Title: Reconstructing quantum states from local data

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Abstract: We consider the problem of reconstructing global quantum states from local data. Because the reconstruction problem has many solutions in general, we consider the reconstructed state of maximum global entropy consistent with the local data. We show that unique ground states of local Hamiltonians are exactly reconstructed as the maximal entropy state. More generally, we show that if the state in question is a ground state of a local Hamiltonian with a degenerate space of locally indistinguishable ground states, then the maximal entropy state is close to the ground state projector. We also show that local reconstruction is possible for thermal states of local Hamiltonians. Finally, we discuss a procedure to certify that the reconstructed state is close to the true global state. We call the entropy of our reconstructed maximum entropy state the "reconstruction entropy", and we discuss its relation to emergent geometry in the context of holographic duality. This is a joint work with Brian Swingle.

Reconstructing quantum states from local data

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August 12th, 2014

* Joint work with Brian Swingle(Harvard)

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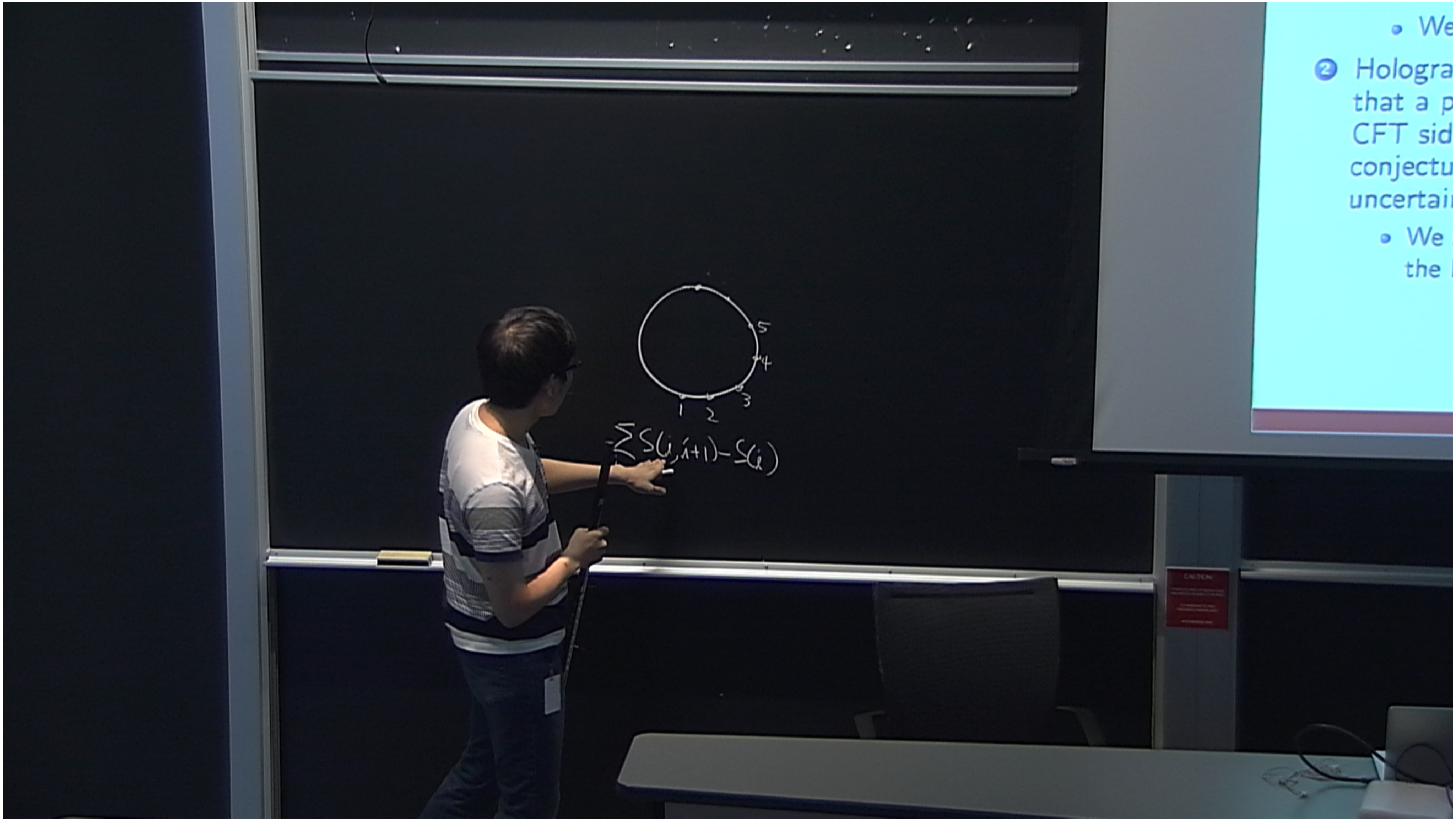
What this talk is about

1. Given a set of local reduced density matrices, we establish a procedure to reconstruct a global state that is consistent with the local data.
2. We use the reconstruction procedure to resolve some open problems in topological order and holographic duality.

