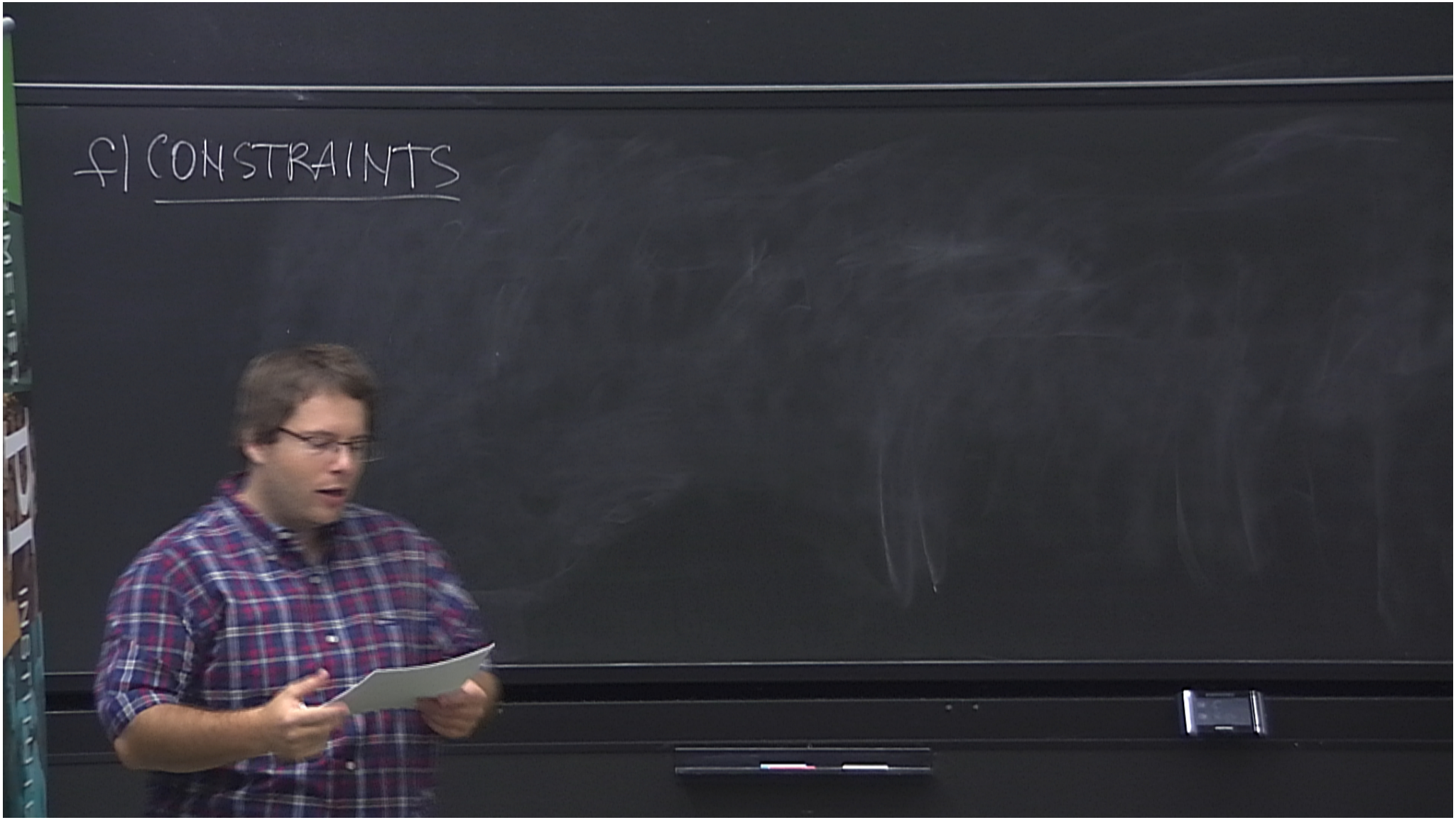
Title: 14/15 PSI - Classical Mechanics

Date: Aug 15, 2014 10:30 AM

URL: http://pirsa.org/14080018

Abstract:


f) CONSTRAINTS

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LET NOT ALL q^i & p_i INDEPENDENT, I.E., AT ANY INSTANT OF TIME WE HAVE A CONSTRAINT

$$\phi(q^i, p_i) = 0$$

- ANY P-S FUNCTION $f \equiv$ "HAMILTONIAN"
  & GENERATES A FLOW ("FICTICIOUS TIME λ")

$$\frac{dq^i}{d\lambda} = \{q^i, f\}, \quad \frac{dp_i}{d\lambda} = \{p_i, f\}$$

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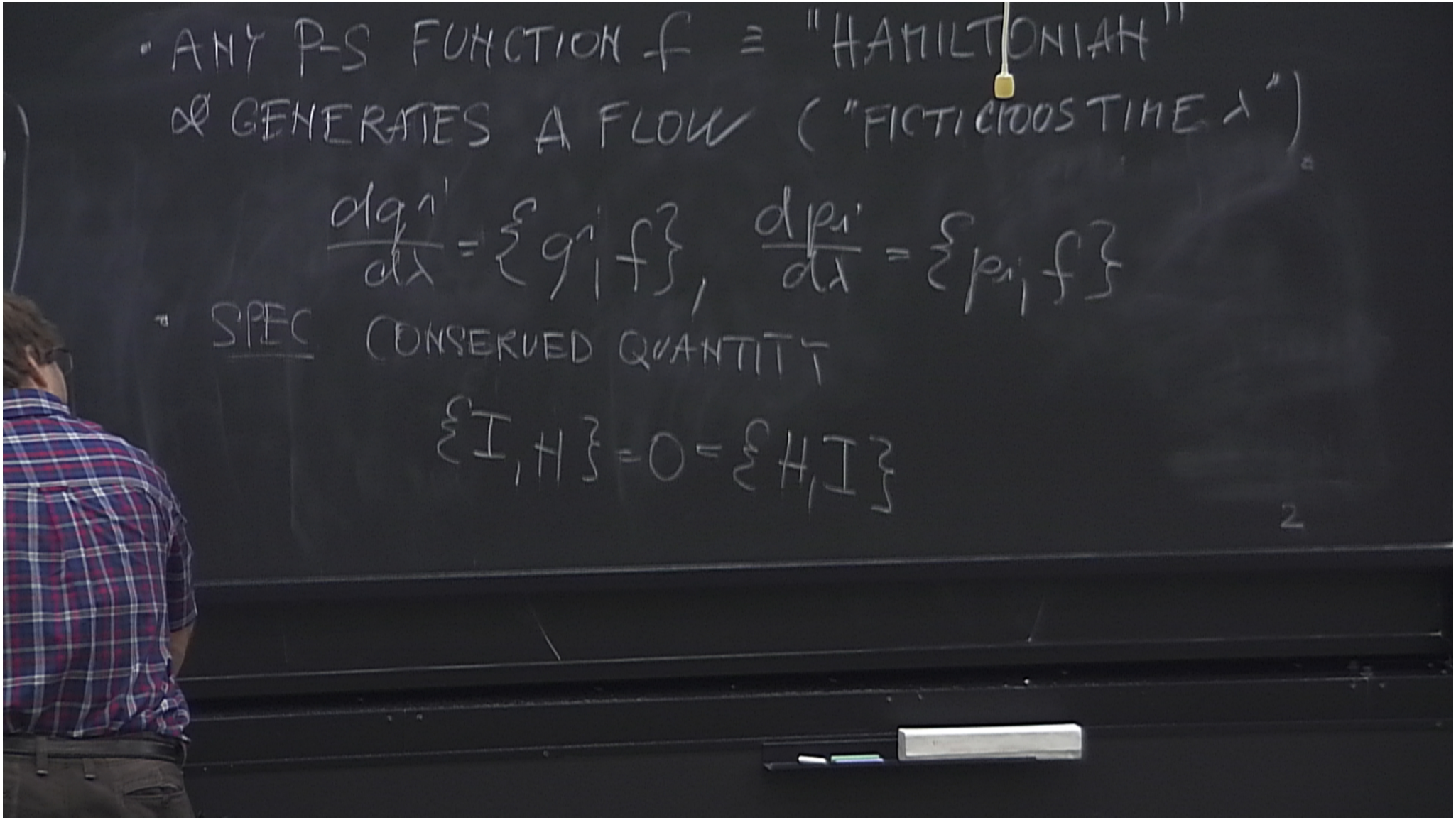
- SPEC CONSERVED QUANTITY

2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$\{I, H\} = 0 = \{H, I\}$
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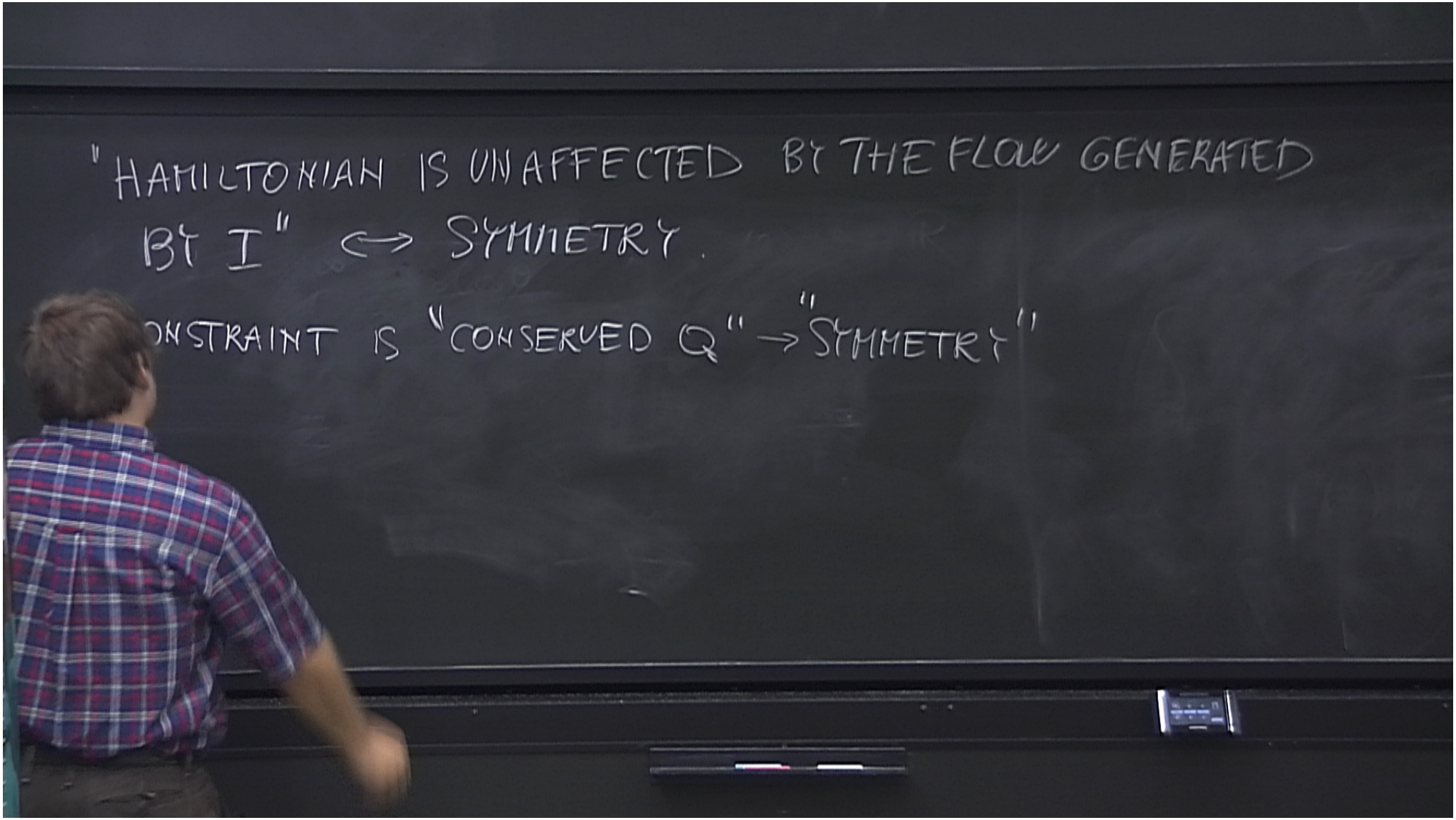
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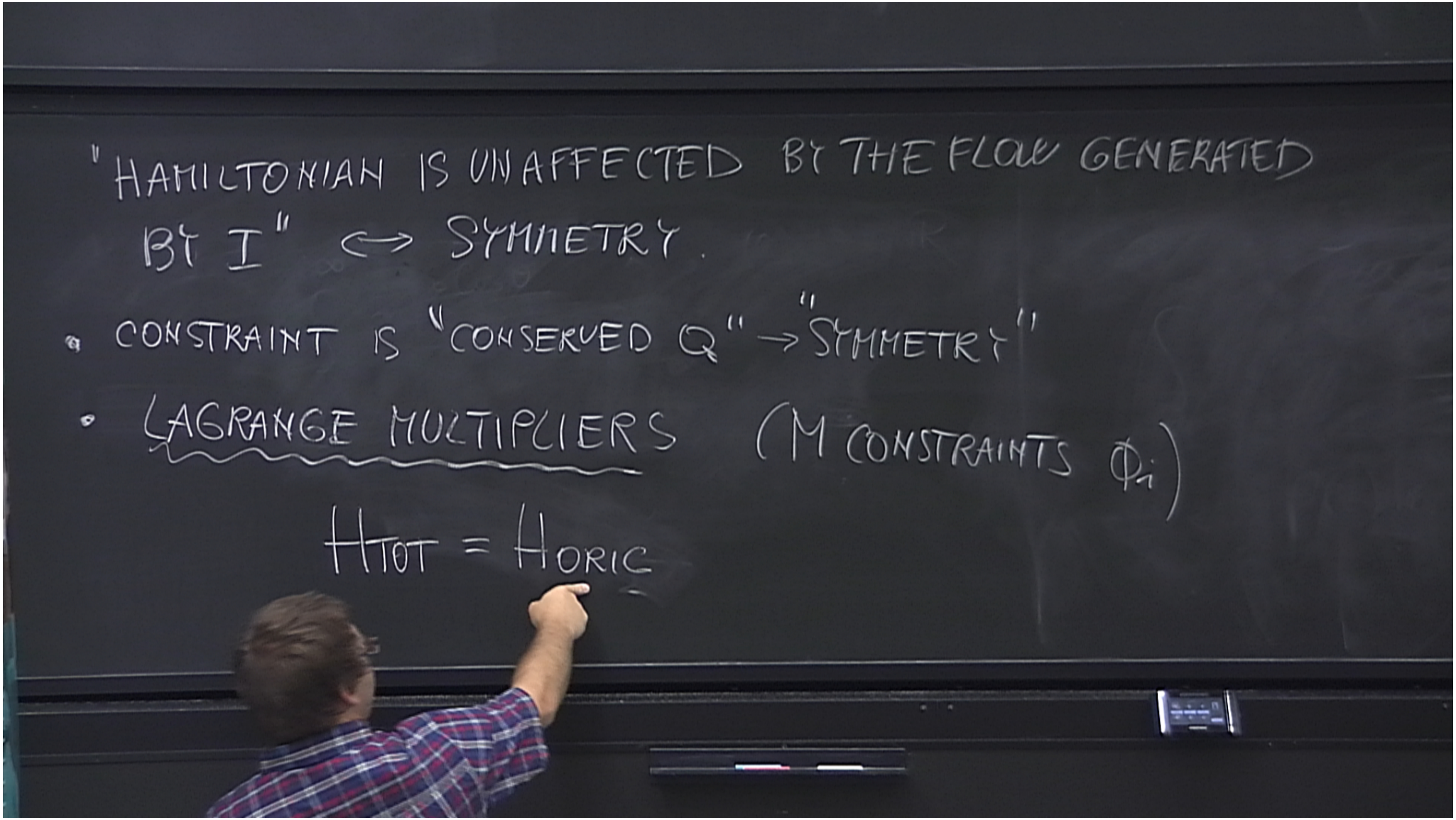
ONSTRAINT IS "CONSERVED Q" → "SYMMETRY"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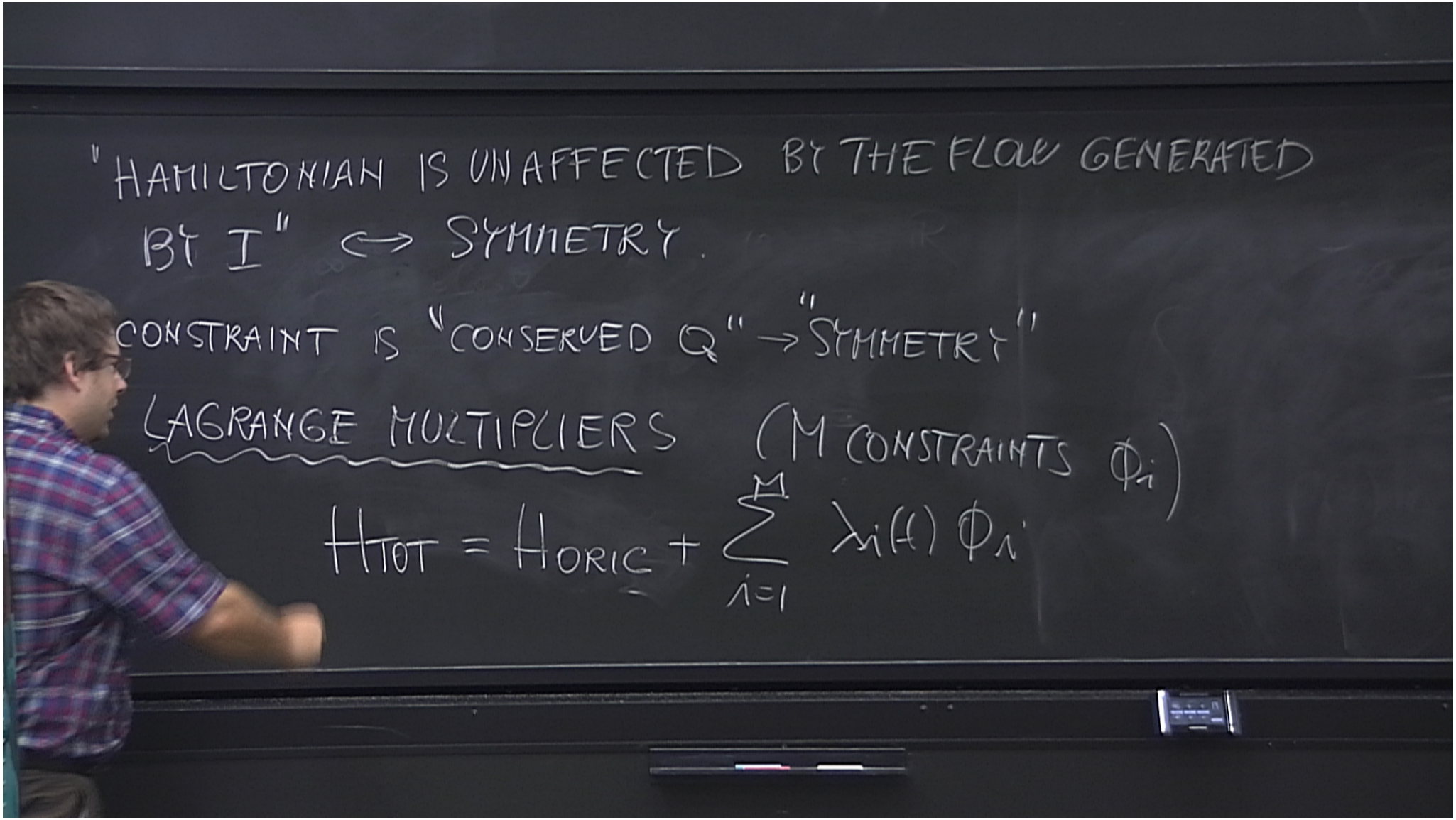
• LAGRANGE MULTIPLIERS (M CONSTRAINTS Φ_i)
$H_{TOT} = H_{ORIG}$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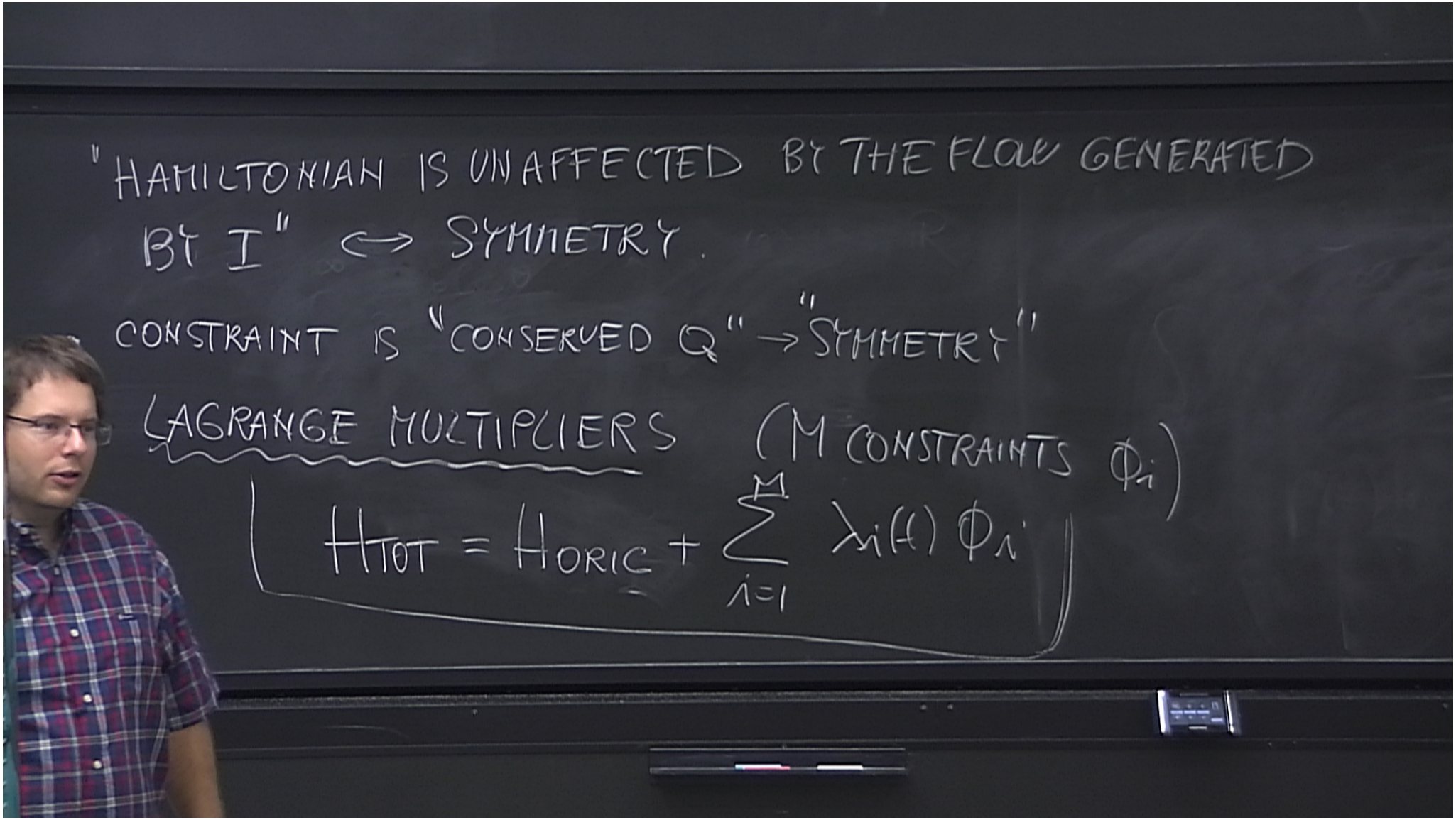
$H_{TOT} = H_{ORIG} + \sum_{i=1}^{M} \lambda_i(t)\,\phi_i$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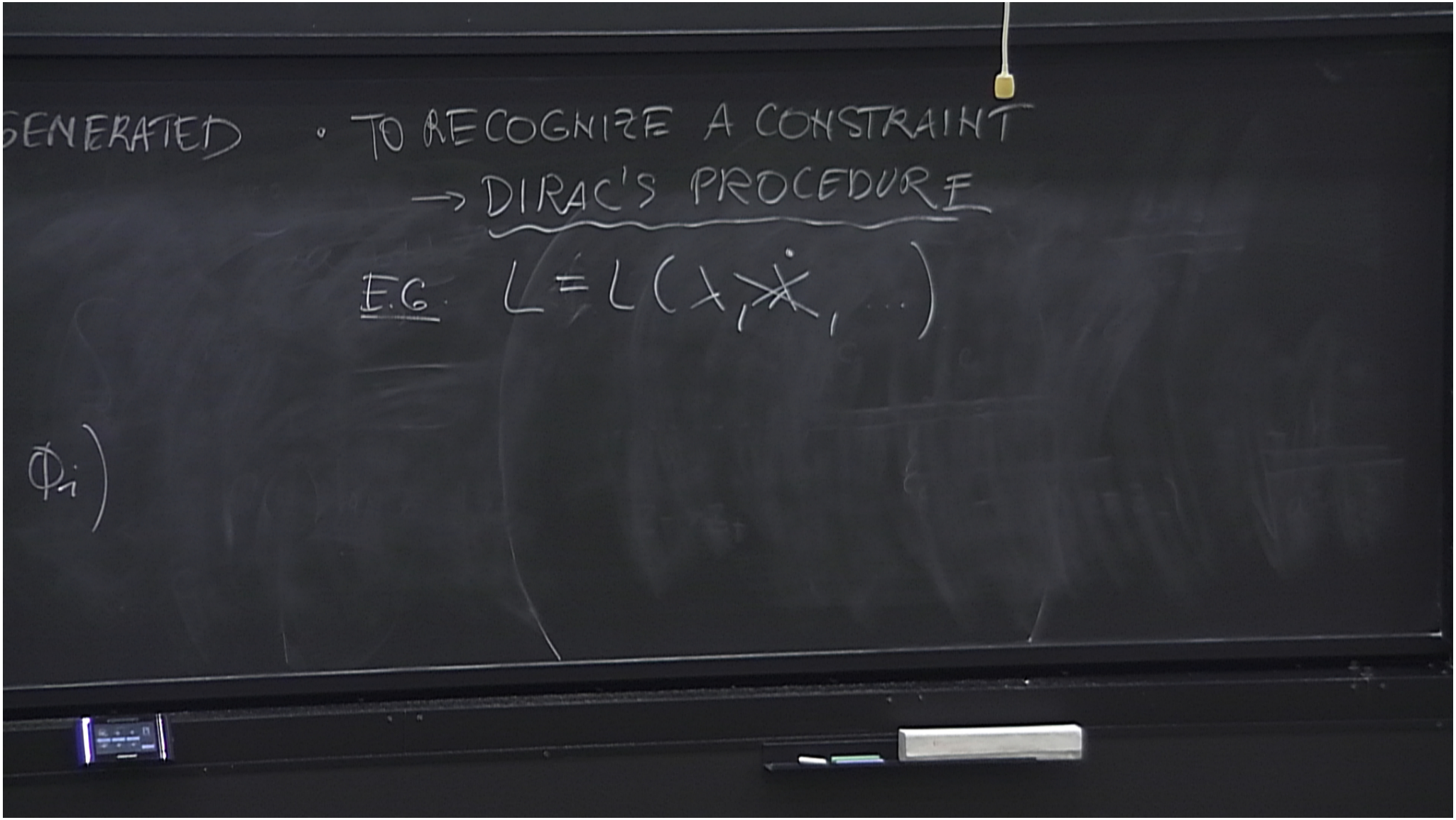
GENERATED
• TO RECOGNIZE A CONSTRAINT
→ DIRAC'S PROCEDURE
E.G. L = L(
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E.G. $L = L(\lambda, \dot{\lambda}, \ldots) \Rightarrow \lambda$ IS NOT DYNAMICAL

∴ LAGRANG MULT.

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E.G. $L = L(\lambda, \dot{\lambda}, \ldots) \Rightarrow \lambda$ IS NOT DYNAMICAL
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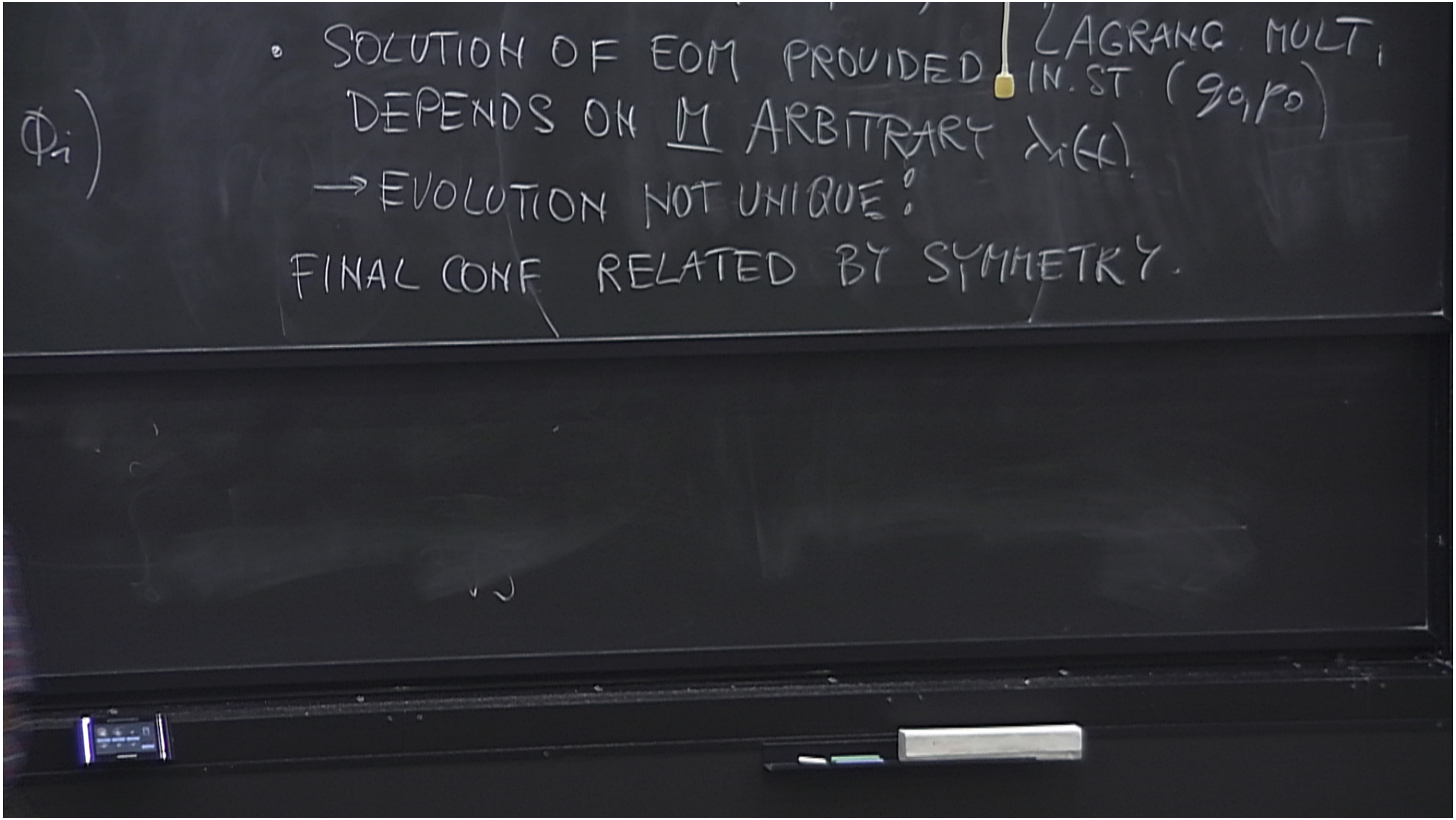
- SOLUTION OF EOM PROVIDED IN. ST (q_0, p_0) DEPENDS ON M ARBITRARY $\lambda_i(t)$.
  → EVOLUTION NOT UNIQUE!

Φ_i)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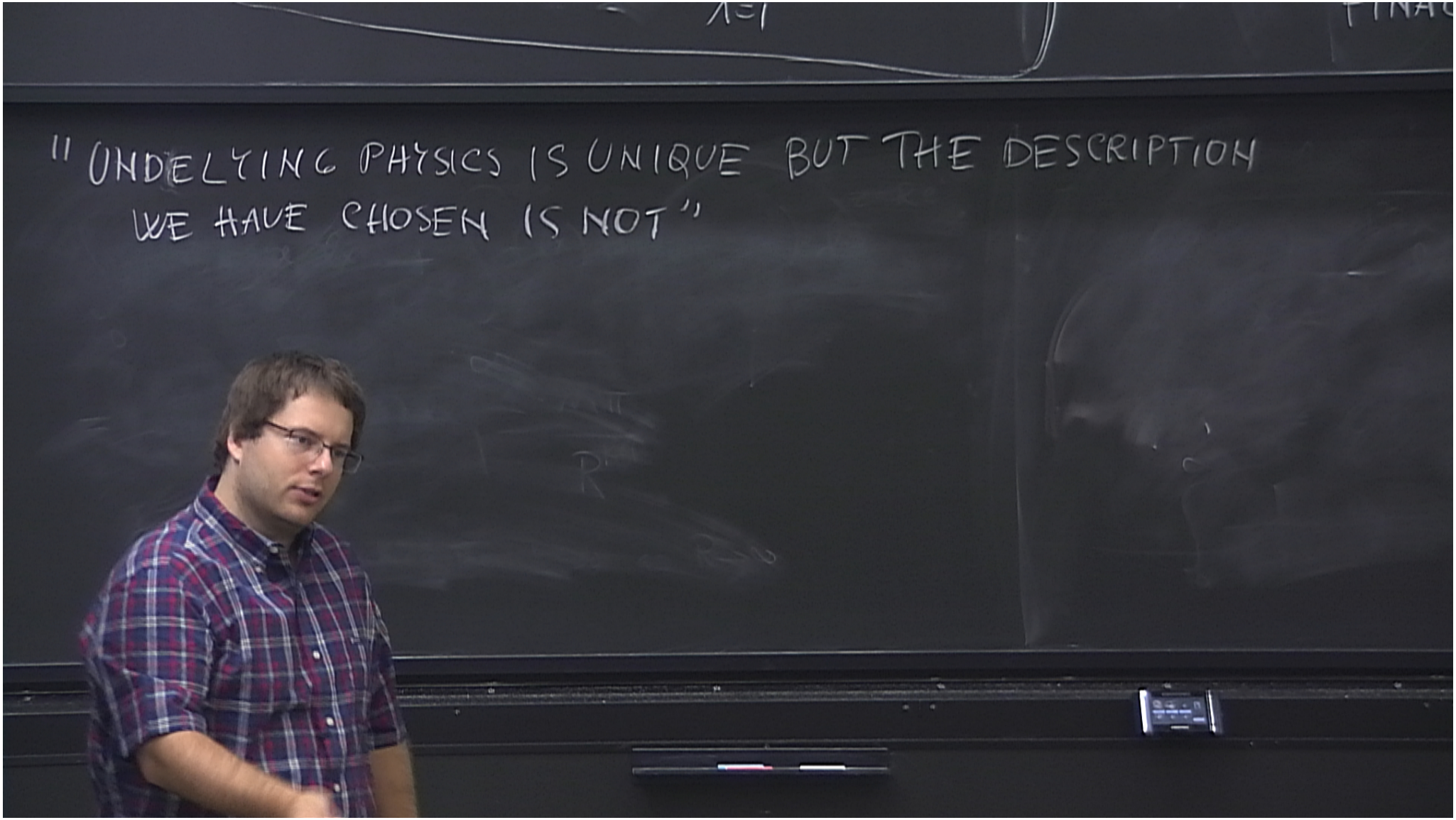
FINAL CONF RELATED BY SYMMETRY.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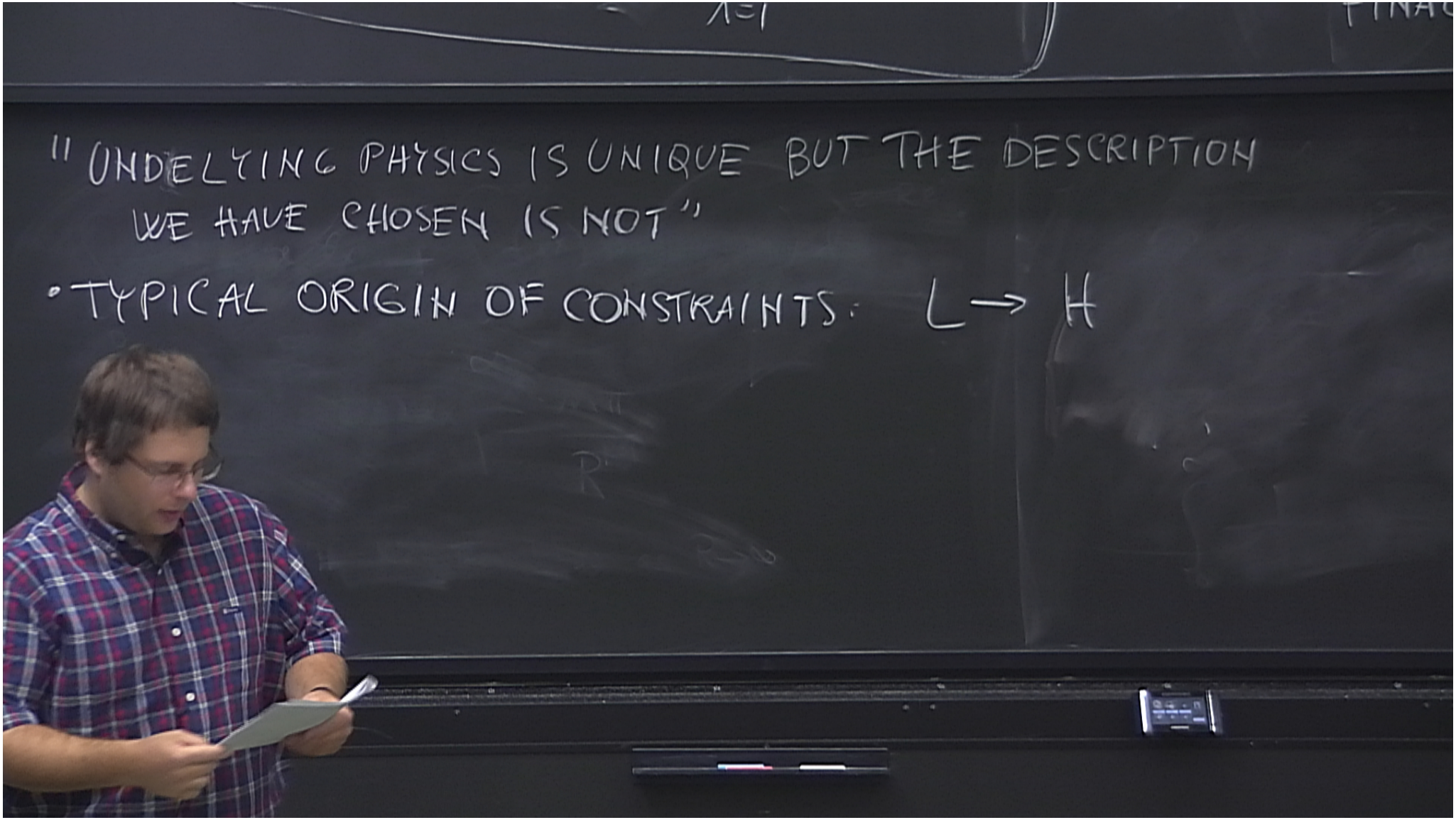
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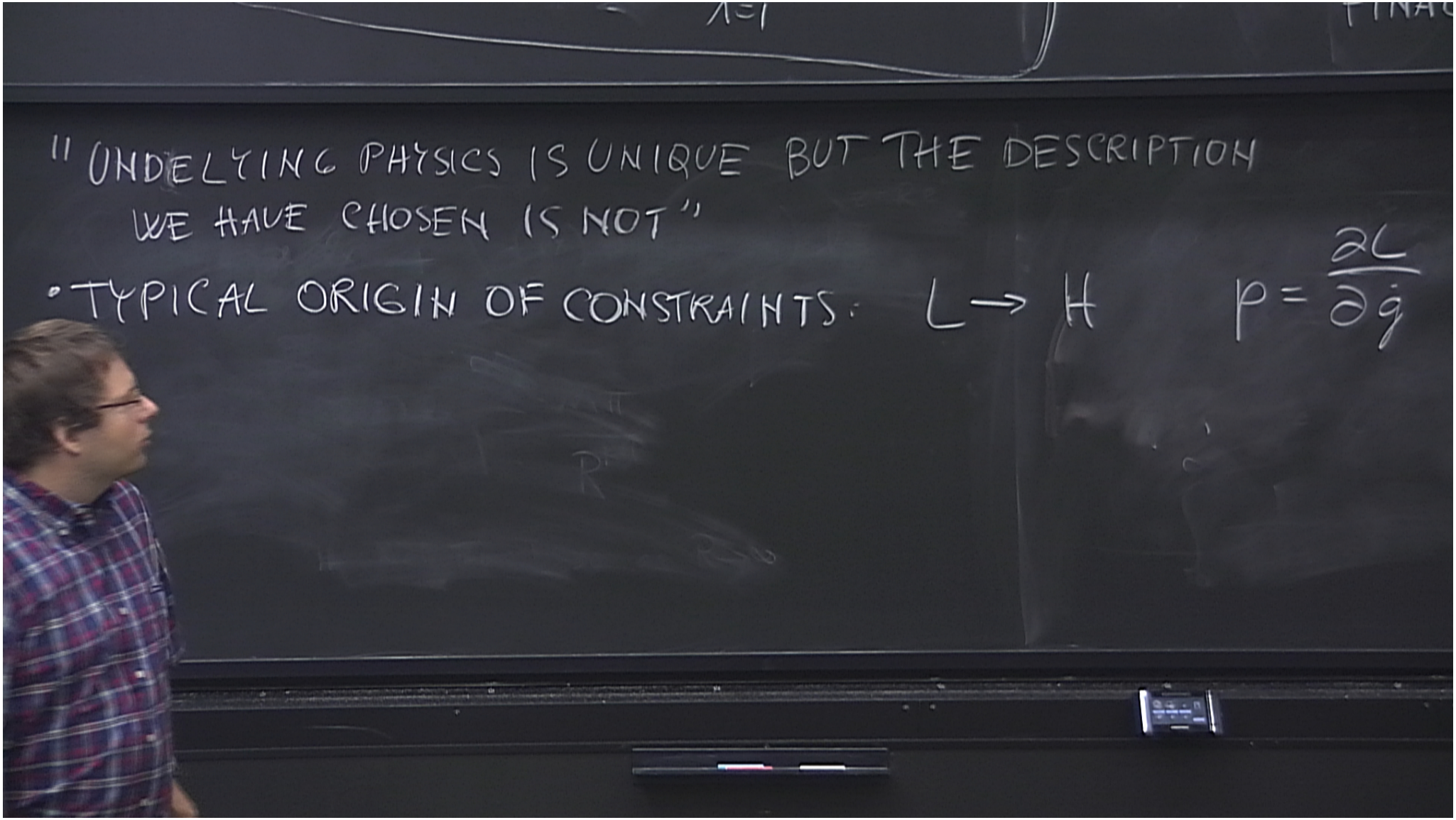
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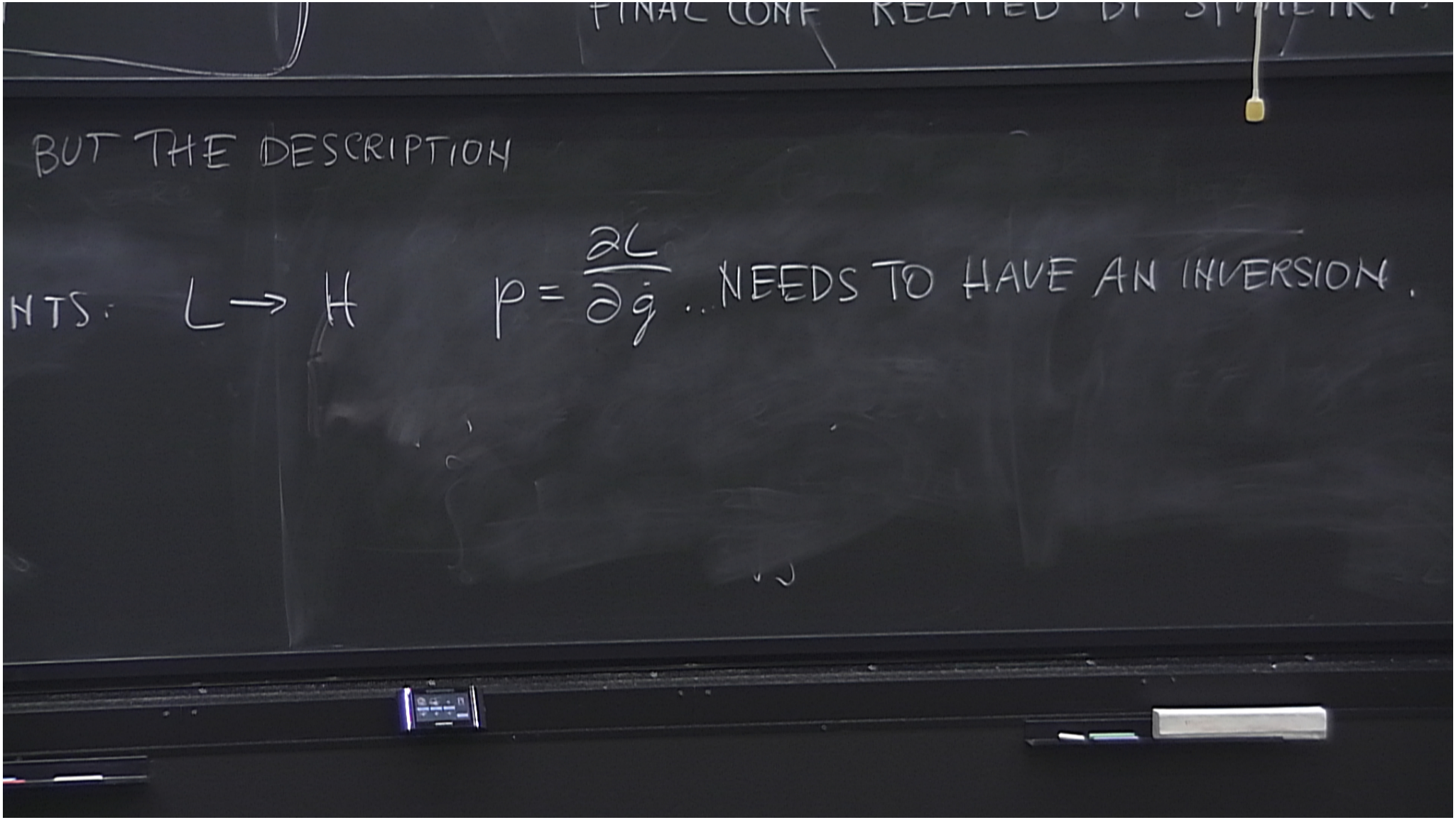
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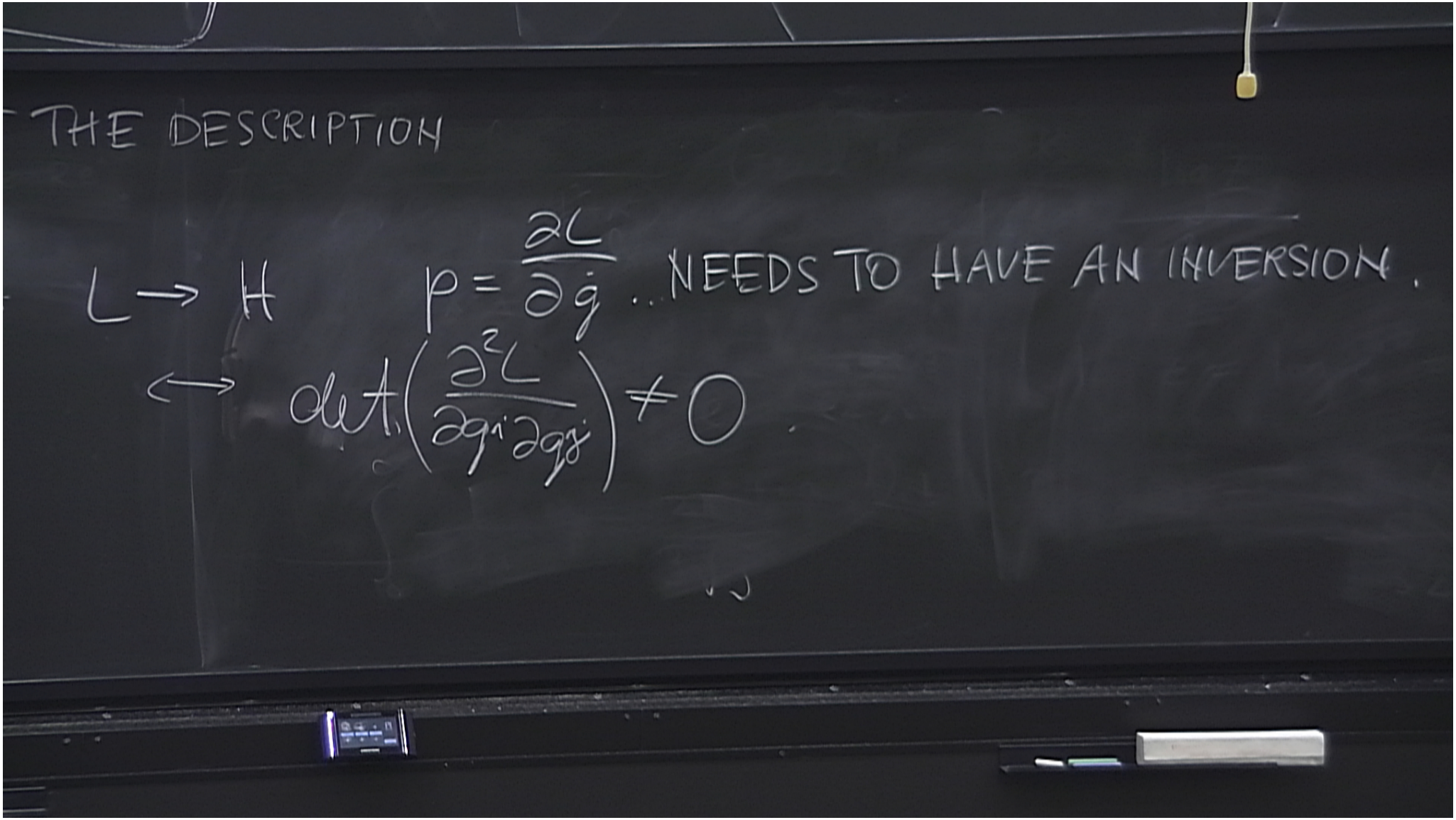
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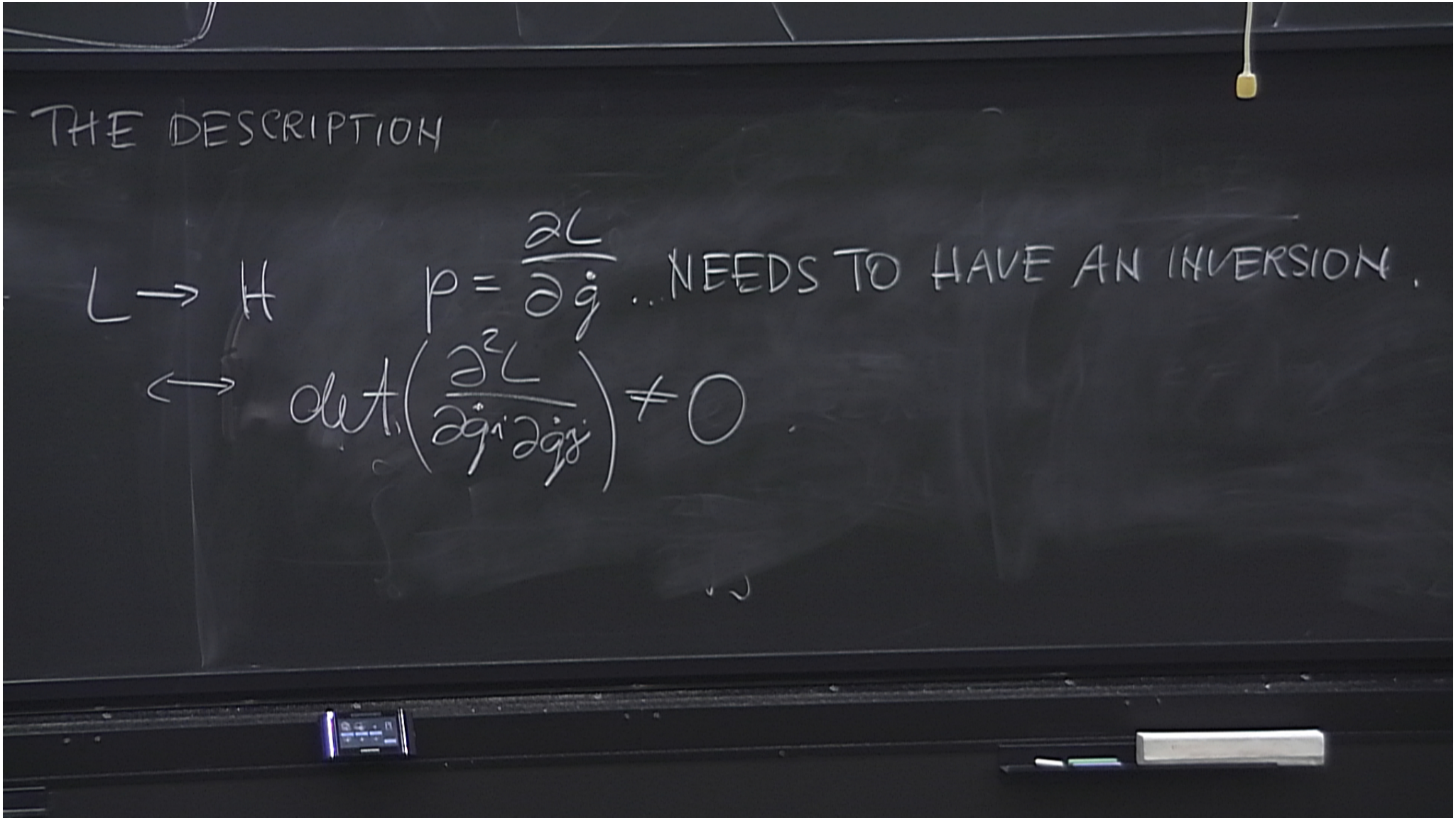
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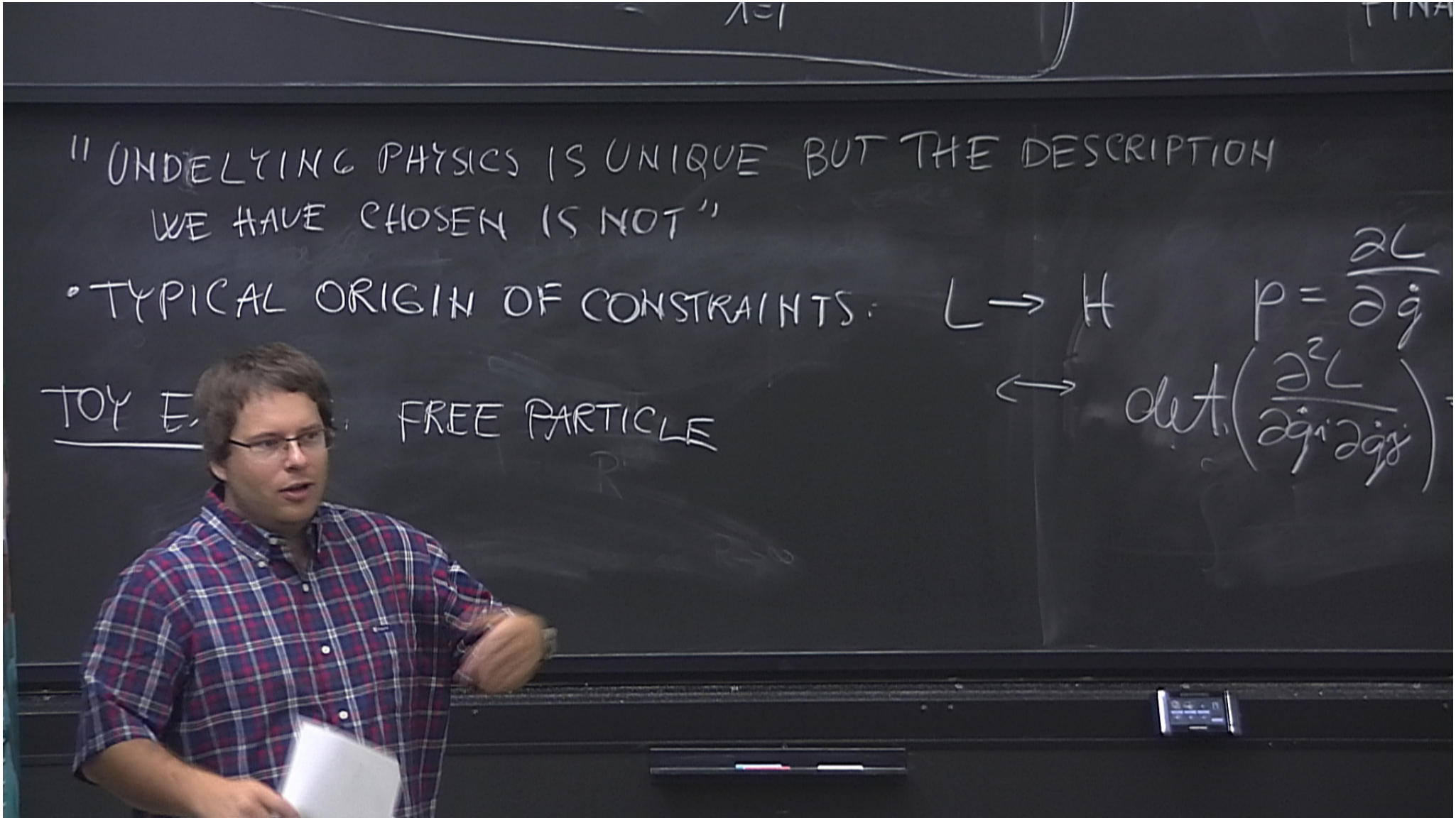
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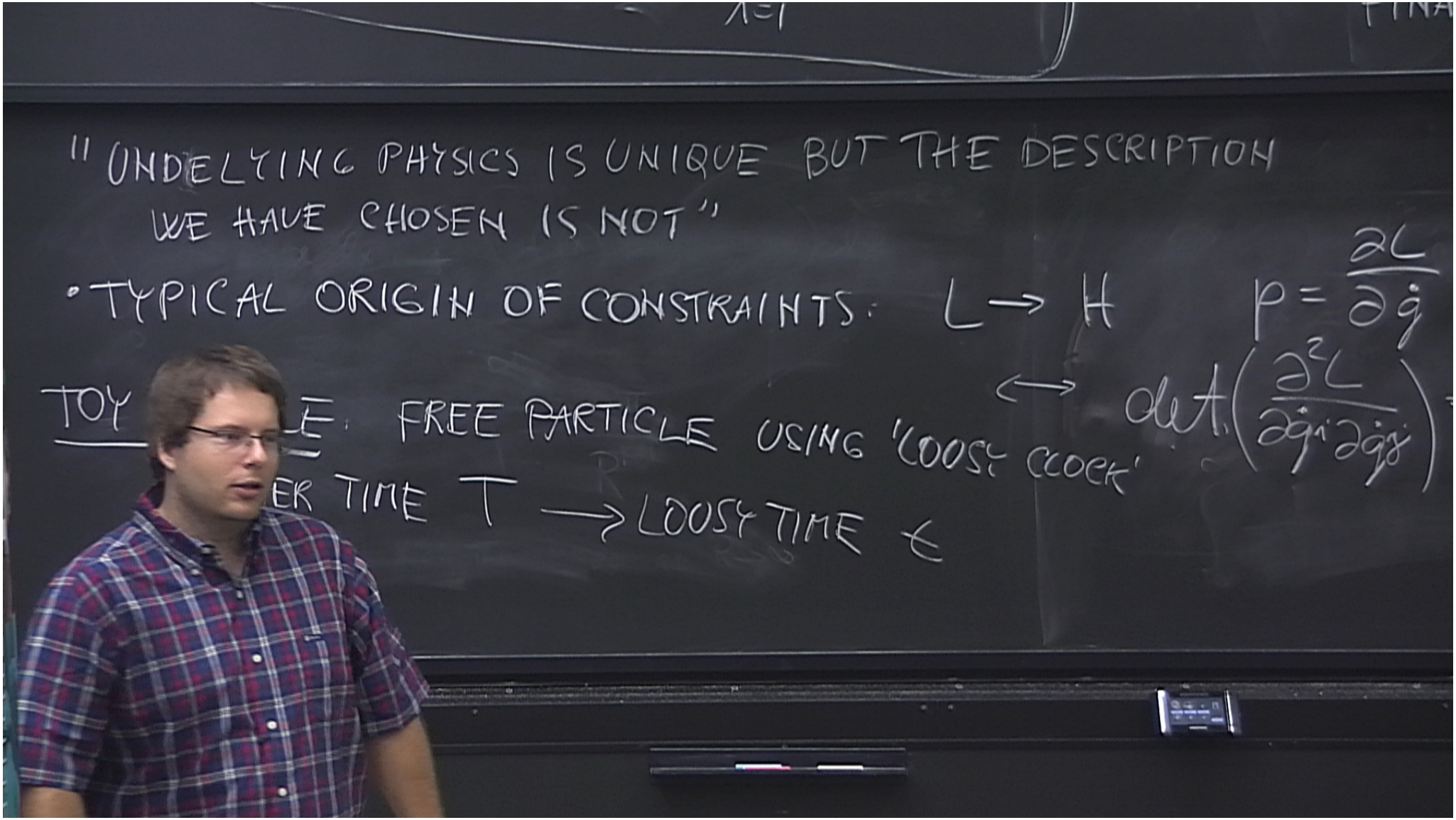
$\Leftrightarrow$ det. $\left(\frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j}\right)$
TOY E
FREE PARTICLE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TOY E: FREE PARTICLE USING 'LOOSE CLOCK'
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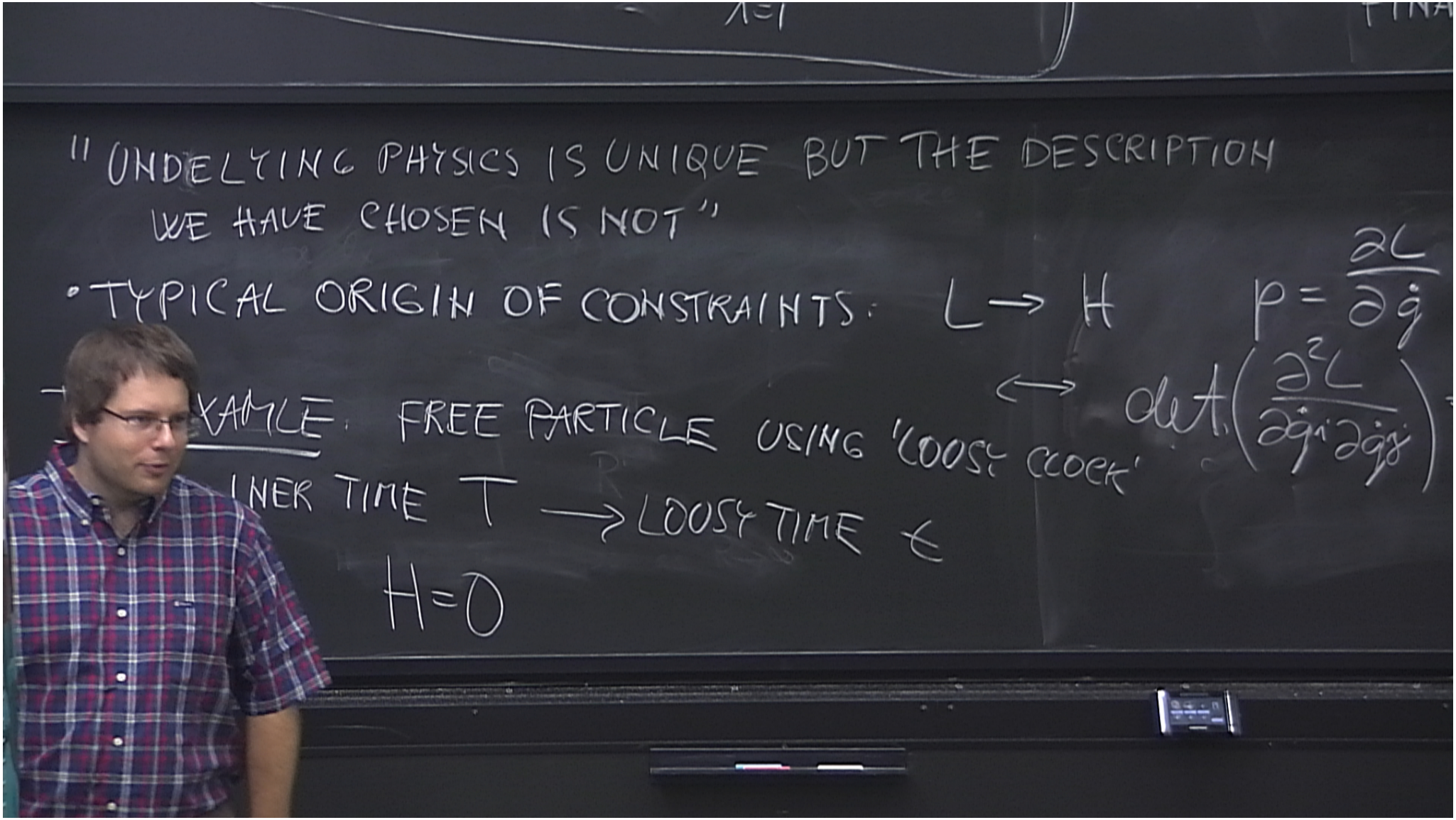
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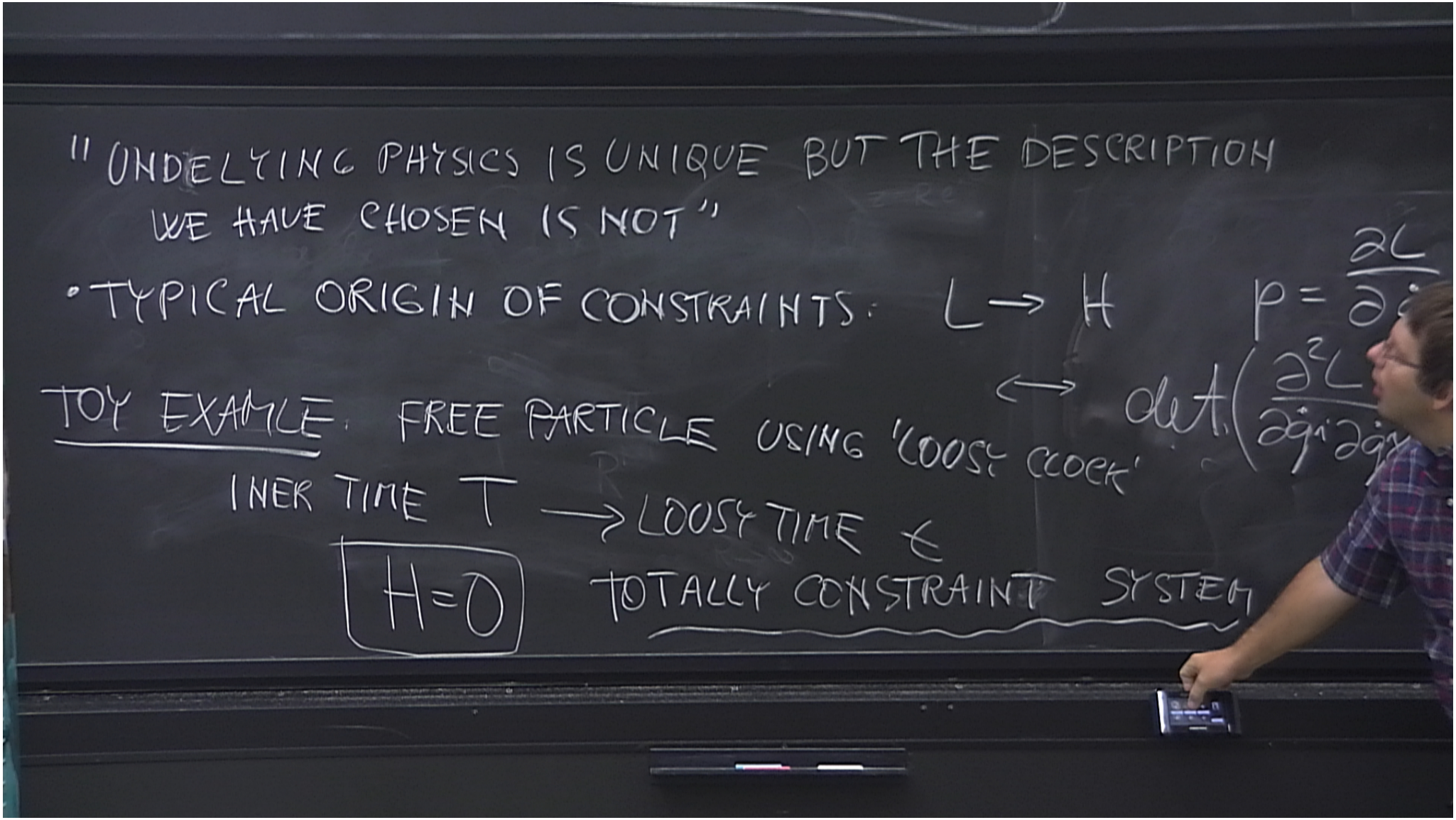
XAMLE: FREE PARTICLE USING 'LOOSY CLOCK'
INER TIME T → LOOSY TIME
H=0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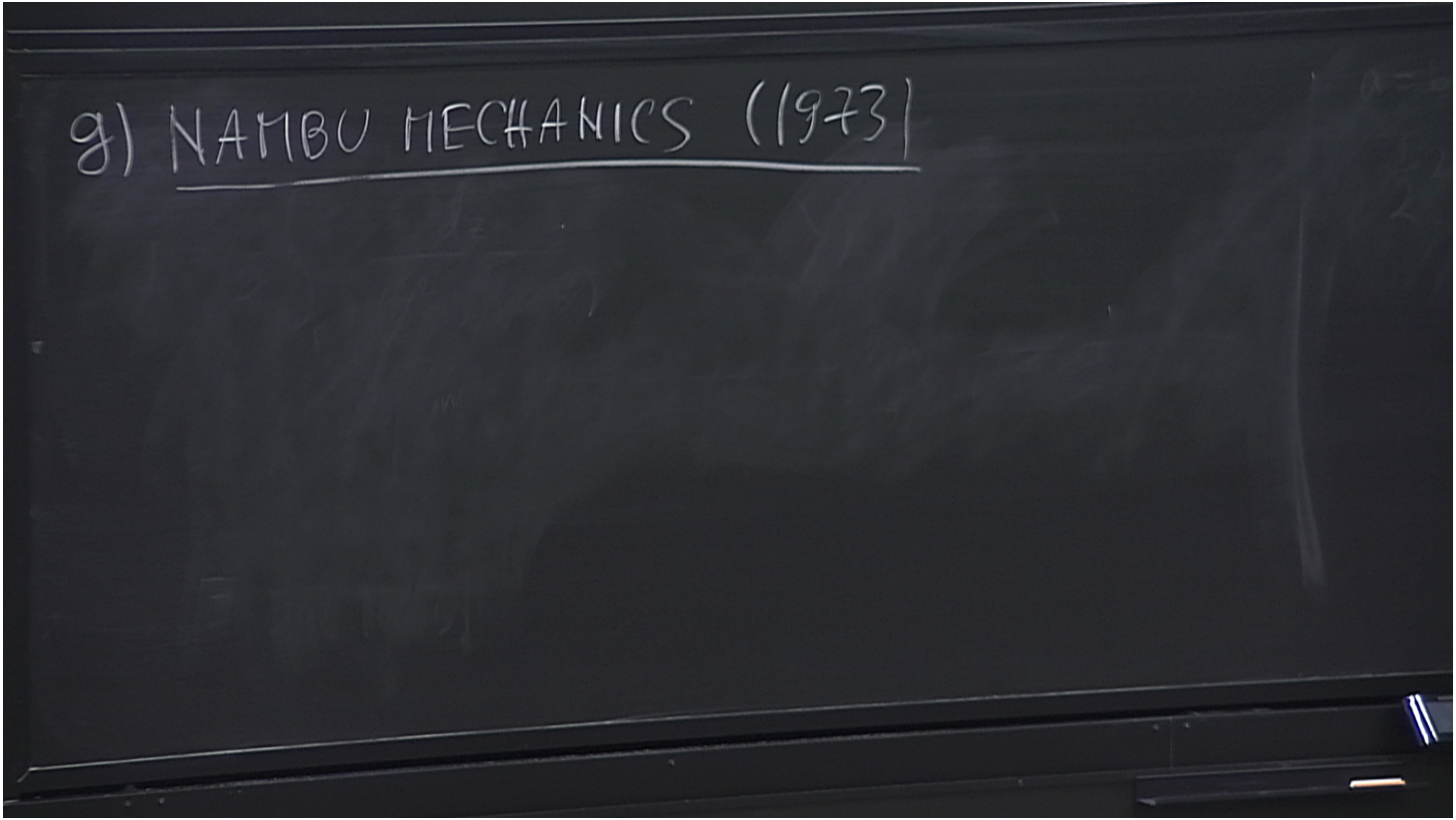
• TYPICAL ORIGIN OF CONSTRAINTS: L → H
TOY EXAMPLE: FREE PARTICLE USING 'LOOSE CLOCK'
INER TIME T → LOOSE TIME ε
H=0
TOTALLY CONSTRAINT SYSTEM

