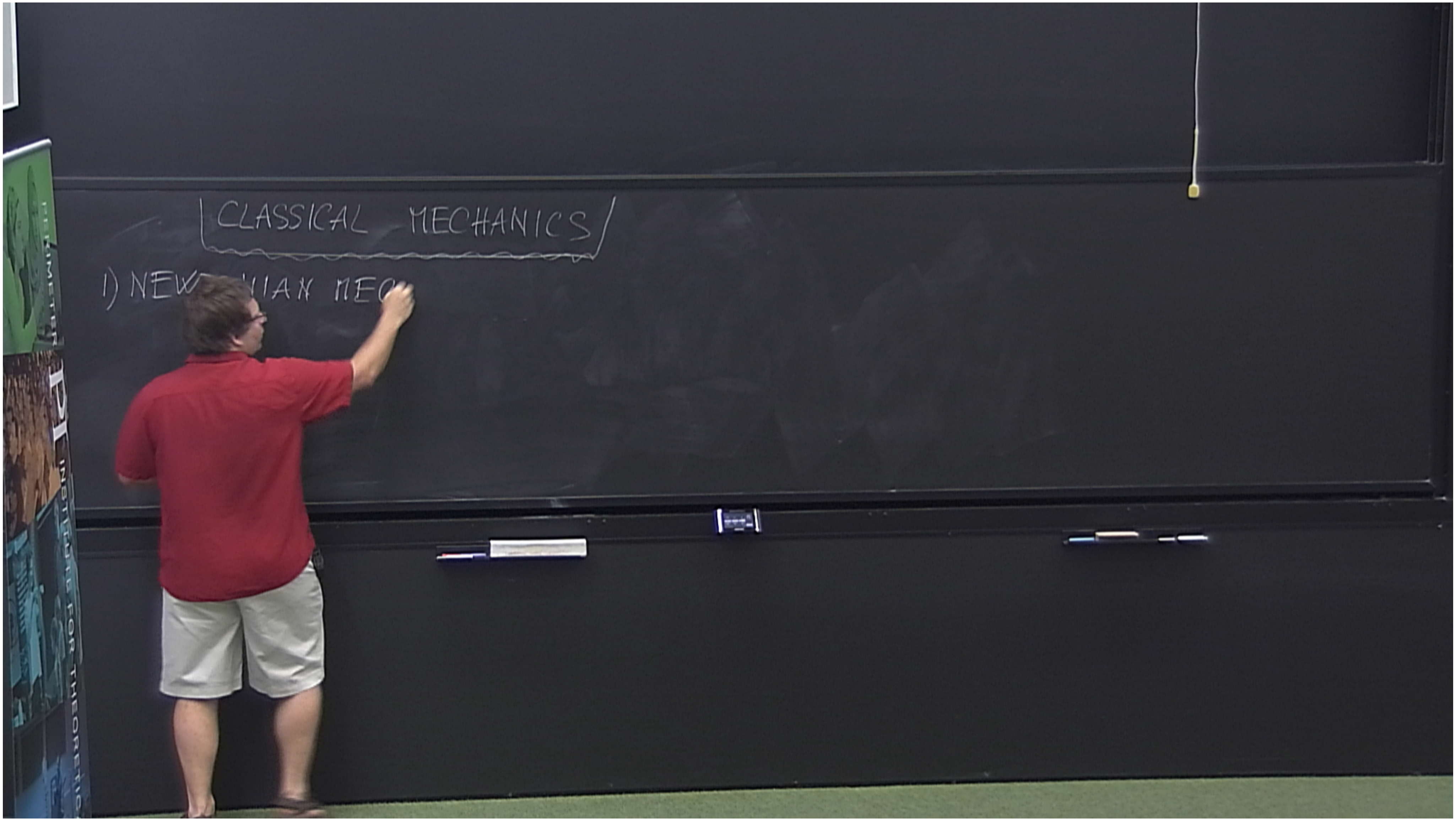
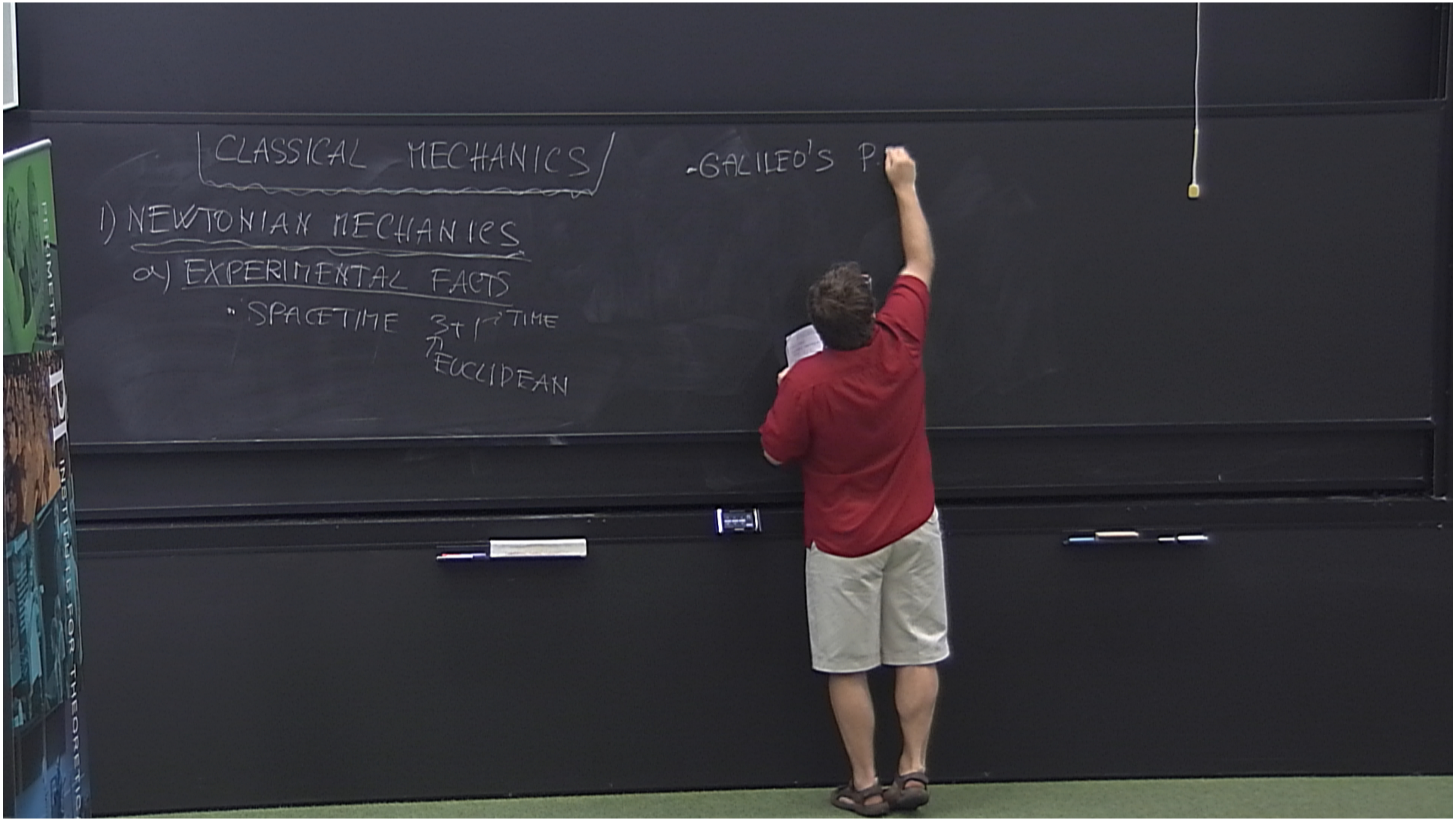
Title: PSI 14/15 - Classic Mechanics

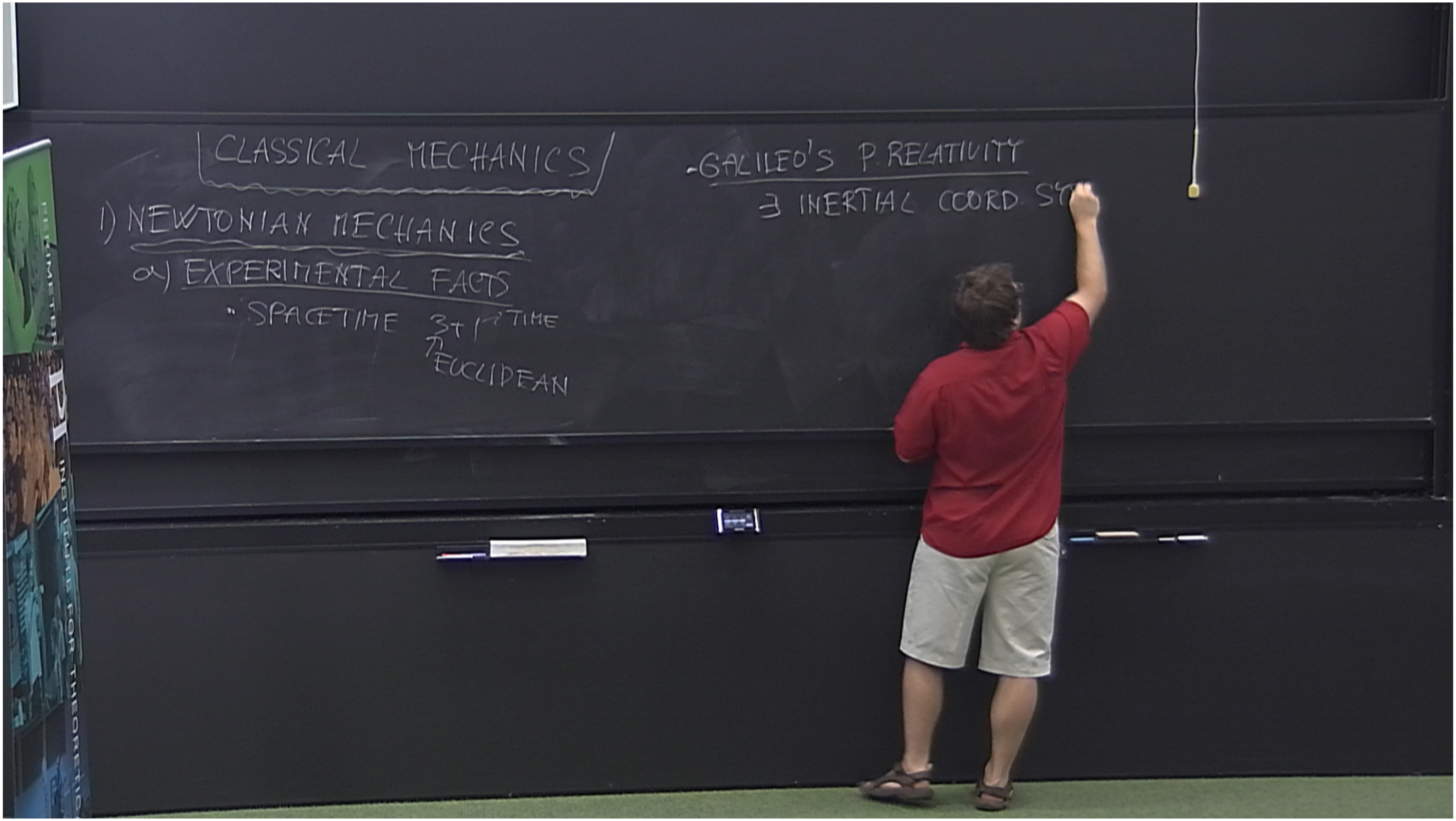
Date: Aug 12, 2014 10:30 AM

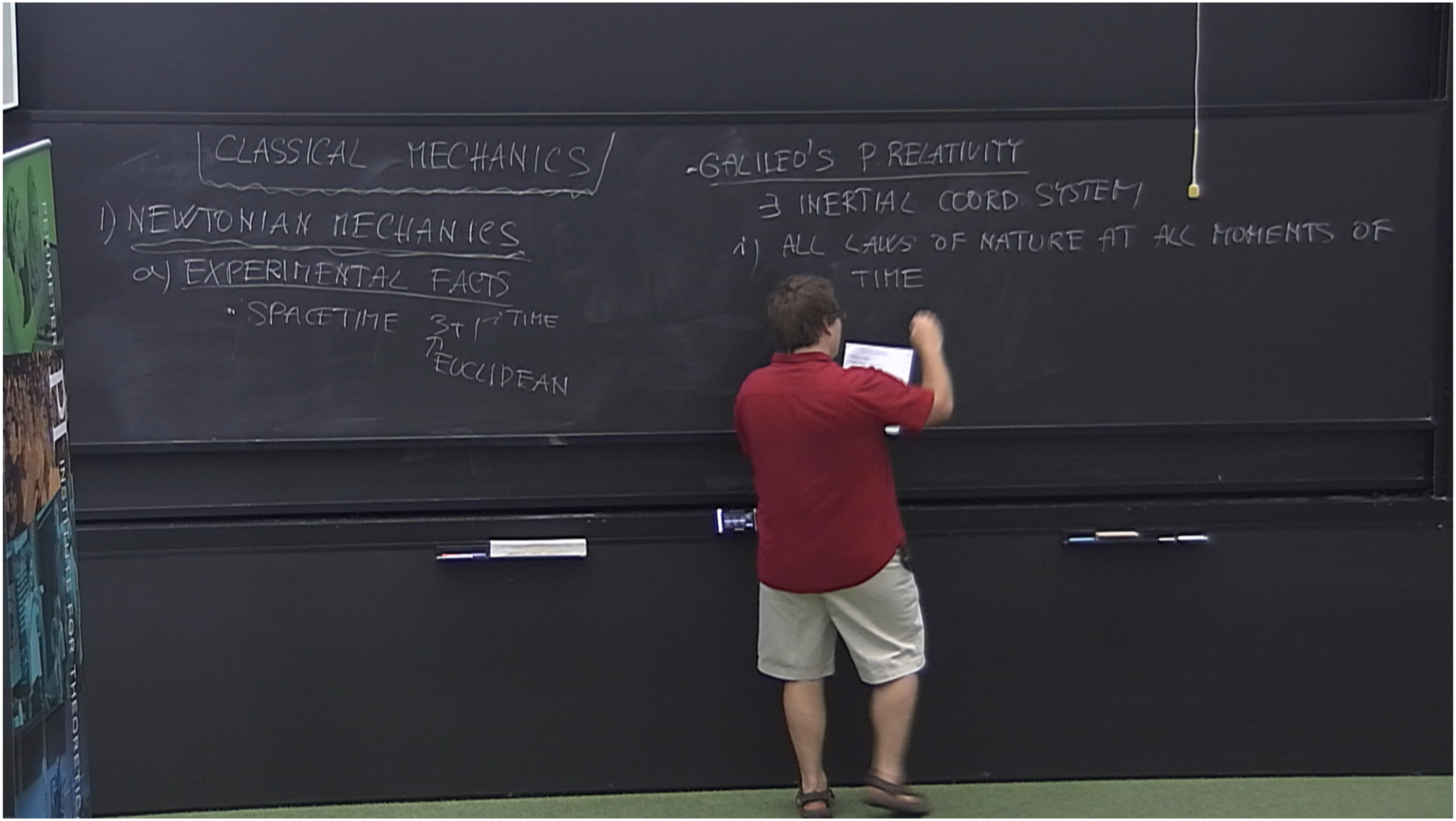
URL: http://pirsa.org/14080009

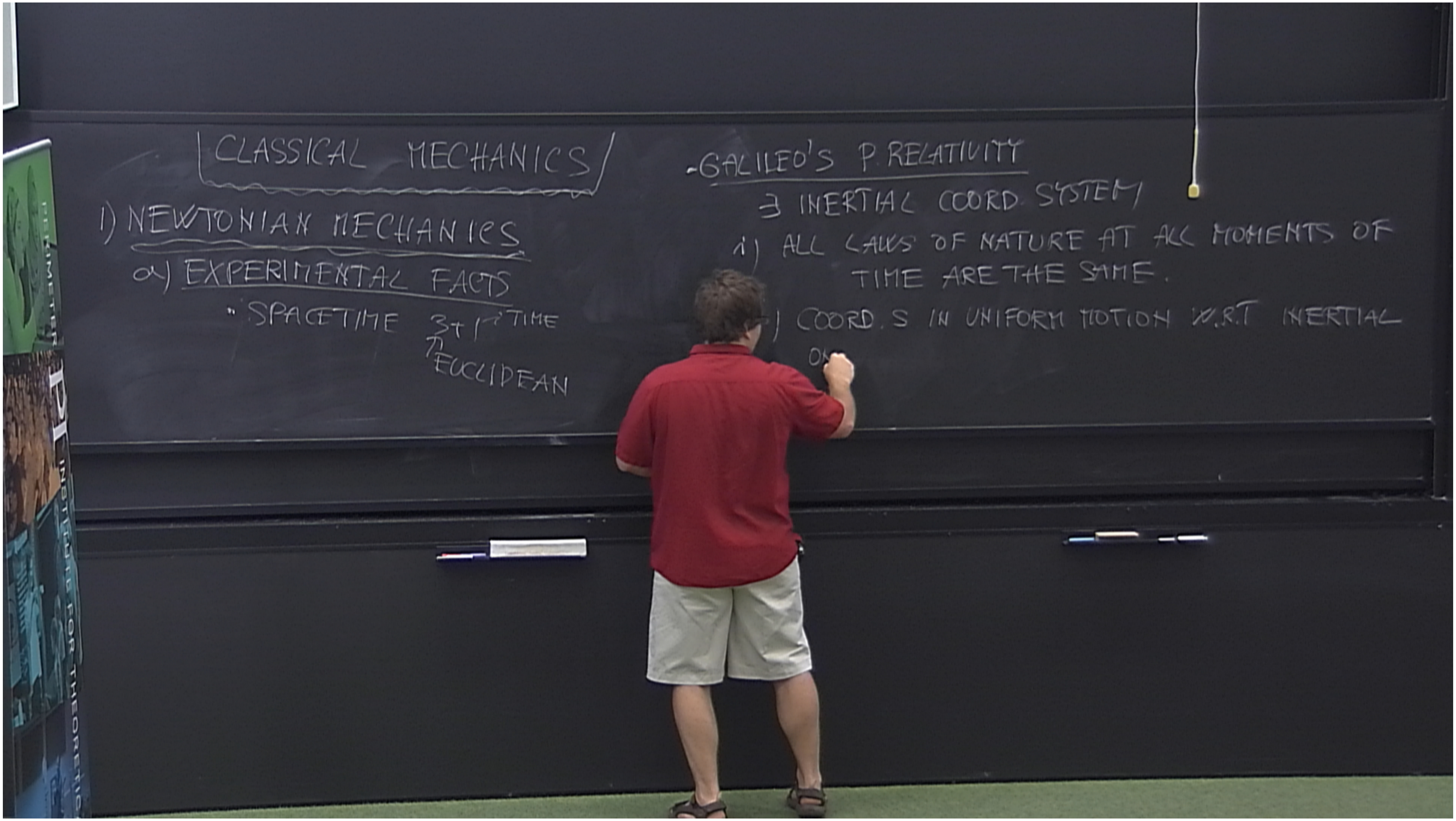
Abstract:

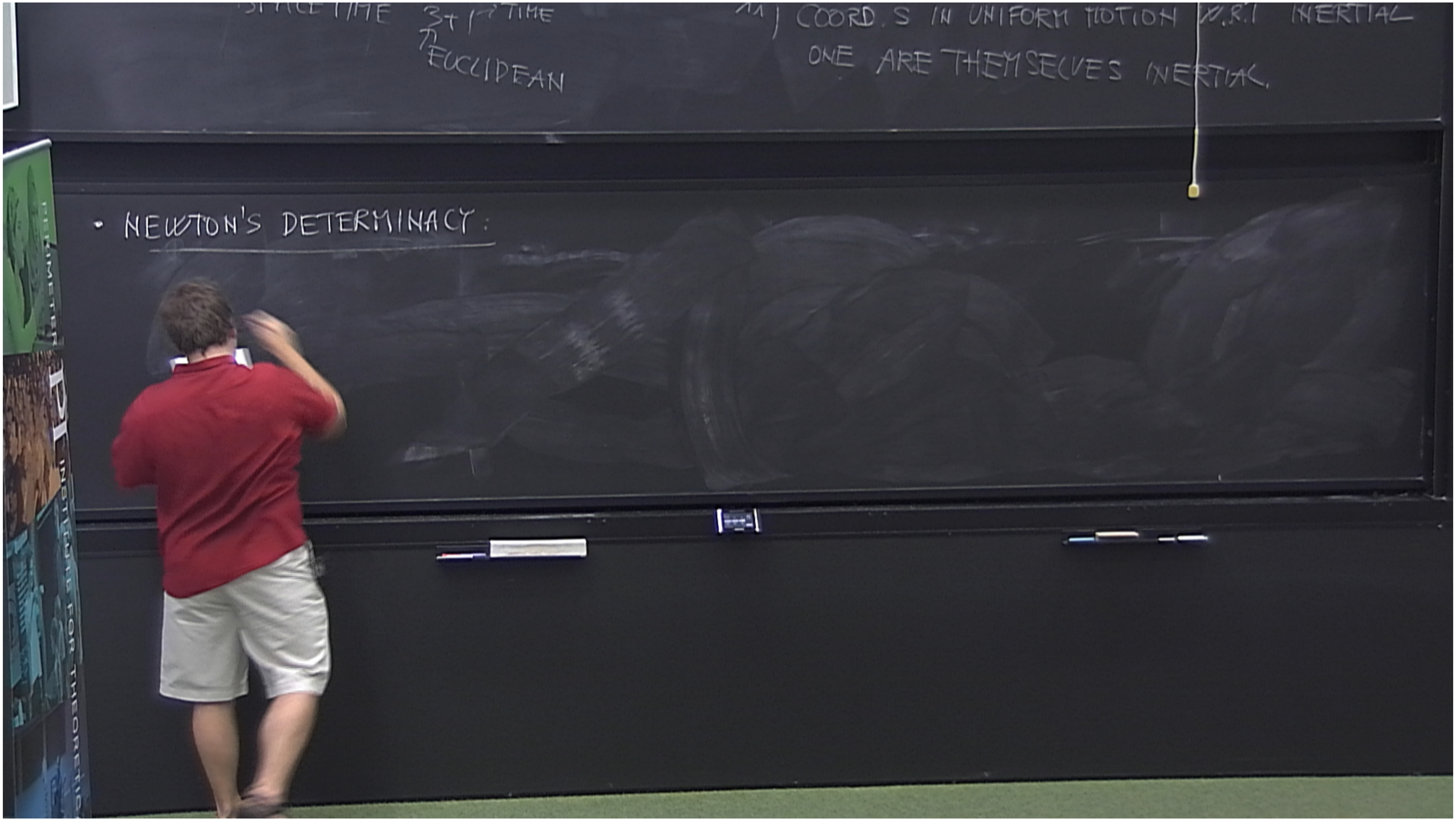

CLASSICAL MECHANICS

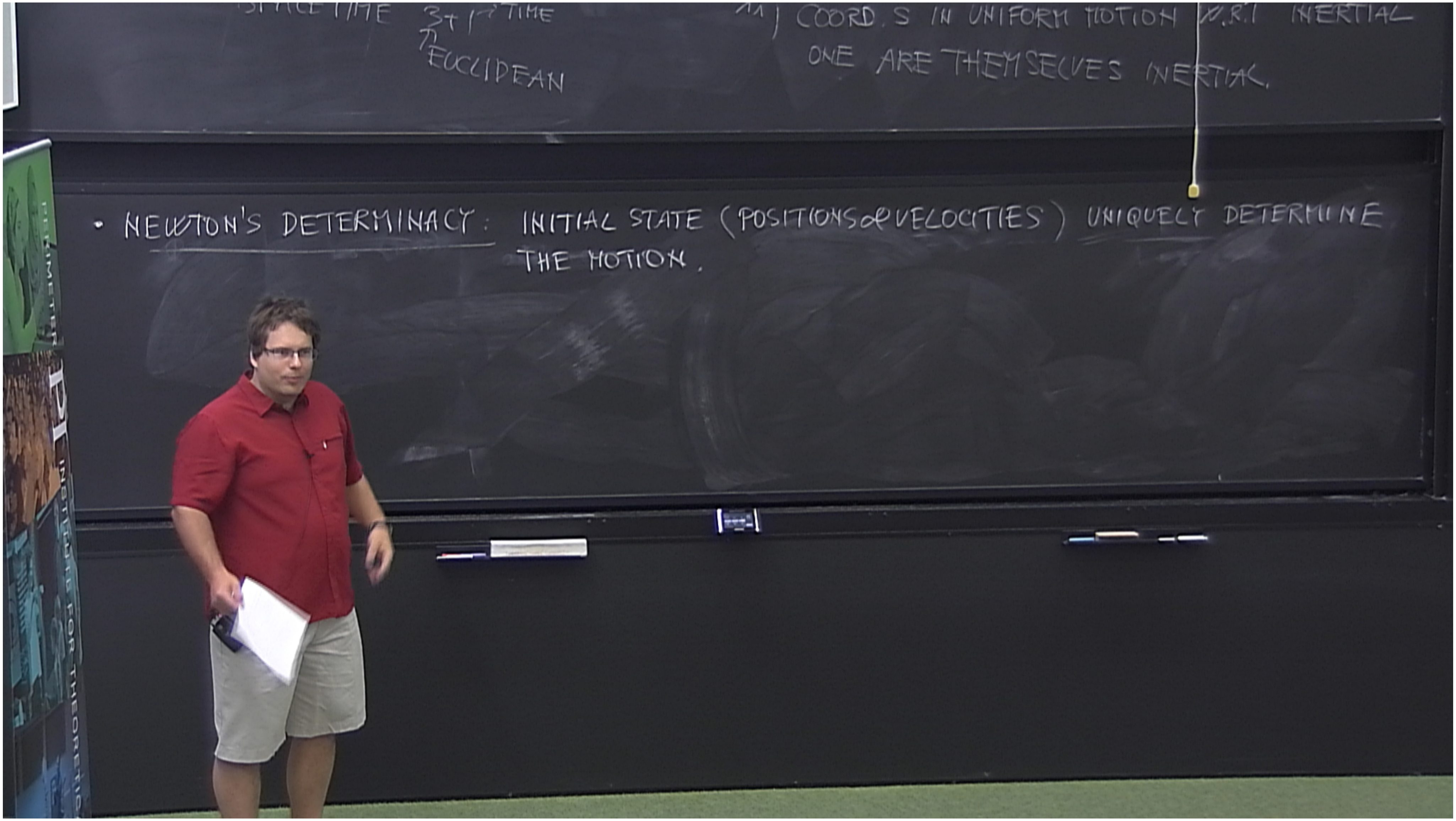

CLASSICAL MECHANICS
-GALILEO'S P.
1) NEWTONIAN MECHANICS
a) EXPERIMENTAL FACTS
• SPACETIME 3+1 TIME
EUCLIDEAN

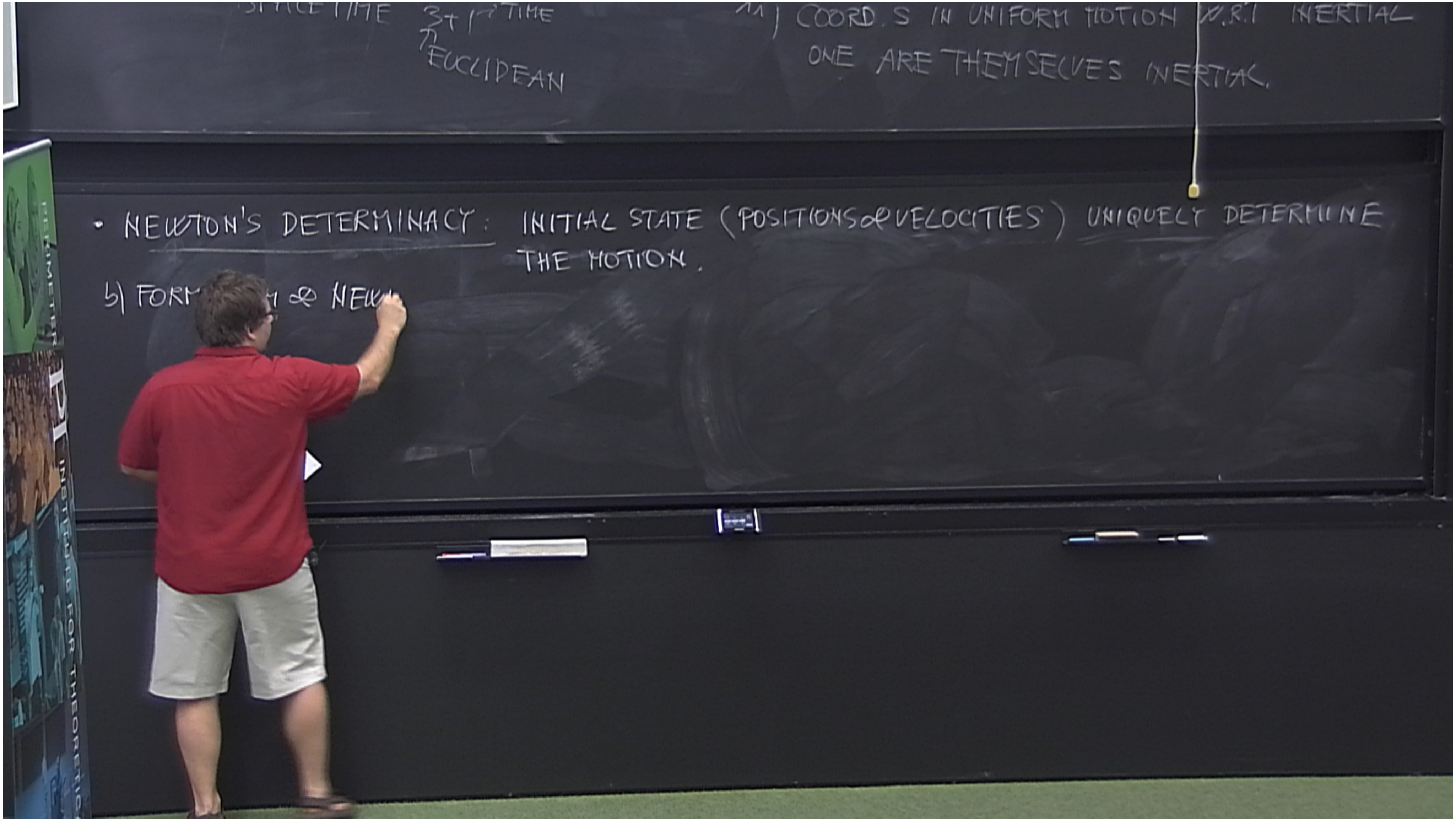

CLASSICAL MECHANICS
1) NEWTONIAN MECHANICS
a) EXPERIMENTAL FACTS
• SPACETIME 3+1 TIME
EUCLIDEAN
- GALILEO'S P. RELATIVITY
∃ INERTIAL COORD.