Teaser trailer for the open problems

1. Topological order: given a set of local reduced density matrices for the ground state of some anyon model, can one determine the universal properties of the low-energy excitations?
   - We show that this is possible, with some caveats.
2. Holographic duality(AdS_3/CFT_{1+1}): Balasubramanian et al. showed that a particular linear combination of entanglement entropy on the CFT side coincides with the length of the bulk curves in AdS_3. They conjectured that the length of the bulk curves are related to some uncertainty in reconstructing the global state from the local data.
   - We show that this is not always possible, assuming our knowledge on the local data is sufficiently accurate.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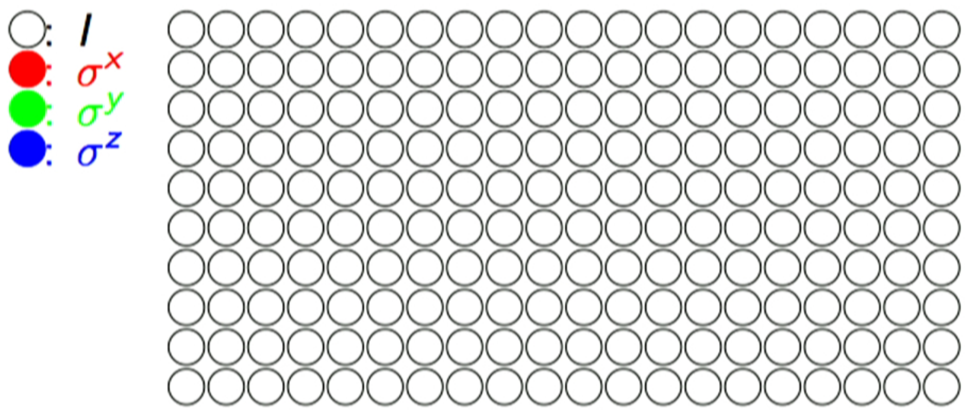
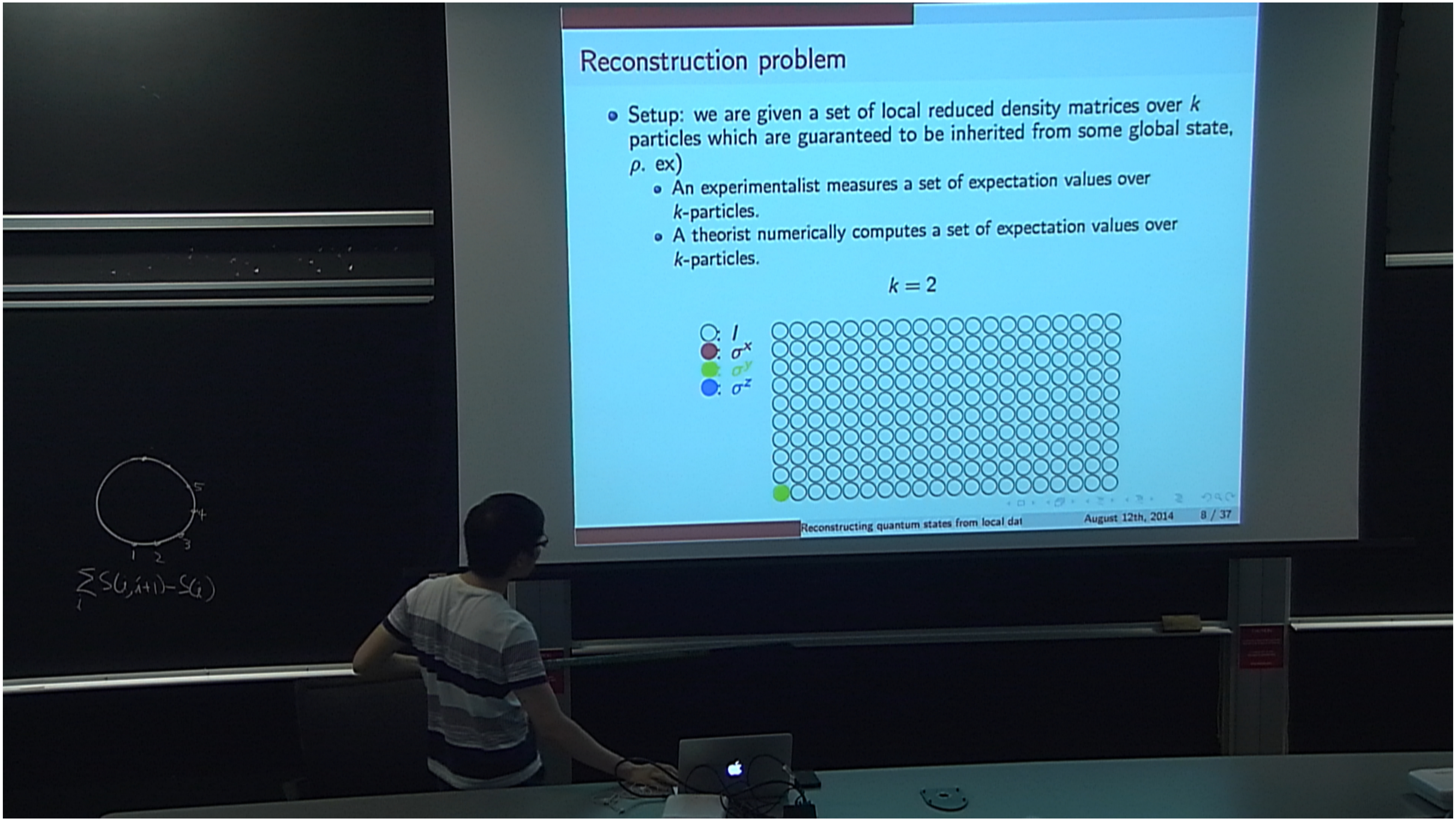
Key questions

