

Title: 14/15 PSI - Classical Mechanics

Date: Aug 13, 2014 10:30 AM

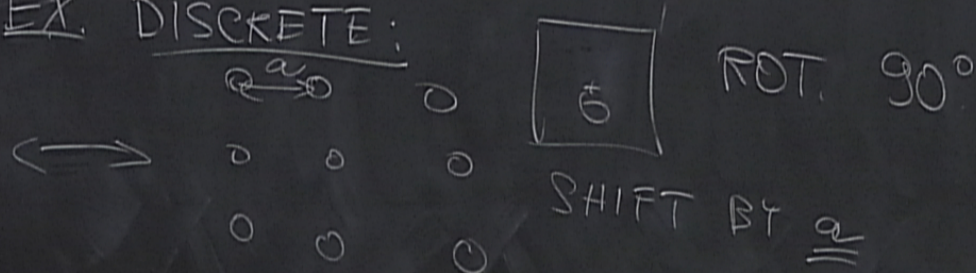
URL: <http://pirsa.org/14080014>

Abstract:

REMARK: $\epsilon \neq \epsilon(t)$ GLOBAL SYMM.

ϵ ... ARBITRARILY SMALL... CONTINUOUS S.

EX. DISCRETE:



3) HAMILTONIAN MECHANICS

a) HAMILTON'S

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a) HAMILTON'S CANONICAL EQS (1834)

• CANONICAL MOMENTUM

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a) HAMILTON'S CANONICAL EQS. (1834)

• CANONICAL MOMENTUM

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad (*)$$

(1834)

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CONFIGURATION S.
= "PHOTO" OF A SYSTEM
 $\{q_j\}$ dim N

PHASE SPACE

CONFIGURATIONS,

= "PHOTO" OF A SYSTEM

$\{q_j\}$ dim N

(*)

PHASE SPACE

..DESCRIBES PHYSICAL
STATES

$\{p_j\}$

3) HAMILTONIAN MECHANICS

a) HAMILTON'S CANONICAL EQS. (1834)

• CANONICAL MOMENTUM

$$p_j = \frac{\partial L}{\partial \dot{q}_j} (*)$$

• HAMILTONIAN = "LEGENDRE TRANSFORM OF L"

$$H(q, p, t) = \sum_j p_j \dot{q}_j - L$$

CONFIGURATION S.

= "PHOTO" OF A SYSTEM

$$\{q_j\} \text{ dim } N$$

PHASE SPACE

• DESCRIBES PHYSICAL STATES

$$\{q, p\} \text{ dim } = 2N$$

NICS

L EQS. (1834)

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad (*)$$

CONFIGURATIONS,

$\equiv$ "PHOTO" OF A SYSTEM

$\{q_j\}$ dim N

PHASE

DESCR

STATE

$\{q_j, p_j\}$

LEGE TRANSFORM OF L

$$H(t) = \sum_j p_j \dot{q}_j - L / \dot{q}_j = (*)$$

CONFIGURATION S.

= "PHOTO" OF A SYSTEM

$$\{q_j\} \dots \dim N$$

$$\left[\begin{array}{c} \vdots \\ q_j \\ \vdots \end{array} \right] (*)$$

TRANSFORM OF L''

$$\dot{q}_j - L / \dot{q}_j \stackrel{(*)}{=} \dot{q}_j (q^i, p_i, t)$$

PHASE SPACE

.. DESCRIBES PHYSICAL STATES

$$\{q^i, p_j\} \dots \dim = 2N$$

CONFIGURATION S.

= "PHOTO" OF A SYSTEM

$$\{q_j\} \quad \dim \underline{N}$$

$$\boxed{\dot{q}_j} \quad (*)$$

FORM OF L''

$$\dot{q}_j \dot{q}_j - L / \dot{q}_j \stackrel{(*)}{=} \dot{q}_j (q^i, p_i, t)$$

PHASE SPACE

.. DESCRIBES PHYSICAL STATES

$$\{q^i, p_j\} \dots \dim = 2N$$

$$H(q^i, p_i, t) = \sum_j p_j \dot{q}^j - L / \dot{q}^j$$

THEOREM: THE SYSTEM OF N (E-L) EQS. IS EQUIVALENT
TO THE SYSTEM OF $2N$ FIRST-ORDER HAMILTON'S EQS.

$$\dot{p}_j =$$

$$H(q^i, p_i, t) = \sum_j p_j \dot{q}^j - L / \dot{q}^j$$

THEOREM. THE SYSTEM OF N (E-L) EQS. IS EQUIVALENT
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$$\dot{p}_j = -\frac{\partial H}{\partial q^j}, \quad \dot{q}^j = \frac{\partial H}{\partial p_j}$$

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THEOREM: THE SYSTEM OF N (E-C) EQS. IS EQUIVALENT
TO THE SYSTEM OF $2N$ FIRST-ORDER HAMILTON'S EQS.

$$\dot{p}_j = -\frac{\partial H}{\partial q_j}, \quad \dot{q}_j = \frac{\partial H}{\partial p_j} \quad (HE)$$

