

Title: 14/15 PSI - Classical Mechanics

Date: Aug 14, 2014 10:30 AM

URL: <http://pirsa.org/14080016>

Abstract:

• $S[q, t]$ FUNCTIONAL
 $S = S(q_0, t_0, q_1, t) \dots$ ON SHELL

$$\frac{\partial S}{\partial q^i} = p_i, \quad \frac{\partial S}{\partial t} = -H$$

A) HAMILTON-JACOBI THEORY

INGENIOUS IDEA, LET'S FIND A SPECIAL GEN.F. S

S.T. $H' = 0$

$\Rightarrow$

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$$\Rightarrow (HE) \quad Q = \alpha = \text{CONST}, \quad P = -\beta = \text{CONST}.$$

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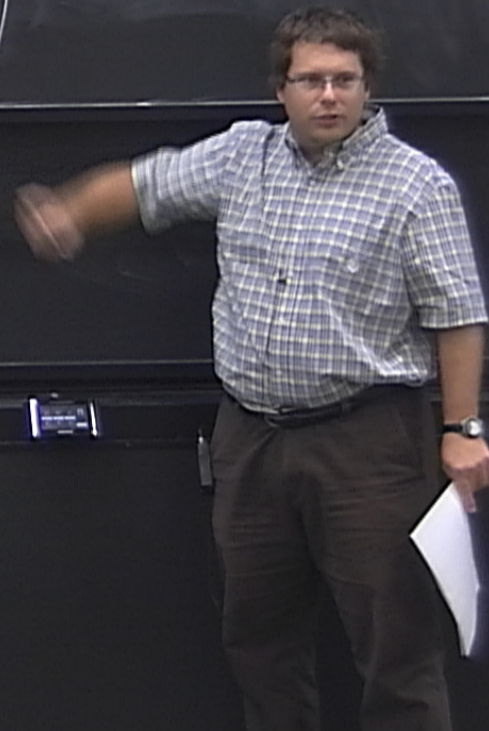
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$$\Rightarrow \bullet (HE) \quad Q = \alpha = \text{CONST}, \quad P = -\beta = \text{CONST}$$
$$\bullet (C) \quad P = -\frac{\partial \bar{F}}{\partial Q} \quad \boxed{\beta = \frac{\partial S(q, \alpha, t)}{\partial \alpha}}$$



② HOW TO FIND S ?

$$(c) \quad p = \frac{\partial S}{\partial q} \quad \& \quad H' = 0 = H +$$

$q = q(t, \alpha, \beta)$ ✓
VERSION

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$$(c) \quad p = \frac{\partial S}{\partial q} \quad \& \quad H' = 0 = H + \frac{\partial S}{\partial t}$$

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✓
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$$\left[\frac{\partial S}{\partial q} \right]_{q_1, \left(\frac{\partial S}{\partial q} \right)_1, t} = 0$$

