Title: Classical statistics: resolving paradoxes since 1749

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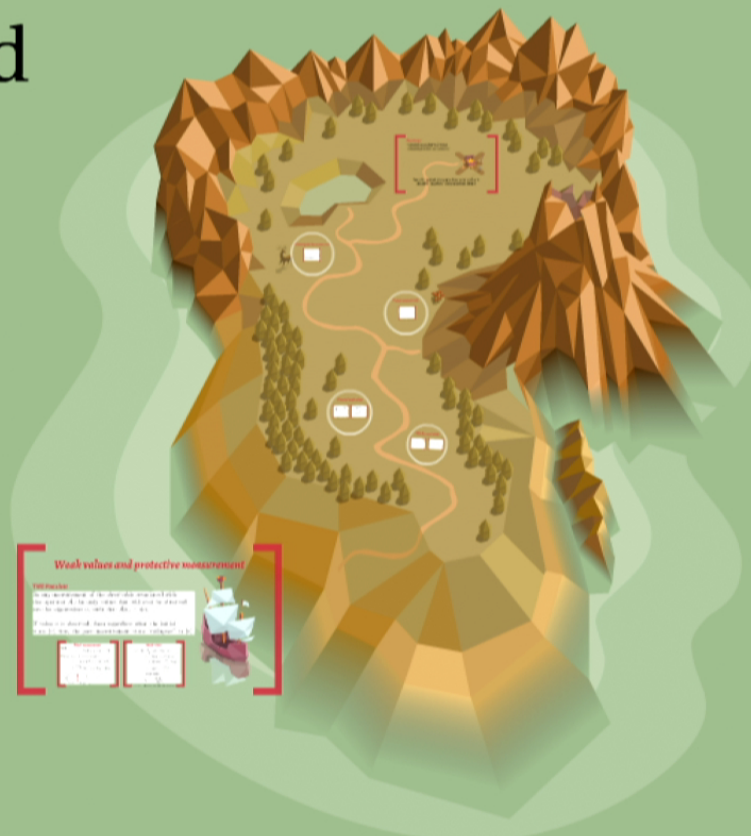
URL: http://pirsa.org/14080005

Abstract: There has been renewed interest in the effect that pre and postselection has on the foundations of quantum theory. Often, but not solely, in conjunction with weak measurement, pre and postselection scenarios are said to simultaneous create and resolve paradoxes. These paradoxes are said to be profound quandaries which bring us closer to the resolving the mysteries of the quantum. Here I was show that the same effects are present in classical physics when postselection and disturbance are allowed. In particular, I will demonstrate that anomalous weak values and protective measurements are already present in classical theory, thereby showing that these effects do not represent something uniquely quantum nor something that ought to be thought of as paradoxical. This is joint work with Josh Combes and Matt Leifer.

Classical postselection and disturbance "paradoxes"

Chris Ferrie (CQuIC/UNM)

Joint work with:
Josh Combes
Matt Leifer



Weak values and protective measurement

THE Postulate

In any measurement of the observable associated with the operator $\hat{A}$, the only values that will ever be observed are the eigenvalues a, such that $\hat{A}|a\rangle = a|a\rangle$.

If value a is observed, then regardless what the initial state $|\psi\rangle$ was, the post-measurement state "collapses" to $|a\rangle$.

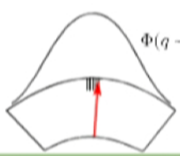
Weak measurement

Recall:

$$U|\psi\rangle|\Phi\rangle = \sum_a |a\rangle\langle a|\psi\rangle e^{-iaP}|\Phi\rangle.$$

What about some other final state $|\phi\rangle|q\rangle$?

$$\langle\phi|\langle q|U|\psi\rangle|\Phi\rangle = \sum_a \langle\phi|a\rangle\langle a|\psi\rangle\Phi(q-a).$$



$$\Phi(q-a) \approx \Phi(q) - a\Phi'(q)$$

Weak values

$$\begin{aligned}\langle\phi|\langle q|U|\psi\rangle\Phi\rangle &= \sum_a \langle\phi|a\rangle\langle a|\psi\rangle\Phi(q-a) \\ &\approx \langle\phi|\psi\rangle\Phi(q) - \langle\phi|A|\psi\rangle\Phi'(q) \\ &= \langle\phi|\psi\rangle\left(\Phi(q) - \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}\Phi'(q)\right) \\ &= \langle\phi|\psi\rangle\Phi\left(q - \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}\right)\end{aligned}$$

weak value:

$$a_w = \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}$$

Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. 60, 1351 (1988)

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To measure $\hat{A}$, couple the system to a "meter" via the interaction:

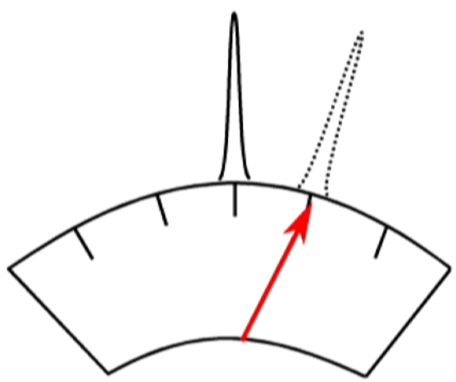
$$U = e^{-i\hat{A}\otimes\hat{P}}.$$

The initial state of the system and meter $|\psi\rangle|\Phi\rangle$ becomes:

$$U|\psi\rangle|\Phi\rangle = \sum_a |a\rangle\langle a|\psi\rangle e^{-ia\hat{P}}|\Phi\rangle.$$

We want to understand the transition to some other state $|a'\rangle|q\rangle$:

$$\langle a'|\langle q|U|\psi\rangle|\Phi\rangle = \langle a'|\psi\rangle\Phi(q-a').$$



$$A = \sum a |a\rangle\langle a|$$

$$f(A) = \sum f(a) |a\rangle\langle a|$$

Weak measurement

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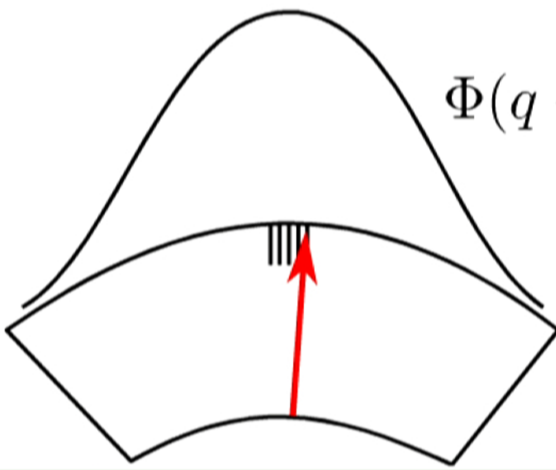