PROOF: • DIRECT CALC. E.G. $\frac{\partial H}{\partial p_j} \stackrel{H(E-C)}{=} \dot{q}_j$

• VAR. PRINCIPLE.

$$0 = \delta S = \delta \int_1^2 (p_i \dot{q}^i - H) dt$$

$$= \int_1^2 \left(p_i \underbrace{\delta \dot{q}^i}_{\frac{d}{dt} \delta q^i} + \dot{q}^i \delta p_i - \frac{\partial H}{\partial q^i} \delta q^i - \frac{\partial H}{\partial p_i} \delta p_i \right) dt$$

$$= \left[p_i \delta q^i \right]_1^2 + \int_1^2 \left[\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right) \delta p_i - \left(\dot{p}_i + \frac{\partial H}{\partial q^i} \right) \delta q^i \right] dt$$

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$$= \left[p_i \delta q^i \right]_1^2 + \int_1^2 \left[\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right) \delta p_i + \left(\dot{p}_i + \frac{\partial H}{\partial q^i} \right) \delta q^i \right] dt$$

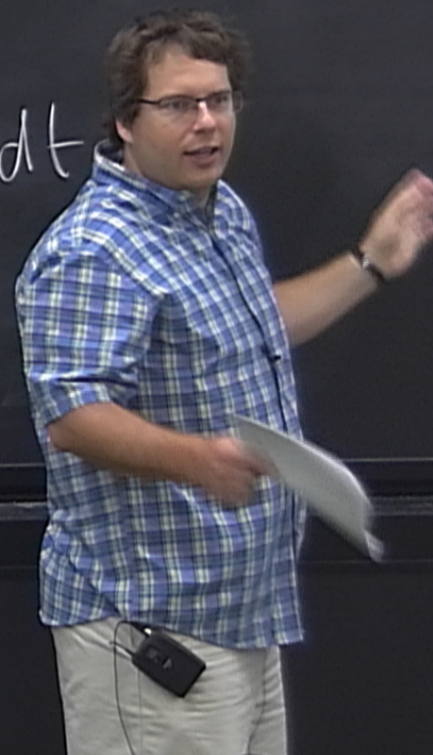
R. PRINCIPLE.

$$0 = \delta S = \delta \int_1^2 (p_i \dot{q}^i - H) dt$$

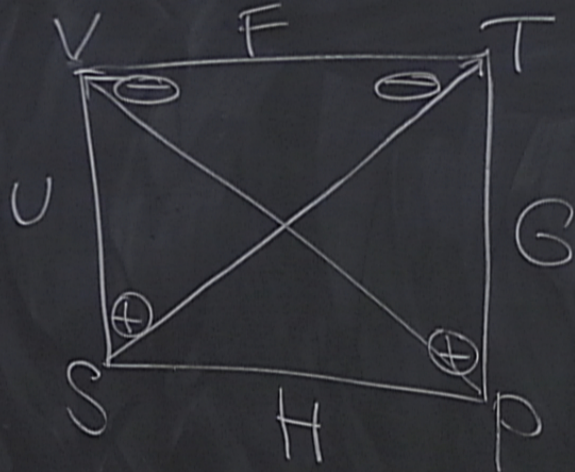
$$\delta \dot{q}^i + \dot{q}^i \delta p_i - \frac{\partial H}{\partial q^i} \delta q^i - \frac{\partial H}{\partial p_i} \delta p_i) dt$$

NOTE: $\frac{dH}{dt} = \frac{\partial H}{\partial t}$

$$\delta \dot{q}^i + \dot{q}^i \delta p_i - \frac{\partial H}{\partial q^i} \delta q^i - \frac{\partial H}{\partial p_i} \delta p_i) dt + \int_1^2 \left[\underbrace{\left(\dot{q}^i - \frac{\partial H}{\partial p_i} \right)}_{=0} \delta p_i - \underbrace{\left(p_i + \frac{\partial H}{\partial q^i} \right)}_{=0} \delta q^i \right] dt$$



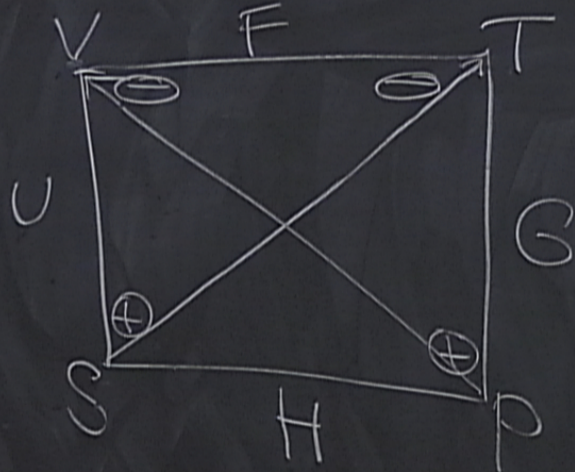
LEGENDRE TRANSF IN THERMODYNAMICS



INFORMATION

- ① TD POTENTIALS ARE TOTAL DIFF. WHEN EXPRESSED "IN THEIR VARIABLES"