- $\boxed{\text{What does reconstruction mean in general?}}$
- What is our reconstruction procedure?
- When does it work?
- Why is it useful?

Reconstruction problem

- Setup: we are given a set of local reduced density matrices over k particles which are guaranteed to be inherited from some global state, ρ. ex)
  - An experimentalist measures a set of expectation values over k-particles.
  - A theorist numerically computes a set of expectation values over k-particles.





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Reconstruction problem

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  - An experimentalist measures a set of expectation values over k-particles.
  - A theorist numerically computes a set of expectation values over k-particles.
- Question: determine a global property of the system from the local reduced density matrices. ex)
  - Determine the global state.
  - Determine whether the prepared state is close to some ideal state they wanted to create, e.g., Haldane chain.
  - Determine whether the system supports topological ground state degeneracy or not.

Reconstruction problem

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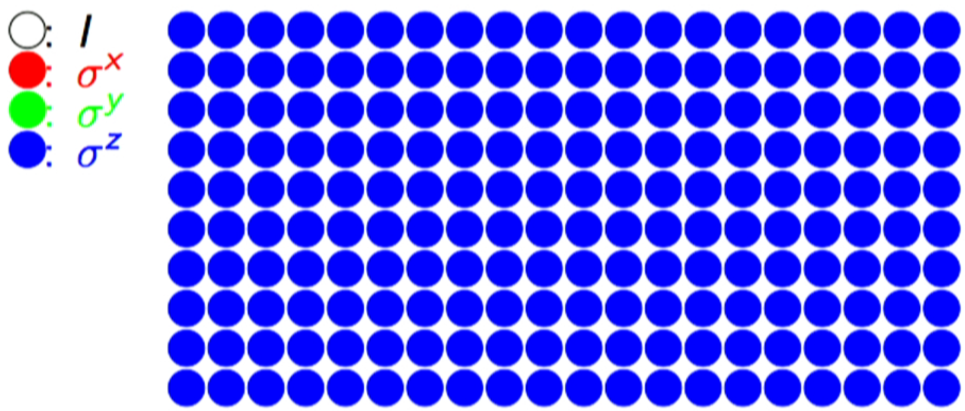
* This may seem like a hopeless task. ex)



Figures of merit for reconstruction procedures

1. Generality: for which states does the reconstruction procedure works well?
2. Certifiability: is it possible to "prove" that the reconstructed state is close to the true global state?
3. Efficiency: how fast can you reconstruct the global state?

