Title: Criterion for non-Fermi Liquid behavior in a metal with broken symmetry

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Abstract:

When do interactions with Goldstone bosons lead to non-Fermi liquids?

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When do interactions with Goldstone bosons lead to non-Fermi liquids?

Answer: When $$\left[Q_a,\ \vec{P}\right] \neq 0$$

Goldstone mode generator: Q_a

Generator of translations: $\vec{P}$

Acknowledgements

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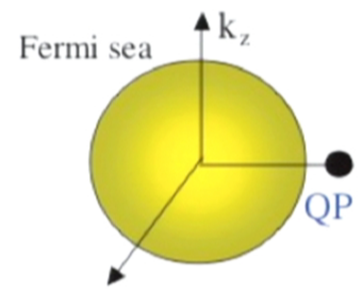
Reference: Haruki Watanabe and AV, arXiv:1404.3728

Special thanks to: Max Metlitski (KITP)

Review – Fermi Liquids

- Dispersion:

$$E(\vec{p}) = v_F p_{||}$$



- Damping: $\Gamma(E) \approx E^2/E_F$

Well defined quasi particle excitations on approaching the Fermi surface.

Fundamental theory describing the metallic state.

Are the metals that are not Fermi liquids?

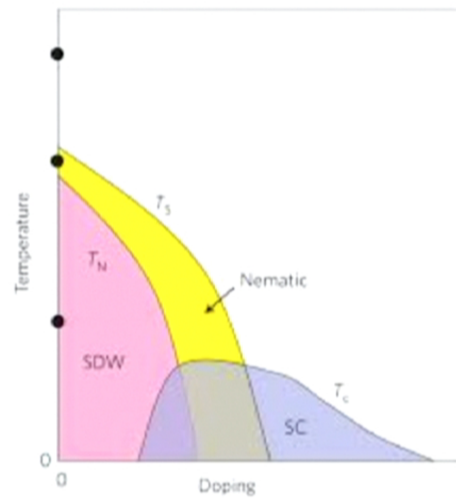
Non-Fermi Liquid Metals?

- Gapless boson coupled to Fermi sea at a critical point.

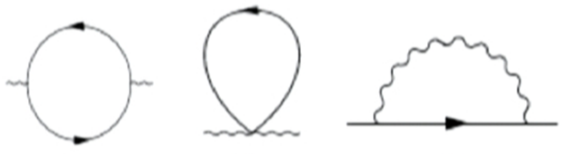
$$\mathcal{L} = L_{fermi} + L_\phi + L_{int}$$

$$L_\phi = (\partial_\mu \phi)^2 + m\phi^2 + \lambda\phi^4$$

$$L_{int} = \phi\psi^\dagger\psi$$



Non-Fermi Liquid Metals?



- Landau Damping of boson:

$$\Pi_{ab}(\nu, \vec{q}) = -i\pi \frac{\nu}{|\vec{q}|}$$

- Non-Fermi liquid:

$$\Gamma(E) \approx E^{2/3}$$

- Caveat – other instabilities might intervene eg. Superconductivity. (...S.S. Lee, Metlitsky-Sachdev, Mross et al.)

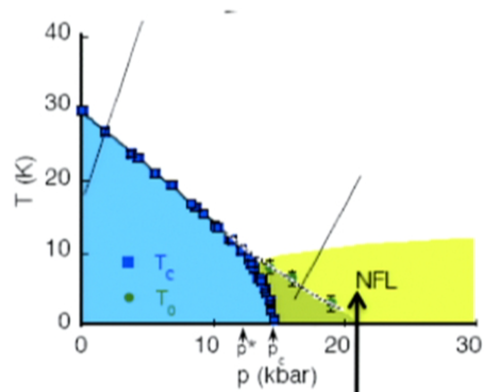
- Non-Fermi liquid phases (rather than critical points)?