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Weak values

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$$= \langle\phi|\psi\rangle\left(\Phi(q) - \frac{\langle\phi|A|\psi\rangle}{\langle\phi|\psi\rangle}\Phi'(q)\right)$$
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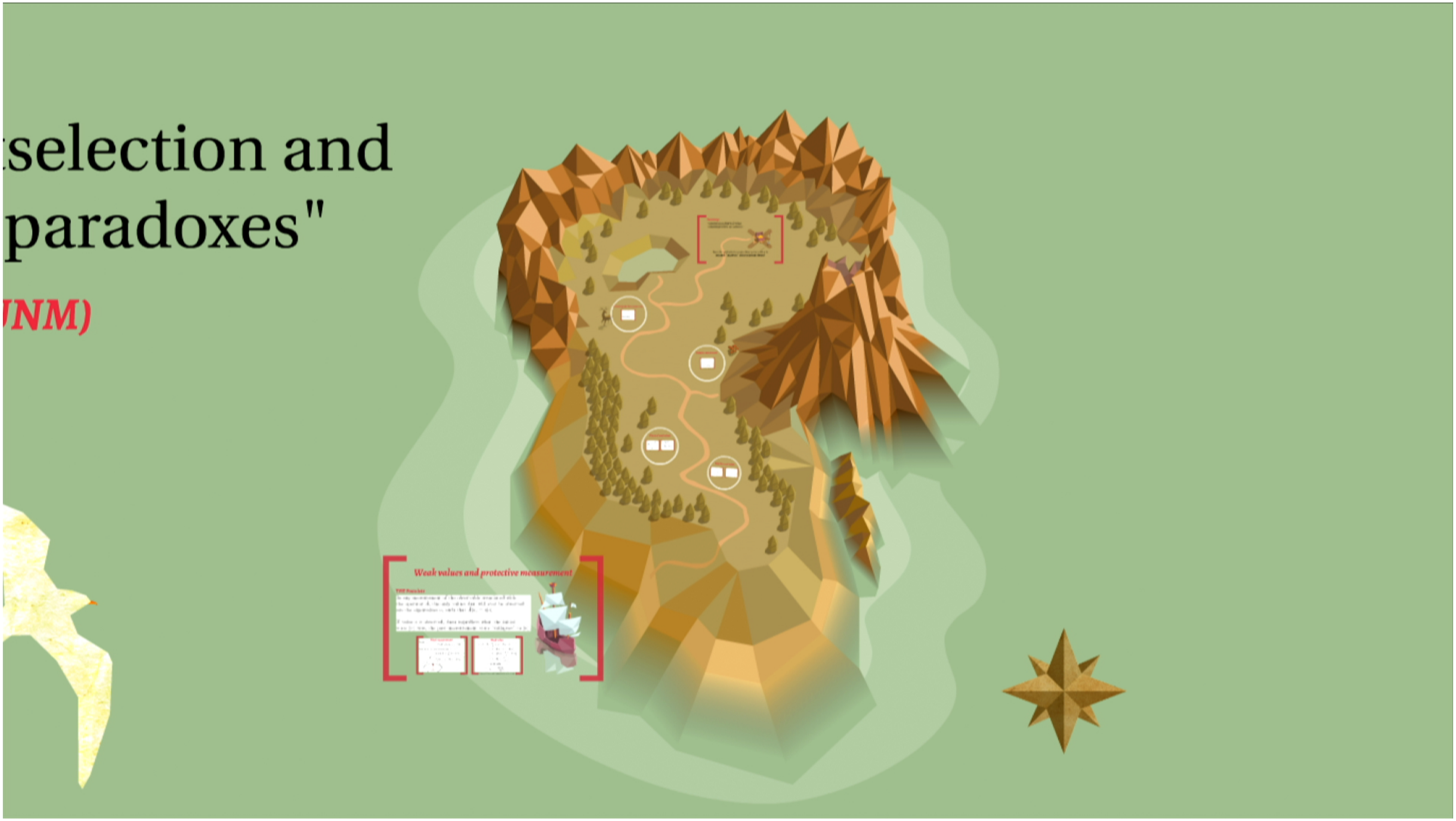
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Weak values and protective measurement

With $s \in \{\pm 1\}$ and $\lambda \in [0, 1]$ consider the POVM elements:

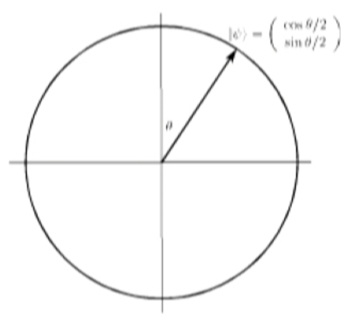
$$E_s = \frac{1}{2}(\mathbb{I} + s\lambda\sigma_z).$$

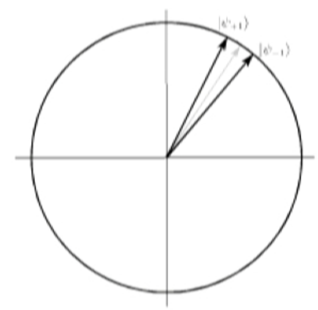
Check:

$$\lambda = 0 \Rightarrow E_s = \mathbb{I}/2,$$
$$\lambda = 1 \Rightarrow E_s = |s\rangle\langle s|.$$

With $\lambda \approx 0$, we use the Kraus operators:

$$M_s = \sqrt{E_s} = \sqrt{\frac{1}{2}}\left(\mathbb{I} + \frac{s\lambda}{2}\sigma_z\right) + O(\lambda^2).$$





J. L. Garretson, H. M. Wiseman, D. T. Pope and D. T. Pegg,
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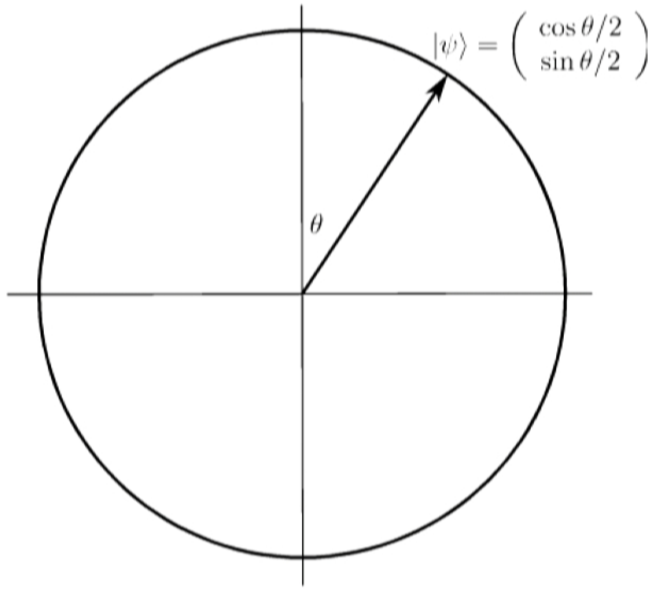

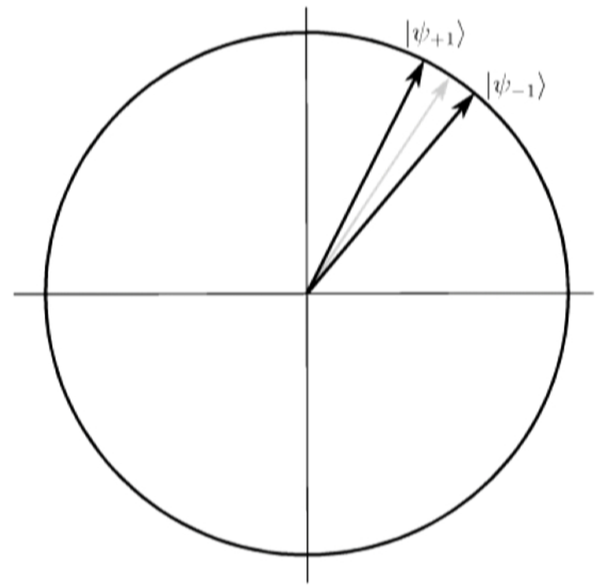

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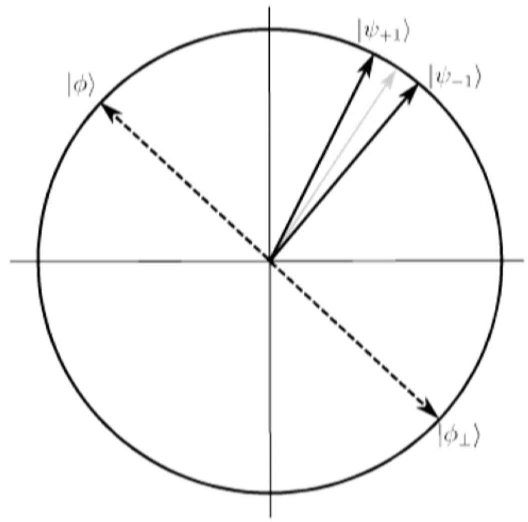
After the weak measurement, let us do a second strong measurement in the basis $\{|\phi\rangle, |\phi_\perp\rangle\}$.