Where our work stands

Reconstruction procedure	Generality	Certifiability	Efficiency
Cramer et al.(2011)	MPS	Yes	Yes
Landon-Cardinal&Poulin(2012)	1D MERA	No	Yes
Baumgratz et al.(2013)	MPO	No	Yes
Kim (2014)	2D gapped[1]	Yes	Yes
Maximum entropy method[2]	Unique GS	Yes	No
Maximum entropy method	TO-GS	No	No
Maximum entropy method	$T > 0$	No	No

Our contributions

There are subtleties all over the place. Please ask if you are in doubt!

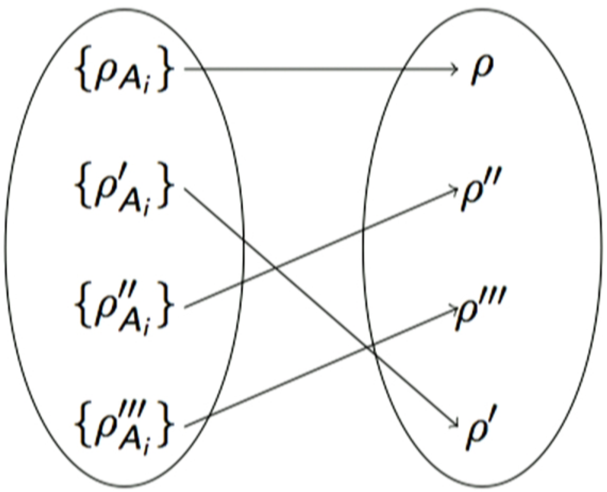
[1] Needs area law assumption.
[2] Jaynes(1957), Chen et al.(2012)

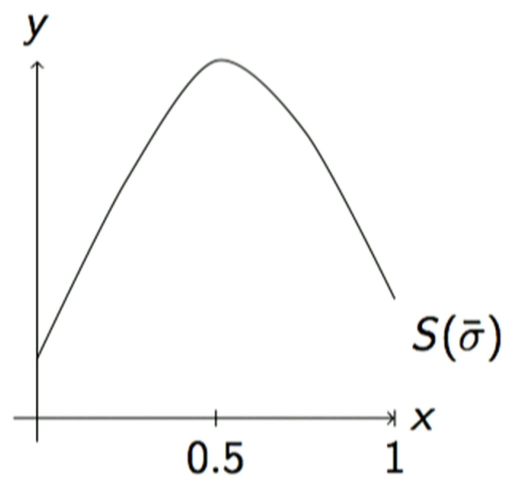
Maximum entropy reconstruction is a function

$$\mathcal{R}(\{\rho_{A_i} | i = 1, \cdots, N\}) \equiv \arg\max_{\sigma_{A_i} = \rho_{A_i}} S(\sigma)$$

$\mathcal{R}$ is well-defined.

Because $S(\bar{\sigma}) = S(x\sigma_1 + (1-x)\sigma_2)$ is strictly concave.





Maximum entropy reconstruction is certifiable

Theorem (Swingle and Kim (2014))

$$\frac{1}{8}\|\mathcal{R}(\{\rho_{A_i}\}) - \rho\|_1^2 \leq S(\mathcal{R}(\{\rho_{A_i}\})) - S(\rho).$$

Clearly, $S(\mathcal{R}(\{\rho_{A_i}\}))$ has a special meaning.

1. It is defined completely by the local data.
2. If it is $\approx S(\rho)$, the reconstructed state is close to the original state(ρ)!
3. If it is ≈ 0, we can be sure that the reconstructed state is close to the original state without knowing anything about the original state!

So, we call

$$S_{rec}(\{\rho_{A_i}\}) = S(\mathcal{R}(\{\rho_{A_i}\}))$$

as the **reconstruction entropy.**

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Maximum entropy reconstruction is robust

Theorem (Swingle and Kim (2014))

$$\frac{1}{8}\|\mathcal{R}^{\epsilon}(\{\rho_{A_i}\}) - \rho\|_1^2 \leq S(\mathcal{R}^{\epsilon}(\{\rho_{A_i}\})) - S(\rho).$$

Clearly, $S(\mathcal{R}^{\epsilon}(\{\rho_{A_i}\}))$ has a special meaning.

1. It is defined completely by the local data.
2. If it is $\approx S(\rho)$, the reconstructed state is close to the original state(ρ)!
3. If it is ≈ 0, we can be sure that the reconstructed state is close to the original state without knowing anything about the original state!