LEGENDRE TRANSF IN THERMODYNAMICS



INFORMATION

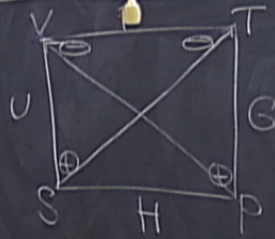
- ① TD POTENTIALS ARE TOTAL DIFF.
WHEN EXPRESSED "IN THEIR VARIABLES"

$$dU = TdS - PdV$$

EINSTEIN

SO IF $H \neq H(T)$
 $p = \text{CONST}$

LEGENDRE TRANS IN THERMODYNAMICS



INFORMATION

① TD POTENTIALS ARE TOTAL DIFF
WHEN EXPRESSED "IN THEIR VARIABLES"

$$dU = TdS - PdV$$

$$dG = VdP - SdT$$



$$dU = TdS - PdV$$

$$dG = VdP - SdT \quad (3)$$

$$\left(\frac{\partial V}{\partial S} \right)_P = \left(\frac{\partial T}{\partial P} \right)_S$$

PROOF: $dH = TdS + VdP$

$$T = \left(\frac{\partial H}{\partial S} \right)_P, \quad V = \left(\frac{\partial H}{\partial P} \right)_S$$

$$\frac{\partial^2 H}{\partial^2}$$



$$dU = TdS - PdV$$

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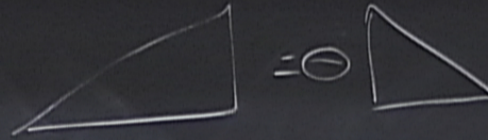


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③ MAXWELL R

$$\oplus \left(\frac{\partial V}{\partial S} \right)_P = \ominus \left(\frac{\partial T}{\partial P} \right)_S$$



PROOF: $dH = TdS + VdP$

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• VAR. PRINCIPLE.

$$0 = \delta S = \delta \int_1^2 (p_i \dot{q}^i - H) dt$$

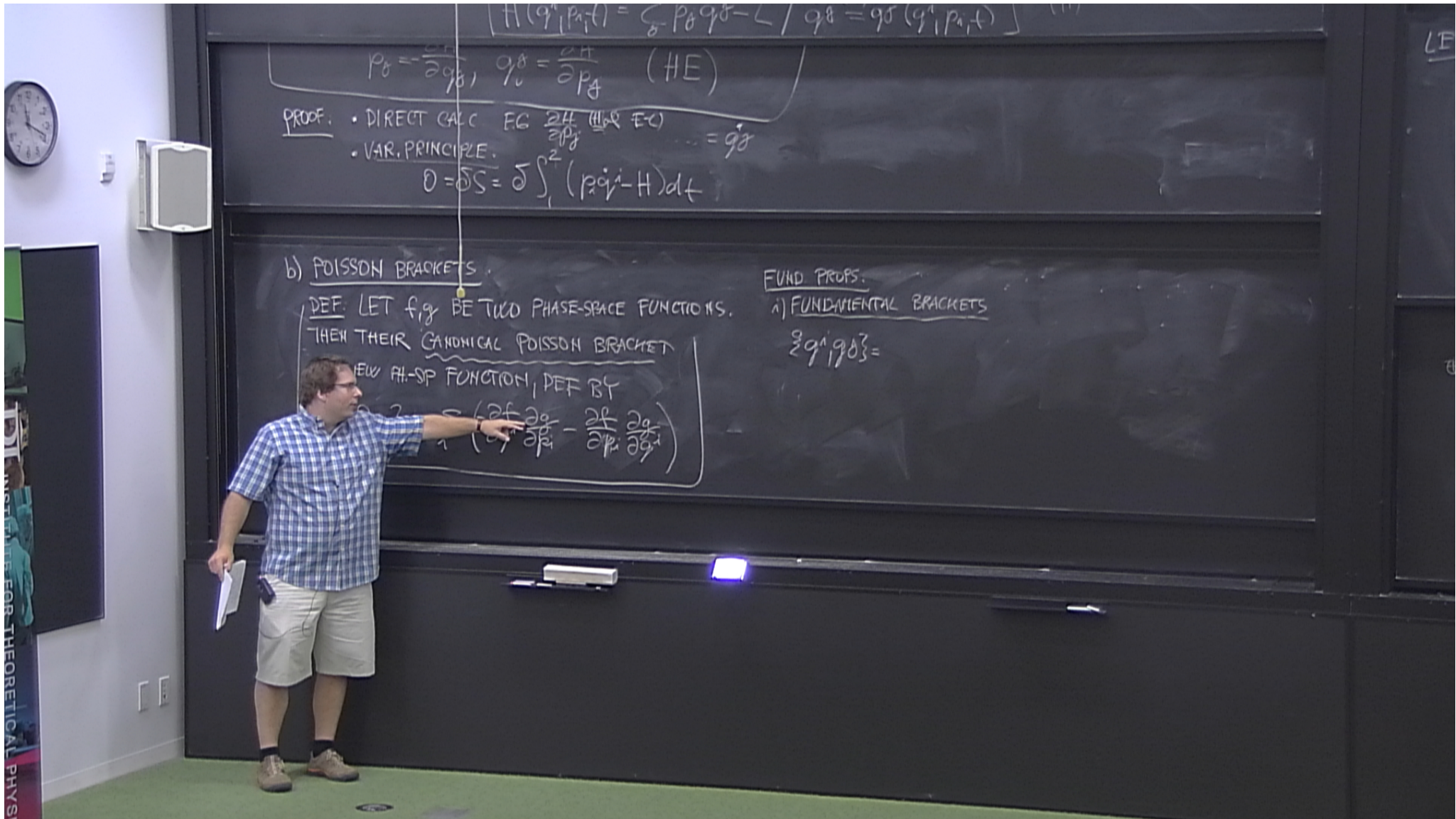
b) POISSON BRACKETS.