$p = \frac{\partial L}{\partial \dot{q}}$ .. NEEDS TO HAVE AN INVERSION.

$\det\left(\frac{\partial^2 L}{\partial \dot{q}^i \partial \dot{q}^j}\right) \neq 0$.

SYSTEM $\exists$ CONSTRAINT $\phi = p_T + \frac{p_x^2}{2m} = 0$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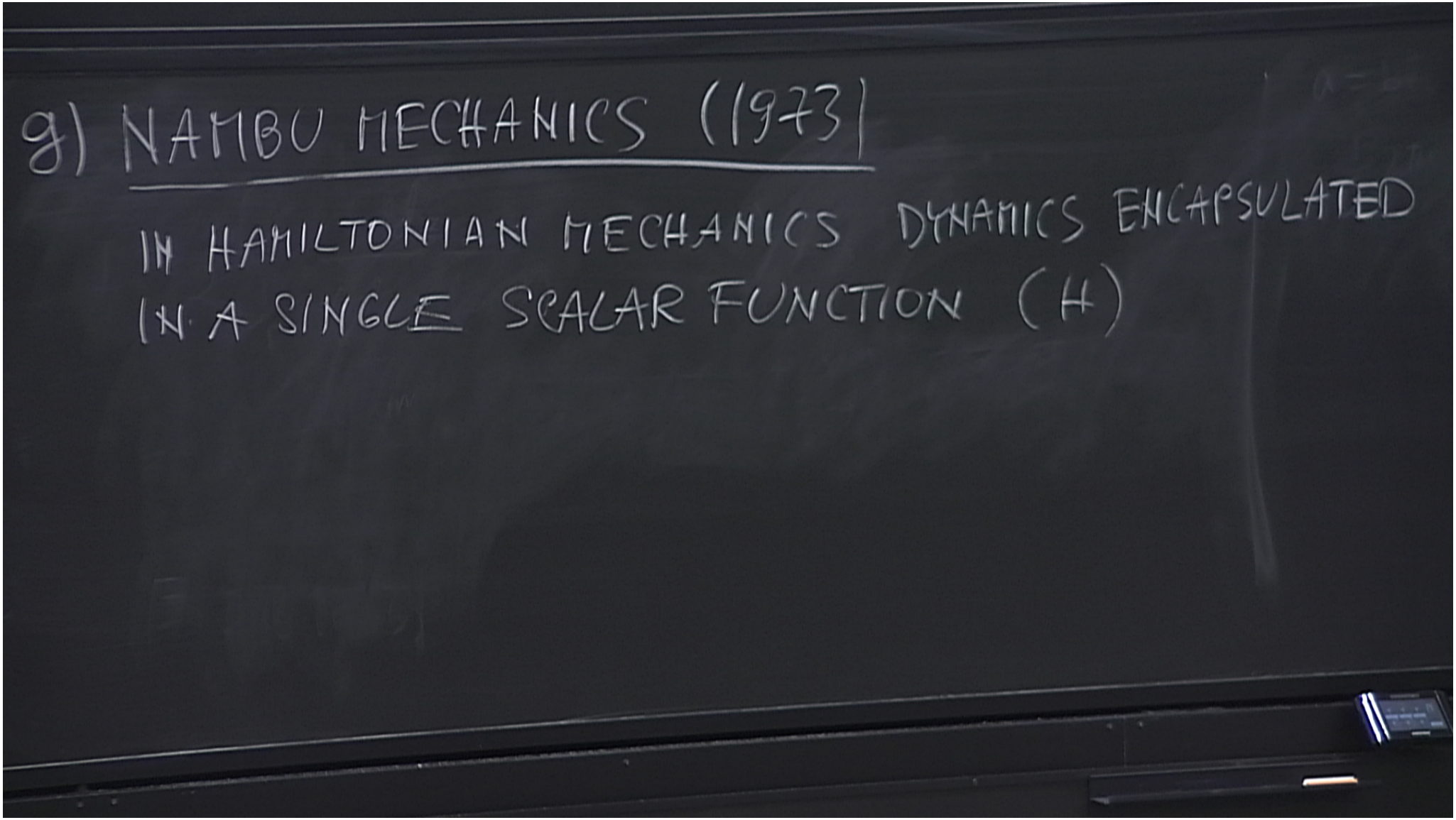
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- MEANS: PHASE-SPACE IS EVEN-DIMENSIONAL.

IN A SINGLE SCALAR FUNCTION (H)

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IN A SINGLE SCALAR FUNCTION (H)

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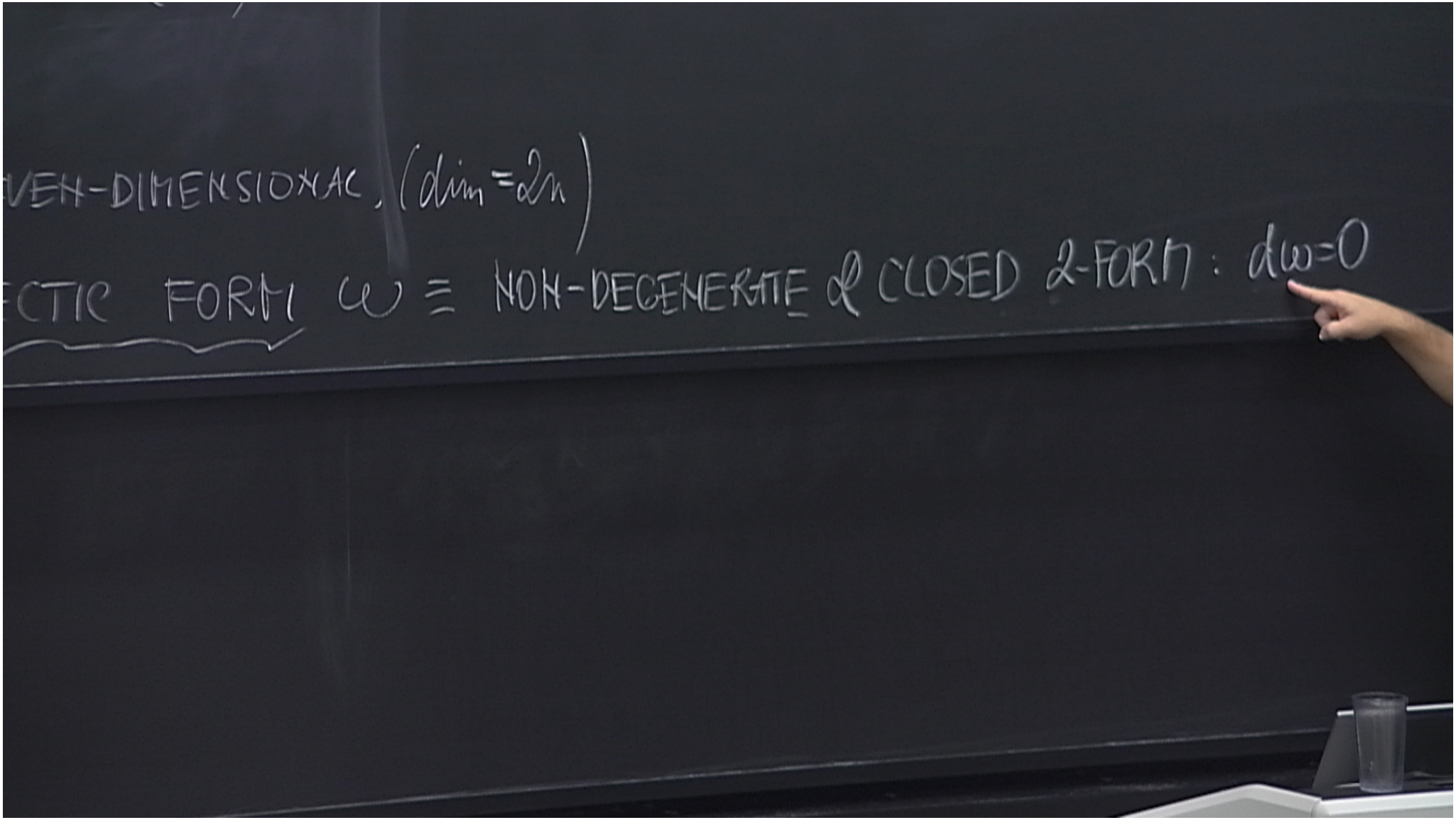
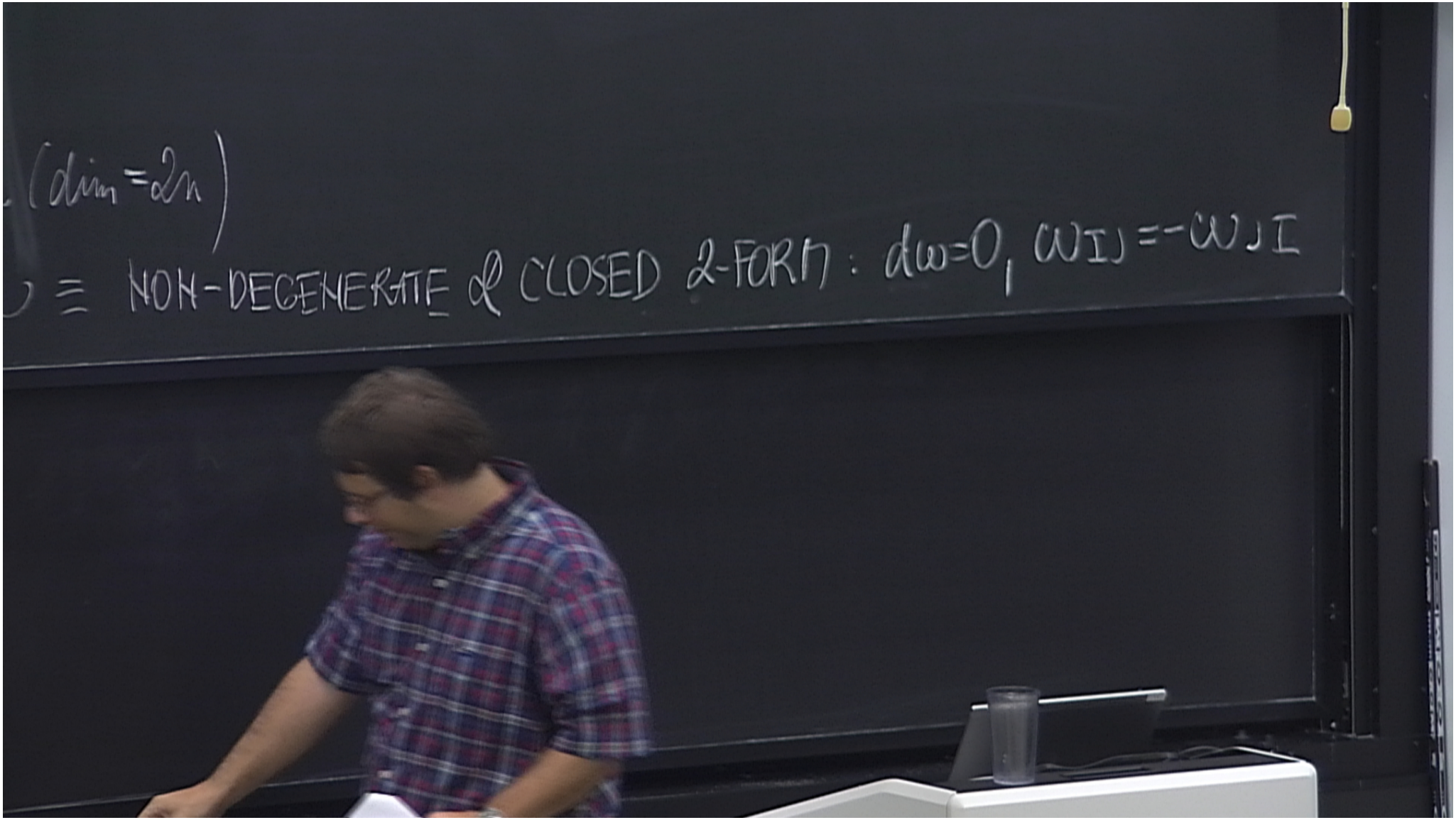
- MEANS: PHASE-SPACE IS EVEN-DIMENSIONAL, (dim = 2n

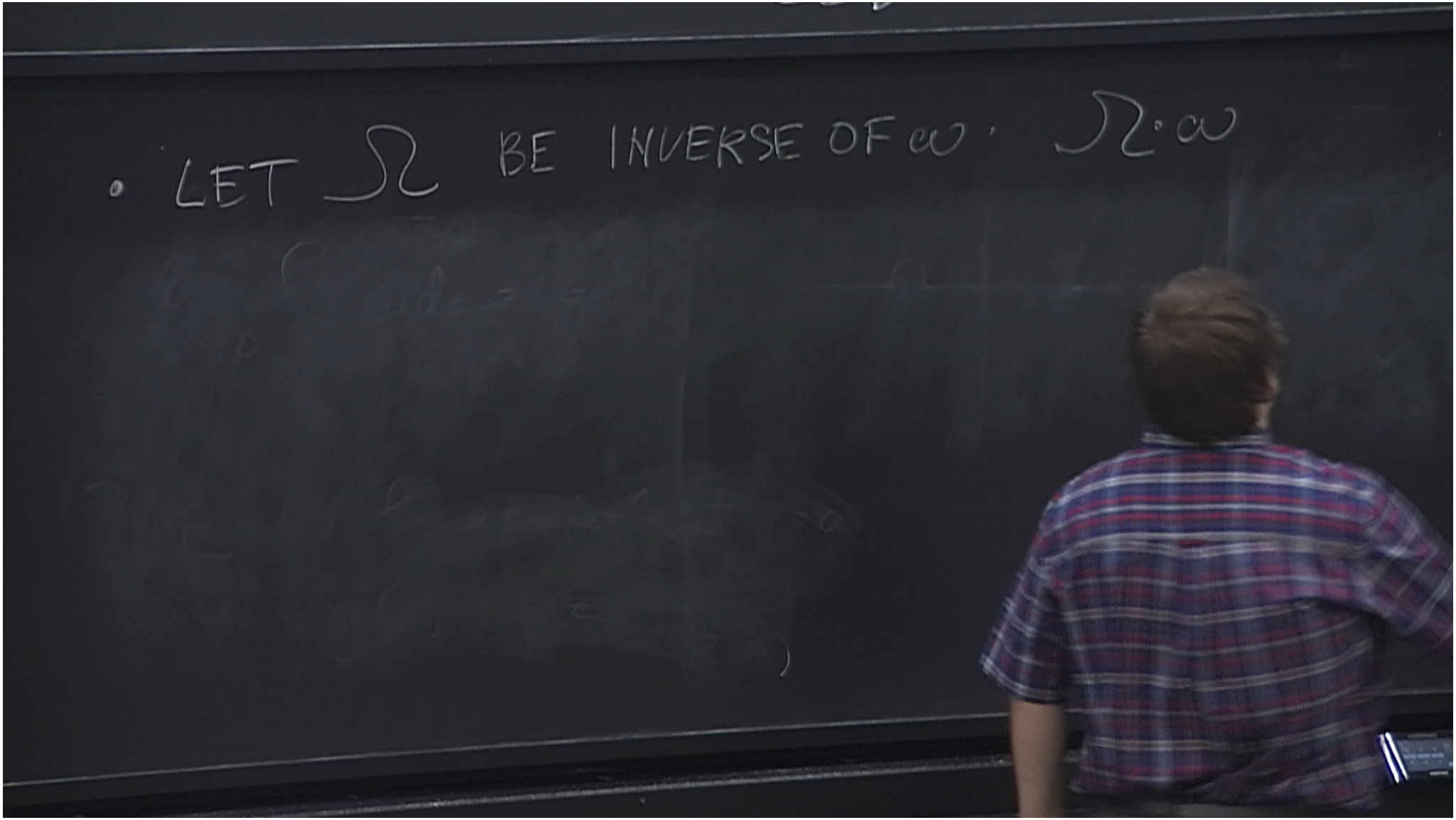
EQUIPED WITH SYMPLECTIC FORM ω

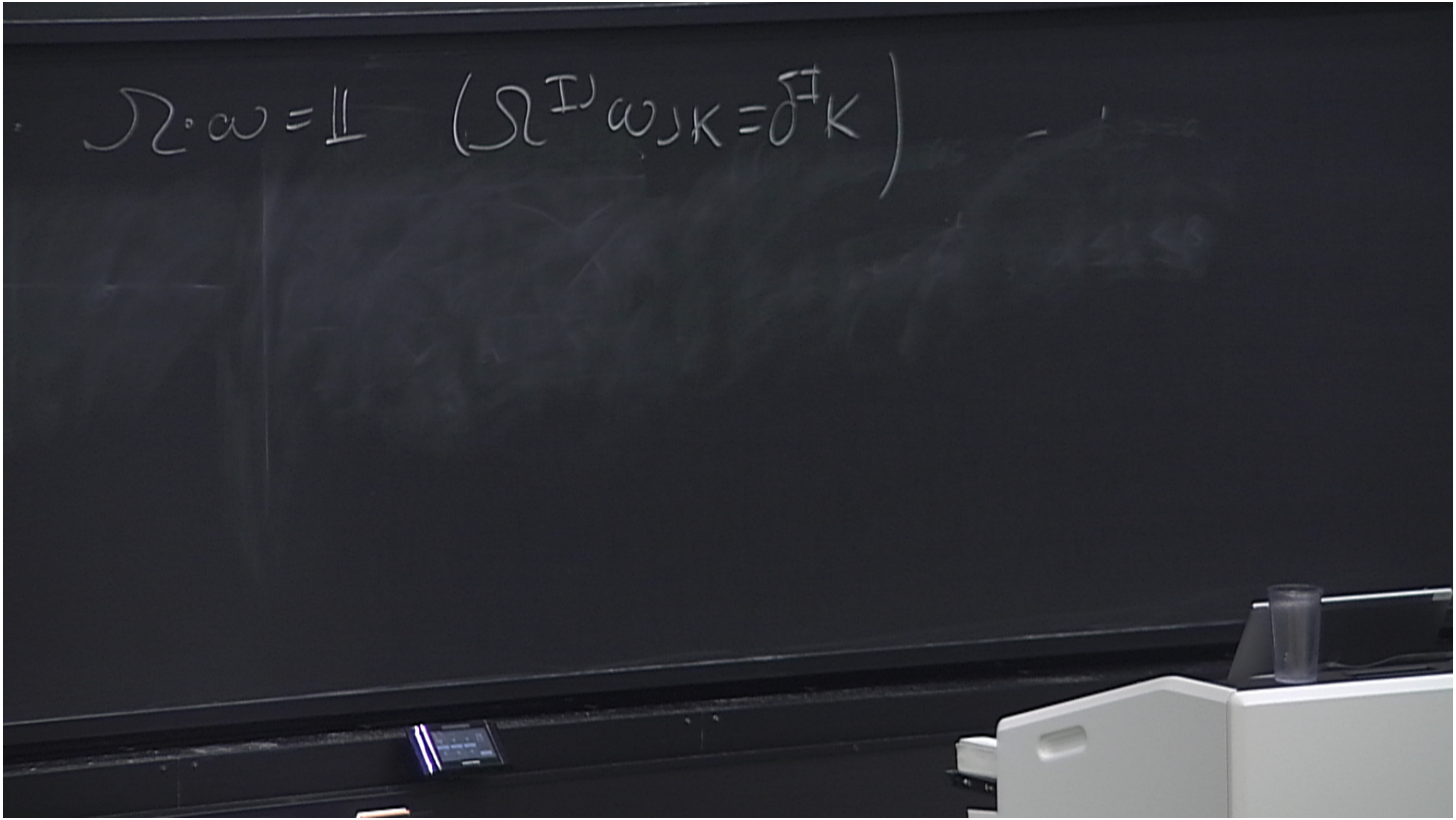
IS EVEN-DIMENSIONAL. $(\dim = 2n)$

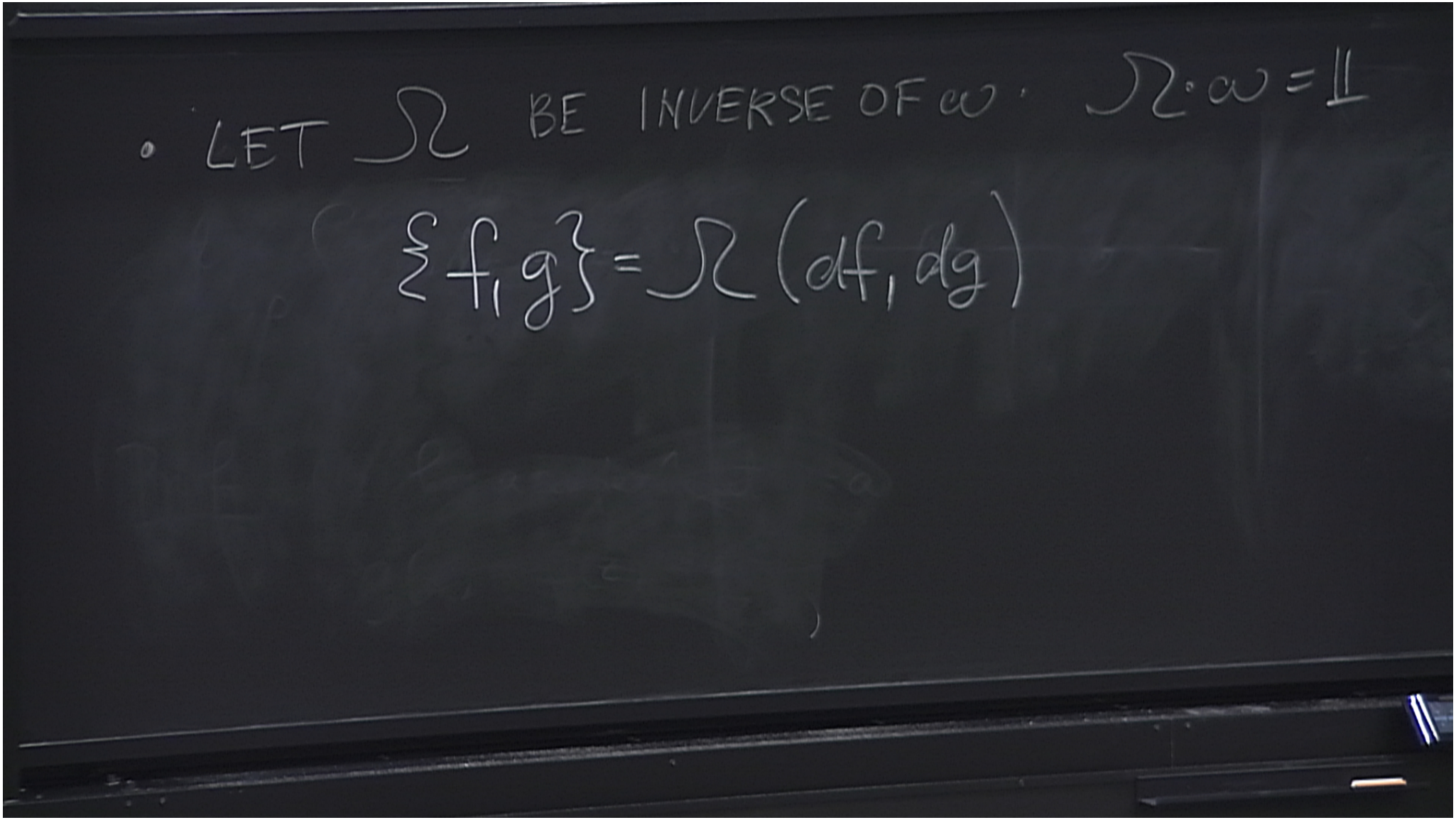
SYMPLECTIC FORM $\omega \equiv$ NON-DEGENERATE & CLOSED 2-FORM

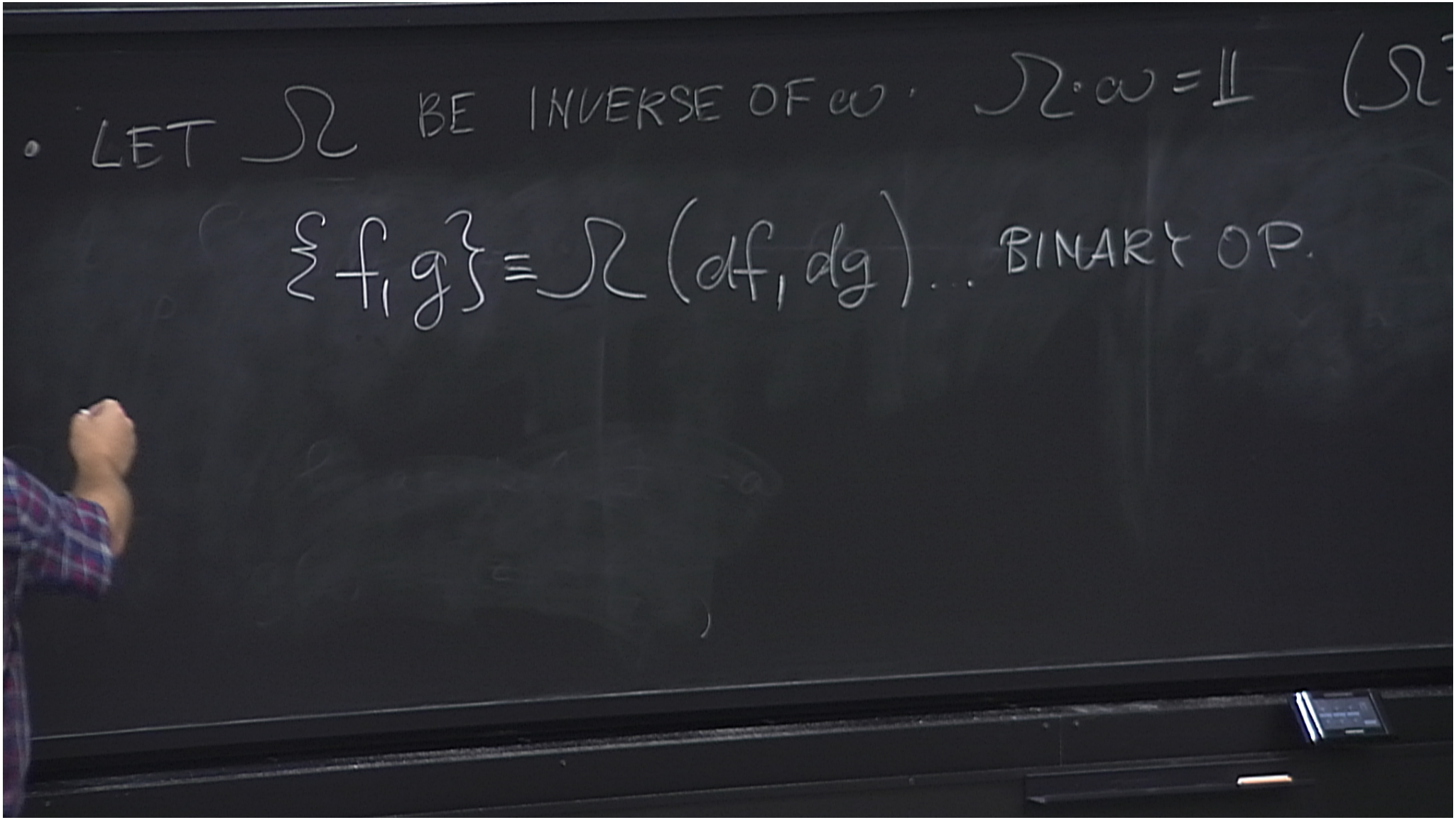

VEN-DIMENSIONAL, (dim = 2n)
ECTIC FORM ω ≡ NON-DEGENERATE & CLOSED 2-FORM : dω = 0

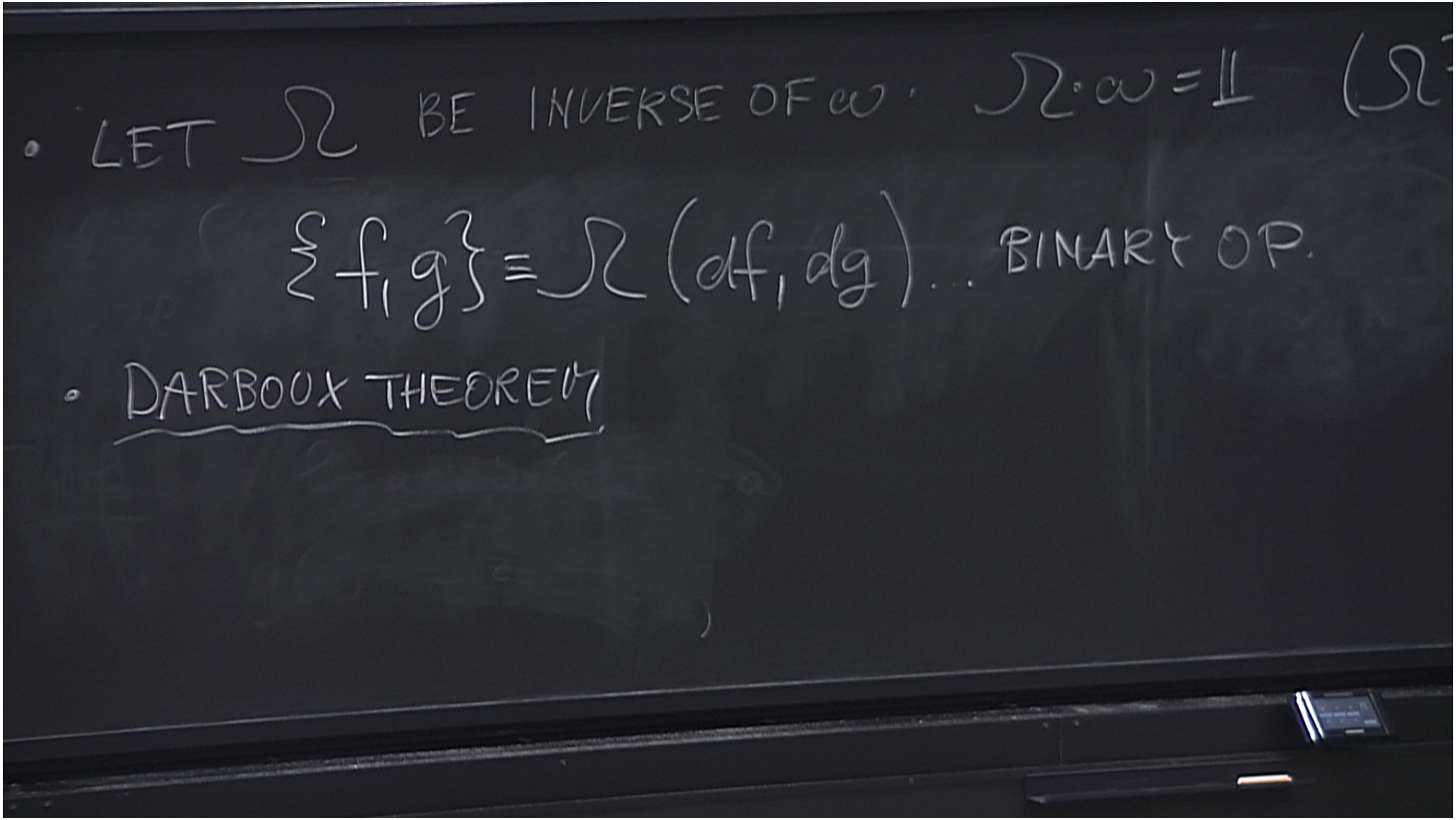

(dim = 2n)
≡ NON-DEGENERATE & CLOSED 2-FORM : dω = 0, ω_IJ = −ω_JI


• LET Ω BE INVERSE OF ω · Ω·ω


$\Omega \cdot \omega = \mathbb{1}$ $(\Omega^{IJ}\omega_{JK} = \delta^{I}{}_{K})$


• LET Ω BE INVERSE OF ω. Ω·ω = 𝟙
{f, g} = Ω(df, dg)