$$\beta = \text{CONST}$$
$$\frac{\partial S(q, \alpha, t)}{\partial \alpha}$$

GIVES

BY INVERSION

SPECIAL GEN. F. S

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HAMILTON-J

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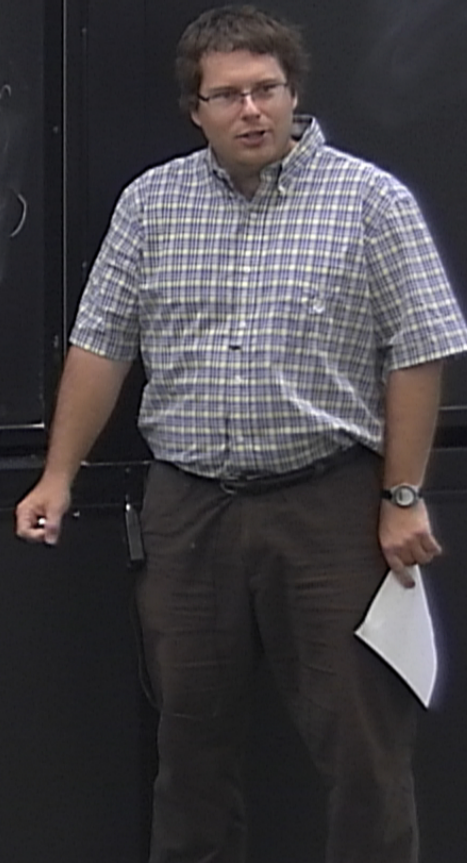
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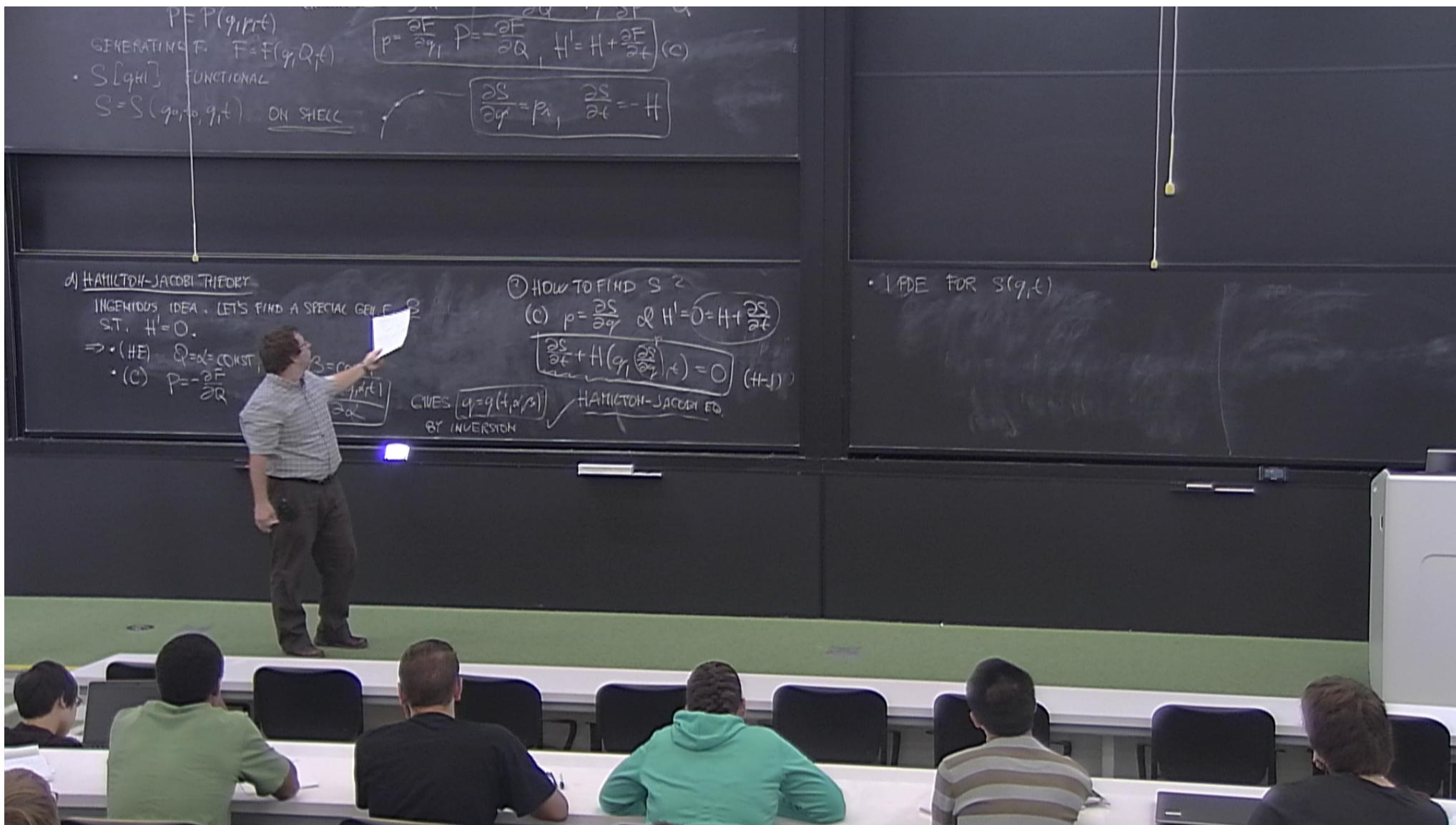