Then, we ask what the value of weak measurement was, conditional on the final state being $|\phi\rangle$ (i.e. we *postselect*): $\mathbb{E}_{s|\phi,\psi}[s]$.

$$
\begin{aligned}
\mathbb{E}_{s|\phi,\psi}[s] &= \sum_{s=\pm 1} s\,\Pr(s|\phi,\psi) \\
&= \sum_{s=\pm 1} s\,\frac{\Pr(\phi|s,\psi)\,\Pr(s|\psi)}{\Pr(\phi|\psi)} \\
&= \frac{1}{\Pr(\phi|\psi)} \sum_{s=\pm 1} s \left| \frac{\langle \phi | M_s | \psi \rangle}{\sqrt{\Pr(s|\psi)}} \right|^2 \Pr(s|\psi) \\
&= \frac{1}{\Pr(\phi|\psi)} \sum_{s=\pm 1} s \left| \langle \phi | \frac{\mathbb{I} + \frac{1}{2} s \lambda \sigma_z}{\sqrt{2}} | \psi \rangle \right|^2 \\
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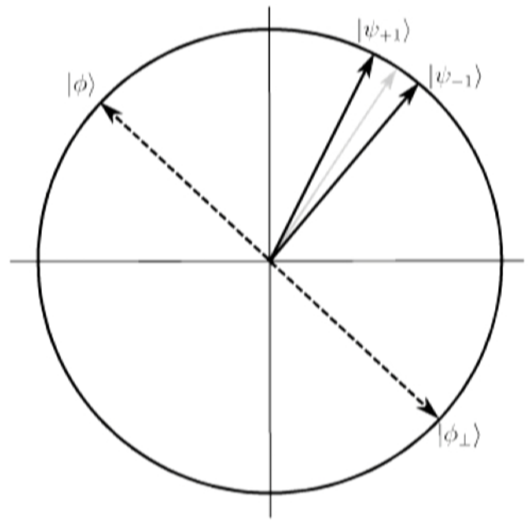

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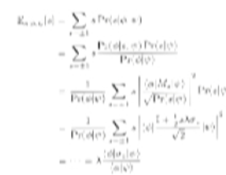
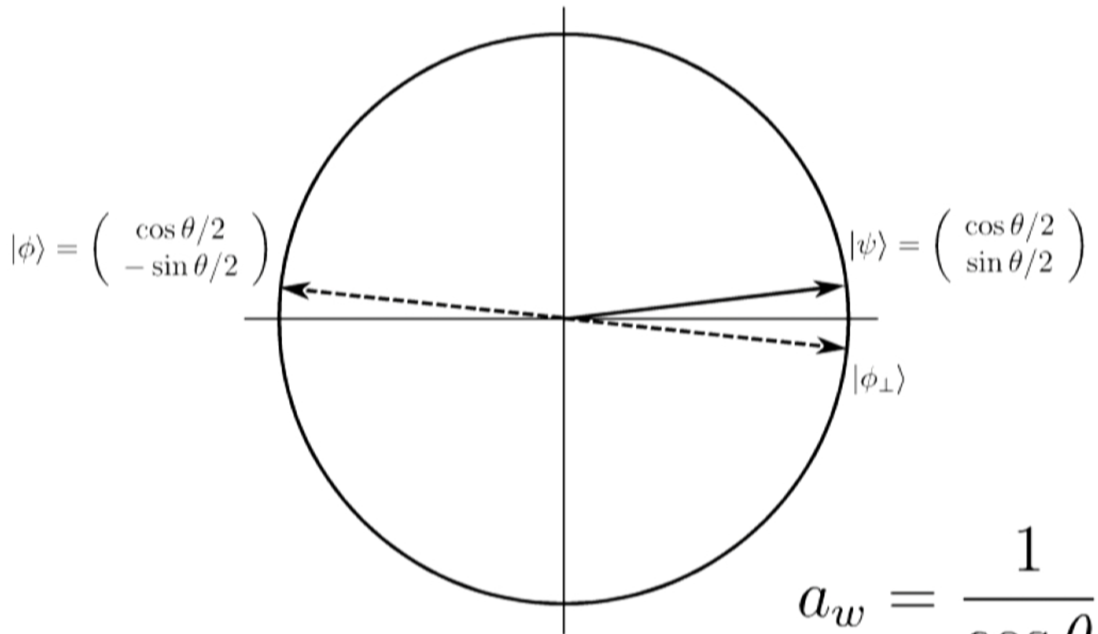
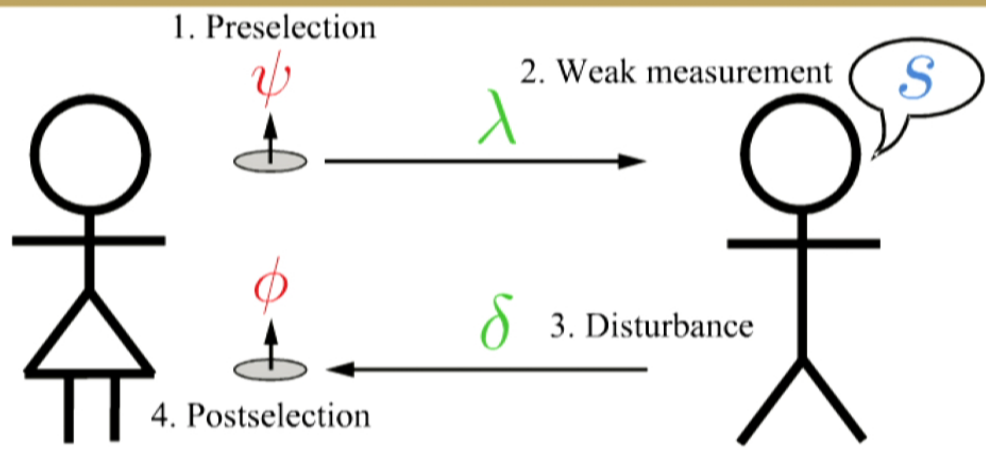
$$\lim_{\lambda \to 0} \mathbb{E}_{s|\phi,\psi}\left[\frac{s}{\lambda}\right] = a_w$$





$$a_w = \frac{1}{\cos\theta}$$



1 **Preselection**: Alice tosses the coin, the outcome ψ is recorded, and she passes it to Bob.

2 **Weak measurement**: Bob reports s with the probability

$$\Pr(s|\psi) = \frac{1}{2}(1 + s\lambda\psi).$$

3 **Classical disturbance**: Bob returns it to Alice after flipping it with probability

$$\frac{(1 + s\lambda)\psi - \delta}{1 + s\lambda\psi}.$$

4 **Postselection**: Alice looks at the coin and records the outcome ϕ.

C. Ferrie and J. Combes, arXiv:1403.2362

Preselect on $\psi = +1$

Postselect on $\phi = -1$

$$\begin{aligned}
\mathbb{E}_{s|\phi,\psi}\left[\frac{s}{\lambda}\right] &= \sum_{s=\pm 1} \frac{s}{\lambda} \Pr(s|\phi,\psi) \\
&= \sum_{s=\pm 1} \frac{s}{\lambda} \frac{\Pr(\phi|s,\psi)\Pr(s|\psi)}{Pr(\phi|\psi)} \\
&= \frac{1}{\lambda \Pr(\phi|\psi)} \sum_{s=\pm 1} s \left(\frac{1+s\lambda-\delta}{1+s\lambda}\right)\left(\frac{1+s\lambda}{2}\right) \\
&= \frac{1}{\Pr(\phi|\psi)} \\
&= \frac{1}{1-\delta}
\end{aligned}$$

$$
\begin{aligned}
\Pr(\phi|\psi) &= \sum_{s=\pm 1} \Pr(\phi, s|\psi) \\
&= \sum_{s=\pm 1} \Pr(\phi|s,\psi)\Pr(s|\psi) \\
&= \sum_{s=\pm 1} \left(\frac{1+s\lambda-\delta}{1+s\lambda}\right)\left(\frac{1+s\lambda}{2}\right) \\
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arXiv.org > quant-ph > arXiv:1403.2362