DEF. LET f, g BE TWO PHASE-SPACE FUNCTIONS.
THEN THEIR CANONICAL POISSON BRACKET

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DEF. LET f, g BE TWO PHASE-SPACE FUNCTIONS.
THEN THEIR CANONICAL POISSON BRACKET
IS A NEW PH-SP FUNCTION, DEF BY

$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$



CE FUNCTIONS.

RACKET

T

$$\left(\frac{\partial g}{\partial p_i} \right)$$

FUND. PROPS.

i) FUNDAMENTAL BRACKETS

$$\{q^i, q^j\} = 0 = \{p_i, p_j\}, \quad \{q^i, p_j\} = \delta^i_j$$

ii) ANTISYMMETRY $\{f, g\} = -\{g, f\}$

FUND. PROPS.

i) FUNDAMENTAL BRACKETS

$$\{q^i, q^j\} = 0 = \{p_i, p_j\}, \quad \{q^i, p_j\} = \delta^i_j$$

ii) ANTISYMMETRY $\{f, g\} = -\{g, f\}$

iii) LINEARITY

$$\{\alpha f + \beta g, h\}$$

$$\sum_i g_i \dot{S} = \sum_i \left(\frac{\partial g_i}{\partial q^i} \frac{\partial \dot{S}}{\partial p_i} - \frac{\partial p_i}{\partial q^i} \frac{\partial \dot{S}}{\partial g_i} \right)$$

$$2\alpha + 1/3 g_{,h}$$

iv) LEIBNITZ $\{fg, h\} = f\{g, h\} + \{f, h\}g$

v) JACOBI ID. $\{ \{f, g\}, h \} + \{ \{g, h\}, f \} + \{ \{h, f\}, g \} = 0$

$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$

$$\{ \alpha f + \beta g, h \} = \alpha \{f, h\} + \beta \{g, h\}$$

$\alpha, \beta \in \mathbb{R}$

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REMARKS: • FORM A LIE ALGEBRA

$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$

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RE: • FORM A LIE ALGEBRA; i-v) CAN BE USED AS DEFINITION.

$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$

$$\{ \alpha f + \beta g, h \} = \alpha \{f, h\} + \beta \{g, h\}$$

$\alpha, \beta \in \mathbb{R}$

iv) LEIBNITZ $\{fg, h\} = f\{g, h\} + \{f, h\}g$

v) JACOBI ID $\{\{f, g\}, h\} + \{\{h, f\}, g\} + \{\{g, h\}, f\} = 0$

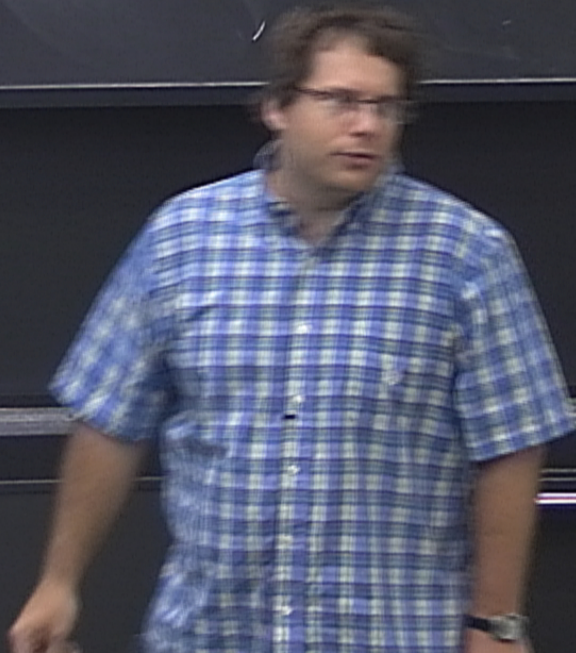
REMARKS FORM A LIE ALGEBRA; i-v) CAN BE USED AS DEFINITION.
PB. DEFINE TIME EVOLUTION:

$$\{ \{ f, g \}, h \} = \{ \{ g, h \}, f \} + \{ \{ h, f \}, g \}$$

V) JACOBI ID: $\{ \{ f, g \}, h \} + \{ \{ h, f \}, g \} + \{ \{ g, h \}, f \} = 0$