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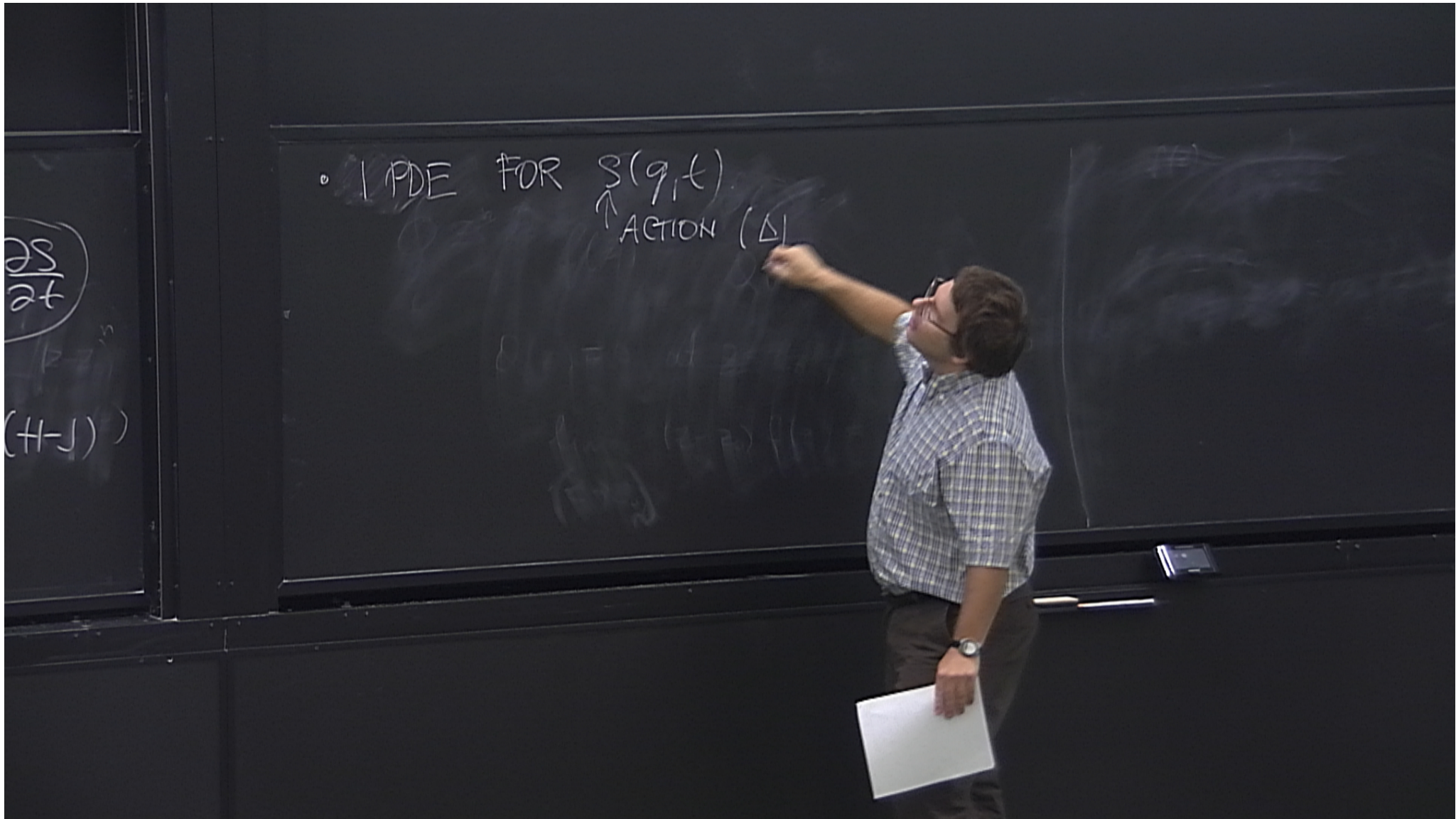
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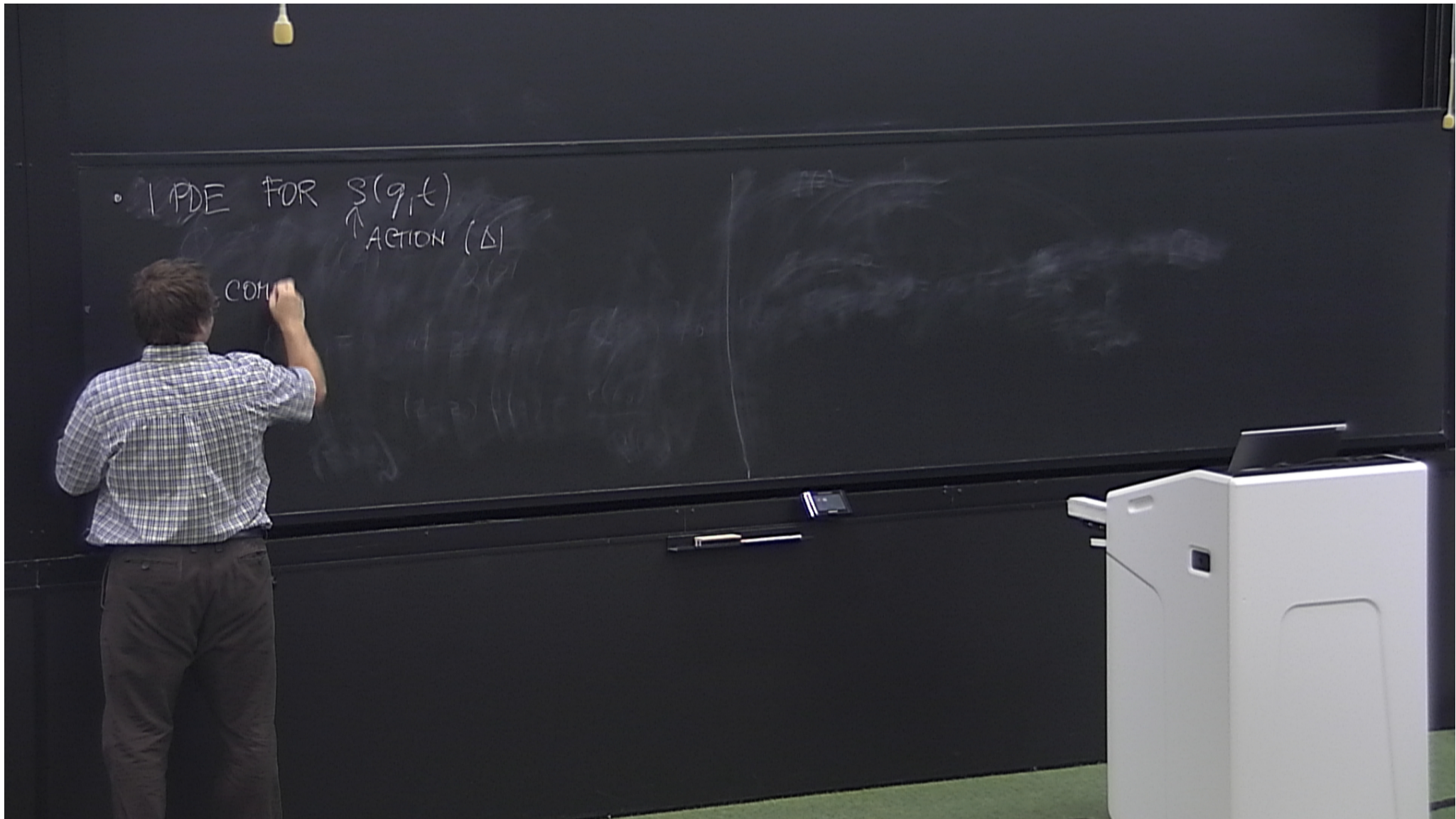