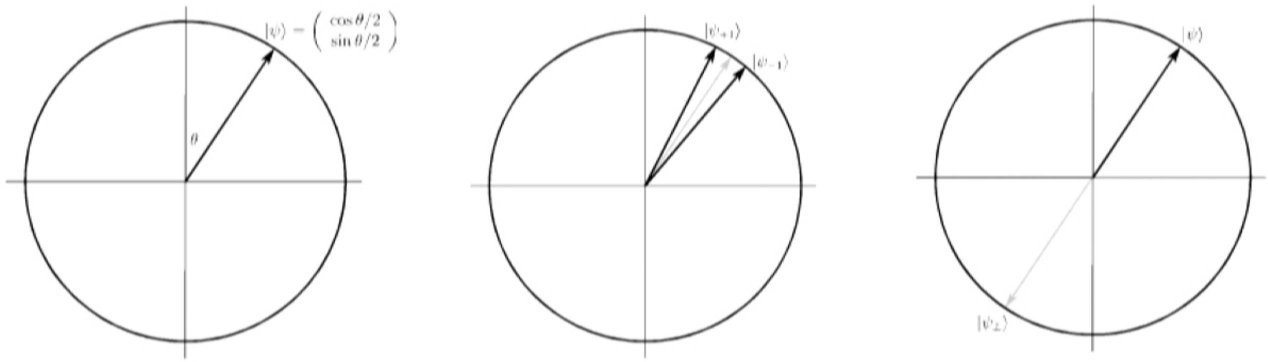
Quantum Physics

How the result of a single coin toss can turn out to be 100 heads

Christopher Ferrie, Joshua Combes

(Submitted on 10 Mar 2014 (v1), last revised 15 Mar 2014 (this version, v2))

We show that the phenomenon of anomalous weak values is not limited to quantum theory. In particular, we show that the same features occur in a simple model of a coin subject to a form of classical backaction with pre- and post-selection. This provides evidence that weak values are not inherently quantum, but rather a purely statistical feature of pre- and post-selection with disturbance



$\Pr(s|\psi) = \frac{1}{2}(1 + s\lambda\cos\theta)$

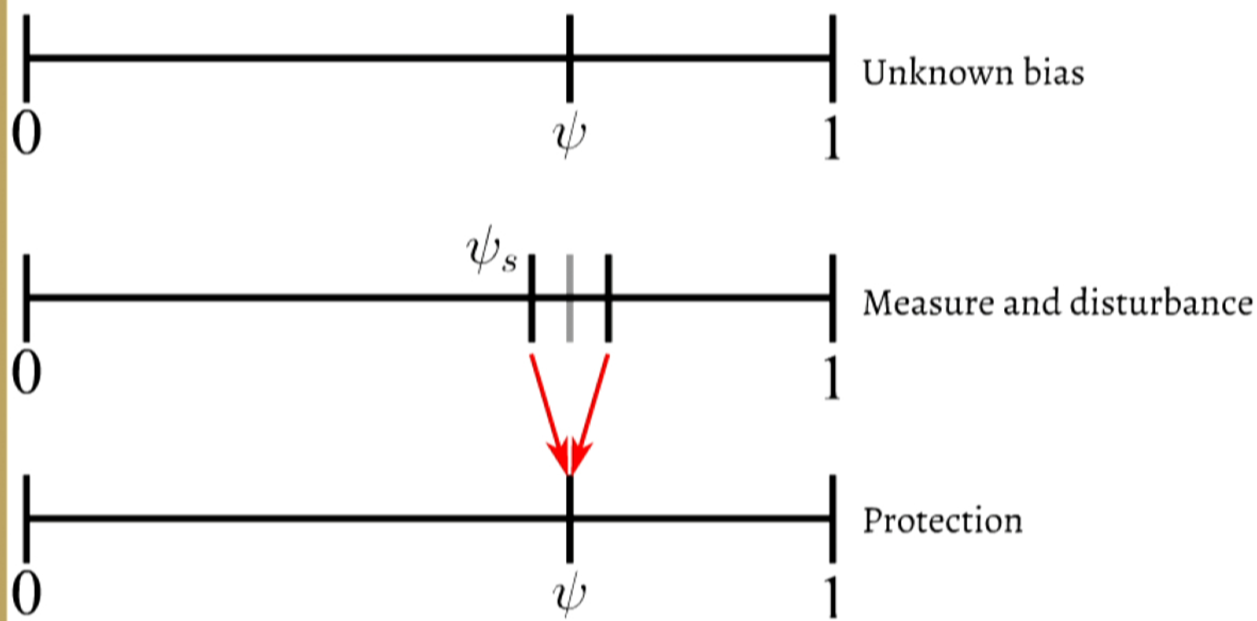
$$|\psi_s\rangle = \frac{M_s|\psi\rangle}{\sqrt{\Pr(s|\psi)}}$$