• LET Ω BE INVERSE OF ω. Ω·ω = 𝟙
{f, g} ≡ Ω(df, dg) ... BINARY OP.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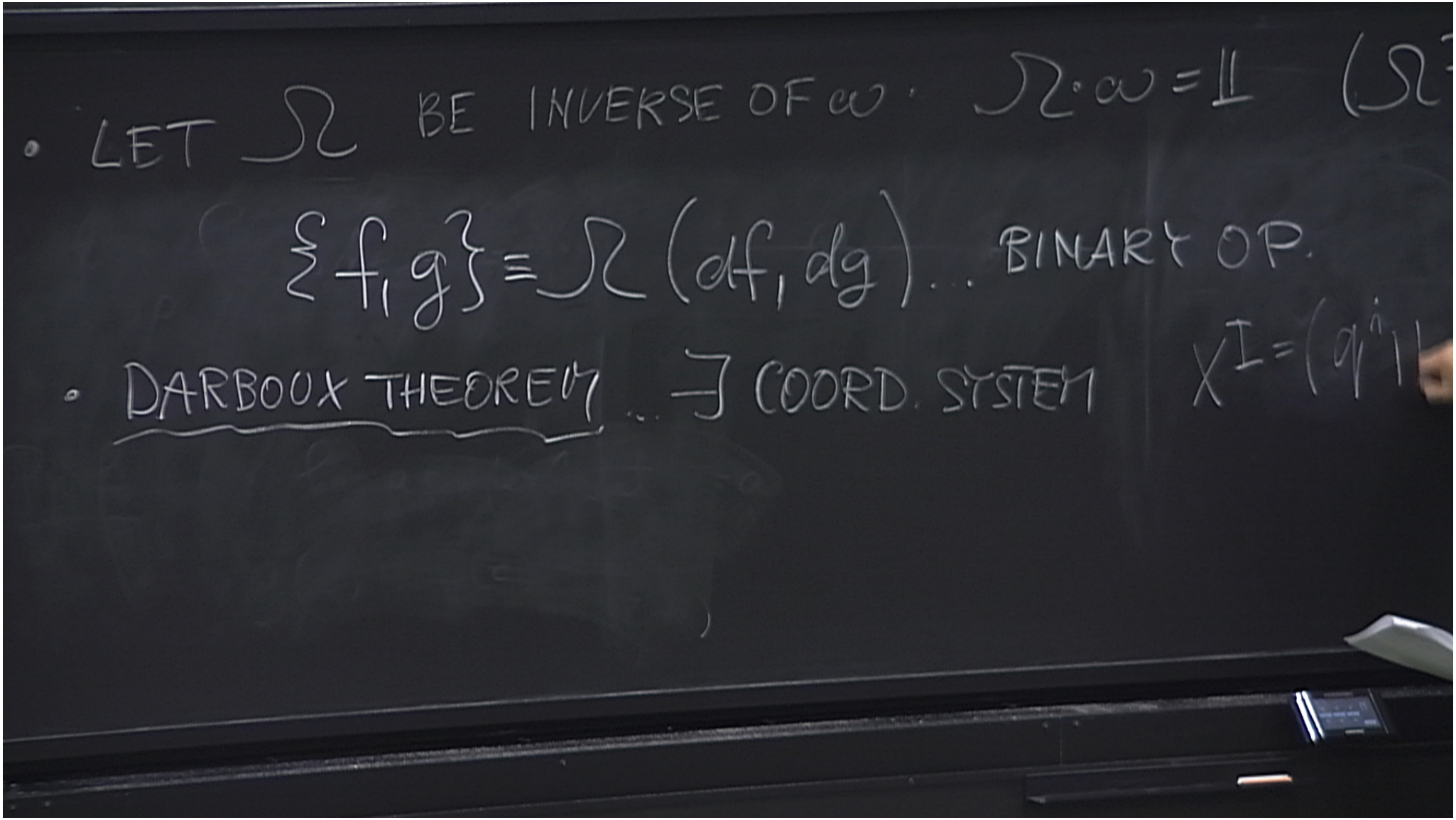
• DARBOUX THEOREM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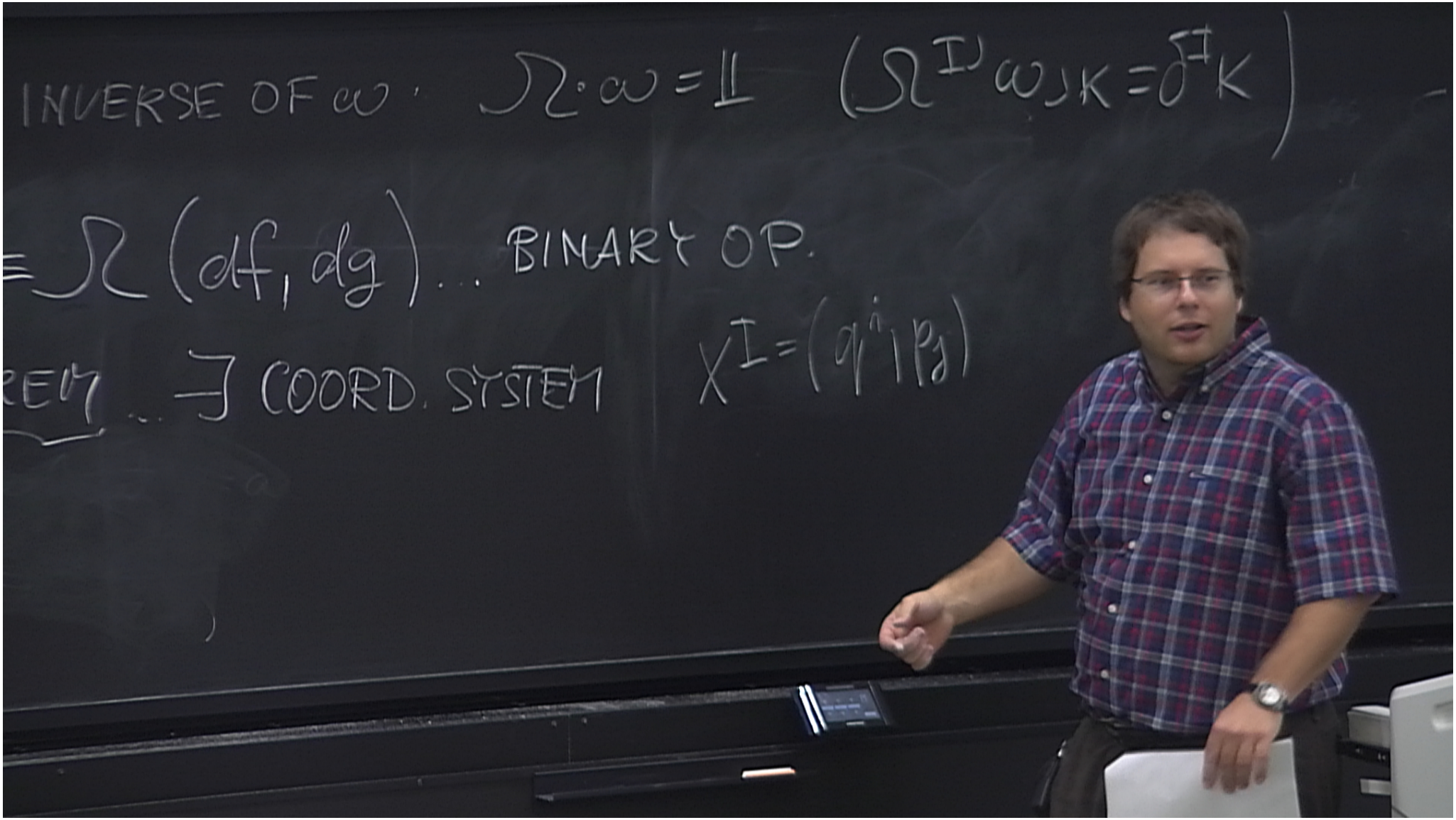
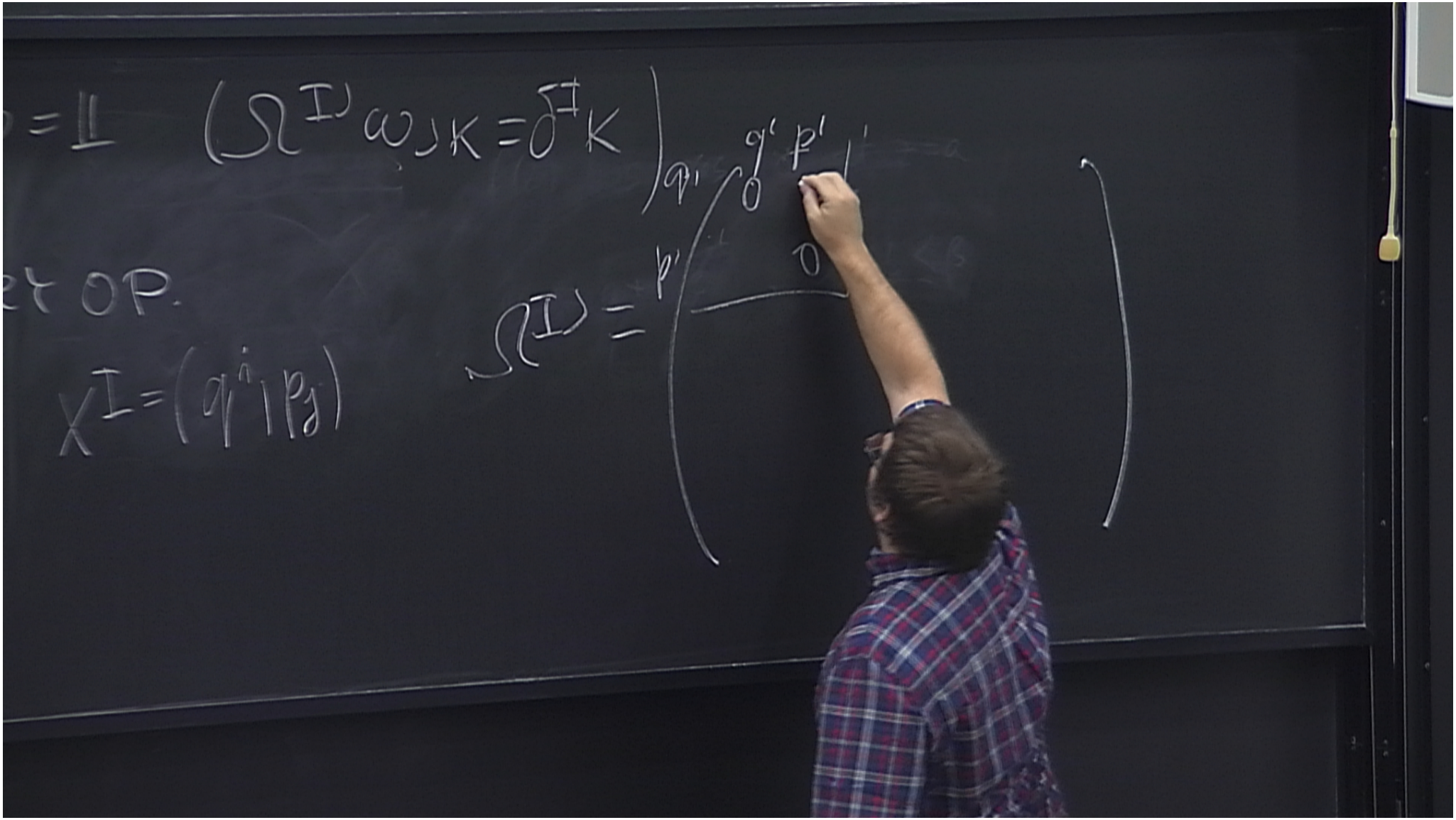
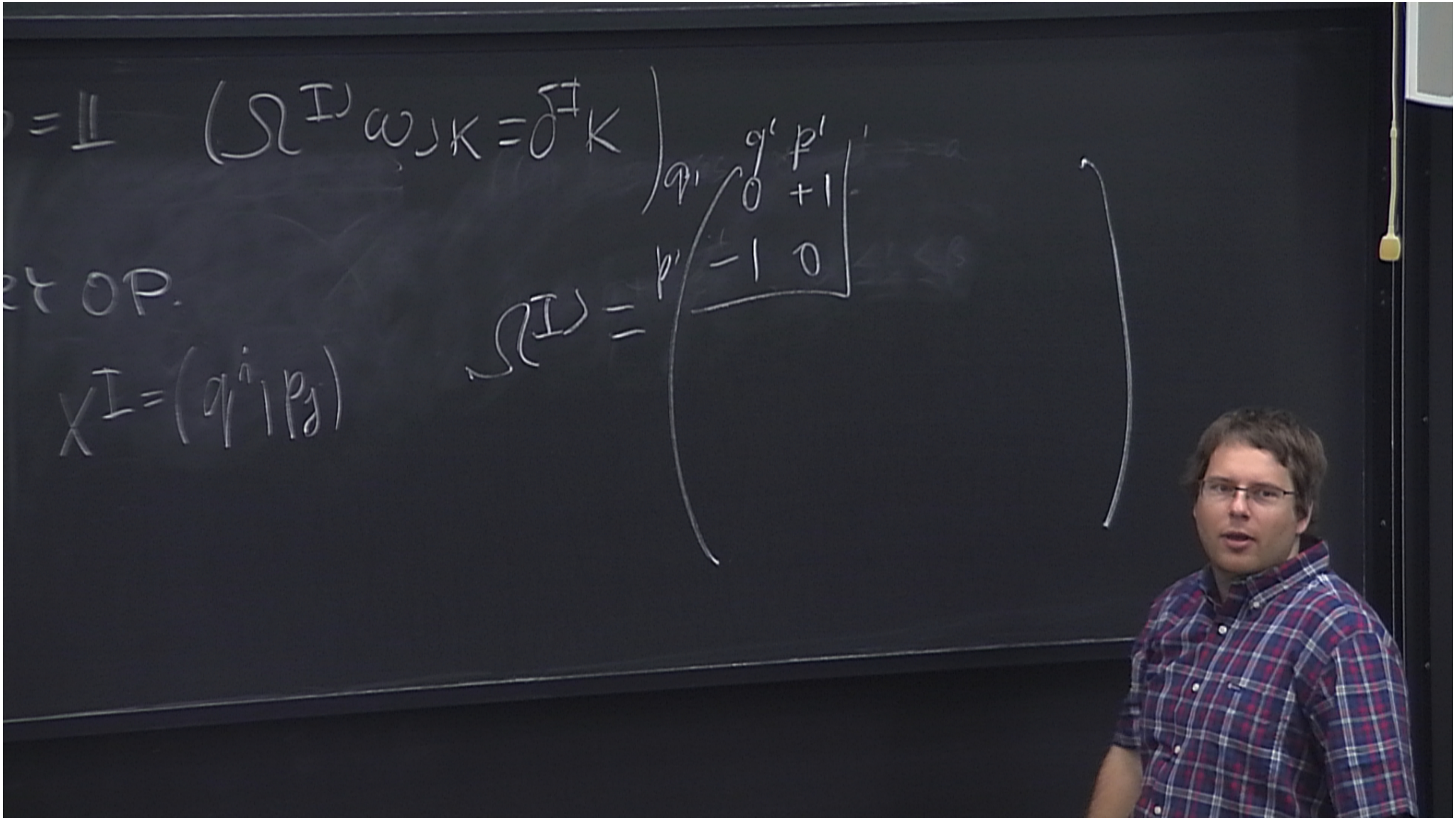
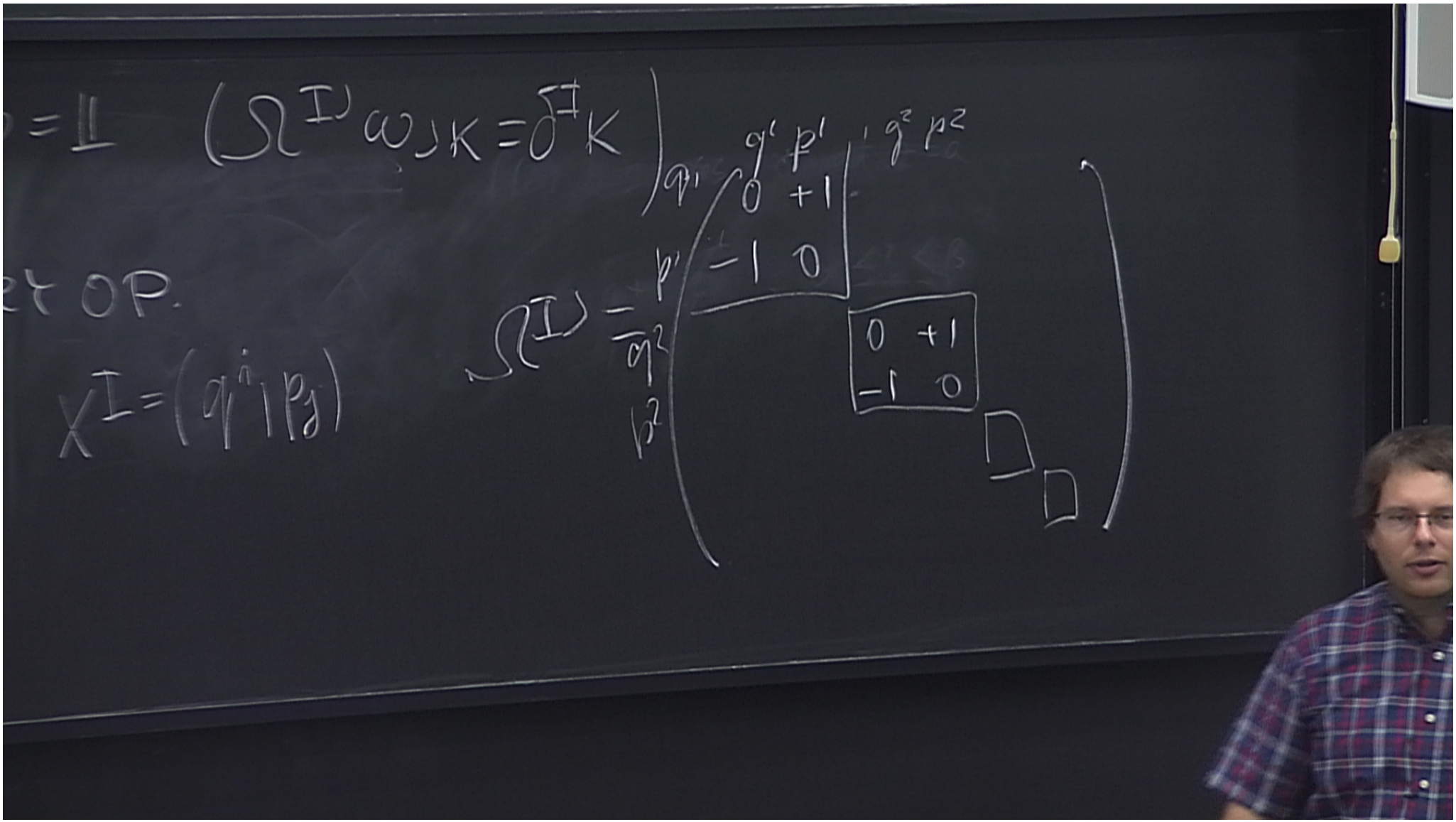
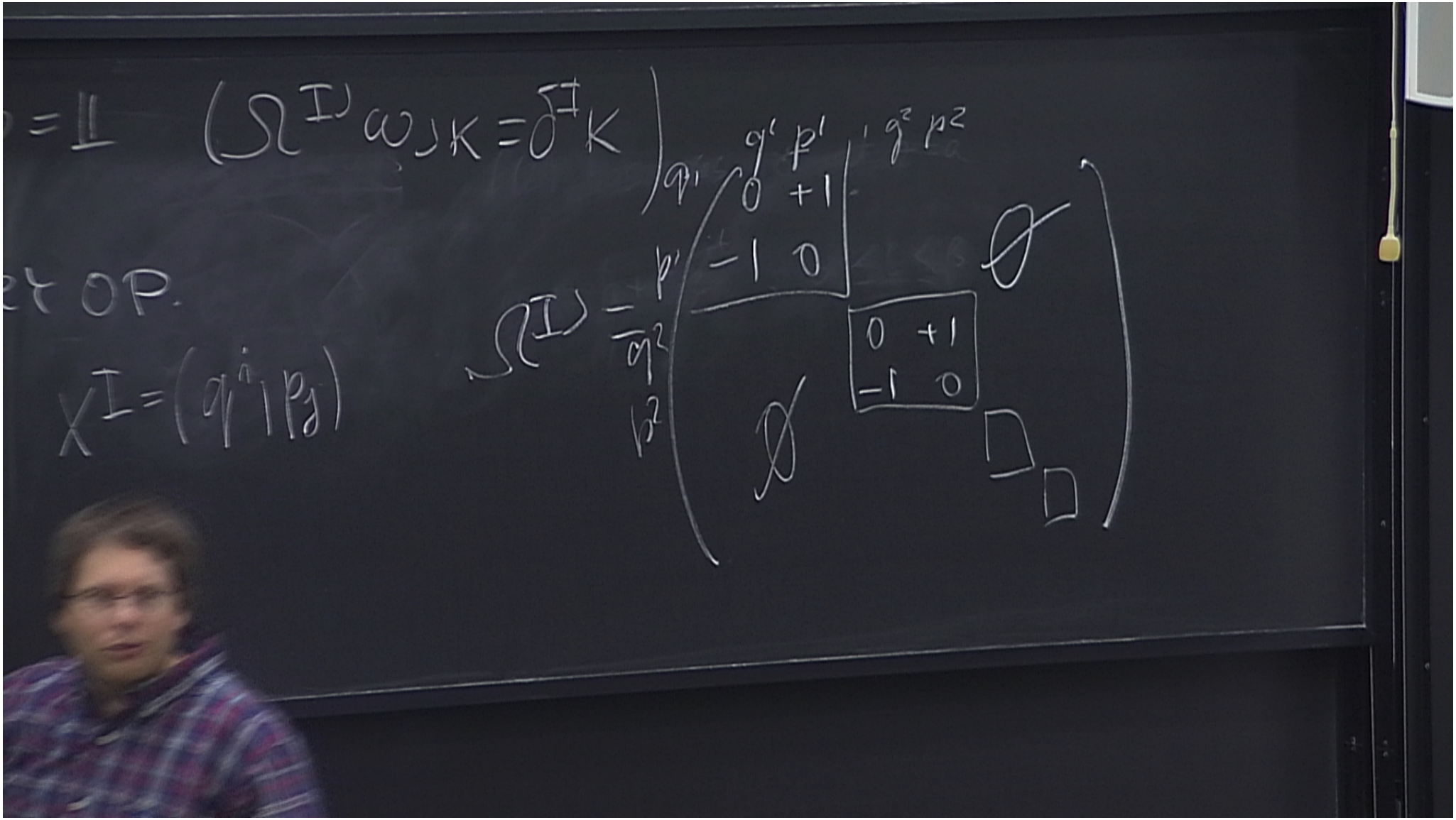
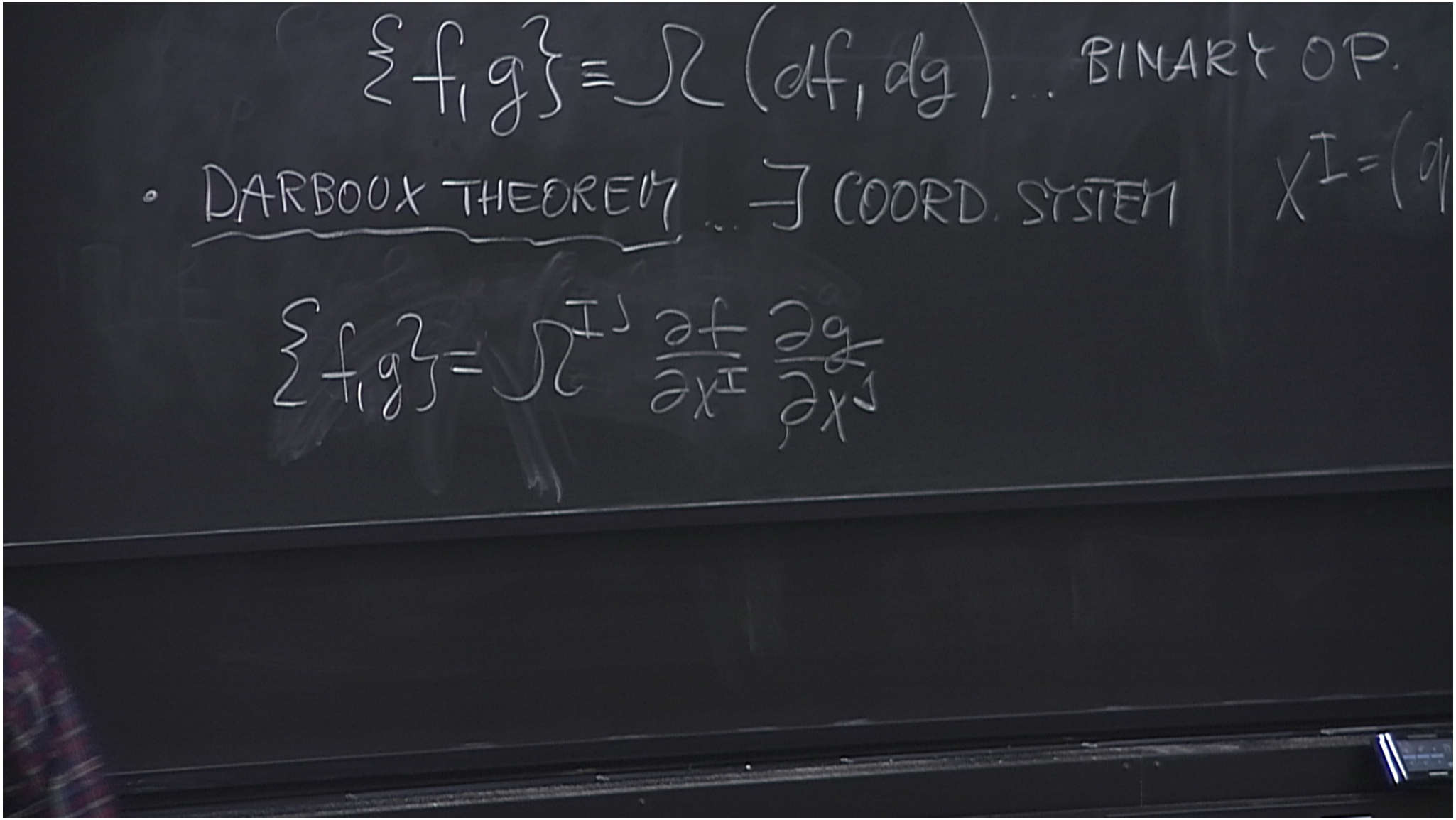

INVERSE OF ω · Ω·ω = 𝟙 (Ω^IJ ω_JK = δ^I_K)
= Ω(df, dg) ... BINARY OP.
REM ... ∃ COORD. SYSTEM X^I = (q^i, P_j)


$\Omega^{IJ}\omega_{JK} = \delta^{I}{}_{K}$
$X^I = (q^i, p_j)$
$\Omega^{(I)} =$


$= \mathbb{1}$
$(\Omega^{IJ}\omega_{JK} = \delta^I{}_K)$
q^i p^i
q^i 0 $+1$
p^i -1 0
$\Omega^{IJ} =$
$X^I = (q^i, p_j)$


$\Omega^{IJ}\omega_{JK} = \delta^I{}_K$
$X^I = (q^i, p_j)$
$\Omega^{(I)}$
q^1 q^2 p^1 p^2
0 +1
−1 0
0 +1
−1 0


$\Omega^{IJ}\omega_{JK} = \delta^I{}_K$
$X^I = (q^i, p_j)$
$\Omega^{(I)}$
q^1 q^2 p^1 p^2
0 +1
−1 0
0 +1
−1 0

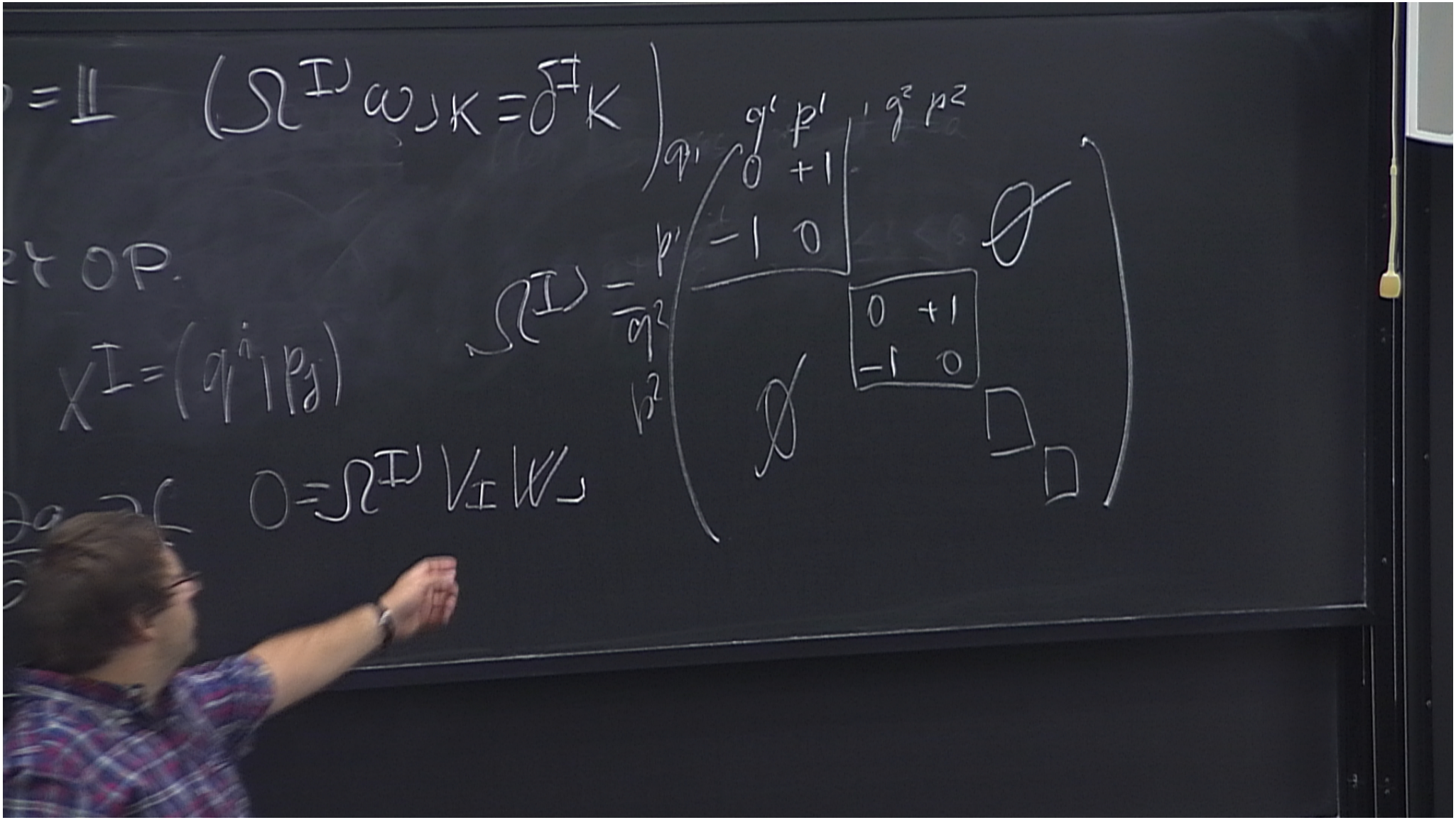

$\{f,g\} \equiv \Omega(df, dg)$ ... BINARY OP.
• DARBOUX THEOREM ... $\exists$ COORD. SYSTEM $X^I = (q$
$\{f,g\} = \Omega^{IJ} \frac{\partial f}{\partial x^I} \frac{\partial g}{\partial x^J}$

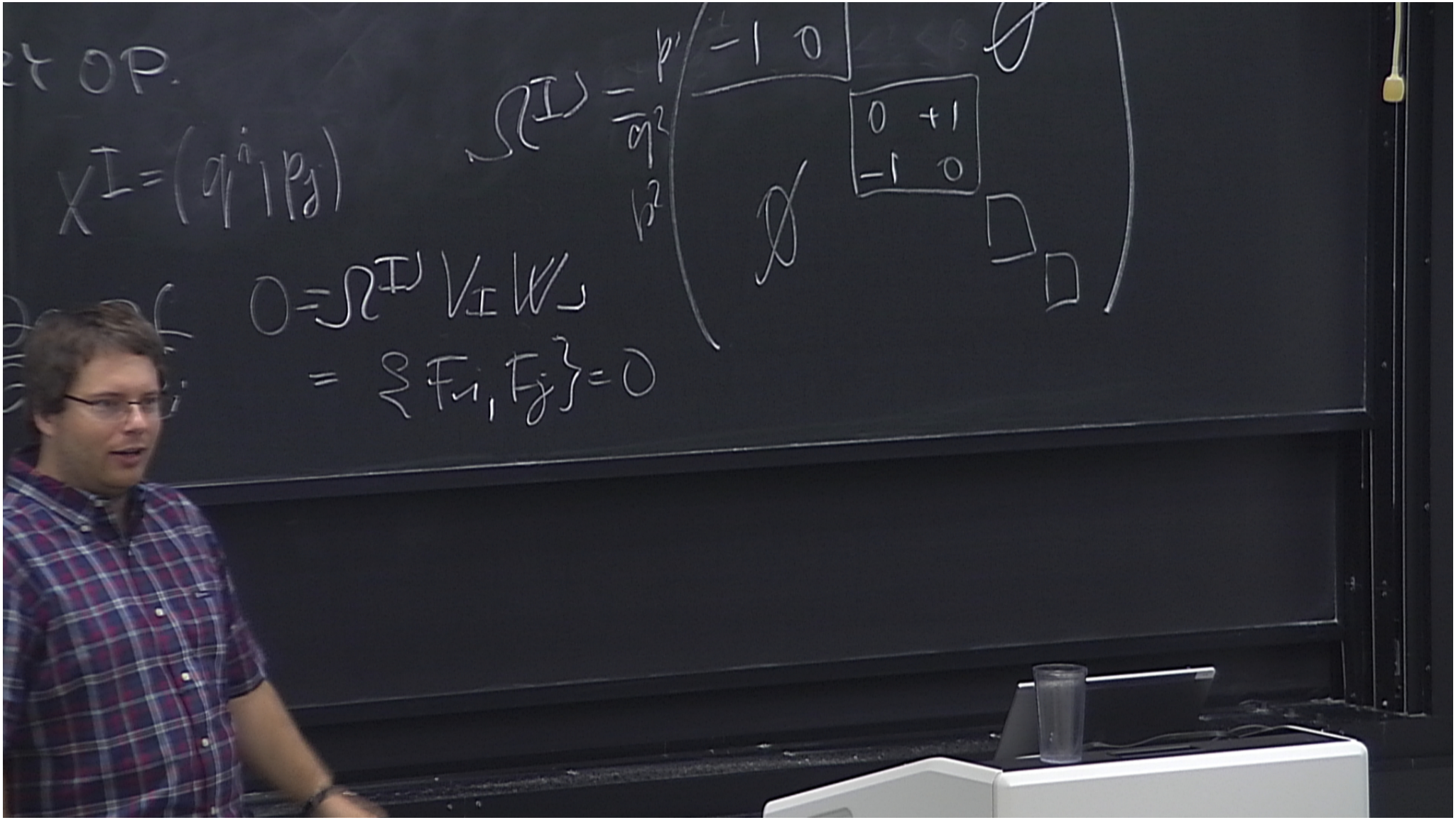
LET Ω BE INVERSE OF ω : $\Omega \cdot \omega = \mathbb{1}$ $(\Omega^{I}$

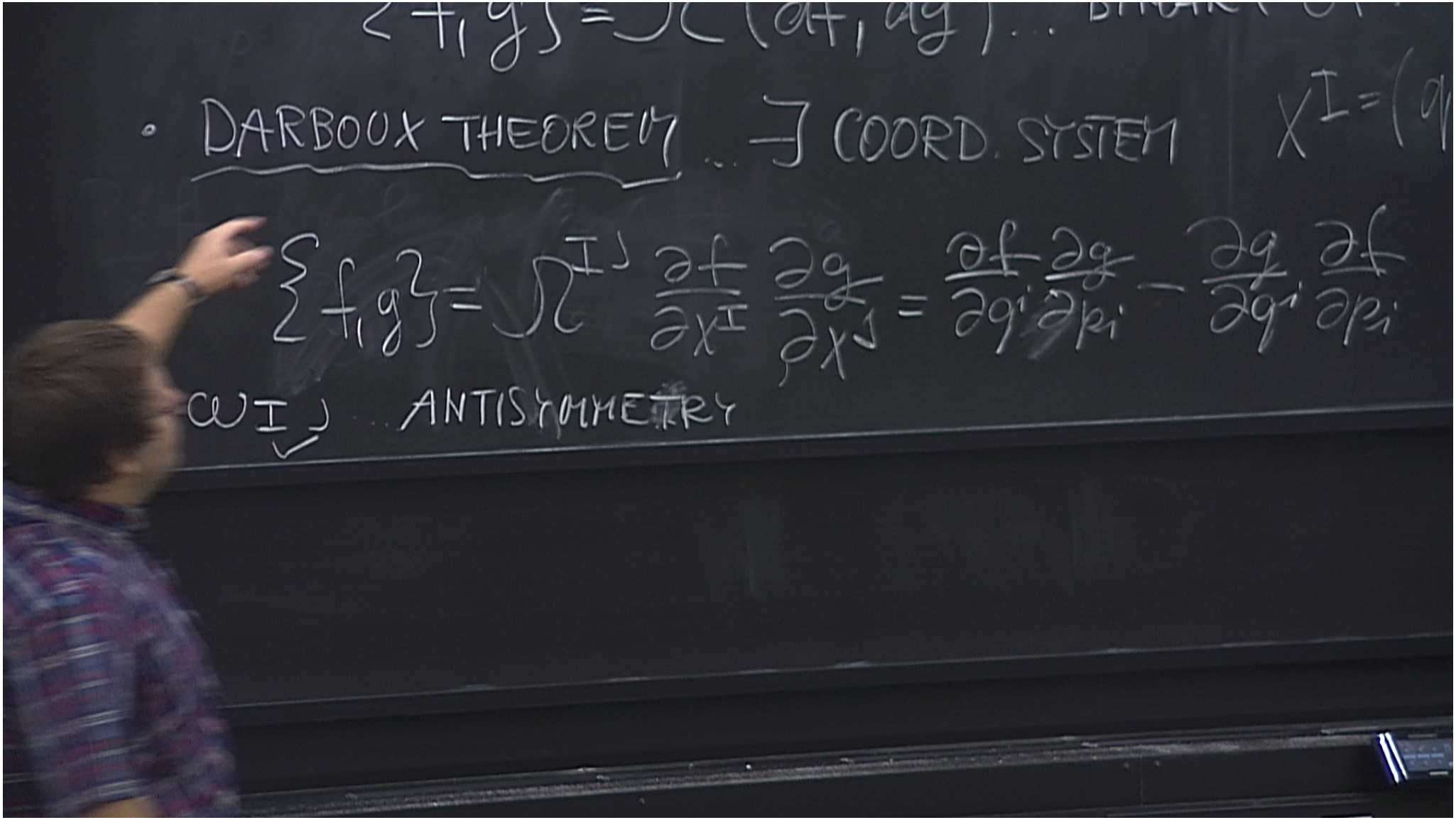
$$\{f, g\} \equiv \Omega(df, dg) \ldots \text{BINARY OP.}$$

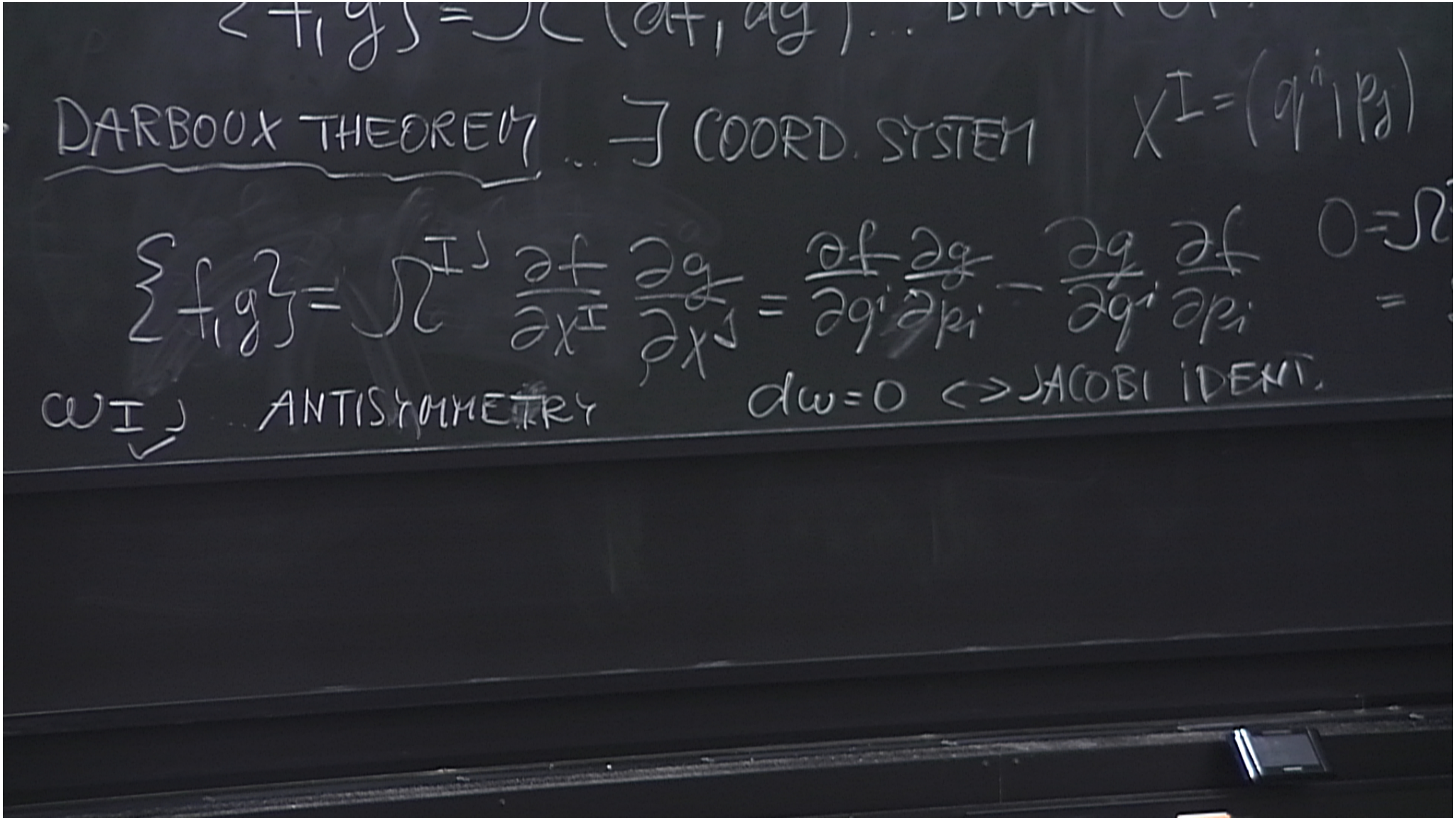
- DARBOUX THEOREM ... $\exists$ COORD. SYSTEM $X^{I} = (q^{i}, p_{j})$

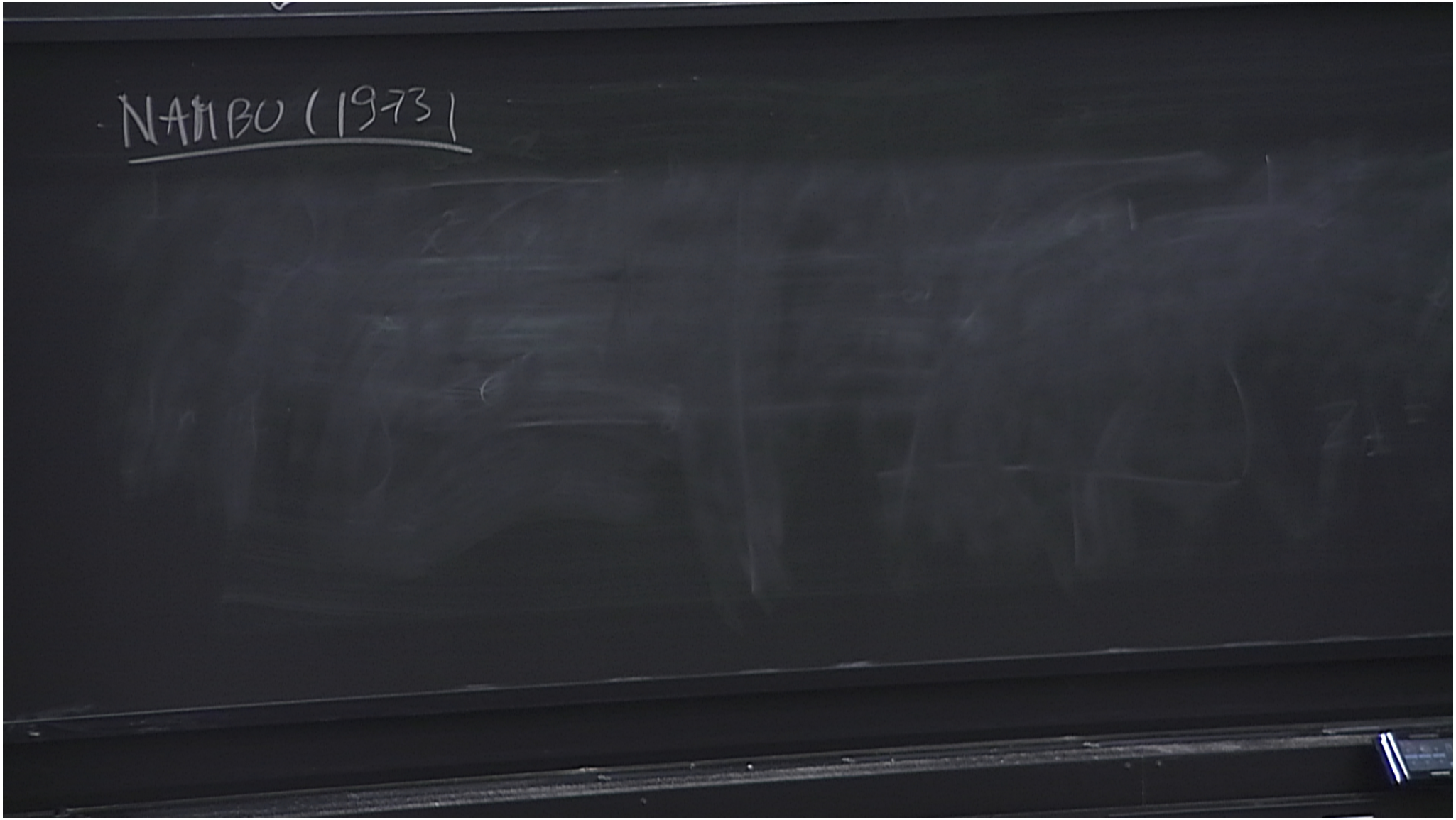
$$\{f, g\} = \Omega^{IJ} \frac{\partial f}{\partial x^{I}} \frac{\partial g}{\partial x^{J}} = \frac{\partial f}{\partial q^{i}} \frac{\partial g}{\partial p_{i}} - \frac{\partial g}{\partial q^{i}} \frac{\partial f}{\partial p_{i}}$$

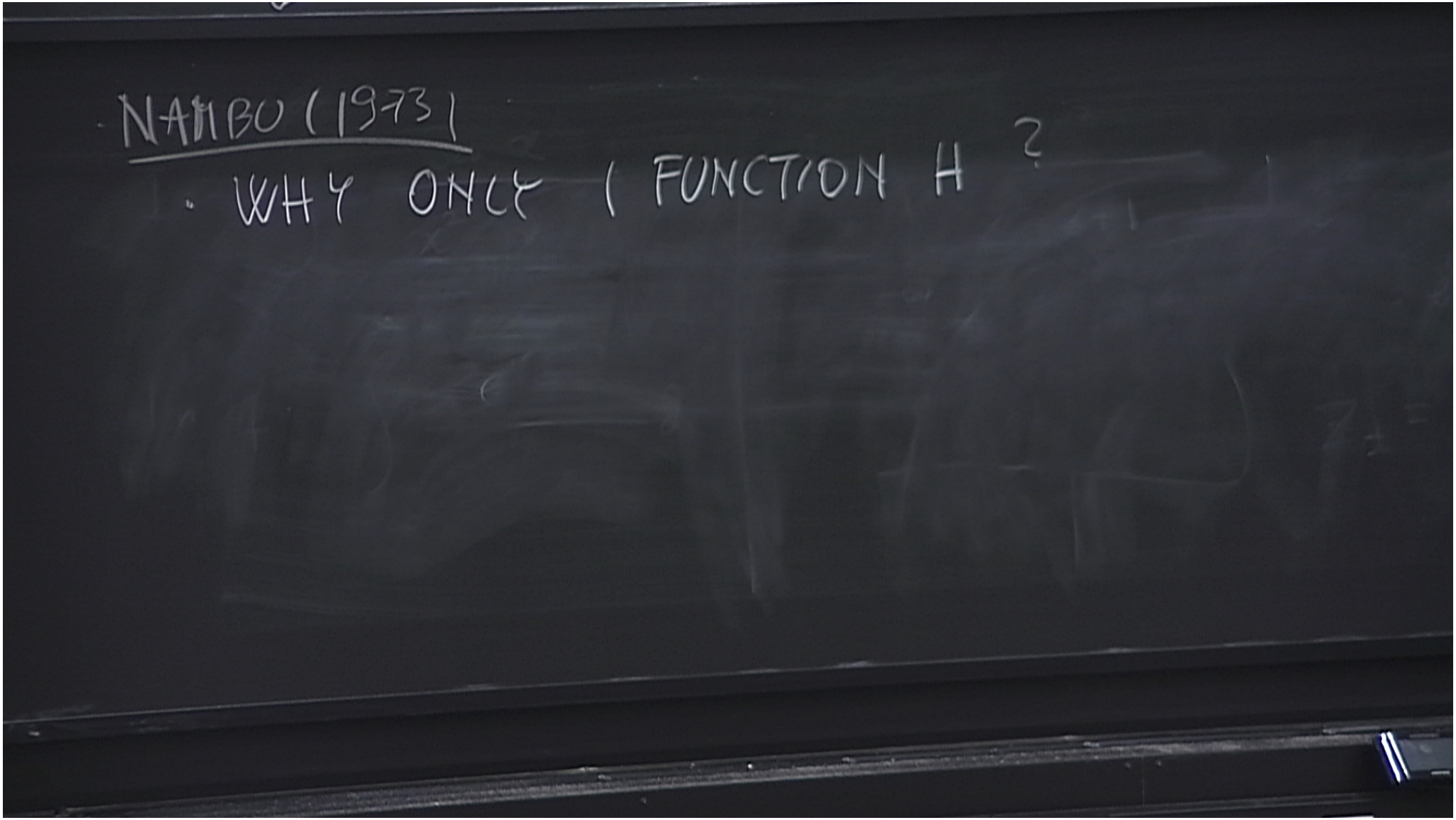

$\Omega^{IJ}\omega_{JK} = \delta^I_K$
$X^I = (q^i | p_j)$
$\Omega^{IJ} =$
$0 = \Omega^{IJ} V_I W_J$

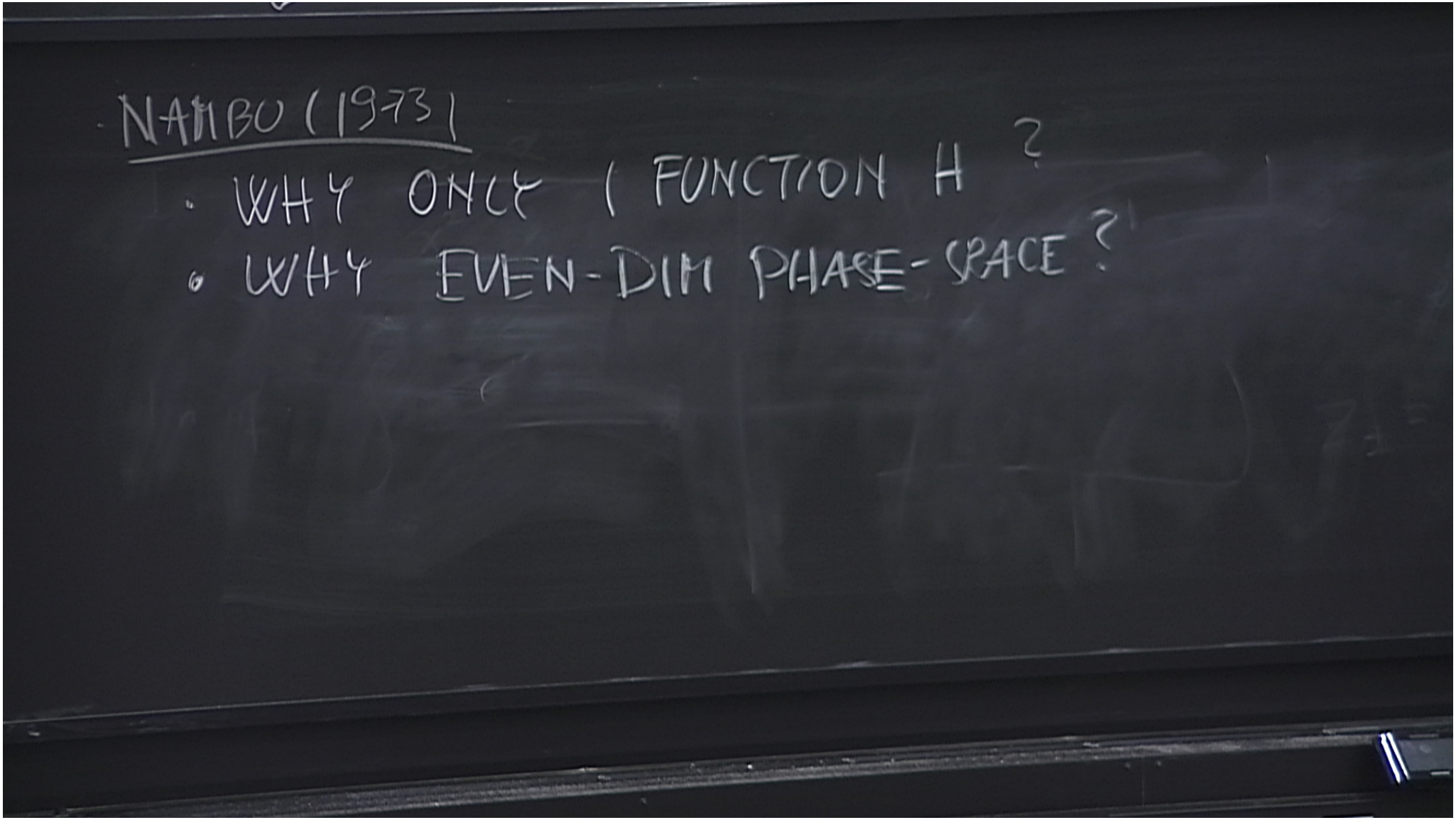

OP.
$X^I = (q^i, p_j)$
Ω^{IJ}
$0 = \Omega^{IJ} V_I W_J$
$= \{F_i, F_j\} = 0$
$-1\ 0$
$0\ +1$
$-1\ 0$