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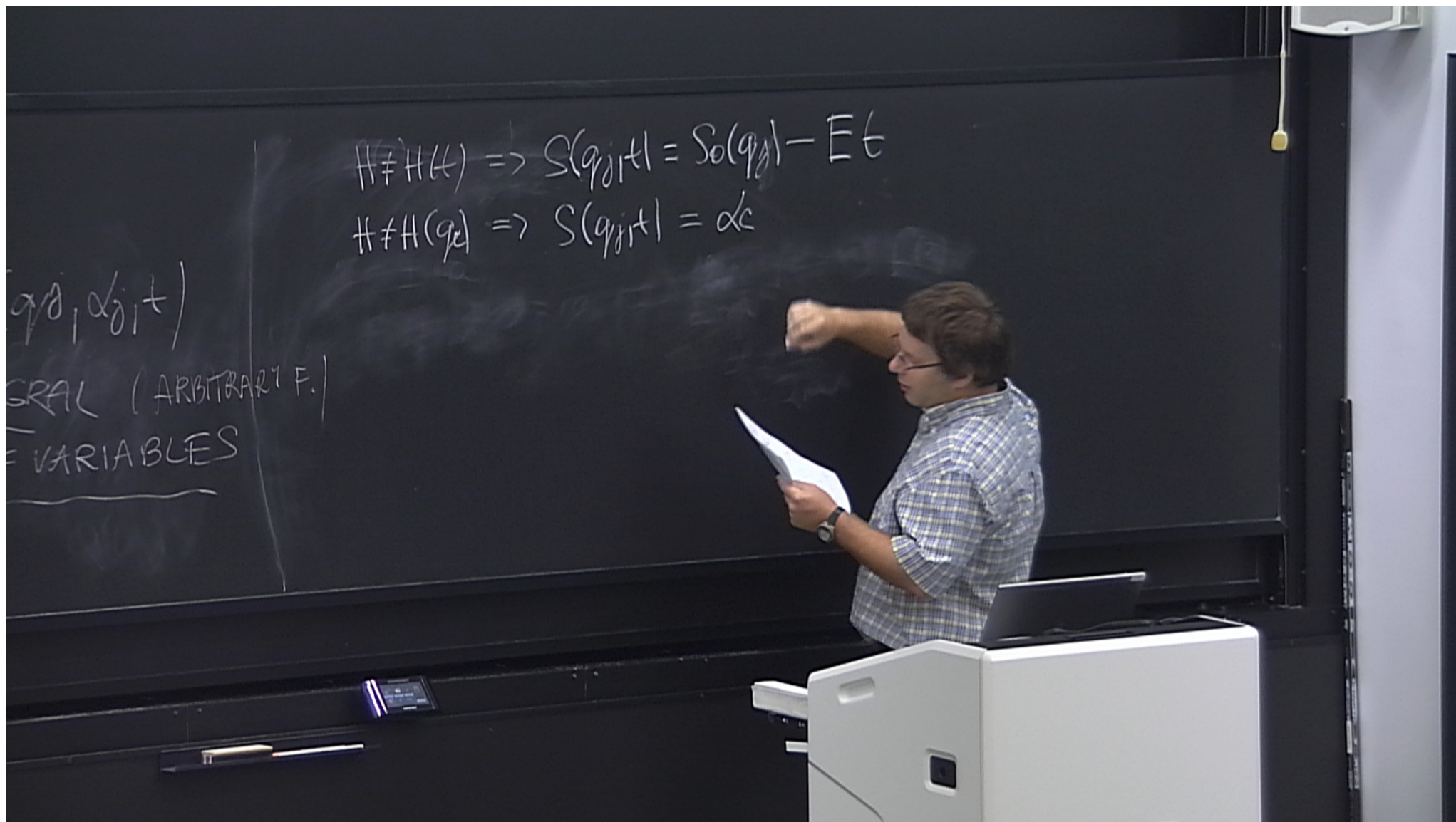
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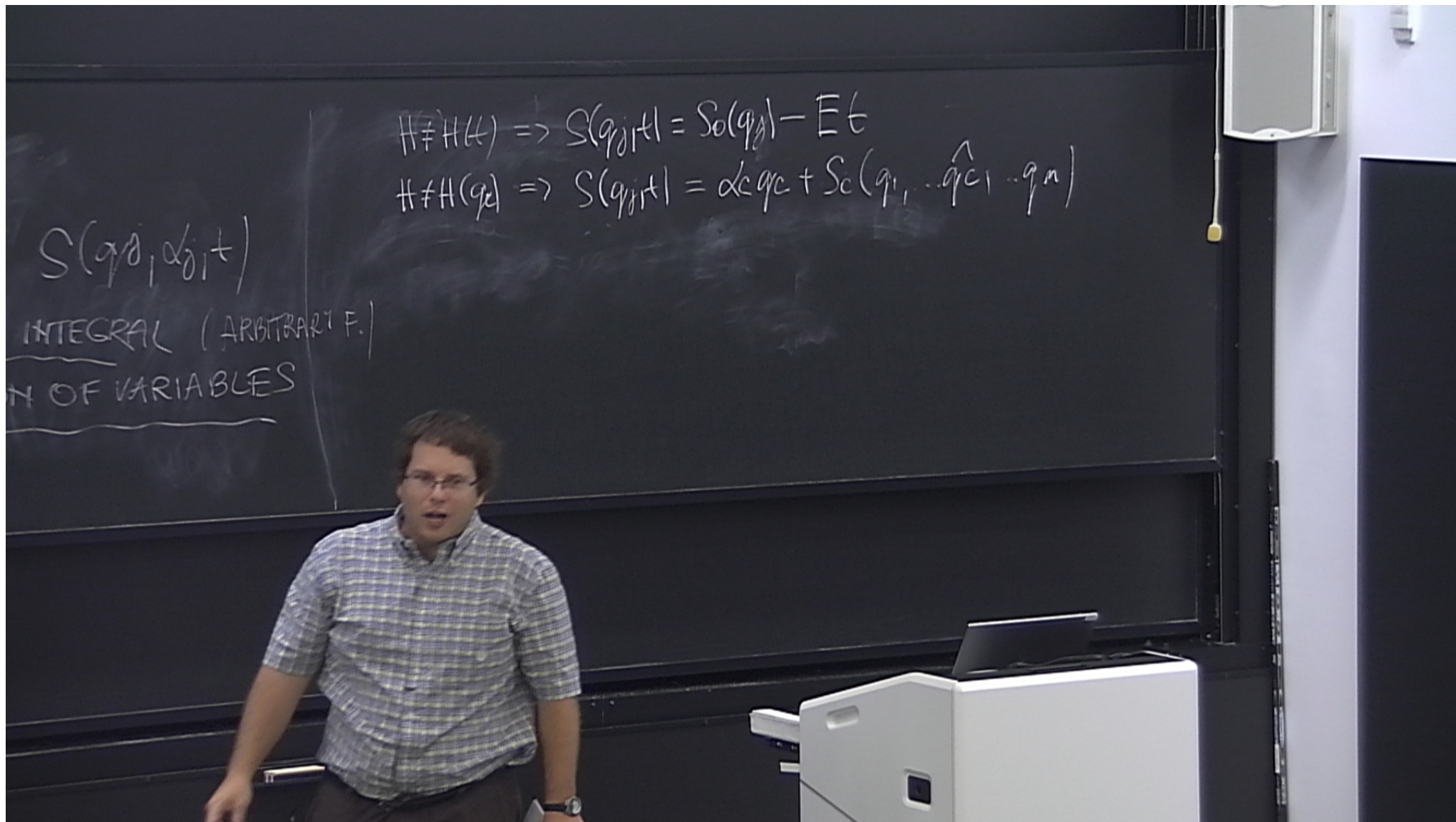
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