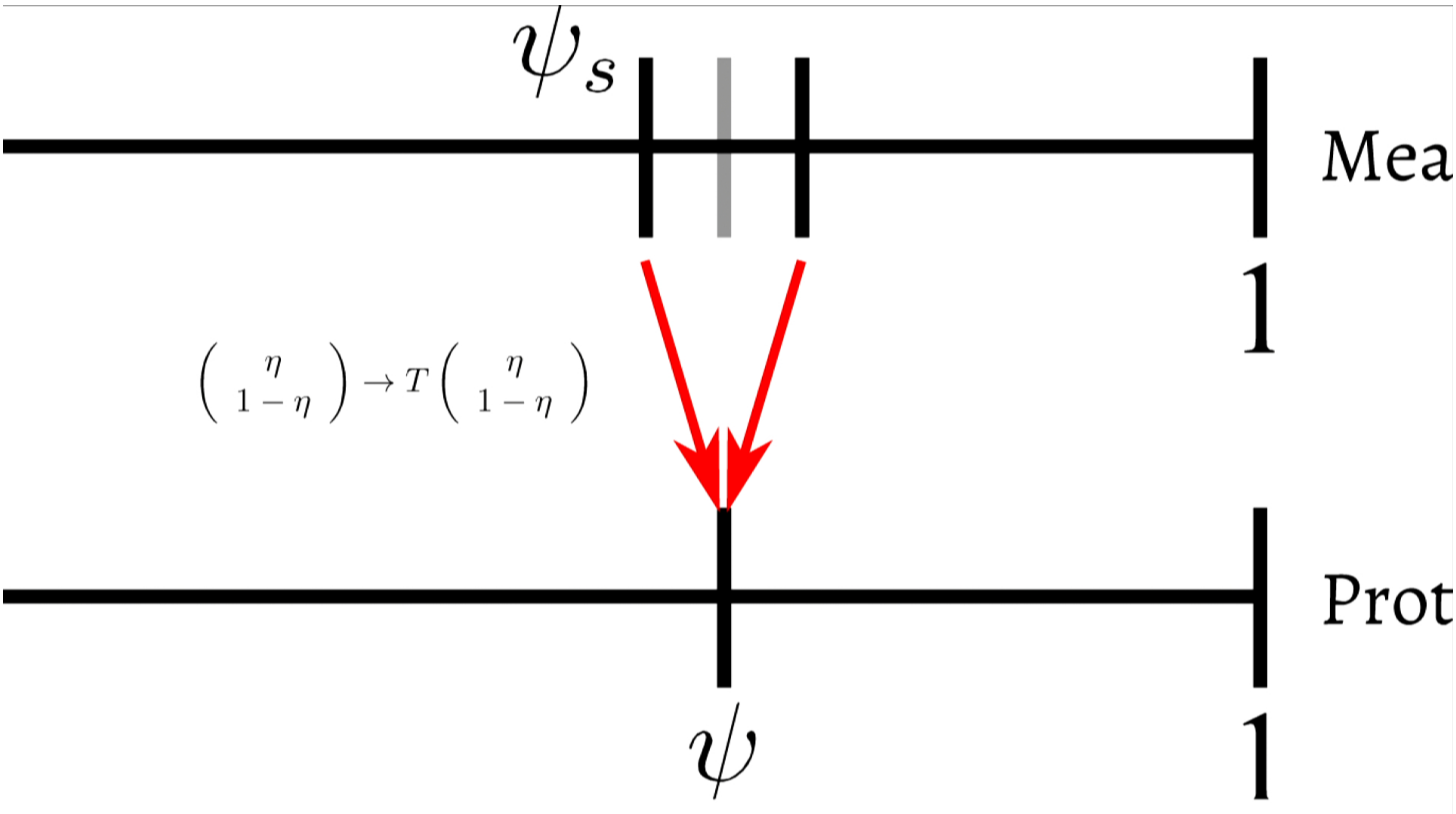
$\Pr(\psi_\perp|\psi_s) = O(\lambda^2)$

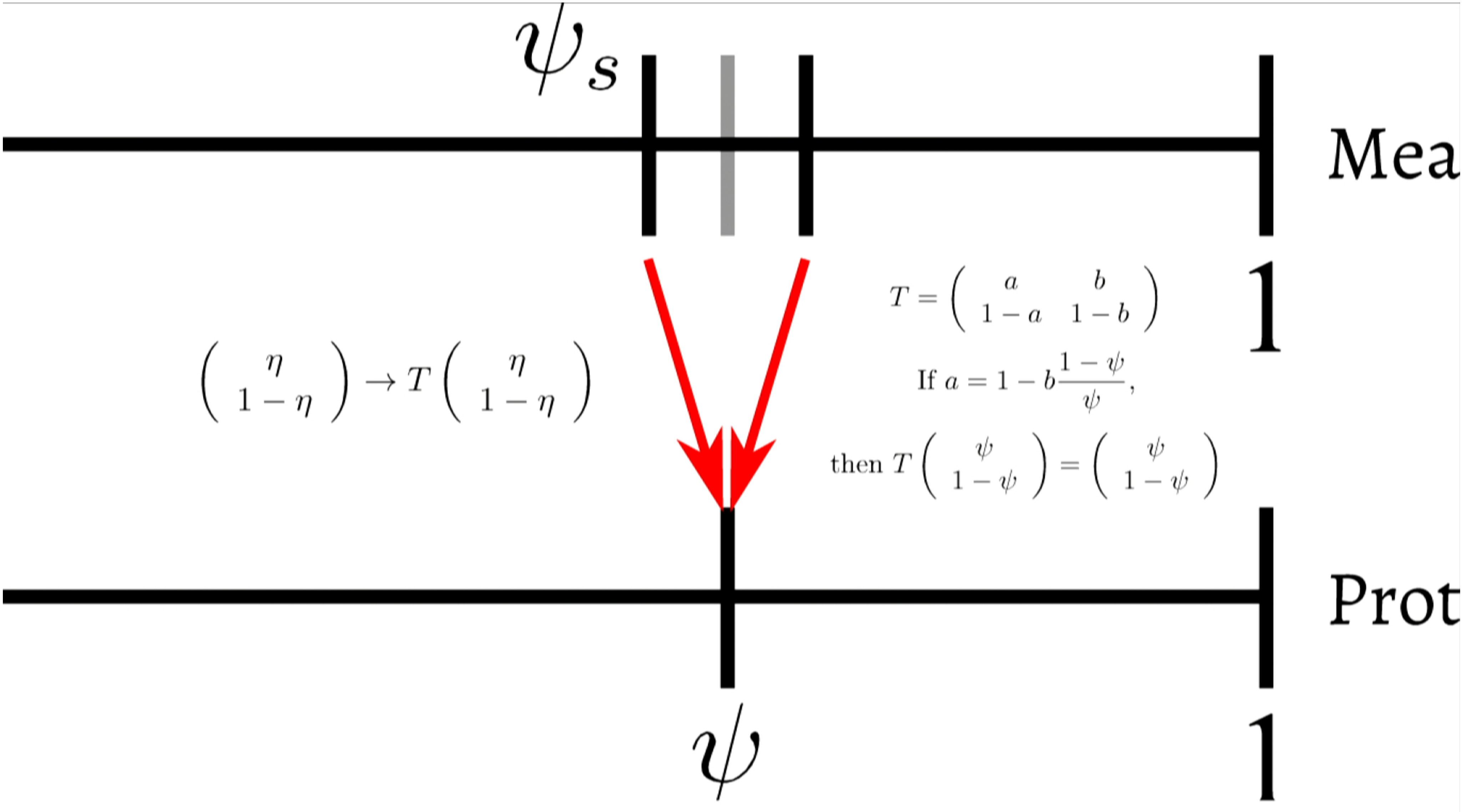
If we perform such an operator for complete set of bases, we can determine the state with a ***single*** copy.

Y. Aharonov and L. Vaidman, Phys. Lett. A 178, 38 (1993).

Protecting the bias of a coin




ψ_s
Mea
1
$\begin{pmatrix} \eta \\ 1-\eta \end{pmatrix} \to T \begin{pmatrix} \eta \\ 1-\eta \end{pmatrix}$
Prot
ψ
1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$T = \begin{pmatrix} a & b \\ 1-a & 1-b \end{pmatrix}$
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If $a = 1 - b\frac{1-\psi}{\psi}$,
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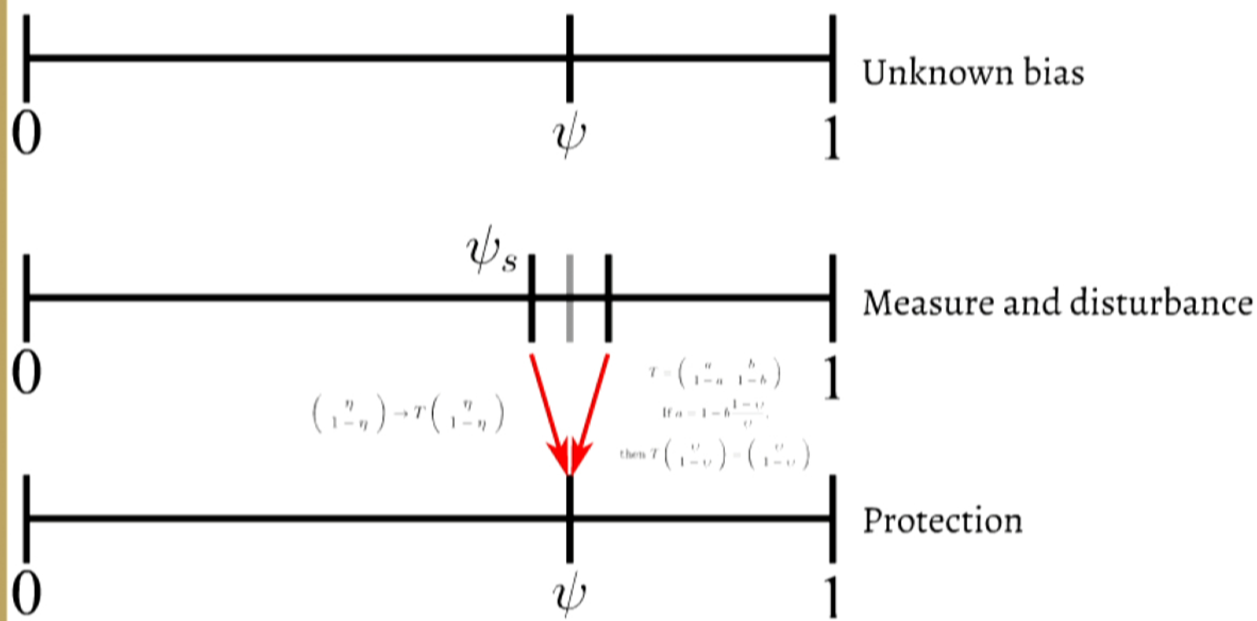

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1

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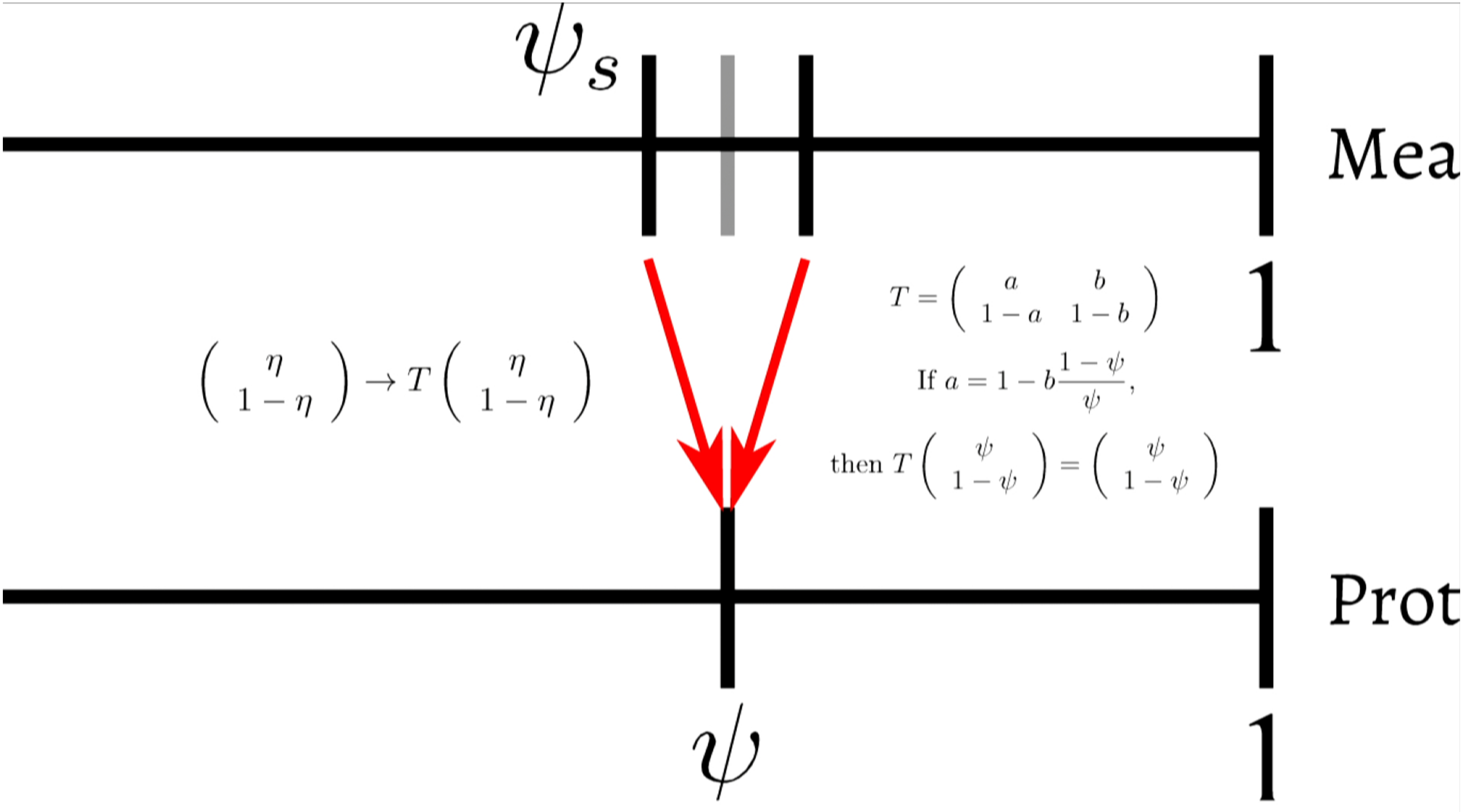
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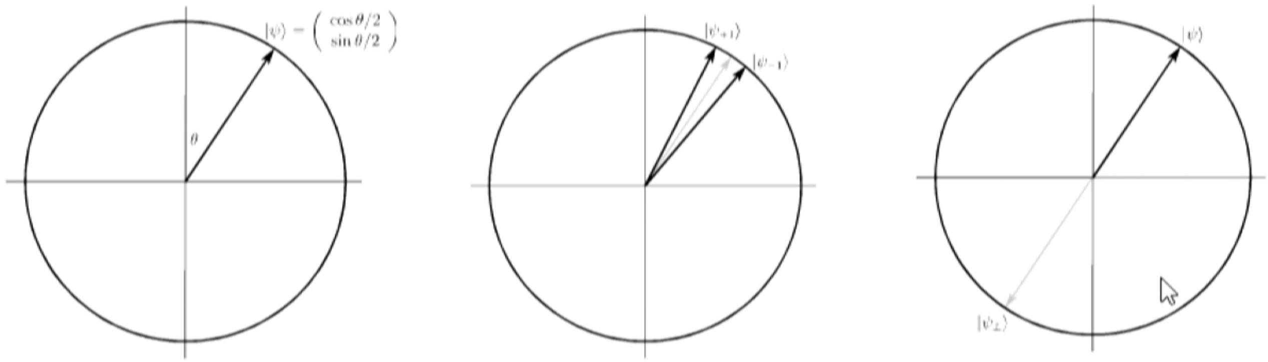
Summary:

- Classical anomalous weak values
- Classical protective measurement

Does the postselection formalism point to what is uniquely "quantum" about quantum theory?



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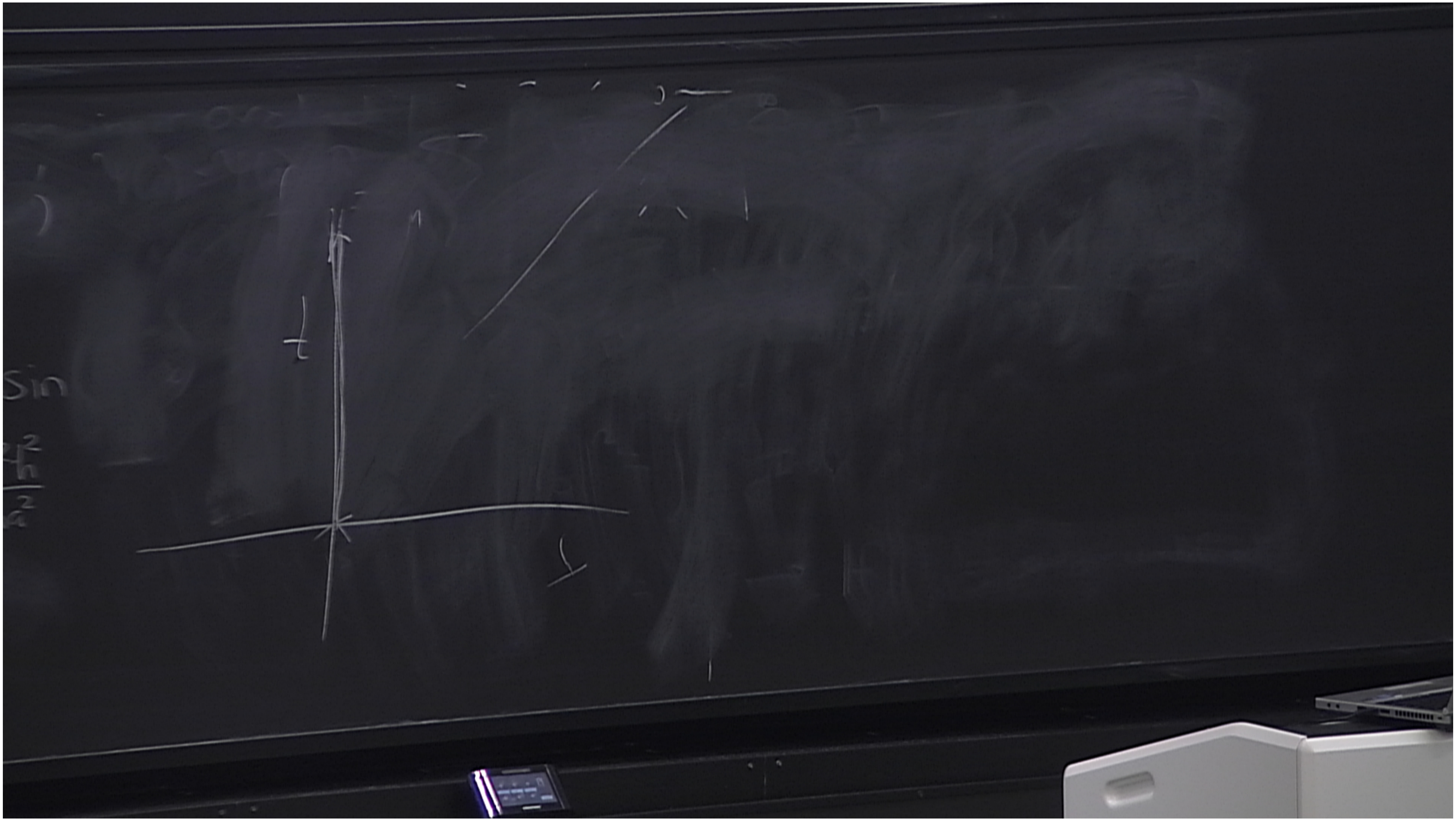