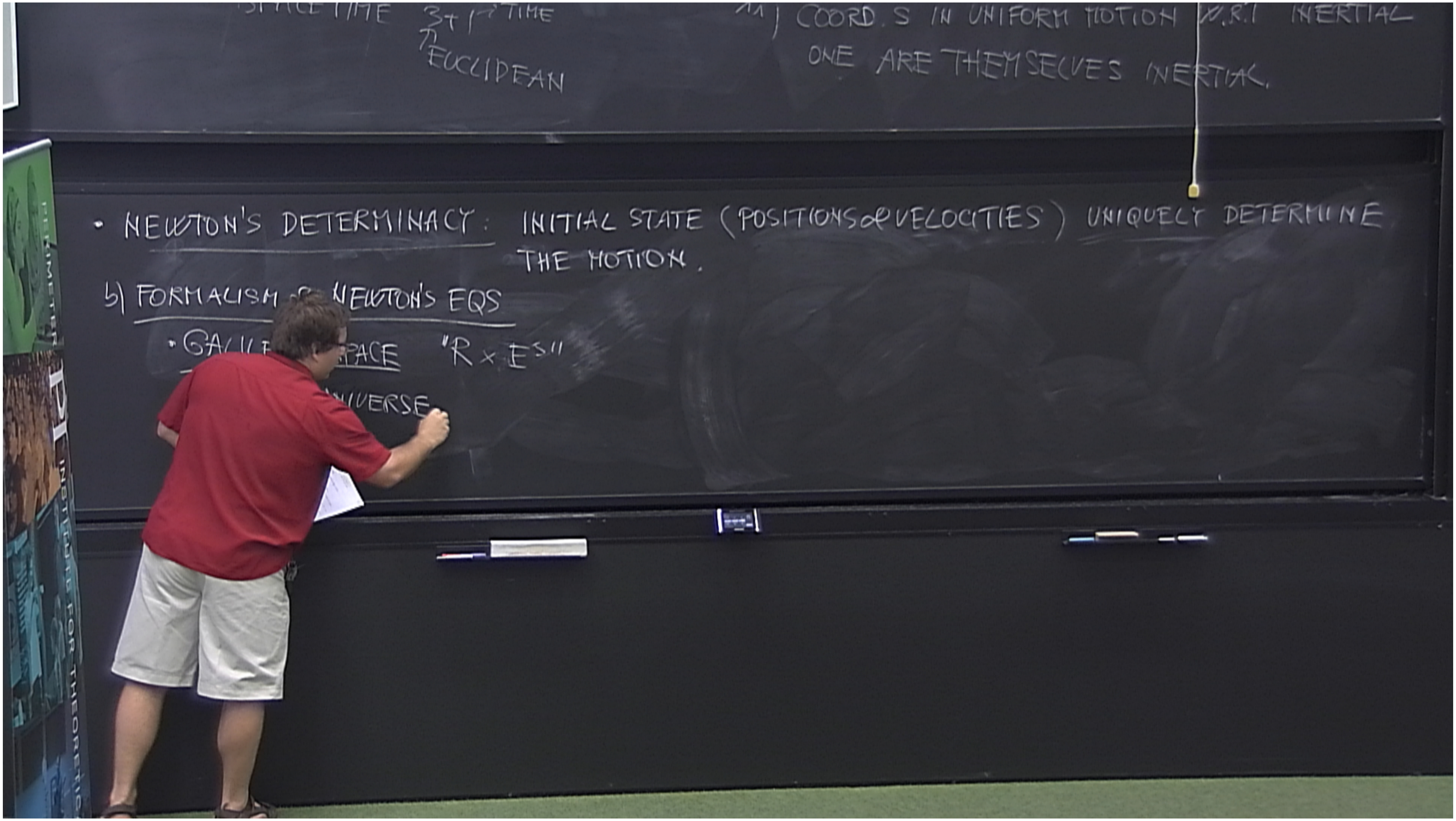

CLASSICAL MECHANICS
1) NEWTONIAN MECHANICS
α) EXPERIMENTAL FACTS
• SPACETIME 3+1 TIME
EUCLIDEAN
- GALILEO'S P. RELATIVITY
∃ INERTIAL COORD. SYSTEM
i) ALL LAWS OF NATURE AT ALL MOMENTS OF
TIME


CLASSICAL MECHANICS
1) NEWTONIAN MECHANICS
a) EXPERIMENTAL FACTS
SPACETIME 3+1 TIME
EUCLIDEAN
GALILEO'S P. RELATIVITY
∃ INERTIAL COORD. SYSTEM
i) ALL LAWS OF NATURE AT ALL MOMENTS OF
TIME ARE THE SAME.
COORD.S IN UNIFORM MOTION W.R.T INERTIAL

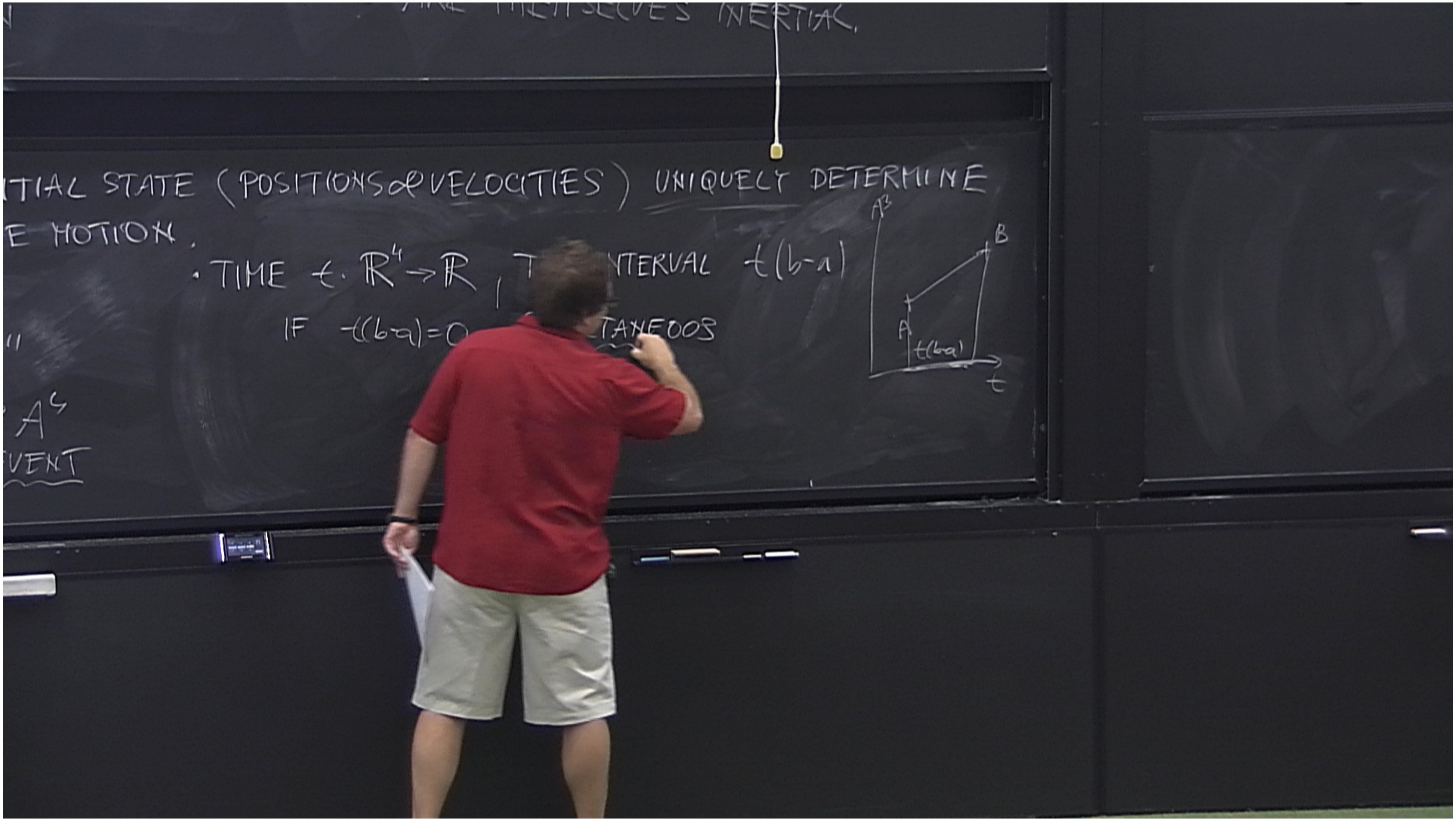

EUCLIDEAN
COORD.S IN UNIFORM MOTION W.R.T INERTIAL
ONE ARE THEMSELVES INERTIAL.
• NEWTON'S DETERMINACY:

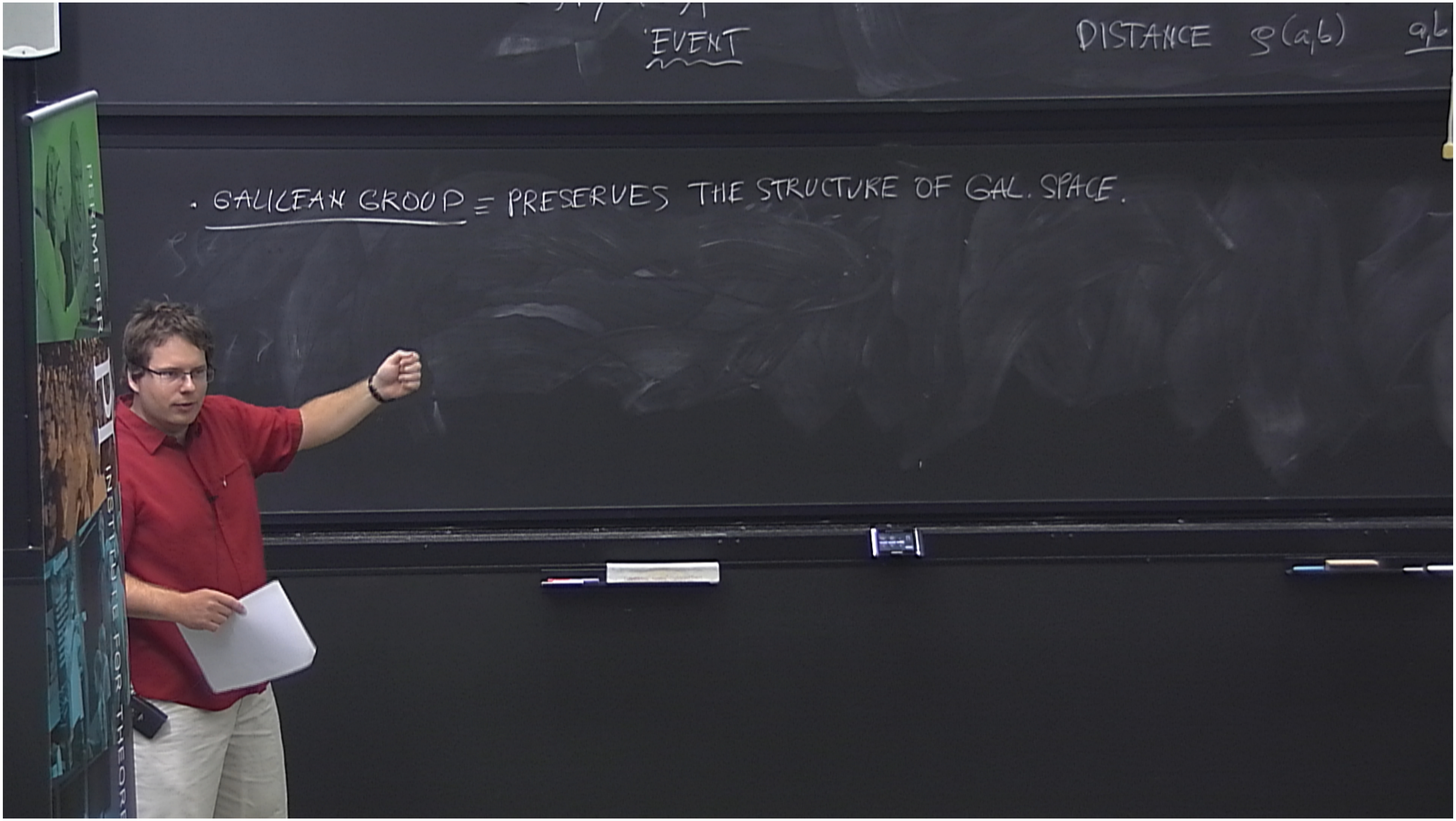

3+1 ← TIME
EUCLIDEAN
COORD.S IN UNIFORM MOTION W.R.T INERTIAL
ONE ARE THEMSELVES INERTIAL.
• NEWTON'S DETERMINACY: INITIAL STATE (POSITIONS & VELOCITIES) UNIQUELY DETERMINE
THE MOTION.

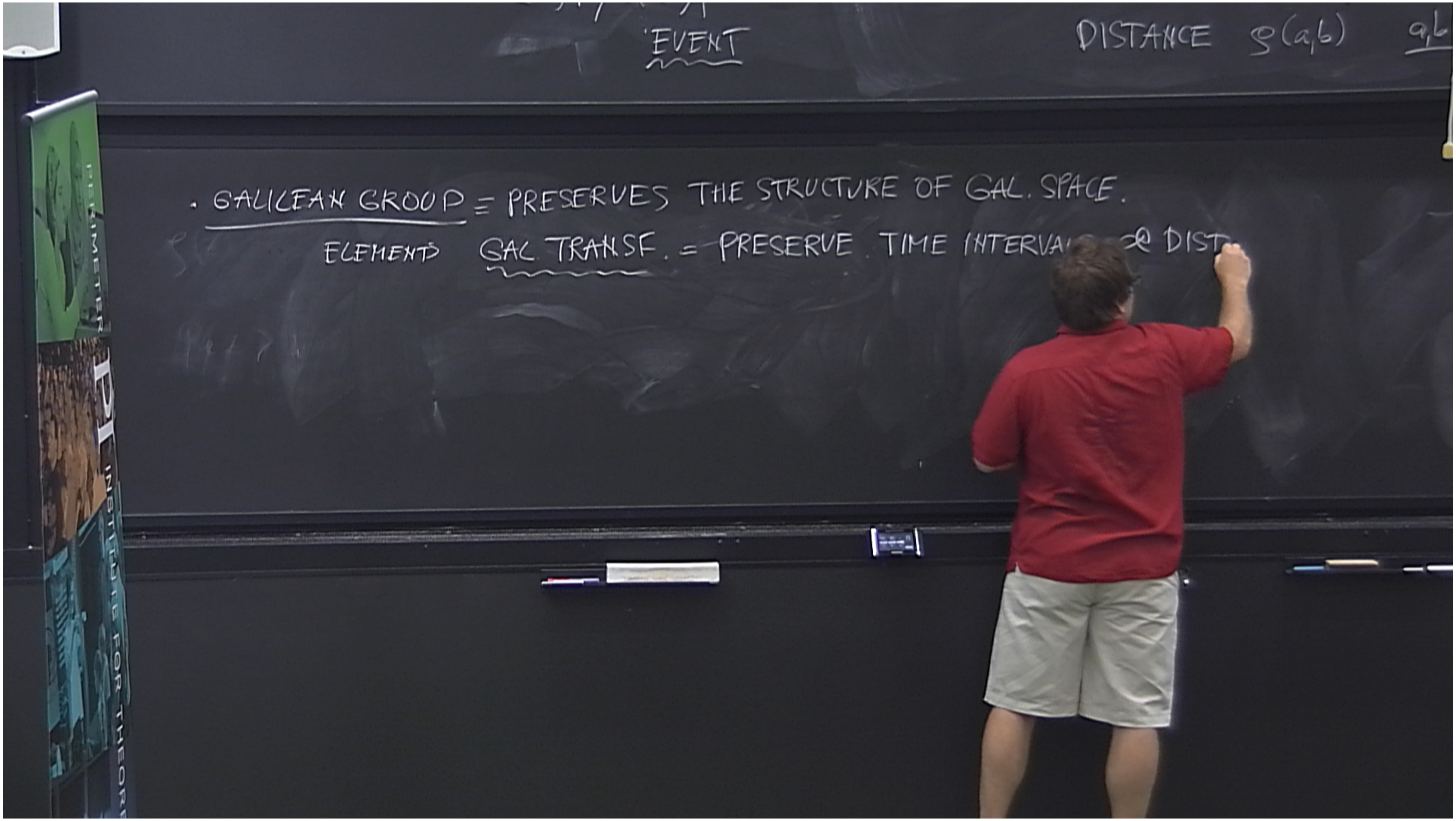

SPACETIME 3+1 TIME
EUCLIDEAN
iii) COORD.S IN UNIFORM MOTION W.R.T INERTIAL ONE ARE THEMSELVES INERTIAL.
• NEWTON'S DETERMINACY: INITIAL STATE (POSITIONS & VELOCITIES) UNIQUELY DETERMINE THE MOTION.
b) FORM & NEW

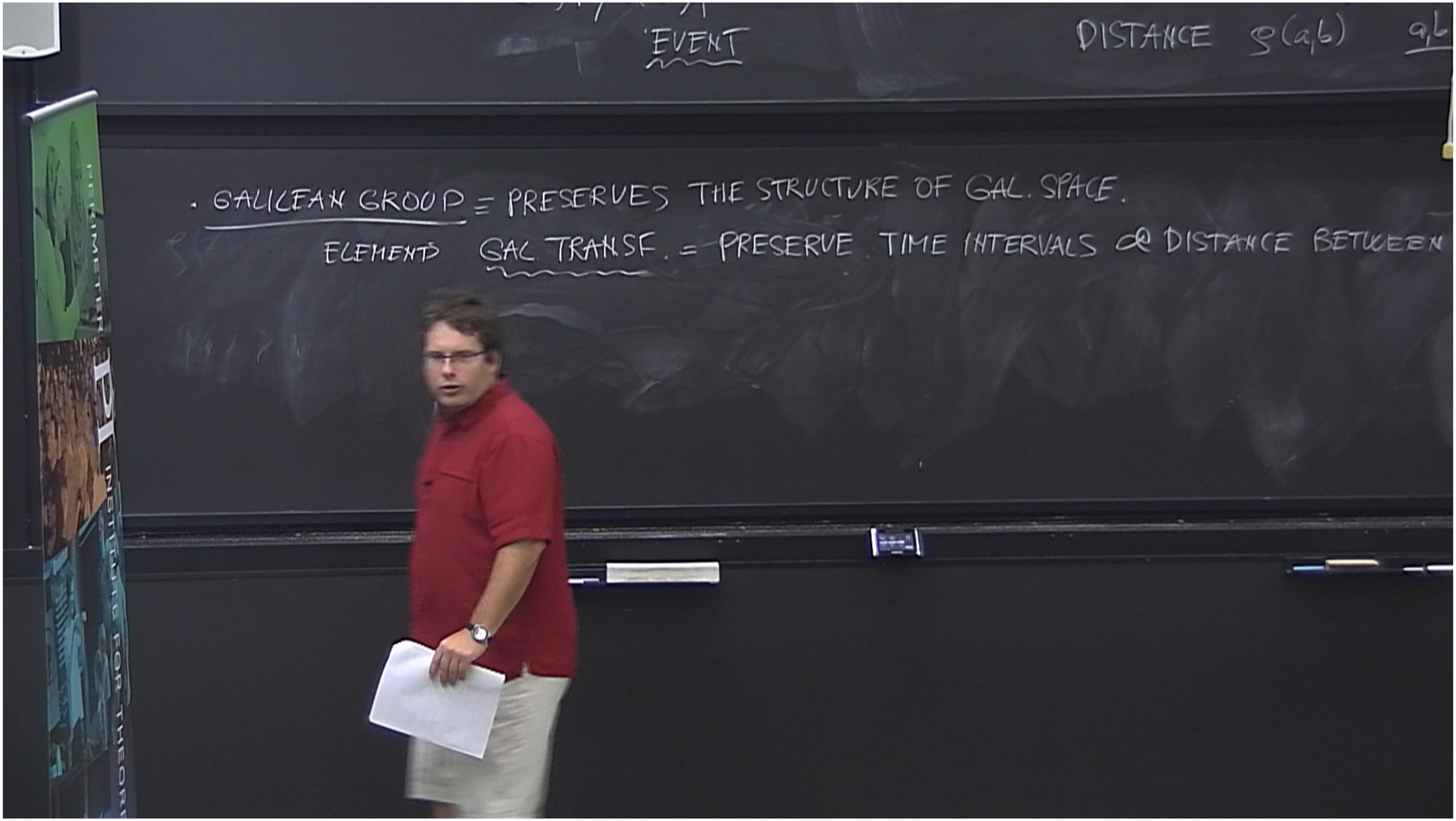


ii) COORD.S IN UNIFORM MOTION W.R.T INERTIAL ONE ARE THEMSELVES INERTIAL.
EUCLIDEAN
• NEWTON'S DETERMINACY: INITIAL STATE (POSITIONS & VELOCITIES) UNIQUELY DETERMINE THE MOTION.
b) FORMALISM OF NEWTON'S EQS
"R × E³"

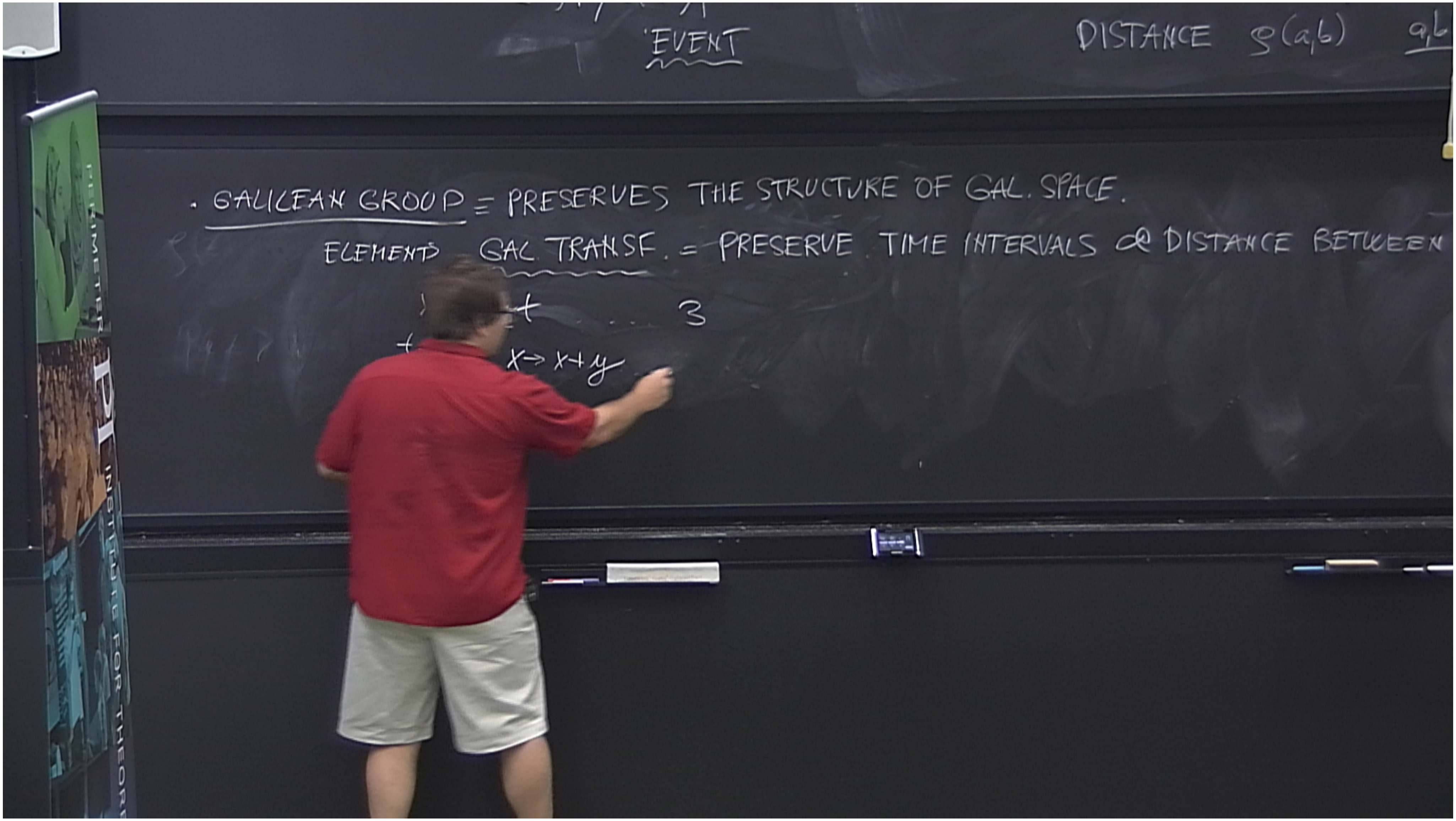

ARE THEMSELVES INERTIAL.
TIAL STATE (POSITIONS & VELOCITIES) UNIQUELY DETERMINE
E MOTION.
· TIME t: R⁴ → R
NTERVAL t(b-a)
IF t(b-a)=0
TANEOUS
A⁴
VENT
A³
B
A
t(b-a)
t


EVENT
DISTANCE ρ(a,b)
• GALILEAN GROUP = PRESERVES THE STRUCTURE OF GAL. SPACE.