REMARKS:

- FORM A LIE ALGEBRA; i-v) CAN BE USED AS DEFINITION.
- PB. DEFINE TIME EVOLUTION: $\frac{df}{dt} = \{ f, H \} + \frac{\partial f}{\partial t}$



$$\{f, g\} = f\{g, h\} + \{f, h\}g$$

$$\{g, h\} + \{ \{h, f\}, g \} + \{ \{g, h\}, f \} = 0$$

• INTEGRALS OF MOTION

ALGEBRA; (i-v) CAN BE USED AS DEFINITION.

TIME EVOLUTION: $\boxed{\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}}$

$$\{f, g\} = f\{g, h\} + \{f, h\}g$$

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TIME EVOLUTION: $\boxed{\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}}$

• INTEGRALS OF MOTION

$$\boxed{\{I, H\} = 0}$$

$$\{f, g\} = f\{g, h\} + \{f, h\}g$$

$$\{f, h\} + \{ \{h, f\}, g \} + \{ \{g, h\}, f \} = 0$$

ALGEBRA; (i-v) CAN BE USED AS DEFINITION.

TIME EVOLUTION: $\boxed{\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}}$

• INTEGRALS OF MOTION

$$\boxed{\{I, H\} = 0}$$

$$\{I_1, I_2\}$$



$$\{g, h\} + \{f, h\}g$$

$$\sum \{ \{h, f\}, g \} + \sum \{ \{g, h\}, f \} = 0$$

$i-v$) CAN BE USED AS DEFINITION.

UTION: $\boxed{\frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}}$

• INTEGRALS OF MOTION

$$\boxed{\{I, H\} = 0}$$

$$\{I, T\} = I_3$$

ALSO INTEGRAL
OF



$$dU = TdS - PdV$$

$$dG = VdP - SdT$$

③ MAXWELL

c) CANONICAL TRANSFORMATIONS

DEF. CAN TRANSF

$$Q' = Q'(q, p, t)$$

$$P' = P'(q, p, t)$$



$$dU = TdS - PdV$$

$$dG = VdP - SdT$$

③ MAXWELL

c) CANONICAL TRANSFORMATIONS

DEF. CAN TRANSF

$$\left[\begin{array}{l} Q = Q(q, p, t) \\ P = P(q, p, t) \end{array} \right] (QP)$$

$$dU = TdS - PdV$$

$$dG = VdP - SdT$$

$$H = U + PV$$

③ MAXWELL RELATIONS

$$\Delta = \Delta$$

TRANSFORMATIONS

$$\left[\begin{array}{l} Q^{\delta} = Q^{\delta}(q^i, p_i, t) \\ P_j = P_j(q^i, p_i, t) \end{array} \right] (QP)$$

THAT PRESERVES THE FORM OF (HE)