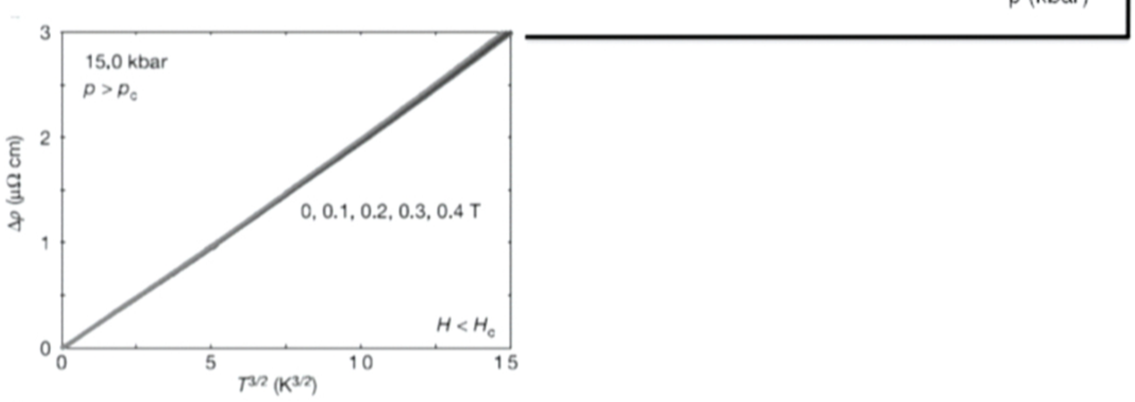
A non-Fermi liquid Phase - Experimental Indications

Experiments: Electrical transport in MnSi

[Pfleiderer et al. `03]:

- Seen 0.05K to 5.0K $\Delta\rho = T^{3/2}$
- Seen p=15kbar to 80kbar [a phase]

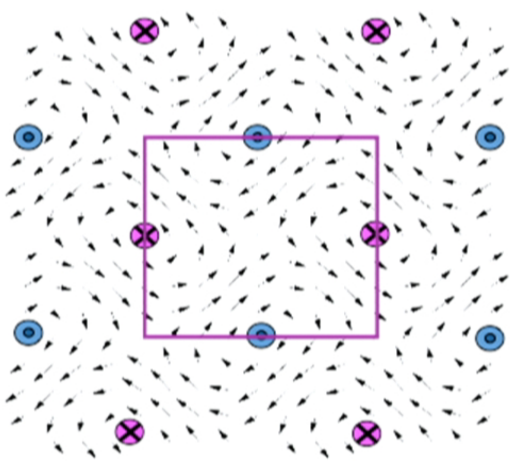




Route to Non-Fermi Liquid Phases

- **Gapless bosons across an entire phase?**
  - Gauge bosons (real or emergent photons).
    - Either too weak or requires exotic ingredient.

  - *Goldstone modes?*

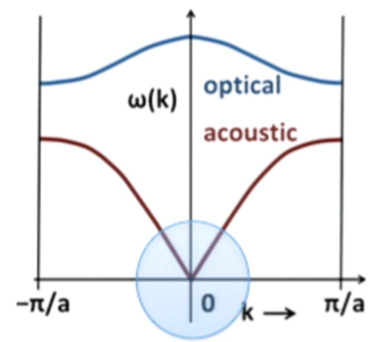


Potential Goldstone modes of exotic order – eg. incommensurate Skyrmion crystal?

Coupling Goldstone Modes to Electrons

- Spontaneous breaking of a continuous symmetry – gapless Nambu-Goldstone modes.

- Examples –
  - Broken translation symmetry: Phonons.
    - Generator $Q_a = P_a$
    - Goldstone mode field: u_a

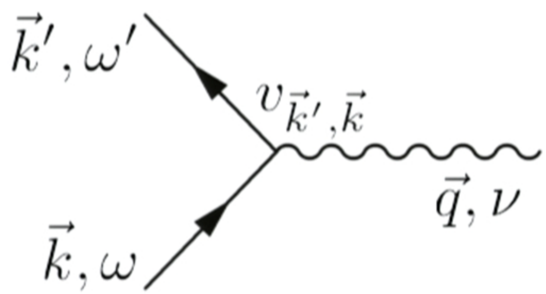


  - Broken `gauge’ symmetry: Superfluid phonons
    - Generator $Q= N$: number operator
    - Goldstone mode field: ϕ

Coupling Goldstone Modes to Electrons

- Couplings constrained by symmetry. Typically, vanishing couplings at low energies (Adler Zeros)
- Example:
  - electron phonon interaction $H_{\text{int}} = \nabla \cdot \mathbf{u}\, \psi^\dagger \psi$

Typically for Goldstone bosons $\lim_{\vec{k}' \to \vec{k}} v_{\vec{k}',\vec{k}} = 0$



v.s.