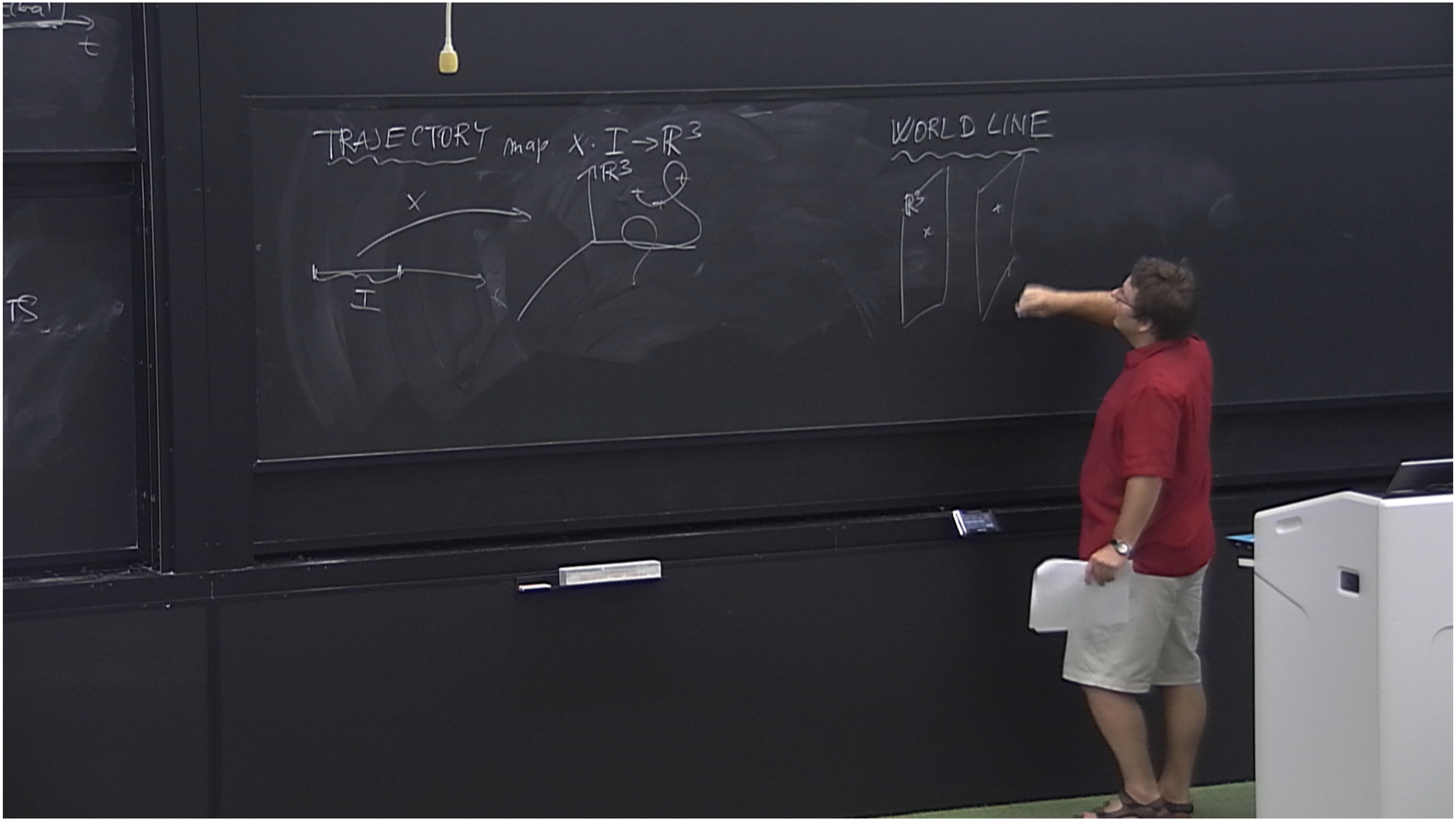

EVENT
DISTANCE ρ(a,b)
· GALILEAN GROUP ≡ PRESERVES THE STRUCTURE OF GAL. SPACE.
ELEMENTS GAL. TRANSF. = PRESERVE TIME INTERVA
& DIST
PI
INSTITUTE FOR

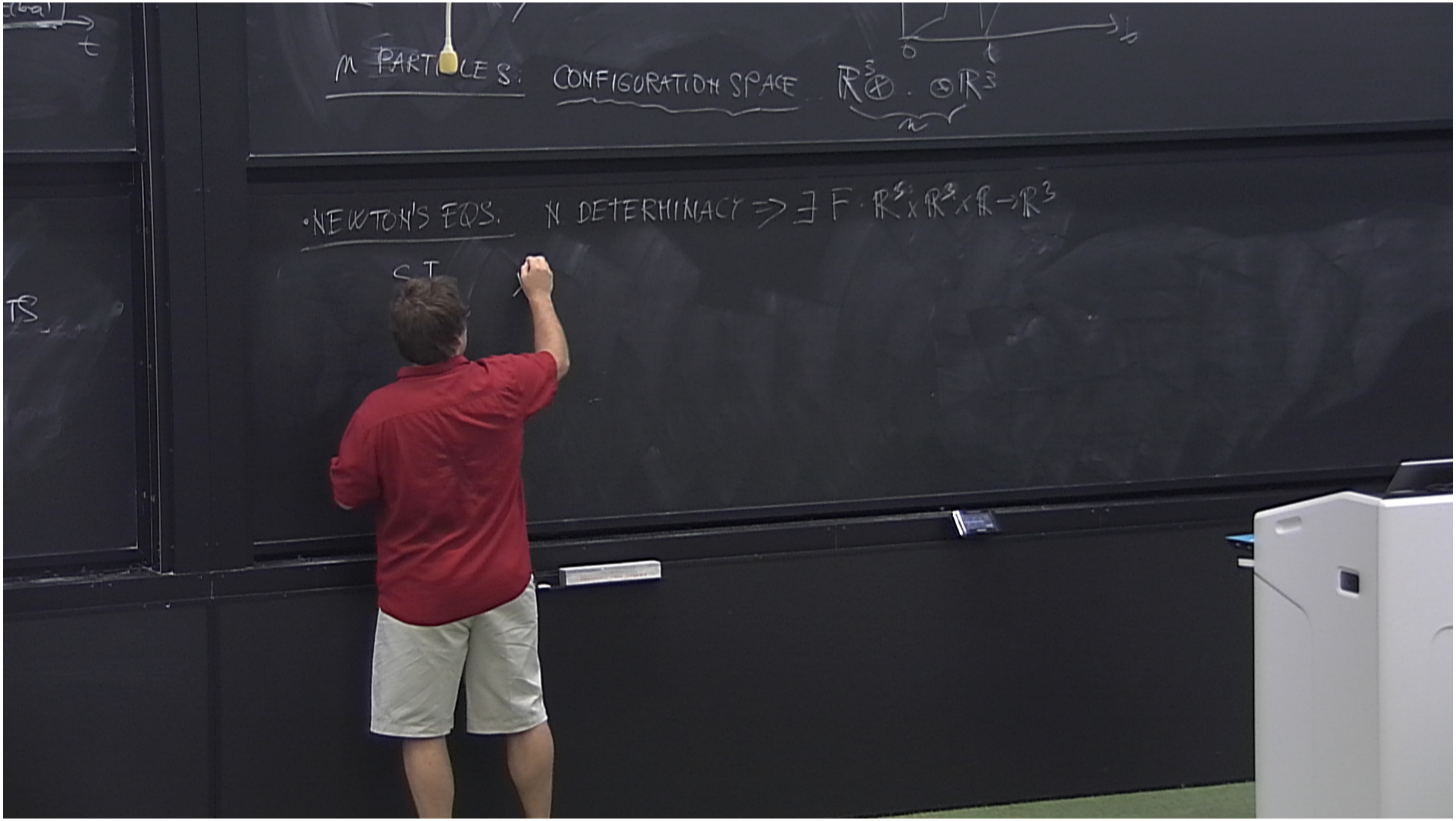


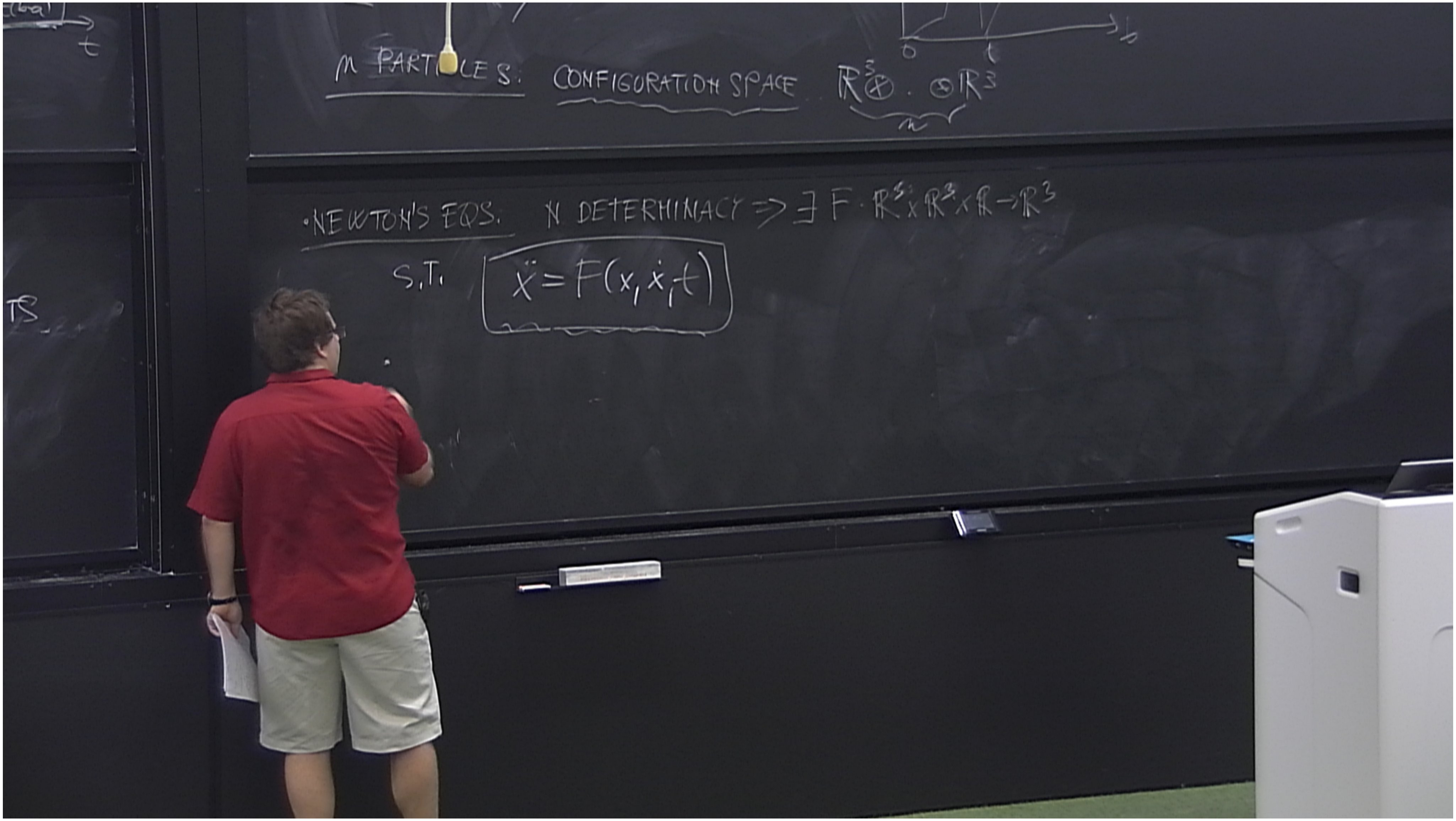
EVENT
DISTANCE ϱ(a,b)
· GALILEAN GROUP = PRESERVES THE STRUCTURE OF GAL. SPACE.
ELEMENTS GAL. TRANSF. = PRESERVE TIME INTERVALS & DISTANCE BETWEEN

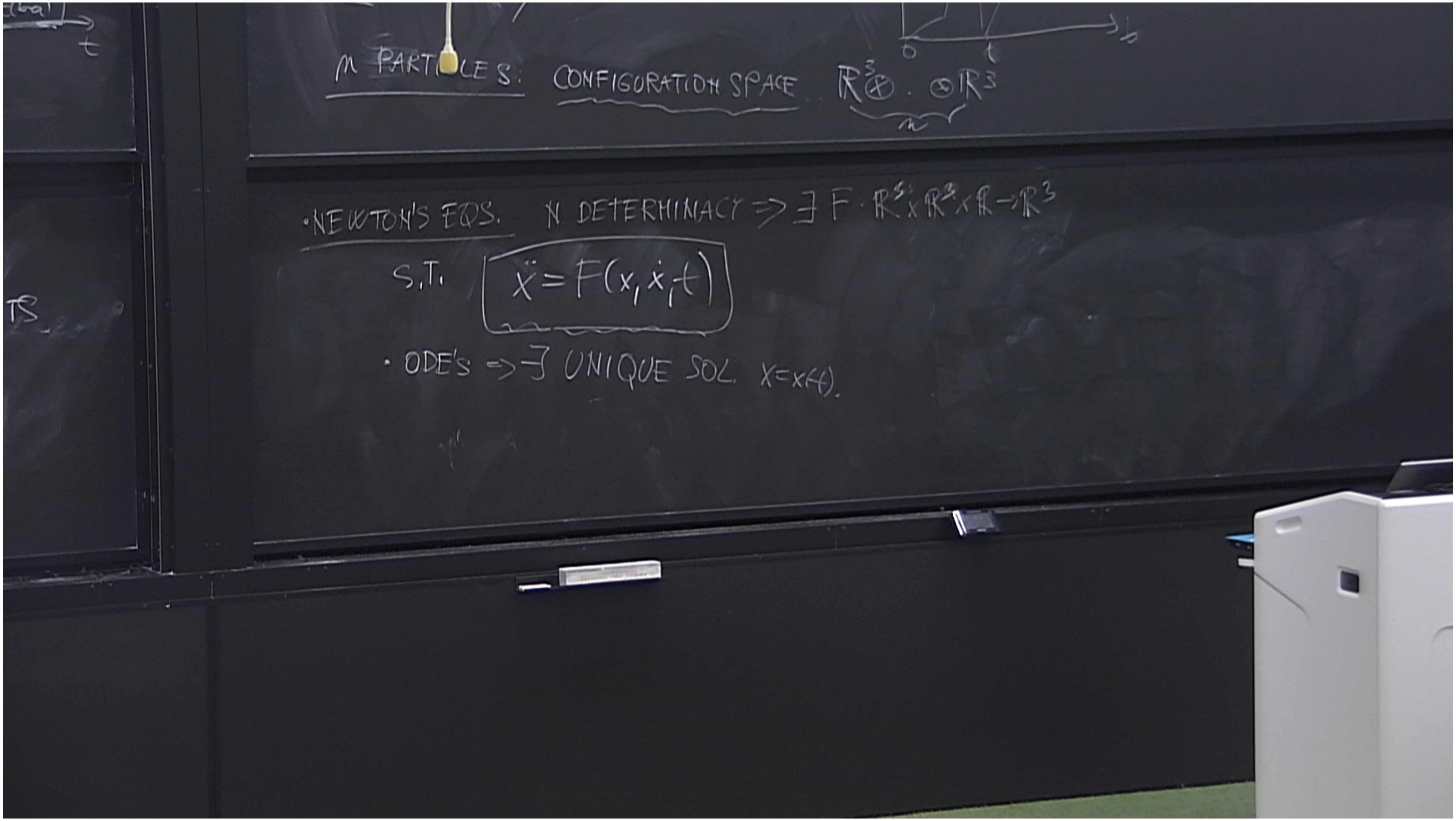


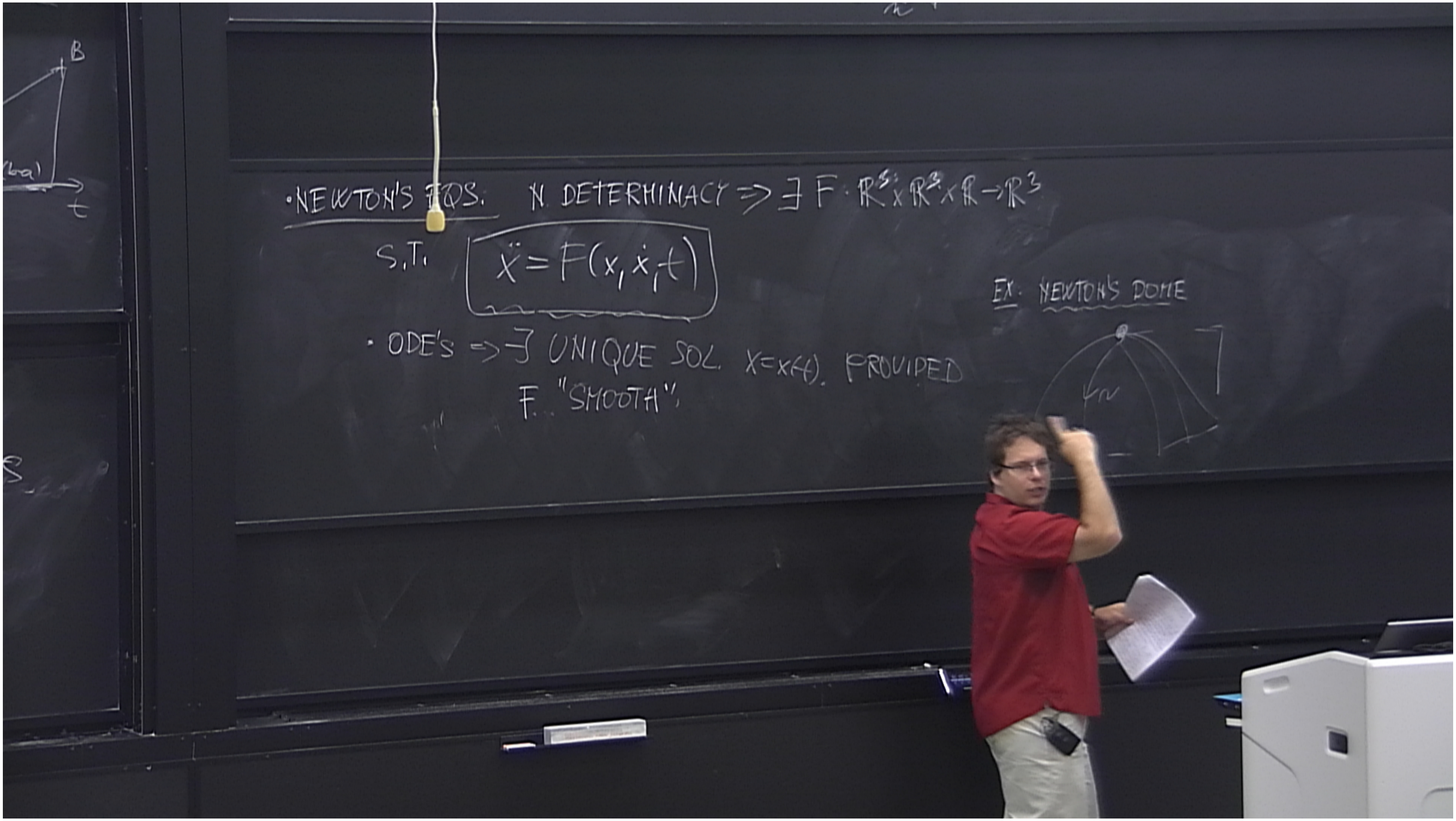
EVENT
DISTANCE ϱ(a,b)
. GALILEAN GROUP = PRESERVES THE STRUCTURE OF GAL. SPACE.
ELEMENTS GAL. TRANSF. = PRESERVE TIME INTERVALS & DISTANCE BETWEEN
3
x → x + y

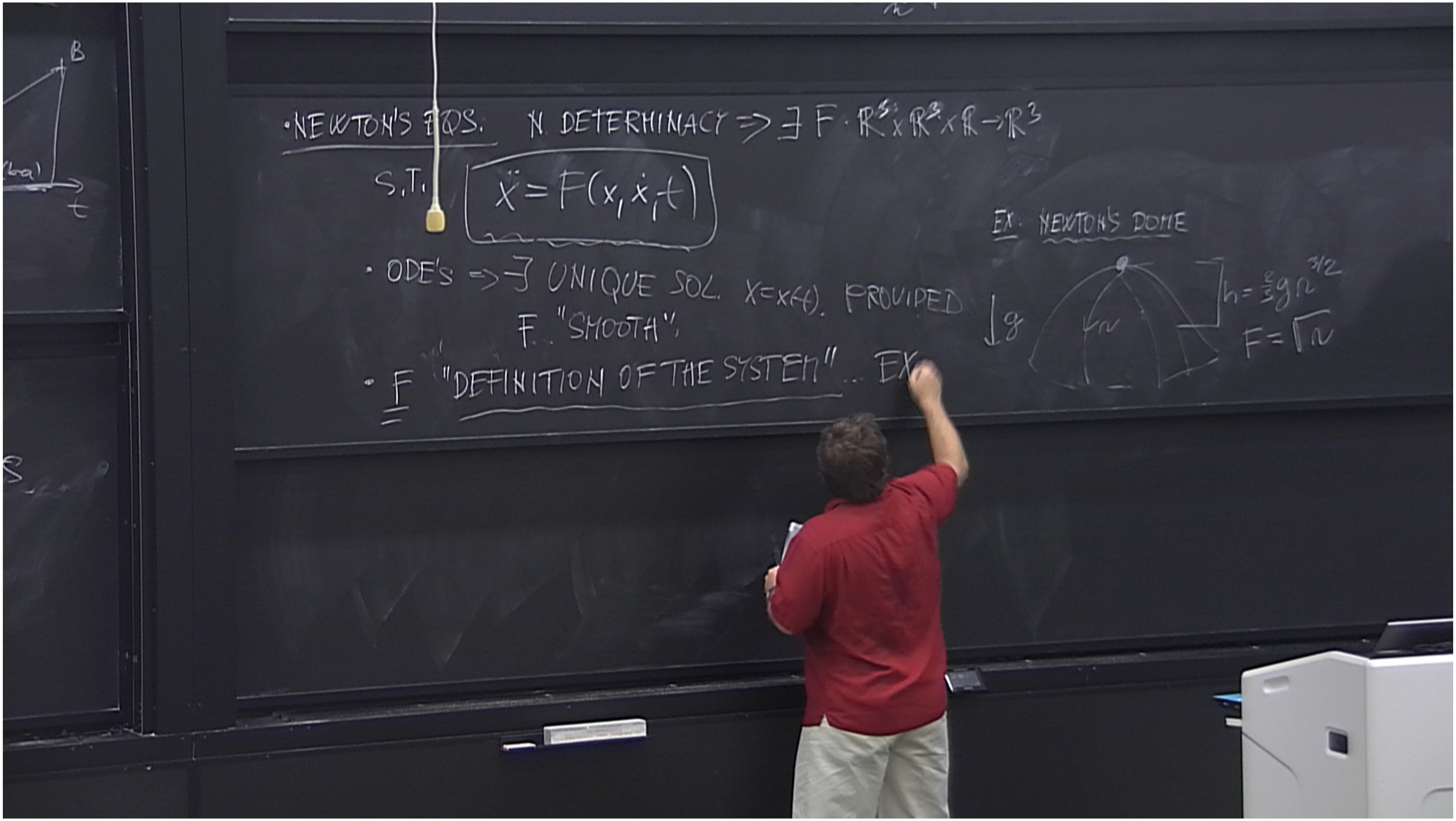

TRAJECTORY map X : I → R^3
X
I
R^3
WORLD LINE
R^3


n PARTICLES:
CONFIGURATION SPACE
•NEWTON'S EQS.
N DETERMINACY

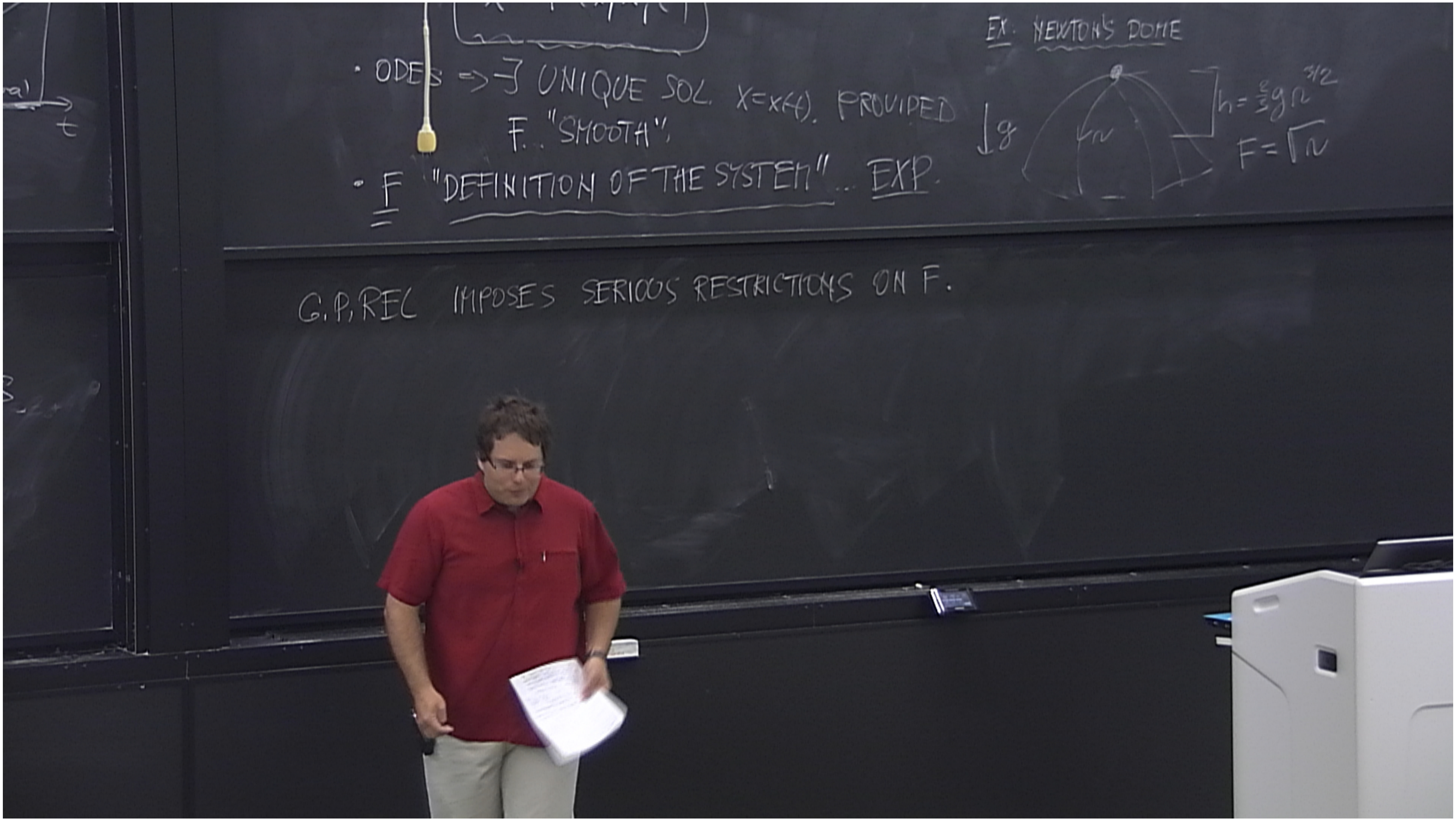

n PARTI LES: CONFIGURATION SPACE $\mathbb{R}^3 \otimes \cdots \otimes \mathbb{R}^3$
n
• NEWTON'S EQS. N. DETERMINACY $\Rightarrow \exists F: \mathbb{R}^{3} \times \mathbb{R}^{3} \times \mathbb{R} \to \mathbb{R}^3$
S.T. $\ddot{x} = F(x, \dot{x}, t)$

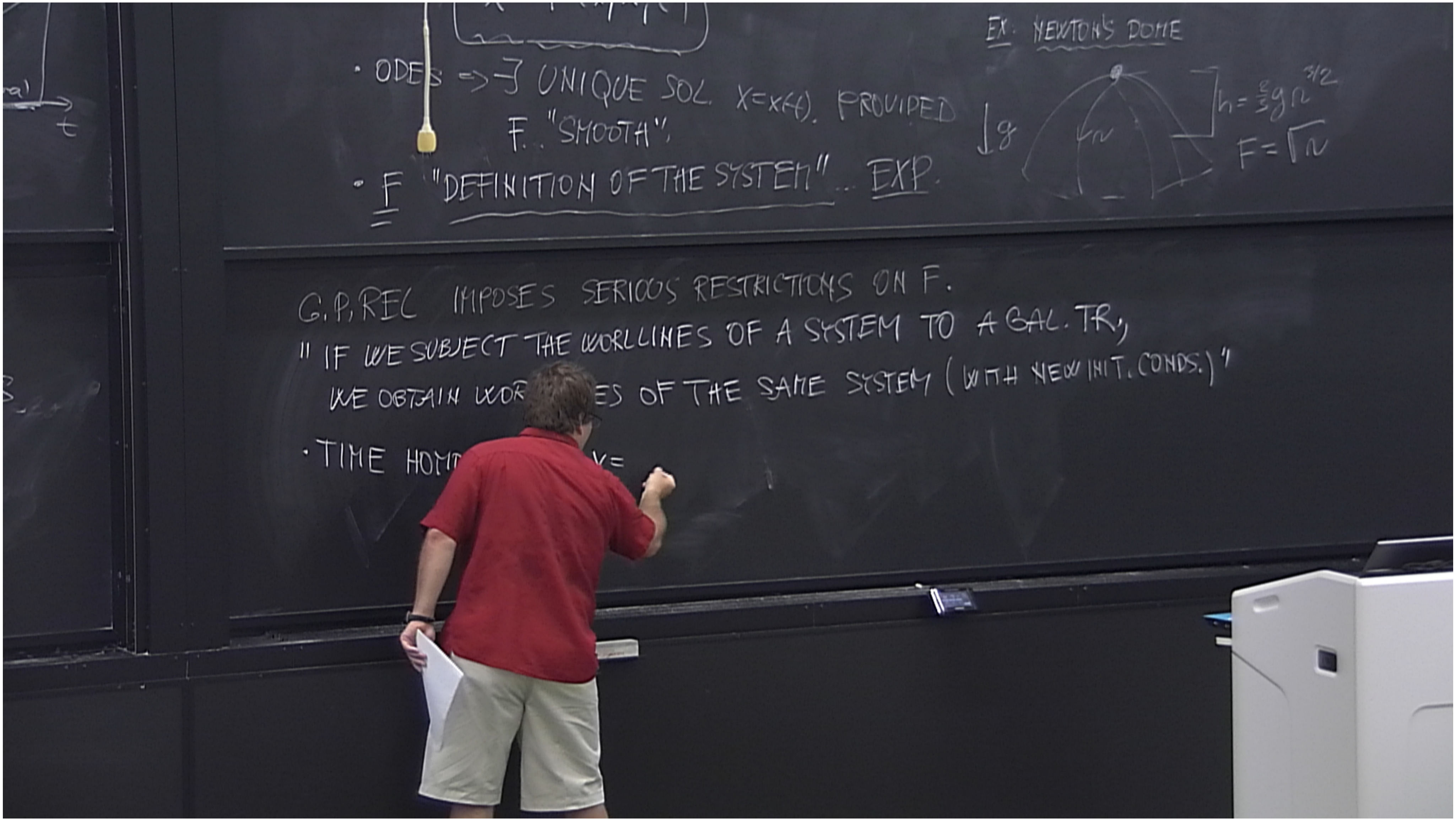

n PARTICLES: CONFIGURATION SPACE ℝ³⊗ . ⊗ℝ³
n
•NEWTON'S EQS. N. DETERMINACY ⇒ ∃ F : ℝ³ⁿ x ℝ³ⁿ x ℝ → ℝ³
S.T. ẍ = F(x, ẋ, t)
• ODE's ⇒ ∃ UNIQUE SOL. x = x(t).