$$TdS = PdV$$

$$VdP = SdT$$

$$H = U + PV$$

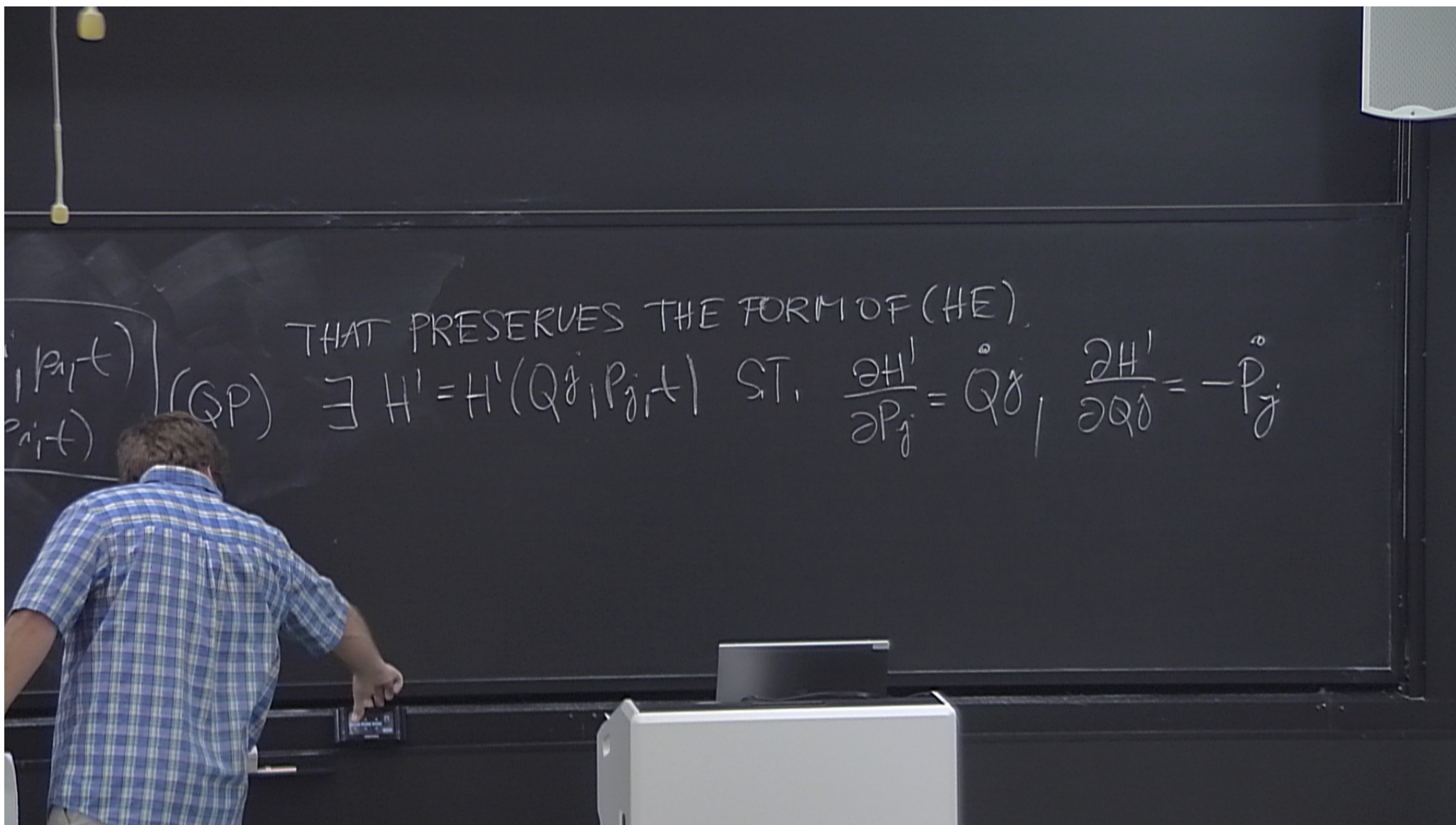
③ MAXWELL RELATIONS

$$\Delta = \Delta$$

THAT PRESERVES THE FORM OF (HE)

$$\left. \begin{matrix} p_i(t) \\ p_i(t) \end{matrix} \right\} (Q, P)$$

$$\exists H' = H'(Q, p_i, t) \text{ s.t. } \frac{\partial H'}{\partial p_j} = \dot{Q}_j, \quad \frac{\partial H'}{\partial Q_j} =$$



c) CANONICAL TRANSFORMATIONS

DEF. CAN TRANSF $\left[\begin{array}{l} Q_j = Q_j(q^i, p_i, t) \\ P_j = P_j(q^i, p_i, t) \end{array} \right] (QP)$ THAT PRESERVES $\exists H' = H'(Q_j, P_j, t)$

WARNING: NOT EVERY T. IS CANONICAL!
FREE PARTICLE

CANONICAL TRANSFORMATIONS

CAN TRANSF

$$\left[\begin{array}{l} Q^j = Q^j(q^i, p_i, t) \\ P_j = P_j(q^i, p_i, t) \end{array} \right] (QP)$$

THAT PRESERVES THE FORM OF
 $\exists H' = H'(Q^j, P_j, t)$ ST. $\frac{\partial H'}{\partial P_j}$

WARNING: NOT EVERY T. IS CANONICAL!

FREE PARTICLE

$$H(q, p) = \frac{p^2}{2m}$$

$$H'(Q, P) = \frac{P^2}{2m}$$

$$Q = \sqrt{q}, \quad P = p$$

NONICAL TRANSFORMATIONS

CAN TRANSF

$$\left[\begin{array}{l} Q^j = Q^j(q^i, p_i, t) \\ P_j = P_j(q^i, p_i, t) \end{array} \right] (QP)$$

THAT PRESERVES THE FORM OF
 $\exists H' = H'(Q^j, P_j, t)$ ST. $\frac{\partial H'}{\partial P_j}$

ARNING: NOT EVERY T. IS CANONICAL?

FREE PARTICLE

$$H(q, p) = \frac{p^2}{2m}$$

$$H'(Q, P) = \frac{P^2}{2m} \Rightarrow \frac{\partial H'}{\partial P} = \frac{P}{m}$$

$$Q = \sqrt{q}, \quad P = p$$

$$(q^i, p_i) \rightarrow (Q, P) \quad \exists H' = H'(Q, P, t) \text{ s.t. } \frac{\partial H'}{\partial P_j} = \dot{Q}_j, \quad \frac{\partial H'}{\partial Q_j} = -\dot{P}_j$$

CANONICAL:

$$q, p = \frac{p^2}{2m}$$

$$Q, P = \frac{P^2}{2m} \Rightarrow \frac{\partial H'}{\partial P} = \frac{P}{m} = \frac{p}{m} = \frac{\partial H}{\partial p} = \dot{q} = 2Q\dot{Q}$$



