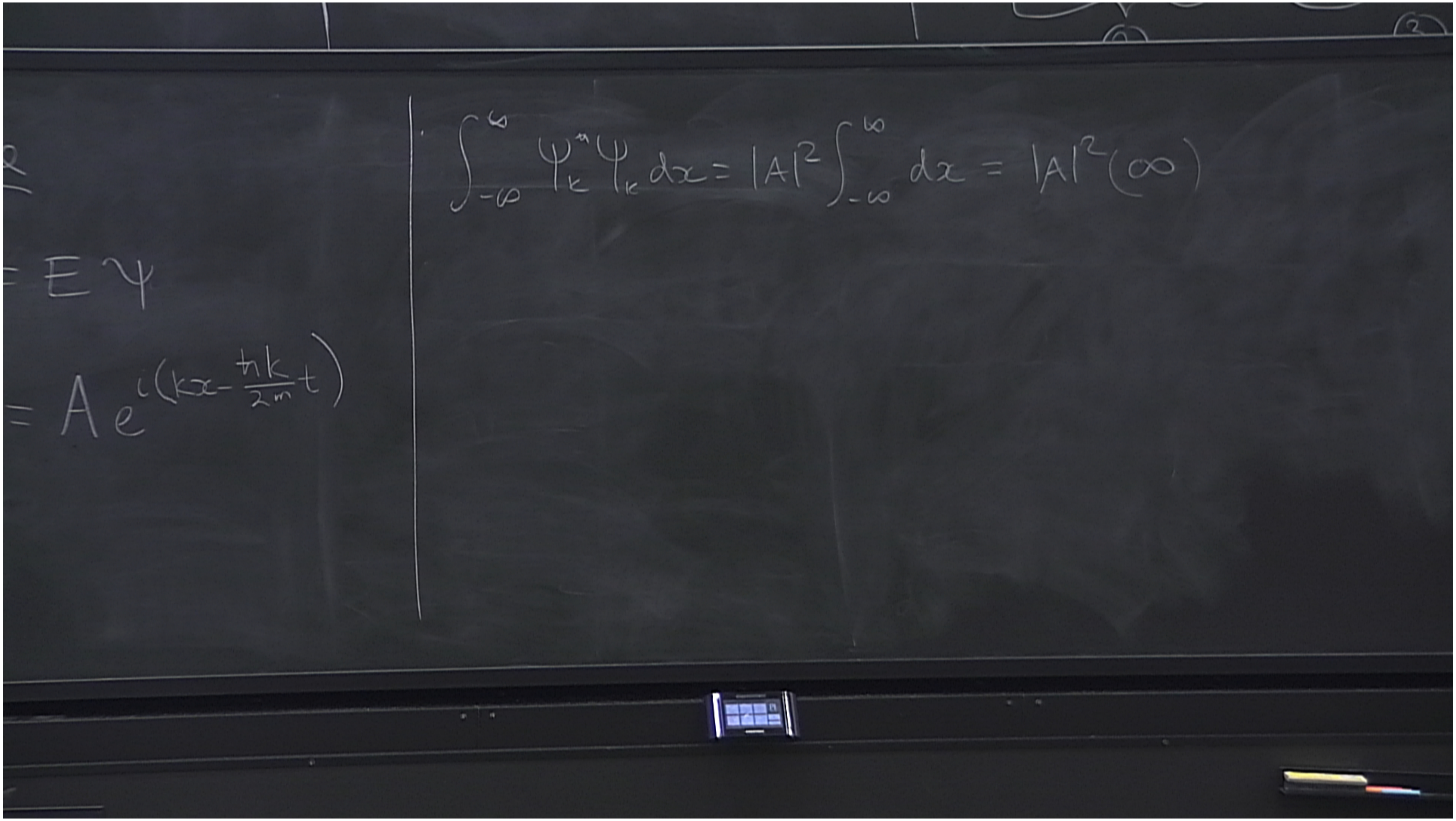


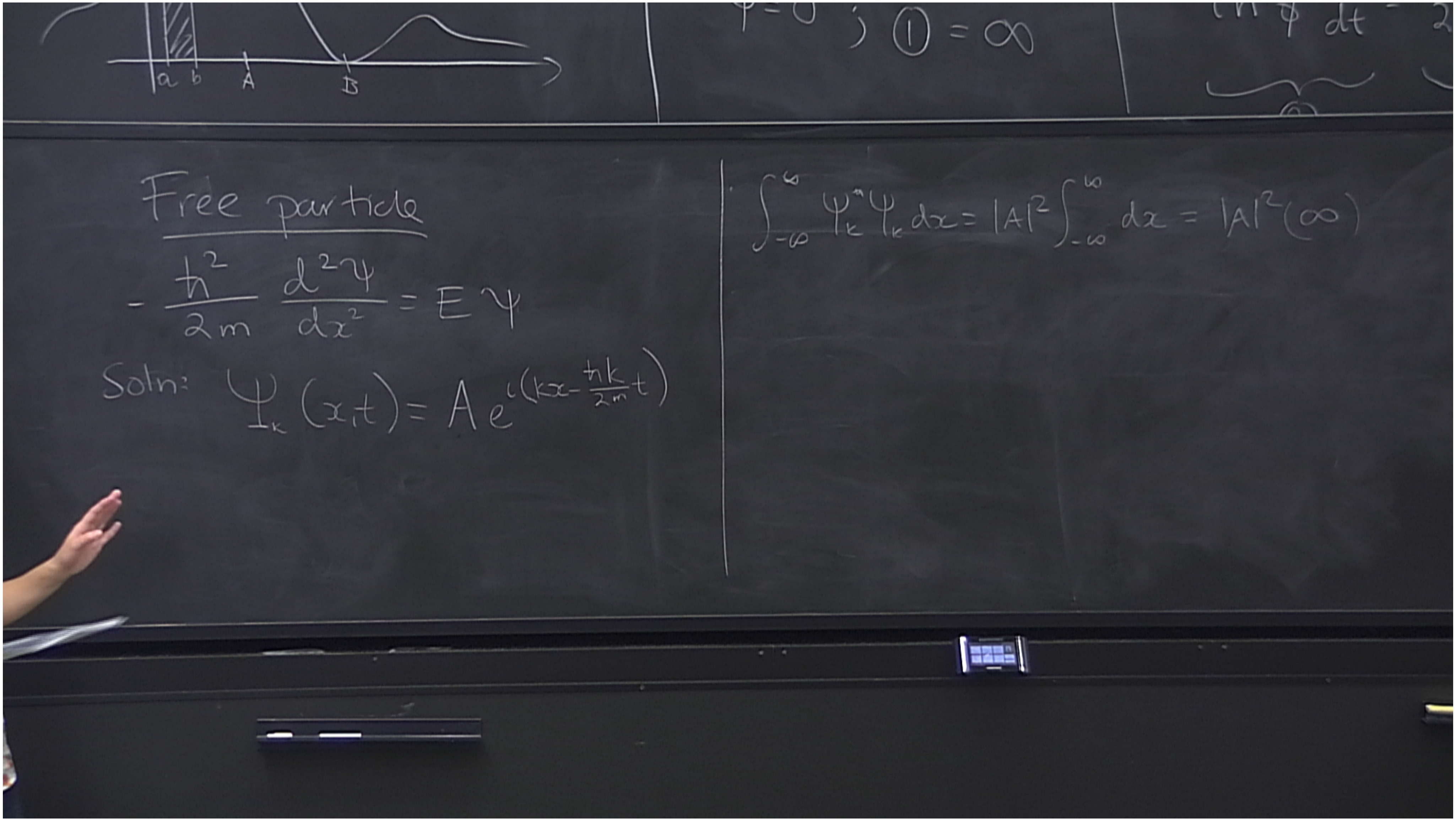
factorise e.g.
$u^2 + v^2 = (iu + v)(-iu + v)$
consider operators:
$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$
now consider
$a_- a_+ = \frac{1}{\hbar\omega} H + \frac{1}{2}$
$[a_-, a_+] = 1$; $[x, p] = i\hbar$