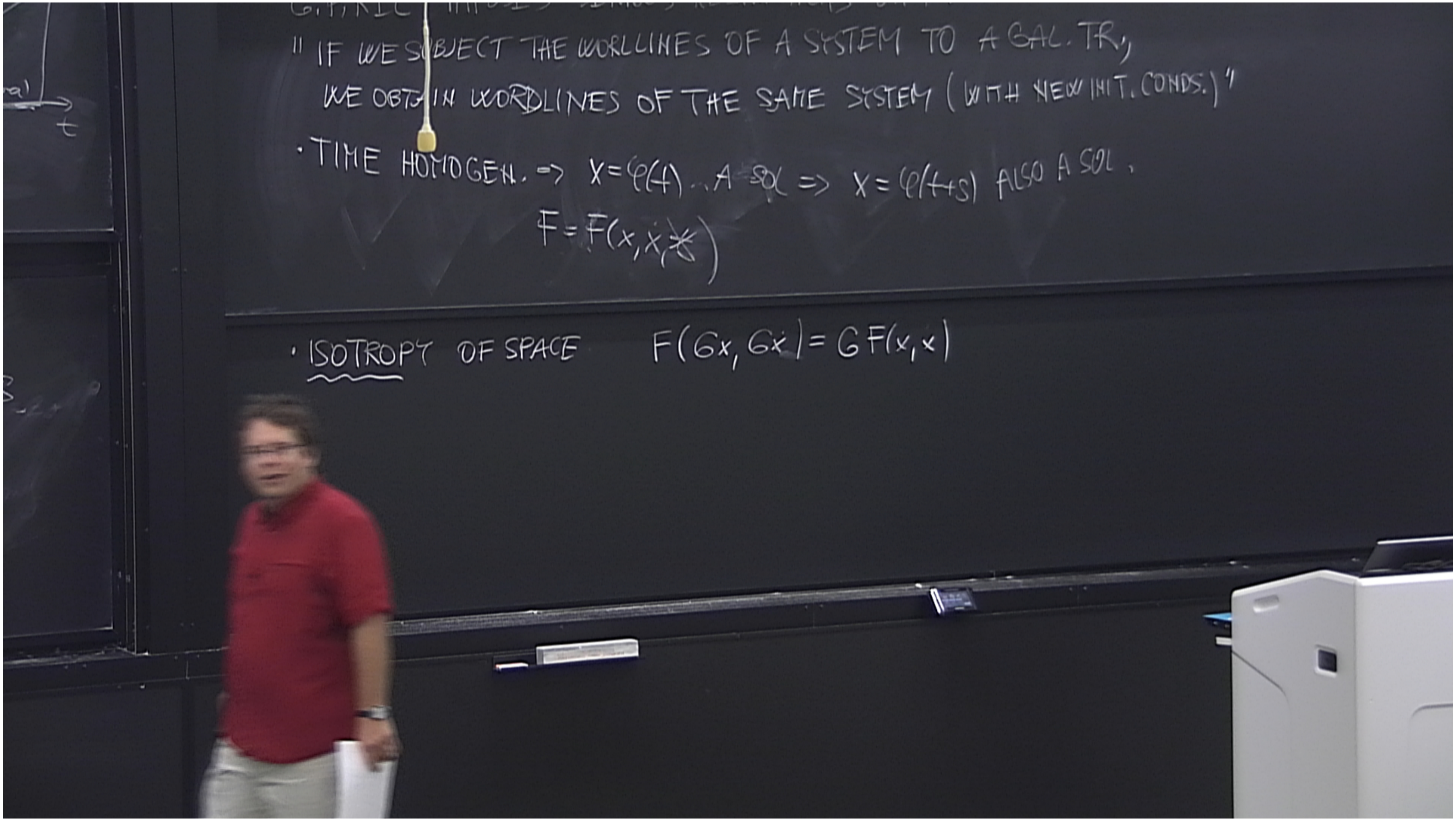

•NEWTON'S EQS: N. DETERMINACY => ∃ F : R³ x R³ x R → R³
S.T. ẍ = F(x, ẋ, t)
• ODE's => ∃ UNIQUE SOL. x=x(t), PROVIDED
F "SMOOTH".
EX: NEWTON'S DOME

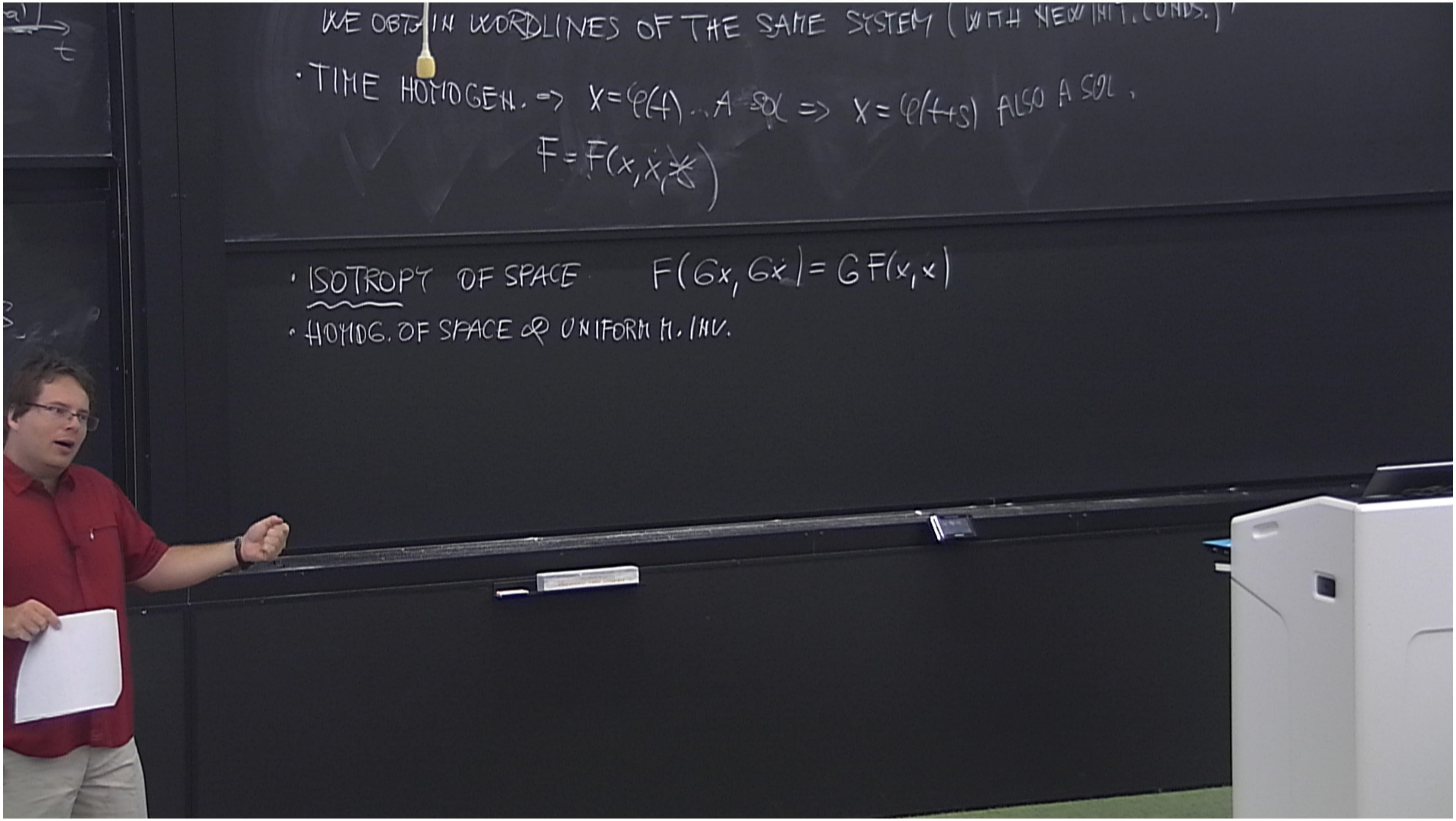


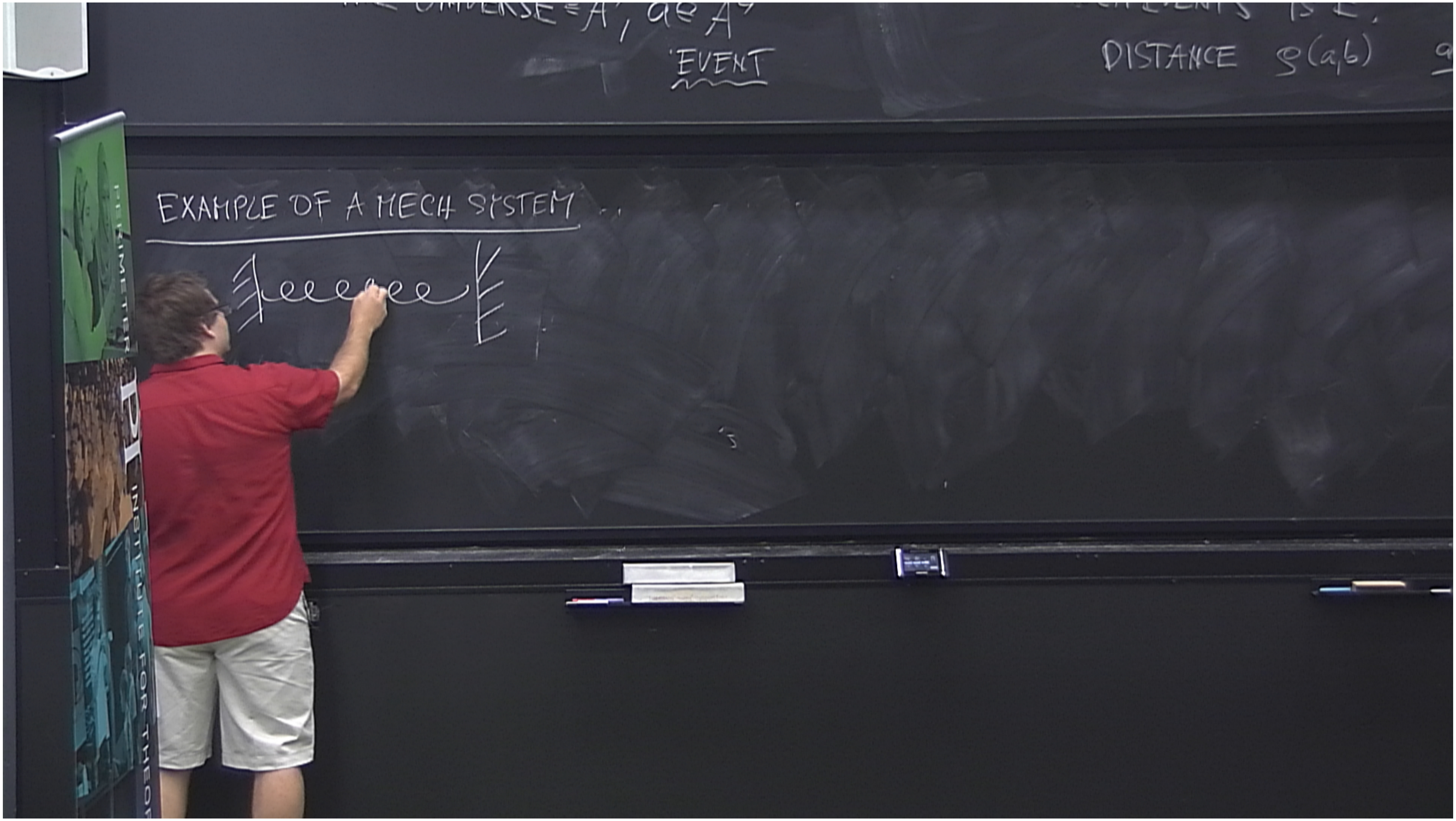
• NEWTON'S EQS. N. DETERMINACY ⇒ ∃ F : R³ × R³ × R → R³
S.T. ẍ = F(x, ẋ, t)
• ODE'S ⇒ ∃ UNIQUE SOL. X=x(t), PROVIDED
F "SMOOTH",
• F "DEFINITION OF THE SYSTEM"... EX
EX: NEWTON'S DOME
g
h = (2/3) g r^{3/2}
F = √r


• ODEs ⇒ ∃ UNIQUE SOL. X=x(t). PROVIDED
F.. "SMOOTH".
• F "DEFINITION OF THE SYSTEM"... EXP.
EX. NEWTON'S DOME
G.P.REC IMPOSES SERIOUS RESTRICTIONS ON F.

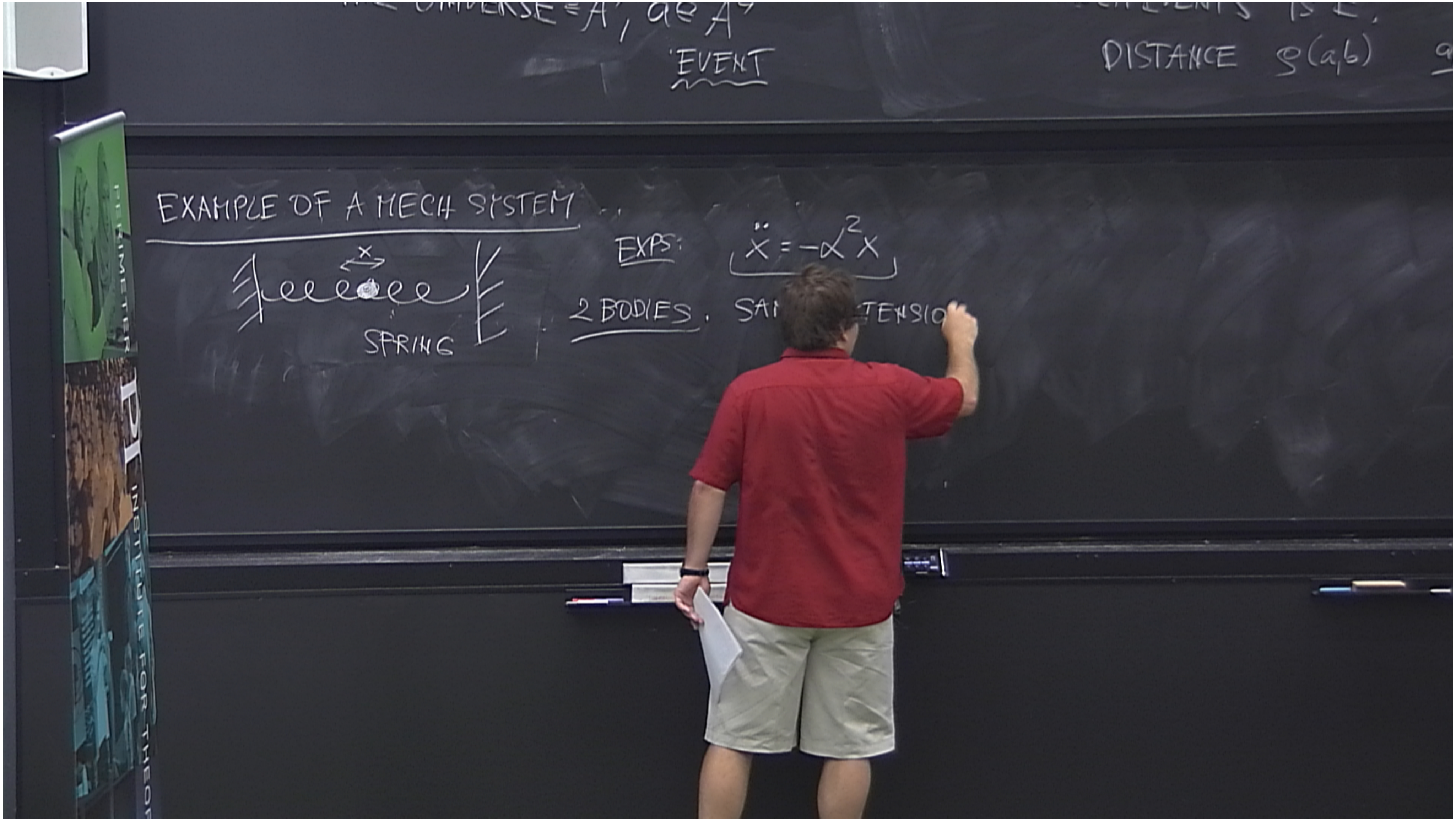

• ODEs => ∃ UNIQUE SOL. X=X(t), PROVIDED
F.. "SMOOTH".
• F "DEFINITION OF THE SYSTEM".. EXP.
EX. NEWTON'S DOME
h = 2/3 g r^3/2
F = √r
G.P.REL IMPOSES SERIOUS RESTRICTIONS ON F.
" IF WE SUBJECT THE WORLDLINES OF A SYSTEM TO A GAL. TR.,
WE OBTAIN WOR ES OF THE SAME SYSTEM (WITH NEW INIT. CONDS.)"
• TIME HOM

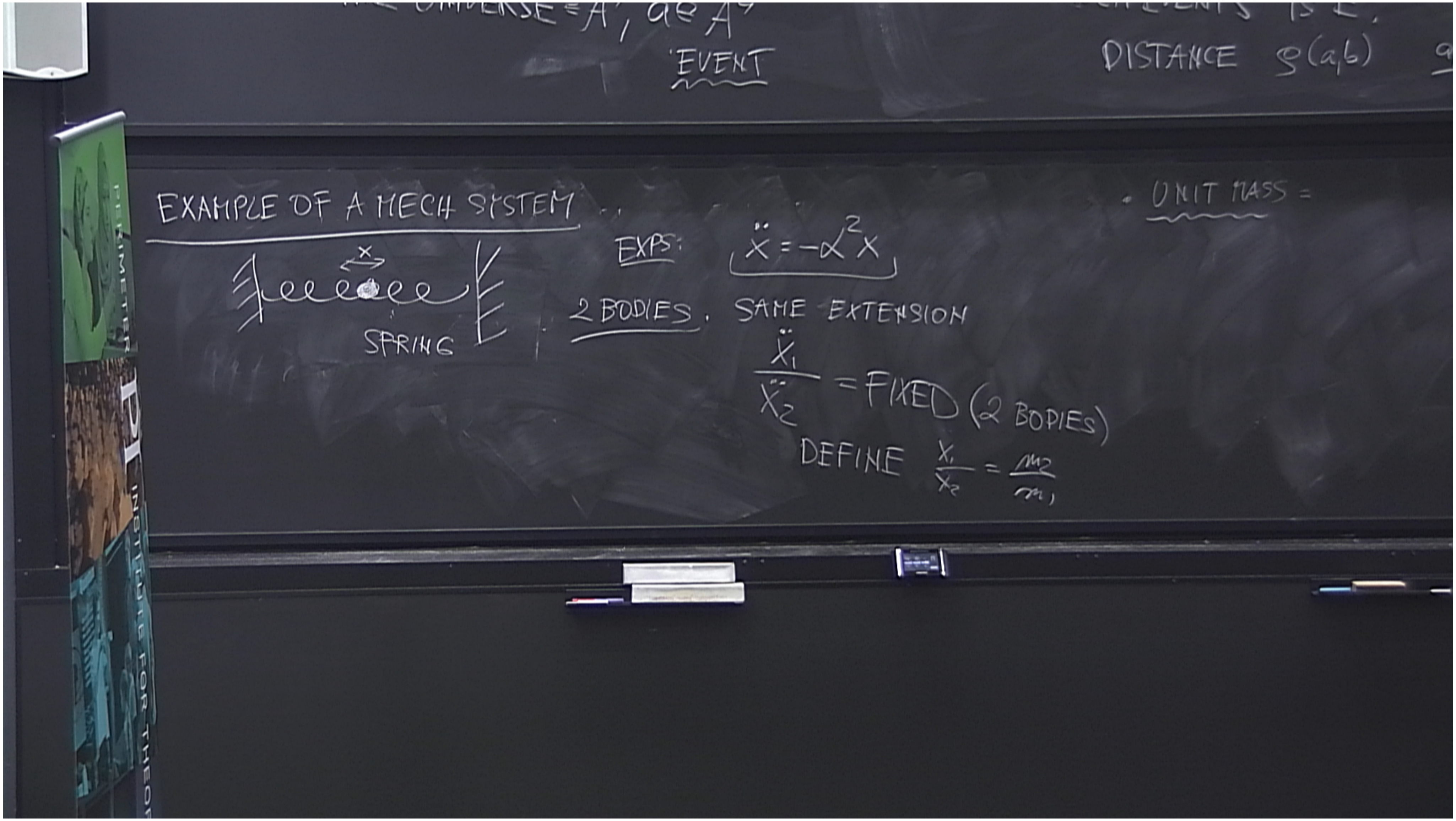

"IF WE SUBJECT THE WORLLINES OF A SYSTEM TO A GAL. TR.,
WE OBTAIN WORDLINES OF THE SAME SYSTEM (WITH NEW INIT. CONDS.)"
• TIME HOMOGEN. => x = φ(t) .. A SOL => x = φ(t+s) ALSO A SOL.
F = F(x, ẋ, ~~t~~)
• ISOTROPY OF SPACE F(Gx, Gẋ) = GF(x, ẋ)

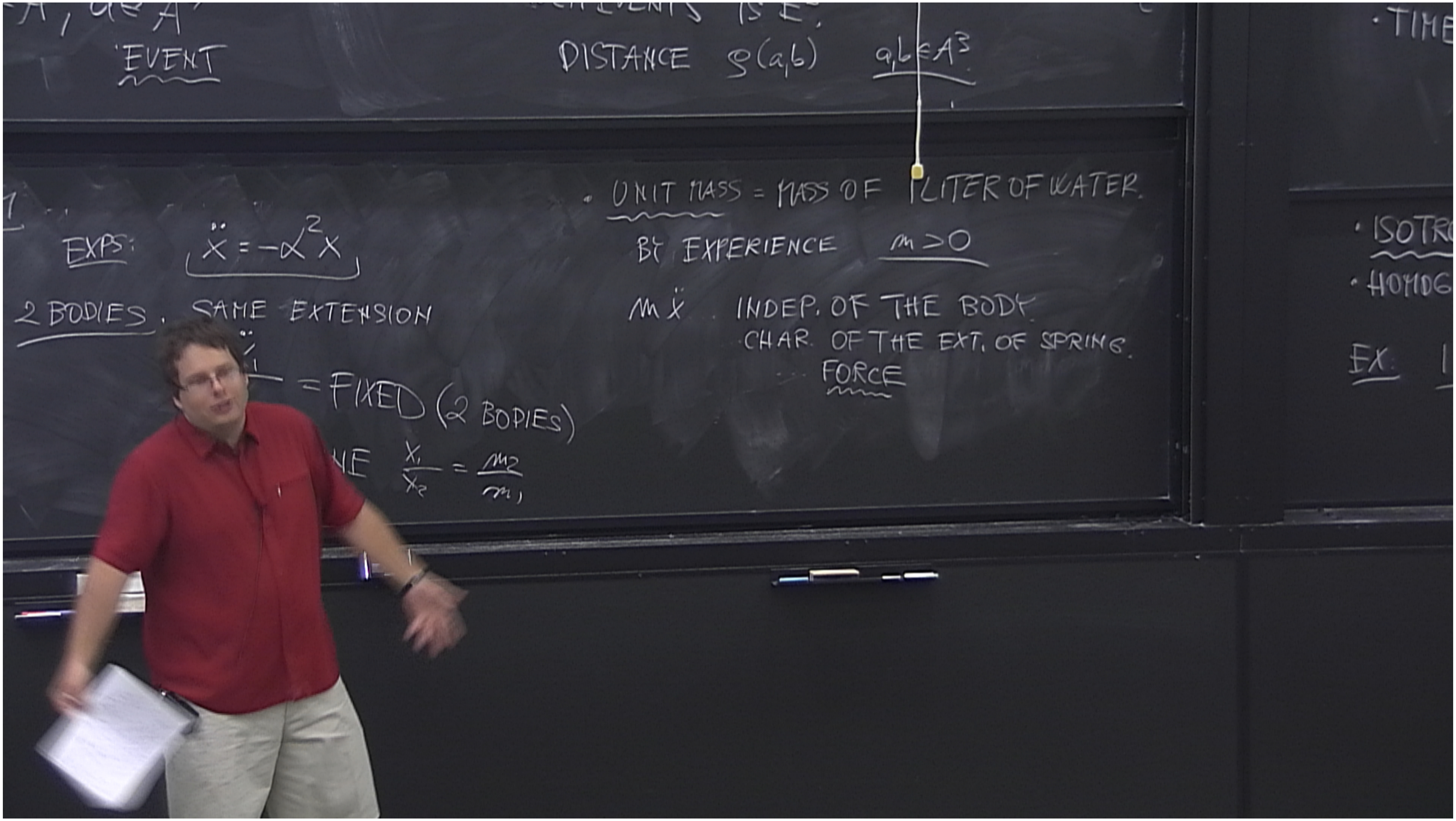

WE OBTAIN WORLDLINES OF THE SAME SYSTEM (WITH NEW INIT. CONDS.)
· TIME HOMOGEN. => X = φ(t) .. A SOL => X = φ(t+s) ALSO A SOL.
F = F(x, ẋ, ✱)
· ISOTROPY OF SPACE F(Gx, Gẋ) = G F(x, ẋ)
· HOMOG. OF SPACE & UNIFORM M. INV.