- Critical bosons $v_{\vec{k}',\vec{k}} = 1 \neq 0$
- Gauge bosons $\vec{v}_{\vec{k}',\vec{k}} = \dfrac{\vec{k} + \vec{k}'}{2m}$

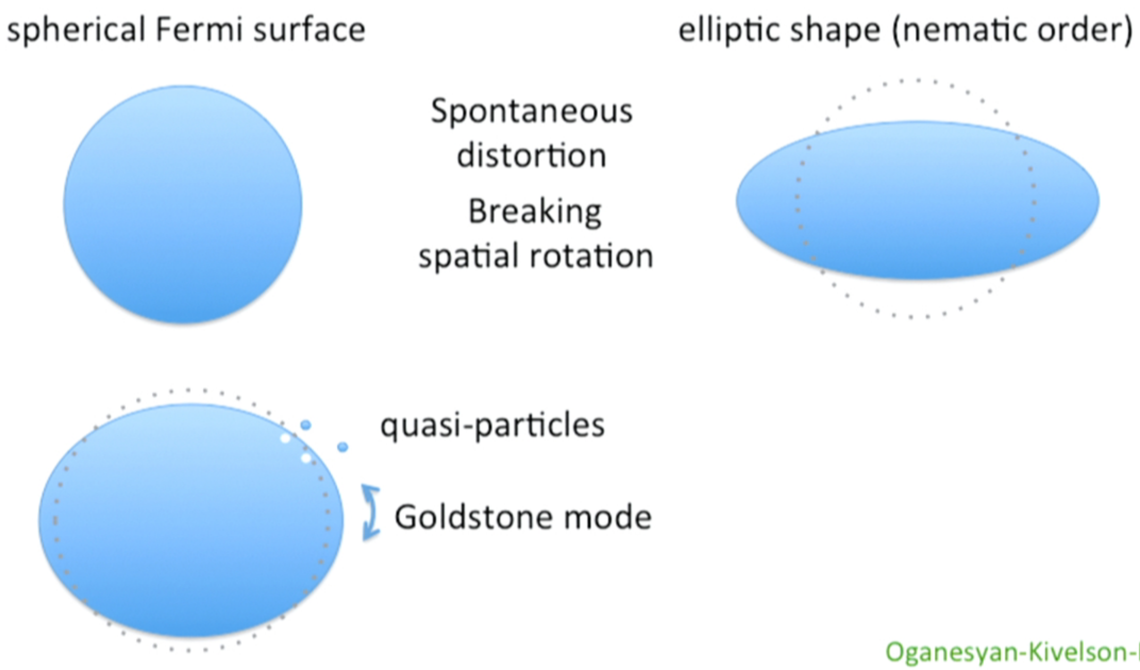
Coupling Goldstone Modes to Electrons

- Vanishing couplings render electron-Goldstone mode interactions irrelevant at low energies.
  - Consistent to have decoupled systems:
    - Stable Fermi liquid+ underdamped Goldstone modes.

- Is this inevitable? Oganesyan-Kivelson-Fradkin (2001)
  - One known exception –
    - Fermi fluid with broken rotation symmetry (eg. nematic order). Non vanishing coupling.