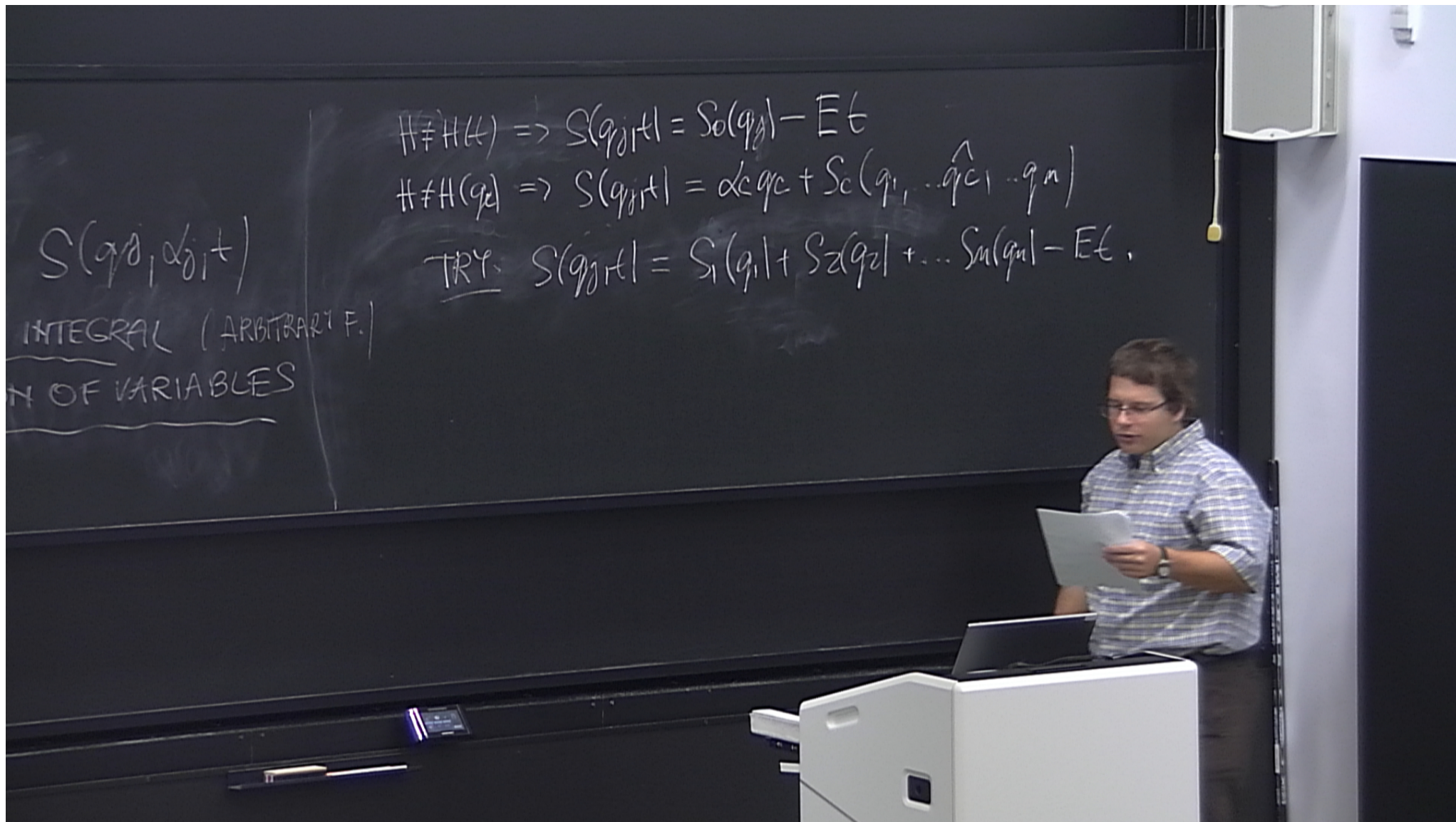
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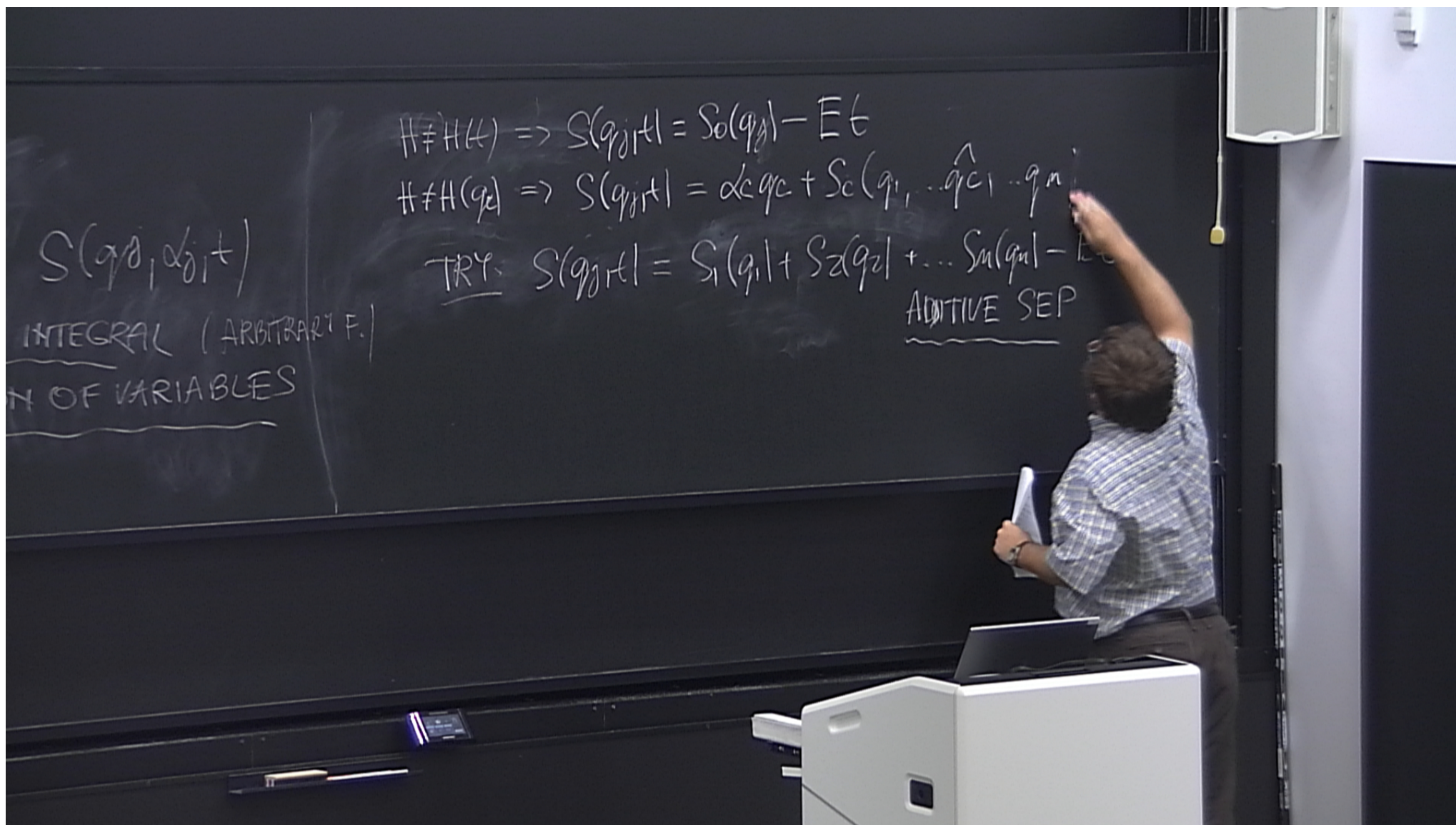
- USEFUL TRICK. SEPARATION OF VARIABLES