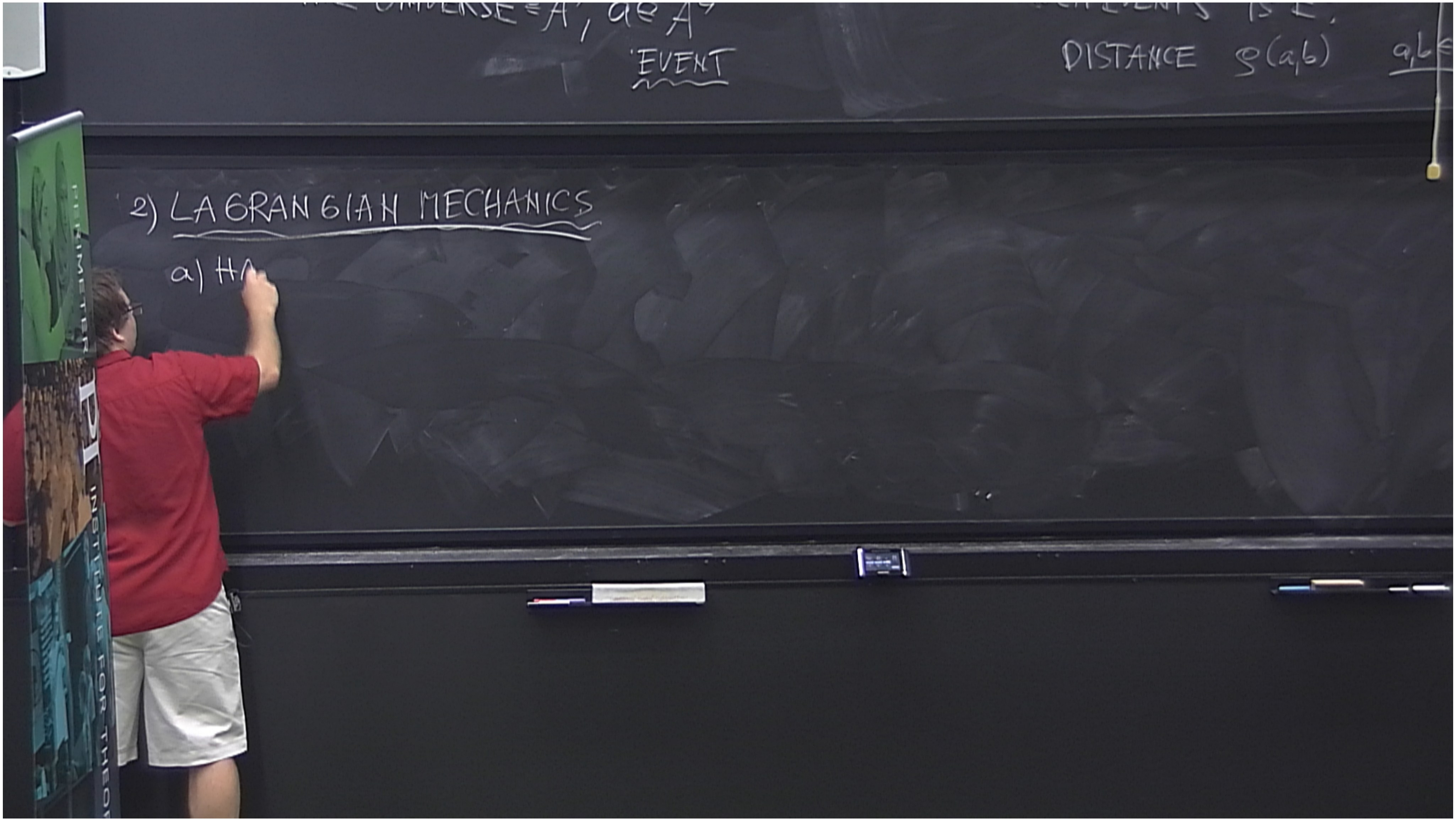


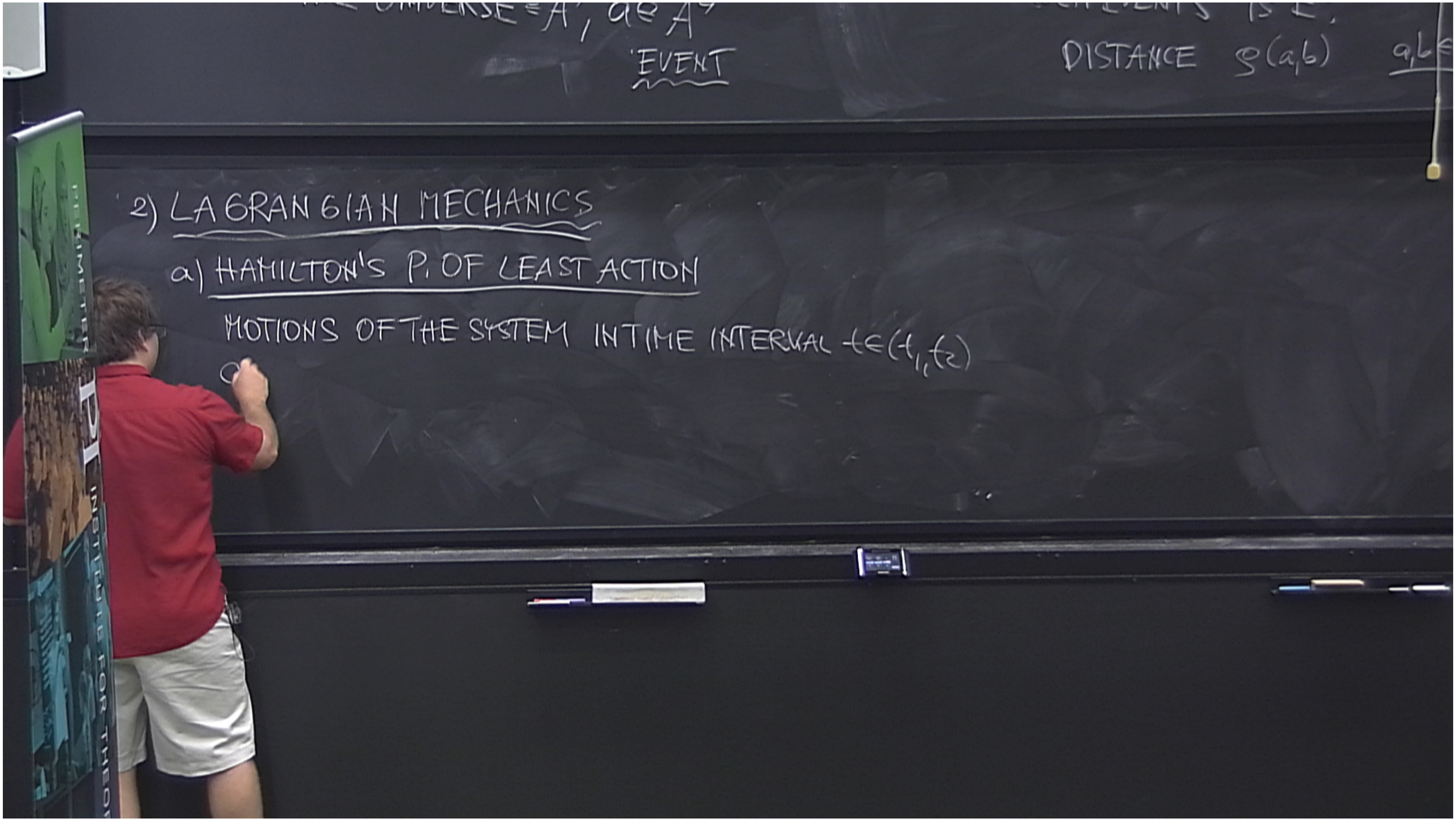
EVENT
DISTANCE ρ(a,b)
EXAMPLE OF A MECH SYSTEM
PERIMETER
PI
INSTITUTE FOR THEOR

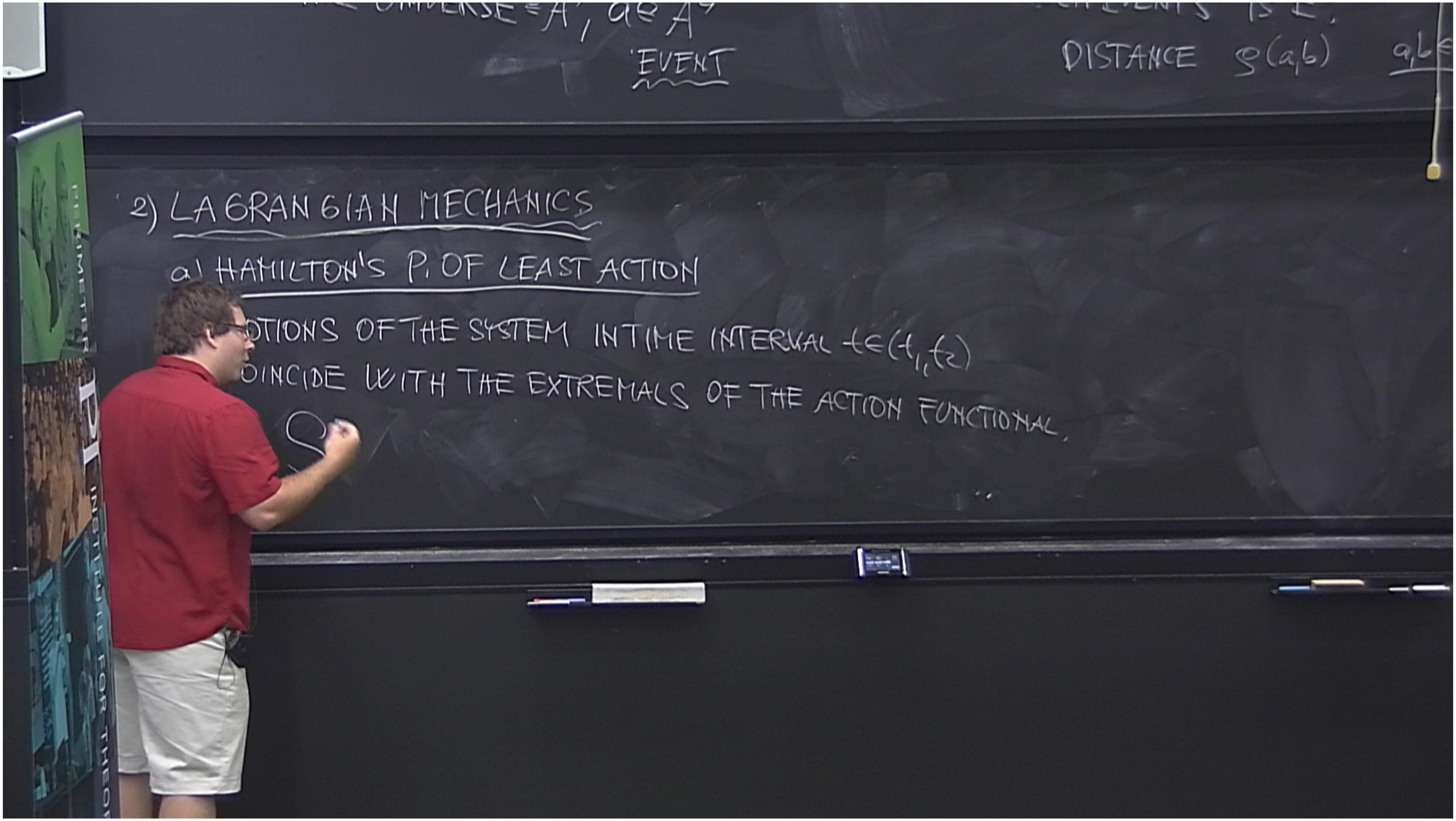

EVENT
DISTANCE ϱ(a,b)
EXAMPLE OF A MECH SYSTEM
x
SPRING
EXPS:
ẍ = −α²x
2 BODIES.

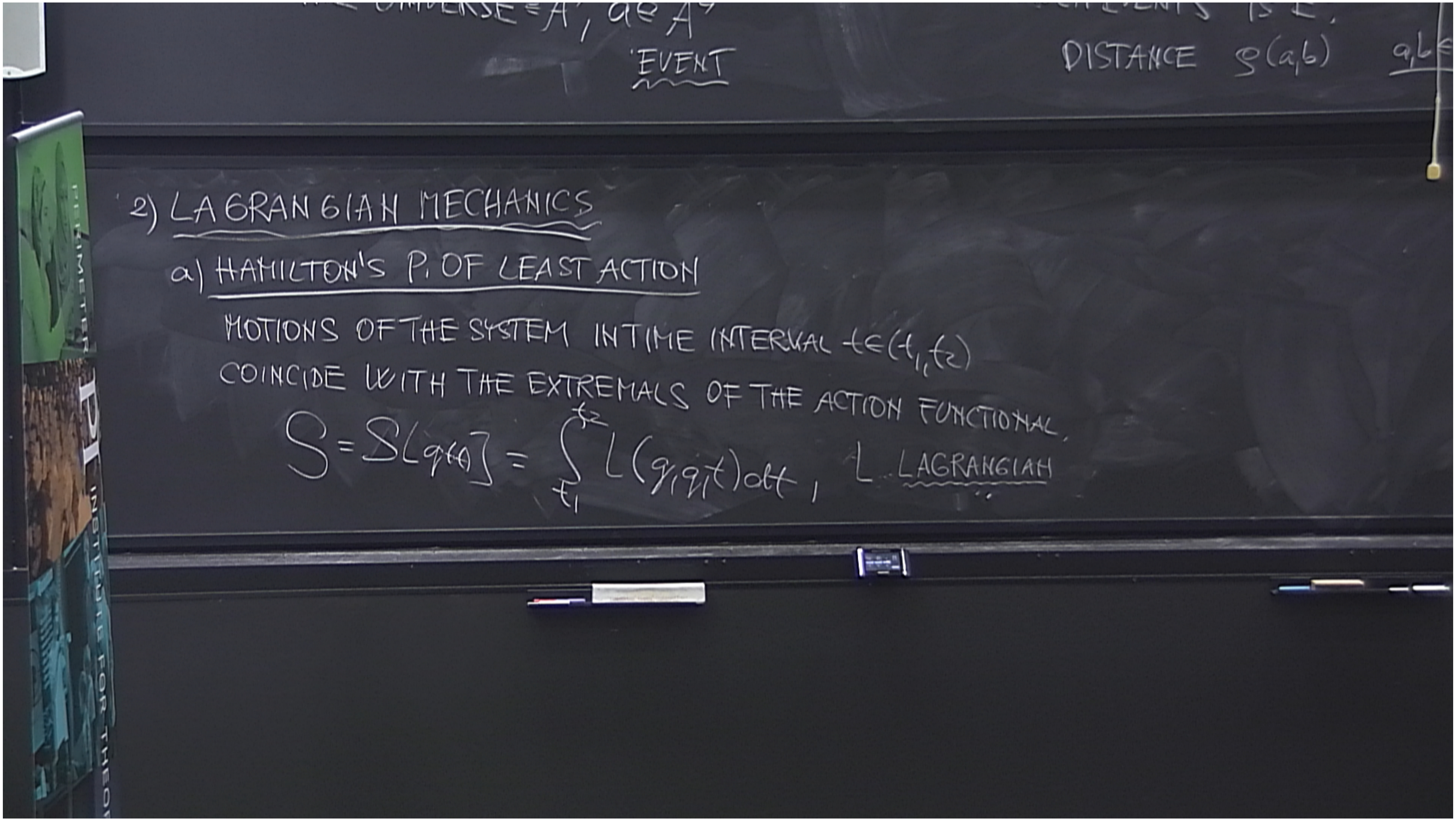

EVENT
DISTANCE ϱ(a,b)
EXAMPLE OF A MECH SYSTEM
x
SPRING
UNIT MASS =
EXPS: $\ddot{x} = -\alpha^2 x$
2 BODIES. SAME EXTENSION
$\frac{\ddot{X}_1}{\ddot{X}_2}$ = FIXED (2 BODIES)
DEFINE $\frac{X_1}{X_2} = \frac{m_2}{m_1}$
PERIMETER
PI
INSTITUTE FOR THEO

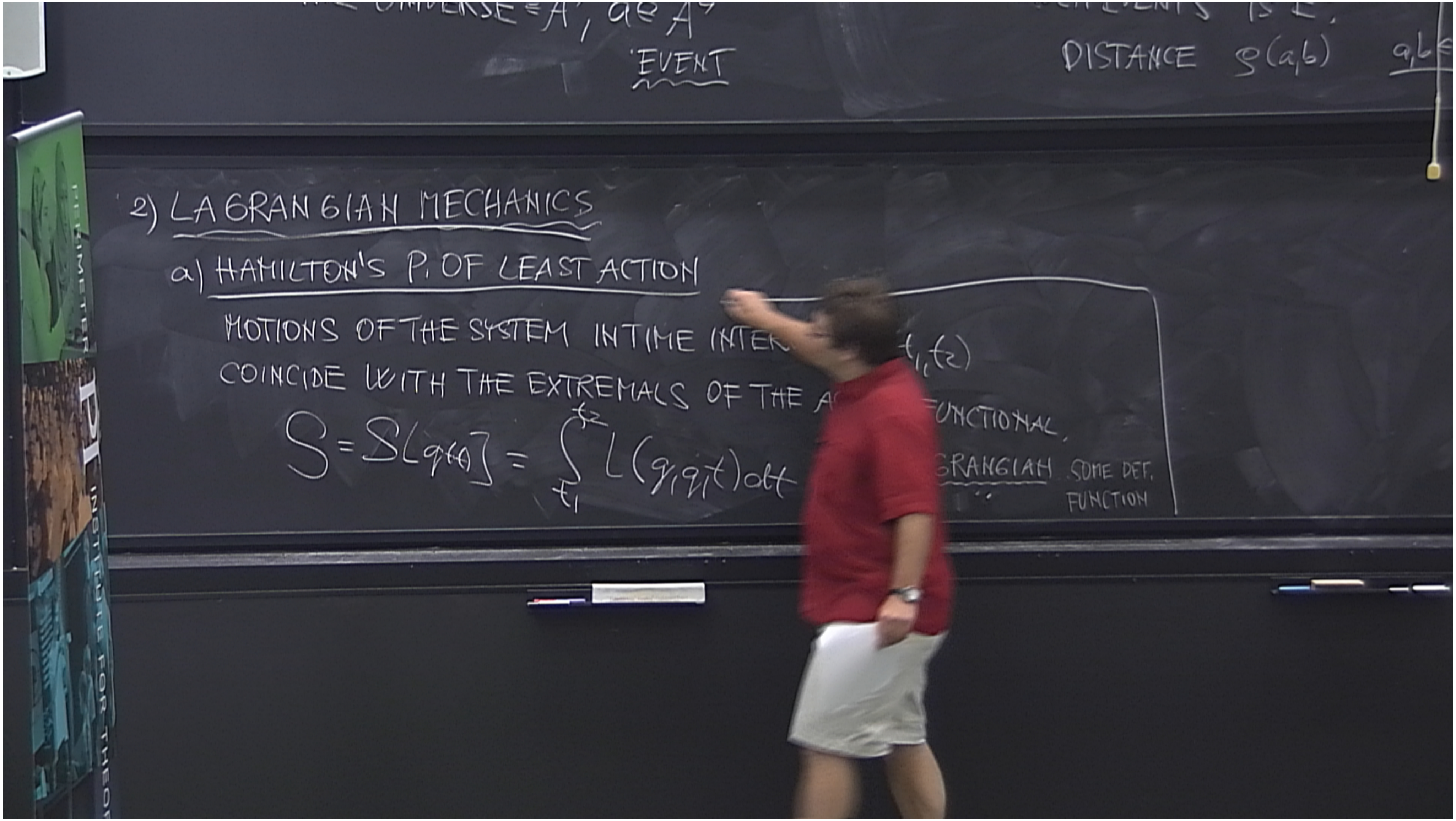

EVENT
DISTANCE ρ(a,b) a,b∈A³
UNIT MASS = MASS OF 1LITER OF WATER.
BY EXPERIENCE m>0
m ẍ INDEP. OF THE BODY
CHAR. OF THE EXT. OF SPRING
FORCE
EXPS: ẍ = −α²x
2 BODIES. SAME EXTENSION
= FIXED (2 BODIES)
x₁/x₂ = m₂/m₁
TIME
ISOTR
HOMOG
EX:

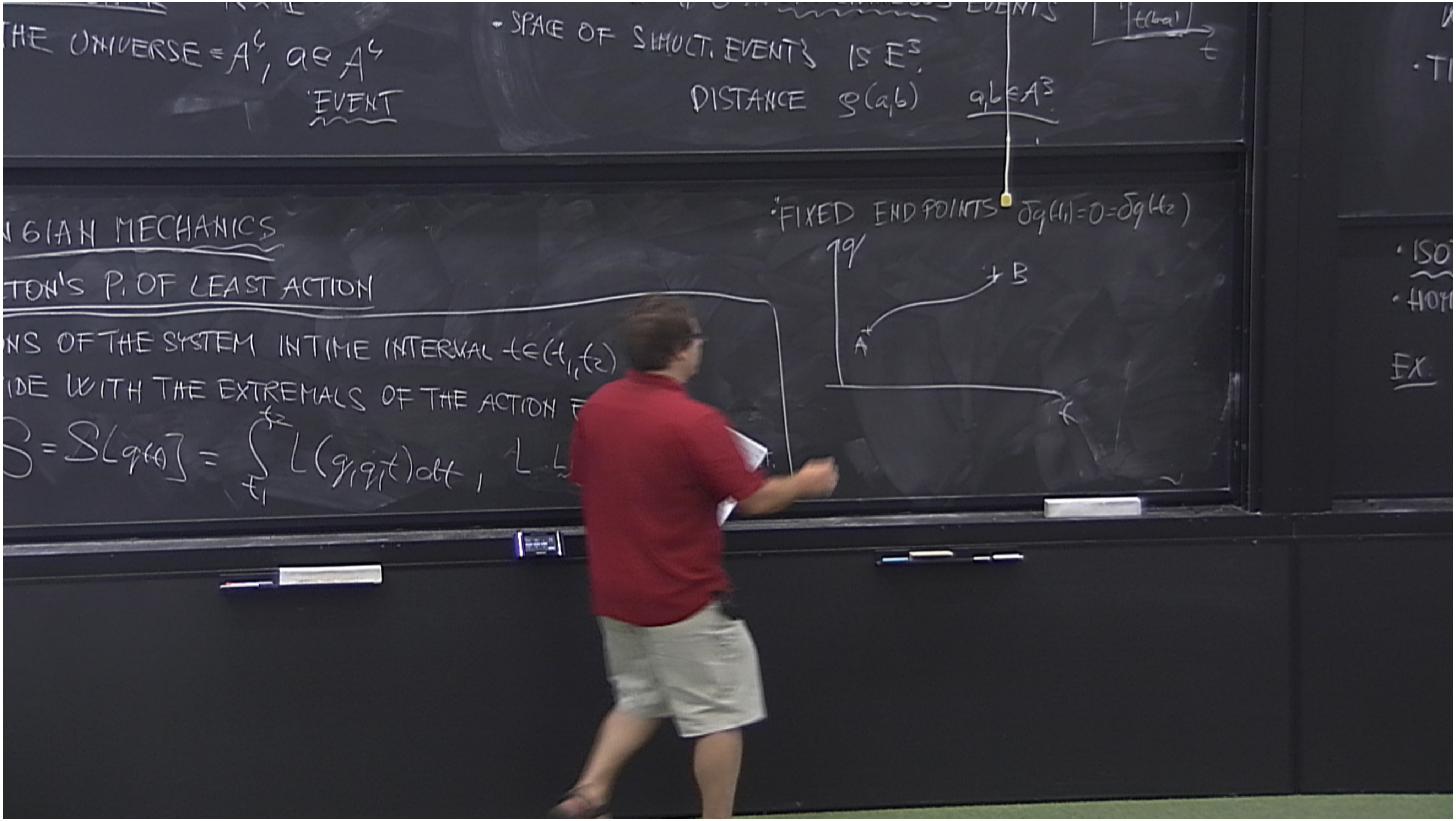

EVENT
DISTANCE ρ(a,b)
2) LAGRANGIAN MECHANICS
a) HA


EVENT
DISTANCE ρ(a,b)
2) LAGRANGIAN MECHANICS
a) HAMILTON'S P. OF LEAST ACTION
MOTIONS OF THE SYSTEM IN TIME INTERVAL t∈(t1,t2)

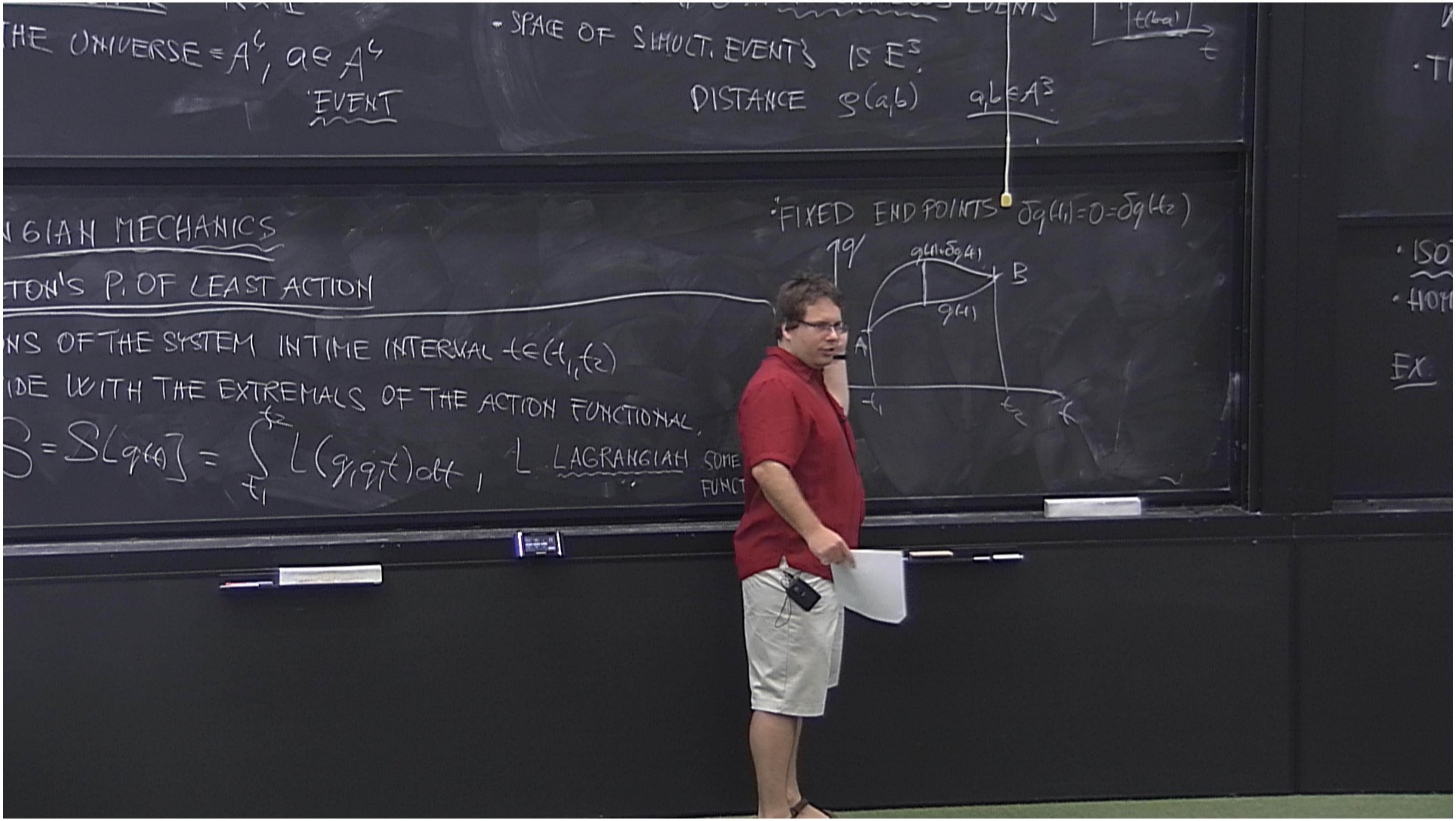

EVENT
DISTANCE ρ(a,b)
2) LAGRANGIAN MECHANICS
a) HAMILTON'S P. OF LEAST ACTION
OTIONS OF THE SYSTEM IN TIME INTERVAL t∈(t1,t2)
OINCIDE WITH THE EXTREMALS OF THE ACTION FUNCTIONAL.