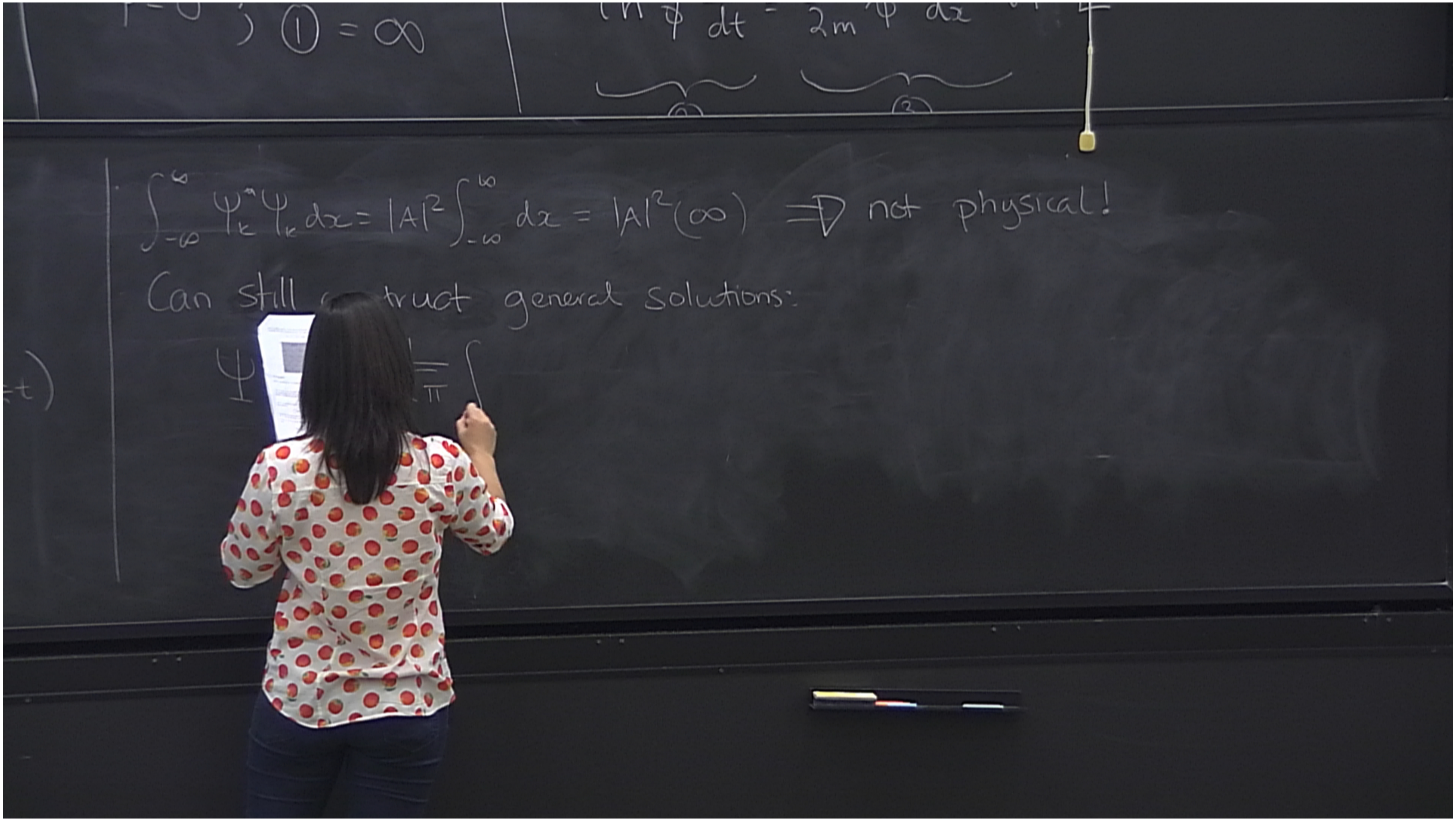

Free particle
$- \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi$
Soln: $\Psi_k(x,t)$ A