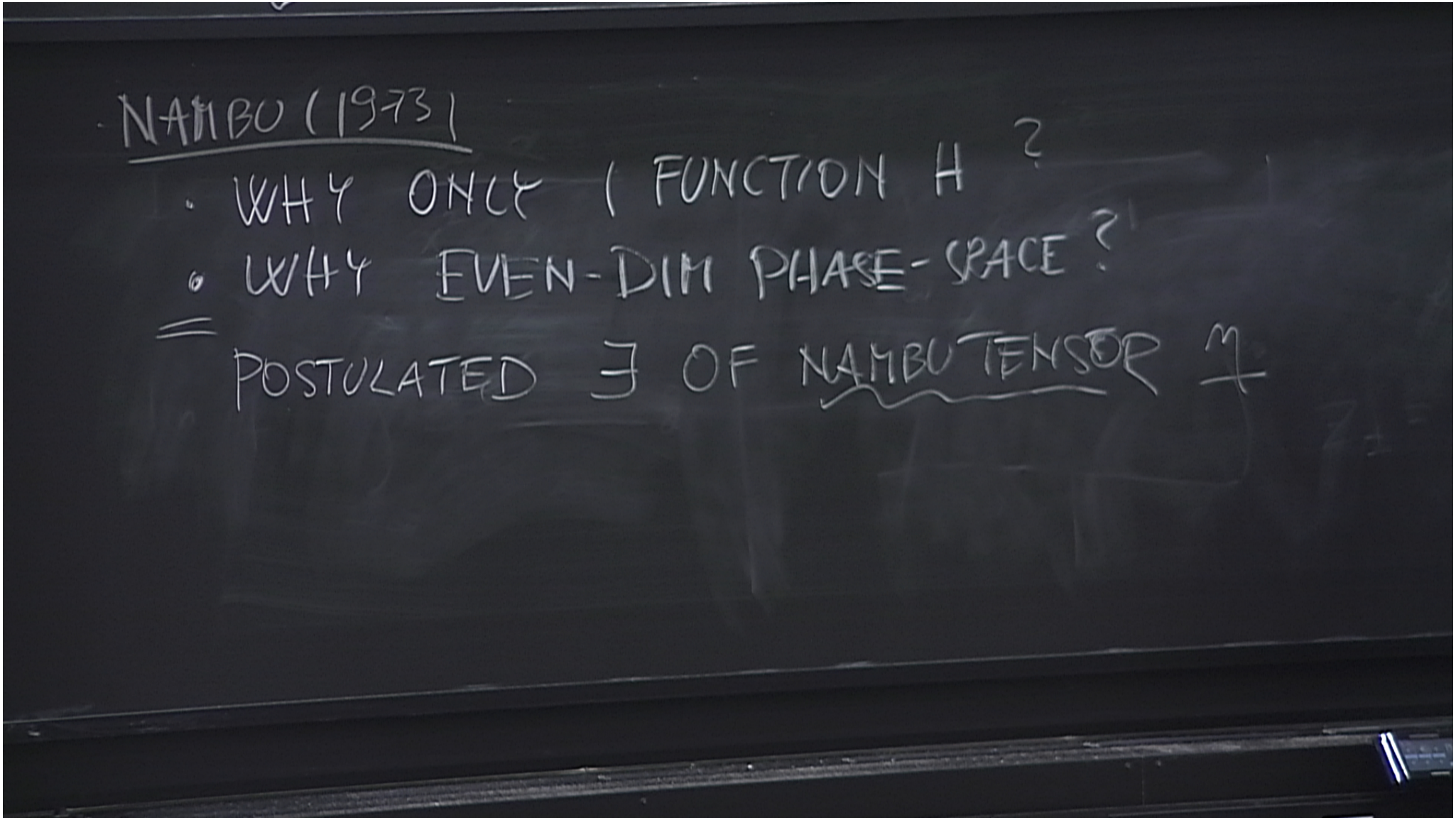

DARBOUX THEOREM ... ∃ COORD. SYSTEM
ANTISYMMETRY

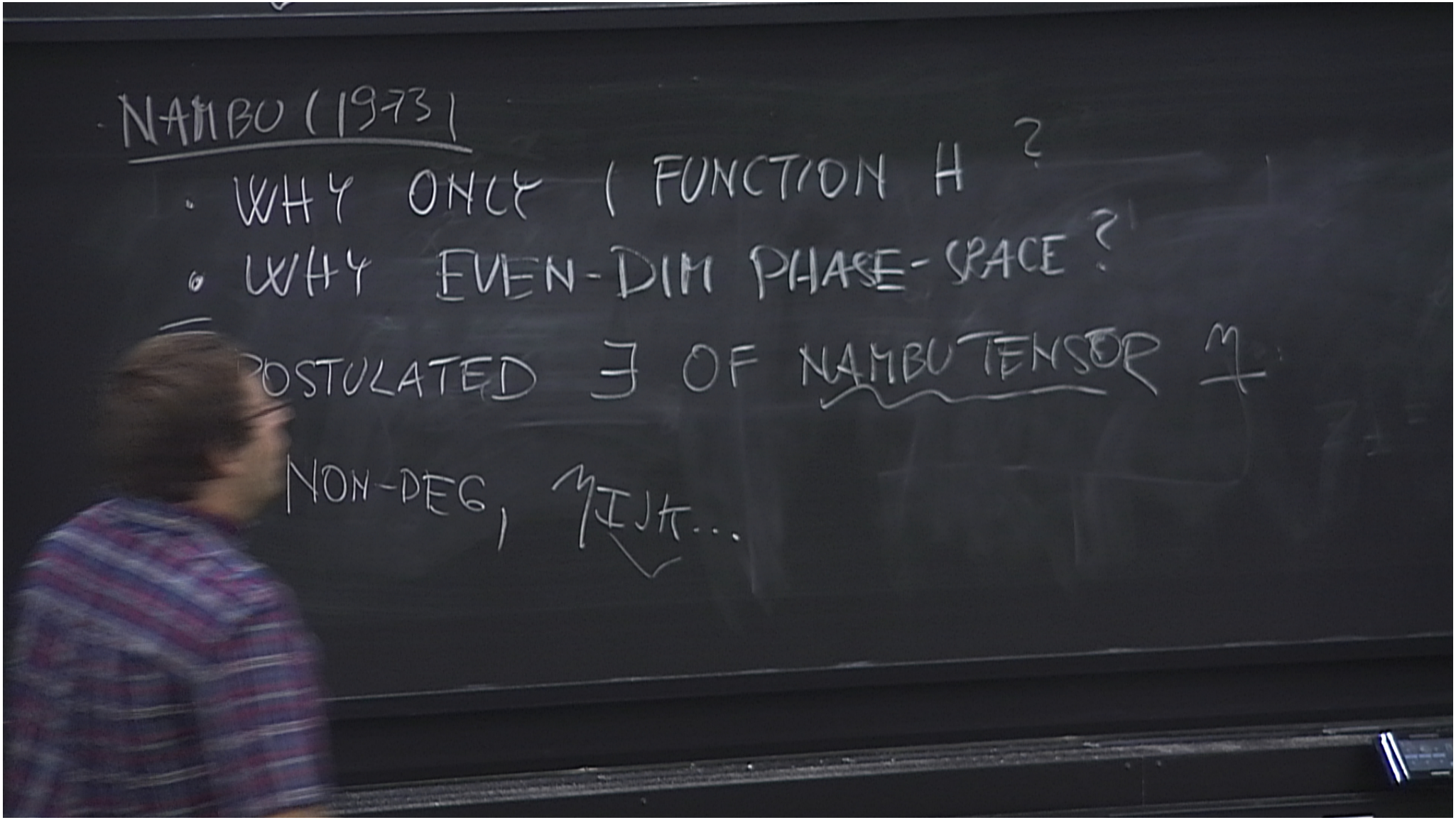

DARBOUX THEOREM ... ∃ COORD. SYSTEM $X^I = (q^i, p_j)$
$\{f,g\} = \Omega^{IJ} \frac{\partial f}{\partial x^I} \frac{\partial g}{\partial x^J} = \frac{\partial f}{\partial q^i}\frac{\partial g}{\partial p_i} - \frac{\partial g}{\partial q^i}\frac{\partial f}{\partial p_i}$
ω_{IJ} ANTISYMMETRY
$d\omega = 0 \Leftrightarrow$ JACOBI IDENT.

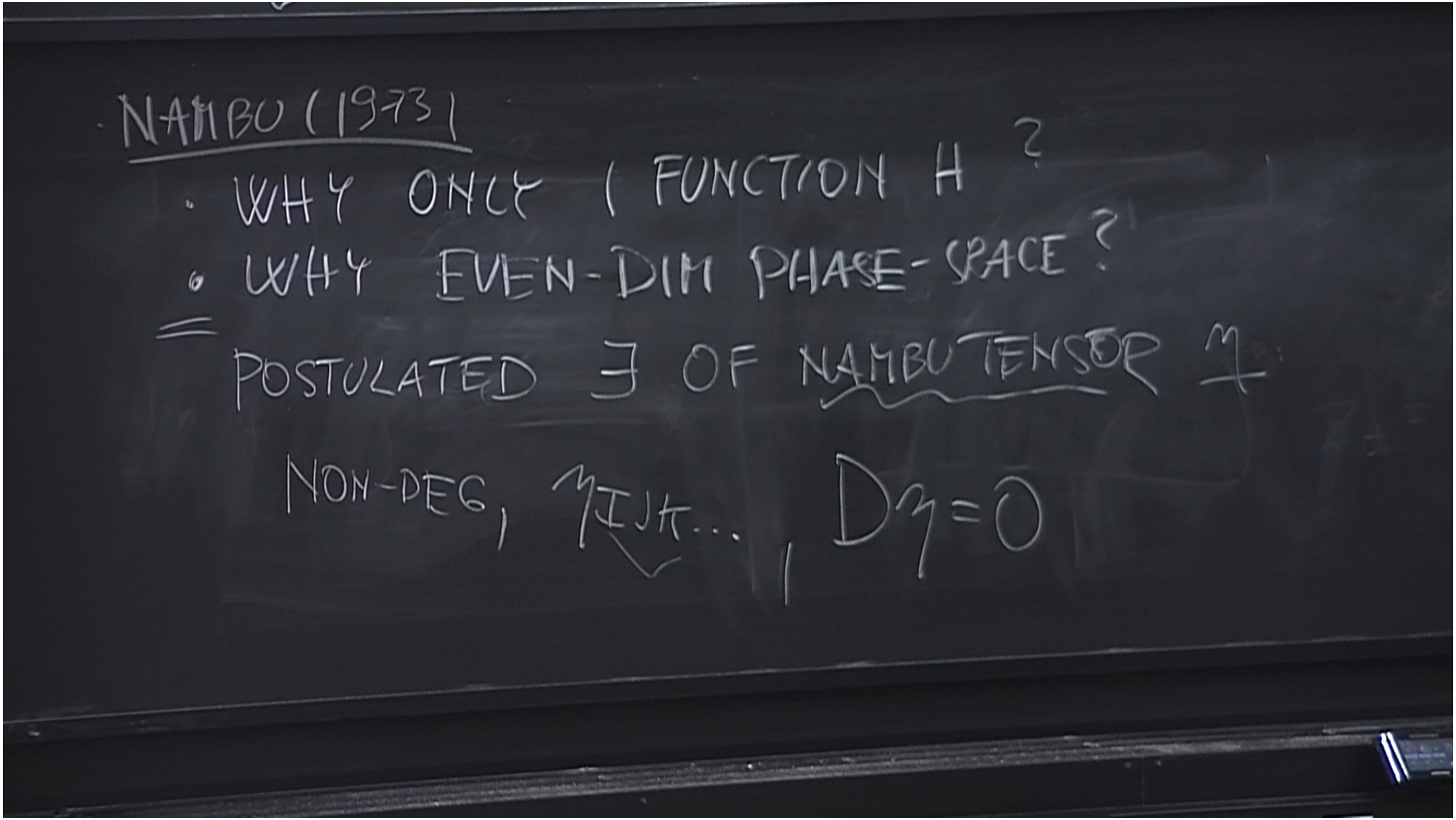

NAMBU (1973)

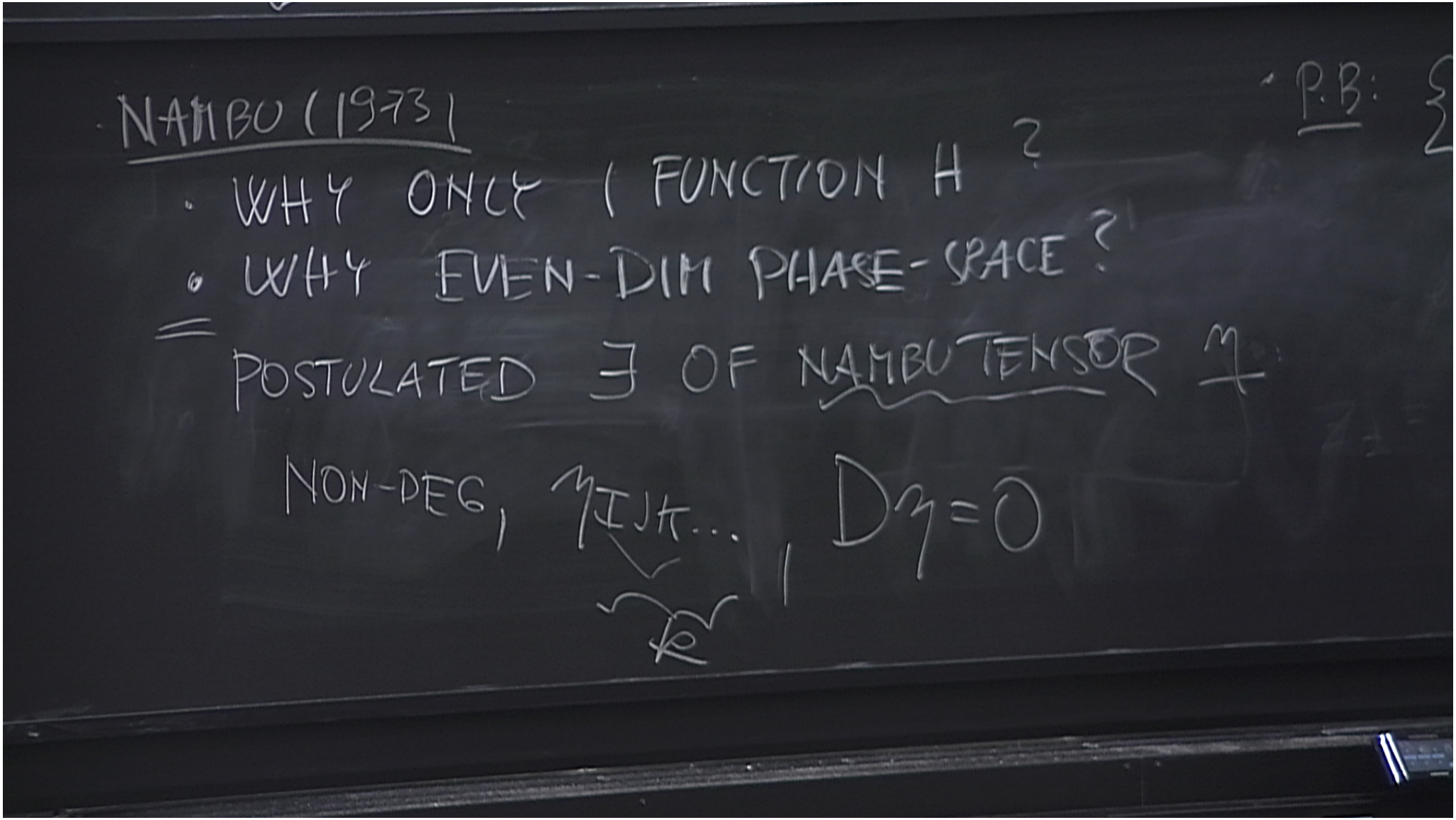

NAMBU (1973)
WHY ONLY 1 FUNCTION H ?

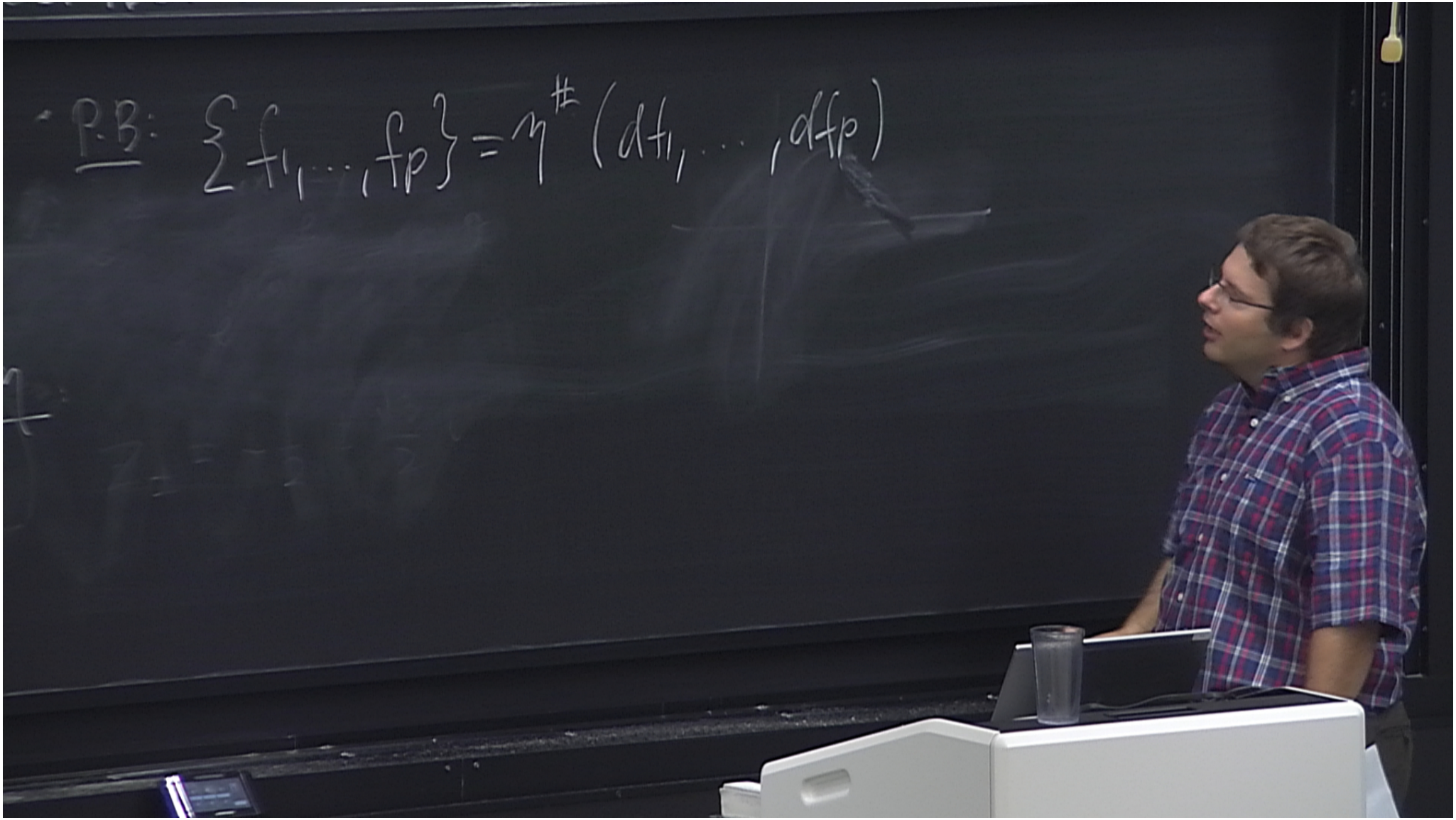

NAMBU (1973)
WHY ONLY 1 FUNCTION H ?
WHY EVEN-DIM PHASE-SPACE ?

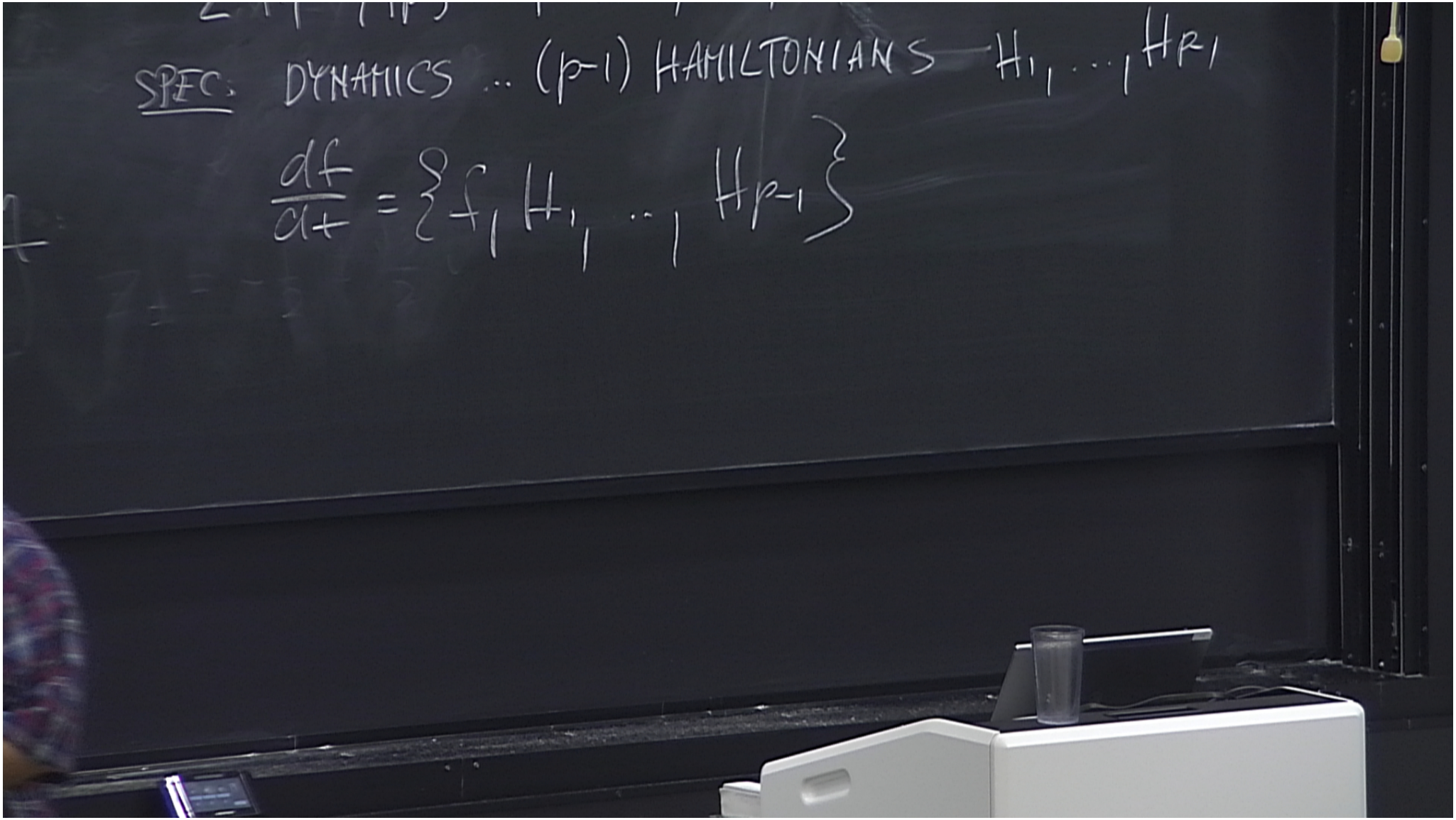

NAMBU (1973)
WHY ONLY 1 FUNCTION H ?
WHY EVEN-DIM PHASE-SPACE ?
= POSTULATED ∃ OF NAMBU TENSOR M

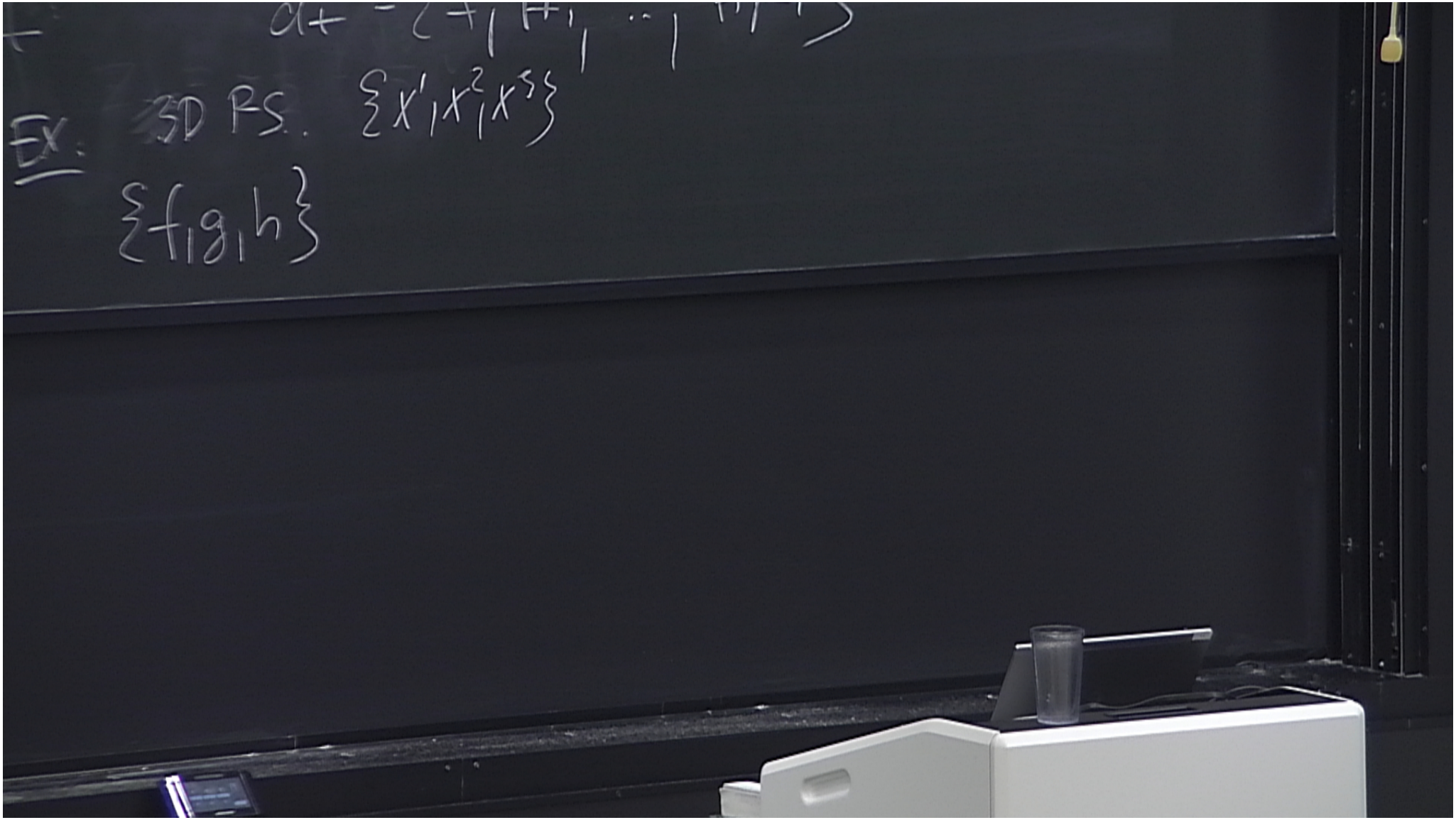

NAMBU (1973)
WHY ONLY 1 FUNCTION H ?
WHY EVEN-DIM PHASE-SPACE ?
POSTULATED ∃ OF NAMBU TENSOR M
NON-DEG, MIJK...

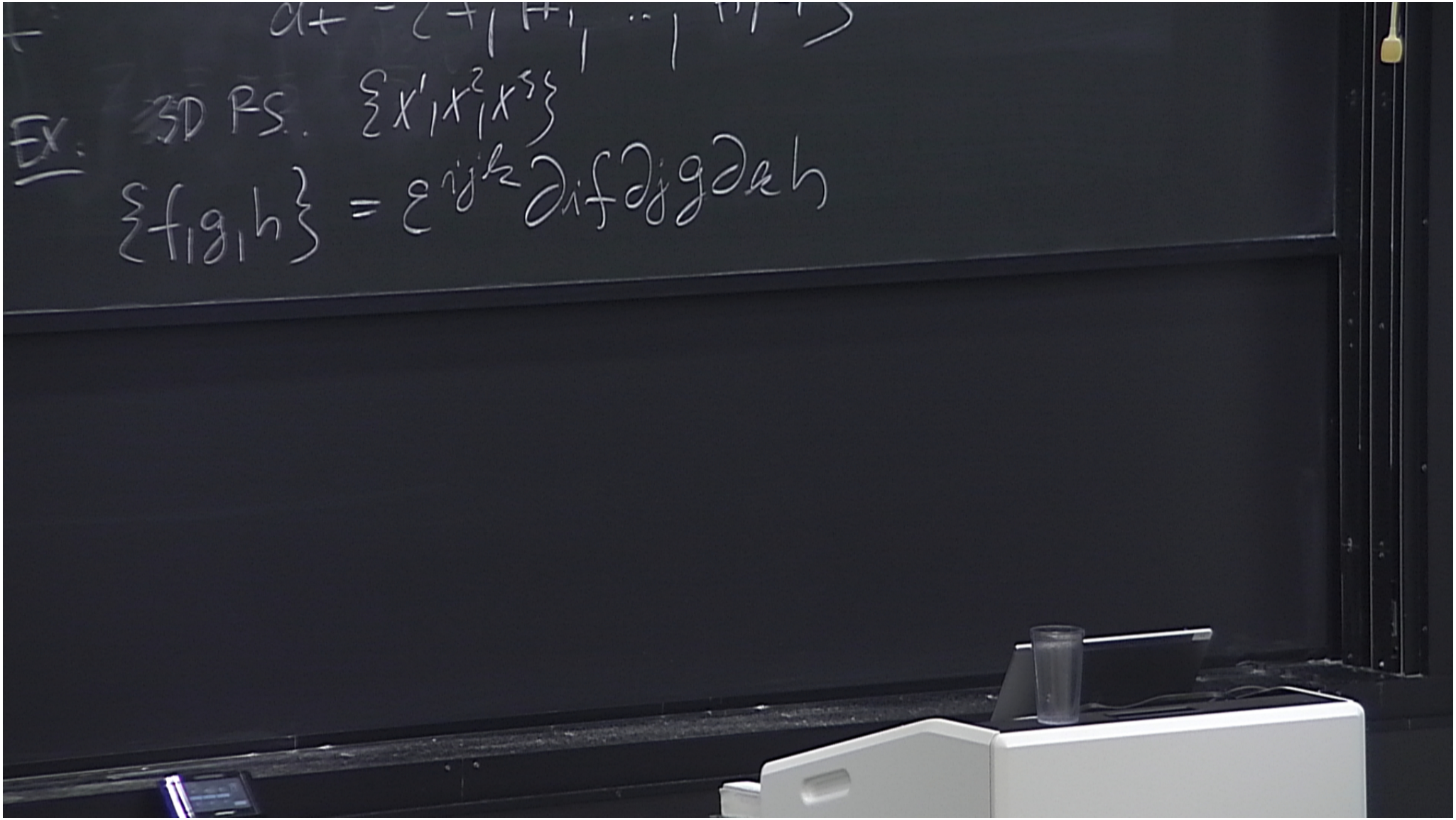

NAMBU (1973)
· WHY ONLY 1 FUNCTION H ?
· WHY EVEN-DIM PHASE-SPACE ?
= POSTULATED ∃ OF NAMBU TENSOR M:
NON-DEG, MIJK..., DM=0

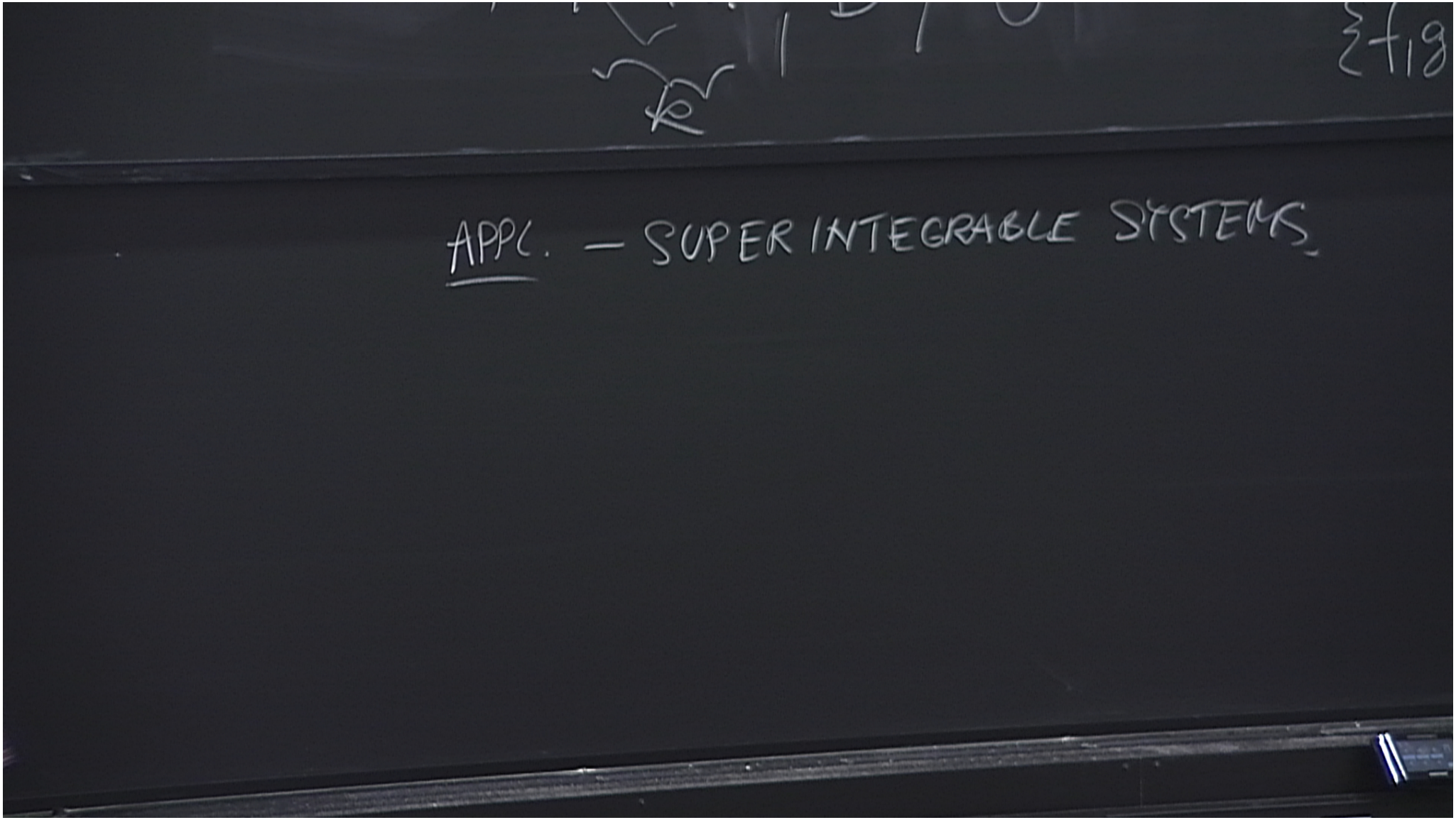

NAMBU (1973)
· WHY ONLY 1 FUNCTION H ?
· WHY EVEN-DIM PHASE-SPACE ?
= POSTULATED ∃ OF NAMBU TENSOR η:
NON-DEG, ηIJK..., Dη=0
P.B:

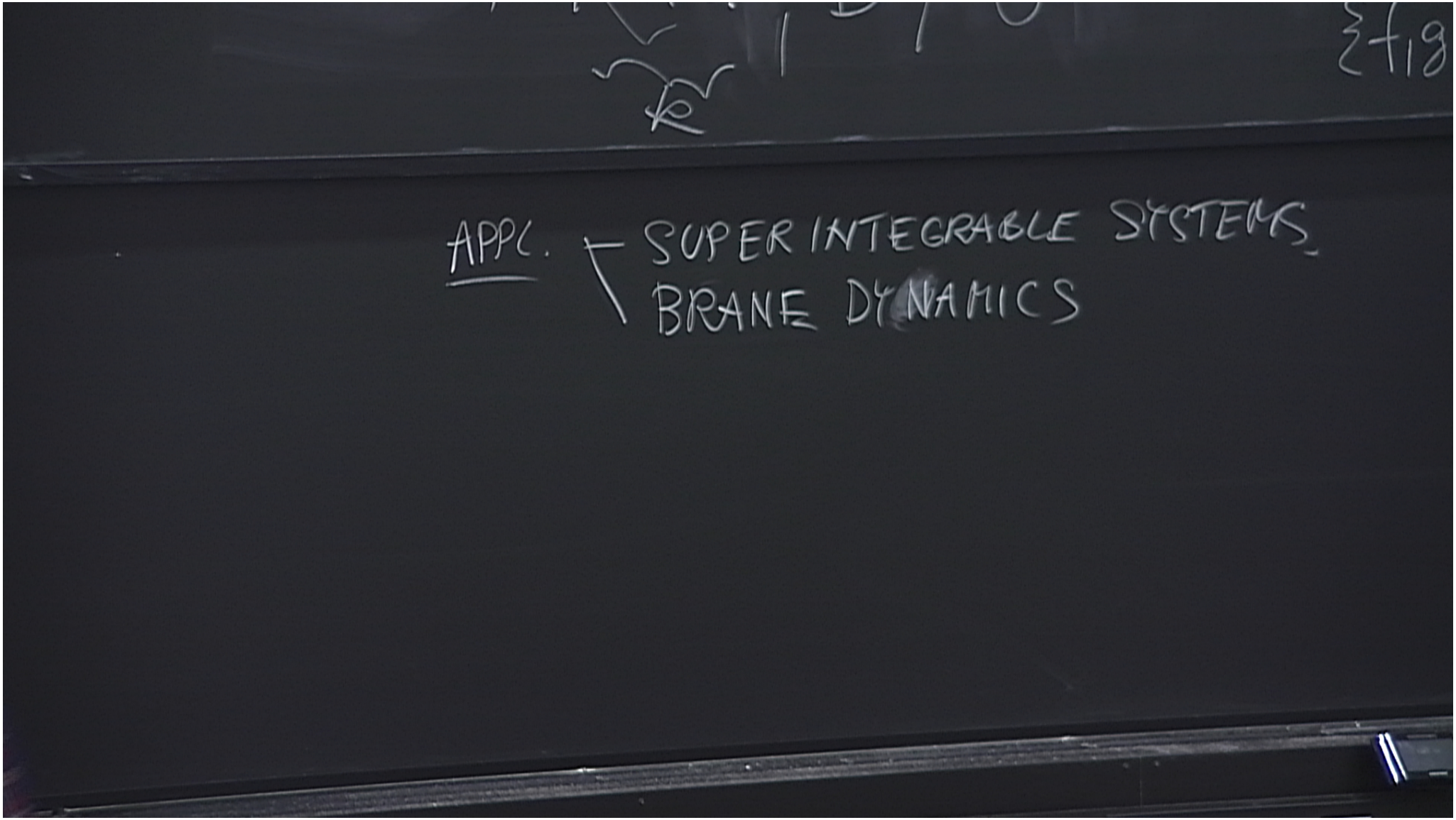

P.B: $\{f_1, \dots, f_p\} = \eta^\sharp(df_1, \dots, df_p)$

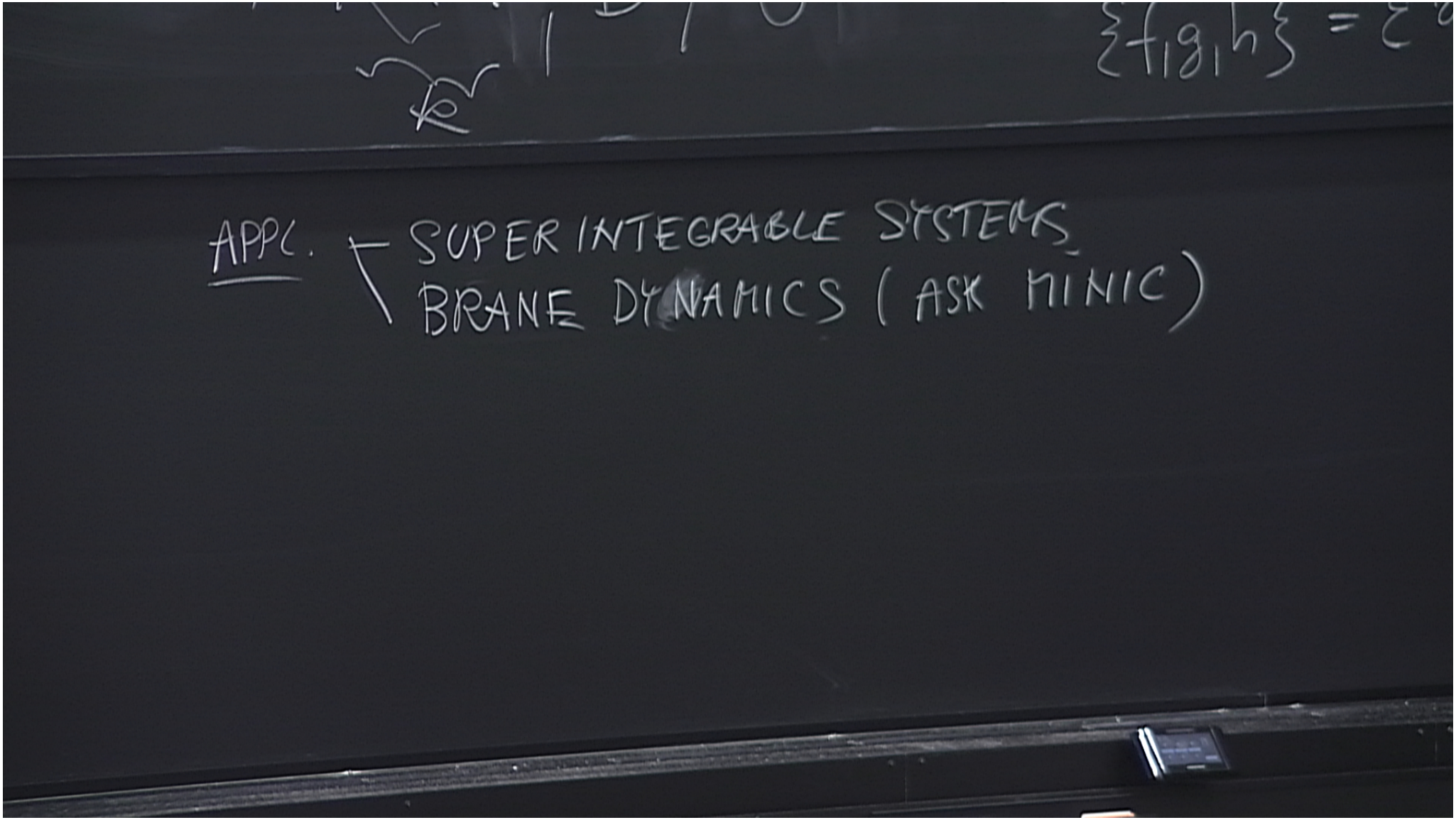

SPEC: DYNAMICS ... (p-1) HAMILTONIANS $H_1, \dots, H_{p-1}$
$\frac{df}{dt} = \{f, H_1, \dots, H_{p-1}\}$

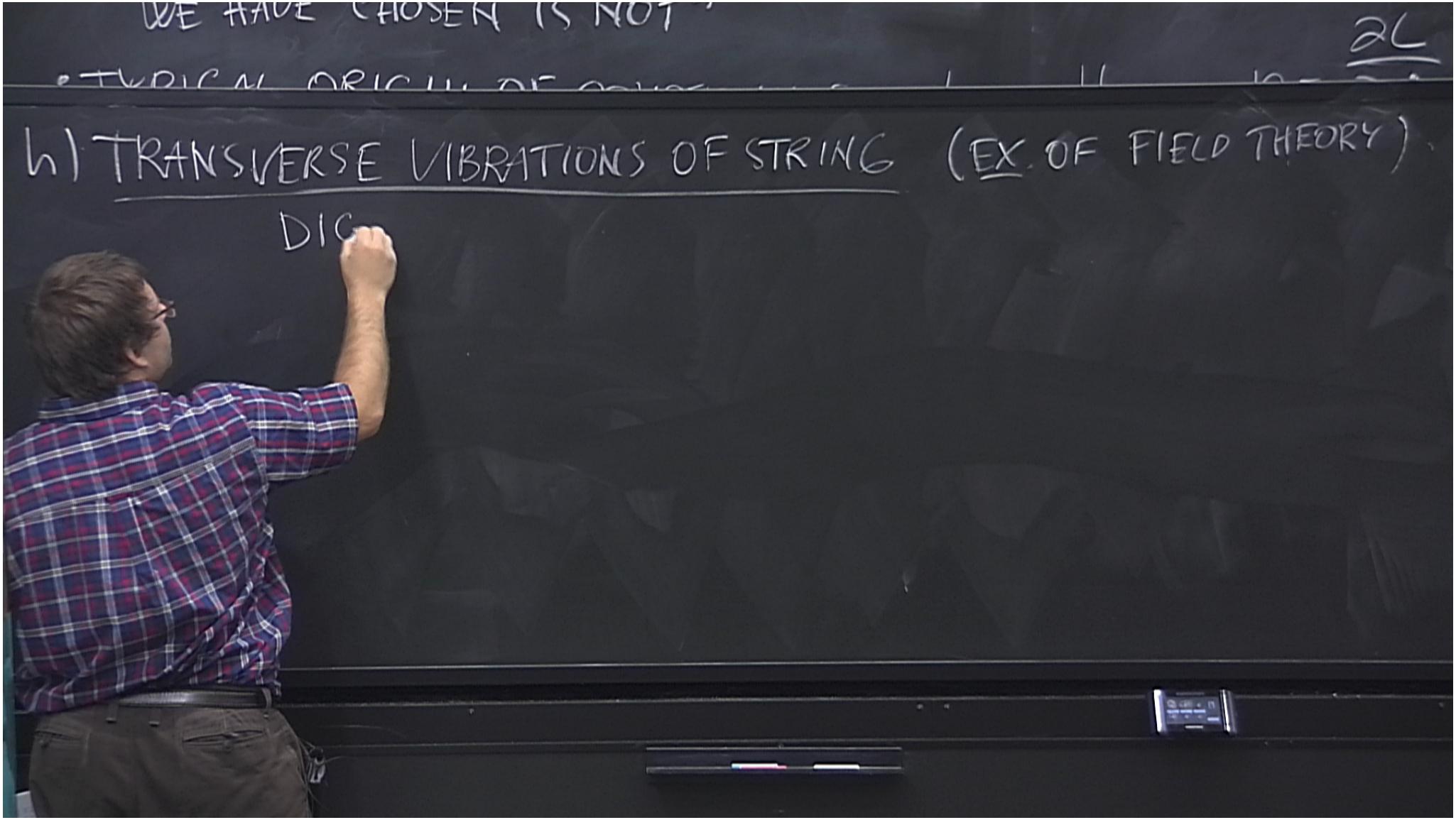

Ex. 3D RS. $\{x^1, x^2, x^3\}$
$\{f, g, h\}$


Ex: 3D RS. $\{x^1, x^2, x^3\}$
$\{f, g, h\} = \varepsilon^{ijk} \partial_i f \partial_j g \partial_k h$


APPL. – SUPERINTEGRABLE SYSTEMS


APPL.
SUPER INTEGRABLE SYSTEMS
BRANE DYNAMICS


APPL.
SUPER INTEGRABLE SYSTEMS
BRANE DYNAMICS (ASK MINIC)

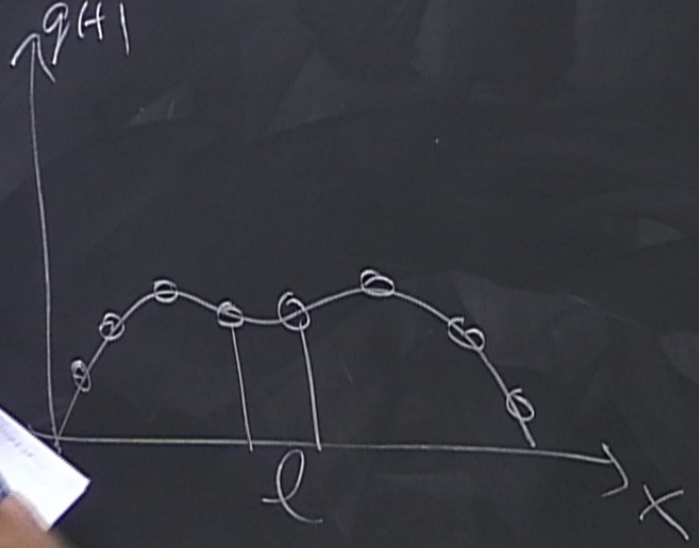

WE HAVE CHOSEN IS NOT
2L
h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)
DIS

WE HAVE CHOSEN IS NOT

2L

h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)

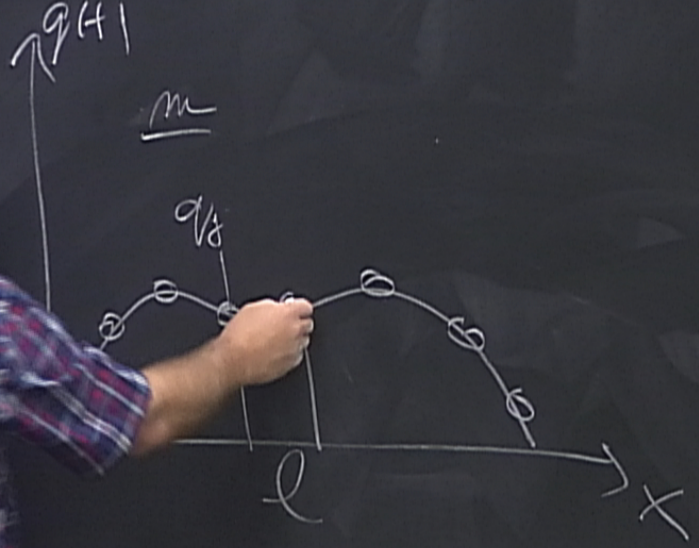
DISCRETE



WE HAVE CHOSEN IS NOT

h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)

DISCRETE



WE HAVE CHOSEN IS NOT

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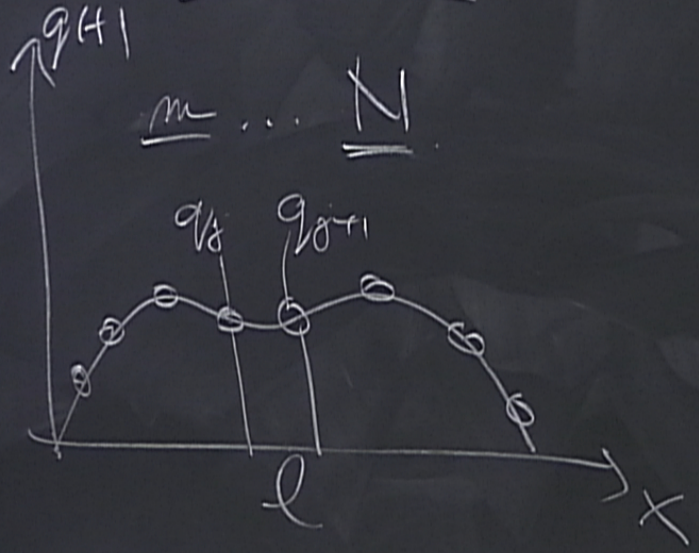
h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)