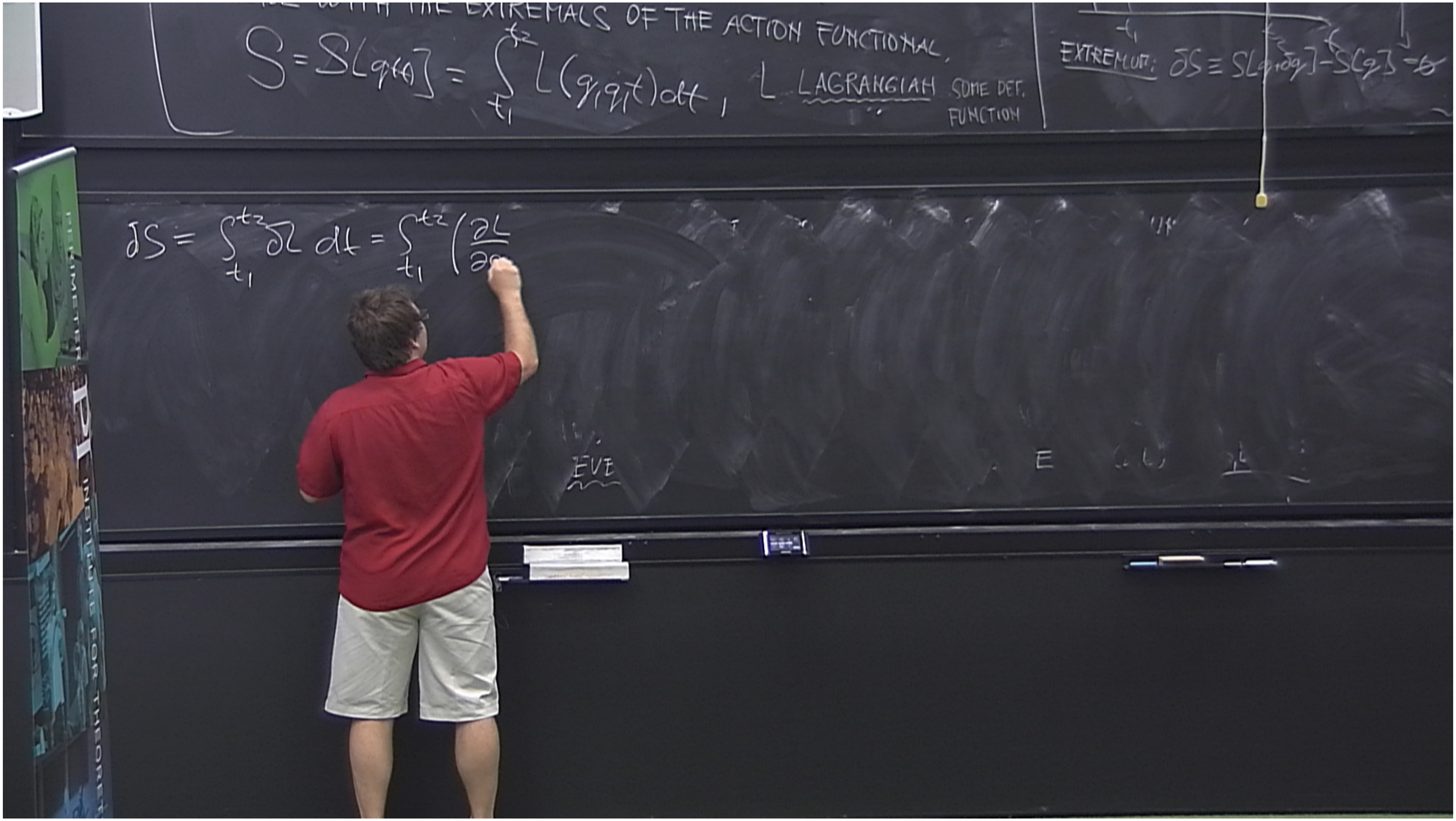

EVENT
DISTANCE $\rho(a,b)$
2) LAGRANGIAN MECHANICS
a) HAMILTON'S P. OF LEAST ACTION
MOTIONS OF THE SYSTEM IN TIME INTERVAL $t \in (t_1, t_2)$
COINCIDE WITH THE EXTREMALS OF THE ACTION FUNCTIONAL.
$S = S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$, L LAGRANGIAN


EVENT
DISTANCE ρ(a,b)
2) LAGRANGIAN MECHANICS
a) HAMILTON'S P. OF LEAST ACTION
MOTIONS OF THE SYSTEM IN TIME
COINCIDE WITH THE EXTREMALS OF THE
S = S[q(t)] = ∫ L(q,q̇,t)dt
GRANGIAN
SOME DEF. FUNCTION

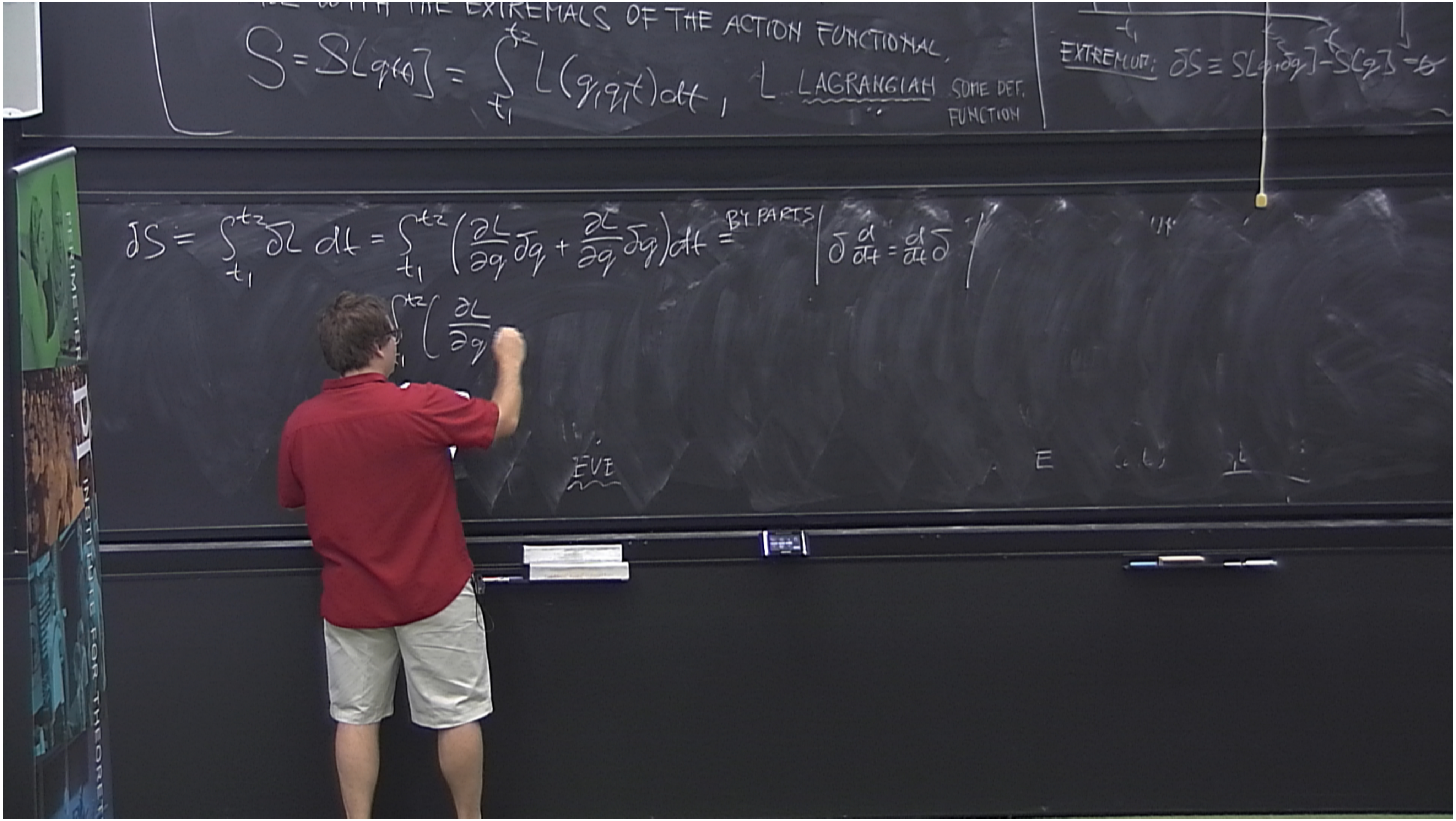


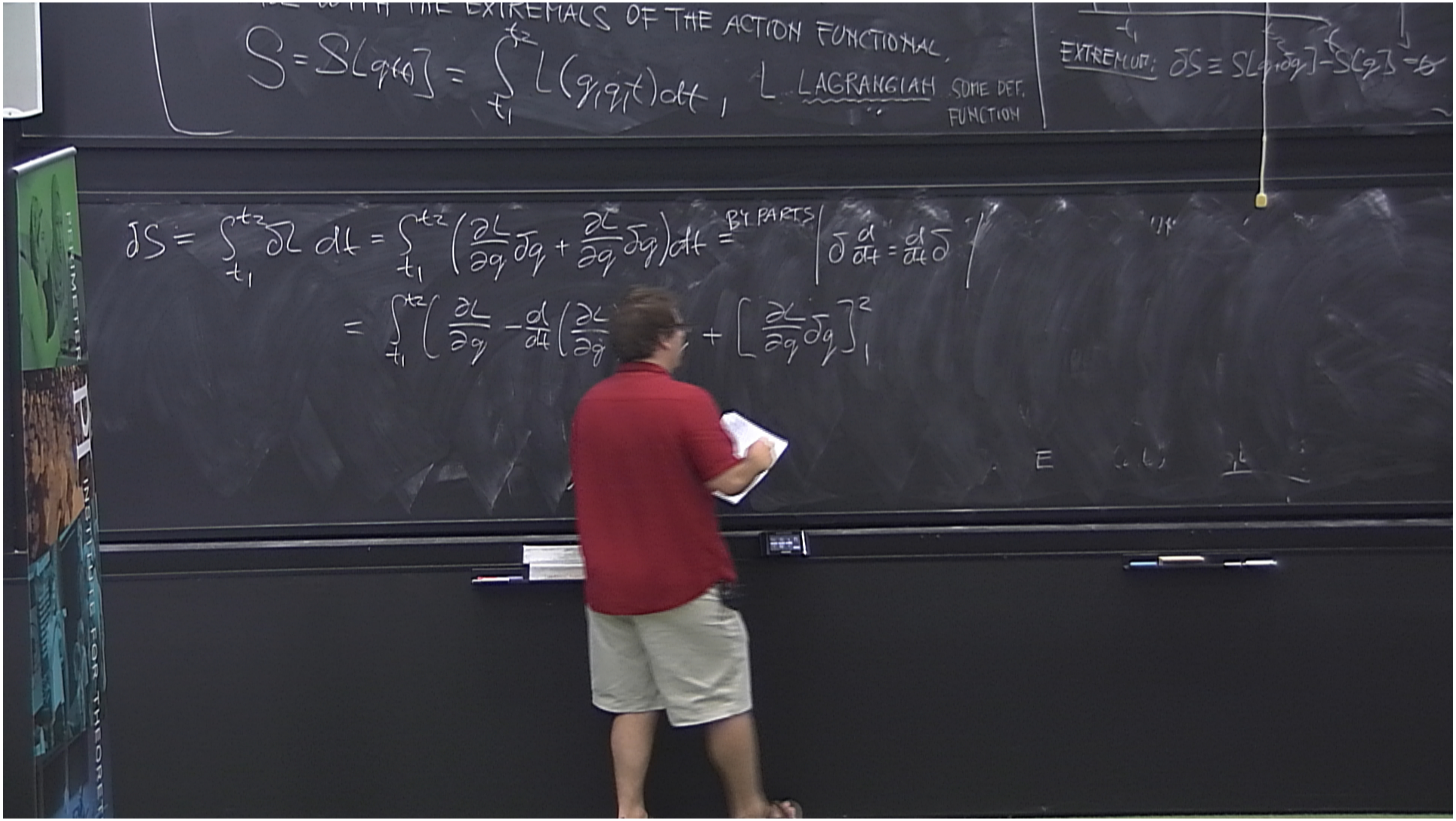
THE UNIVERSE = A^4, $a \in A^4$
EVENT
• SPACE OF SIMULT. EVENTS IS E^3.
DISTANCE $\rho(a,b)$ $a,b \in A^3$
LAGRANGIAN MECHANICS
TON'S P. OF LEAST ACTION
FIXED END POINTS $\delta q(t_1) = 0 = \delta q(t_2)$
B
A
EX.

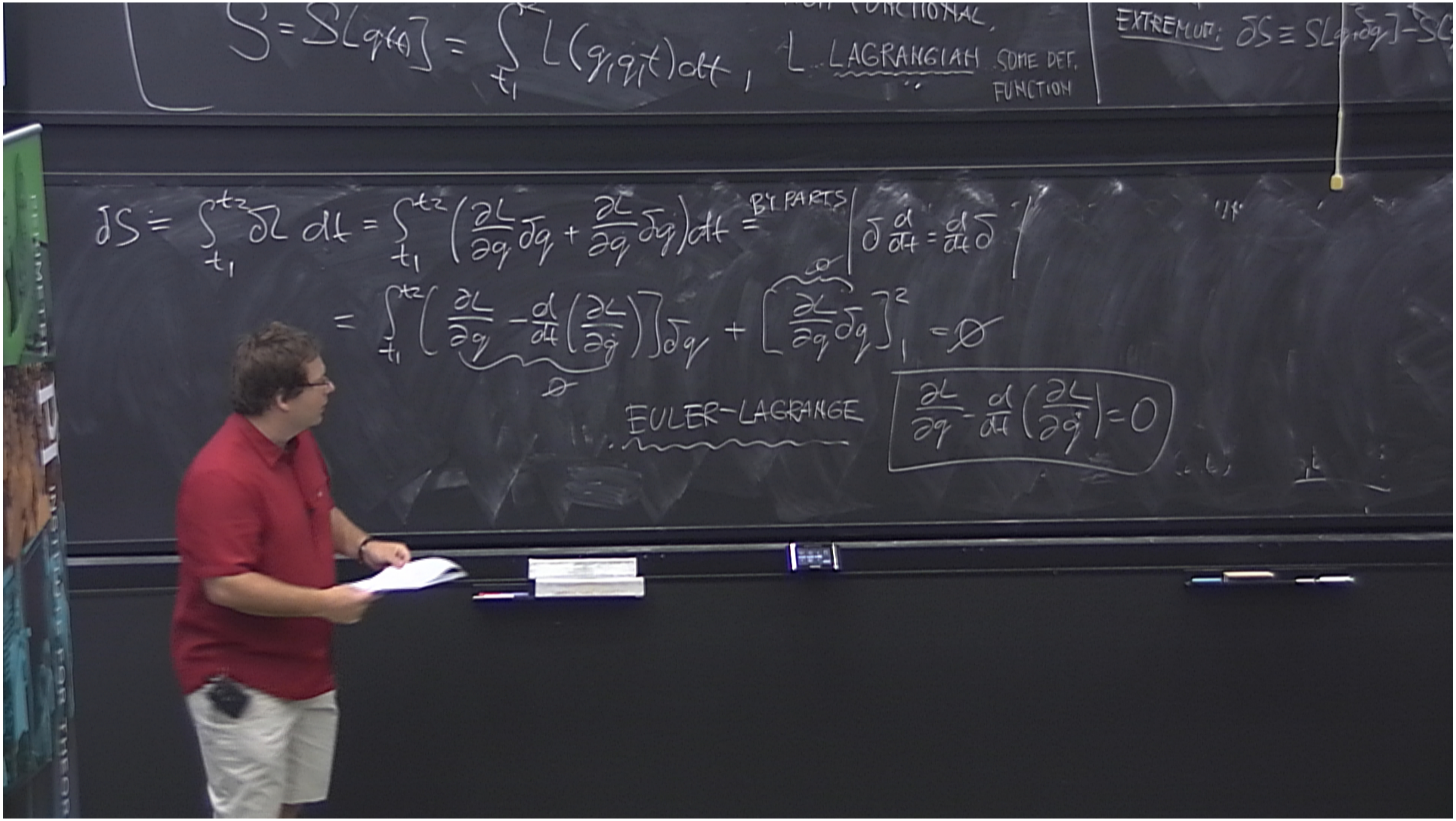

THE UNIVERSE = A^4, $a \in A^4$
EVENT
SPACE OF SIMULT. EVENTS IS E^3.
DISTANCE $\rho(a,b)$ $a,b \in A^3$
NGIAN MECHANICS
TON'S P. OF LEAST ACTION
NS OF THE SYSTEM IN TIME INTERVAL $t \in (t_1, t_2)$
IDE WITH THE EXTREMALS OF THE ACTION FUNCTIONAL.
$S = S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$, L LAGRANGIAN SOME
FUNCT
FIXED END POINTS $\delta q(t_1) = 0 = \delta q(t_2)$
q
$q(t)+\delta q(t)$
B
$q(t)$
A
t_1
t_2
t
EX.

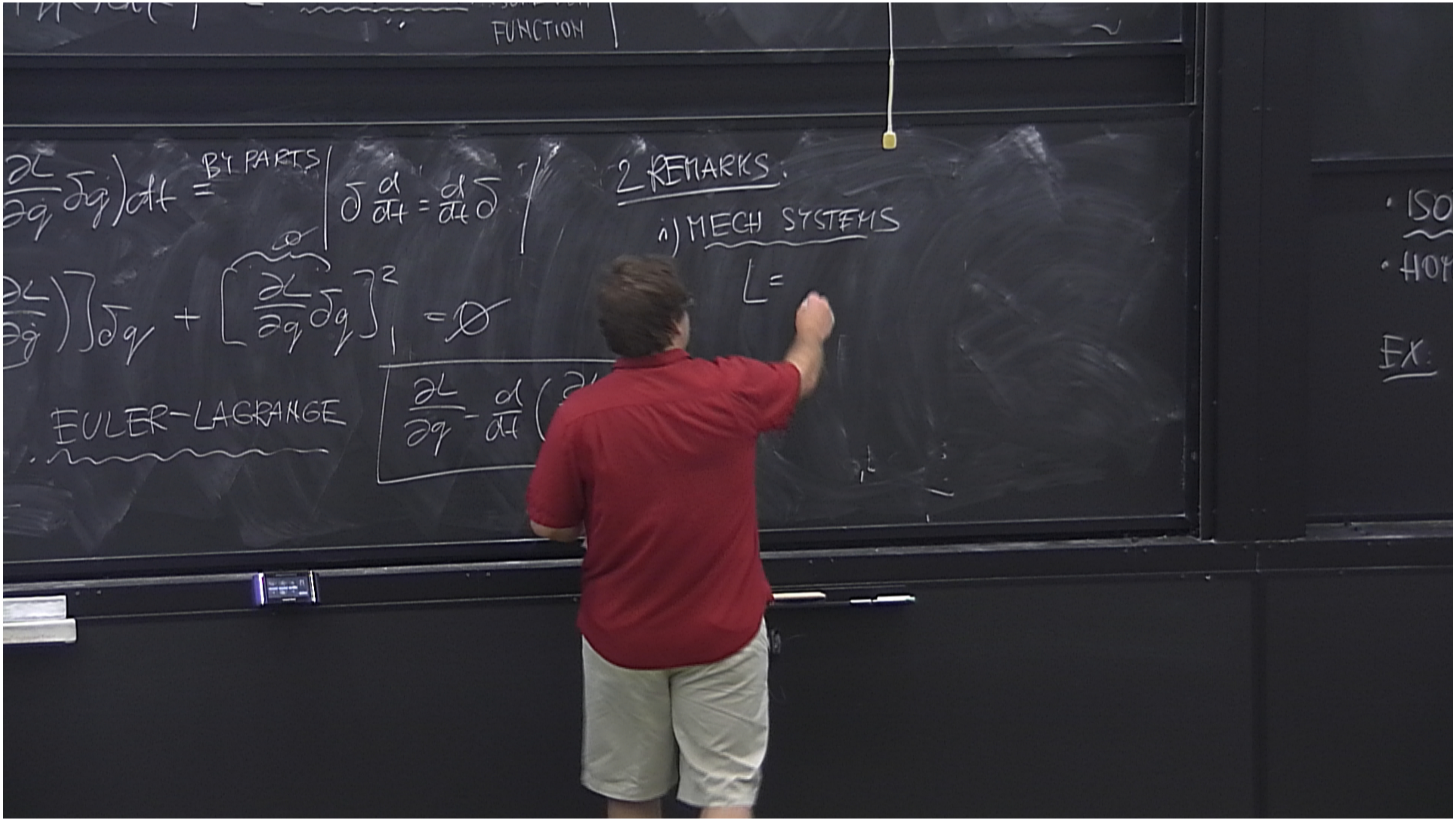


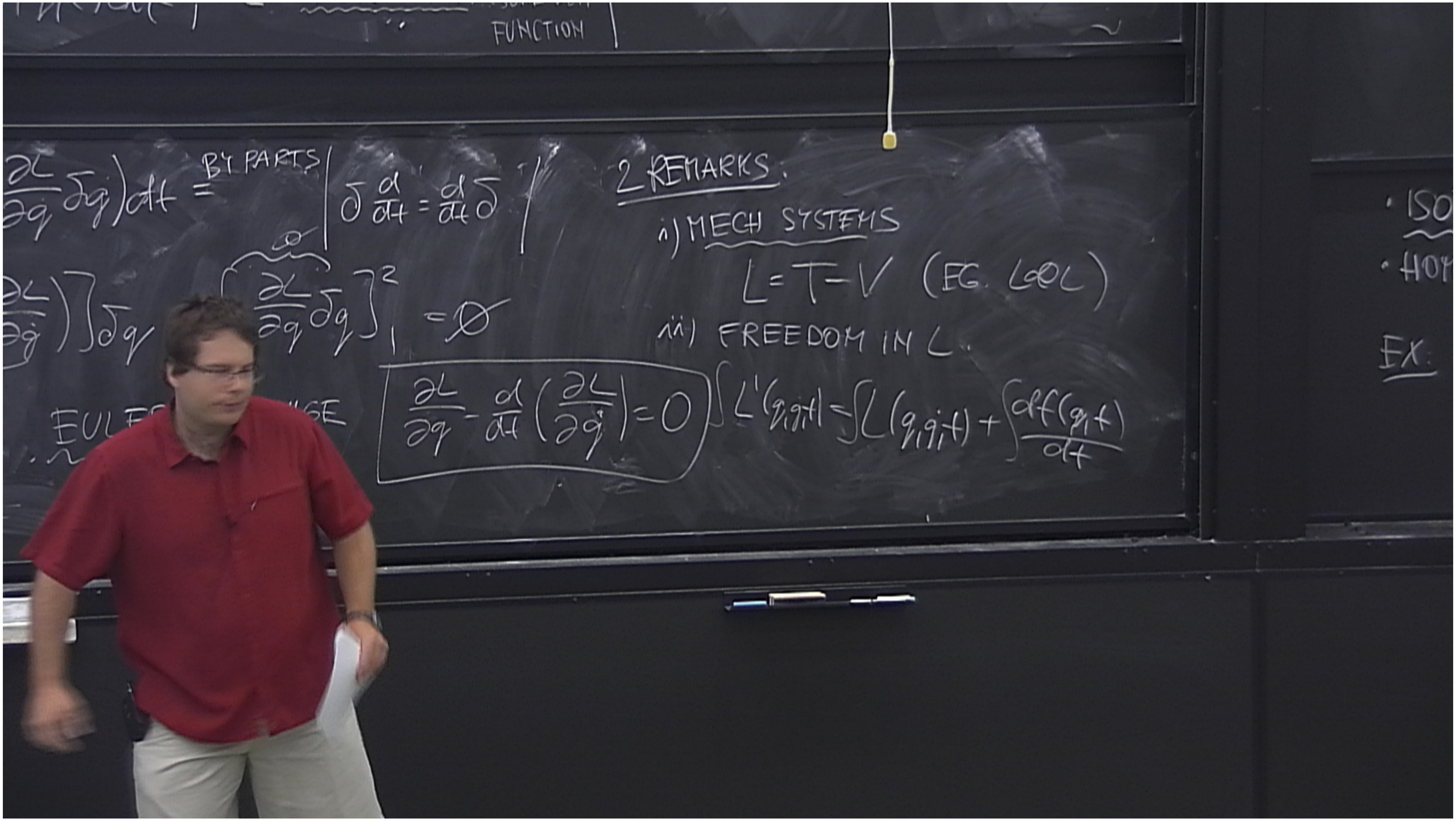
THE EXTREMALS OF THE ACTION FUNCTIONAL,
S = S[q(t)] = ∫ L(q,q̇,t)dt , L LAGRANGIAN
SOME DEF. FUNCTION
EXTREMUM: δS ≡ S[q+δq] − S[q] = 0
δS = ∫ δL dt = ∫ (∂L/∂

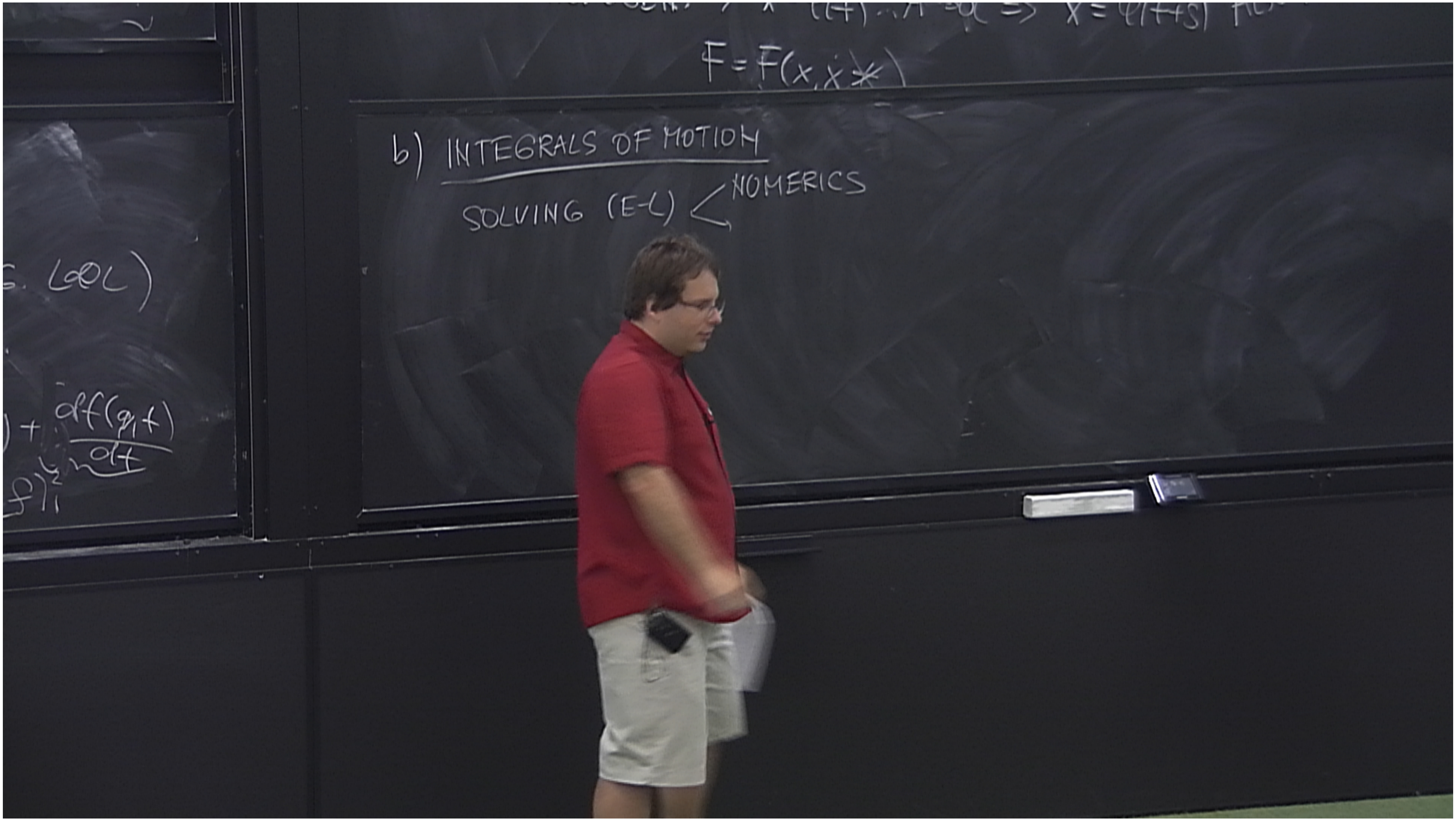

EXTREMALS OF THE ACTION FUNCTIONAL,
S = S[q(t)] = ∫_{t1}^{t2} L(q,q̇,t)dt , L LAGRANGIAN
SOME DEF. FUNCTION
EXTREMUM: δS ≡ S[q+δq] − S[q] = 0
δS = ∫_{t1}^{t2} δL dt = ∫_{t1}^{t2} (∂L/∂q δq + ∂L/∂q̇ δq̇) dt
BY PARTS
δ d/dt = d/dt δ

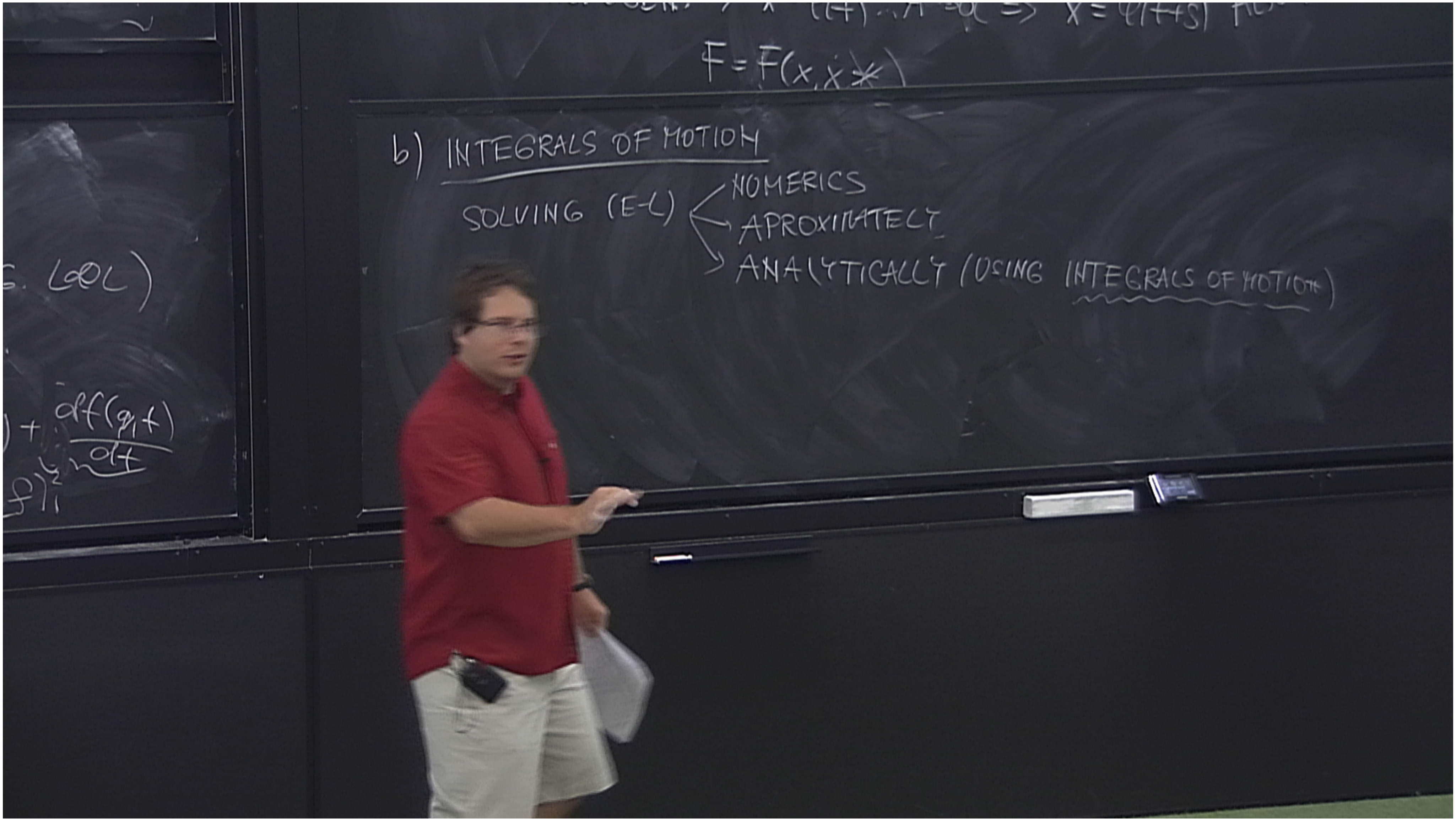

WITH THE EXTREMALS OF THE ACTION FUNCTIONAL.
S = S[q(t)] = ∫ L(q, q̇, t) dt , L LAGRANGIAN
SOME DEF. FUNCTION
EXTREMUM: δS ≡ S[q + δq] − S[q] = 0
δS = ∫ δL dt = ∫ (∂L/∂q δq + ∂L/∂q̇ δq̇) dt = BY PARTS
δ d/dt = d/dt δ