WARNING: NOT EVERY T. IS CANONICAL

FREE PARTICLE

$$H(q,p) = \frac{p^2}{2m}$$
$$H'(Q,P) = \frac{P^2}{2m} \Rightarrow$$

$$Q = \sqrt{q} \quad P = p$$
$$\frac{\partial H'}{\partial P} = \frac{P}{m} = \frac{p}{m} = \frac{\partial H}{\partial p} = v$$

• C.T. ARE GENERATED BY "GENERATING FUNCTION" F

$$H(Q,P) = \frac{1}{2m} \Rightarrow \frac{\partial H}{\partial P} = \frac{P}{m} \quad \frac{\partial H}{\partial Q} = \frac{1}{2m} \frac{\partial P^2}{\partial Q} = \frac{P}{m} \frac{\partial P}{\partial Q}$$

• C.T. ARE GENERATED BY "GENERATING FUNCTION" $F = F(\text{OLD } (q,p))$

EG: $F(q, Q, t)$

$$S = \int (p \dot{q} - H - \left(\frac{dF}{dt} \right)) dt = \text{WANT} \int (P \dot{Q} - H(P, Q, t)) dt$$

$$= \int [$$

$$H(Q,P) = \frac{1}{2m} \Rightarrow \frac{\partial F}{\partial P} = m \quad m \quad \frac{\partial F}{\partial P}$$

• C.T. ARE GENERATED BY "GENERATING FUNCTION" $F = F(\overset{\uparrow}{\text{OLD}}, \overset{\uparrow}{\text{NEW}})$

EG: $F(q, Q, t)$

$$\frac{\partial F}{\partial q} \dot{q} + \frac{\partial F}{\partial Q} \dot{Q} + \frac{\partial F}{\partial t}$$

$$S = \int (p \dot{q} - H - \left(\frac{dF}{dt} \right)) dt = \overset{\text{WANT}}{\int} (P \dot{Q} - H(P, Q, t)) dt$$

$$= \int \left[\dot{q} \left(p - \frac{\partial F}{\partial q} \right) - \right]$$

$$H(Q,P) = \frac{1}{2m} \Rightarrow \frac{\partial F}{\partial P} = m \quad m \quad \frac{\partial F}{\partial P}$$

• C.T. ARE GENERATED BY "GENERATING FUNCTION" $F = F(\overset{\uparrow}{\text{OLD}}, \overset{\uparrow}{\text{NEW}})$

EG: $F(q, Q, t)$

$$\frac{\partial F}{\partial q} \dot{q} + \frac{\partial F}{\partial Q} \dot{Q} + \frac{\partial F}{\partial t}$$

$$S = \int (p \dot{q} - H - \left(\frac{dF}{dt} \right)) dt = \overset{\text{WANT}}{\int (P \dot{Q} - H(P, Q, t)) dt}$$

$$= \int \left[\dot{q} \left(p - \frac{\partial F}{\partial q} \right) - \frac{\partial F}{\partial Q} \dot{Q} - \left(H + \frac{\partial F}{\partial t} \right) \right] dt$$

$$P = -\frac{\partial F}{\partial Q}, \quad p = \frac{\partial F}{\partial q}$$

$$H(Q,P) = \frac{1}{2m} \Rightarrow \frac{\partial F}{\partial P} = m \quad m \quad \frac{\partial F}{\partial P}$$

• C.T. ARE GENERATED BY "GENERATING FUNCTION" $F = F(\overset{\uparrow}{\text{OLD}}, \overset{\uparrow}{\text{NEW}})$

EG: $F(q, Q, t)$

$$\frac{\partial F}{\partial q} \dot{q} + \frac{\partial F}{\partial Q} \dot{Q} + \frac{\partial F}{\partial t}$$

$$S = \int (p \dot{q} - H - \left(\frac{dF}{dt} \right)) dt = \overset{\text{WANT}}{\int (P \dot{Q} - H'(P, Q, t)) dt}$$

$$= \int \left[\dot{q} \left(p - \frac{\partial F}{\partial q} \right) - \frac{\partial F}{\partial Q} \dot{Q} - \left(H + \frac{\partial F}{\partial t} \right) \right] dt$$

$$\boxed{P = -\frac{\partial F}{\partial Q}, p = \frac{\partial F}{\partial q}, H' = H + \frac{\partial F}{\partial t}}$$

$$H(Q,P) = \frac{1}{2m} \Rightarrow \frac{\partial F}{\partial P} = m \quad m \quad \frac{\partial F}{\partial P}$$

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$(q,p) \quad (Q,P)$

$$S = \int (p \dot{q} - H - \left(\frac{dF}{dt} \right)) dt = \overset{\text{WANT}}{\int (P \dot{Q} - H'(P, Q, t)) dt}$$

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$$\boxed{P = -\frac{\partial F}{\partial Q}, \quad p = \frac{\partial F}{\partial q}, \quad H' = H + \frac{\partial F}{\partial t}} \quad \text{REF}$$

$$H(Q,P) = \frac{1}{2m} \Rightarrow \frac{\partial F}{\partial P} = m \quad m \quad \frac{\partial F}{\partial P}$$