CO

DISCRETE



WE HAVE CHOSEN IS NOT

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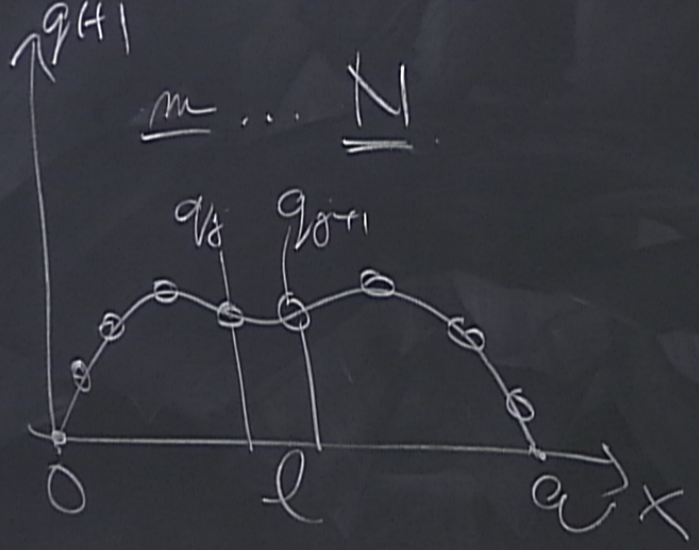
h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)

DISCRETE

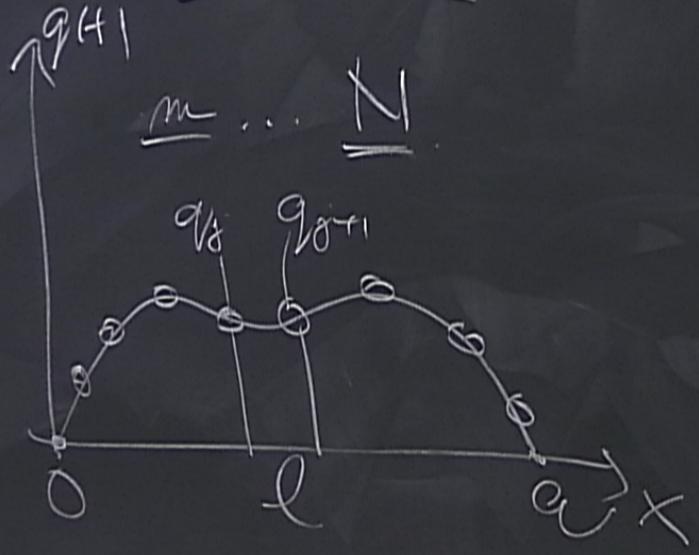


WE HAVE CHOSEN IS NOT

2L

h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)

DISCRETE

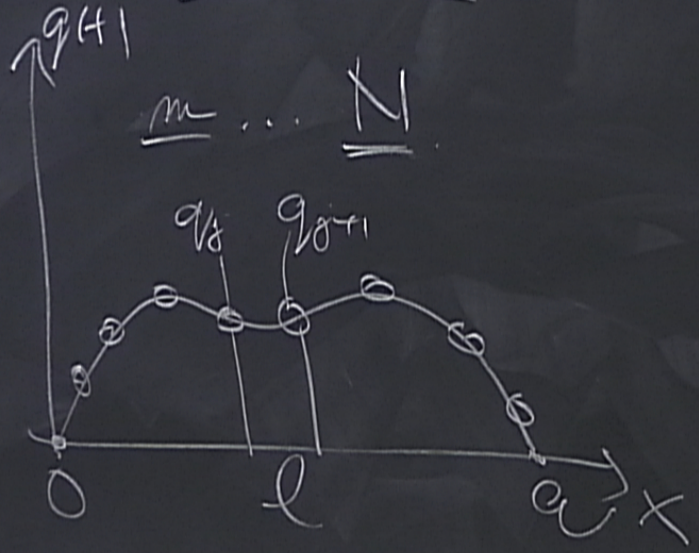


$l \to 0$

WE HAVE CHOSEN IS NOT

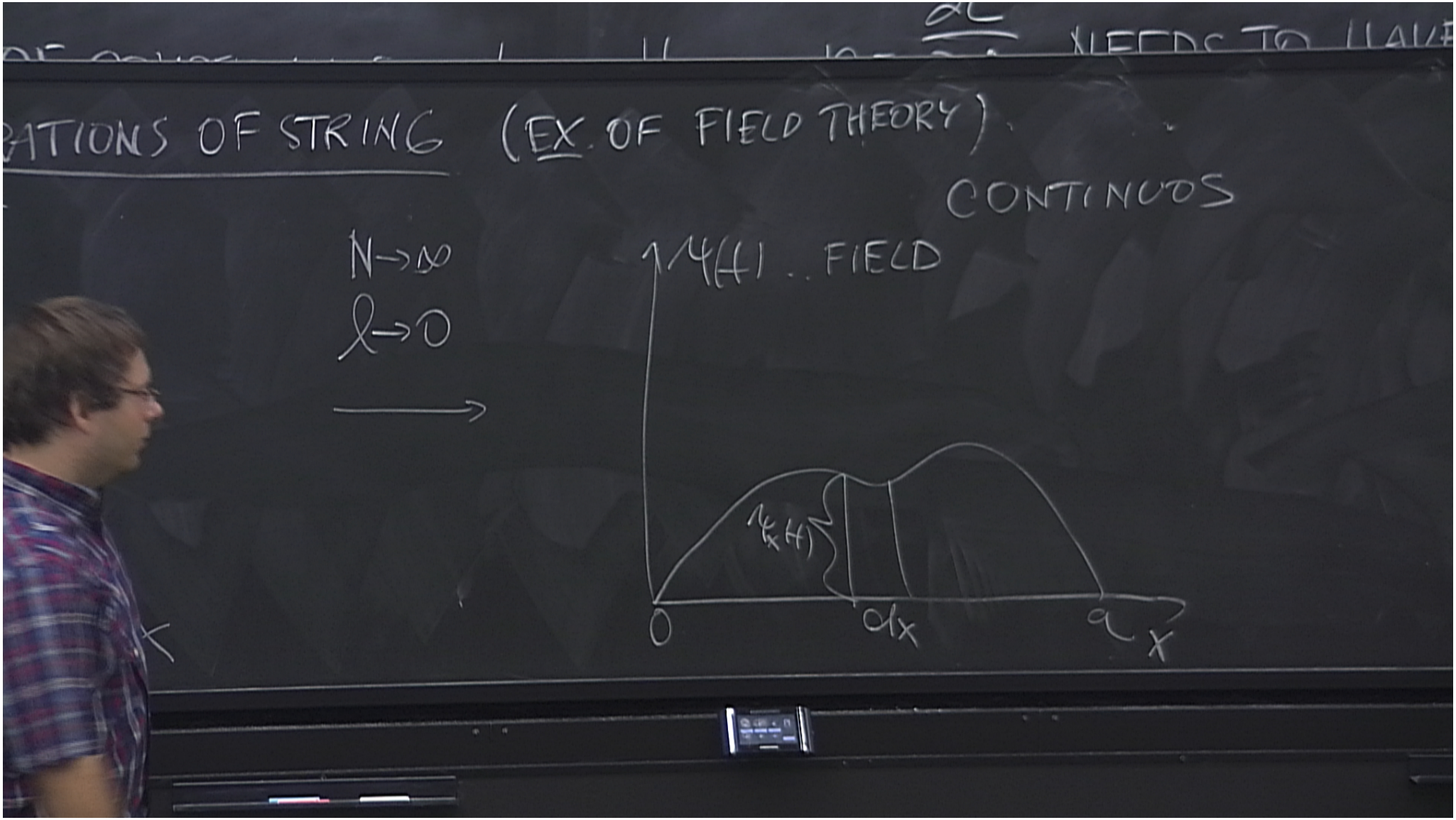
h) TRANSVERSE VIBRATIONS OF STRING (EX. OF FIELD THEORY)

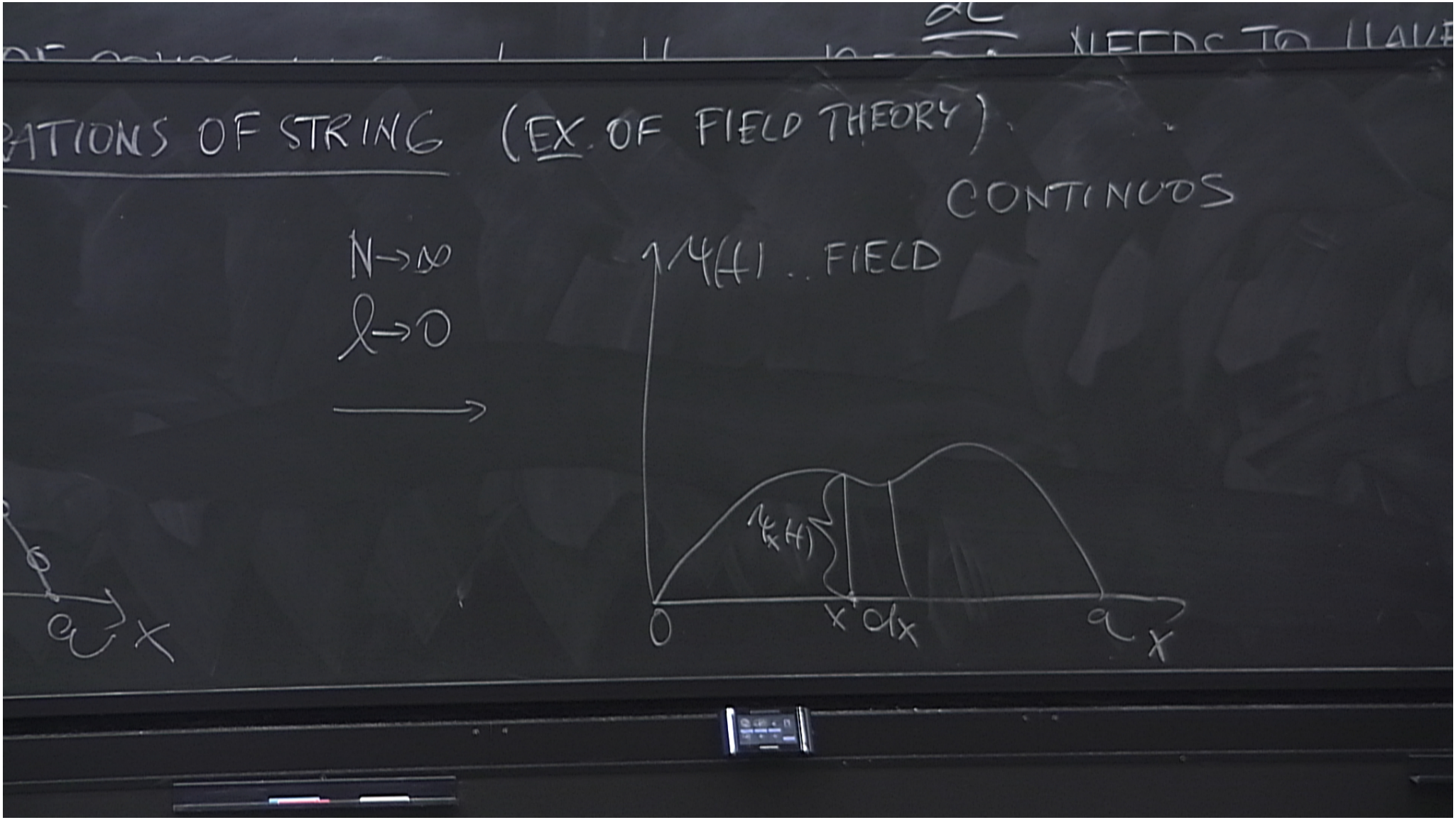
DISCRETE



$N \to \infty$

$l \to 0$


ATIONS OF STRING (EX. OF FIELD THEORY)
CONTINUOS
N→∞
ℓ→0
ψ(t) .. FIELD
ψ(x,t)
0
dx
a
x


NEEDS TO HAVE
ATIONS OF STRING (EX. OF FIELD THEORY).
CONTINUOS
N→∞
ℓ→0
ψ(t) .. FIELD
ψ(x,t)
0
x
dx
a
x

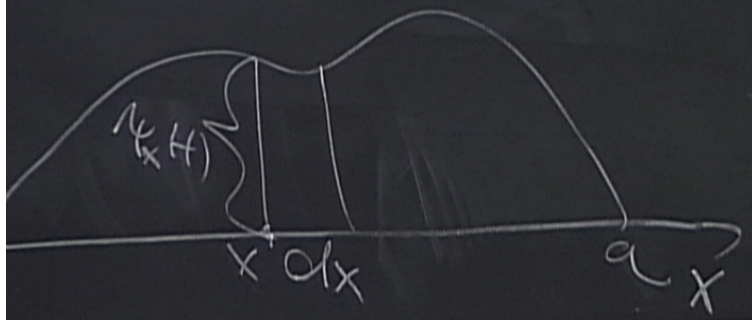
NEEDS TO HAVE AN INVERSION

FIELD THEORY).

CONTINUOS

$q_j(t) \rightarrow \psi(x,t)$... FIELD

$\psi(t)$.. FIELD



2

NEEDS TO HAVE AN INVERSION

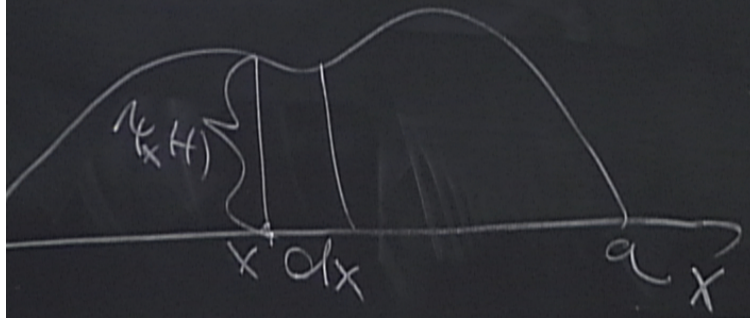
FIELD THEORY)

$q_j(t) \rightarrow \psi(x,t)$ .. FIELD

CONTINUOS

$\psi(t)$ .. FIELD

$T_{KIN} = \frac{1}{2}$



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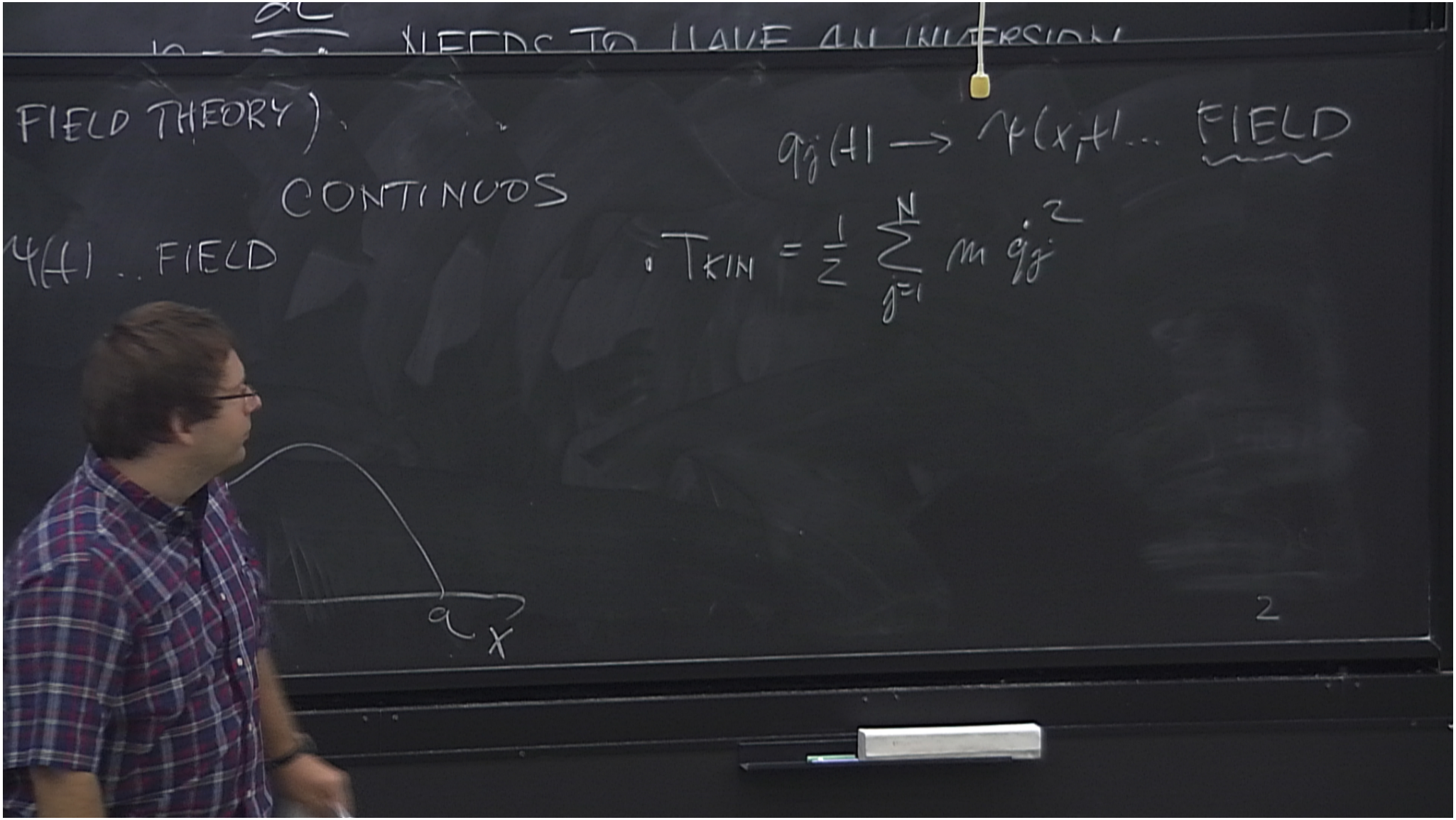

NEEDS TO HAVE AN INVERSION
FIELD THEORY).
CONTINUOS
$q_j(t) \rightarrow \psi(x,t)$... FIELD
$\psi(t)$.. FIELD
$T_{KIN} = \frac{1}{2}\sum_{j=1}^{N} m \dot{q}_j^2$
x
2

NEEDS TO HAVE AN INVERSION

FIELD THEORY)

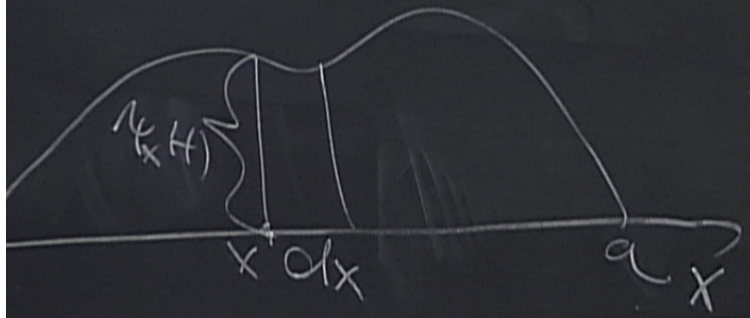
CONTINUOS

$q_j(t) \rightarrow \psi(x,t)$ .. FIELD

$\psi(t)$ .. FIELD

$$T_{KIN} = \frac{1}{2} \sum_{j=1}^{N} m \dot{q}_j^2$$

$$V = \sum_{j=1}^{N} T (\Delta l)_j$$



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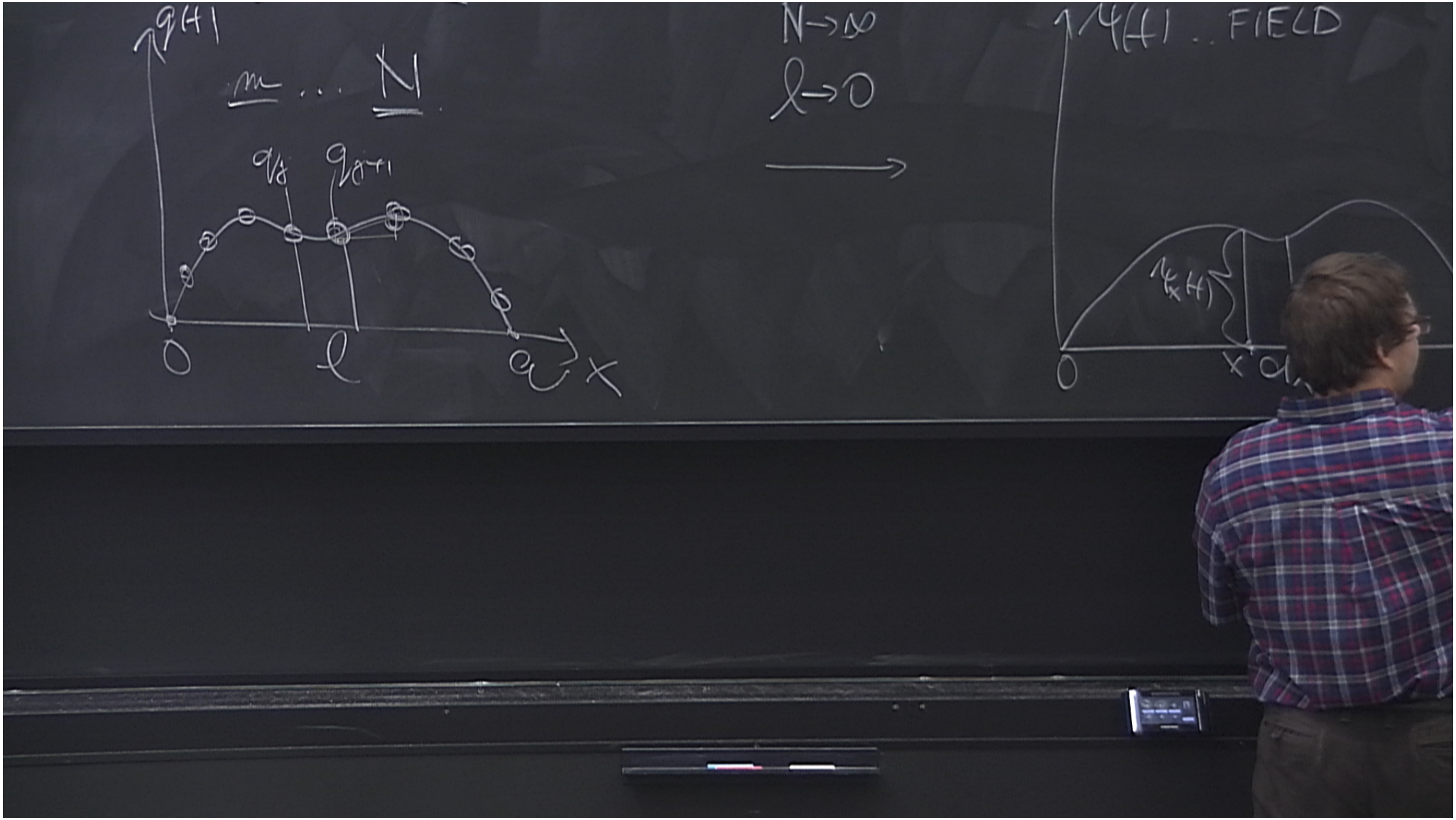
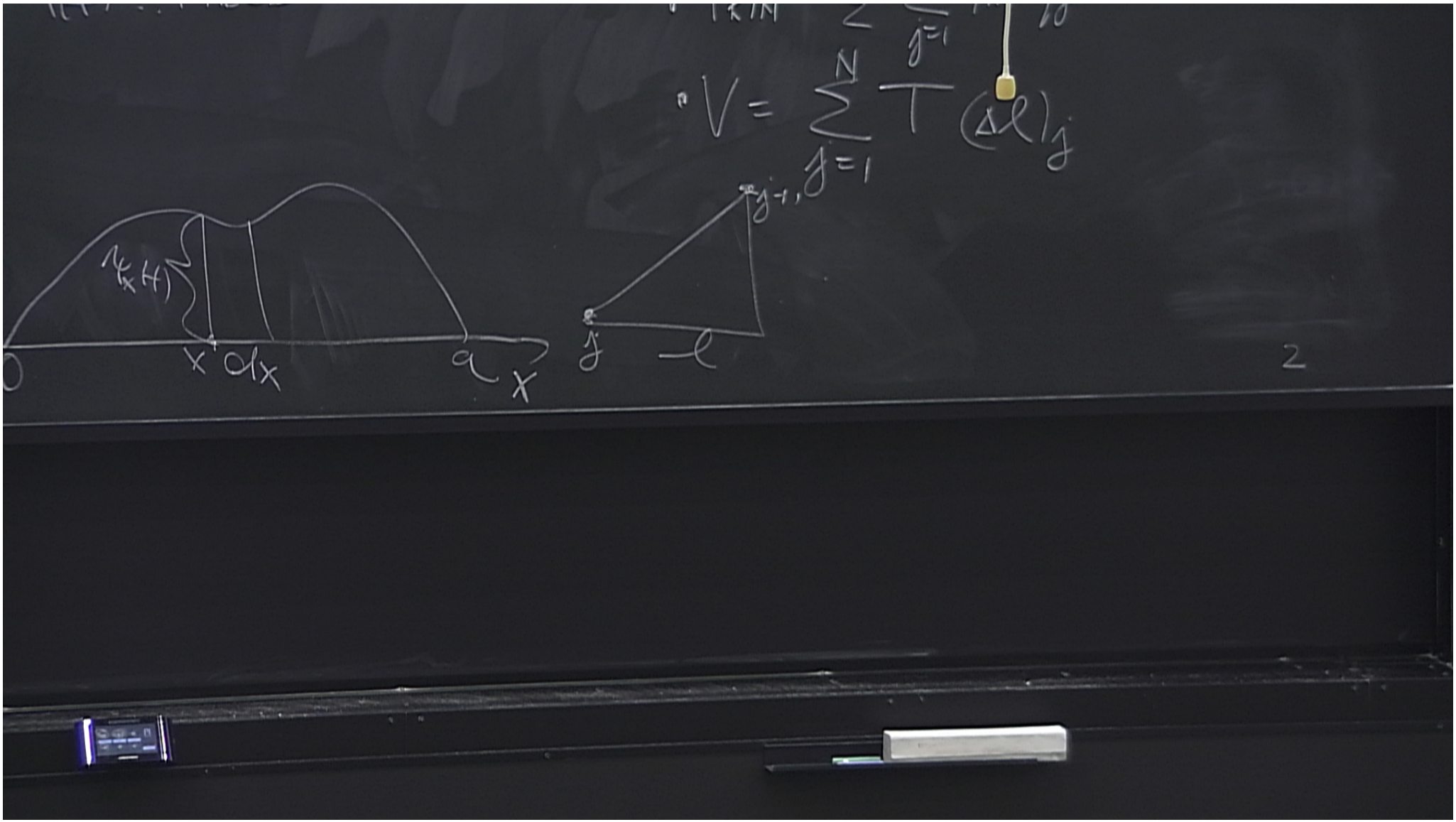
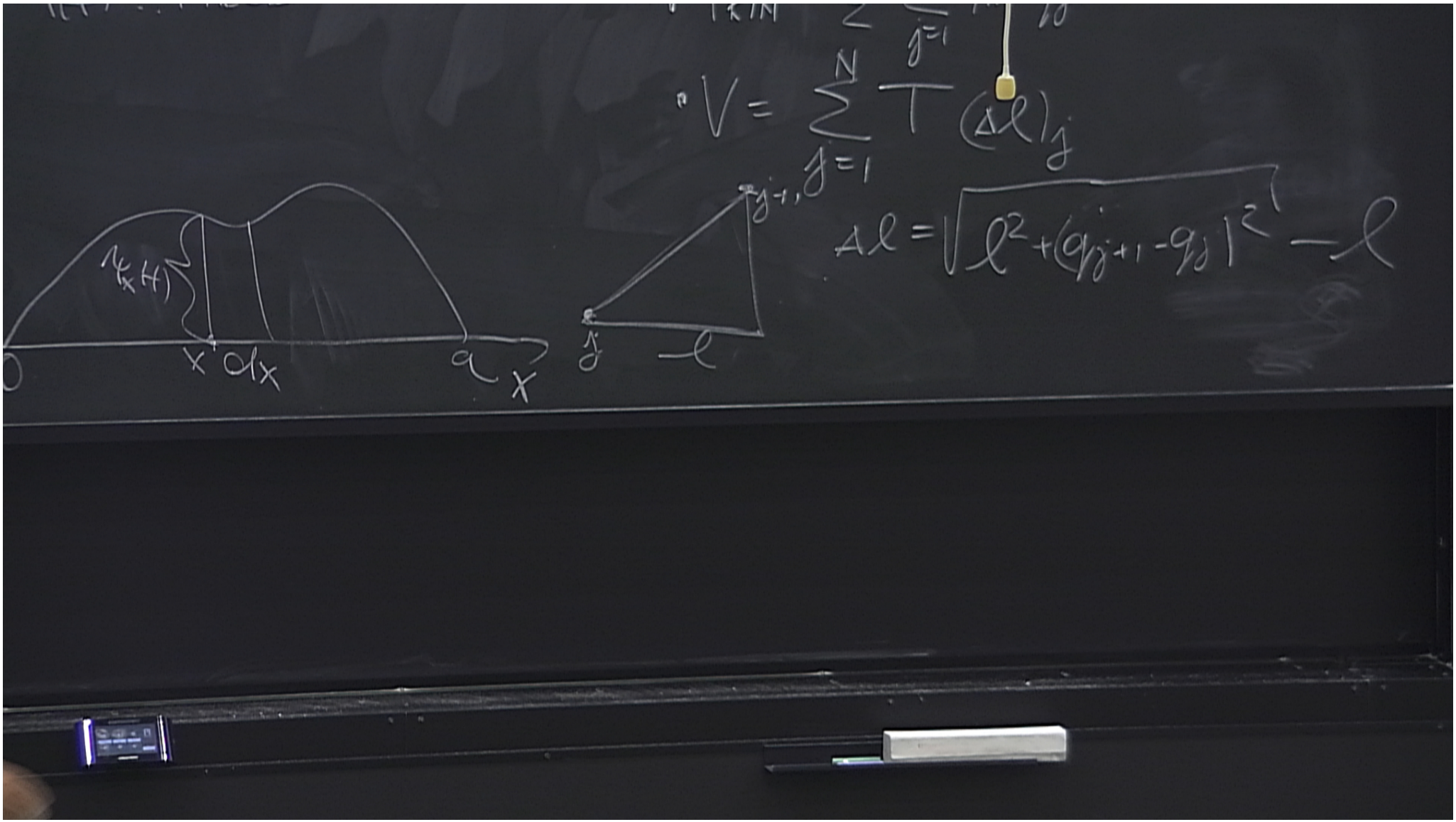
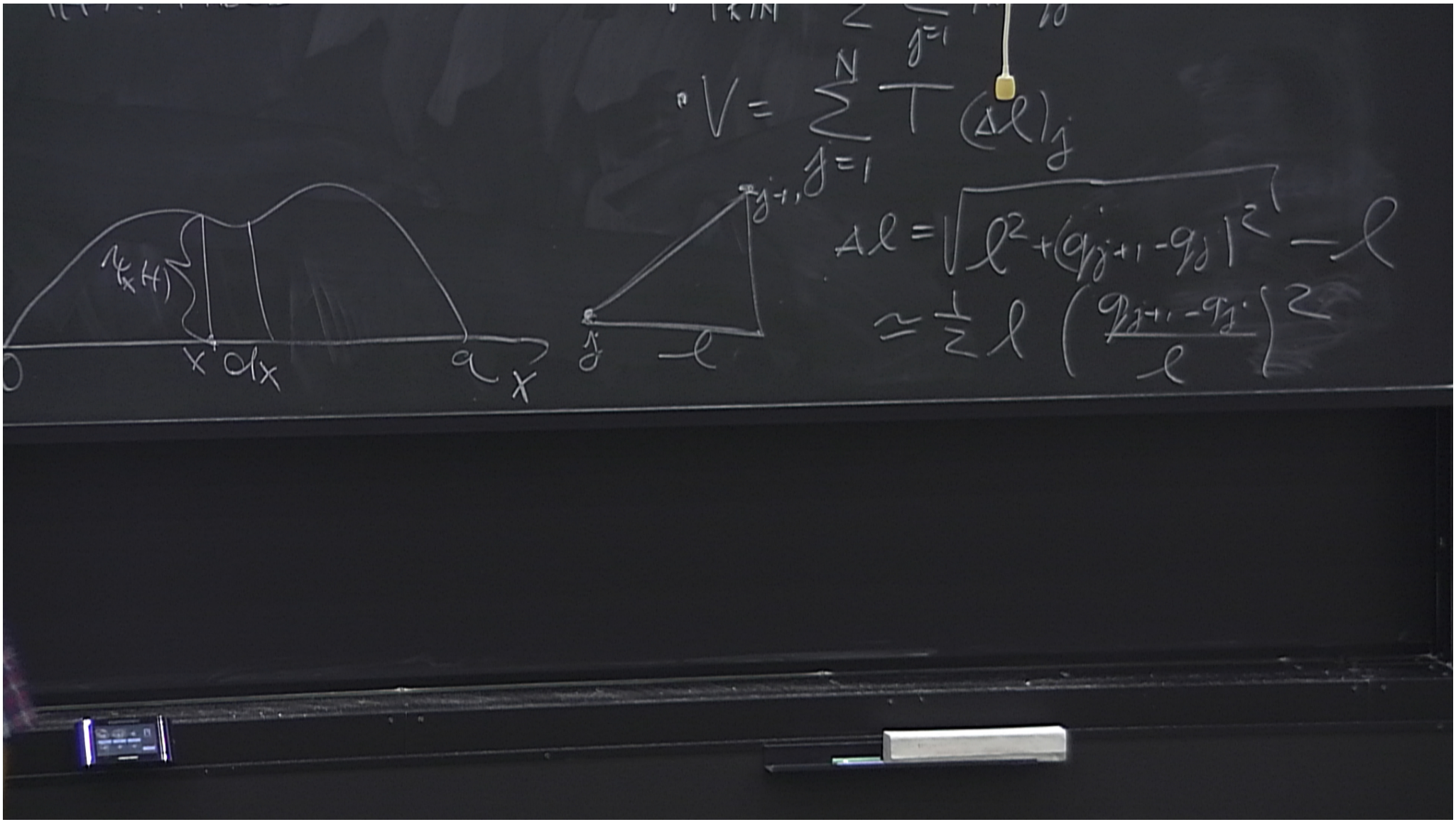
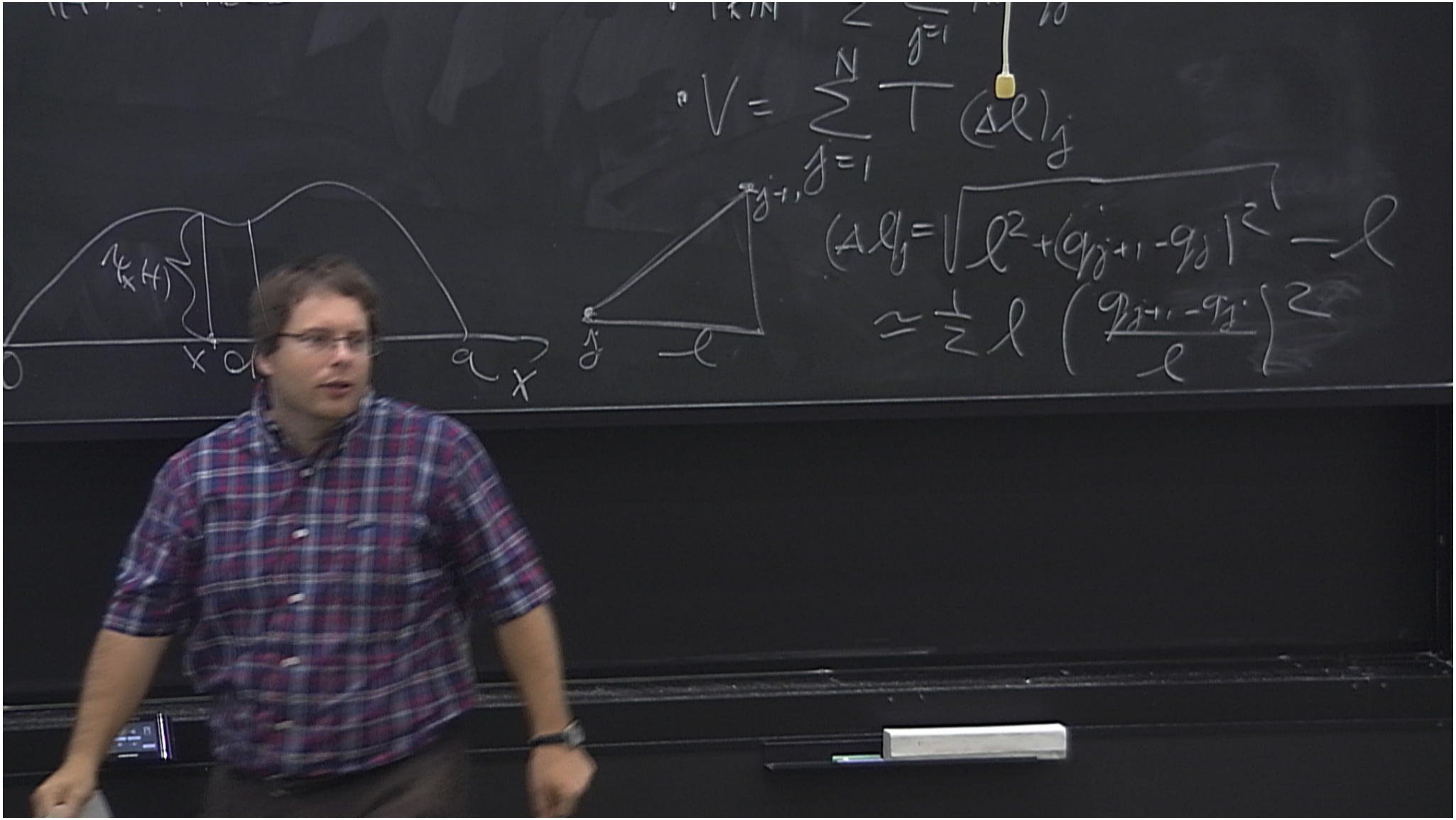

q(t)
m ... N
q_j
q_{j+1}
0
l
a
x
N→∞
l→0
ψ(t) .. FIELD
ψ_x(t)
0
x

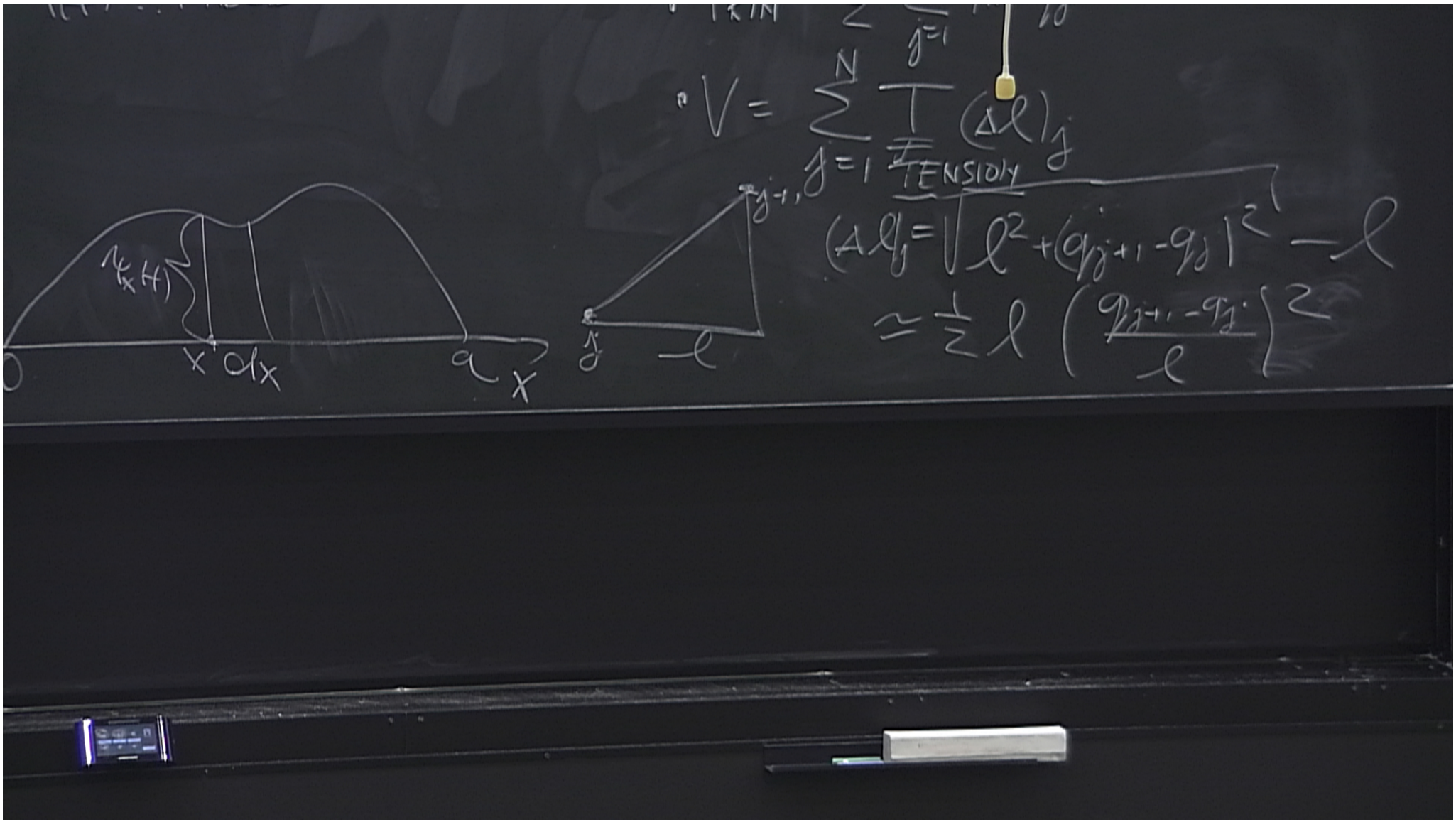

V = Σ_{j=1}^{N} T (Δl)_j
ψ_x(t)
x
dx
a
x
j
j+1
l
2



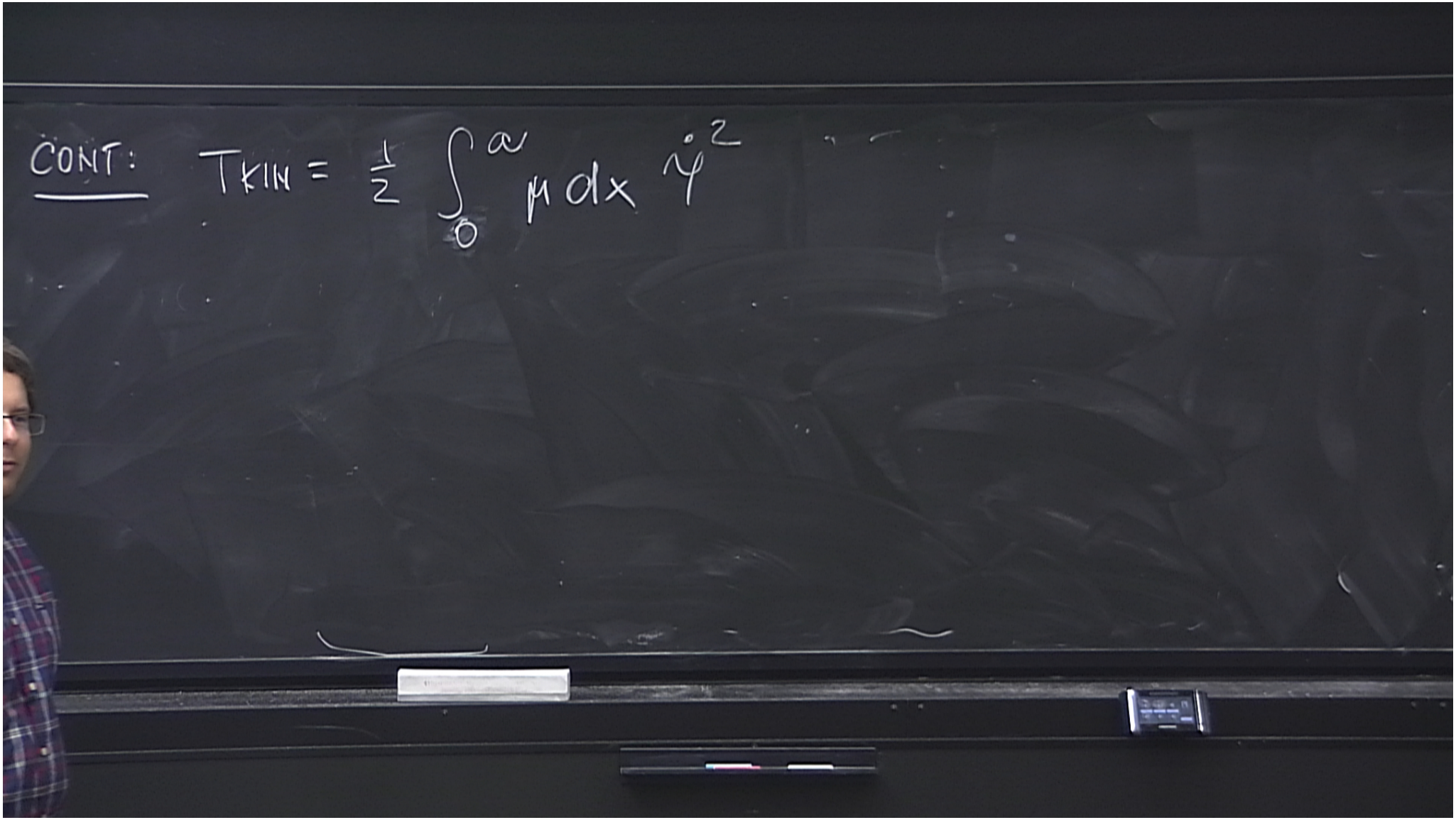
$V = \sum_{j=1}^{N} T (\Delta l)_j$
$\Delta l = \sqrt{l^2 + (q_{j+1} - q_j)^2} - l$
$\psi_x(t)$
x
dx
a
x
j+1
j
l

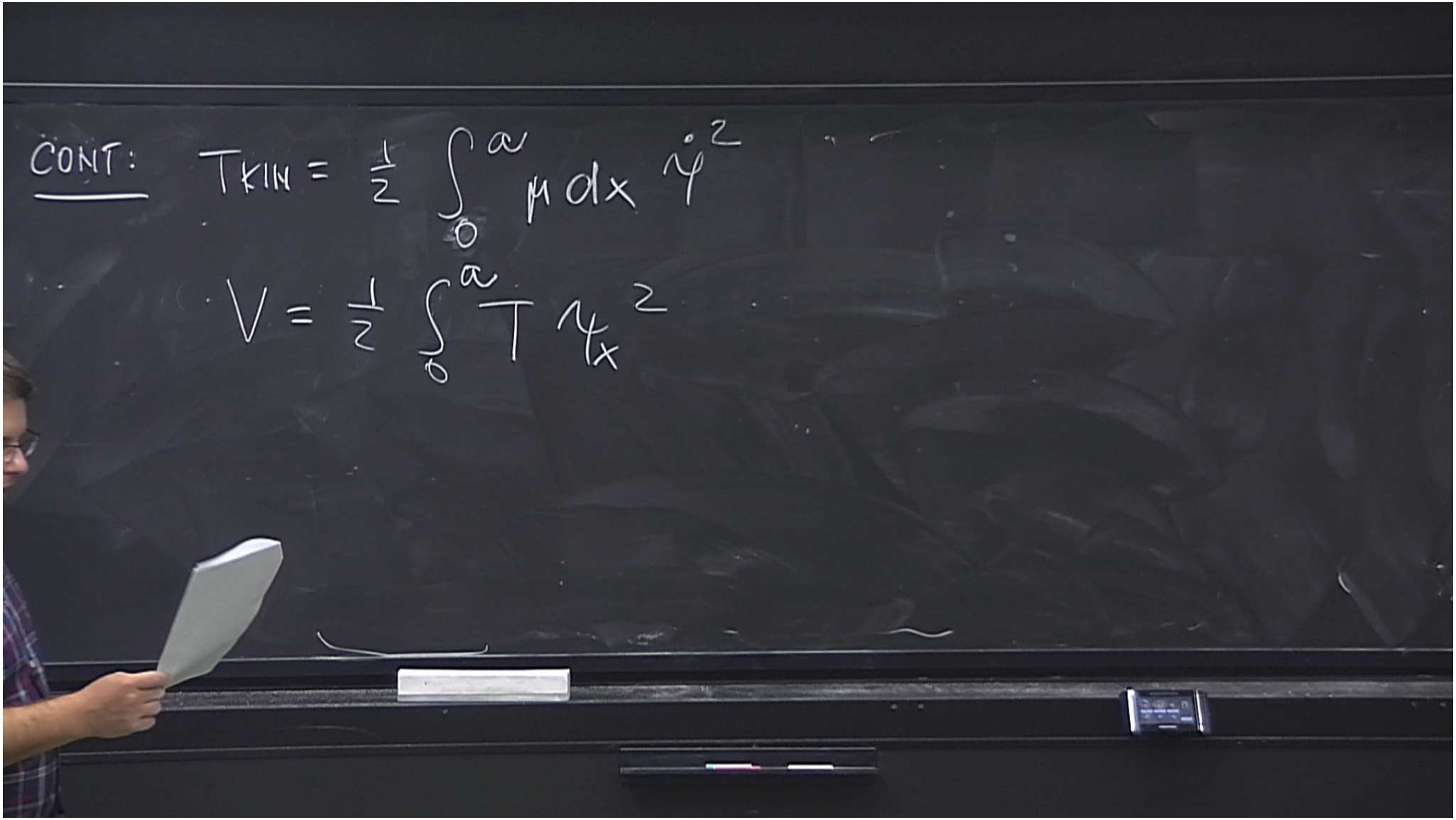

$V = \sum_{j=1}^{N} T (\Delta l)_j$
$\Delta l = \sqrt{l^2 + (q_{j+1} - q_j)^2} - l$
$\simeq \frac{1}{2} l \left(\frac{q_{j+1} - q_j}{l} \right)^2$
$\psi_x(t)$
x
dx
a
x
j
$j+1$
l

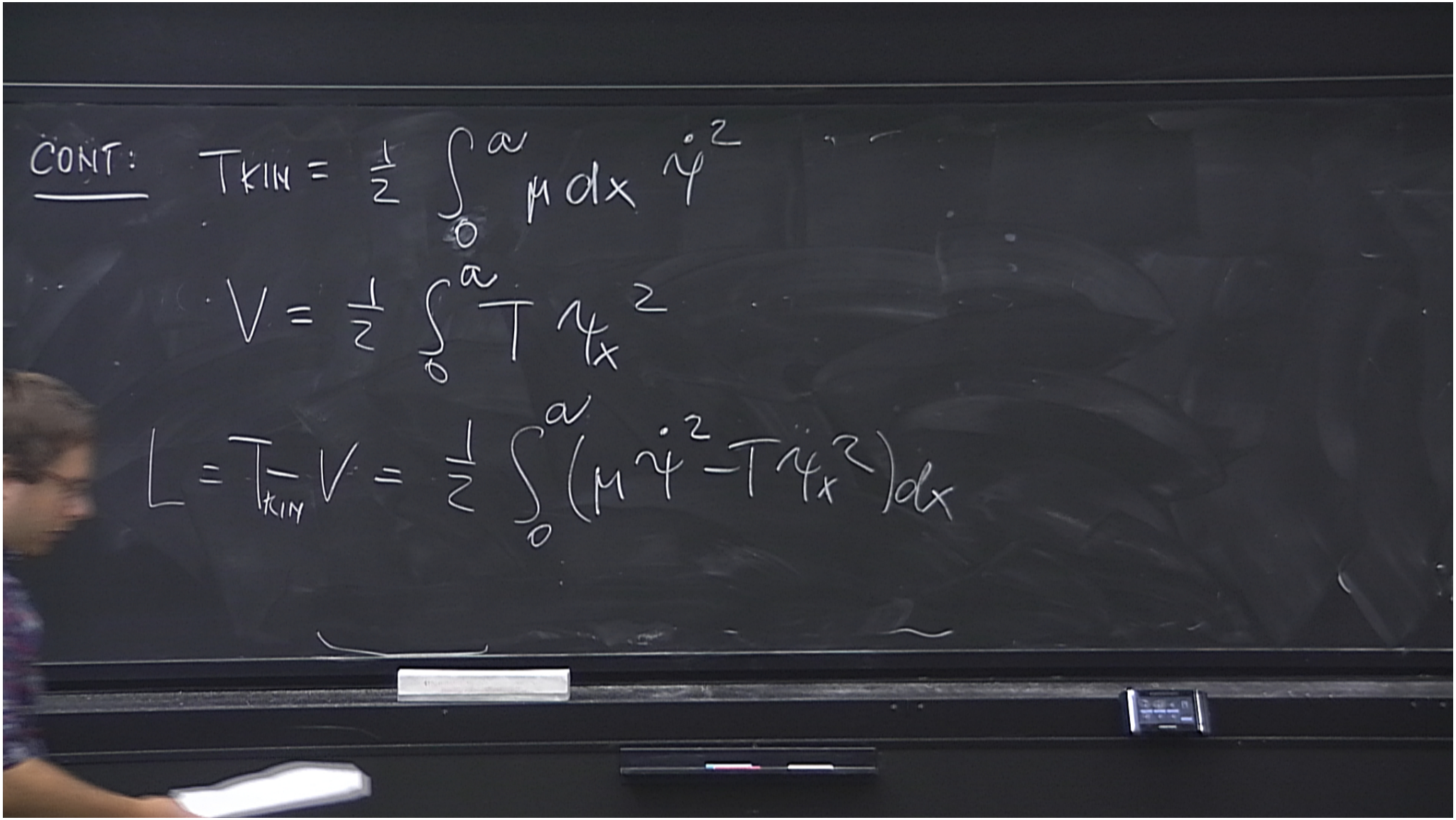

V = Σ T (Δl)_j
Δl_j = √(l² + (q_{j+1} − q_j)²) − l
≃ ½ l ((q_{j+1} − q_j)/l)²
q
x