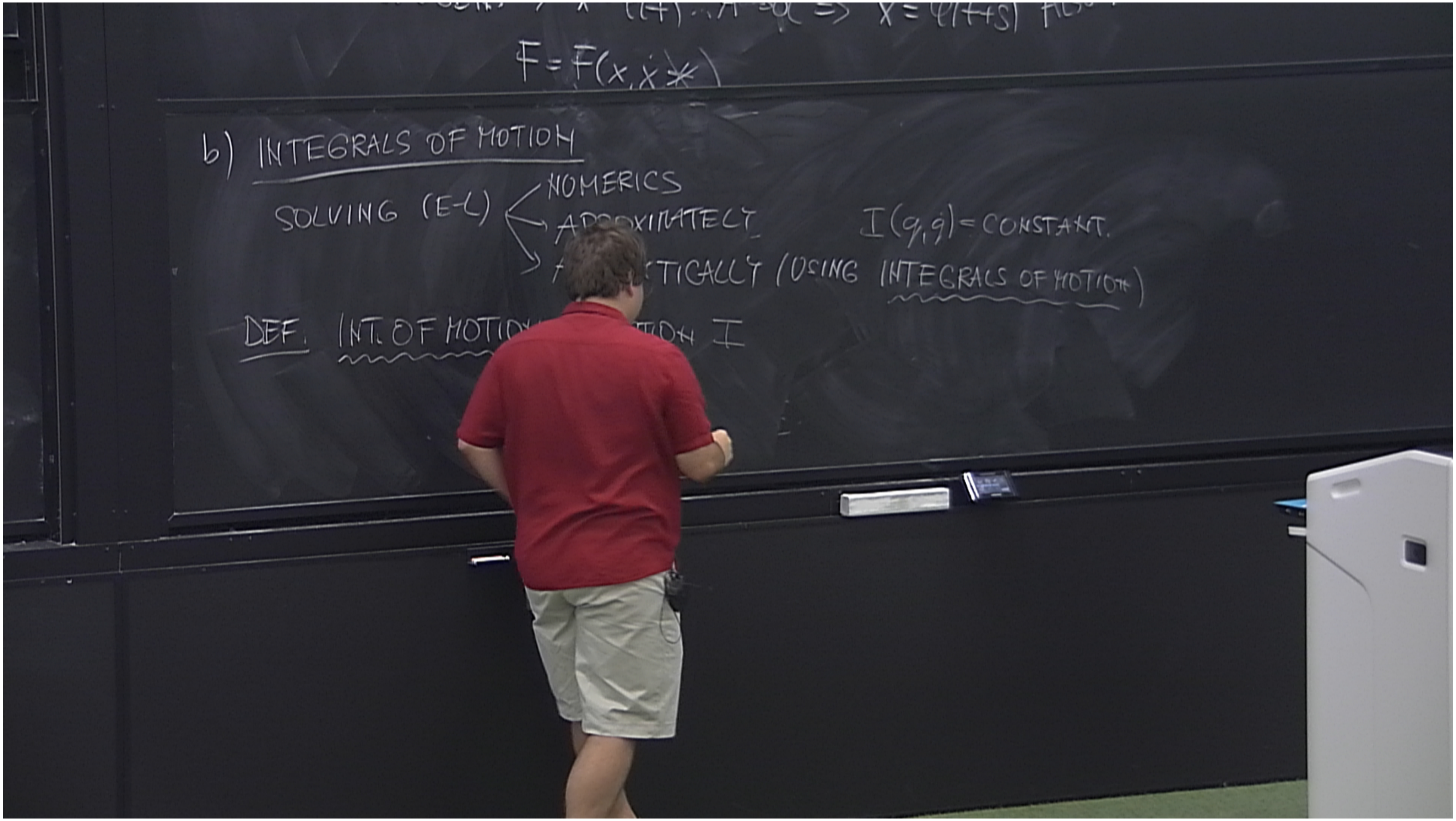

FUNCTIONAL
LAGRANGIAN
SOME DEF. FUNCTION
EXTREMUM:
BY PARTS
EULER-LAGRANGE

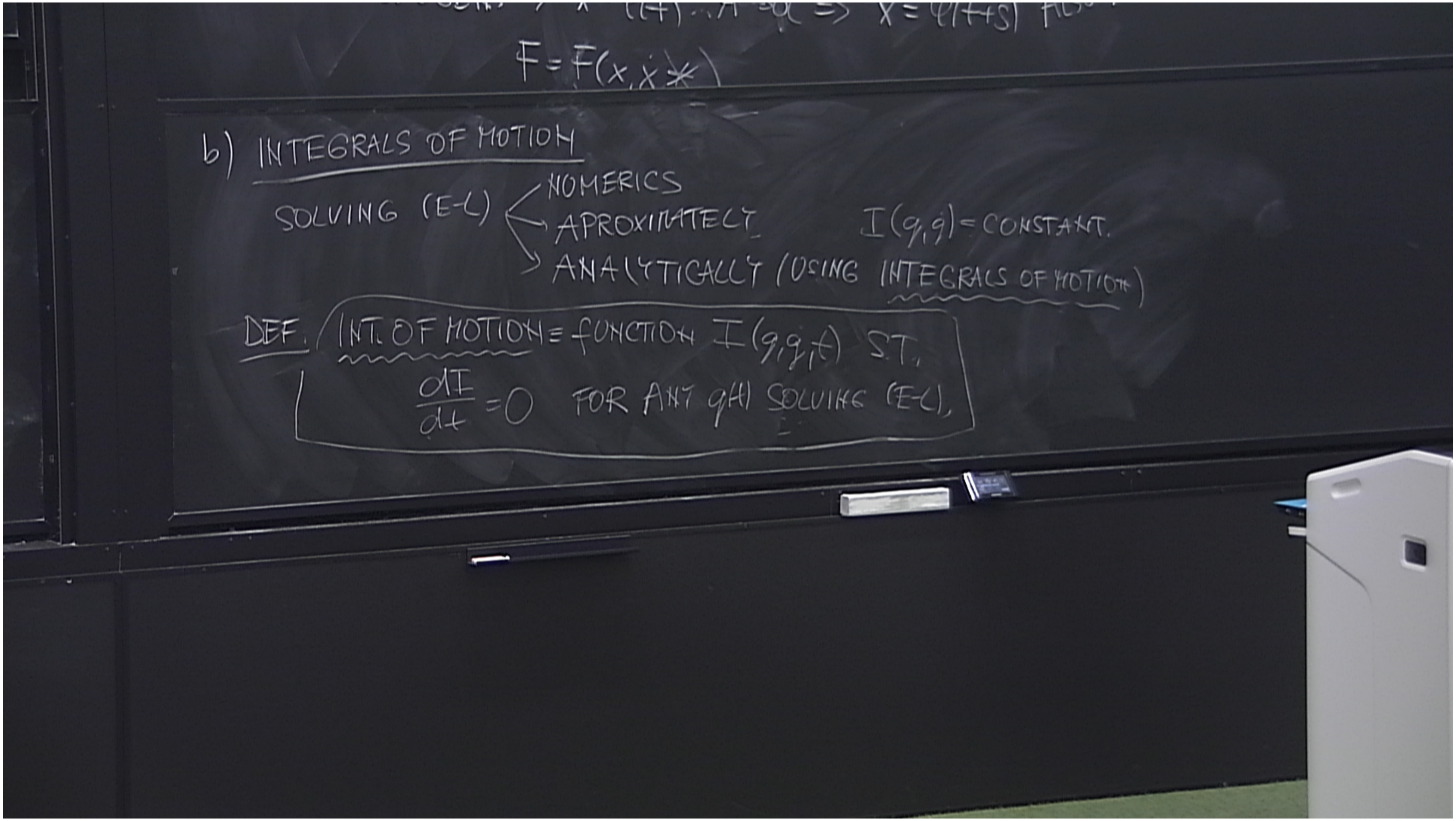

FUNCTION
BY PARTS
2 REMARKS
i) MECH SYSTEMS
L=
EULER-LAGRANGE
EX:

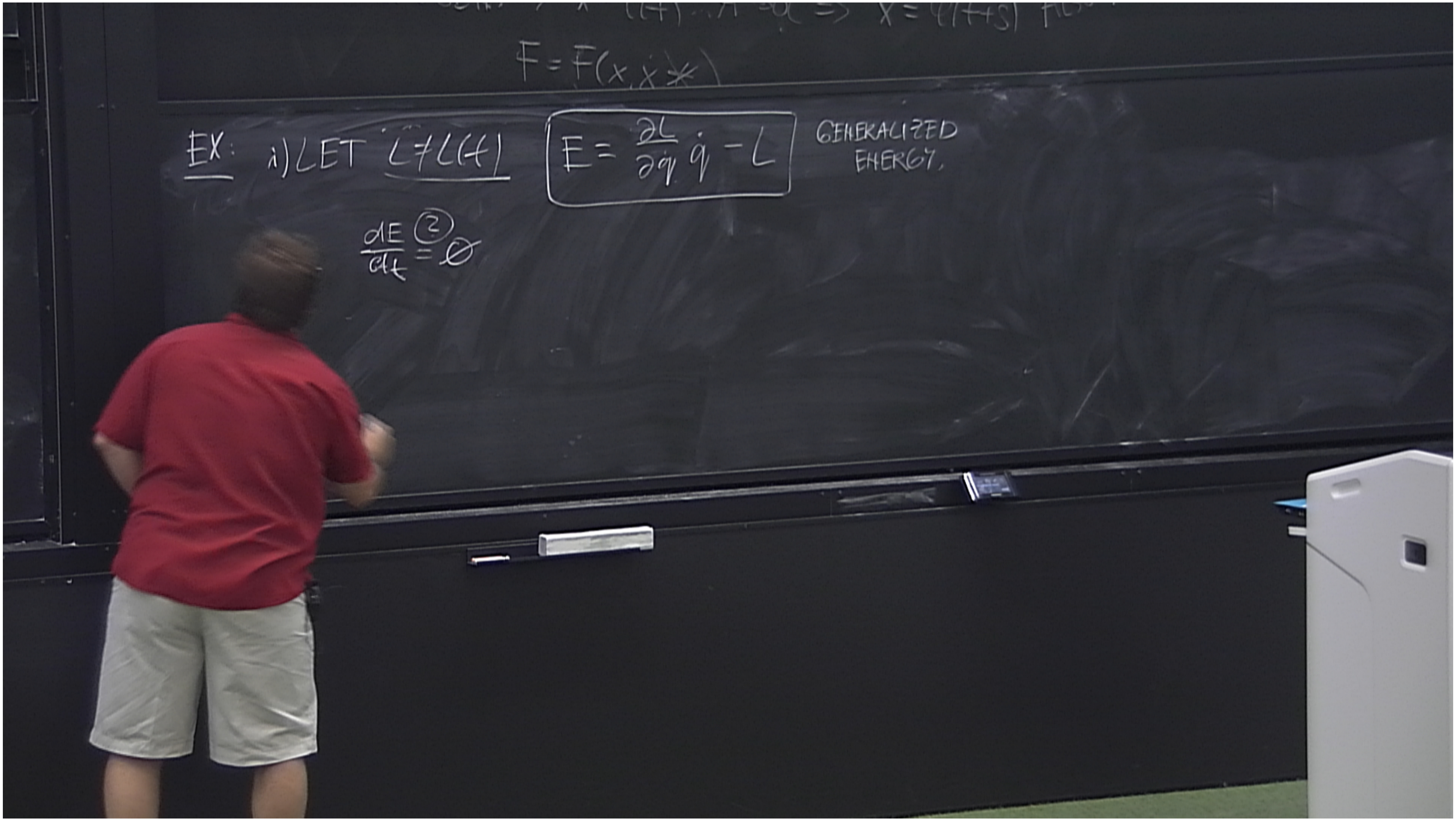

FUNCTION
BY PARTS
2 REMARKS.
i) MECH SYSTEMS
L = T − V (EG. LOOL)
ii) FREEDOM IN L.
EX:

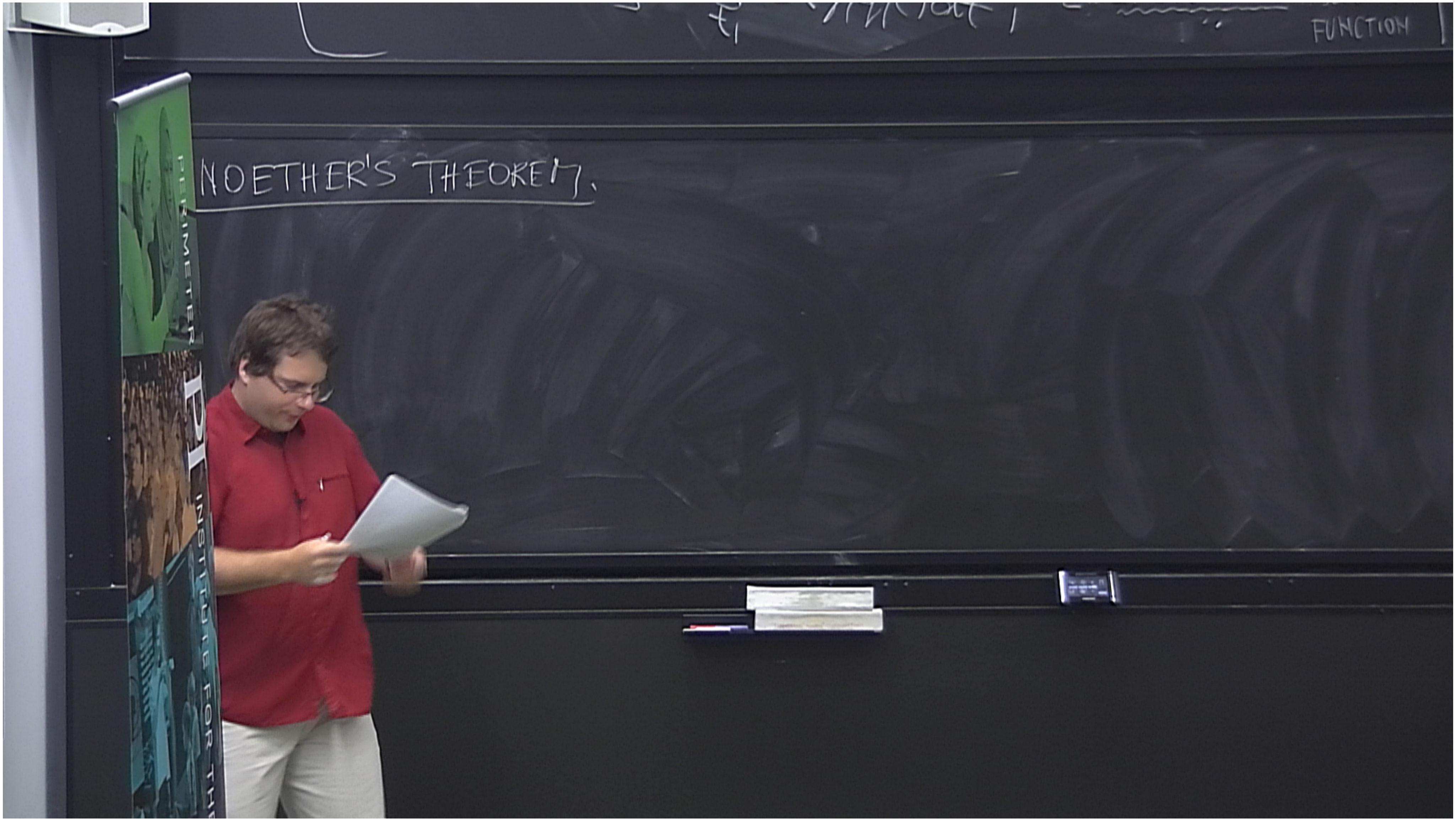

b) INTEGRALS OF MOTION
SOLVING (E-L)
NUMERICS

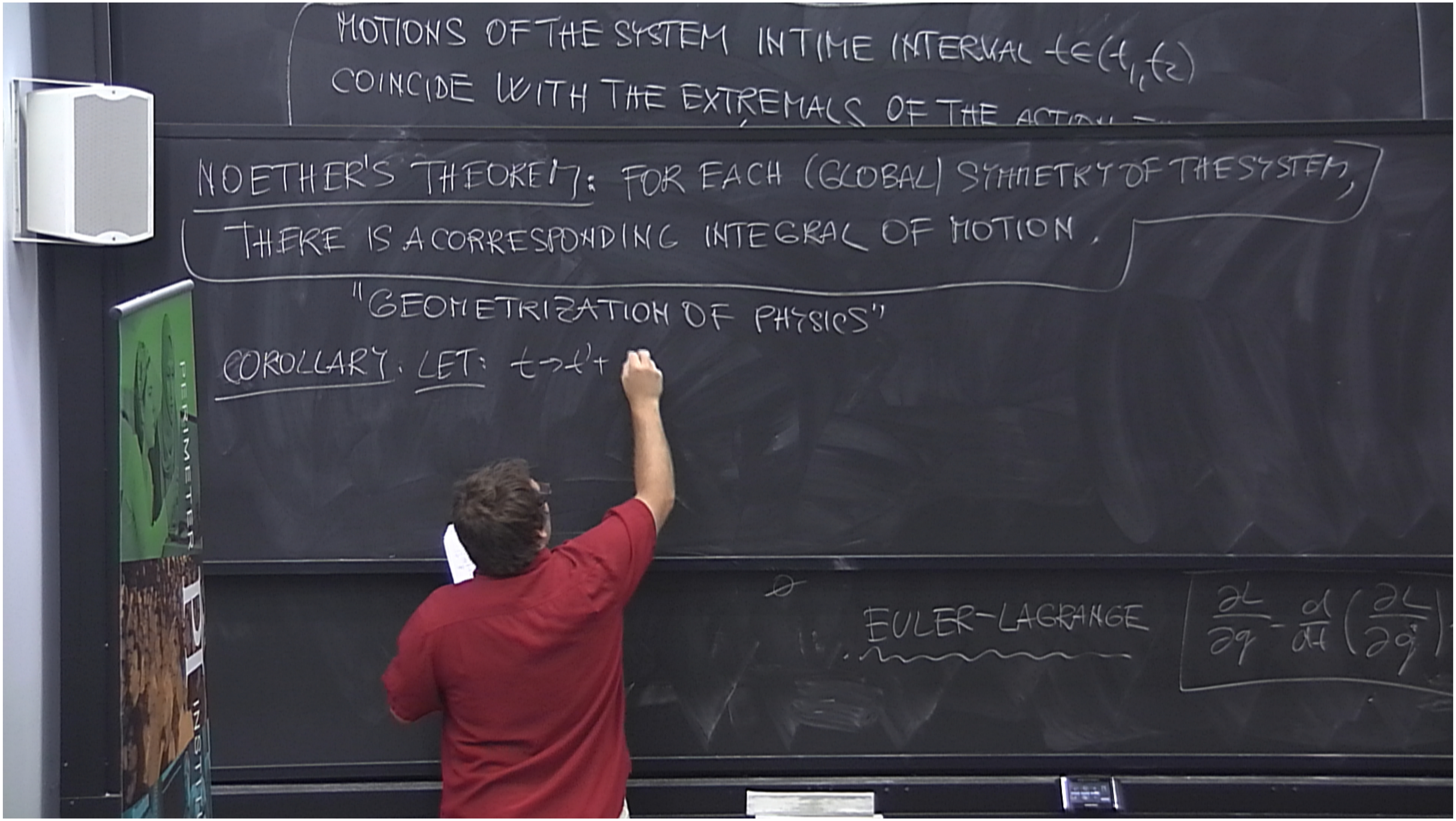

b) INTEGRALS OF MOTION
SOLVING (E-L)
NUMERICS
APROXIMATELY
ANALYTICALLY (USING INTEGRALS OF MOTION)

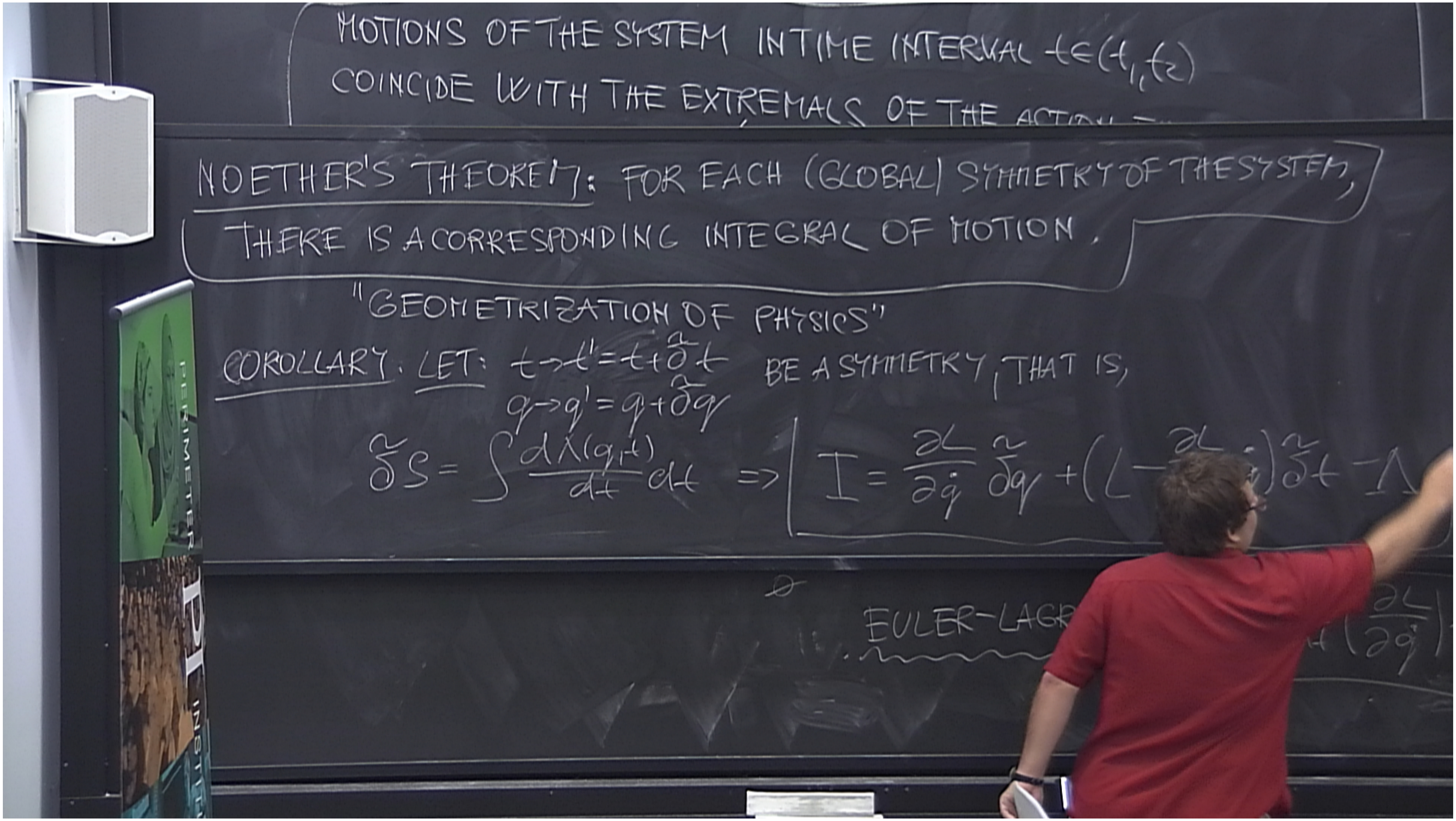

b) INTEGRALS OF MOTION
SOLVING (E-L)
NUMERICS
I(q,q̇) = CONSTANT.
(USING INTEGRALS OF MOTION)
DEF.

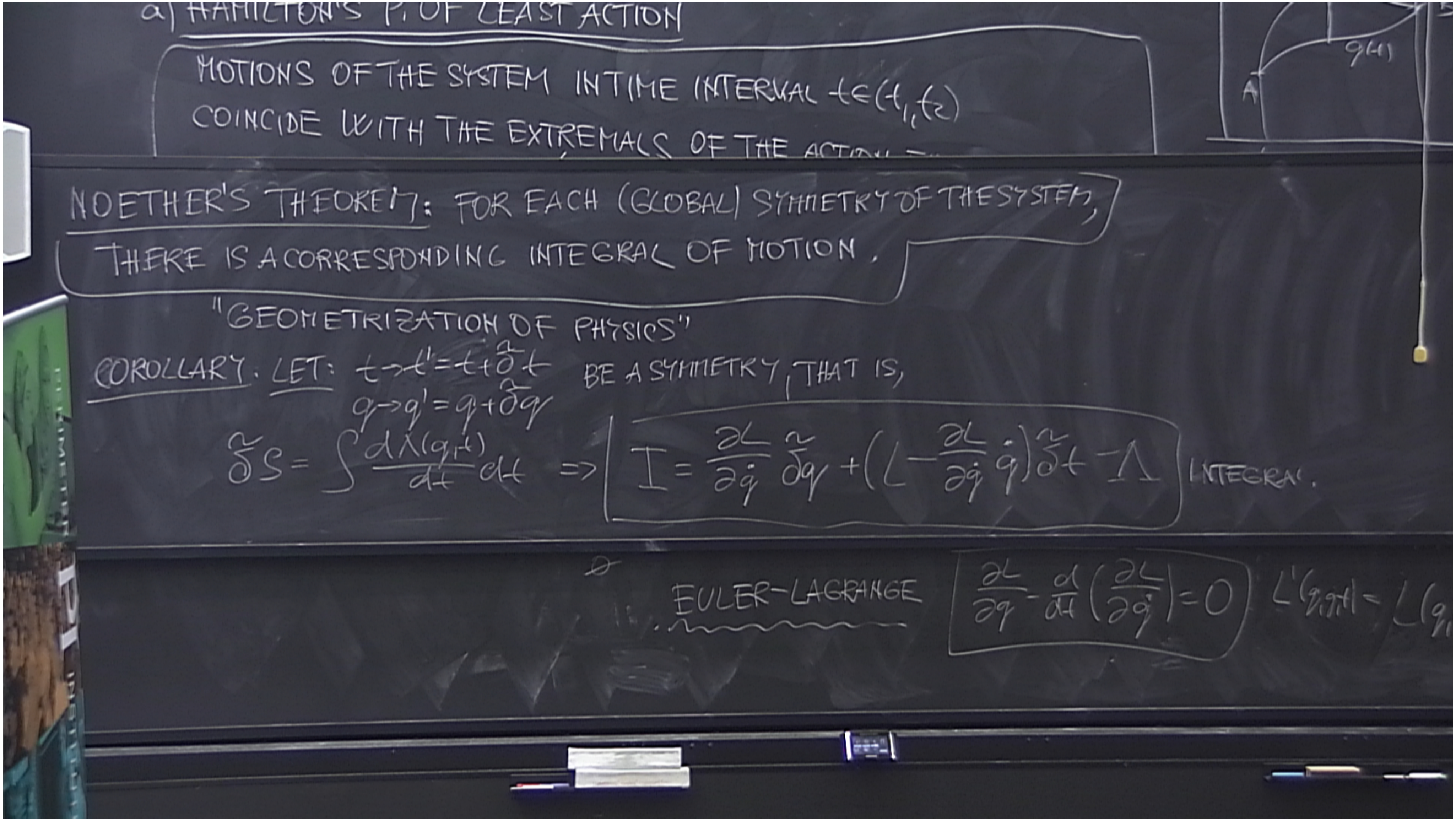

F = F(x, ẋ ✱)
b) INTEGRALS OF MOTION
SOLVING (E-L) < NUMERICS
APROXIMATELY
ANALYTICALLY (USING INTEGRALS OF MOTION)
I(q, q̇) = CONSTANT.
DEF. INT. OF MOTION ≡ FUNCTION I(q, q̇, t) S.T.
dI/dt = 0 FOR ANY q(t) SOLVING (E-L).