So, we call

$$S^{\epsilon}_{rec}(\{\rho_{A_i}\}) = S(\mathcal{R}^{\epsilon}(\{\rho_{A_i}\}))$$

as the $\epsilon-$**reconstruction entropy.**

Unique ground state

For a unique ground state ρ of some local Hamiltonian, $S_{rec}(\{\rho_{A_i}\}) = 0$.

- ex) $H = \sum_{i=1}^{N} \sigma_i^z \sigma_{i+1}^z + J\sigma_i^x + J'\sigma_i^z$

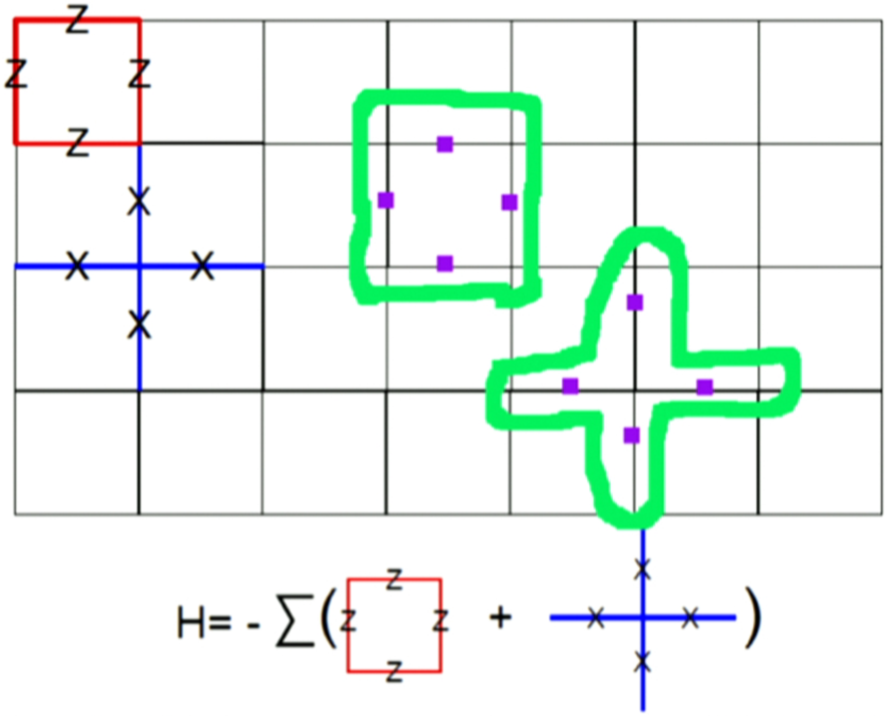
Sketch of the argument: without loss of generality, shift the eigenvalues of H so that the ground state energy is 0. Let $\rho_{i,i+1} = \rho'_{i,i+1}$ for all i,

$$\mathrm{Tr}(\rho' H) = \mathrm{Tr}(\rho H) = 0 \geq \Delta \mathrm{Tr}(\rho' P_e)$$

*P_e : projector onto the excited subspace.
** Δ : energy gap.

Topological ground states

For a ground state of a topologically ordered system, $S_{rec}(\{\rho_{A_i}\}) = \log N$, which is equal to the entropy of the maximally mixed state over the ground state subspace.(N: topological ground state degeneracy.)



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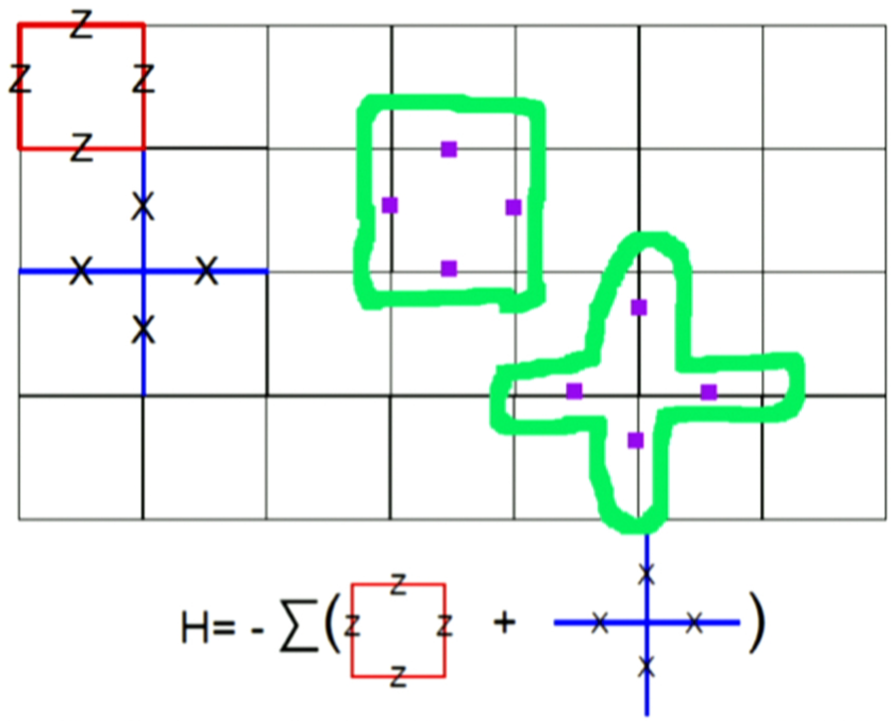