$V = \sum_{j=1}^{N} T (\Delta l)_j$
TENSION
$(\Delta l)_j = \sqrt{l^2 + (q_{j+1} - q_j)^2} - l$
$\simeq \frac{1}{2} l \left(\frac{q_{j+1} - q_j}{l} \right)^2$
$\psi_x(t)$
x
dx
a
x
j
$j+1$
l


CONT: $T_{KIN} = \frac{1}{2}\int_0^a \mu\, dx\, \dot{\psi}^2$

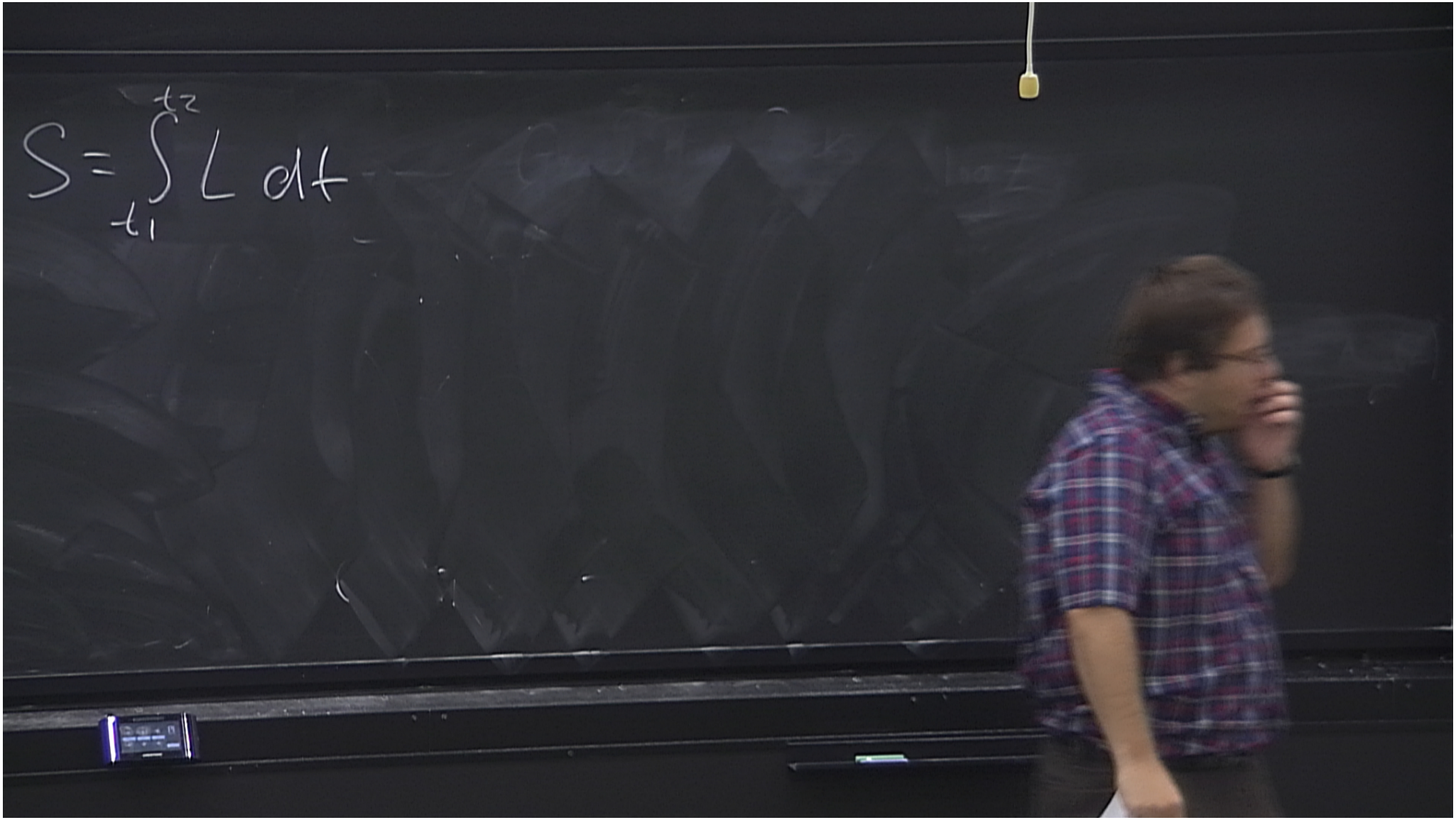

CONT:
$T_{KIN} = \frac{1}{2}\int_0^a \mu\, dx\, \dot{\eta}^2$
$V = \frac{1}{2}\int_0^a T\, \eta_x^2$

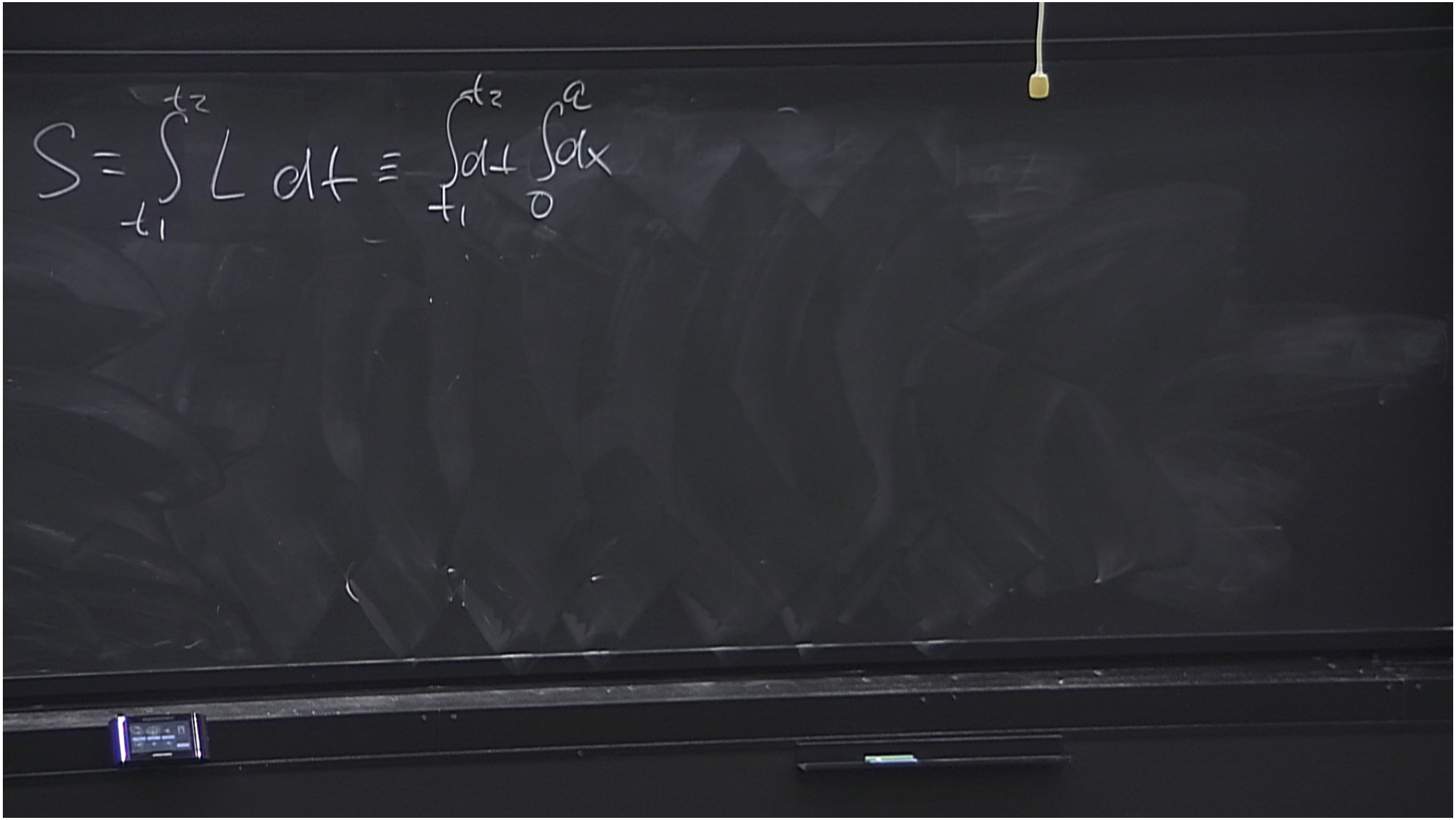

CONT:
$T_{KIN} = \frac{1}{2}\int_0^a \mu\, dx\, \dot{\eta}^2$
$V = \frac{1}{2}\int_0^a T\, \eta_x^2$
$L = T_{KIN} - V = \frac{1}{2}\int_0^a (\mu\dot{\eta}^2 - T\eta_x^2)\, dx$

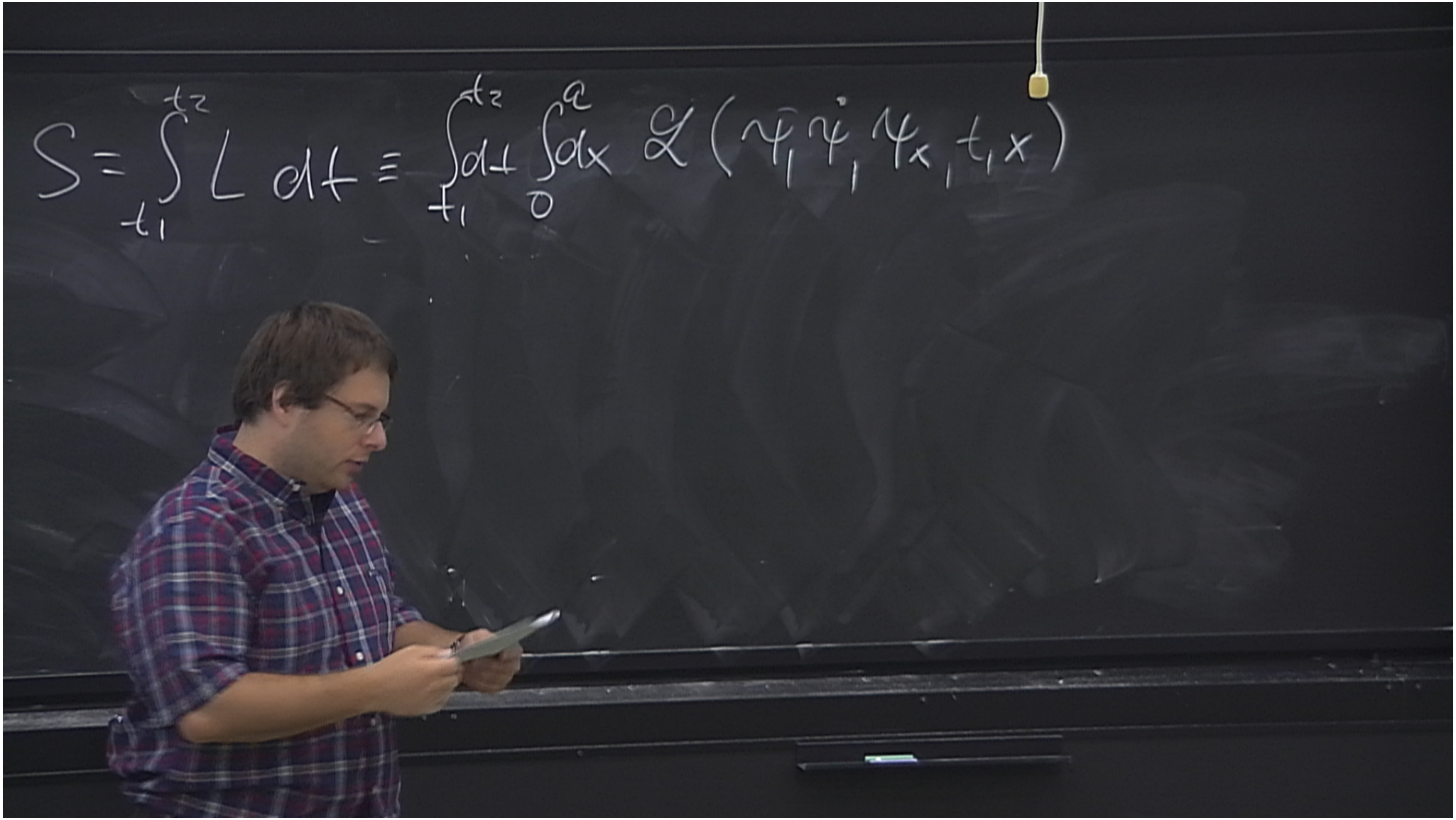
CONT: $T_{KIN} = \frac{1}{2}\int_0^a \mu\, dx\, \dot{\eta}^2$

$V = \frac{1}{2}\int_0^a T\, \eta_x^2$

$L = T_{KIN} - V = \frac{1}{2}\int_0^a (\mu\dot{\eta}^2 - T\eta_x^2)\, dx$


$S = \int_{t_1}^{t_2} L \, dt$


$S = \int_{t_1}^{t_2} L \, dt \equiv \int_{t_1}^{t_2} dt \int_0^a dx$


$S = \int_{t_1}^{t_2} L\, dt \equiv \int_{t_1}^{t_2} dt \int_0^a dx\, \mathcal{L}(\varphi, \dot{\varphi}, \varphi_x, t, x)$

$$S = \int_{t_1}^{t_2} L\, dt \equiv \int_{t_1}^{t_2} dt \int_0^a dx\, \mathcal{L}(\eta, \dot{\eta}, \eta_x, t, x)$$

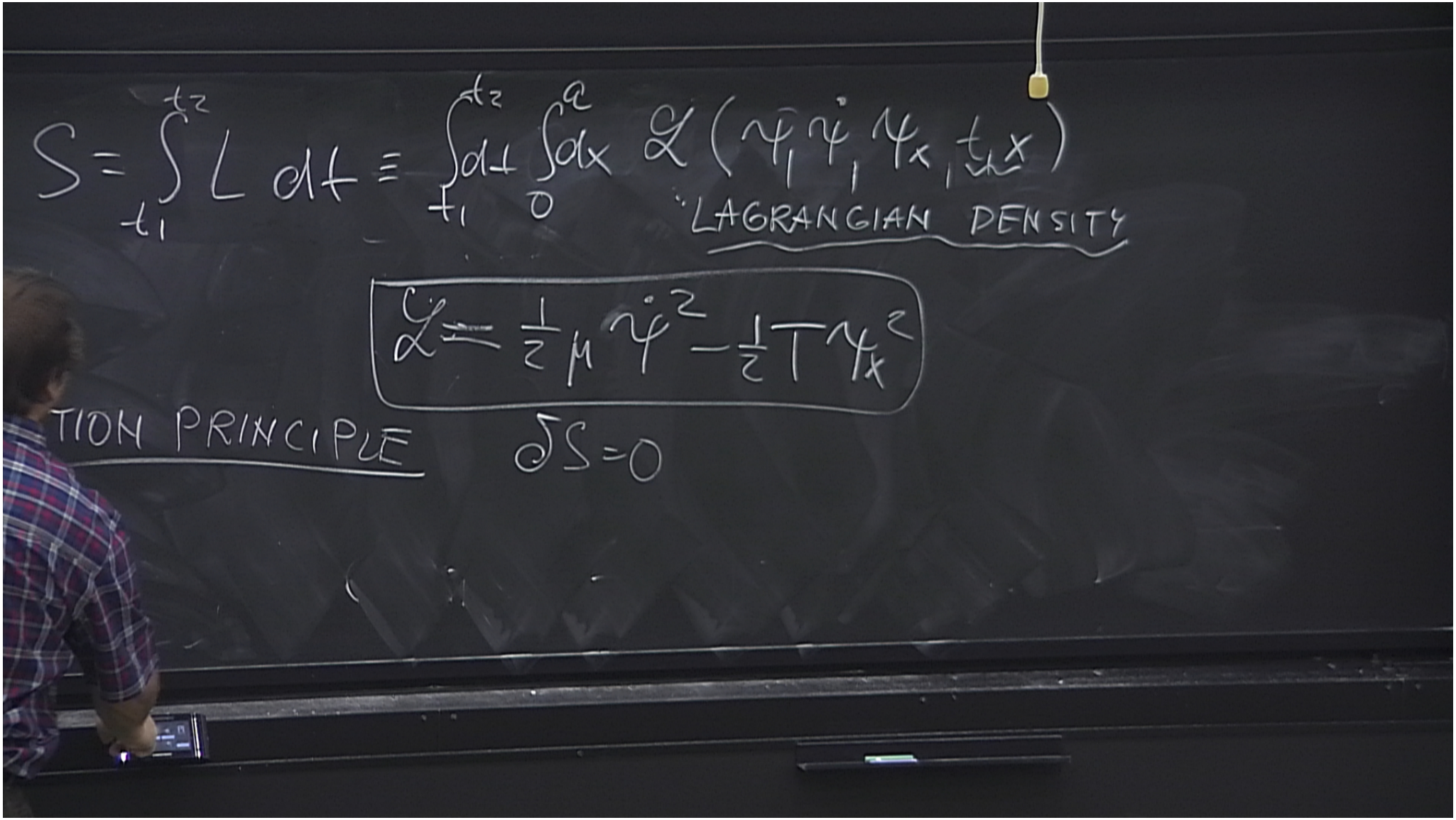
"LAGRANGIAN DENSITY"

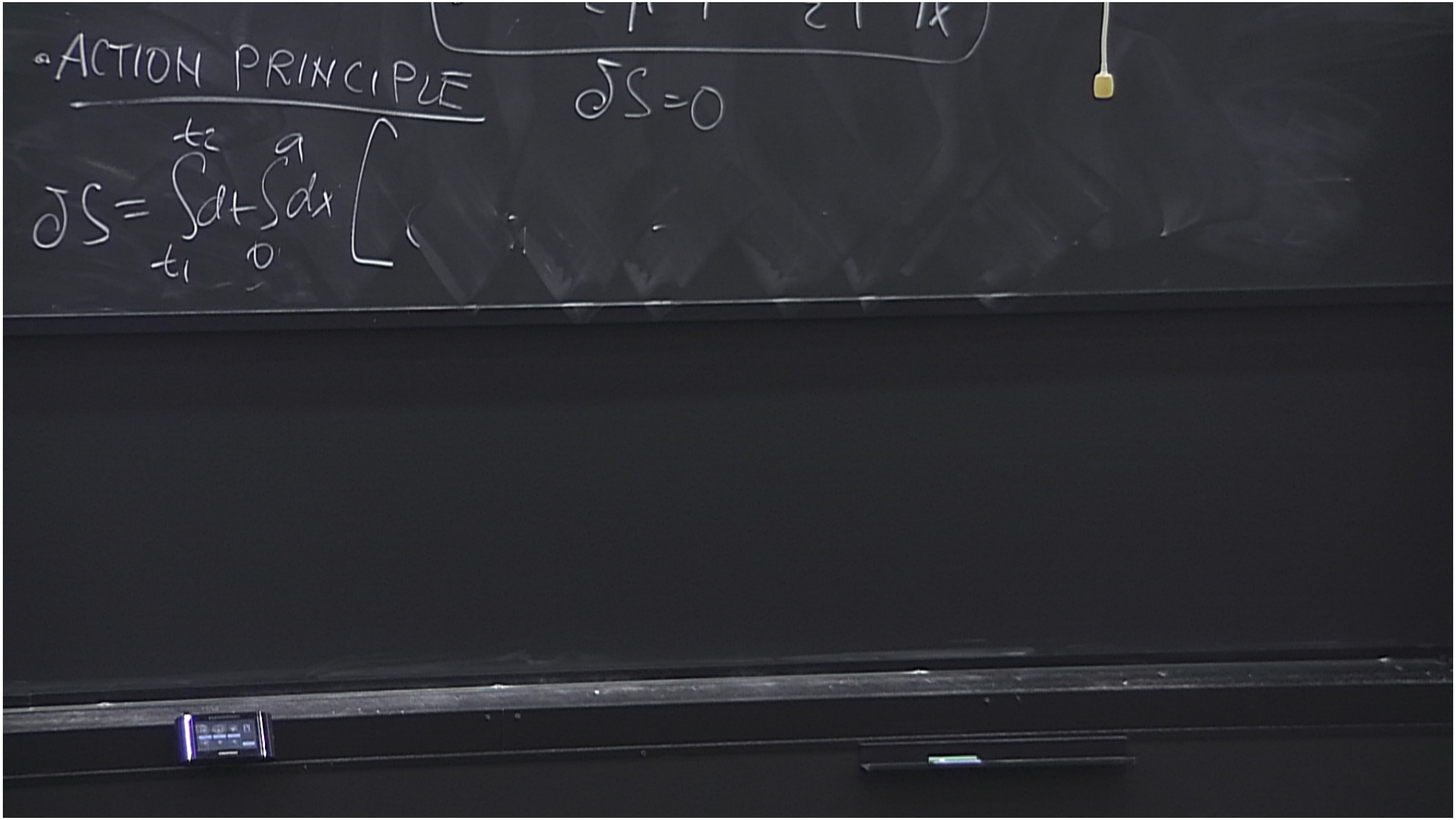
$\mathcal{L} =$

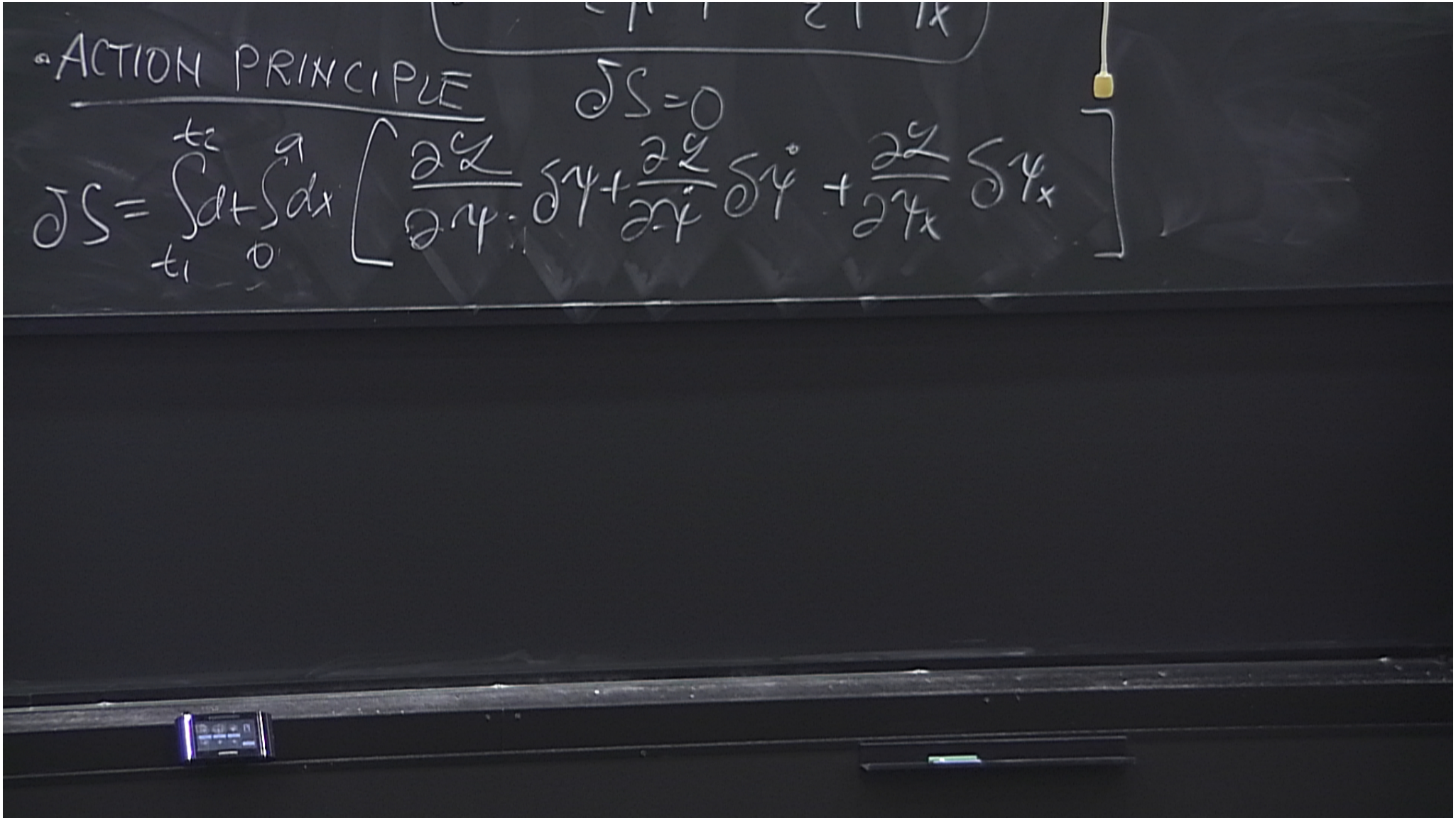
$$S = \int_{t_1}^{t_2} L\, dt \equiv \int_{t_1}^{t_2} dt \int_0^a dx\, \mathcal{L}(\eta, \dot{\eta}, \eta_x, t, x)$$

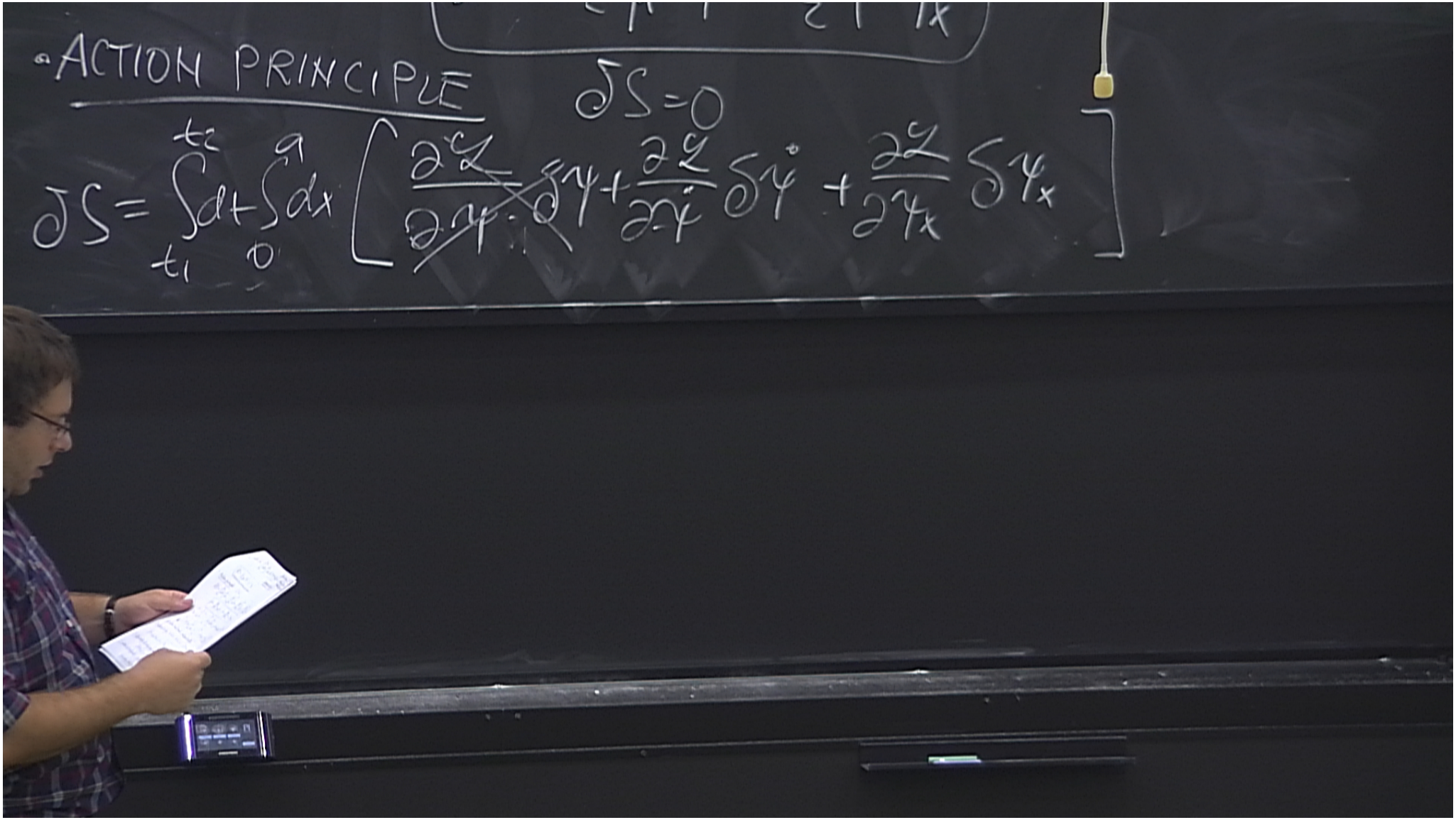
"LAGRANGIAN DENSITY"

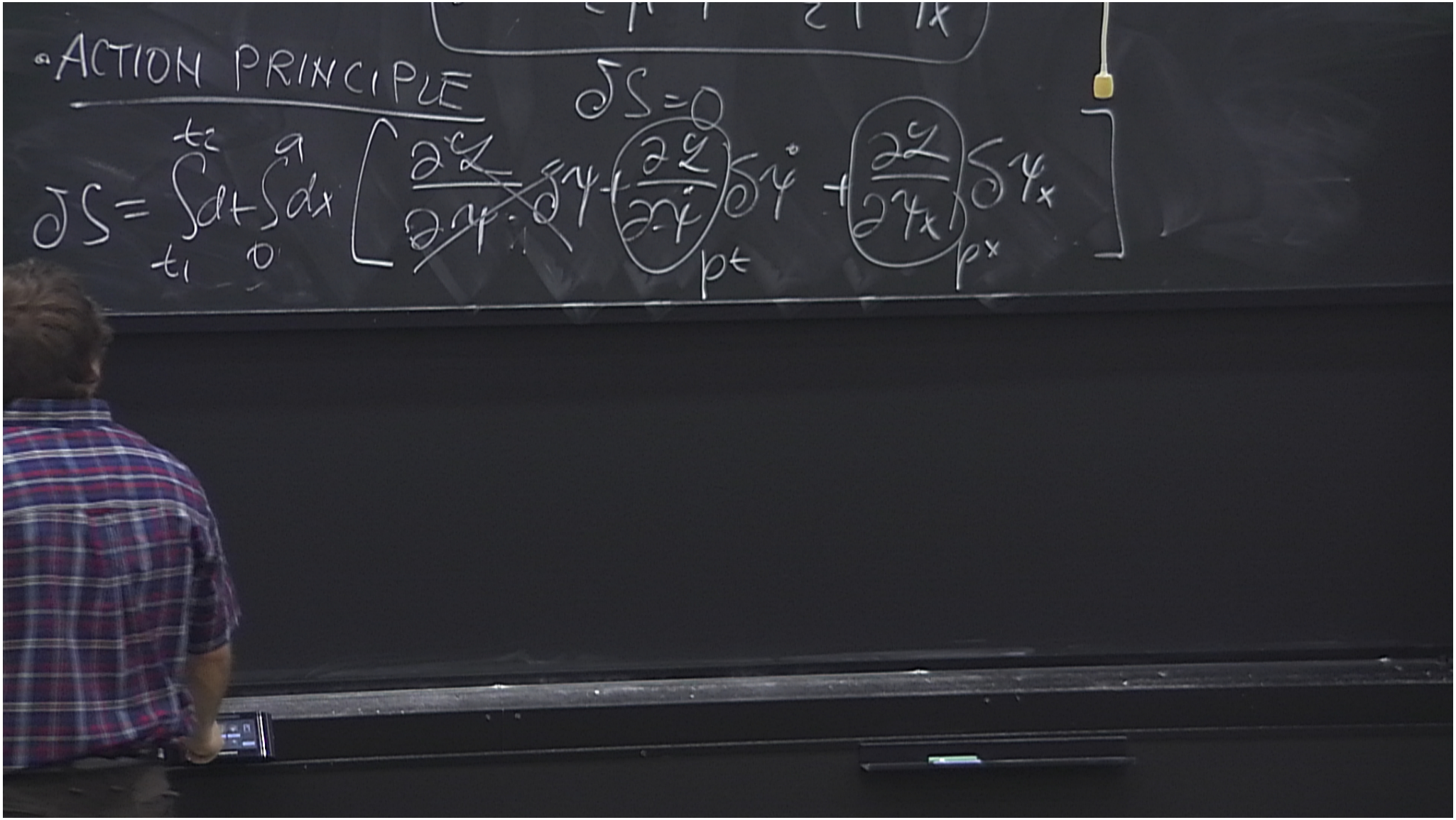
$$\mathcal{L} = \frac{1}{2}\mu \dot{\eta}^2 - \frac{1}{2} T \eta_x^2$$


$S = \int_{t_1}^{t_2} L\, dt \equiv \int_{t_1}^{t_2} dt \int_0^a dx\, \mathcal{L}(\eta, \dot{\eta}, \eta_x, t, x)$
LAGRANGIAN DENSITY
$\mathcal{L} = \frac{1}{2}\mu\dot{\eta}^2 - \frac{1}{2}T\eta_x^2$
TION PRINCIPLE
$\delta S = 0$


ACTION PRINCIPLE
$\delta S = 0$
$\delta S = \int_{t_1}^{t_2} dt \int_0^q dx \left[\right.$


ACTION PRINCIPLE
$\delta S = 0$
$\delta S = \int_{t_1}^{t_2} dt \int_0^q dx \left[\frac{\partial \mathcal{L}}{\partial \psi} \delta\psi + \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \delta\dot{\psi} + \frac{\partial \mathcal{L}}{\partial \psi_x} \delta\psi_x \right]$


ACTION PRINCIPLE
$\delta S = 0$
$\delta S = \int_{t_1}^{t_2} dt \int_0^q dx \left[\frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \delta\dot{\varphi} + \frac{\partial \mathcal{L}}{\partial \varphi_x} \delta\varphi_x \right]$


ACTION PRINCIPLE
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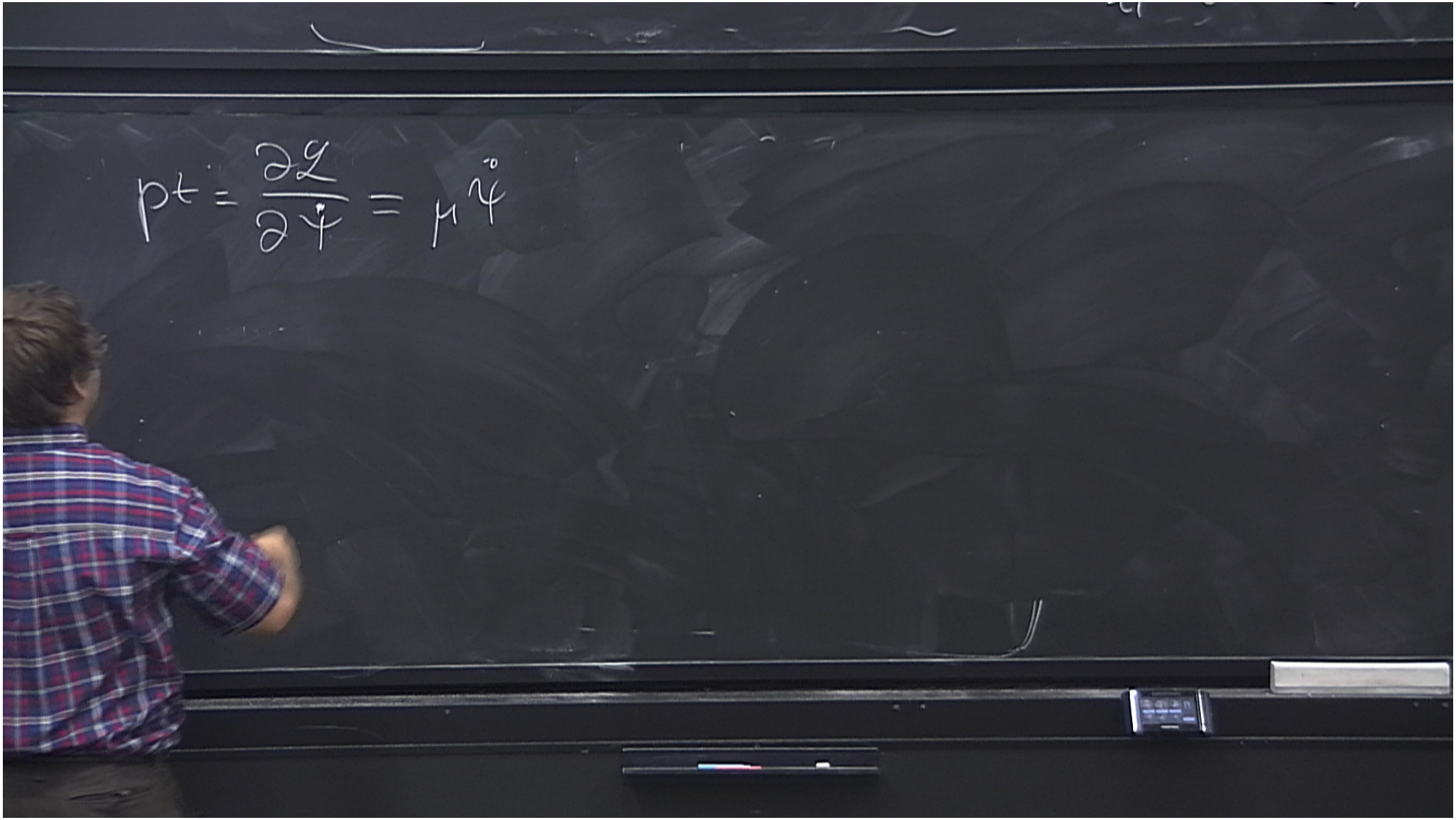
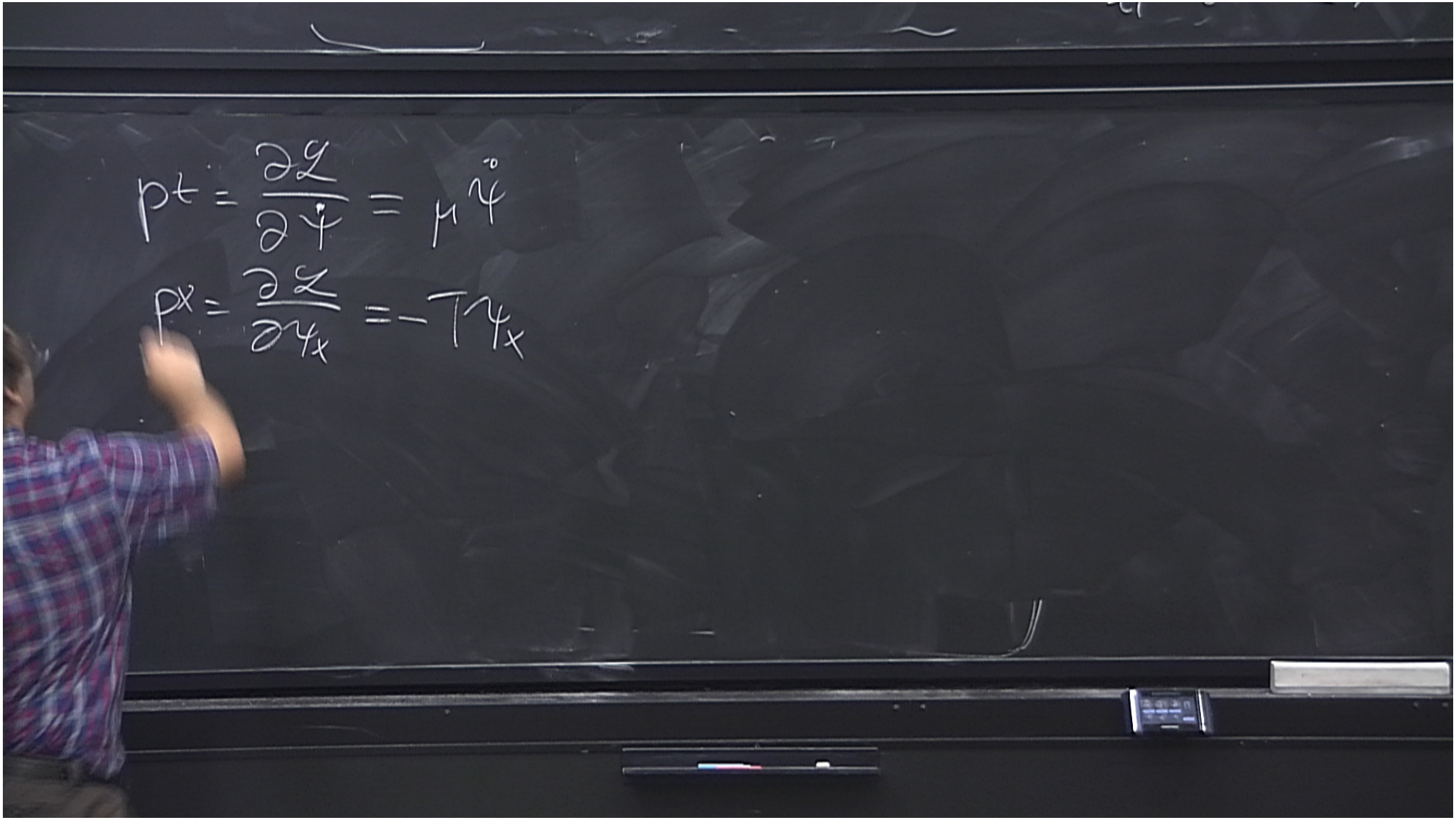
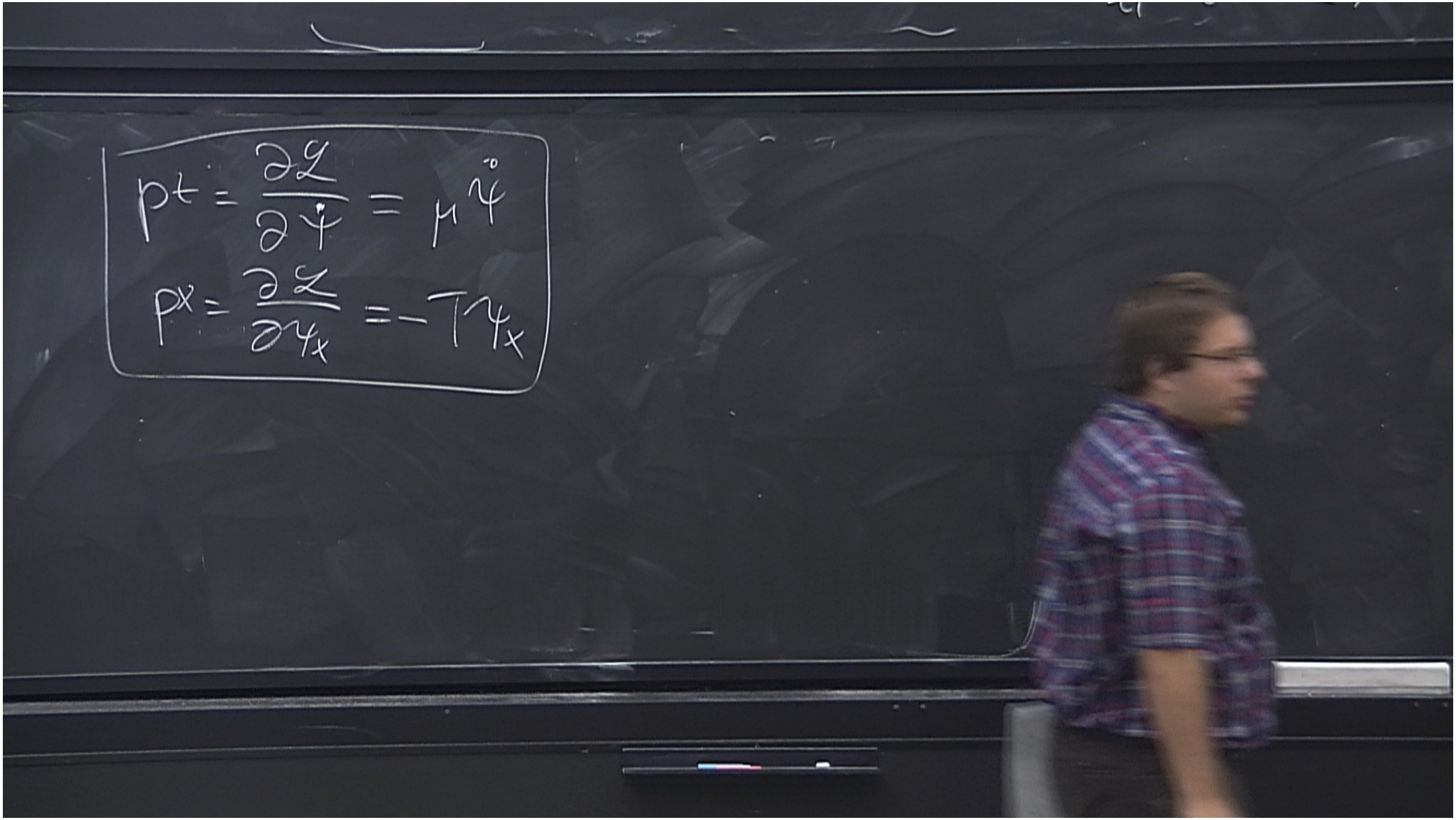
p_t
p_x

$p_t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} =$


$p_t = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \mu \dot{\varphi}$

$p_t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu \dot{\psi}$
$p_x = \frac{\partial \mathcal{L}}{\partial \dot{\psi}_x} =$