Nematic Fermi Fluid

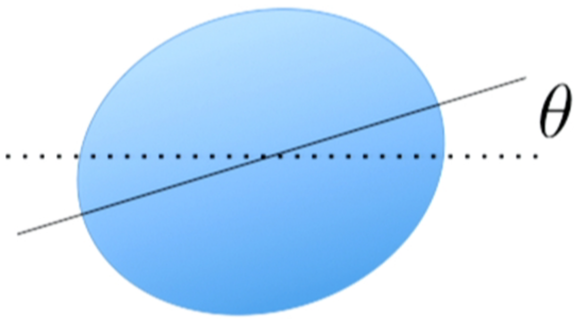
- Ordered *phase* that breaks rotation symmetry.
- No lattice (gapless orientation mode)



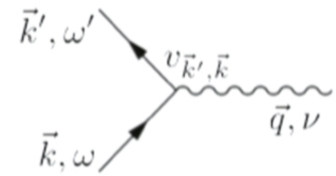
Oganesyan-Kivelson-Fradkin (2001)

Nematic Fermi Fluid

- Non-vanishing coupling from direct calculation



$$H_{\rm int} = (p_x^2 - p_y^2)\cos 2\theta + 2p_xp_y\sin 2\theta$$
$$\approx (p_x^2 - p_y^2) + 4p_xp_y\theta + O(\theta^2)$$



The vertex does not vanish!!

$$\lim_{\vec{k}' \to \vec{k}} v_{\vec{k}',\vec{k}} = 4k_xk_y \neq 0$$ over most of the FS

General criterion will show why this case is special.

Also, can identify a *new* scenario with non vanishing couplings

Coupling Goldstones to Electrons
General Considerations

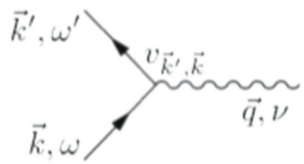
- $\mathcal{H}_0$ Hamiltonian in a symmetry broken vacuum: Symmetry dictates form of interaction.

Symmetry → $\mathcal{H}_{\rm int}^{(1)} = -i[\pi^a Q_a, \mathcal{H}_0]$

So that: $\mathcal{H}_0 + \mathcal{H}_{int}^{(1)}$ Invariant to lowest order

Fermion states:

$$\mathcal{H}_0|n\vec{k}\rangle = \epsilon_{n\vec{k}}|n\vec{k}\rangle$$



Vertex:

$$\langle\vec{k}|\mathcal{H}_{int}^{(1)}|\vec{k}'\rangle$$

Condition for non vanishing coupling

→ For Matrix element. to diverge: $\langle\vec{k}|Q_a|\vec{k}'\rangle \propto \frac{1}{k-k'}$

→ focus on the operator *P*
(need continuous/discrete translations to define Fermi surface)

The General Criterion

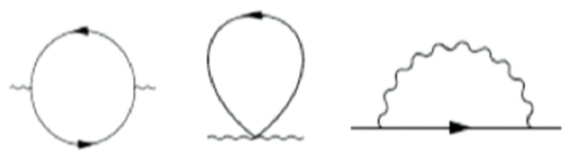
$$[Q_a,\, P_i] = \Lambda_{ai} \neq 0$$ for a generic *k*

Then the vertex diverges: $\langle\vec{k}|Q_a|\vec{k}'\rangle = \frac{\langle k|\Lambda_{ai}|k'\rangle}{k'_i - k_i}$

To give a nonvanishing coupling: $v^a_{k,k'} = -i\sum_j \langle k|\Lambda_{aj}|k'\rangle \nabla_j \epsilon_{\vec{k}}$

Consequences of nonvanishing coupling

- Non-vanishing couplings lead to overdamped Goldstone modes and nonFermi liquid behavior via the usual arguments.



(2a) (2b) (3)

$$\Pi_{ab}(\nu, \vec{q}) = -i\pi \frac{\nu}{|\vec{q}|} \gamma^{ab}(\hat{q})$$

$$\gamma^{ab}(\hat{q}) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} v^a_{\vec{k},\vec{k}} v^b_{\vec{k},\vec{k}} \delta(\epsilon_{\vec{k}}) \delta(\hat{q} \cdot \vec{\nabla}_{\vec{k}} \epsilon_{\vec{k}}).$$

- Other instabilities, such as superconductivity, could intervene.
- However, the decoupled Fermi liquid + Goldstone modes scenario is unstable.