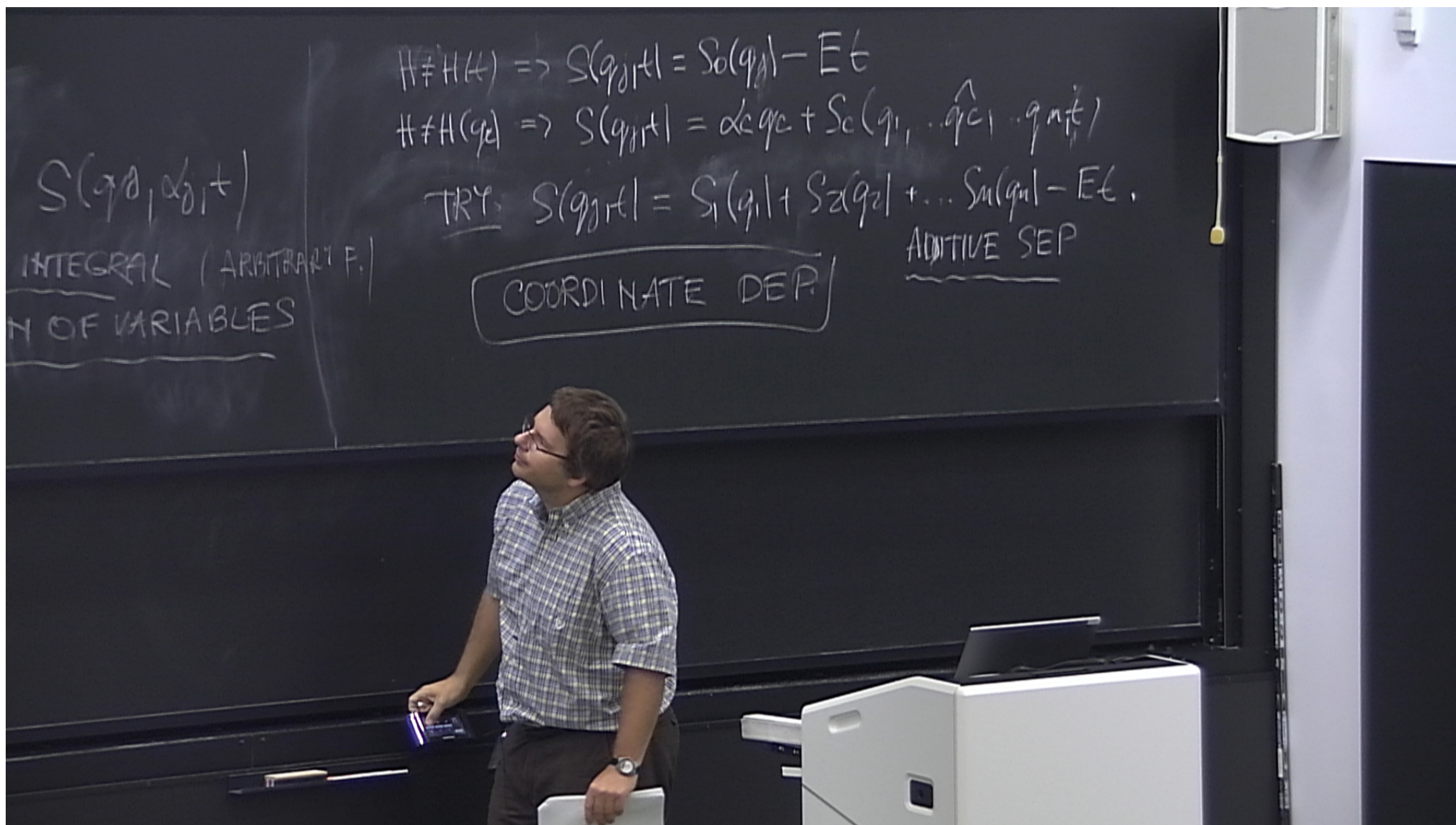
$$H \neq H(t) \Rightarrow C$$











EXAMPLE. FREE FALL

$$H = \frac{p^2}{2m} + V, \quad V = -mgx$$

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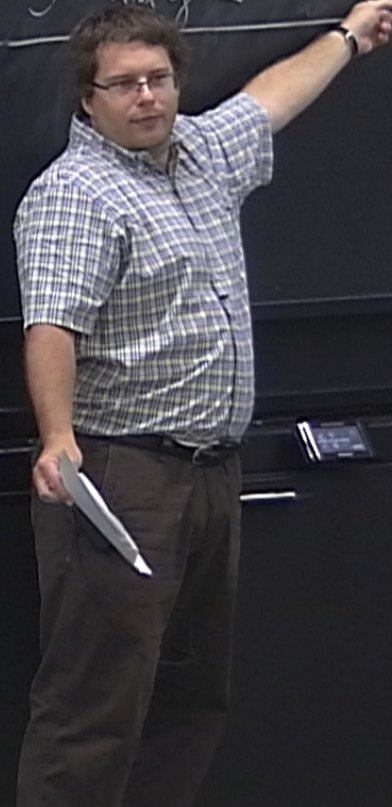
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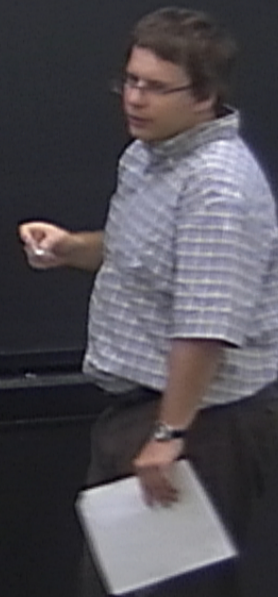
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CONNECTION TO QM:

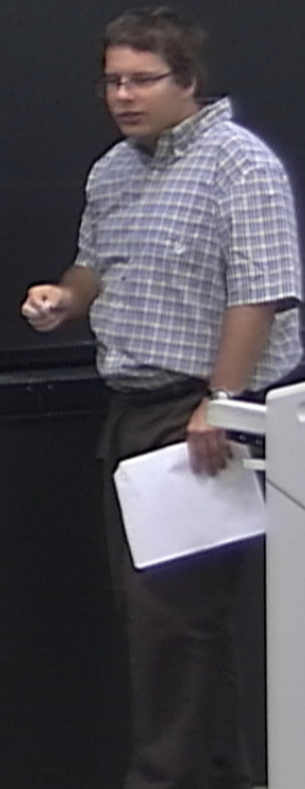
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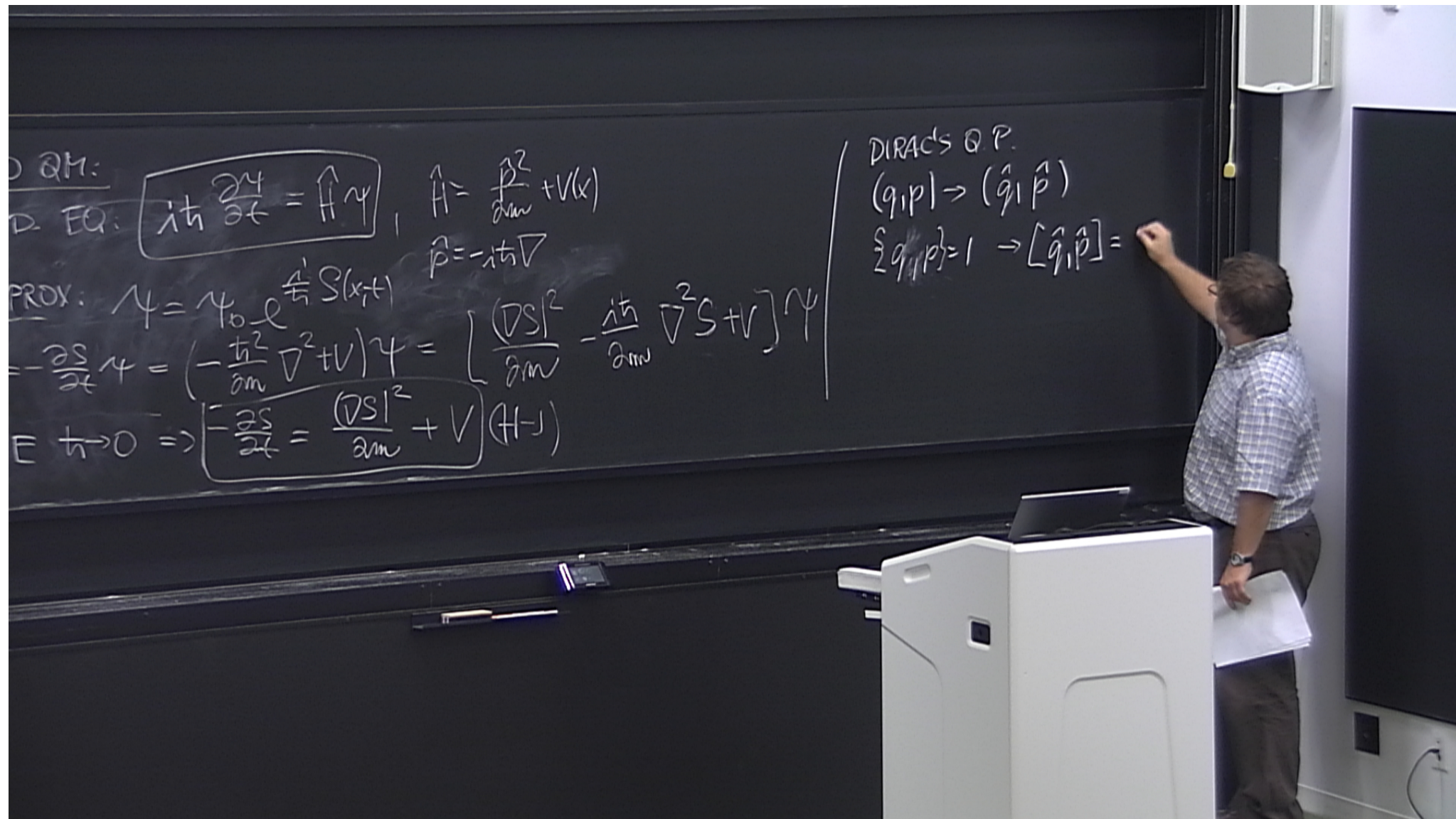
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$$\hbar \rightarrow 0 \Rightarrow \boxed{-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V} \quad (H-J)$$