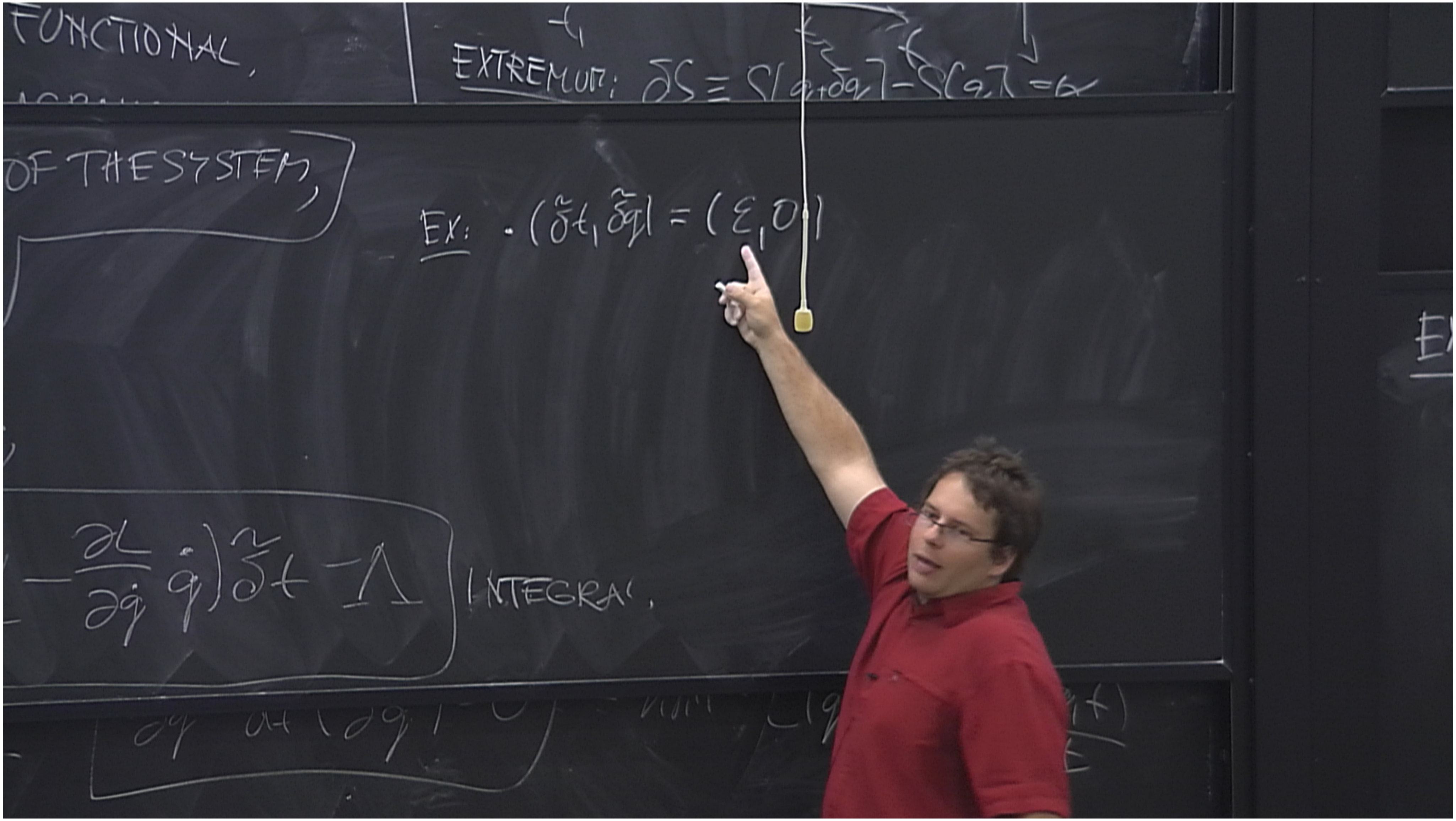

F = F(x, ẋ ✱)
EX: i) LET L ≠ L(t)
E = ∂L/∂q̇ q̇ − L
GENERALIZED ENERGY.
dE/dt = ∅ (?)

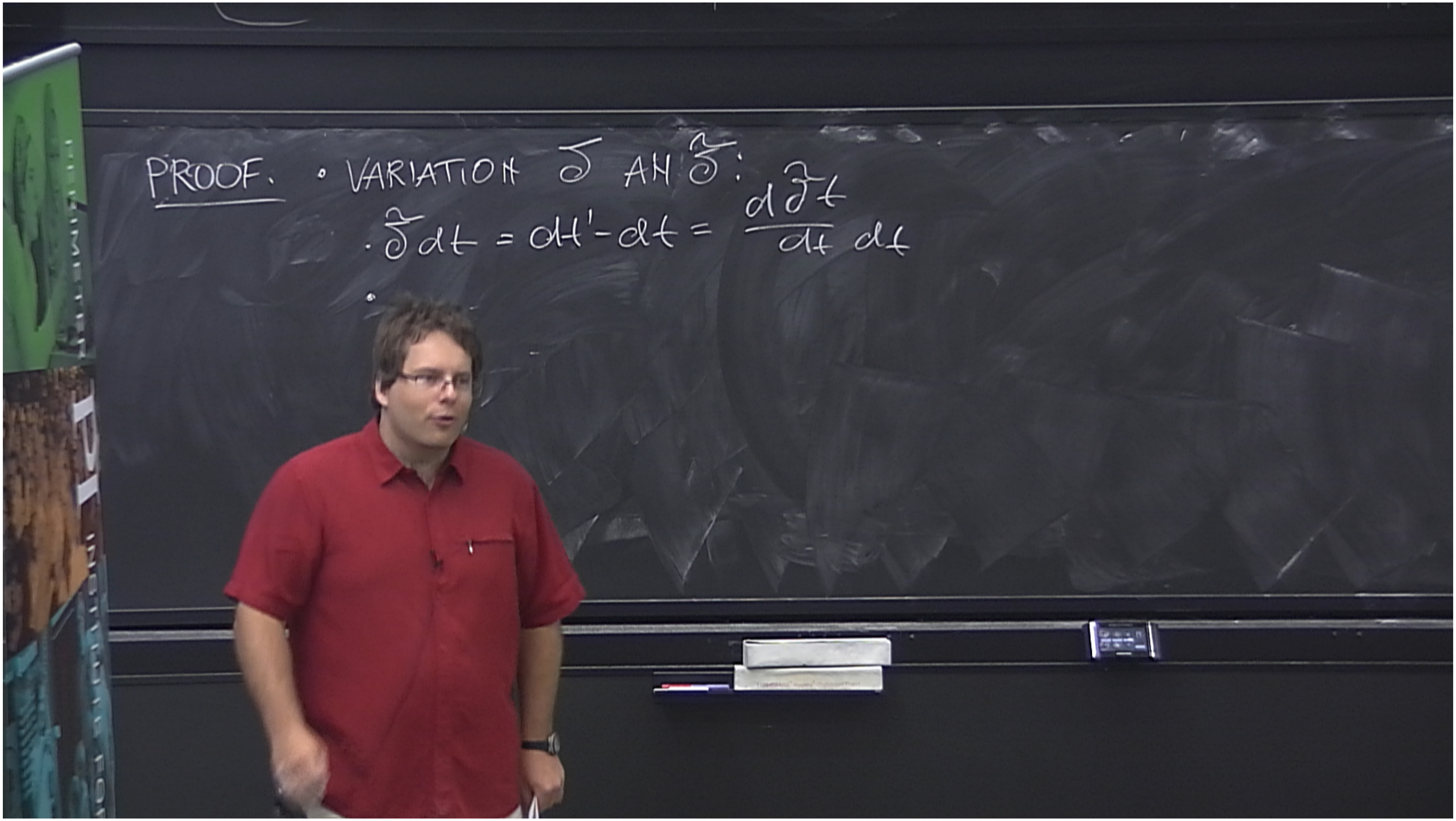

NOETHER'S THEOREM.
FUNCTION
PERIMETER

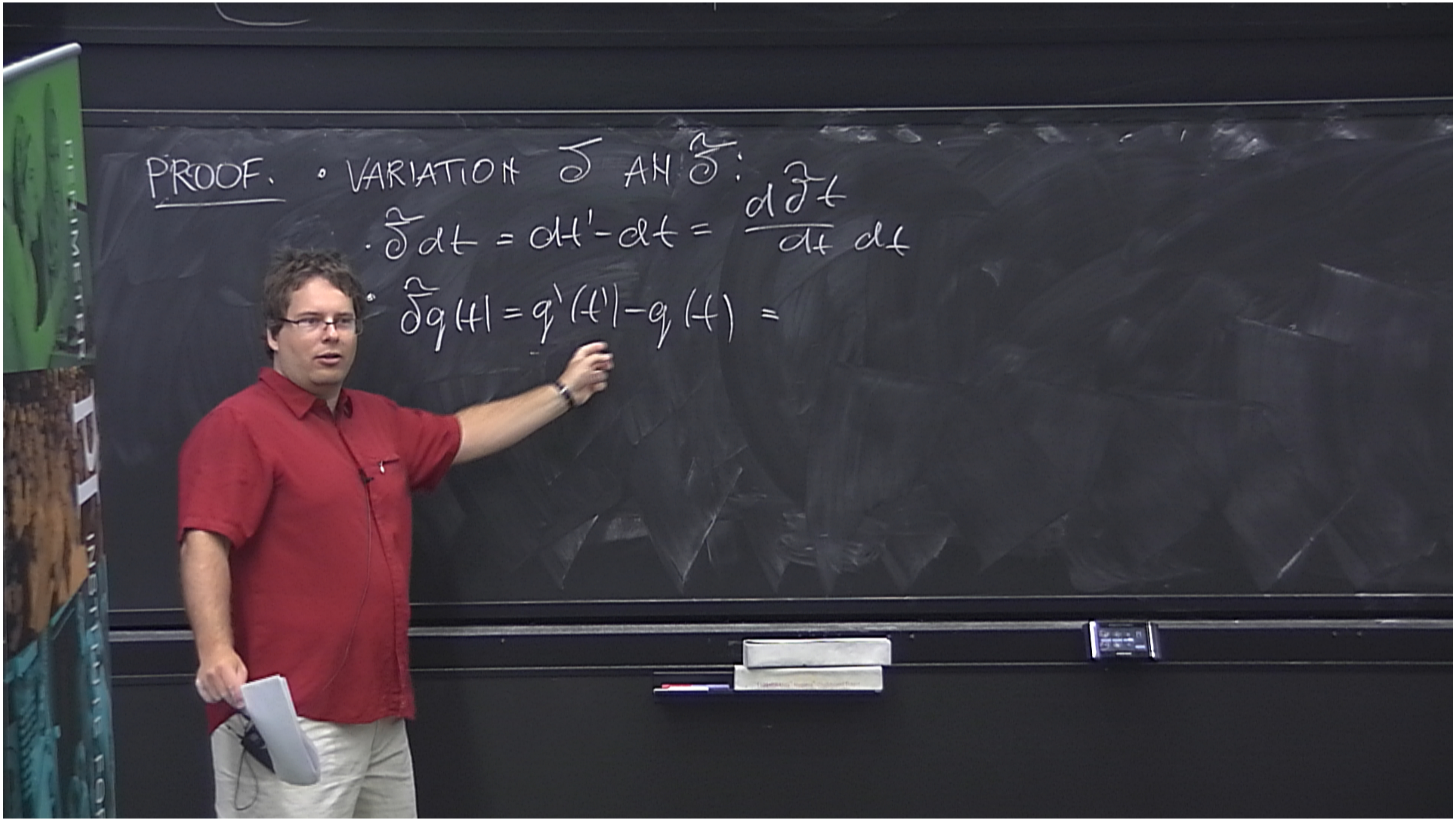

MOTIONS OF THE SYSTEM IN TIME INTERVAL t∈(t1,t2)
COINCIDE WITH THE EXTREMALS OF THE ACTION
NOETHER'S THEOREM: FOR EACH (GLOBAL) SYMMETRY OF THE SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION.
"GEOMETRIZATION OF PHYSICS"
COROLLARY. LET: t→t'+
EULER-LAGRANGE

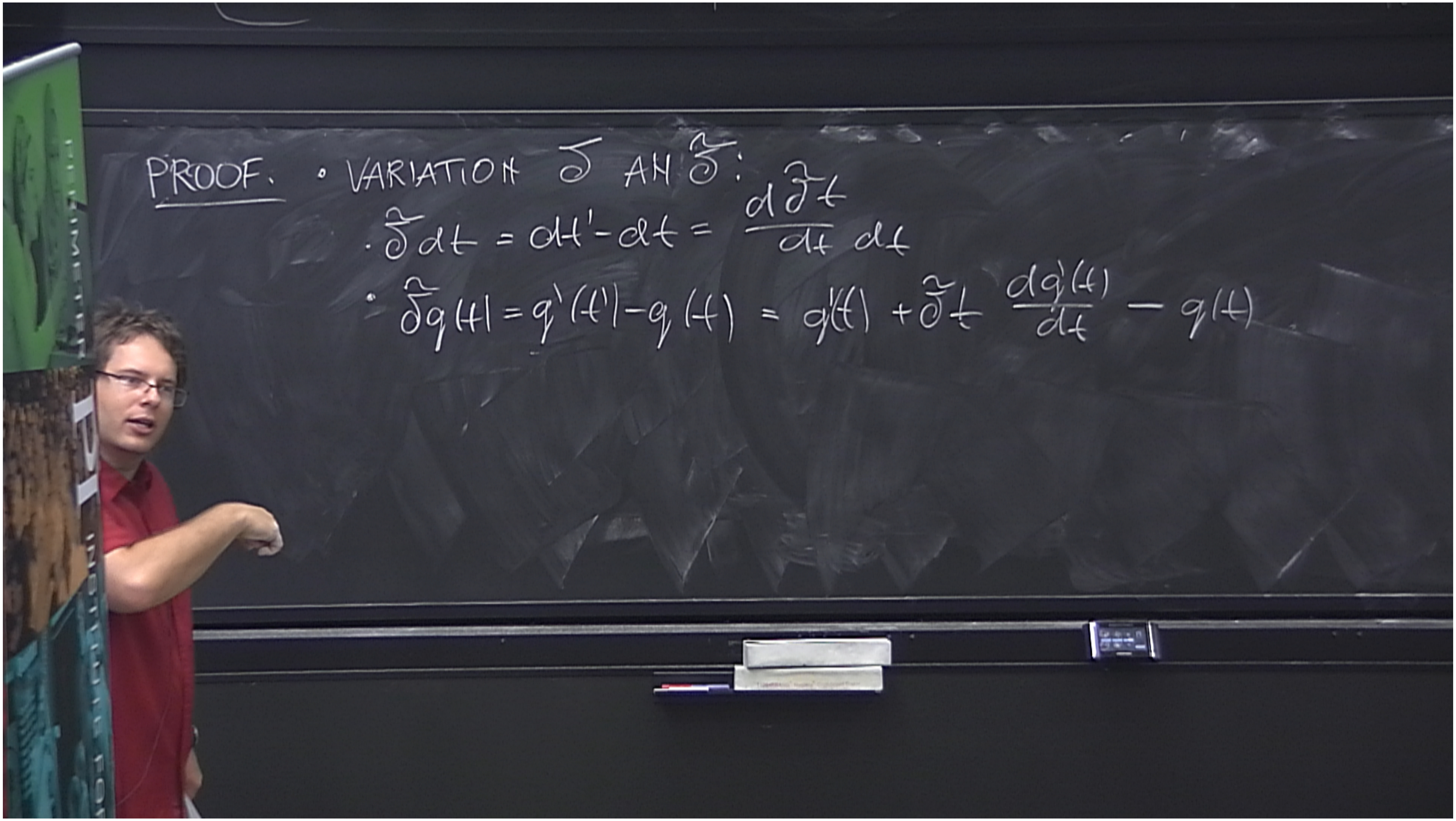

MOTIONS OF THE SYSTEM IN TIME INTERVAL t∈(t1,t2)
COINCIDE WITH THE EXTREMALS OF THE
NOETHER'S THEOREM: FOR EACH (GLOBAL) SYMMETRY OF THE SYSTEM,
THERE IS A CORRESPONDING INTEGRAL OF MOTION.
"GEOMETRIZATION OF PHYSICS"
COROLLARY. LET:
BE A SYMMETRY, THAT IS,
EULER-LAGR

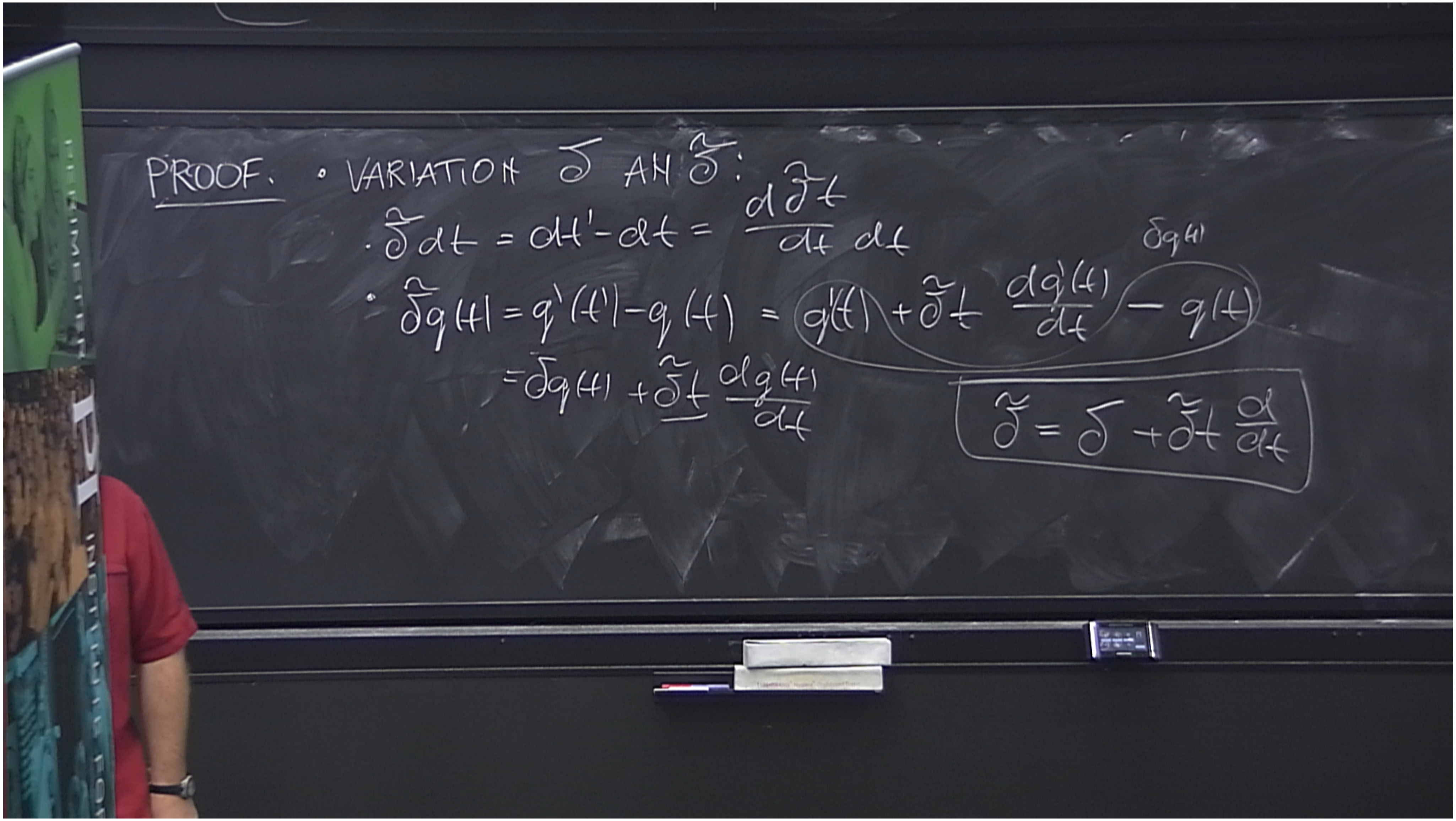

a) HAMILTON'S P. OF LEAST ACTION
MOTIONS OF THE SYSTEM IN TIME INTERVAL $t \in (t_1, t_2)$
COINCIDE WITH THE EXTREMALS OF THE ACTION
q(t)
A
NOETHER'S THEOREM: FOR EACH (GLOBAL) SYMMETRY OF THE SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION.
"GEOMETRIZATION OF PHYSICS"
COROLLARY. LET: $t \to t' = t + \tilde{\delta} t$, $q \to q' = q + \tilde{\delta} q$ BE A SYMMETRY, THAT IS,
$\tilde{\delta} S = \int \frac{d\Lambda(q,t)}{dt} dt \Rightarrow I = \frac{\partial L}{\partial \dot{q}} \tilde{\delta} q + \left(L - \frac{\partial L}{\partial \dot{q}} \dot{q}\right) \tilde{\delta} t - \Lambda$ INTEGRAL.
EULER-LAGRANGE $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$

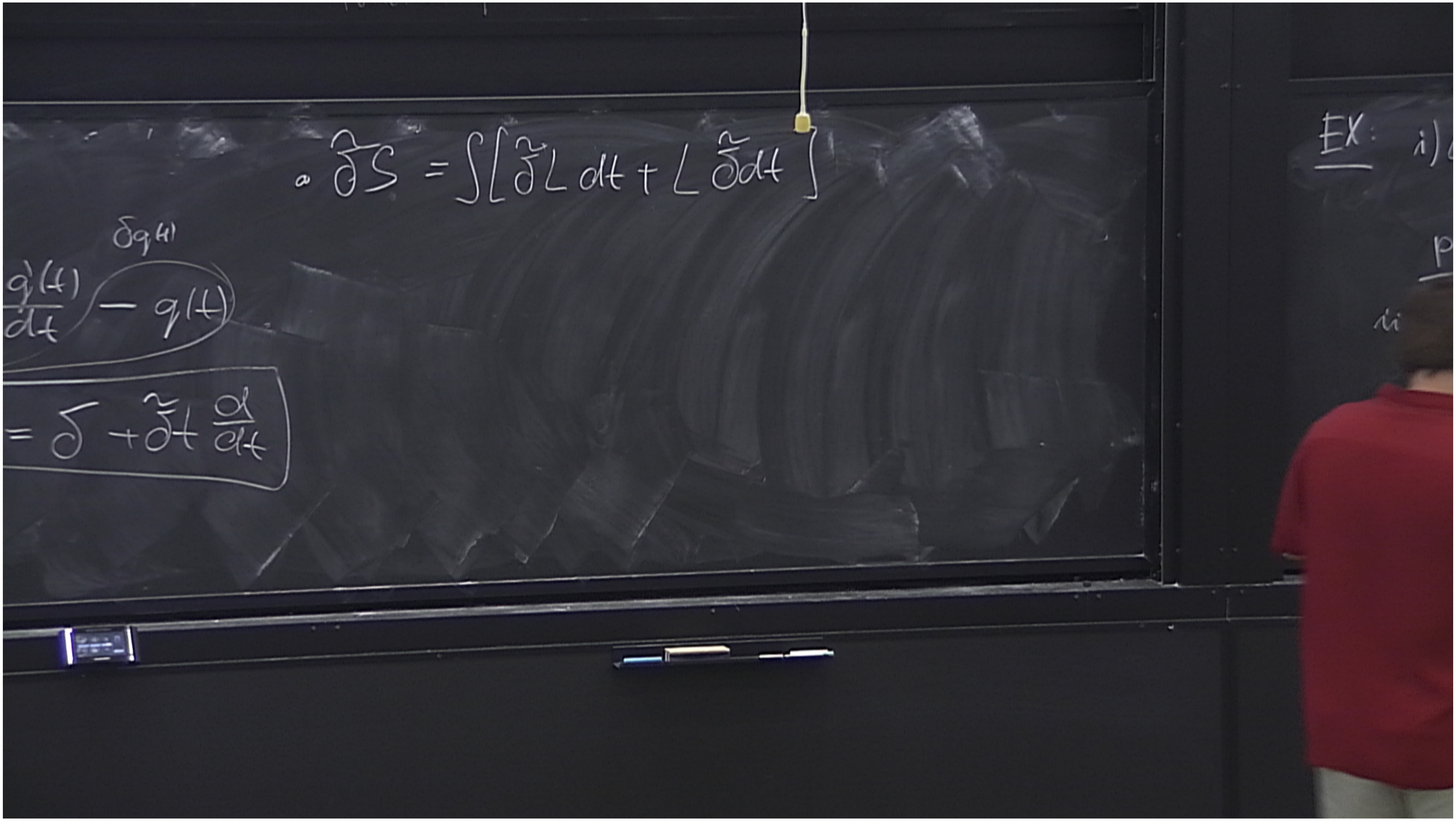

FUNCTIONAL,
EXTREMUM: δS ≡ S[q+δq] − S[q] = 0
OF THE SYSTEM,
Ex: (δ̃t, δ̃q) = (ε, 0)
INTEGRAL.


PROOF. · VARIATION δ ON δ̃:
· δ̃dt = dt' − dt = (dδ̃t / dt) dt

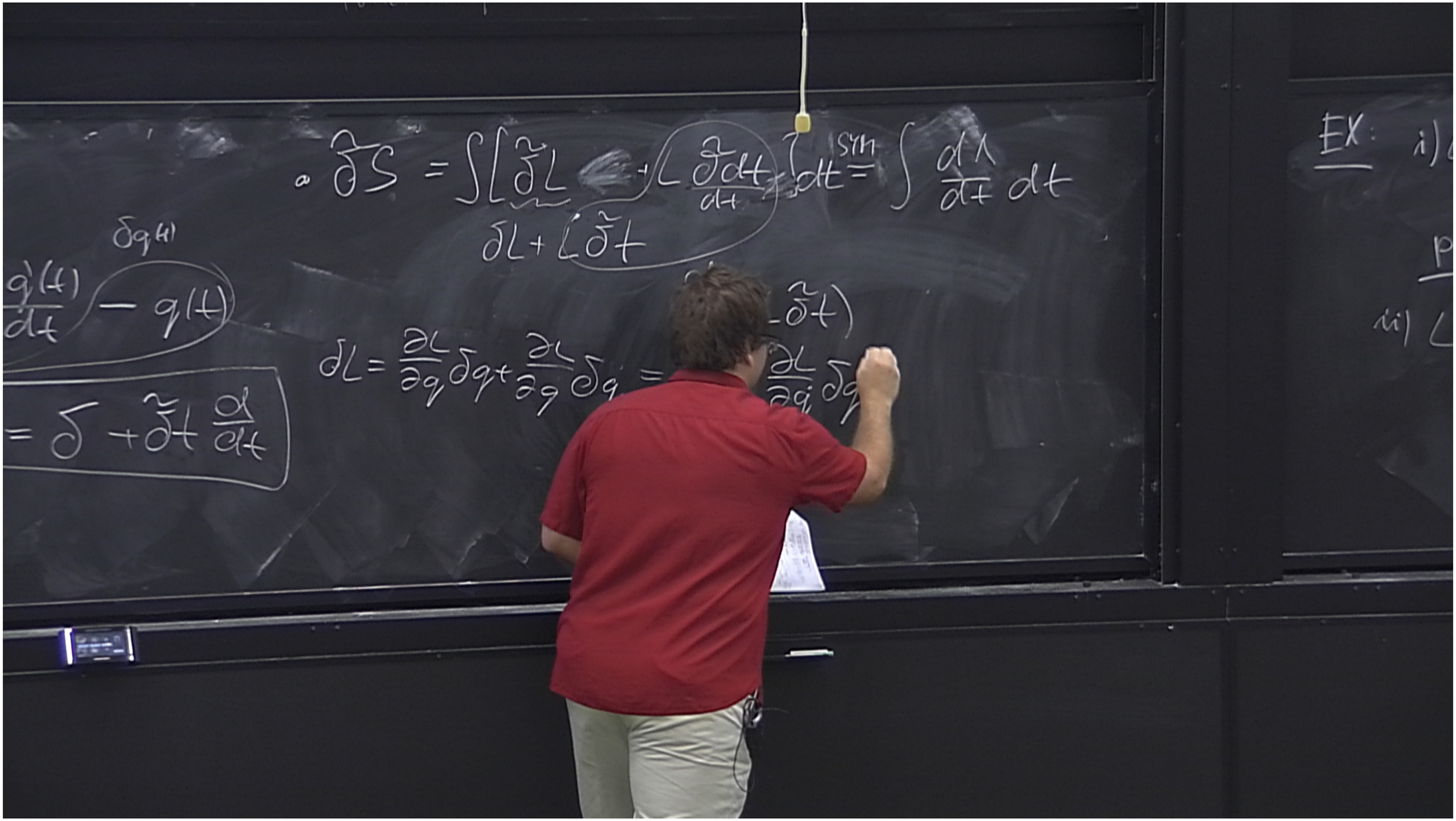

PROOF. • VARIATION δ AN δ̃ :
· δ̃dt = dt' - dt = dδ̃t/dt dt
· δ̃q(t) = q'(t') - q(t) =


PROOF. • VARIATION δ AND δ̃:
· δ̃dt = dt' − dt = (dδ̃t / dt) dt
· δ̃q(t) = q'(t') − q(t) = q'(t) + δ̃t (dq'(t) / dt) − q(t)


PROOF. • VARIATION δ AND $\tilde{\delta}$:
• $\tilde{\delta} dt = dt' - dt = \frac{d\tilde{\delta}t}{dt} dt$
$\delta q(t)$
• $\tilde{\delta} q(t) = q'(t') - q(t) = q'(t) + \tilde{\delta}t \frac{dq'(t)}{dt} - q(t)$
$= \delta q(t) + \tilde{\delta}t \frac{dq(t)}{dt}$
$\tilde{\delta} = \delta + \tilde{\delta}t \frac{d}{dt}$



$\tilde{\delta} S = \int [\tilde{\delta} L \, dt + L \, \tilde{\delta} dt]$
$\delta q(t)$
$- q(t)$
$= \delta + \tilde{\delta} t \frac{d}{dt}$
EX:


EX: