Title: 14/15 PSI - Complex Analysis 1

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Abstract:

Tibra Ali - Complex Analysis

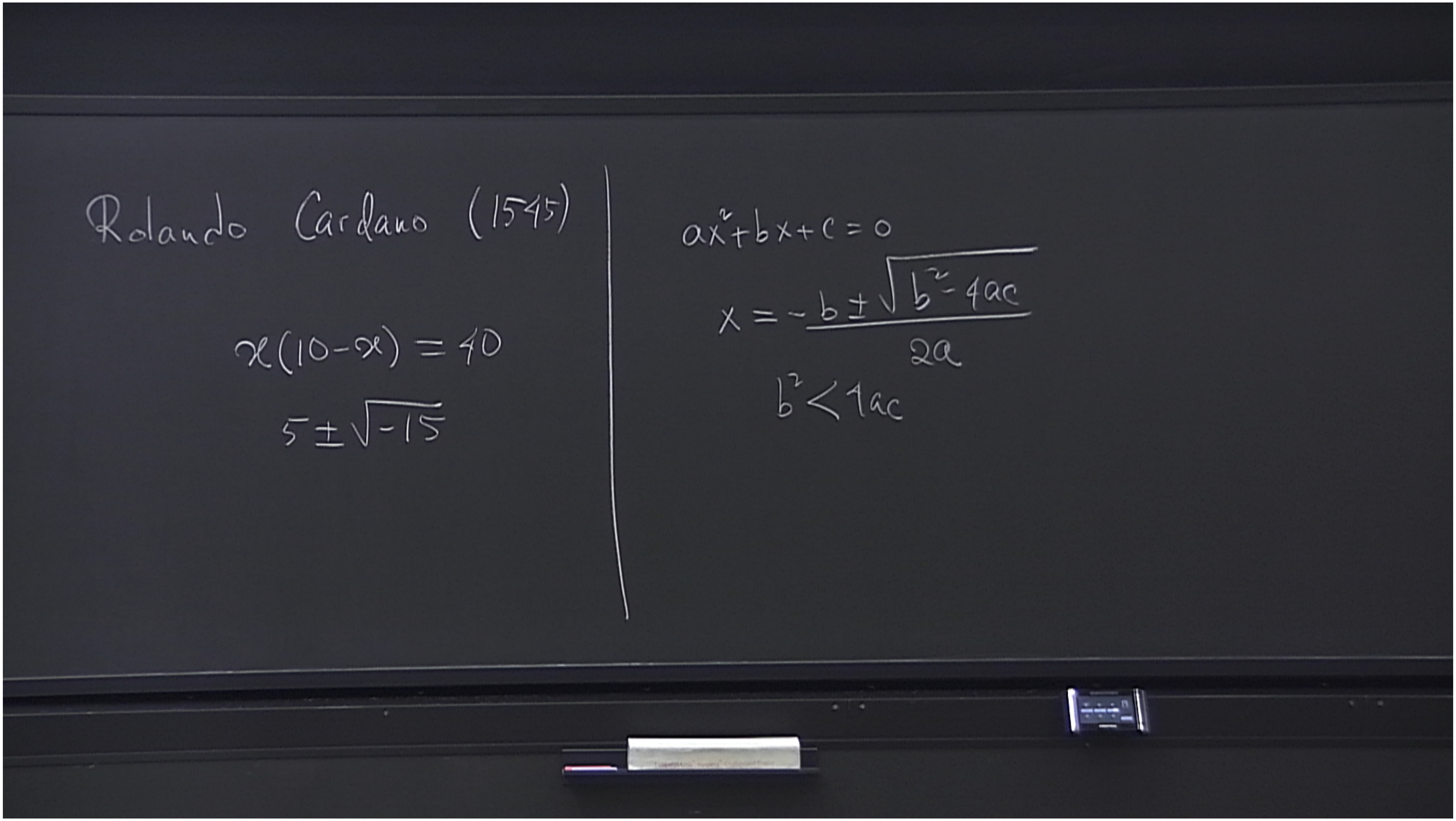
Books

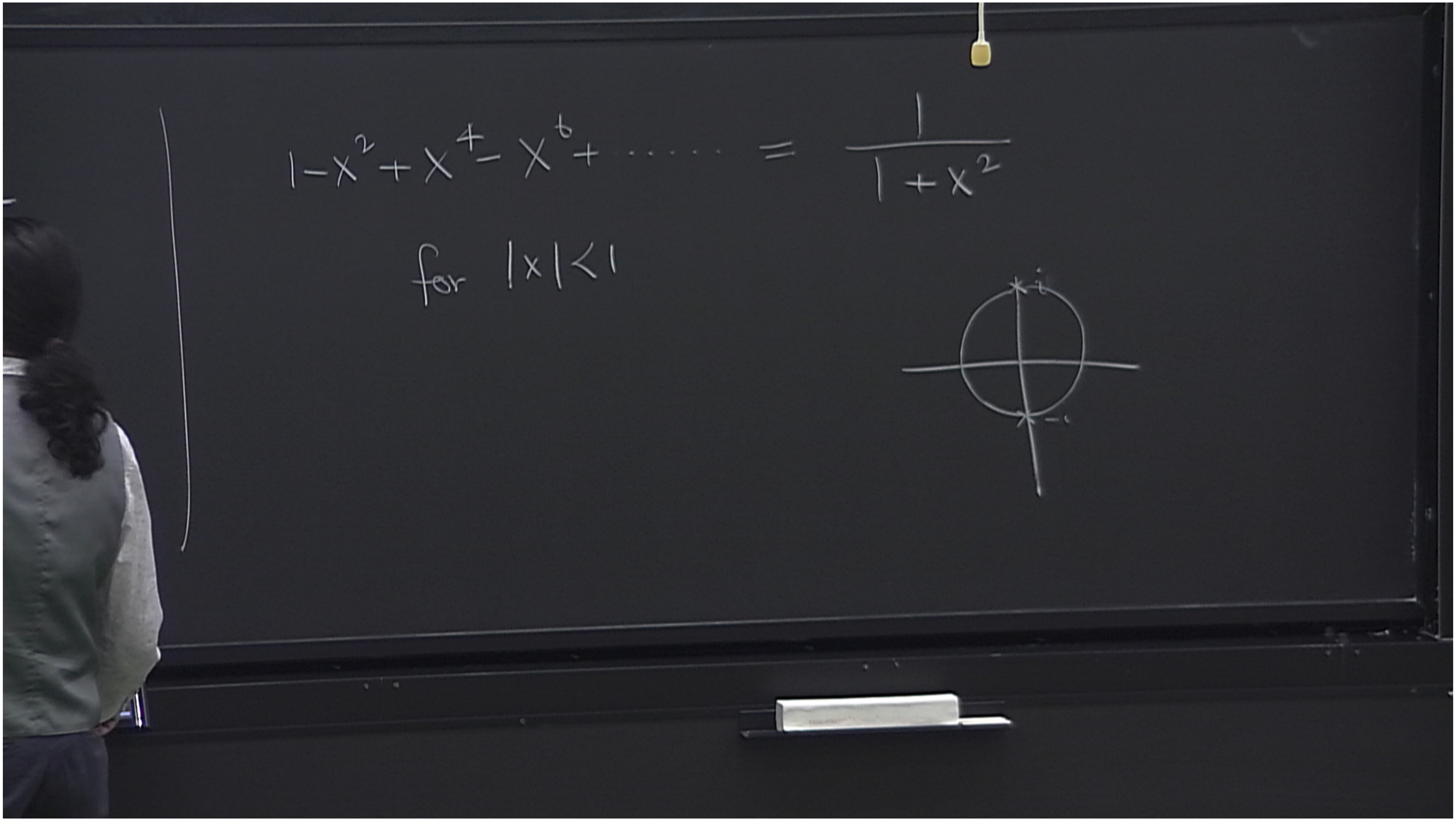
1 Murray R. Speigel ~ Theory and problems of cmplx analysis. Schaum's outline series

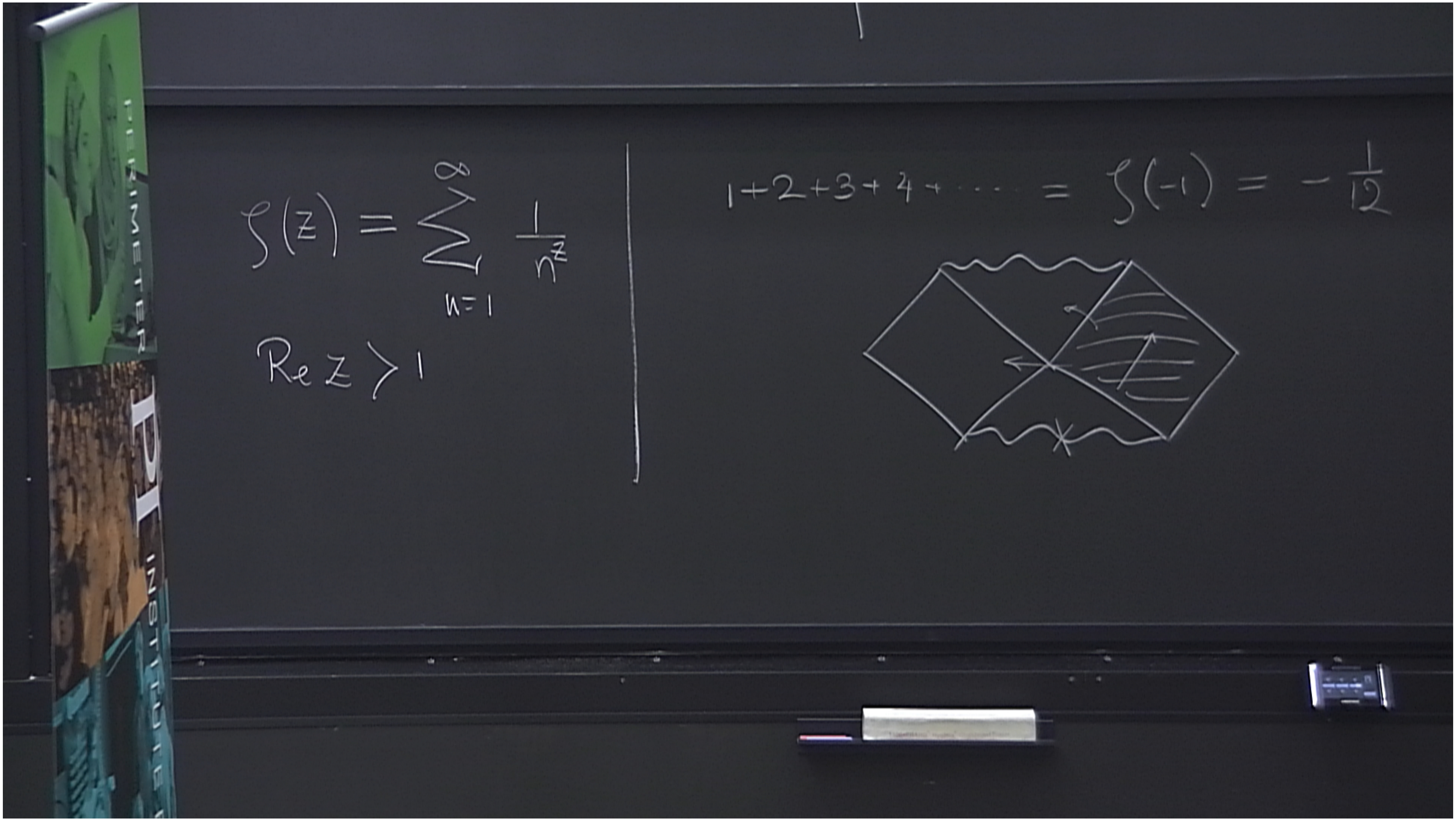
2 Ablowitz & Fokas ~ Complex Analysis and Applications

3 William T. Shaw ~ " with Mathematica

4 Roger Penrose ~ Road to Reality


Rolando Cardano (1545)
$x(10-x) = 40$
$5 \pm \sqrt{-15}$
$ax^2+bx+c=0$
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$b^2 < 4ac$


$1-x^2+x^4-x^6+\cdots\cdots = \frac{1}{1+x^2}$
for $|x|<1$
i
$-i$


$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$
$\mathrm{Re}\, z > 1$
$1+2+3+4+\cdots = \zeta(-1) = -\frac{1}{12}$

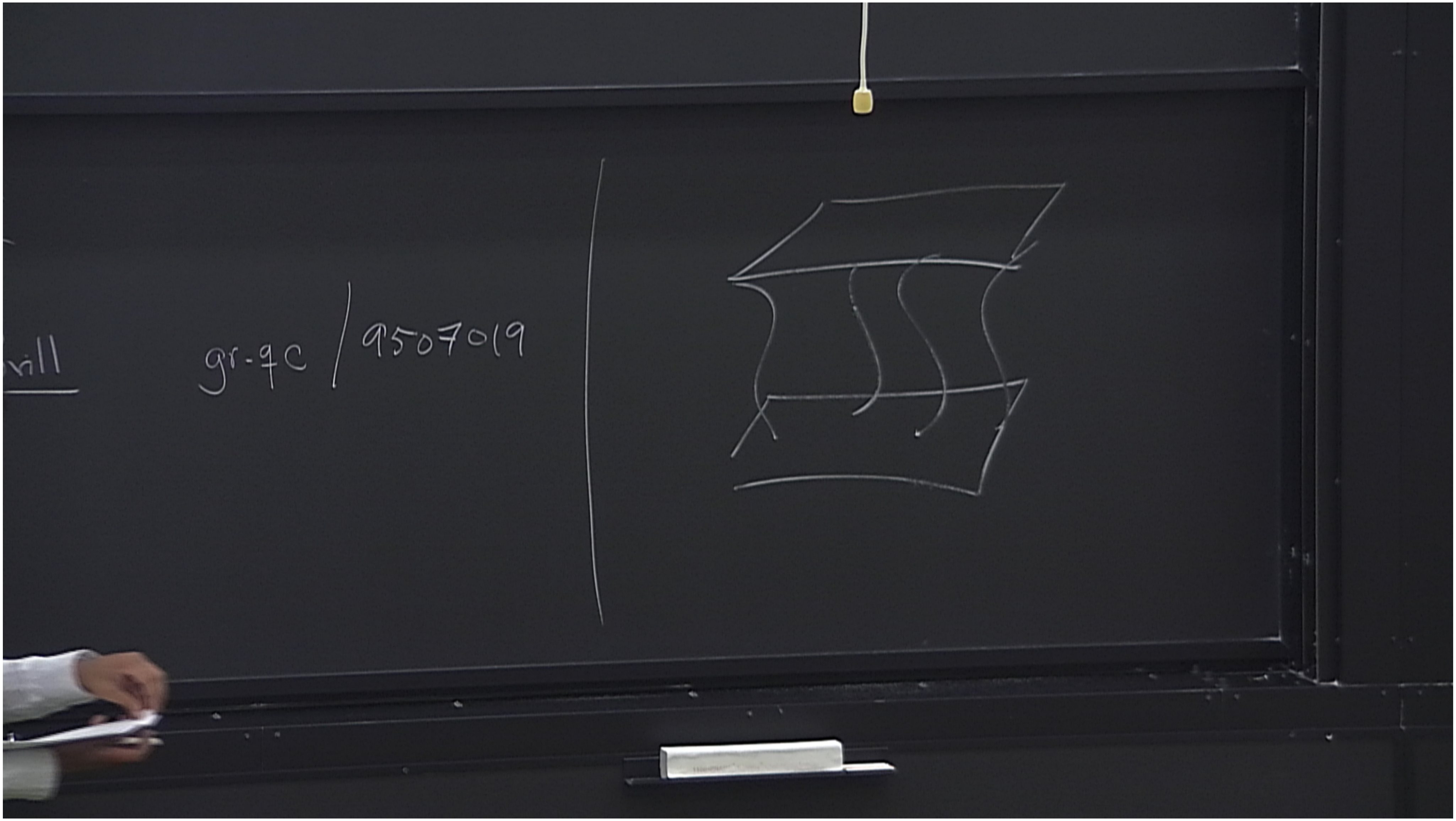
$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$$

$\mathrm{Re}\, z > 1$

$$1+2+3+4+\cdots = \zeta(-1) = -\frac{1}{12}$$

Brill

gr-qc/9507019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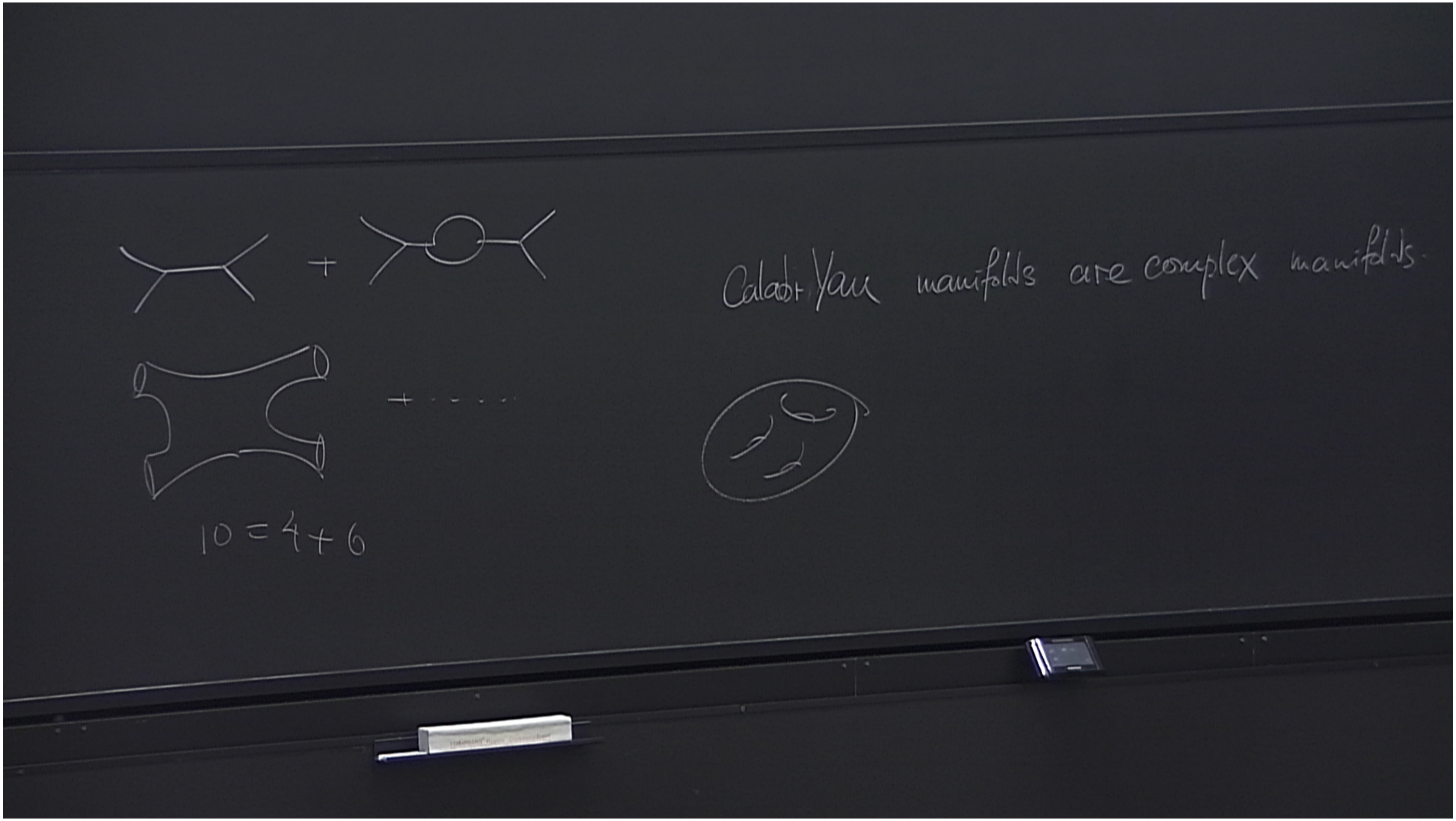
Calabi-Yau manifolds are complex manifolds.
10 = 4 + 6

John Baez

Basics

$(x,y) \longrightarrow x+iy = z$

Commutative, associative div. algebra + $i^2 = -1$

Complex Conjugation $\underset{=z}{(x+iy)} \xrightarrow{*} (x-iy) \equiv \bar{z}$

n Baez

Division

$$\frac{z_1}{z_2} = \frac{x_1 + iy}{x_2 + iy_2} \cdot \frac{x_2 - iy_2}{x_2 - iy_2} = \frac{x_1 x_2 + y_2 y_1}{(x_2^2 + y_2^2)} + i\left(\frac{-x_1 y_2 + y_1 x_2}{(x_2^2 + y_2^2)}\right)$$

John Baez

Basics

$(x, y) \longrightarrow x + iy = z$

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Complex Conjugation $\underbrace{(x+iy)}_{=z} \xrightarrow{*} (x-iy) \equiv \bar{z}$

n Baez

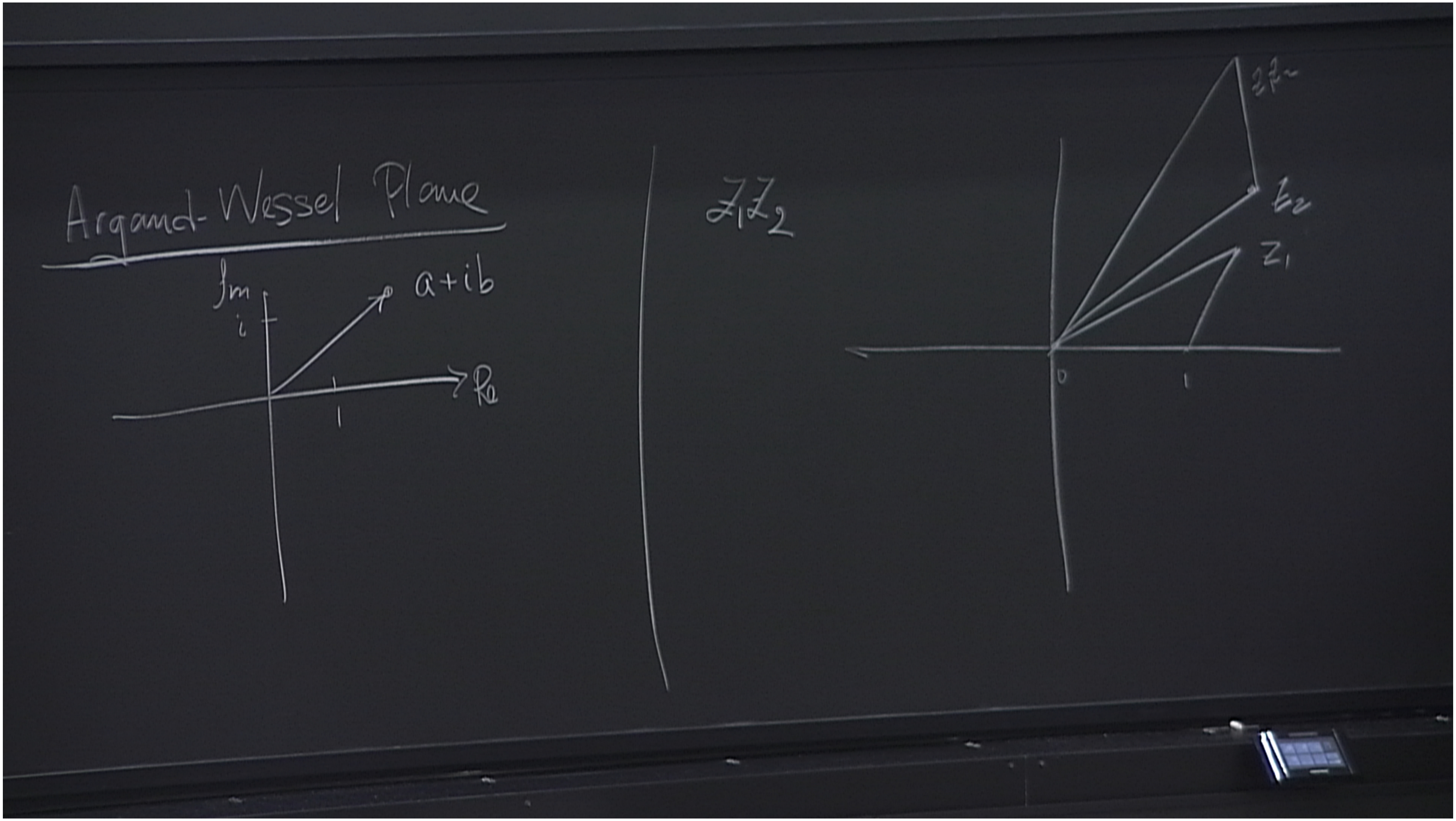
Division

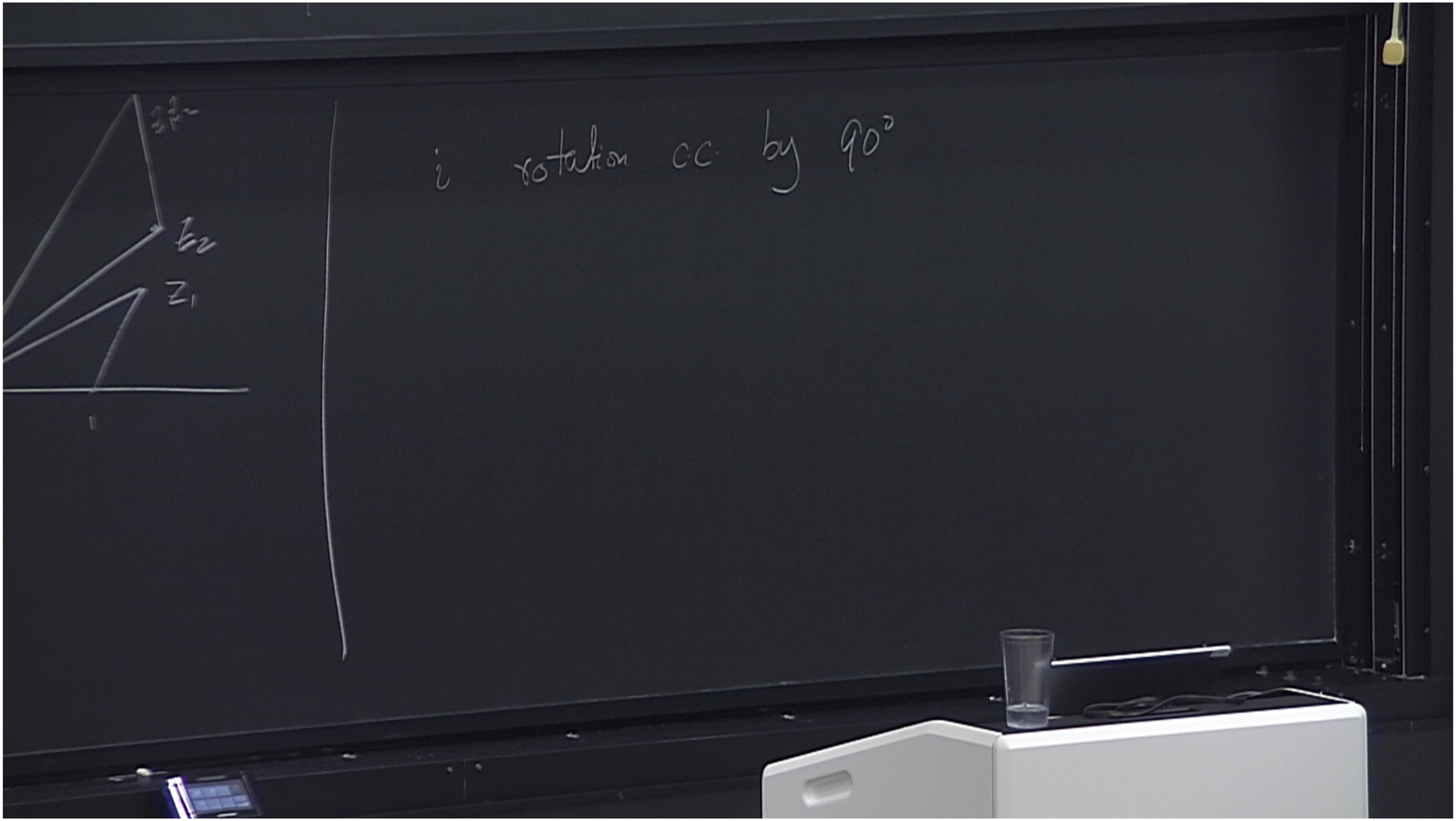
$$\frac{z_1}{z_2} = \frac{x_1+iy}{x_2+iy_2}\cdot\frac{x_2-iy_2}{x_2-iy_2} = \frac{x_1x_2+y_2y_1}{(x_2^2+y_2^2)} + i\,\frac{(-x_1y_2+y_1x_2)}{(x_2^2+y_2^2)}$$

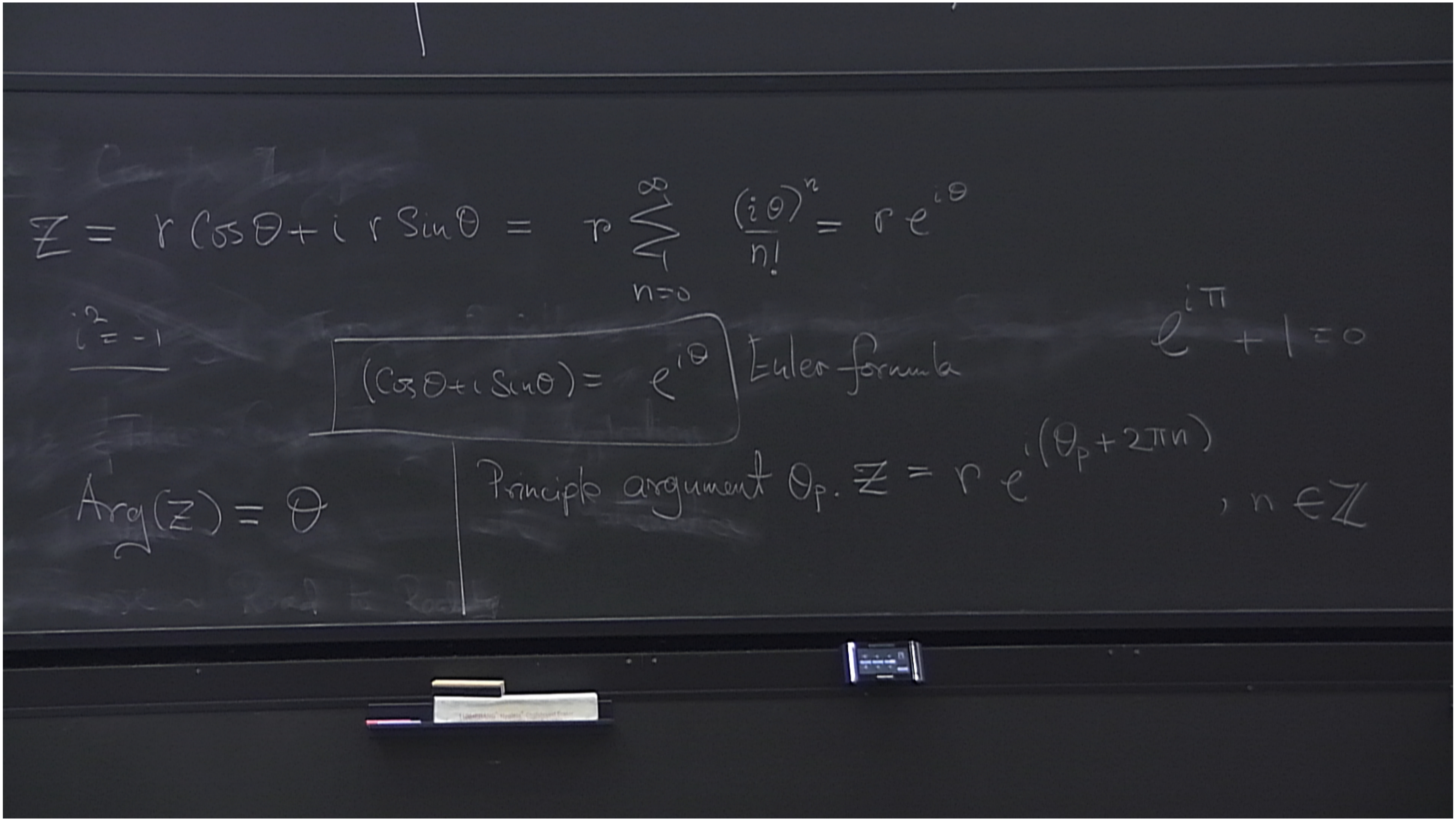
Modulus: $|z| = \sqrt{z\bar{z}} = \sqrt{x^2+y^2}$

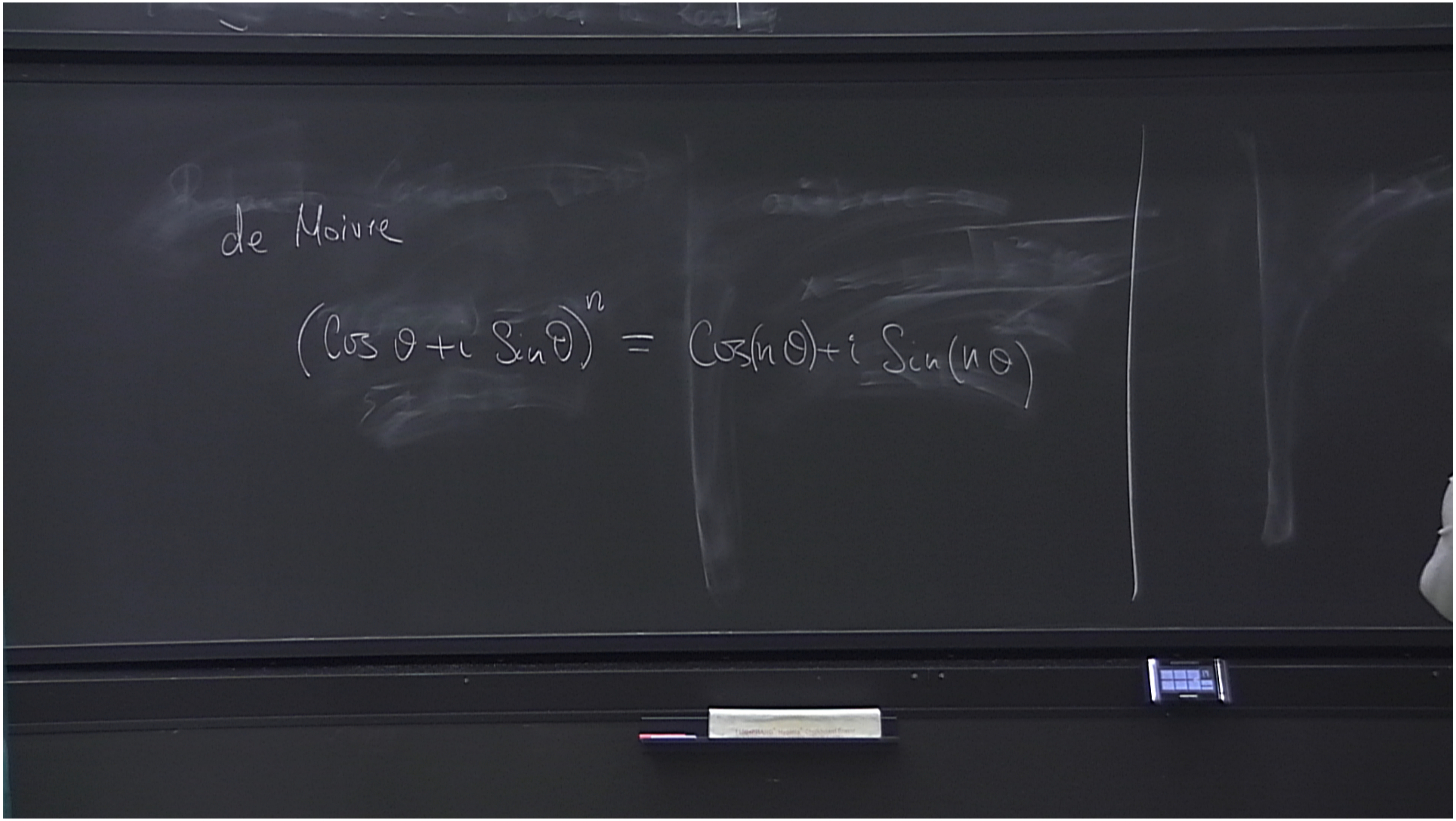
$z_1 = 1+i \quad , \quad z_2 = -1+i$

$|z_1| = |z_2| = \sqrt{2}$


Argand-Wessel Plane
Im
i
a+ib
1
Re
Z_1Z_2
z_1z_2
z_2
z_1
0
1


i rotation cc by 90°
z2
z1
1

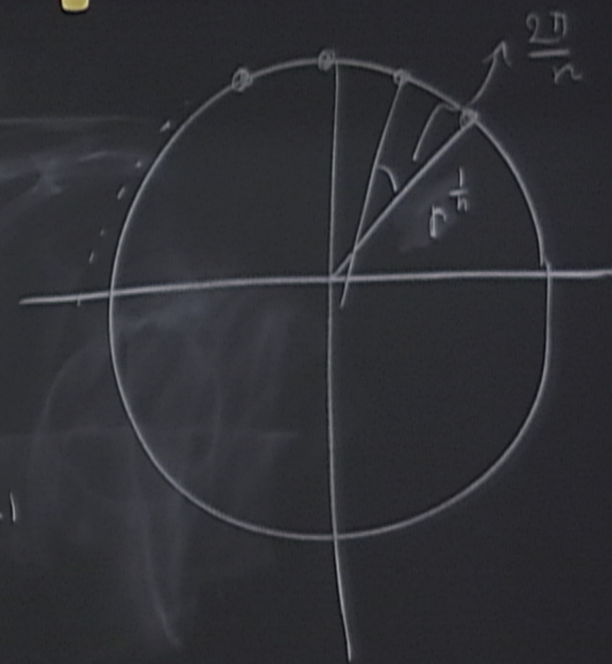

$Z = r\cos\theta + i\, r\sin\theta = r\sum_{n=0}^{\infty}\frac{(i\theta)^n}{n!} = r e^{i\theta}$
$i^2 = -1$
$\boxed{(\cos\theta + i\sin\theta) = e^{i\theta}}$ Euler formula
$e^{i\pi} + 1 = 0$
$\mathrm{Arg}(Z) = \theta$
Principle argument θ_p. $Z = r e^{i(\theta_p + 2\pi n)}$, $n \in \mathbb{Z}$