Thermal states at finite T

Recall that thermal state is a maximum entropy state under an energy constraint $\langle H \rangle = E$.

Theorem (Swingle and Kim (2014))

$$\frac{1}{8}\|\mathcal{R}(\{\rho_{A_i}\}) - \rho\|_1^2 \leq S_{rec}(\{\rho_{A_i}\}) - S(\rho).$$

1. $\mathcal{R}(\{\rho_{A_i}\})$ has the same energy as ρ.
2. Therefore, $S_{rec}(\{\rho_{A_i}\}) \leq S(\rho)$.
3. But $S_{rec}(\{\rho_{A_i}\}) \geq S(\rho)$ by the above theorem.
4. Therefore, $S_{rec}(\{\rho_{A_i}\}) = S(\rho)$ and $\mathcal{R}(\{\rho_{A_i}\}) = \rho$.

For thermal states, reconstruction entropy is equal to the thermal entropy!

What if there is an error?

So far we assumed that we know the density matrices exactly. What if we have a limited precision? ($\mathcal{R}^\epsilon$ as opposed to $\mathcal{R}$). If the error scales as $\sim \frac{1}{\text{poly}(n)}$ (n: number of particles), the aforementioned results remain intact under the following assumptions:

- Locality
- Bounded interaction strength

Key questions

- What does reconstruction mean in general?
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Excited states?

We have so far studied reconstruction entropy for the ground state and the thermal states. It's only natural to study the excited states at this point. Unfortunately, we cannot proceed without extra assumptions. Assuming Eigenstate Thermalization Hypothesis(ETH),[Deutsch (1991), Srednicki (1994)],

$$\| Tr_{A^c}(|E\rangle\langle E|) - Tr_{A^c}(\frac{e^{-H/T(E)}}{Z})\|_1 \leq \epsilon$$

for an eigenstate $|E\rangle$ with energy E. If ϵ is sufficiently small, e.g., $\frac{1}{\text{poly}(n)}$),

$$S_{rec}(\mathcal{R}(\{\rho_{A_i}\})) \approx S(\frac{e^{-H/T(E)}}{Z}))$$

Topological ground states

Given a set of local reduced density matrices for the ground state of some anyon model, can one determine the universal properties of the low-energy excitations?

Yes.

1. Given $\{\rho_{A_i}\}$, construct $\mathcal{R}(\{\rho_{A_i}\}) \approx \frac{1}{\mathrm{Tr}(P)} P$.($P$: projector onto the ground state subspace)
2. Diagonalize the ground state subspace.
3. Obtain the minimum entangled states within the ground state subspace, from which one can deduce the quasi-particles braiding/statistics. [Zhang et al. (2012)]

$$\{\rho_{A_i}\} \xrightarrow{\mathcal{R}} \frac{1}{N} \sum_i |\psi_i\rangle \langle\psi_i| \xrightarrow{\text{Zhang et al.}} S \text{ and } U \text{ matrices.}$$

Quantum critical point

Given a set of local reduced density matrices for the ground state of some 1D critical system, can one determine the central charge?
Yes.

1. Given $\{\rho_{A_i}\}$, construct $\mathcal{R}(\{\rho_{A_i}\}) \approx |\psi_0\rangle\langle\psi_0|.(|\psi_0\rangle$: ground state)
2. Use $S(A) \sim \frac{c}{3}\log l_A$ to extract c.

$$\{\rho_{A_i}\} \xrightarrow{\mathcal{R}} G.S. \xrightarrow{S(A)\sim\frac{c}{3}\log l_A} c.$$

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