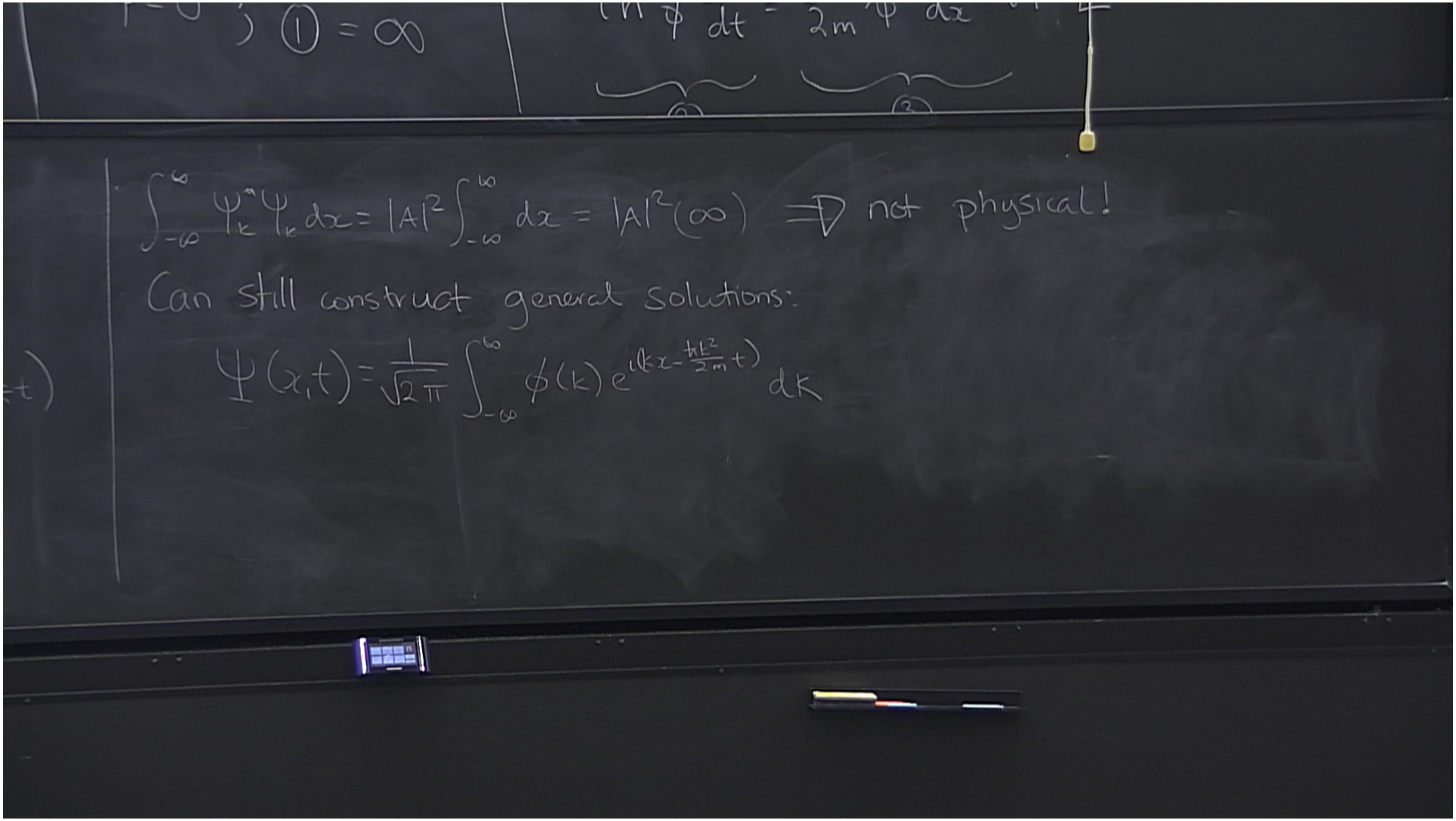

Free particle
$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$
Soln: $\Psi_k(x,t) = Ae^{i(kx - \frac{\hbar k}{2m}t)}$


Free particle
$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$
Soln: $\Psi_k(x,t) = A e^{i(kx - \frac{\hbar k}{2m} t)}$


= E ψ
= A e^{i(kx - ħk/2m t)}
∫_{-∞}^{∞} ψ_k^* ψ_k dx = |A|^2 ∫_{-∞}^{∞} dx = |A|^2 (∞)


a b A B
$\textcircled{1} = \infty$
Free particle
$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$
Soln: $\Psi_k(x,t) = A e^{i(kx - \frac{\hbar k}{2m}t)}$
$\int_{-\infty}^{\infty} \Psi_k^* \Psi_k \, dx = |A|^2 \int_{-\infty}^{\infty} dx = |A|^2(\infty)$


not physical!
Can still
truct general solutions:


$\int_{-\infty}^{\infty} \psi_k^* \psi_k \, dx = |A|^2 \int_{-\infty}^{\infty} dx = |A|^2 (\infty) \Rightarrow$ not physical!
Can still construct general solutions:
$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \frac{\hbar k^2}{2m} t)} dk$