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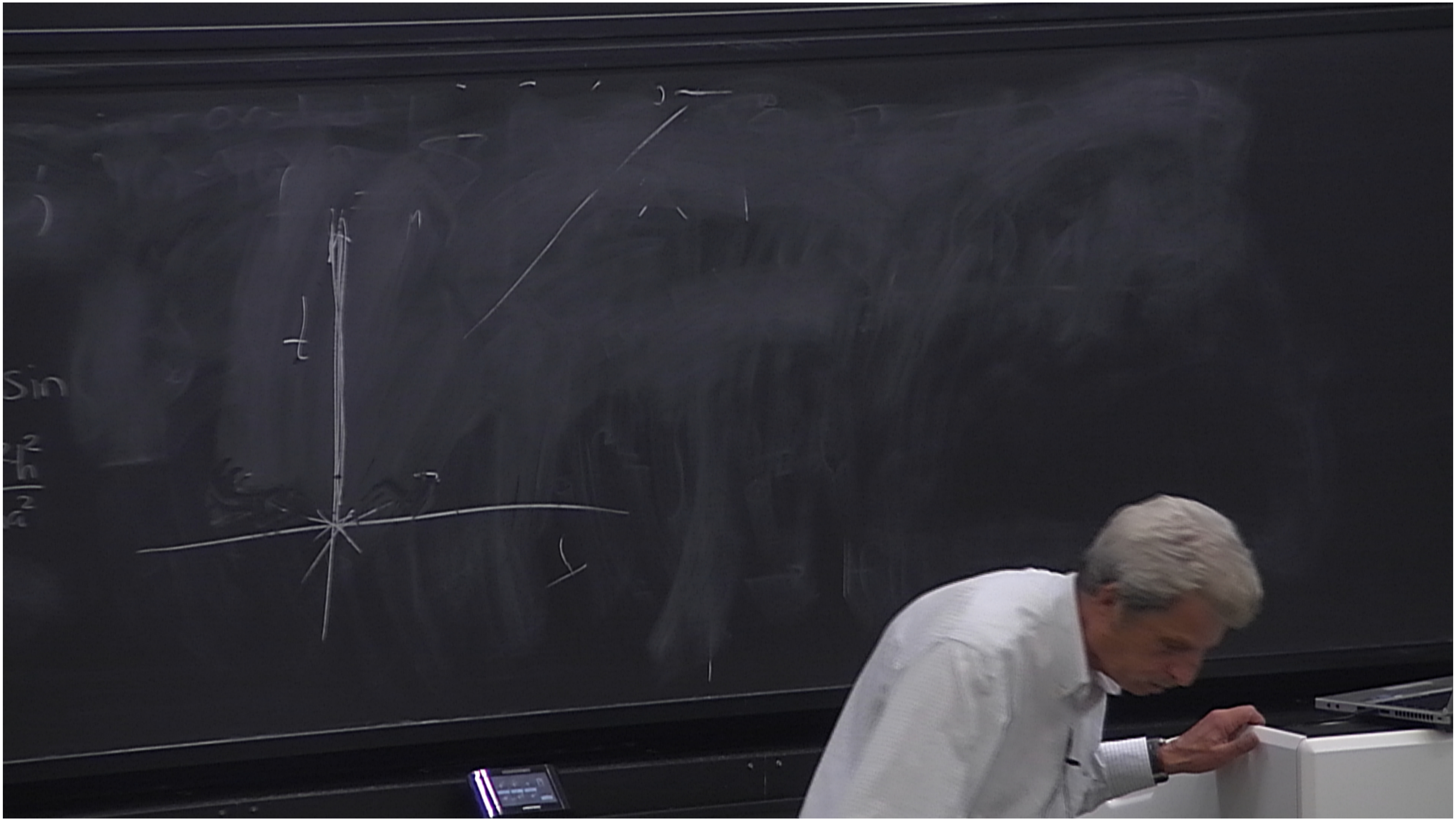
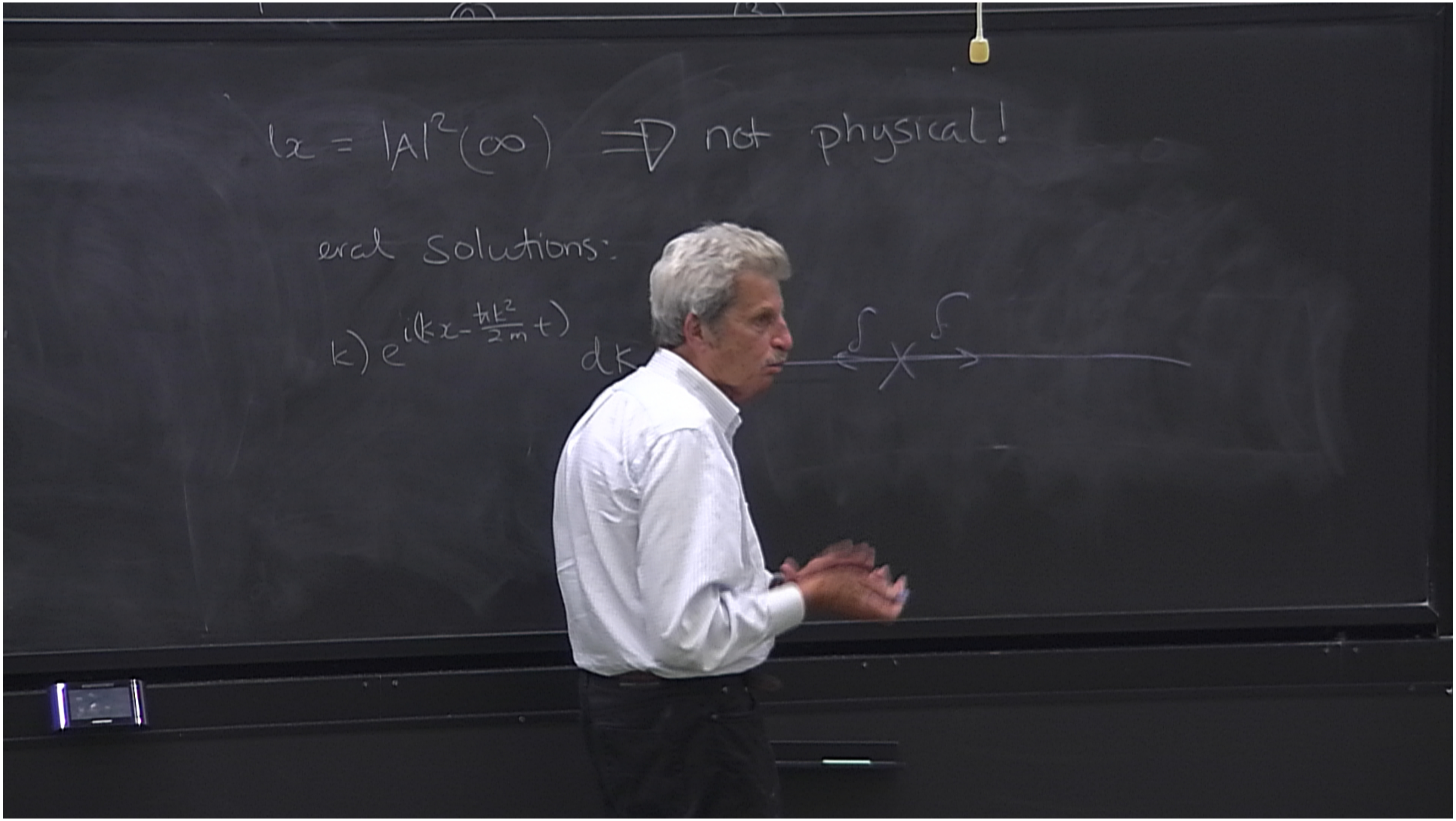
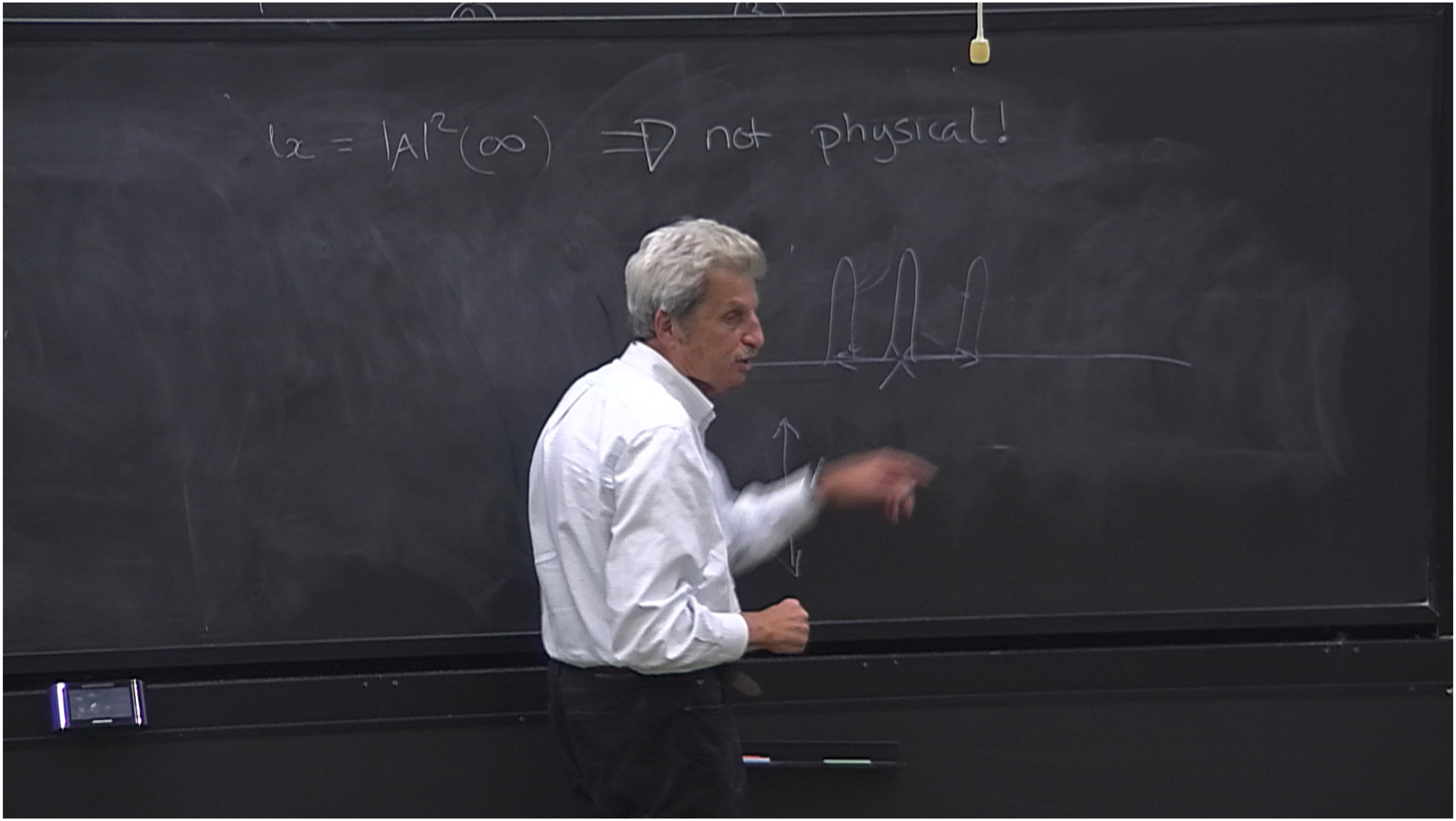