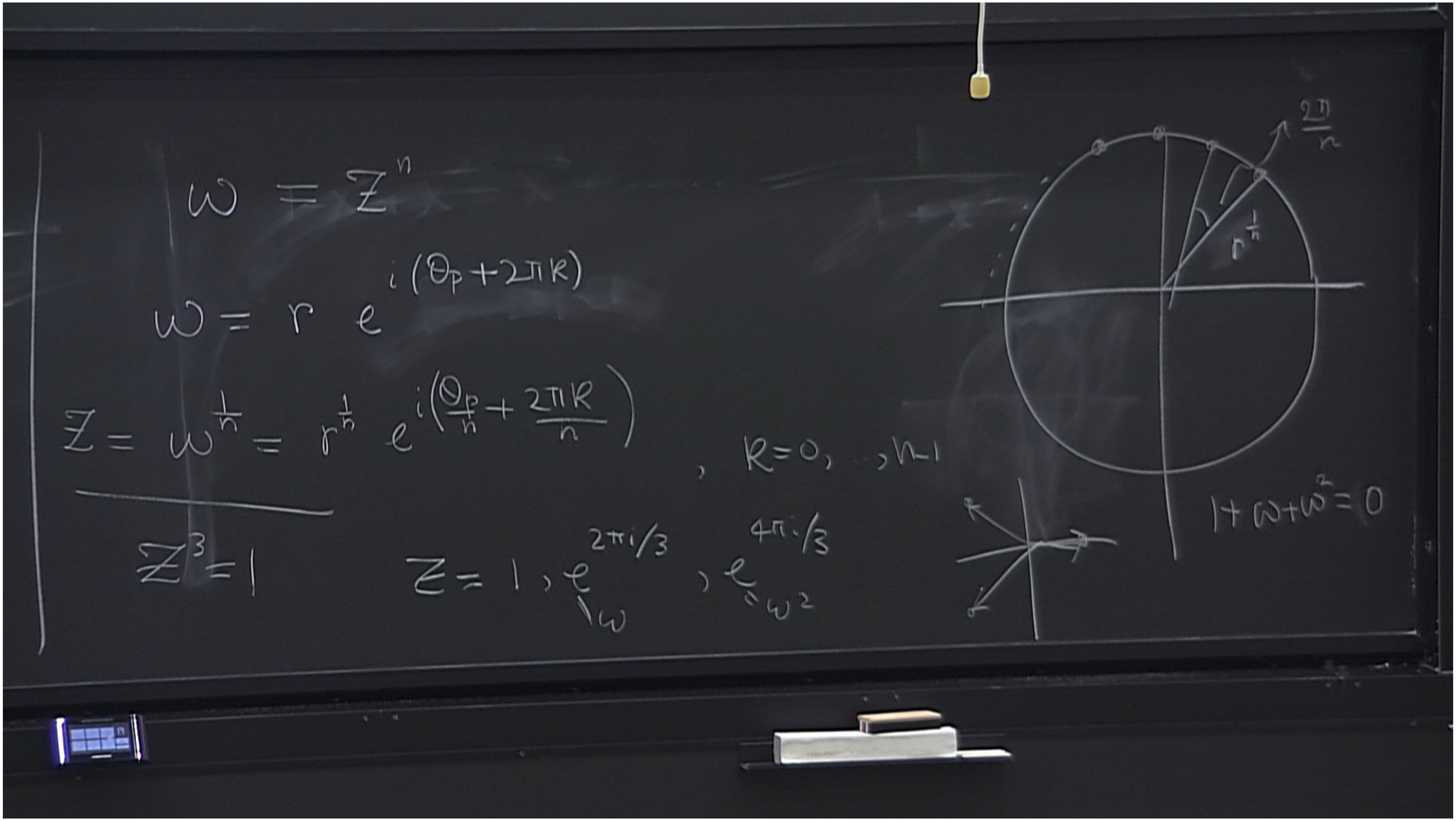

de Moivre
$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

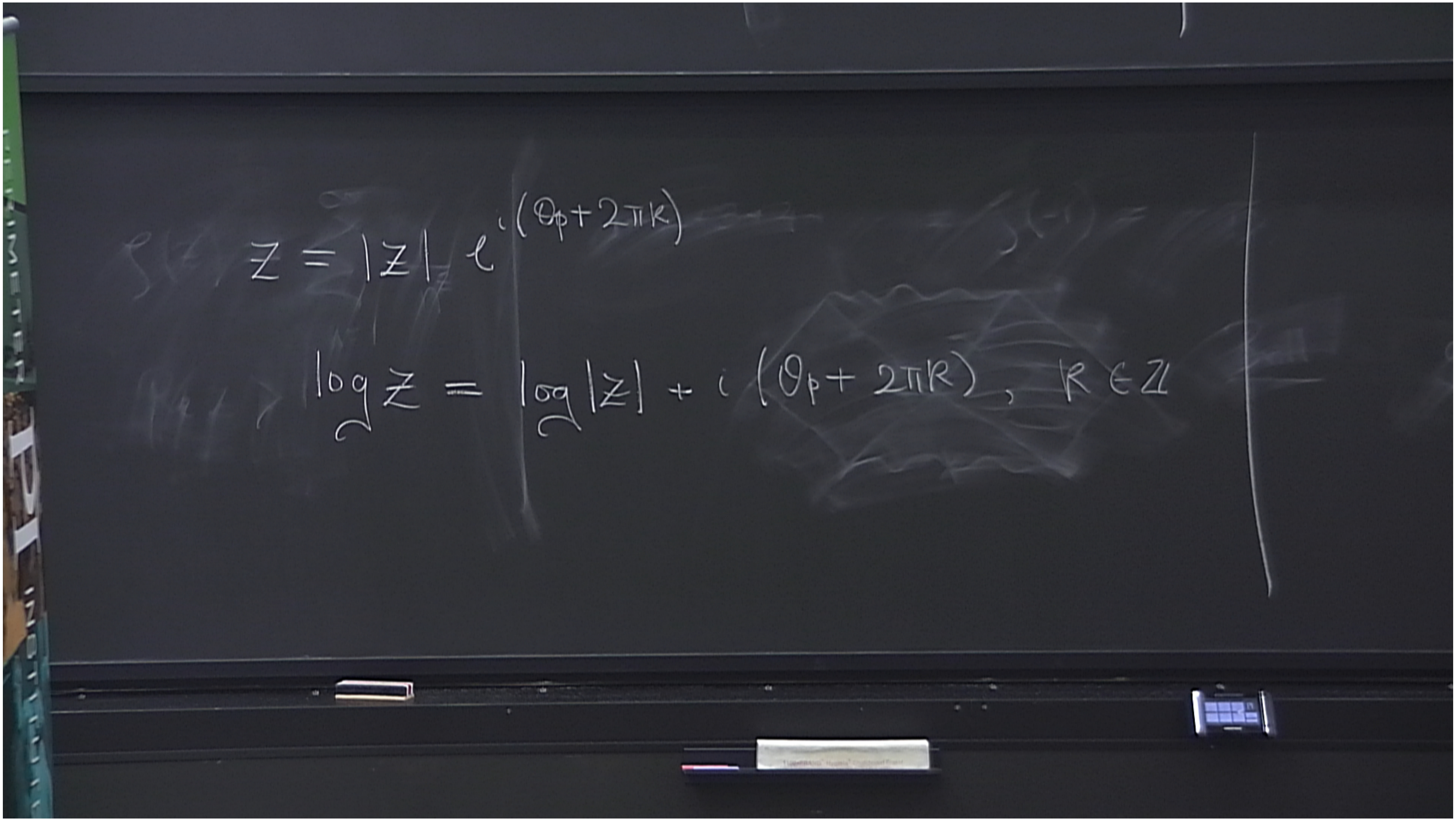
$$\omega = z^n$$

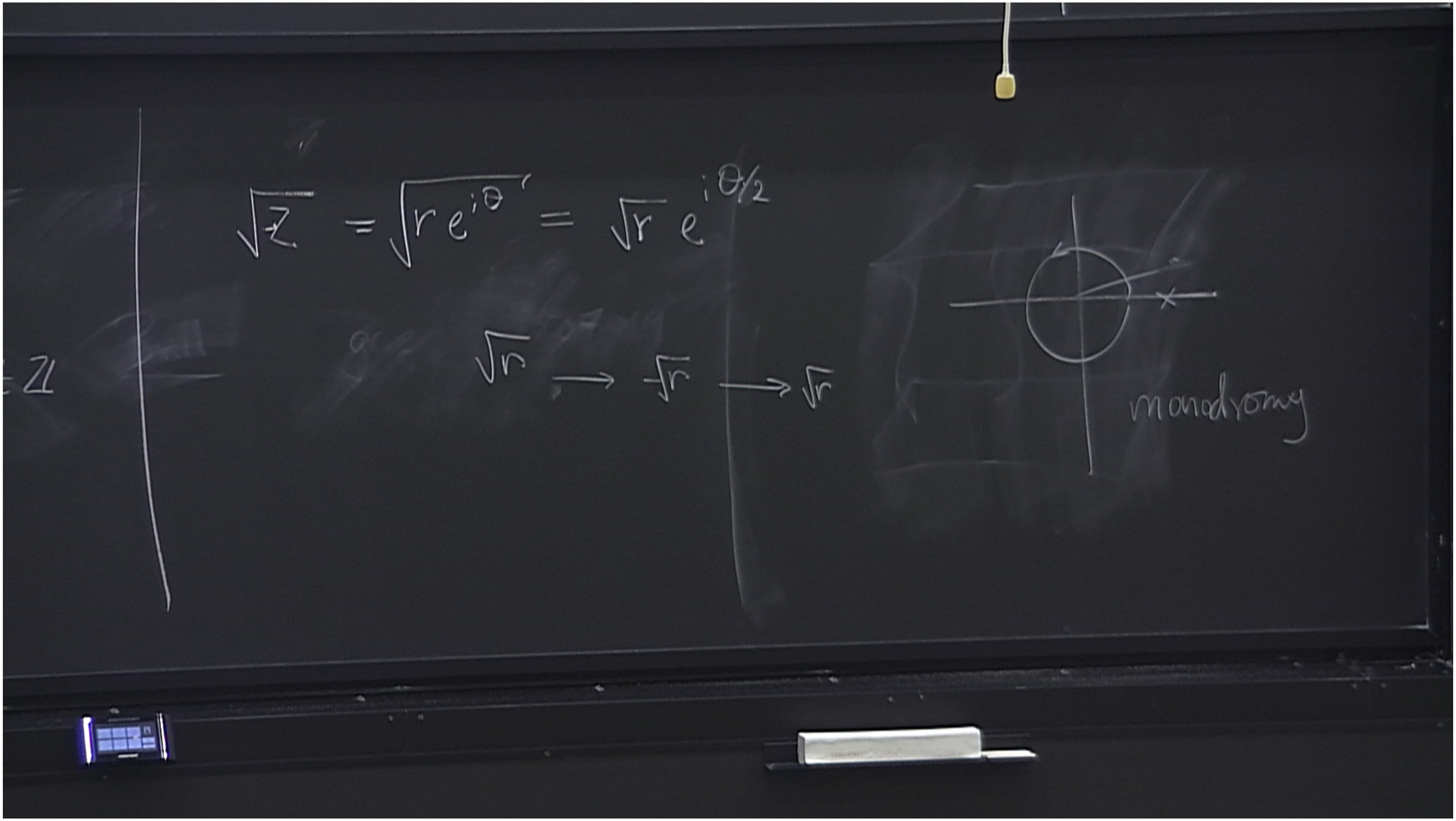
$$\omega = r\, e^{i(\theta_p + 2\pi k)}$$

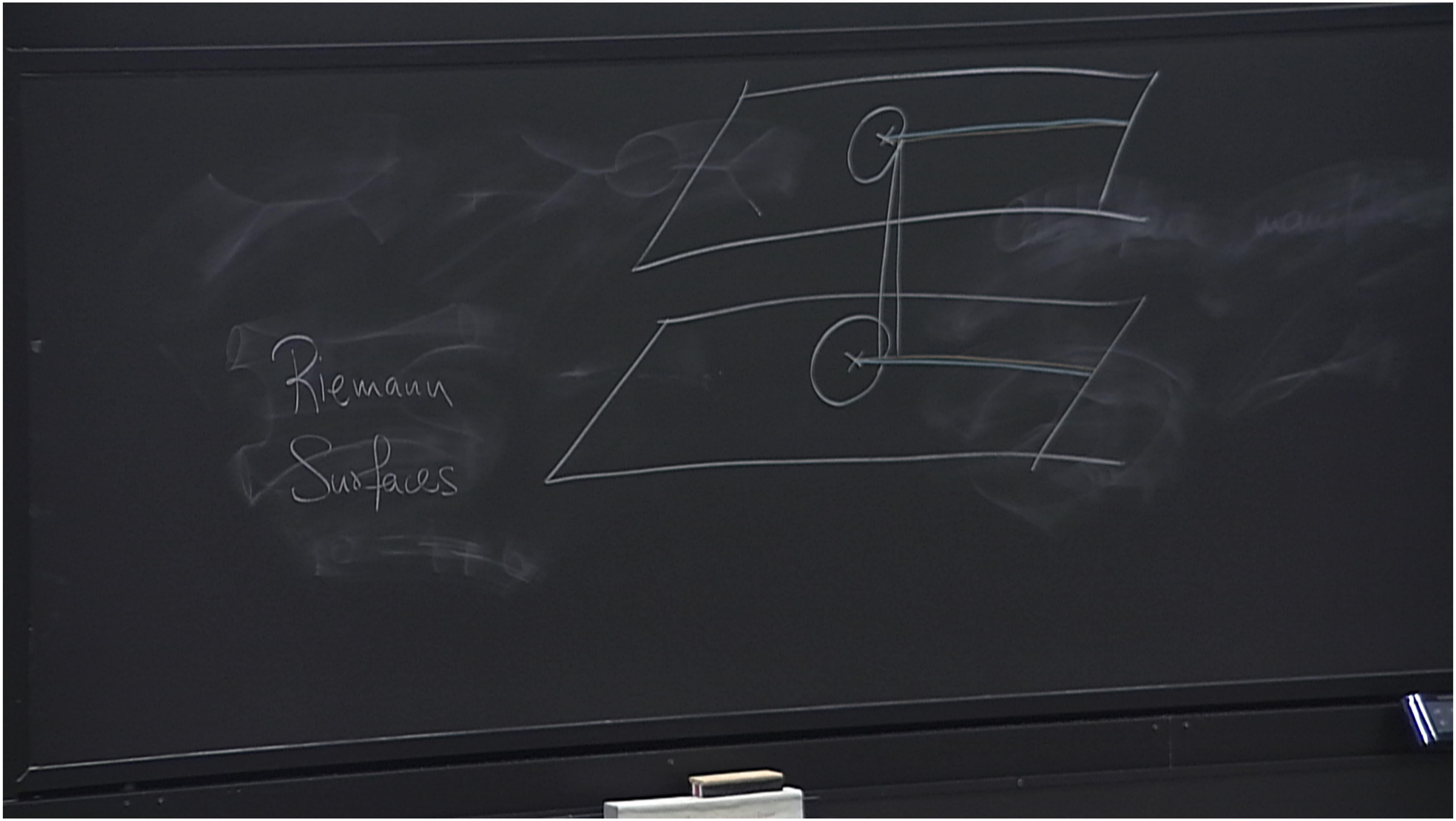
$$z = \omega^{\frac{1}{n}} = r^{\frac{1}{n}}\, e^{i\left(\frac{\theta_p}{n} + \frac{2\pi k}{n}\right)} \quad , \; k = 0, \dots, n-1$$




$\omega = z^n$
$\omega = r\, e^{i(\theta_p + 2\pi k)}$
$z = \omega^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i\left(\frac{\theta_p}{n} + \frac{2\pi k}{n}\right)}$, $k = 0, \dots, n-1$
$z^3 = 1$
$z = 1, e^{2\pi i/3}, e^{4\pi i/3}$
ω
$= \omega^2$
$\frac{2\pi}{n}$
$r^{\frac{1}{n}}$
$1 + \omega + \omega^2 = 0$


$z = |z| \, e^{i(\theta_p + 2\pi k)}$
$\log z = \log|z| + i(\theta_p + 2\pi k), \quad k \in \mathbb{Z}$


$\sqrt{z} = \sqrt{re^{i\theta}} = \sqrt{r}\, e^{i\theta/2}$
$\sqrt{r} \longrightarrow -\sqrt{r} \longrightarrow \sqrt{r}$
monodromy


Riemann
Surfaces