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INVOLVING

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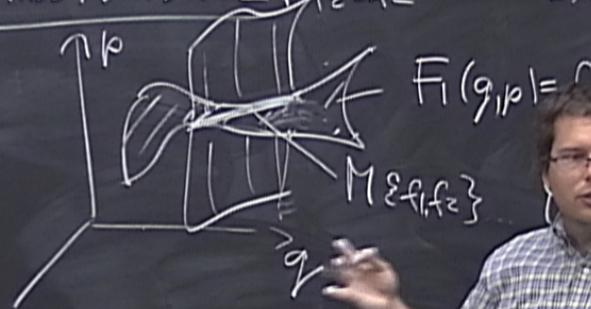
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THEOREM: THE SOLUTION OF EOM OF A COMP INTEG. SYSTEM IS OBTAIN
 -- (FINE # OF INTEGRATIONS & ALGEBRAIC O

ii) INDEPENDENCE $F_2 = f_2$

EACH INTEGRAL F_i DEFINES A HYPERSURFACE
 DYN. TR. MOST REMAIN IN THIS SU



$$\{F_i, F_j\} = 0 \quad \forall i, j.$$

OF EOM OF A COMP INTEG. SYSTEM IS OBTAINED BY "QUADRATURE"
 -- (FINE # OF INTEGRATIONS & ALGEBRAIC OPERATIONS)

EACH INTEGRAL F_i DEFINES A HYPERSURFACE
 DYN. TR. MUST REMAIN IN THIS SURFACE.

F_i
INDEP HYPERSURFACES ARE "NEVER TANGENT"

MOTION.

$M \dot{x}$

GRAV

DEF. THE DYN. SYSTEM IS COMPLETELY (LIOUVILLE) INTEGRABLE IF IT POSSESSES n
INDEPENDENT CONSERVED QUANTITIES $F_i(q, p) = f_i$, $\sum H, F_i = 0$
 IN INVOLUTION $\{F_i, F_j\} = 0 \quad \forall i, j$.

THEOREM: THE SOLUTION OF EOM OF A COMP INTEG. SYSTEM IS OBTAINED BY "QUADRATURE"
 -- (FINITE # OF INTEGRATIONS & ALGEBRAIC OPERATIONS)

IDEA OF PROOF.

- WANT A CAN. TRANSF. $(q^i, p_j) \rightarrow (F^i, \psi_j)$
 $\uparrow$
 CONS. Q

$$= \sum H, F_i$$

DEF. THE DYN. SYSTEM IS COMPLETELY (LIOUVILLE) INTEGRABLE IF IT POSSESSES n
INDEPENDENT CONSERVED QUANTITIES $F_i(q, p) = f_i, \sum H, F_i \} = 0$
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IDEA OF PROOF.

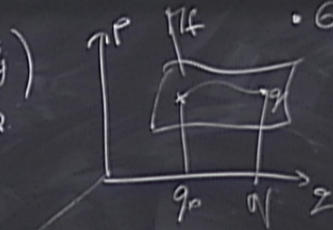
- WANT A CAN. TRANSF.

$$(q, p) \rightarrow (F, \gamma)$$

$\uparrow$
CONS. Q

$$\dot{F} = \{H, F\} = 0$$

$$\dot{\gamma}_i = \{H, \gamma_i\} = \frac{\partial H}{\partial F_i} = \mathcal{L}_i(F_j) = \text{CONST}$$



• GEN. S^1

$$M_f = \{F_i = f_i\}$$

INVERT TO GET $(-p, q)$ ON M_f

$$S(q, F) = \int_{q_0}^q p dq$$

INDEPENDENT CONSERVED QUANTITIES $F_i(q_i, p_i) = f_i, \sum H_i, F_i = 0$
 IN INVOLUTION $\{F_i, F_j\} = 0 \quad \forall i, j.$

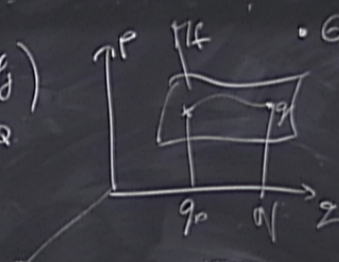
THEOREM: THE SOLUTION OF EOM OF A COMP INTEG. SYSTEM IS OBTAINED BY "QUADRATURE"
 - (FINITE # OF INTEGRATIONS & ALGEBRAIC OPERATIONS)

IDEA OF PROOF.

• WANT A CAN. TRANSF. $(q_i, p_i) \rightarrow (F_i, \gamma_i)$
 ↑
 CONS. Q

$$\dot{F}_i = \{H, F_i\} = 0$$

$$\dot{\gamma}_i = \{H, \gamma_i\} = \frac{\partial H}{\partial F_i} = R_i(F_j) = \text{CONST}$$



• GEN. S

$$M_f = \{F_i = f_i\}$$

INVERT TO GET $p_i = p_i(f_i, q)$

$$S(q, F) = \int_{q_0}^q p_i dq_i$$

IF THIS EXISTS $\Rightarrow \gamma_i = -\frac{\partial S}{\partial F_i}$