$|x = |A|^2(\infty) \Rightarrow$ not physical!
eral solutions:
$k) e^{i(kx - \frac{\hbar k^2}{2m} t)}$
dk



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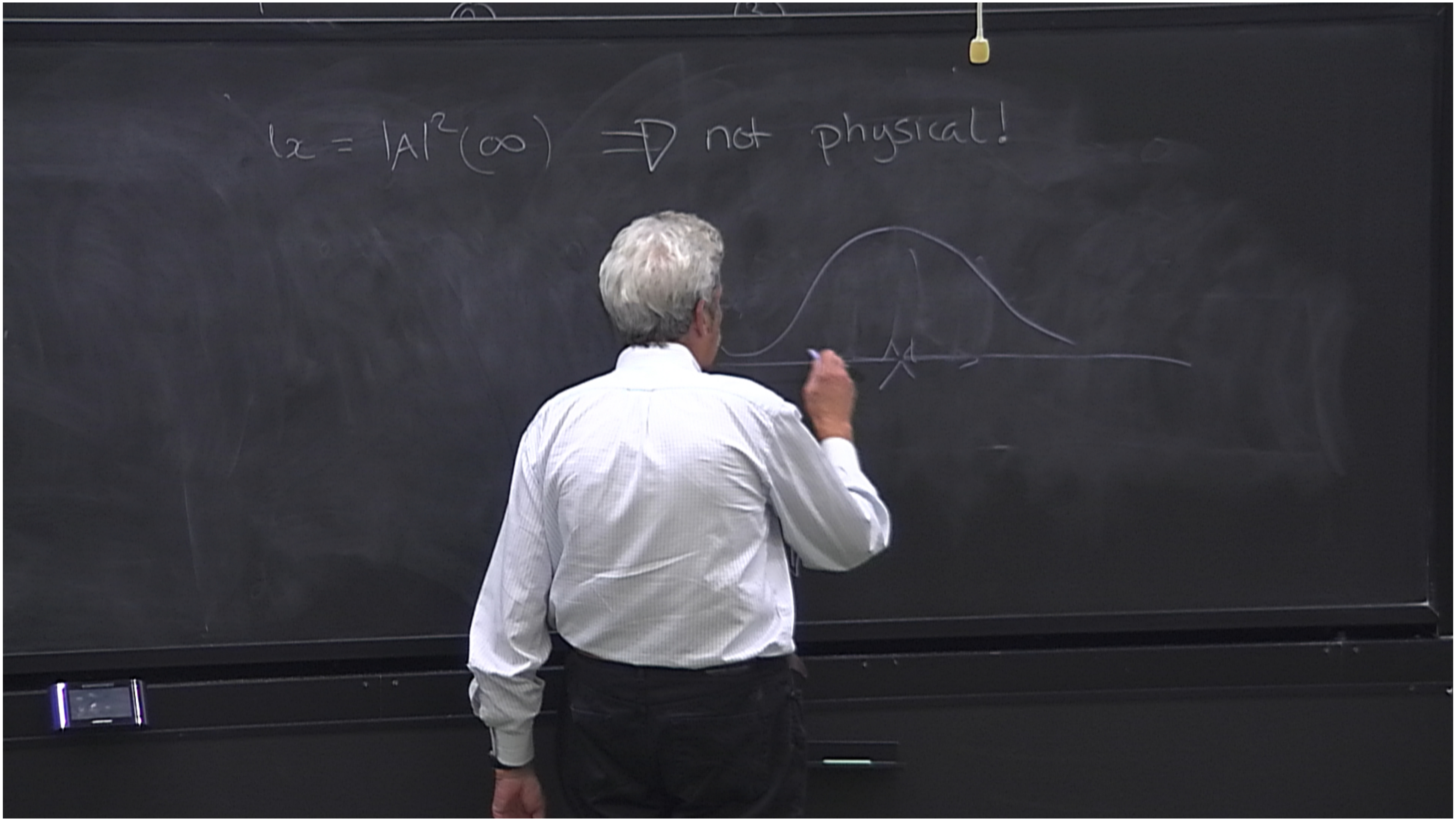

lx = |A|²(∞) ⟹ not physical!