$p_t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu \ddot{\psi}$
$p_x = \frac{\partial \mathcal{L}}{\partial \psi_x} = -T\psi_x$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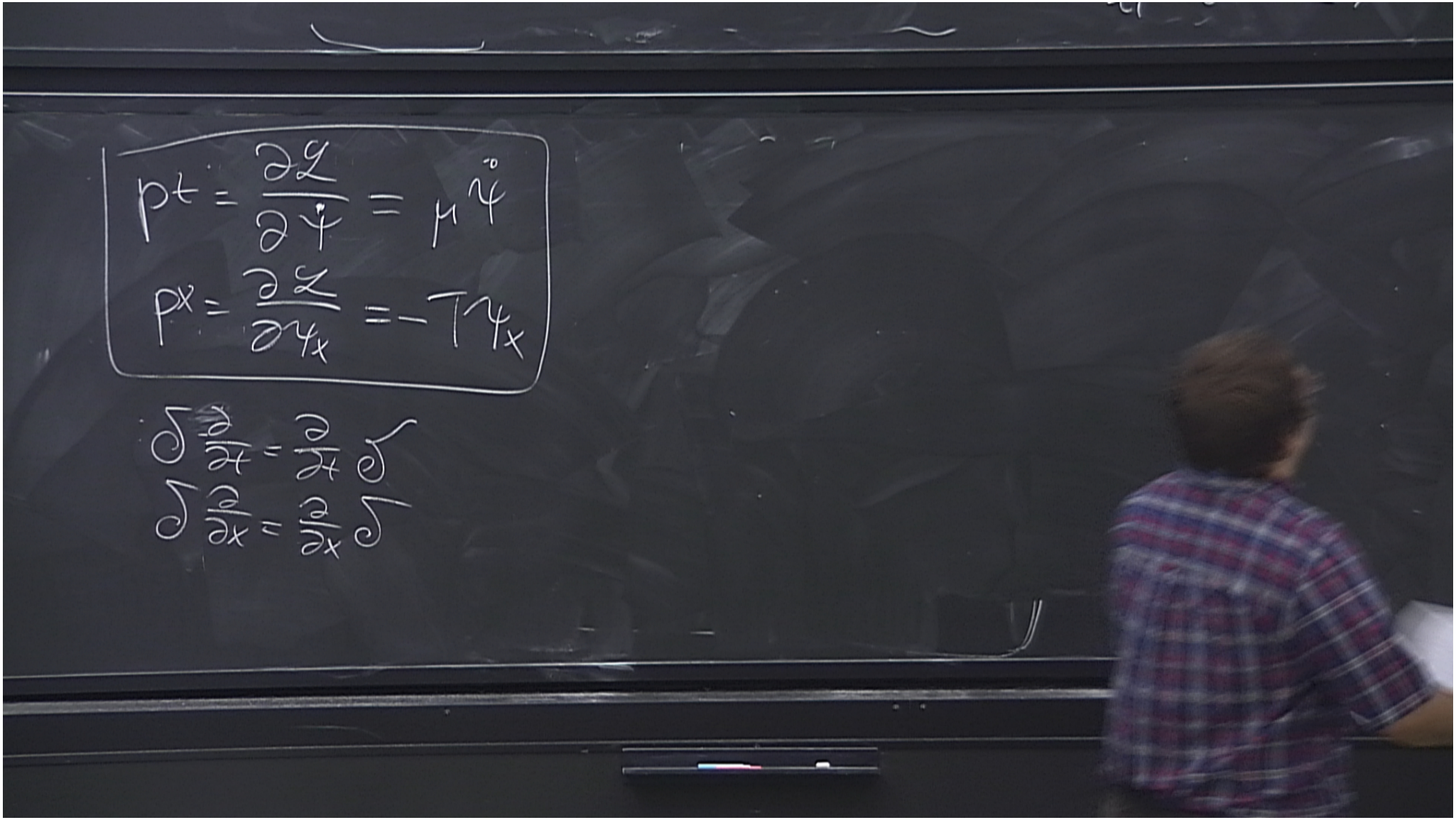
$p^t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu \ddot{\psi}$
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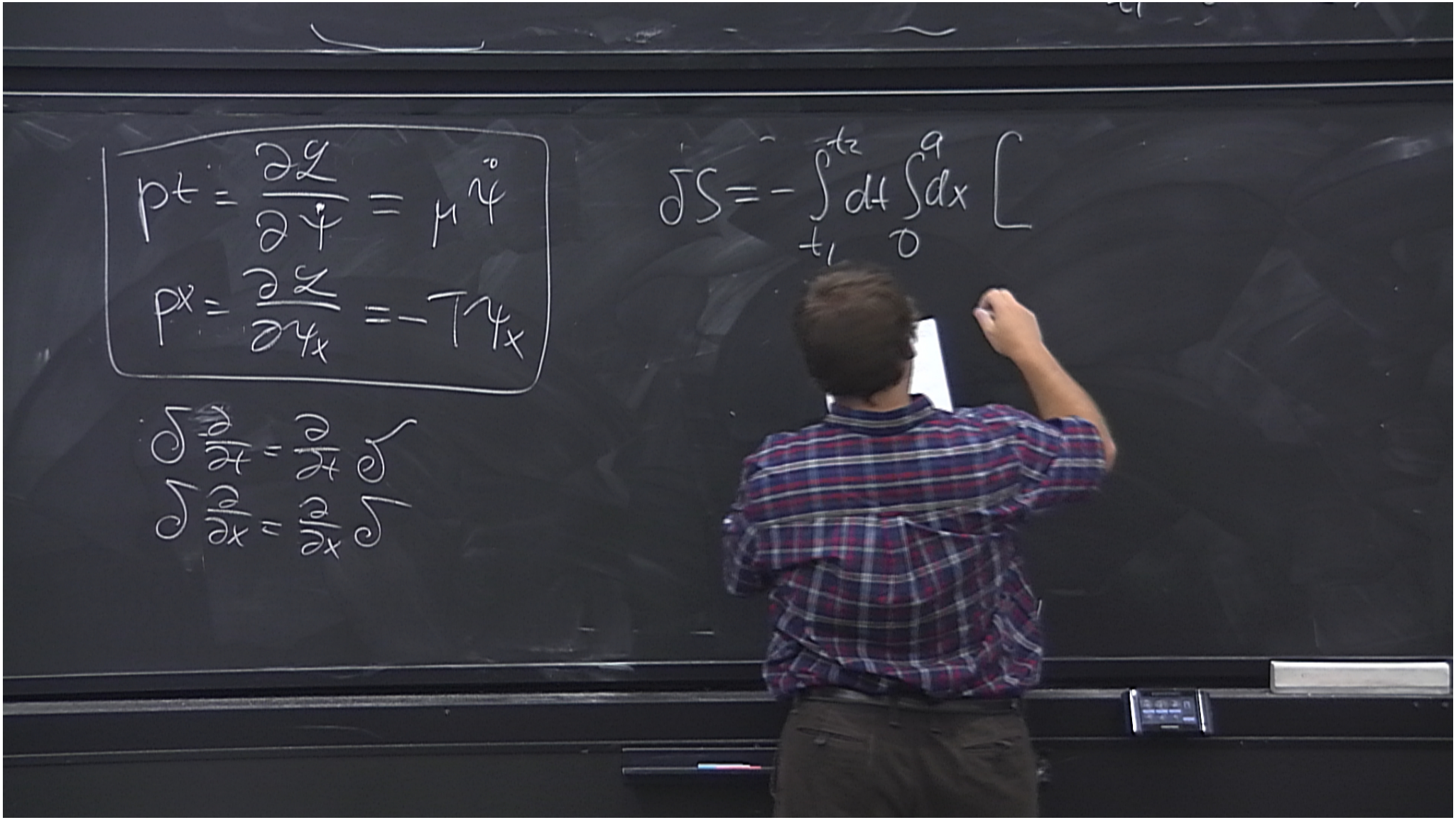
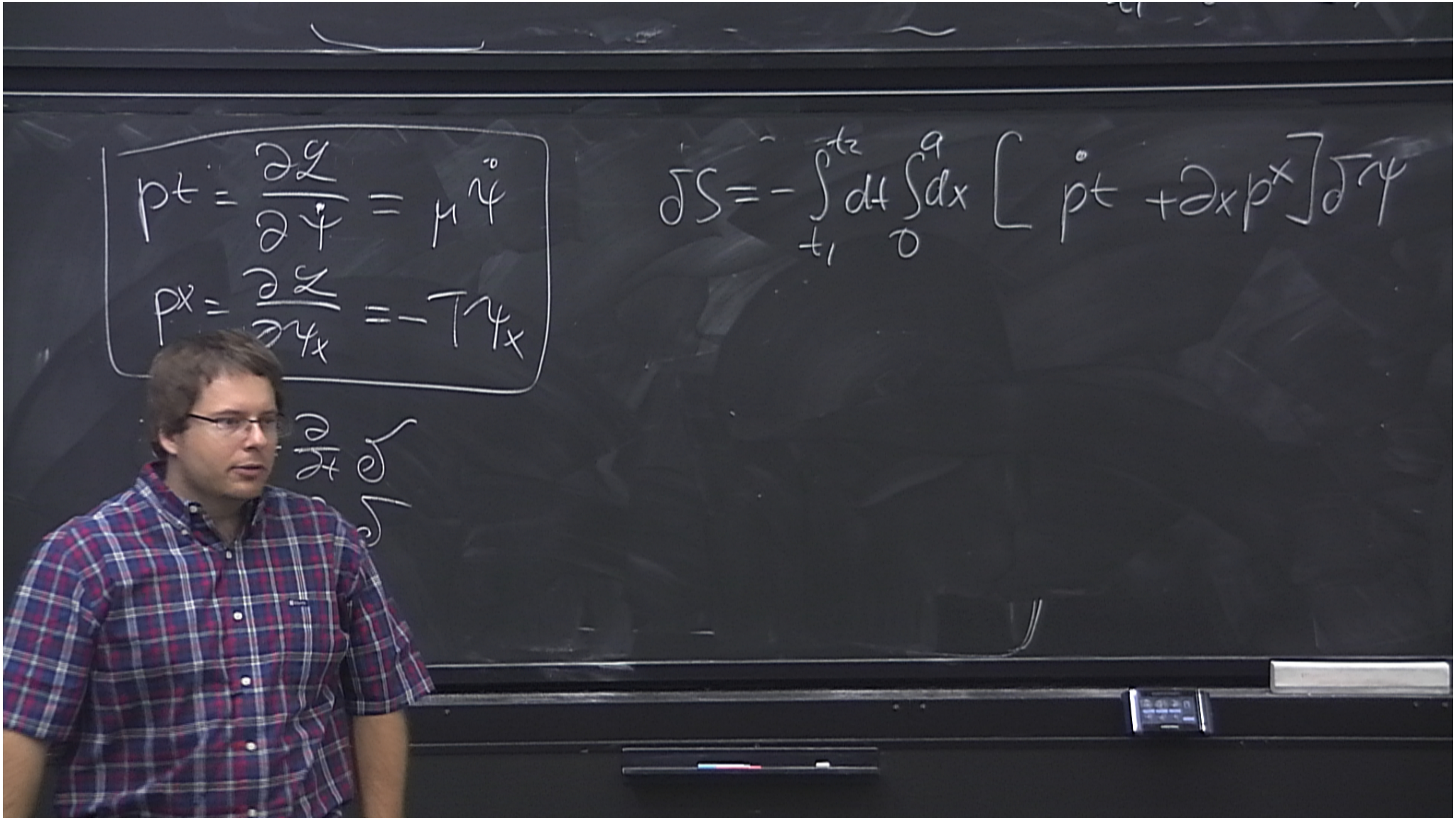
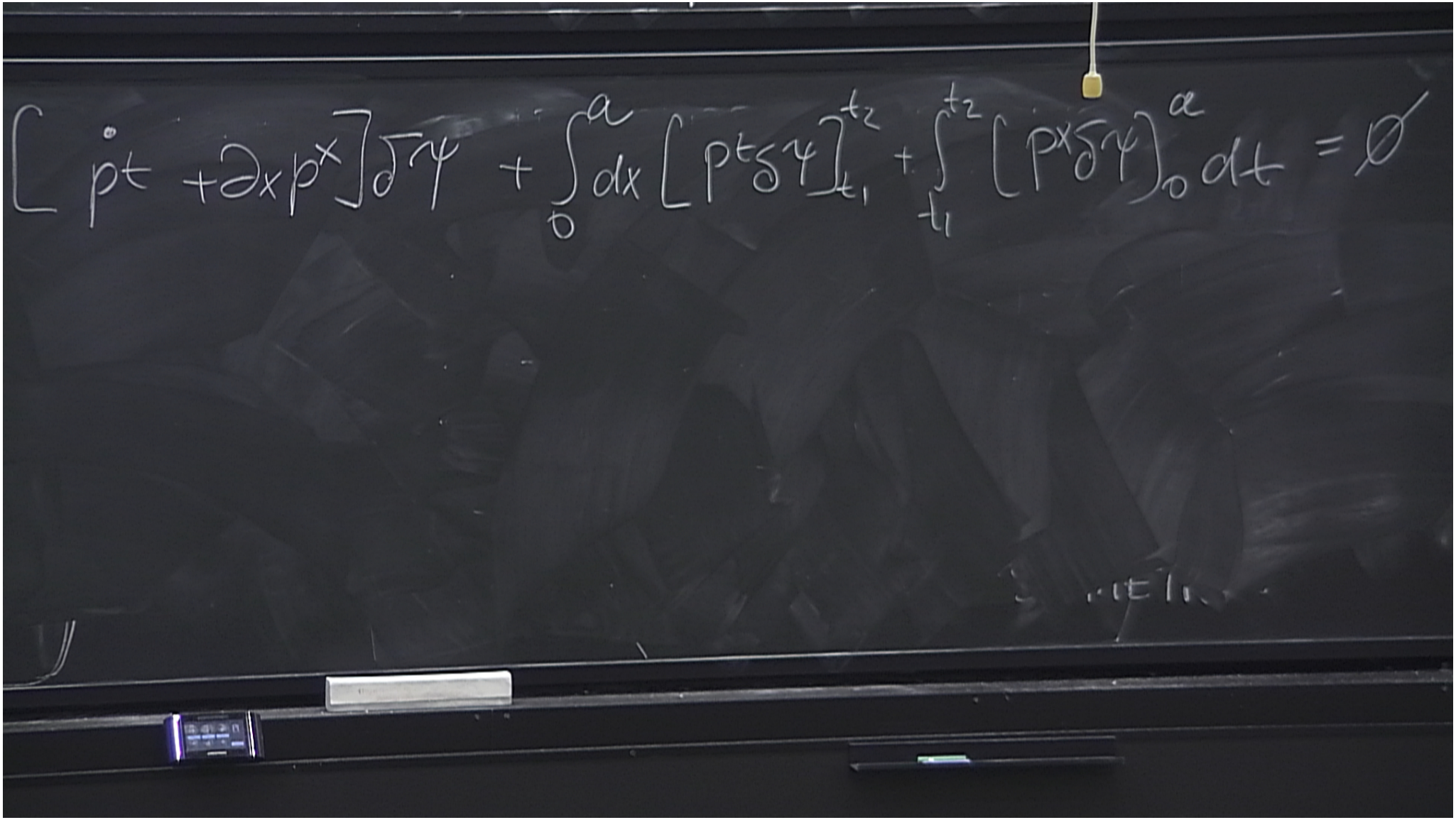
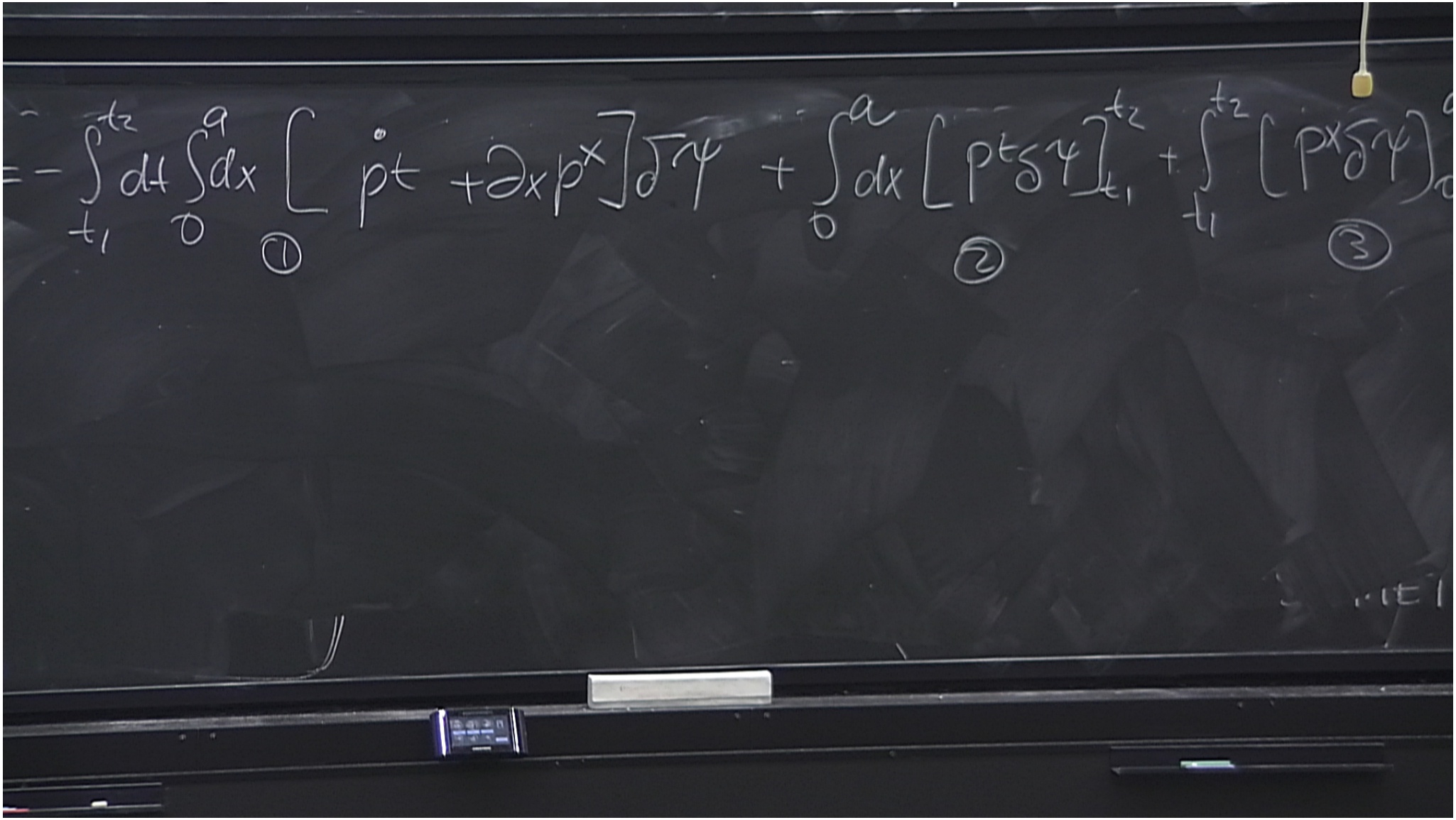
$\int \frac{\partial}{\partial t} = \frac{\partial}{\partial t} \int$
$\int \frac{\partial}{\partial x} = \frac{\partial}{\partial x} \int$


$p_t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu \ddot{\psi}$
$p^x = \frac{\partial \mathcal{L}}{\partial \psi_x} = -T\psi_x$
$\delta \frac{\partial}{\partial t} = \frac{\partial}{\partial t} \delta$
$\delta \frac{\partial}{\partial x} = \frac{\partial}{\partial x} \delta$
$\delta S = -\int_{t_1}^{t_2} dt \int_0^9 dx \, [$


$p^t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \mu \dot{\psi}$
$p^x = \frac{\partial \mathcal{L}}{\partial \psi_x} = -T \psi_x$
$\delta S = -\int_{t_1}^{t_2} dt \int_0^a dx \left[\dot{p}^t + \partial_x p^x \right] \delta \psi$

$$\left[\dot{p}^t + \partial_x p^x\right]\delta\psi + \int_0^a dx \left[p^t \delta\psi\right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left[p^x \delta\psi\right]_0^a dt$$


$[\dot{p}^t + \partial_x p^x]\delta\psi + \int_0^a dx\,[p^t\delta\psi]_{t_1}^{t_2} + \int_{t_1}^{t_2} [p^x\delta\psi]_0^a\,dt = 0$


$= -\int_{t_1}^{t_2} dt \int_0^a dx \,[\dot{p}^t + \partial_x p^x]\,\delta\psi + \int_0^a dx\,[p^t \delta\psi]_{t_1}^{t_2} + \int_{t_1}^{t_2} [p^x \delta\psi]$
①
②
③

$$\delta S = -\int_{t_1}^{t_2} dt \int_0^a dx \underbrace{\left[\dot{p}^t + \partial_x p^x \right] \delta Y}_{①} + \underbrace{\int_0^a dx \left[p^t \delta Y \right]_{t_1}^{t_2}}_{②} + \underbrace{\int_{t_1}^{t_2} \left[p^x \delta \right.}_{③}$$

① MOTION OF STRING $x \in (0,a)$, $t \in (t_1,t_2)$, δY NOT RESTRICTED

$=$

$$\delta S = -\int_{t_1}^{t_2} dt \int_0^a dx \underbrace{\left[\dot{p}^t + \partial_x p^x\right]}_{①} \delta Y + \underbrace{\int_0^a dx \left[p^t \delta Y\right]_{t_1}^{t_2}}_{②} + \underbrace{\int_{t_1}^{t_2} \left[p^x \delta\right.}_{③}$$

① MOTION OF STRING $x \in (0,a)$, $t \in (t_1, t_2)$, δY NOT RESTRICTED

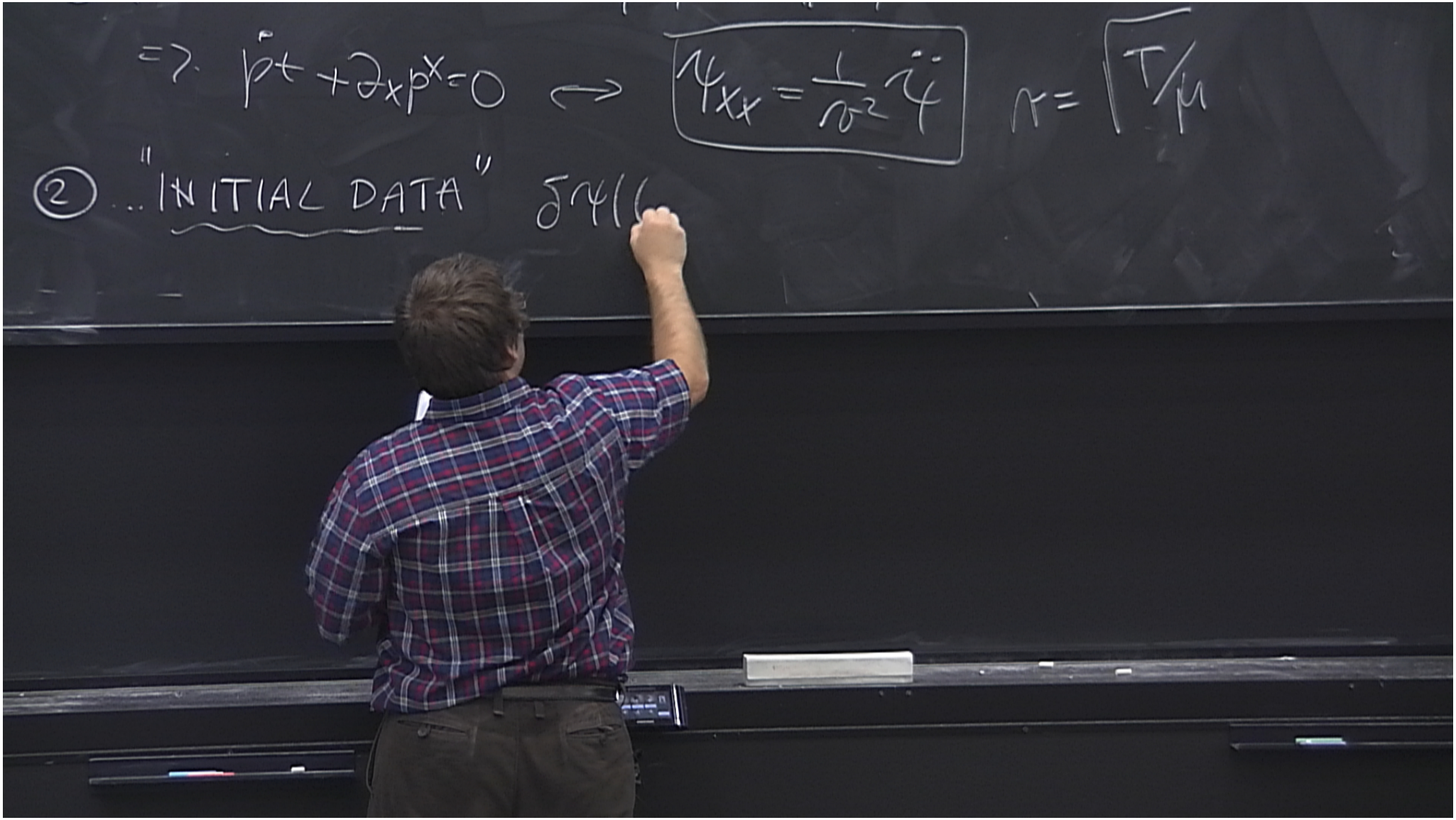
$\Rightarrow \dot{p}^t + \partial_x p^x = 0$

$$\delta S = -\int_{t_1}^{t_2} dt \int_0^a dx \,[\dot{p}^t + \partial_x p^x]\,\delta\psi + \int_0^a dx\,[p^t \delta\psi]_{t_1}^{t_2} + \int_{t_1}^{t_2} [p^x \delta$$

① ② ③

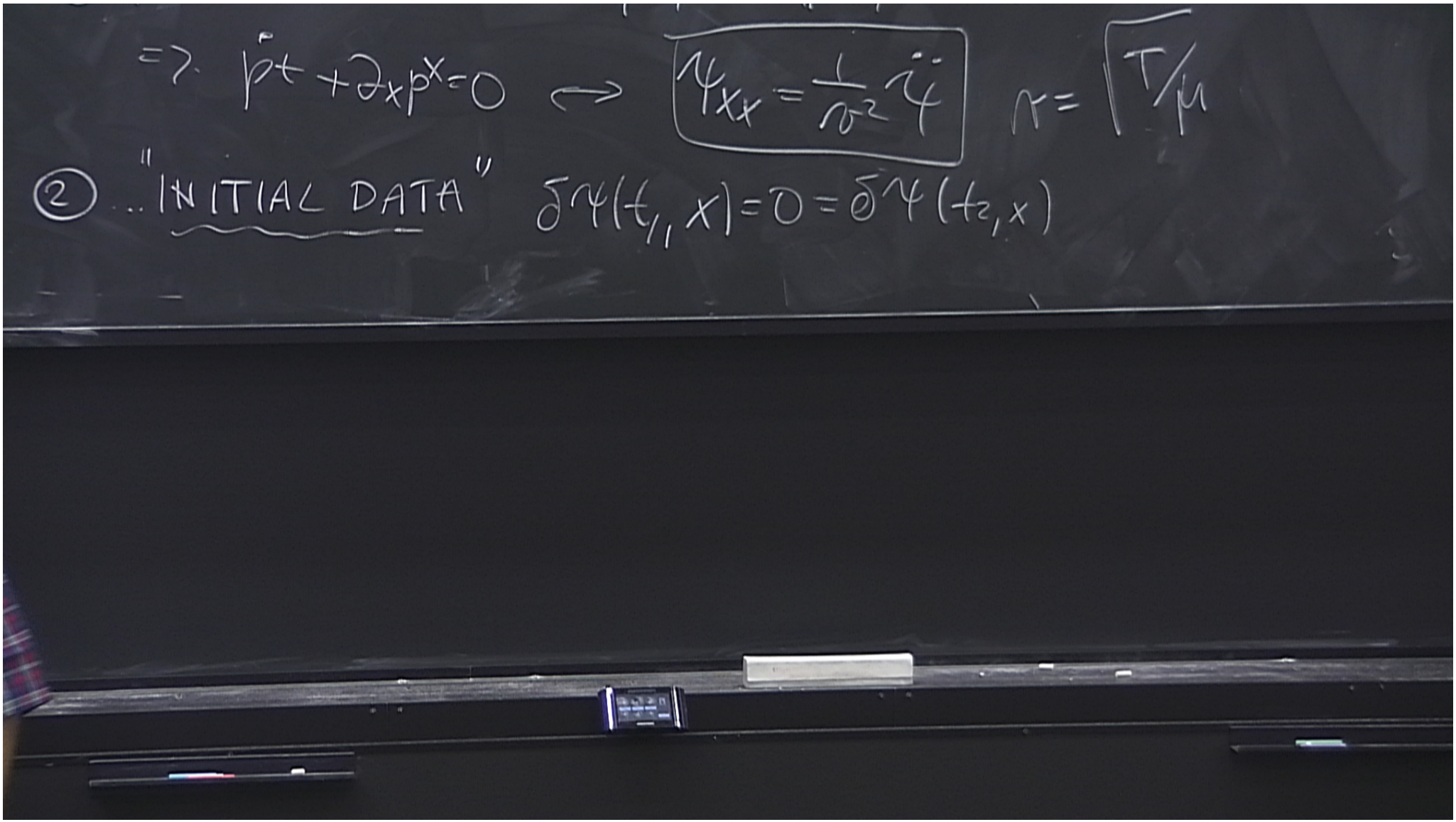
① MOTION OF STRING $x \in (0,a)$, $t \in (t_1,t_2)$, $\delta\psi$ NOT RESTRICTED

$\Rightarrow \dot{p}^t + \partial_x p^x = 0 \iff \boxed{\psi_{xx} = \frac{1}{v^2}\ddot{\psi}}$ $\quad v = \sqrt{T/\mu}$


$\Rightarrow \dot{p}^t + \partial_x p^x = 0 \iff \psi_{xx} = \frac{1}{v^2}\ddot{\psi}$
$v = \sqrt{T/\mu}$
② "INITIAL DATA" $\delta\psi(t_0$

$\Rightarrow \dot{p}^t + \partial_x p^x = 0 \iff \boxed{\psi_{xx} = \frac{1}{v^2}\ddot{\psi}}$ $v = \sqrt{T/\mu}$

② "INITIAL DATA" $\delta\psi(t_1, x) = 0 = \delta\dot{\psi}(t_2, x)$



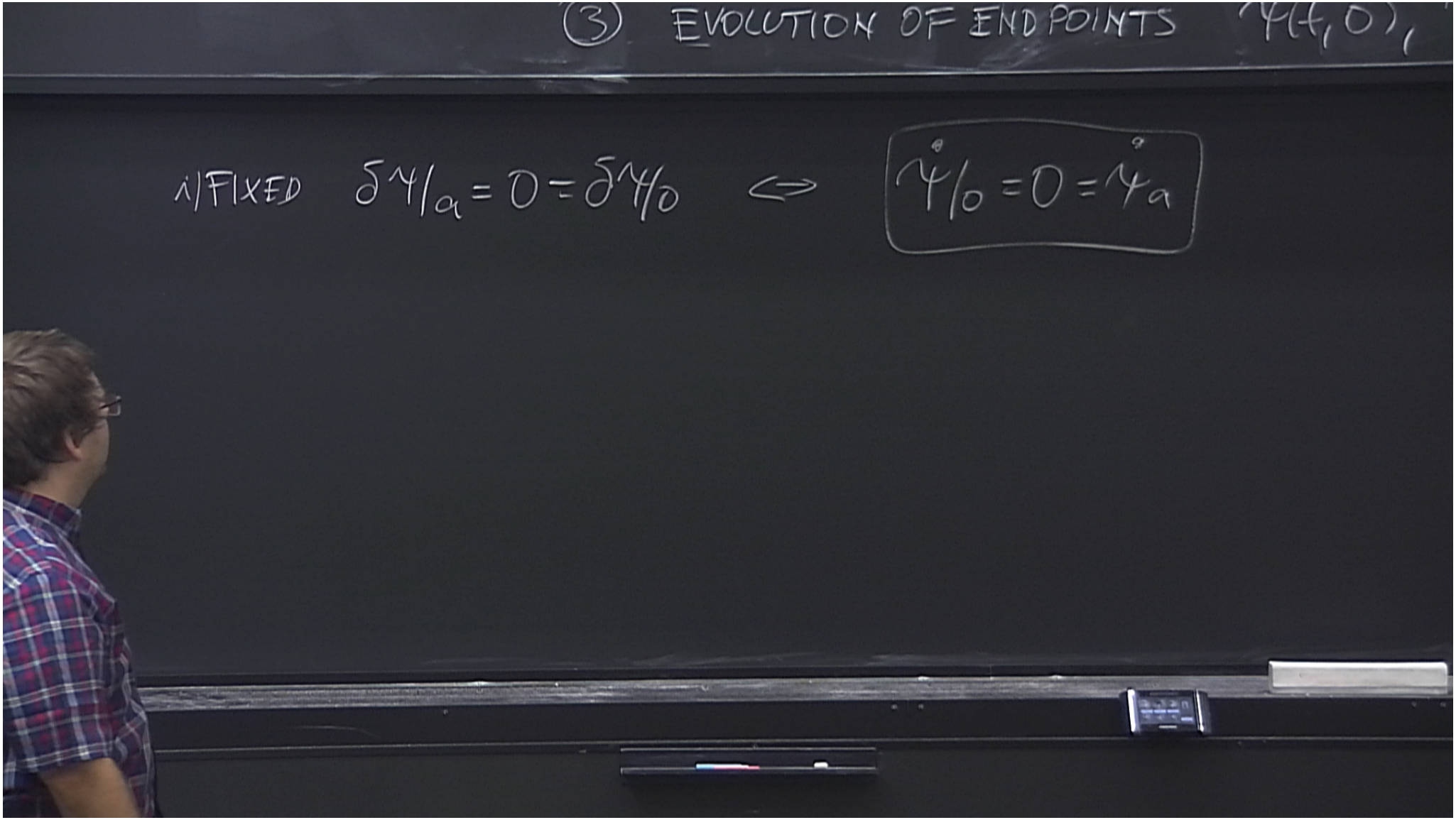
$\Rightarrow \dot{p}^t + \partial_x p^x = 0 \iff \boxed{\psi_{xx} = \frac{1}{v^2}\ddot{\psi}}$ $\quad v = \sqrt{T/\mu}$

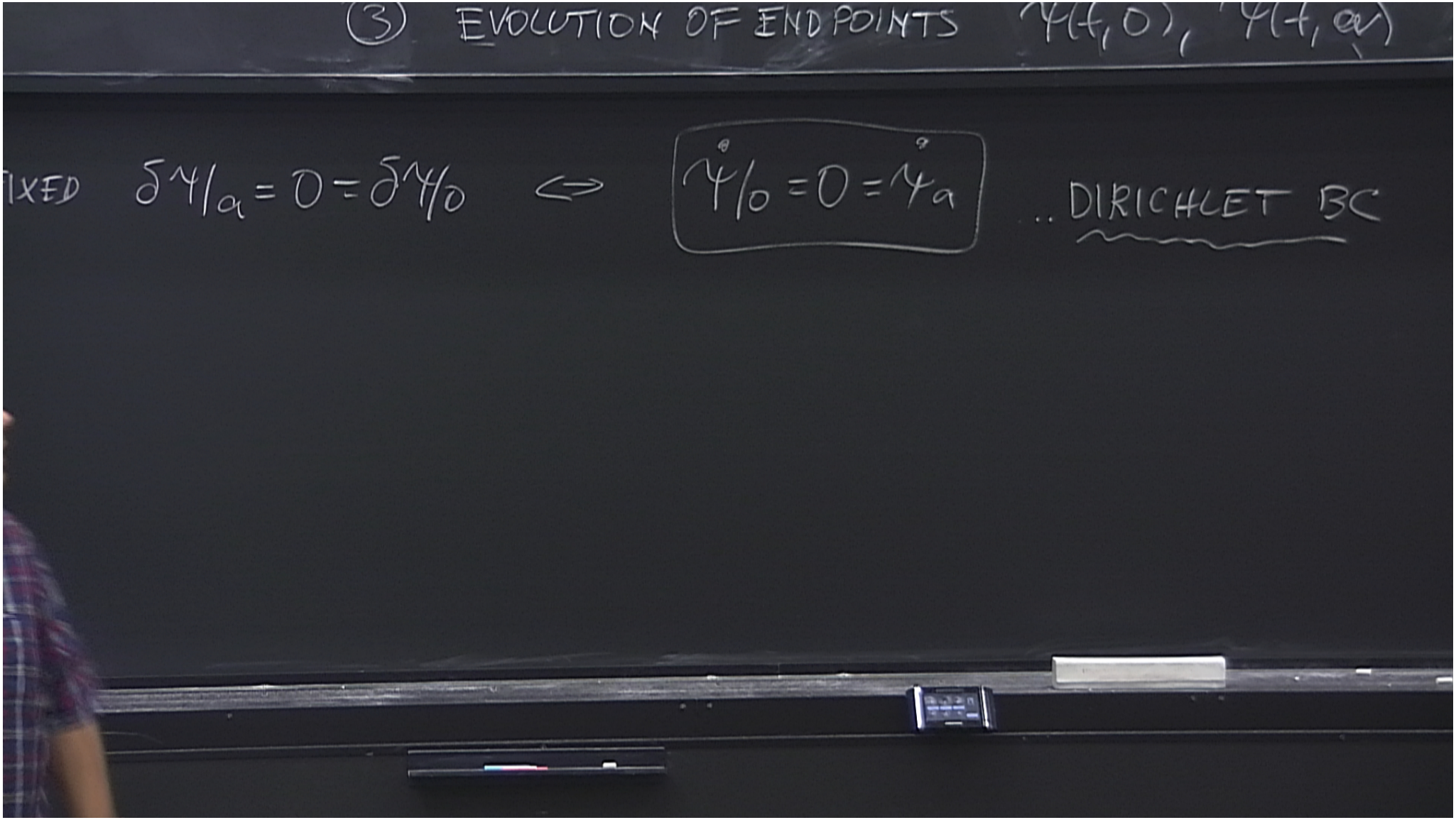
② "INITIAL DATA" $\delta\psi(t_1, x) = 0 = \delta\psi(t_2, x)$

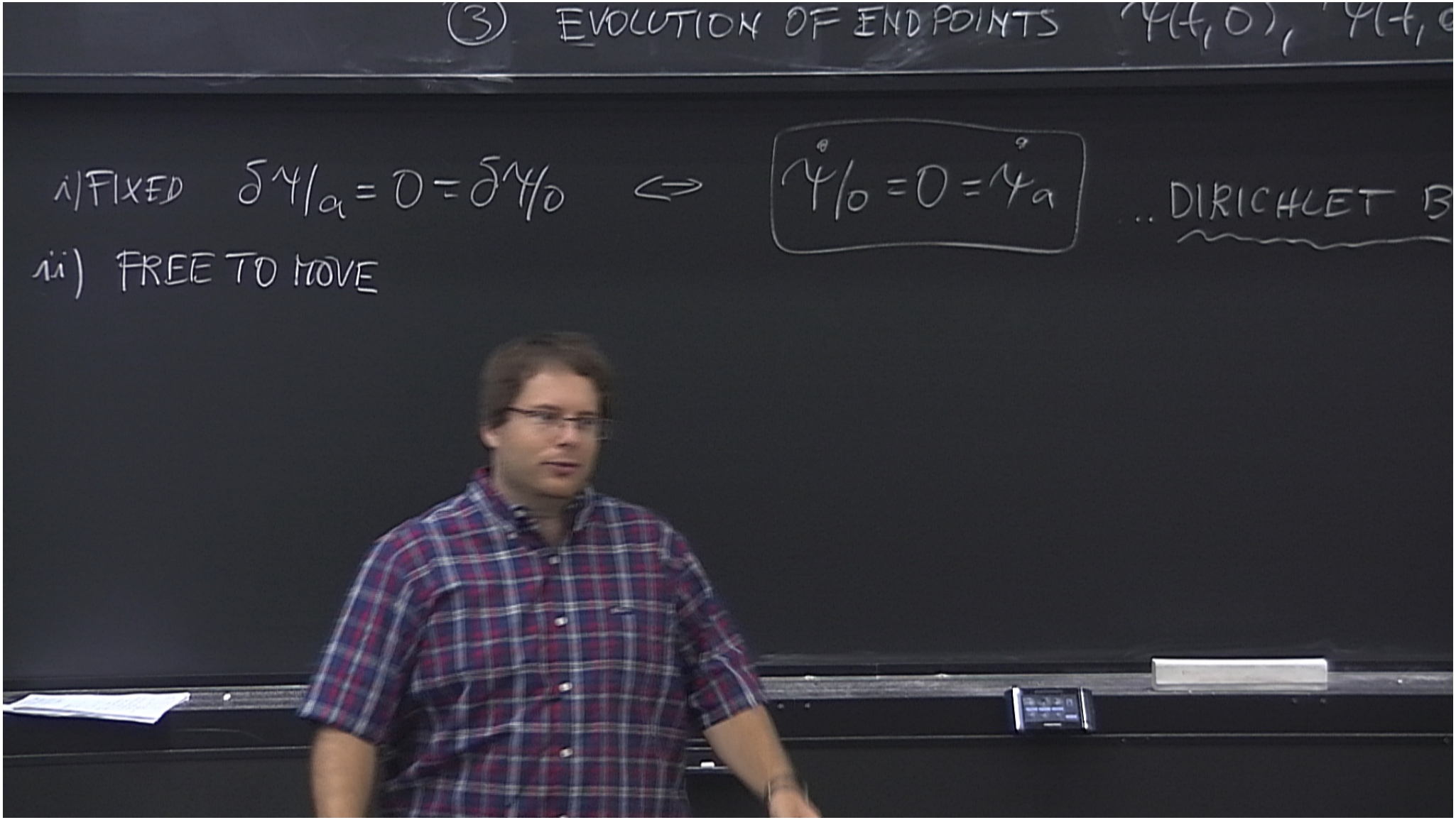
③ EVOLUTION OF END POINTS $\psi(t, 0)$, $\psi(t, a)$

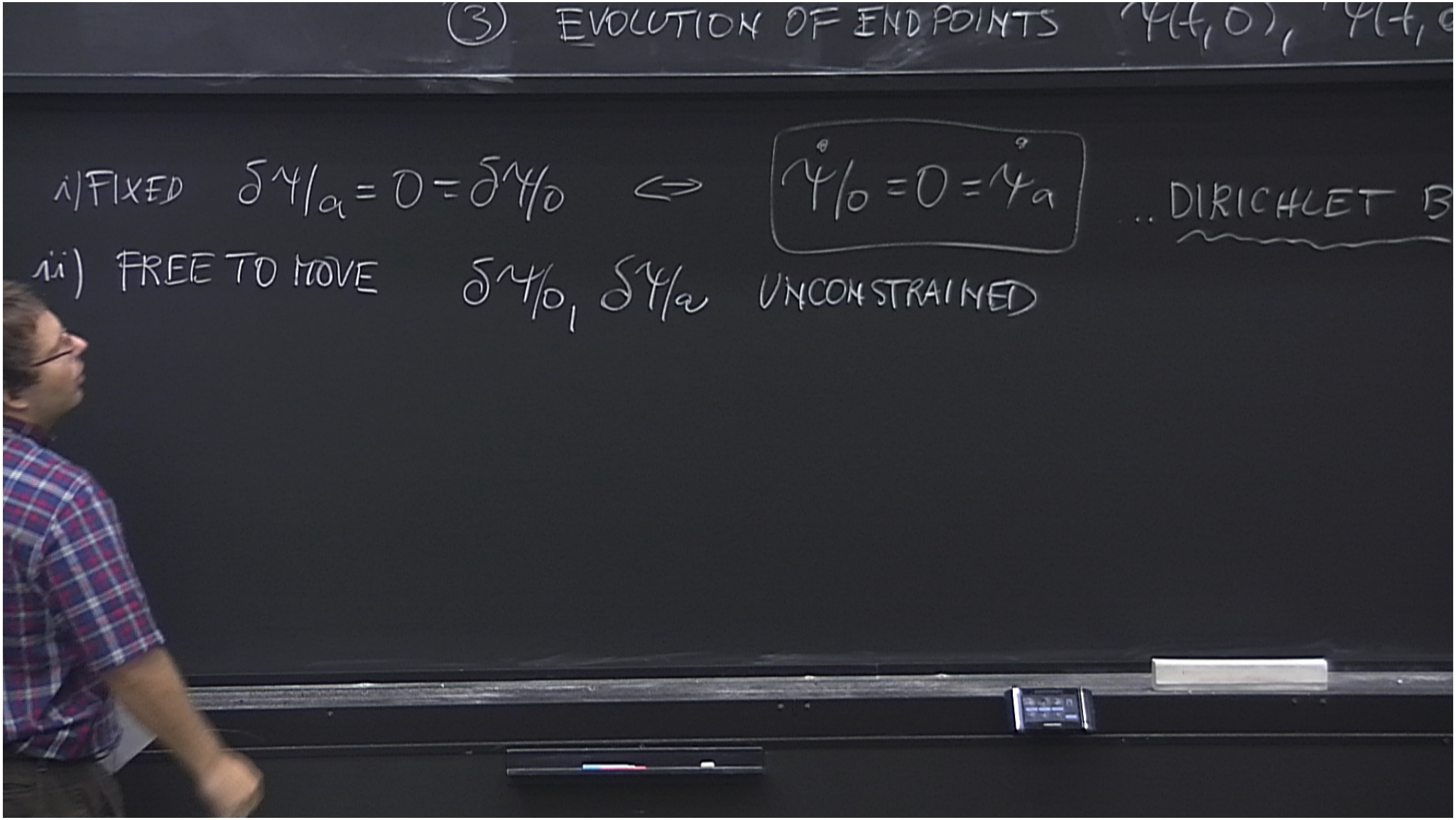
$\delta\psi(t_1, x) = 0 = \delta\psi(t_2, x)$

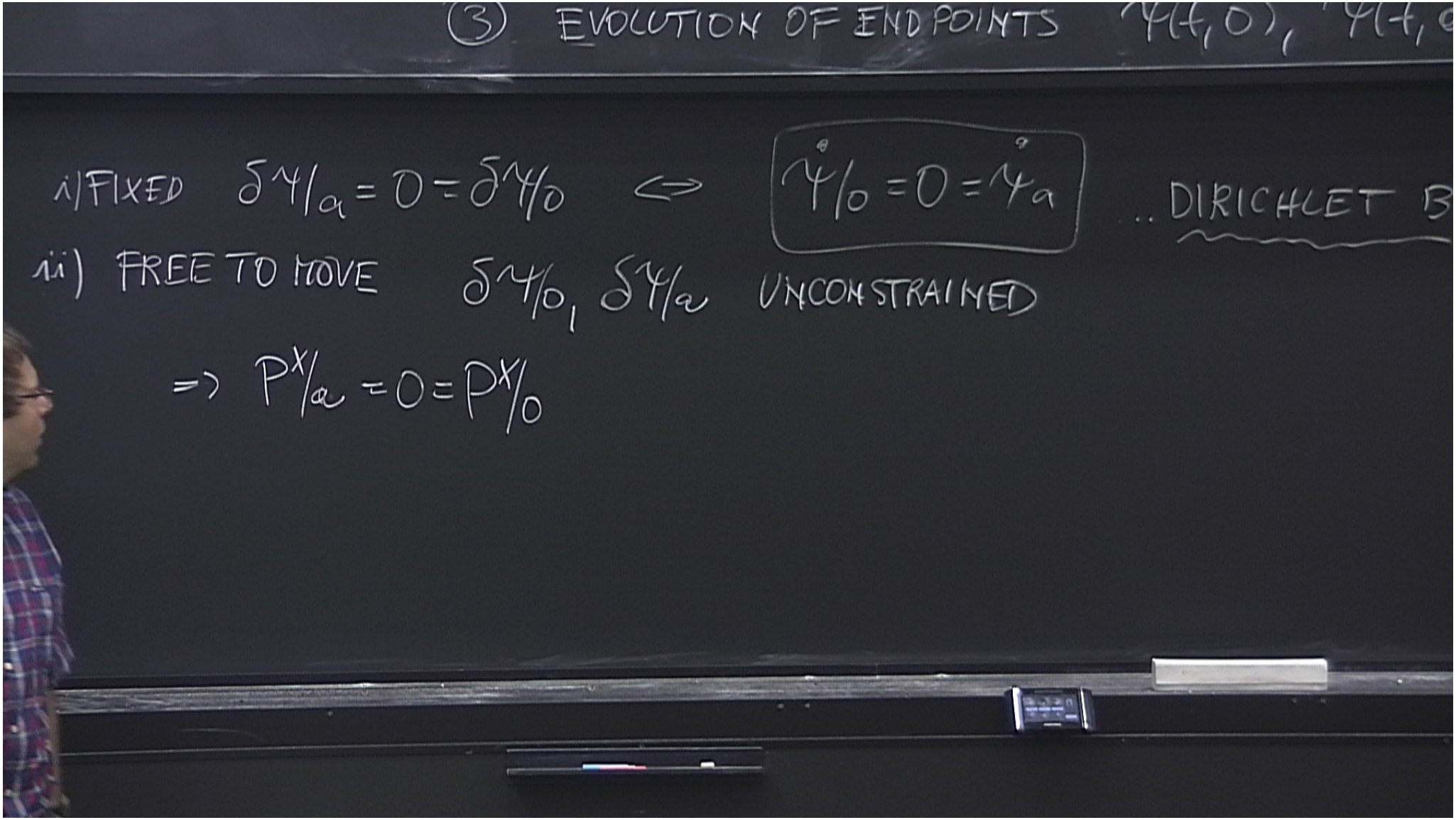
DINTS $\psi(t, 0)$, $\psi(t, \infty)$ .. BOUNDARY COND (BC)

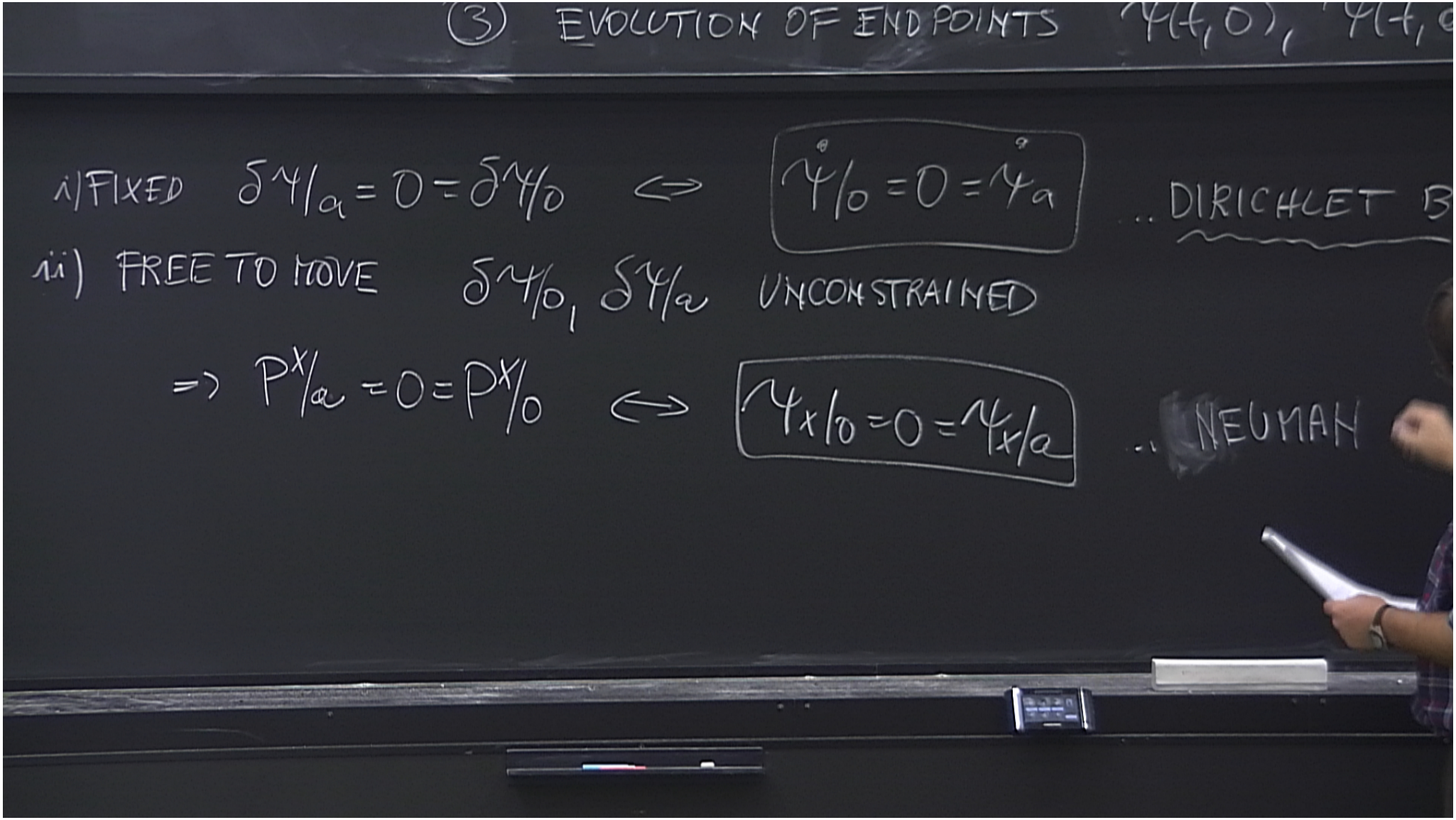

(3) EVOLUTION OF END POINTS
$\psi(t,0)$,
λ) FIXED $\delta\psi|_a = 0 = \delta\psi|_0 \Leftrightarrow \dot{\psi}|_0 = 0 = \dot{\psi}_a$

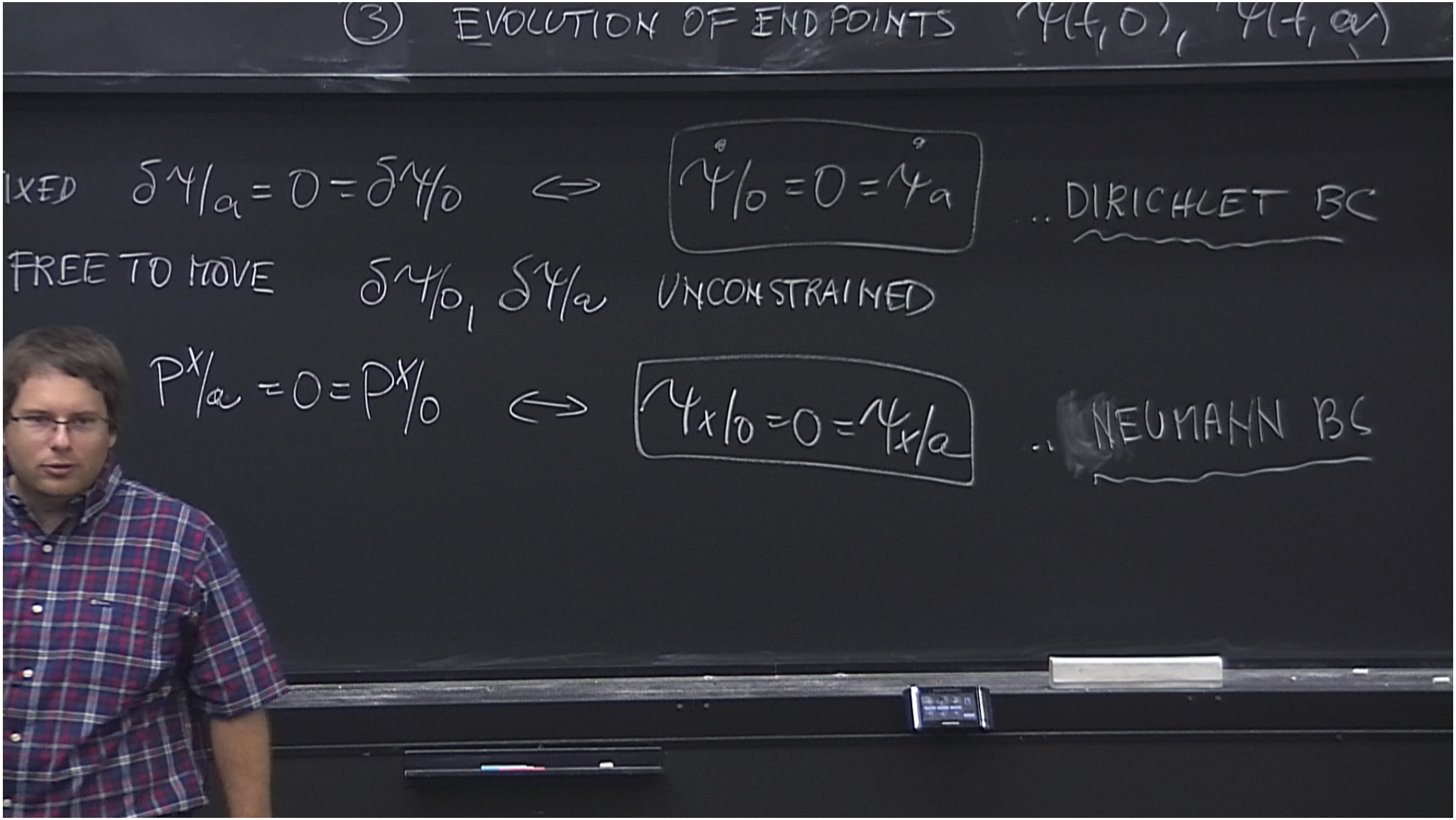

(3) EVOLUTION OF END POINTS
IXED
$\delta\psi|_a = 0 = \delta\psi|_0 \Leftrightarrow \psi|_0 = 0 = \psi_a$
.. DIRICHLET BC


(3) EVOLUTION OF END POINTS
i) FIXED $\delta\psi|_a = 0 = \delta\psi|_b$ $\Leftrightarrow$ $\psi|_b = 0 = \psi_a$
.. DIRICHLET B
ii) FREE TO MOVE