• C.T. ARE GENERATED BY "GENERATING FUNCTION" $F = F(\overset{\uparrow}{\text{OLD}}, \overset{\uparrow}{\text{NEW}})$

EG: $F(q, Q, t)$

$$\overset{\frac{\partial F}{\partial q} \dot{q} + \frac{\partial F}{\partial Q} \dot{Q} + \frac{\partial F}{\partial t}}{\text{WANT}} \quad (q,p) \quad (Q,P)$$

$$S = \int (p \dot{q} - H - \left(\frac{dF}{dt} \right)) dt = \int (P \dot{Q} - H'(P, Q, t)) dt$$

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$$\boxed{P = -\frac{\partial F}{\partial Q}, \quad p = \frac{\partial F}{\partial q}, \quad H' = H + \frac{\partial F}{\partial t}} \quad \text{EQUIRE}$$

v) JACOBI ID. $\sum \{ \{ f, g \}, h \} + \sum \{ h, \{ f, g \} \} + \sum \{ \{ h, f \}, g \} = 0$

REMARKS: i) PB ARE INVARIANT VIRT. CAN. TRANS.

$$\{ f, g \}_{q,p} = \{ f, g \}_{Q,P}$$

$g \equiv H$ $\frac{df}{dt} = \{ f, g \}$ - PHYS. INDEP OF COORDS.

ii) ANTISYMMETRY $\{f, g\} = -\{g, f\}$

IT, CAN, TRANS,

QIP

PHYS. INDEP OF COORDS,

COORDS

$$j, \{Q, Q\} = 0 = \{P, P\}$$

TERMS OF Q&P ... GIVES CONDS. FOR THE TRANSF. TO BE CANONICAL.

$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$

$$\{ \alpha f + \beta g, h \}$$

iii) "TIME EVOLUTION $\equiv$ CHANGE OF COORDS"

TIME EVOLUTION $\equiv$ CANONICAL TRANSF. GENERATED BY

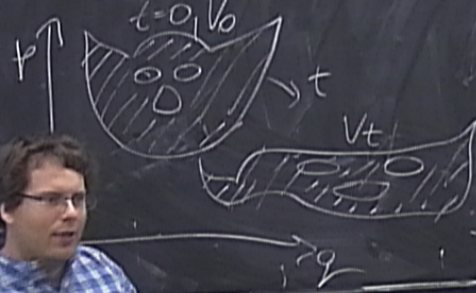
$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} \right)$$

$$\{ \alpha f + \beta g, h \}$$

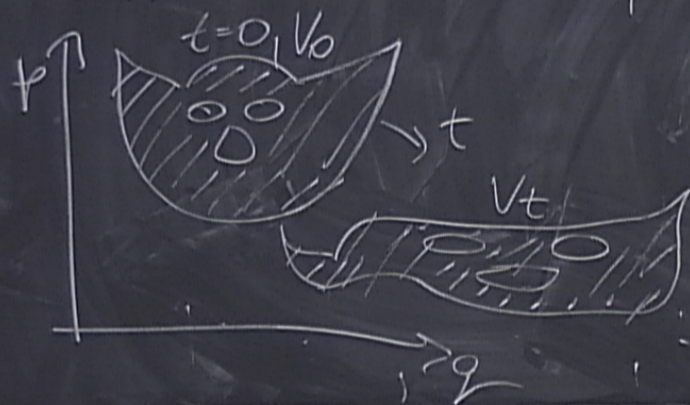
iii) "TIME EVOLUTION $\equiv$ CHANGE OF COORDS"

TIME EVOLUTION $\equiv$ CANONICAL TRANSF. GENERATED BY $F = -S$

LIUVILLE'S THEOREM VOLUME OF THE PHASE-SPACE REMAINS INVARIANT,
UNDER CAN.T. IN PART, PHASE FLOW TIME EVOLUTION PRESERVE THIS VOLUME.



LIOUVILLE'S THEOREM VOLUME OF THE PHASE-SPACE REMAINS INVARIANT,
 UNDER CAN.T. IN PART, PHASE FLOW TIME EVOLUTION PRESERVE THIS



PROOF: $V_t = \int dQ dP = \int JAC dq dp$

$$JAC = \begin{vmatrix} \frac{\partial Q}{\partial q} & \frac{\partial Q}{\partial p} \\ \frac{\partial P}{\partial q} & \frac{\partial P}{\partial p} \end{vmatrix}$$

THEOREM VOLUME OF THE PHASE-SPACE REMAINS INVARIANT,
IN PART, PHASE FLOW TIME EVOLUTION PRESERVES THIS VOLUME.

PROOF: $V_t = \int dQ dP = \int JAC dq dp = \int dq dp = V_0$

$$JAC = \begin{vmatrix} \frac{\partial Q}{\partial q} & \frac{\partial Q}{\partial p} \\ \frac{\partial P}{\partial q} & \frac{\partial P}{\partial p} \end{vmatrix} = \{Q, P\} = 1$$