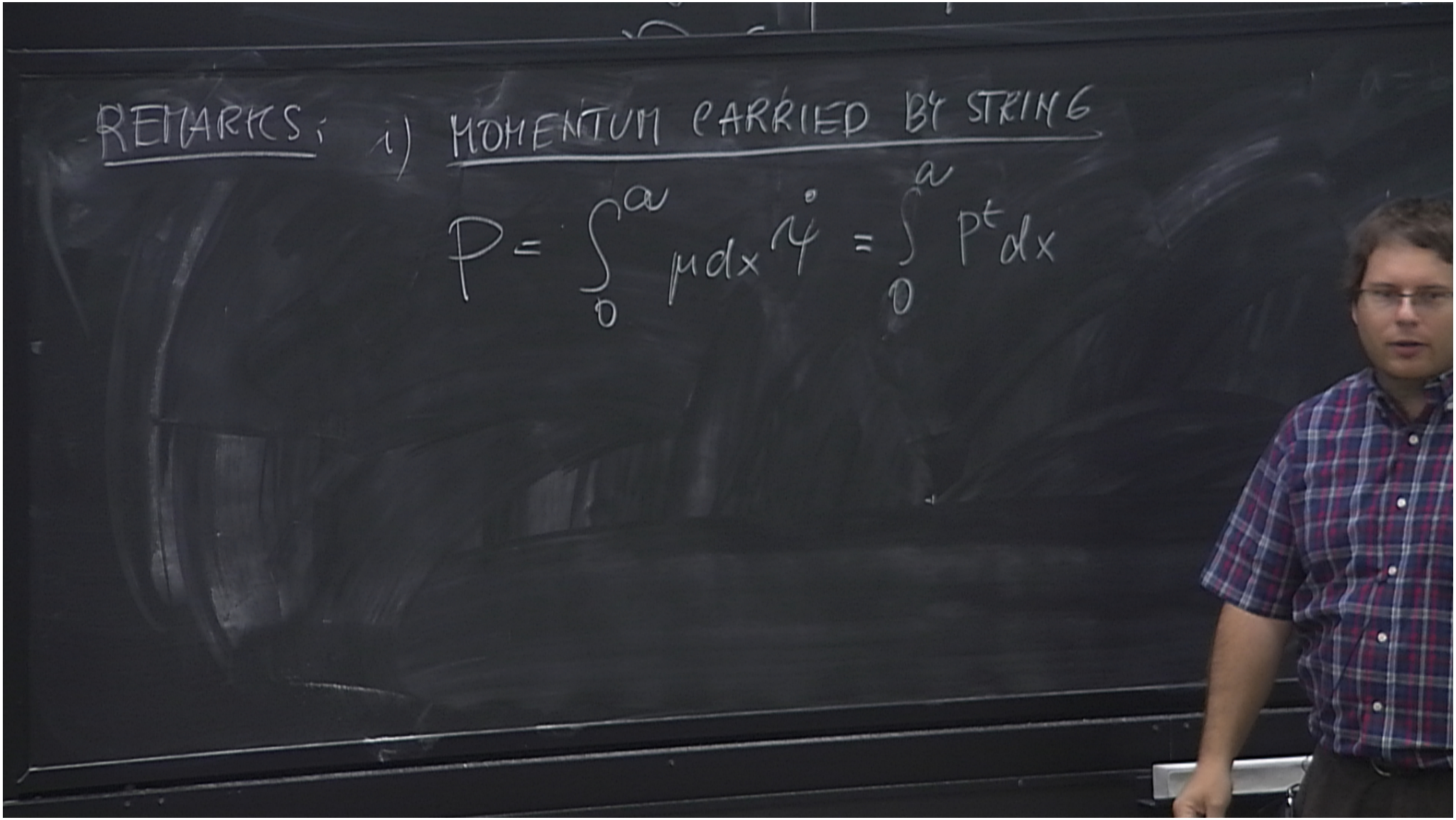

(3) EVOLUTION OF END POINTS
i) FIXED $\delta\psi|_a = 0 = \delta\psi|_0$ $\Leftrightarrow$ $\dot{\psi}|_0 = 0 = \dot{\psi}_a$ .. DIRICHLET B
ii) FREE TO MOVE $\delta\psi|_0, \delta\psi|_a$ UNCONSTRAINED


(3) EVOLUTION OF END POINTS
i) FIXED $\delta\psi|_a = 0 = \delta\psi|_0 \Leftrightarrow \dot{\psi}|_0 = 0 = \dot{\psi}_a$
.. DIRICHLET B
ii) FREE TO MOVE $\delta\psi|_0, \delta\psi|_a$ UNCONSTRAINED
$\Rightarrow P^X|_a = 0 = P^X|_0$


(3) EVOLUTION OF END POINTS
i) FIXED
DIRICHLET B
ii) FREE TO MOVE
UNCONSTRAINED
NEUMAN


(3) EVOLUTION OF END POINTS
IXED $\delta\psi_a = 0 = \delta\psi_0$ $\Leftrightarrow$ $\psi_0 = 0 = \psi_a$ .. DIRICHLET BC
FREE TO MOVE $\delta\psi_0, \delta\psi_a$ UNCONSTRAINED
$P^x_a = 0 = P^x_0$ $\Leftrightarrow$ $\psi_{x0} = 0 = \psi_{xa}$ .. NEUMANN BC

REMARKS: i) MOMENTUM CARRIED BY STRING


REMARKS: i) MOMENTUM CARRIED BY STRING
$P = \int_0^a \mu dx \dot{\eta} = \int_0^a P^t dx$

REMARKS: i) MOMENTUM CARRIED BY STRING

$$P = \int_0^a \mu \, dx \, \dot{\eta} = \int_0^a p^t \, dx$$

$$\frac{dP}{dt} = \int_0^a \dot{p}^t \, dt$$

REMARKS: i) MOMENTUM CARRIED BY STRING

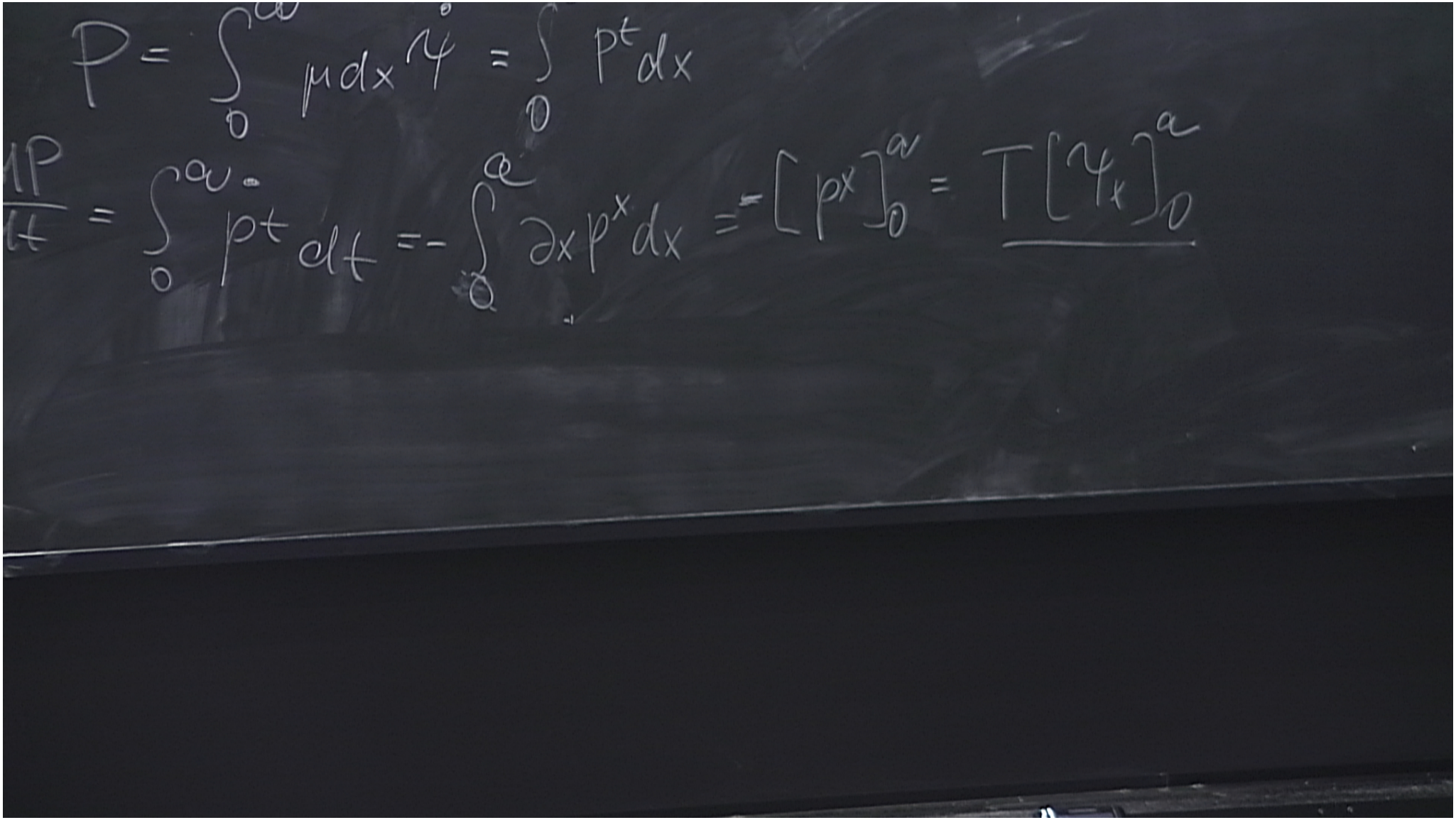
$$P = \int_0^a \mu \, dx \, \dot{Y} = \int_0^a p^t \, dx$$

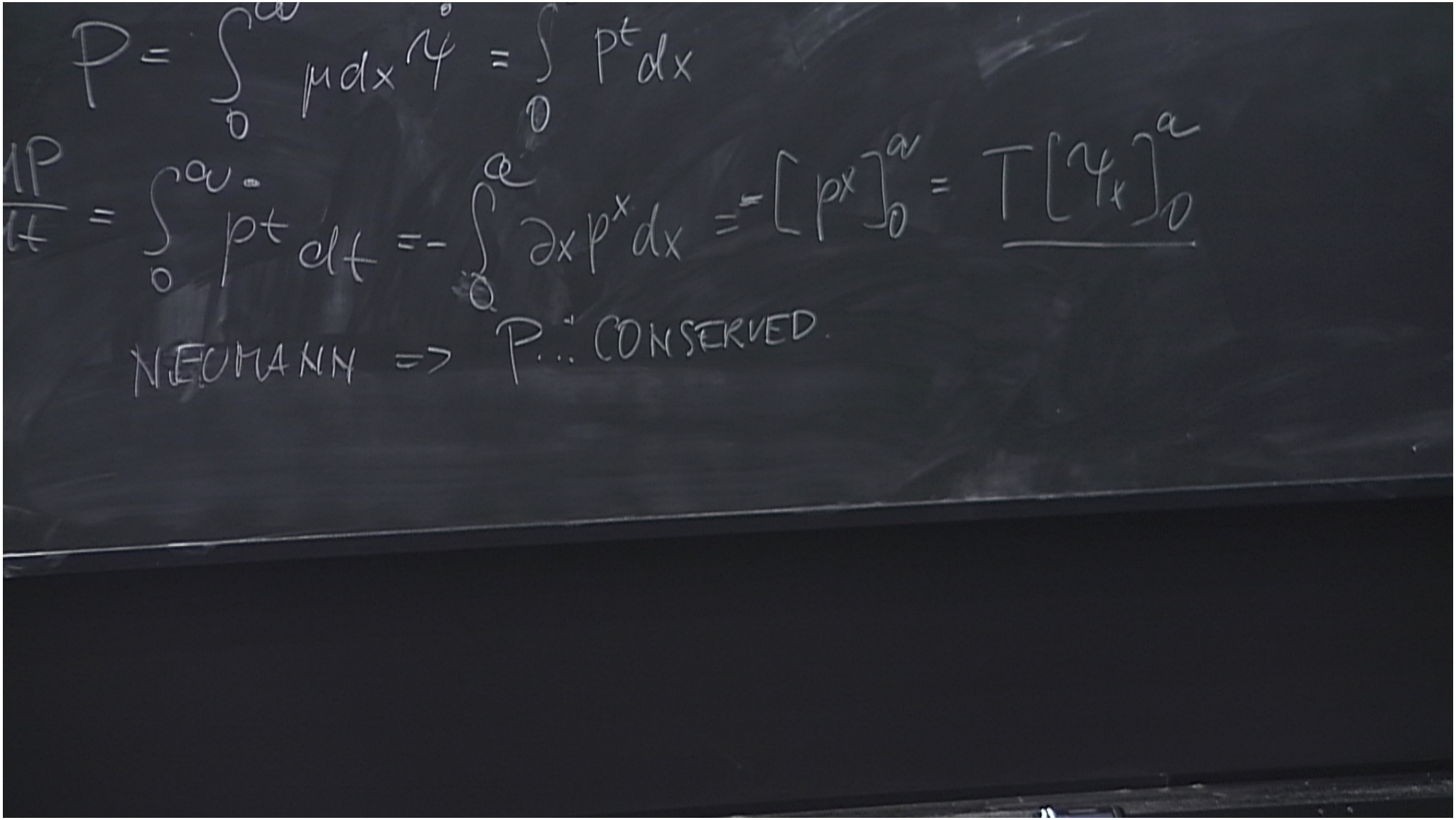
$$\frac{dP}{dt} = \int_0^a \dot{p}^t \, dt = - \int_0^a \partial_x p^x \, dx$$

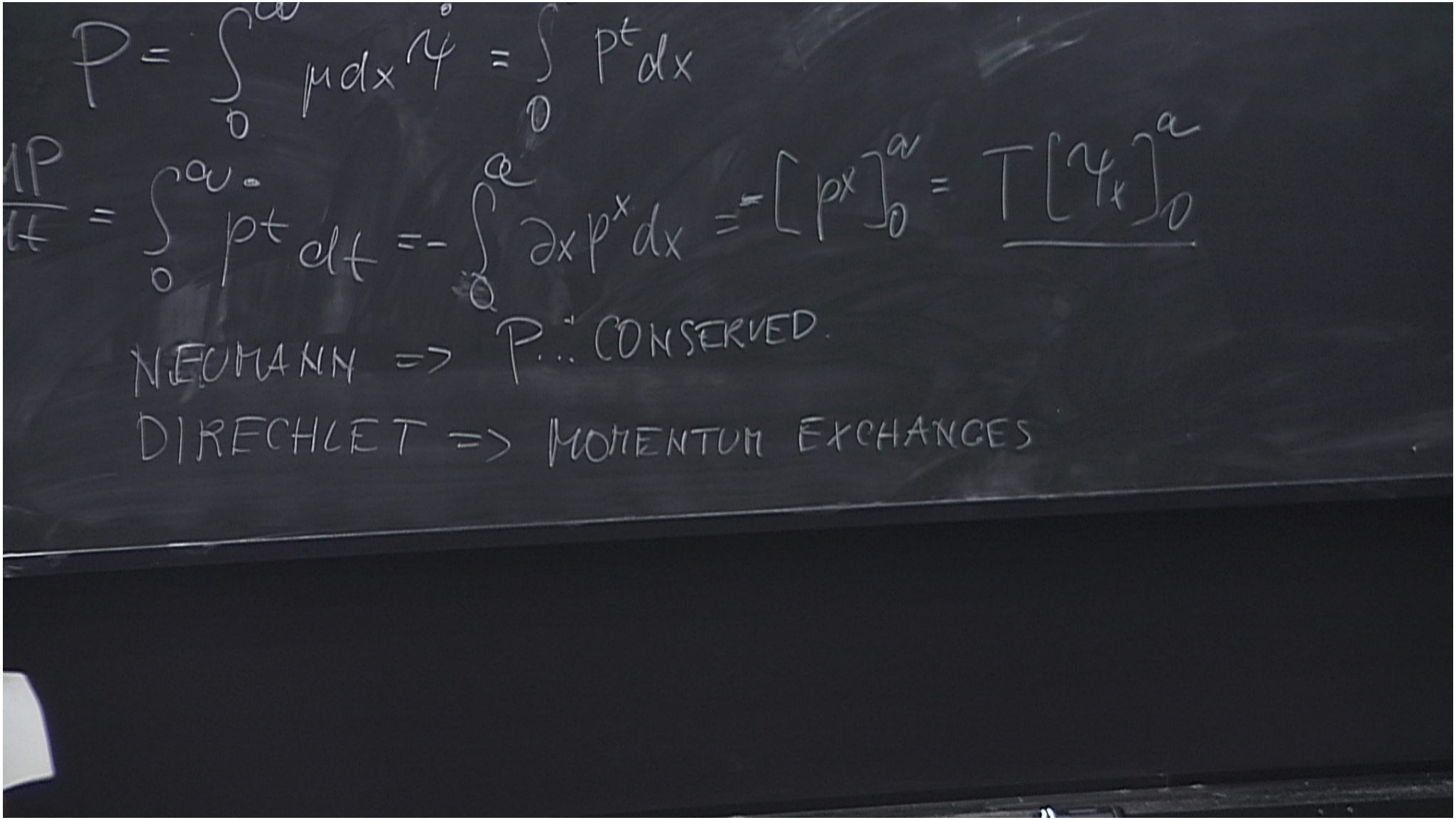
REMARKS: i) MOMENTUM CARRIED BY STRING

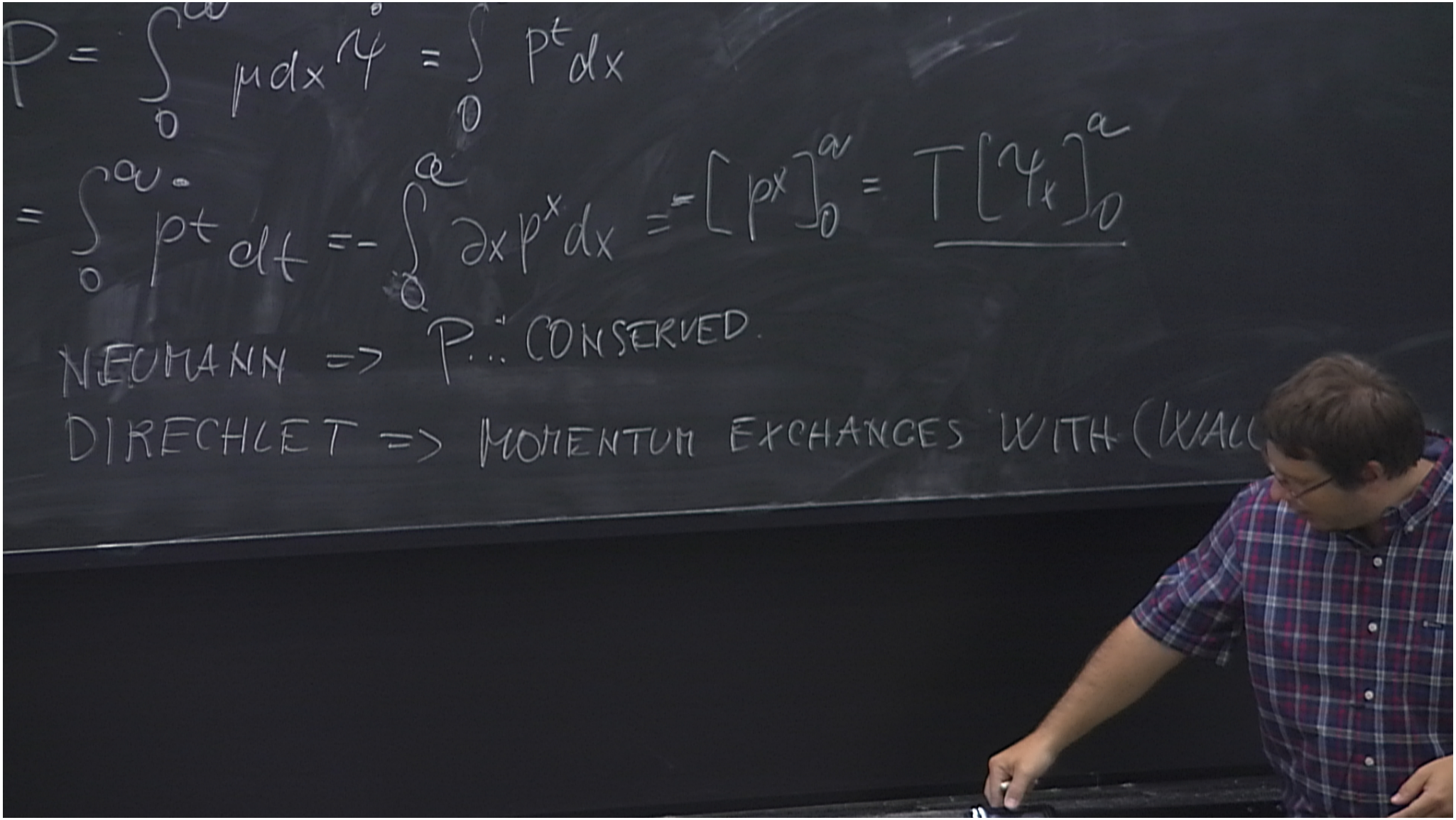
$$P = \int_0^a \mu \, dx \, \dot{\psi} = \int_0^a p^t \, dx$$

$$\frac{dP}{dt} = \int_0^a \dot{p}^t \, dt = -\int_0^a \partial_x p^x \, dx = -\left[p^x \right]_0^a$$


$P = \int_0^{\infty} \mu dx \dot{\psi} = \int_0^{} p^t dx$
$\frac{dP}{dt} = \int_0^{\infty} p^t dt = -\int_0^{a} \partial_x p^x dx = -[p^x]_0^a = \underline{T[\psi_x]_0^a}$


$P = \int_0^a \mu dx \dot{y} = \int_0^a p^t dx$
$\frac{dP}{dt} = \int_0^a \dot{p}^t dt = -\int_0^a \partial_x p^x dx = -[p^x]_0^a = \underline{T[y_x]_0^a}$
NEUMANN => P ∴ CONSERVED


$P = \int_0^{\infty} \mu dx \dot{\gamma} = \int_0^{} p^t dx$
$\frac{dP}{dt} = \int_0^{\infty} \dot{p}^t dt = -\int_0^{a} \partial_x p^x dx = -[p^x]_0^a = T[\gamma_x]_0^a$
NEUMANN => P.. CONSERVED
DIRECHLET => MOMENTUM EXCHANGES

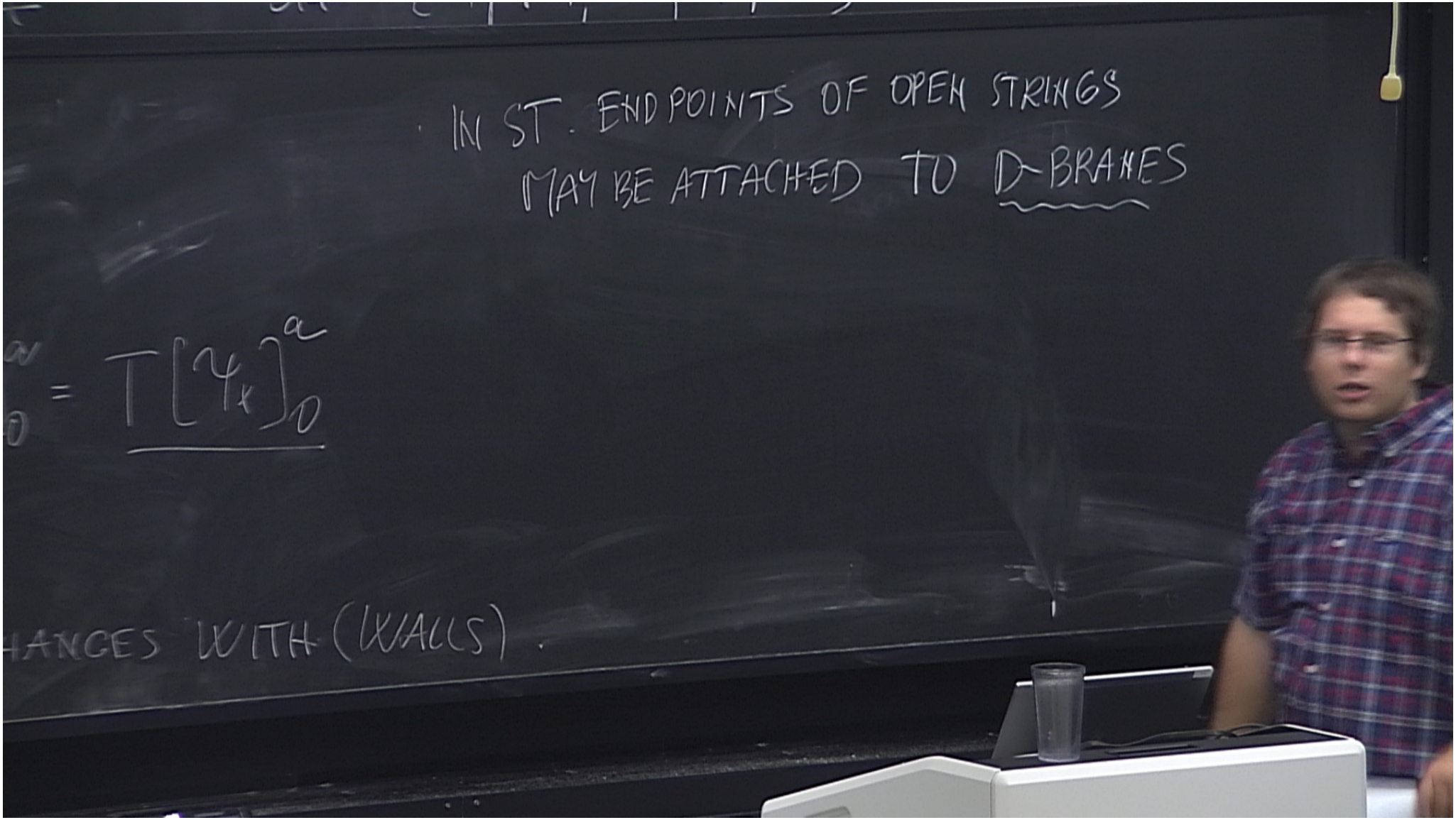

$P = \int_0^{\infty} \mu dx \dot{\gamma} = \int_0^{} p^t dx$
$= \int_0^{\infty} p^t dt = -\int_0^{a} \partial_x p^x dx = -[p^x]_0^a = T[\gamma_x]_0^a$
NEUMANN => P... CONSERVED
DIRECHLET => MOMENTUM EXCHANGES WITH (WALL

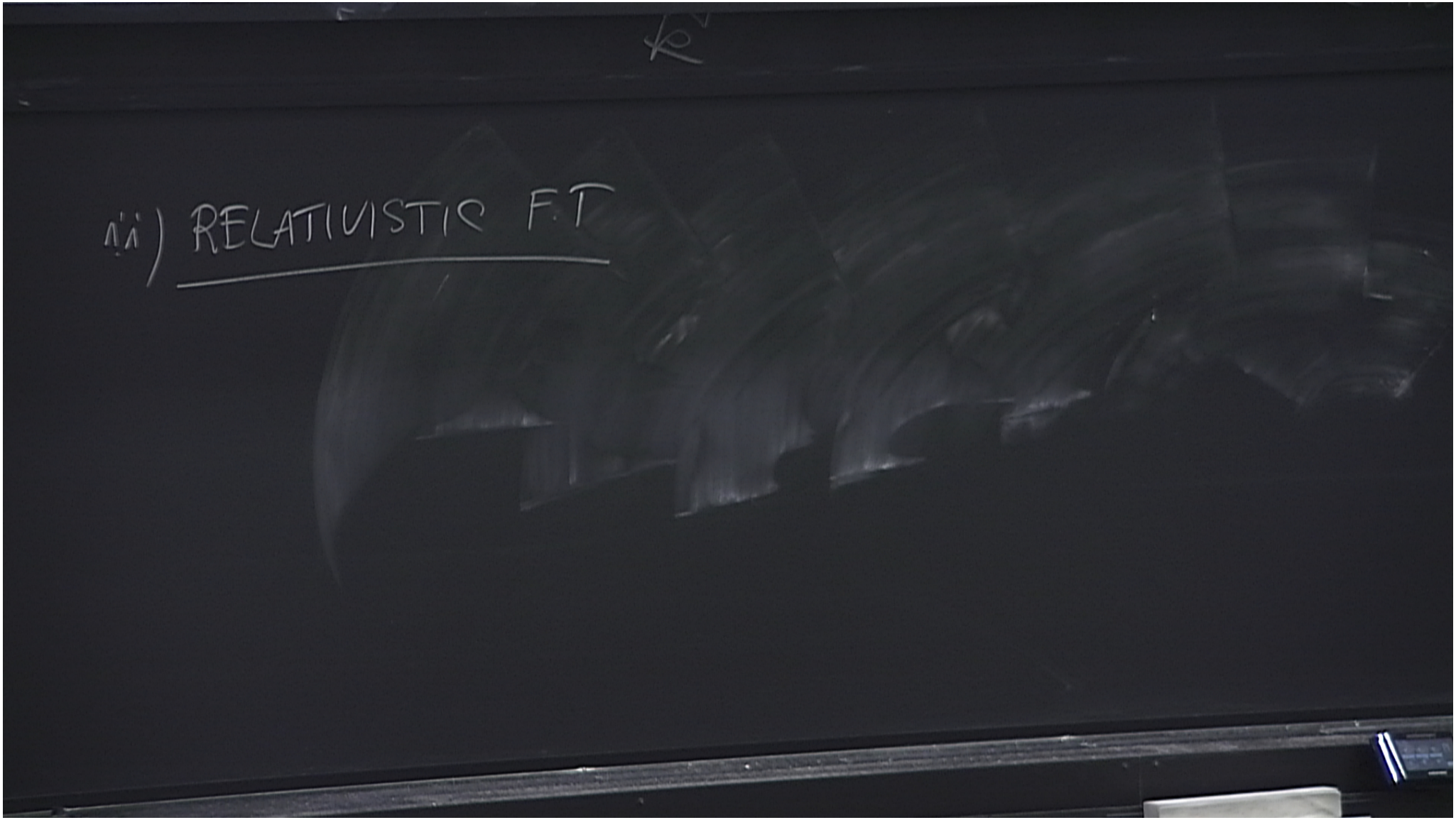
$$P = \int_0^{\infty} \mu \, dx \, \dot{\gamma} = \int_0^{\infty} p^t \, dx$$

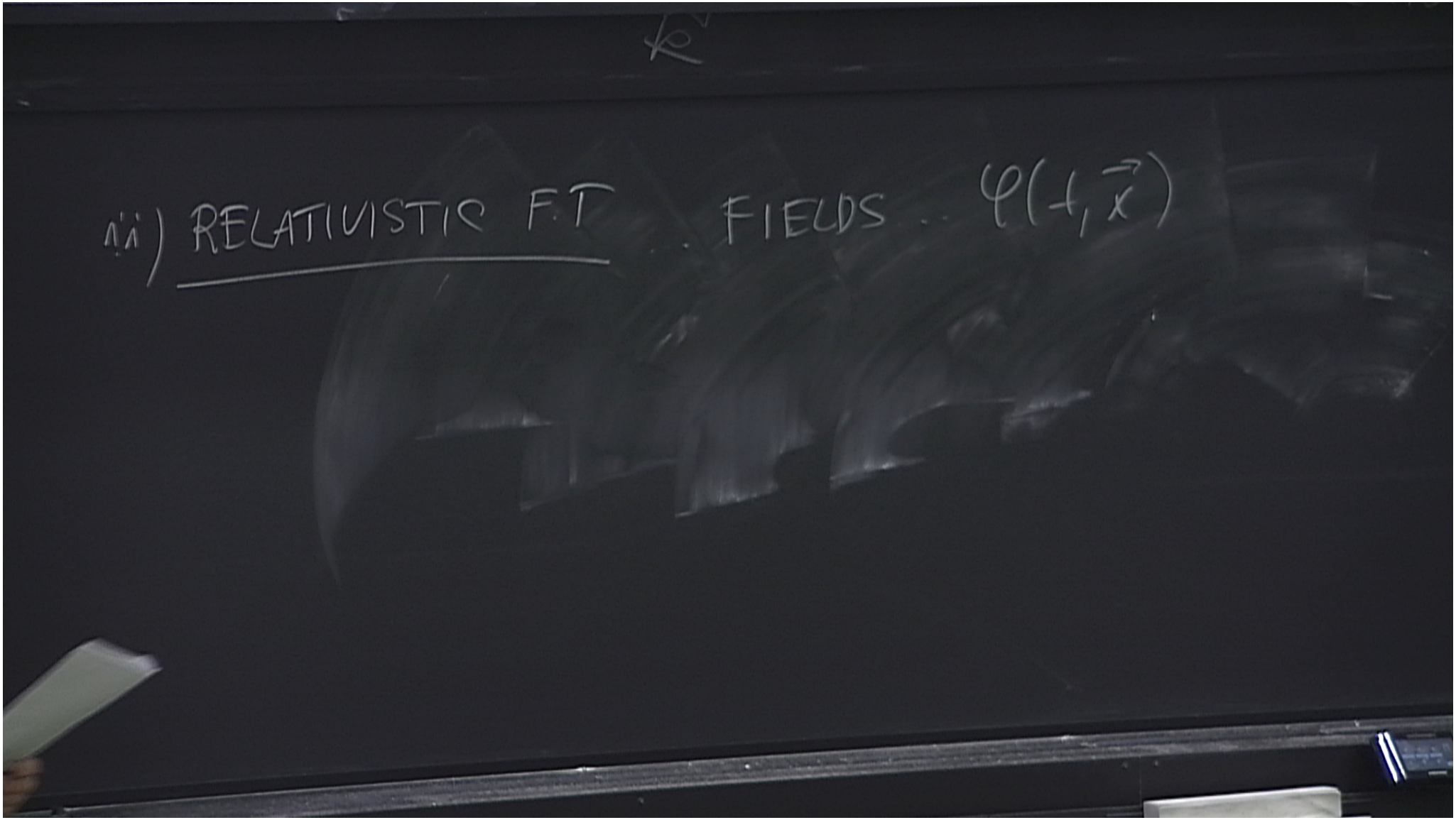
$$= \int_0^{\infty} p^t \, dt = -\int_0^{a} \partial_x p^x \, dx = -\left[p^x\right]_0^a = \underline{T\left[\gamma_x\right]_0^a}$$

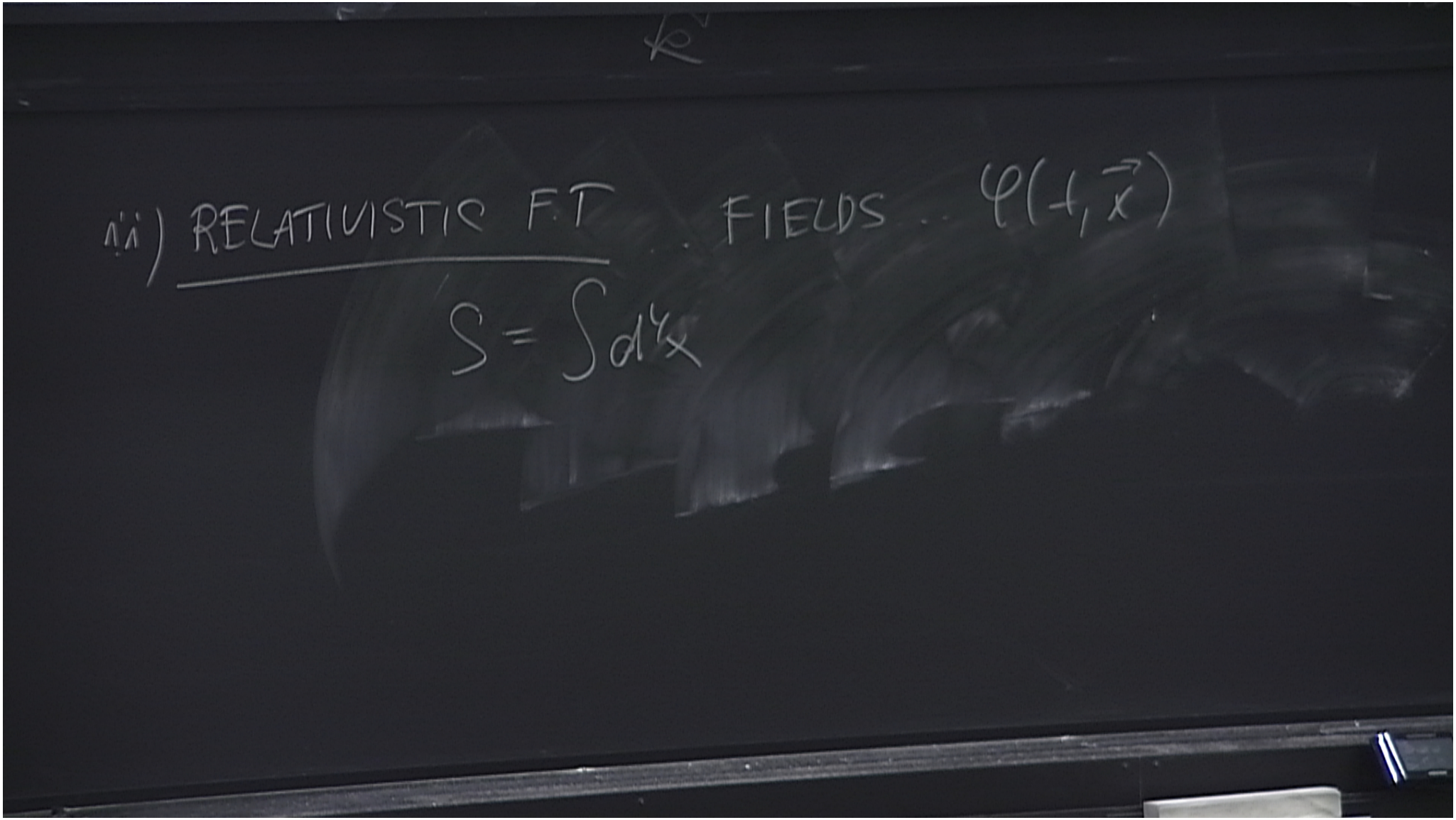
NEUMANN => P ∴ CONSERVED.

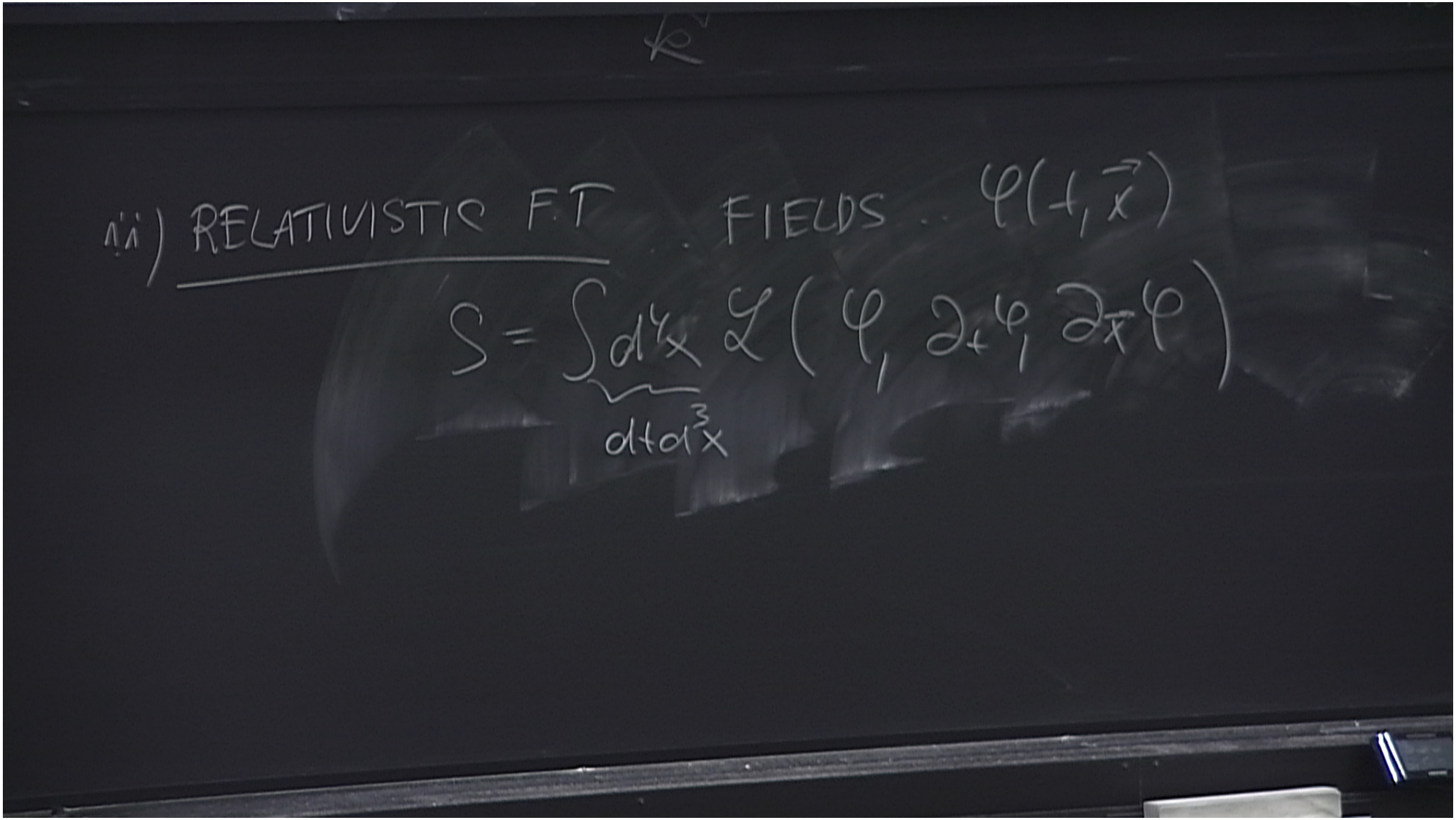
DIRECHLET => MOMENTUM EXCHANGES WITH (WALLS).

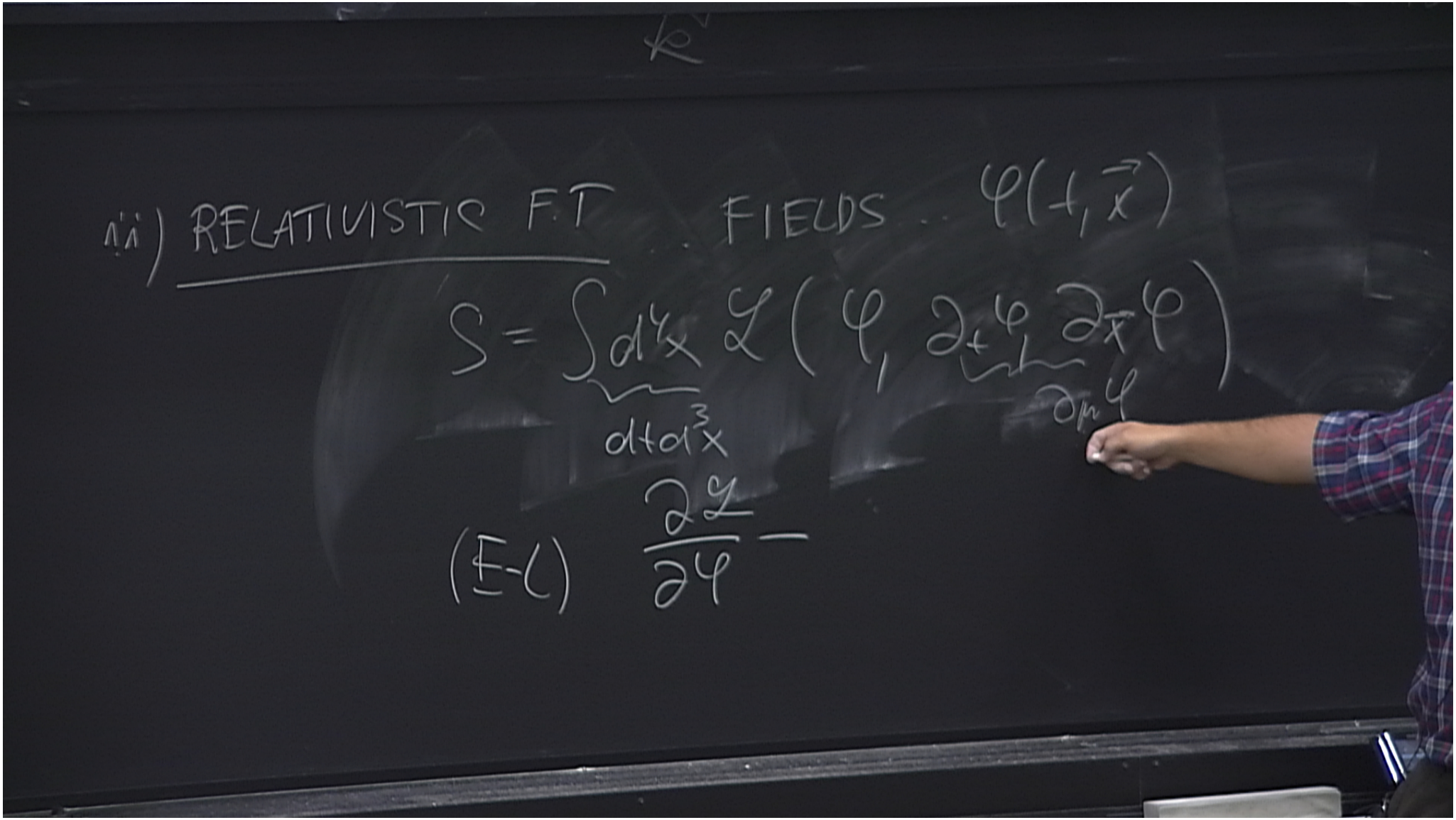

IN ST. END POINTS OF OPEN STRINGS
MAY BE ATTACHED TO D-BRANES
$= T[\gamma_x]_0^a$
HANGES WITH (WALLS).

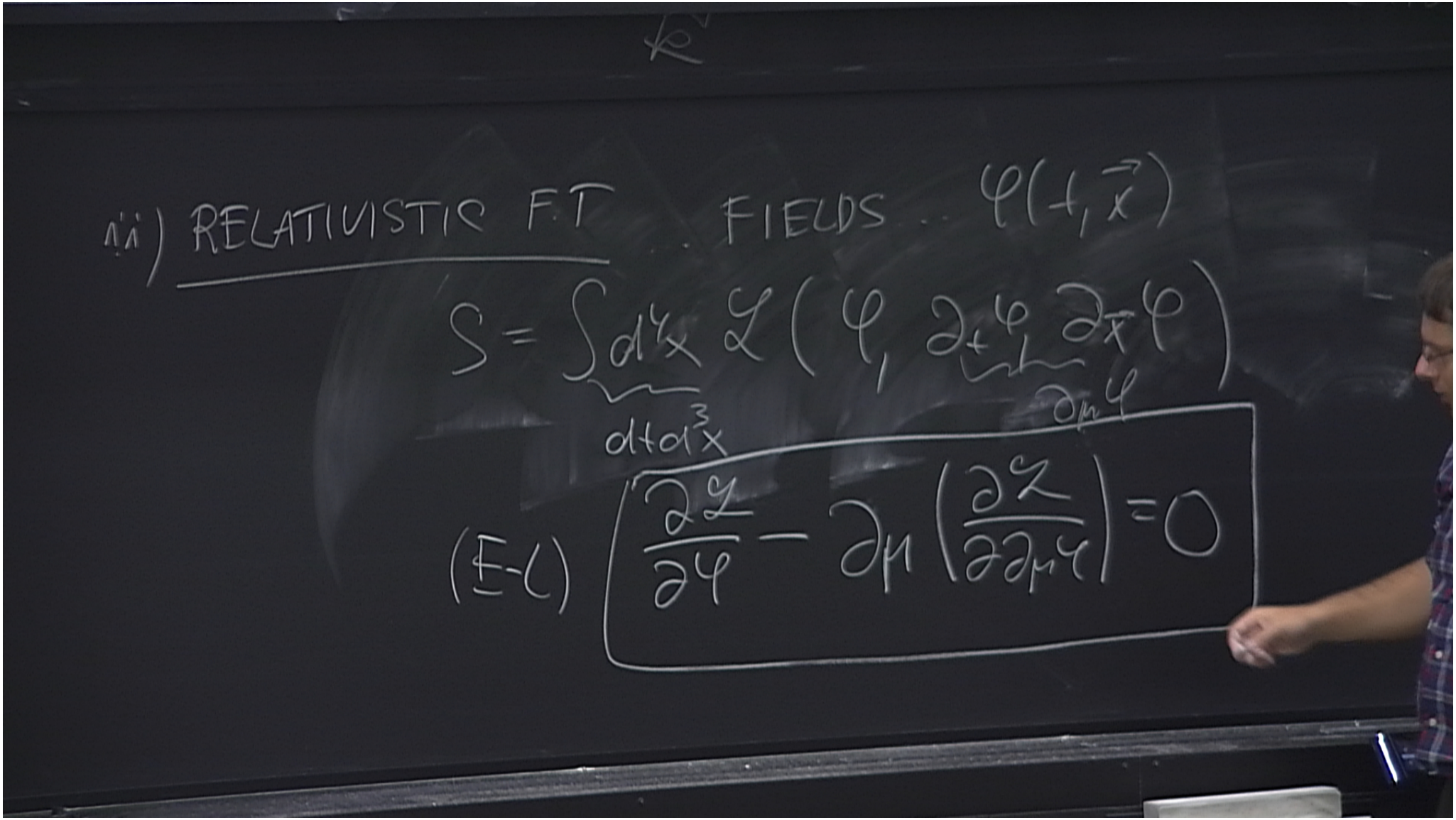

iii) RELATIVISTIC F.T

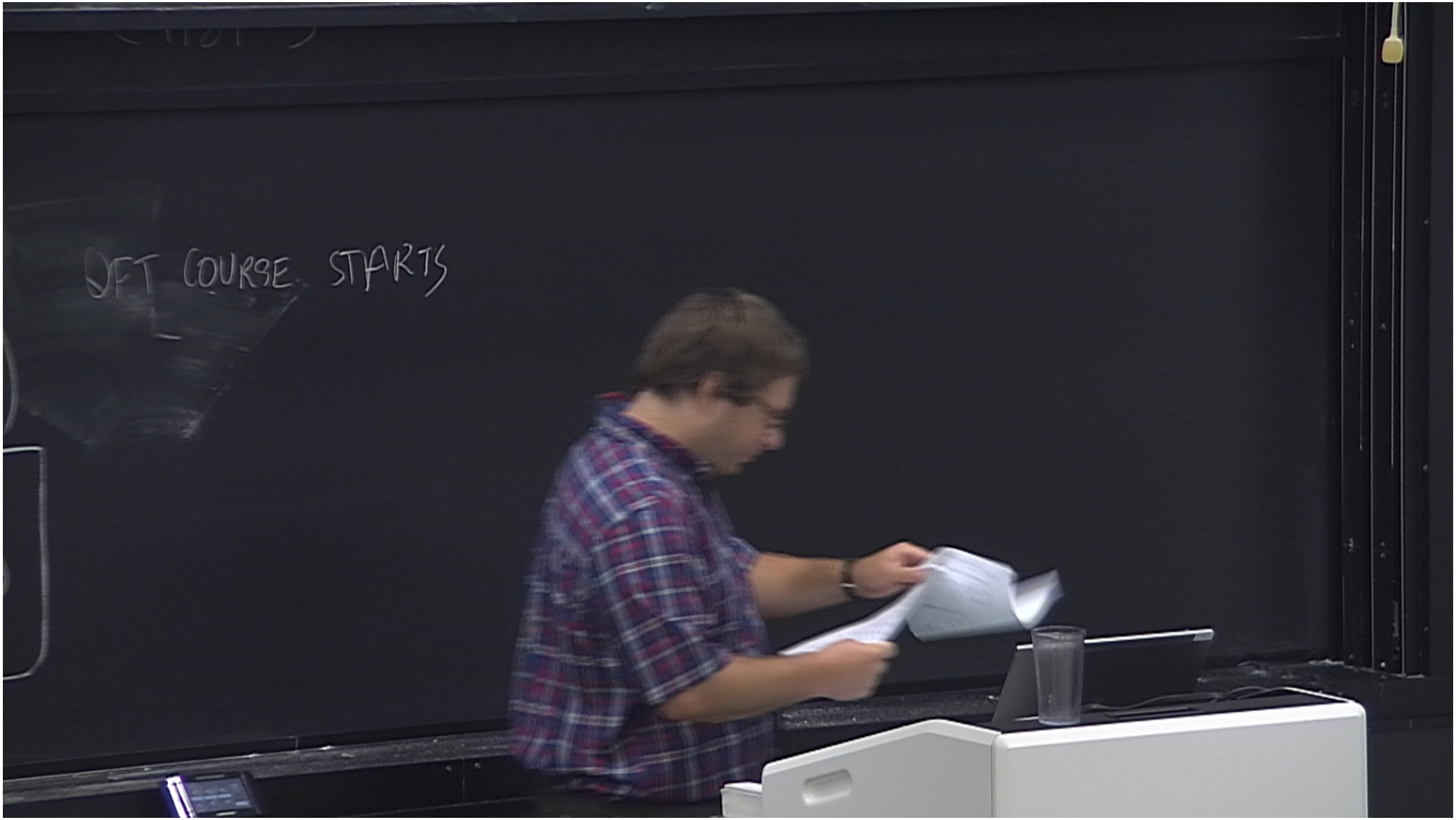

ii) RELATIVISTIC F.T. FIELDS $\varphi(t,\vec{x})$


R^4
ii) RELATIVISTIC F.T. .. FIELDS .. $\varphi(t, \vec{x})$
$S = \int d^4x$


ii) RELATIVISTIC F.T. FIELDS $\varphi(t,\vec{x})$
$S = \int d^4x \, \mathcal{L}(\varphi, \partial_t\varphi, \partial_x\varphi)$
$dt\,d^3x$


ii) RELATIVISTIC F.T. FIELDS $\varphi(t,\vec{x})$
$S = \int d^4x \, \mathcal{L}(\varphi, \partial_\mu \varphi, \partial_\nu \varphi)$
$d^4x = dt\, d^3x$
$\partial_\mu \varphi$
(E-L) $\frac{\partial \mathcal{L}}{\partial \varphi} -$


ii) RELATIVISTIC F.T. ... FIELDS ... $\varphi(t,\vec{x})$
$S = \int d^4x \, \mathcal{L}(\varphi, \partial_t\varphi, \partial_x\varphi)$
$d^4x = dt\,d^3x$
$\partial_\mu\varphi$
(E-L) $\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)}\right) = 0$


OFT COURSE STARTS