Some examples

The General Criterion

$$[Q_a,\, P_i] = \Lambda_{ai} \neq 0$$

- Conventional (Vanishing) Couplings:
  - Internal symmetry, eg. Spin rotation symmetry breaking.
    - $\left[\vec{S},\, P_j\right] = 0$
  - Phonons. Translation Symmetry Breaking
    - $\left[\vec{P},\, P_j\right] = 0$
- Non-Vanishing Couplings:
  - Spatial rotation symmetry breaking (eg. 2D nematic)

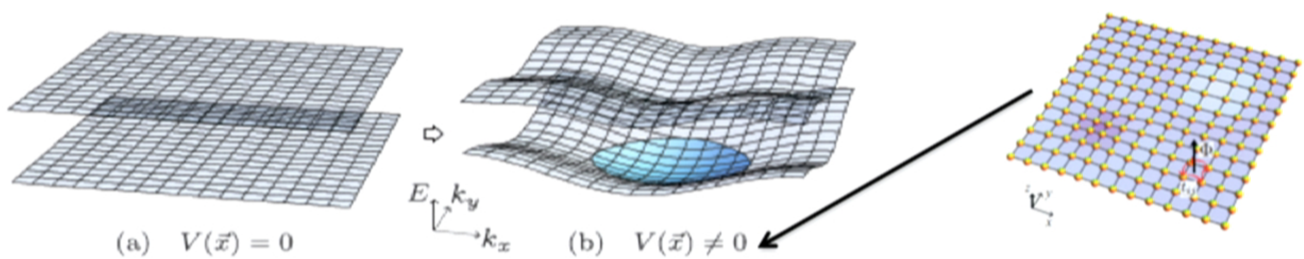
$$[L_z,\, P_j] = \epsilon_{jk} P_k \neq 0$$

New Example of non-vanishing coupling

The General Criterion

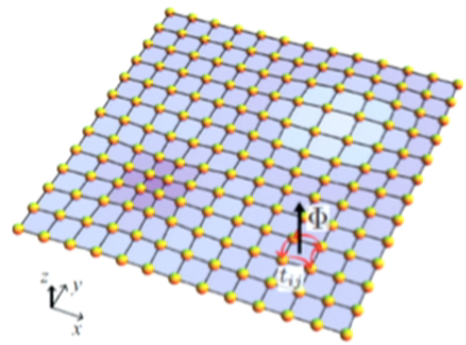
$$[Q_a, P_i] = \Lambda_{ai} \neq 0 \qquad H_{int} = B\mathbf{u} \times \mathbf{j_{el}}$$

- Magnetic translations: $[\mathcal{P}_x, \mathcal{P}_y] = -ieB \neq 0$
  - Electrons in the continuum. Uniform B field. Landau levels
  - Spontaneously break *magnetic translation symmetry* to get a crystal. Electrons acquire a dispersion.
  - Electrons interact with the (magneto) phonon. Nonvanishing coupling! (confirmed by direct calculation)



Physical intuition – nonvanishing coupling from broken magnetic translations

$$H_{int} = B\mathbf{u} \times \mathbf{j_{el}}$$



- The Goldstone mode behaves like a gauge field.
- Compression of the lattice – changes flux.

$$\delta B = \nabla \times \mathcal{A} = B\nabla \cdot \vec{u}$$

$$\mathcal{A} \sim B\hat{z} \times \vec{u}$$

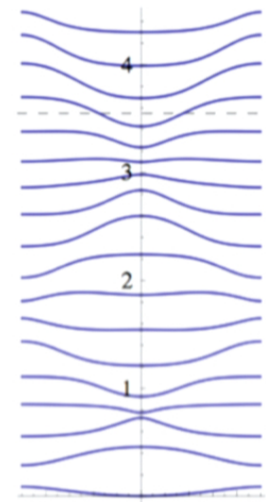
$$H_{int} = j \cdot \mathcal{A}$$

Physical Realization?

Apply a magnetic field to a crystal (first symmetry breaking and then field).
Should lead to a non-Fermi liquid and overdamped phonons!

However the scale is extremely small since

- Magnetic length *l* >> lattice spacing *a*
- *Dispersion of the Landau levels, j~* $e^{-C\frac{l^2}{a^2}}$
- *100 Tesla, magnetic length 25A >> a=2.5A (graphene).*



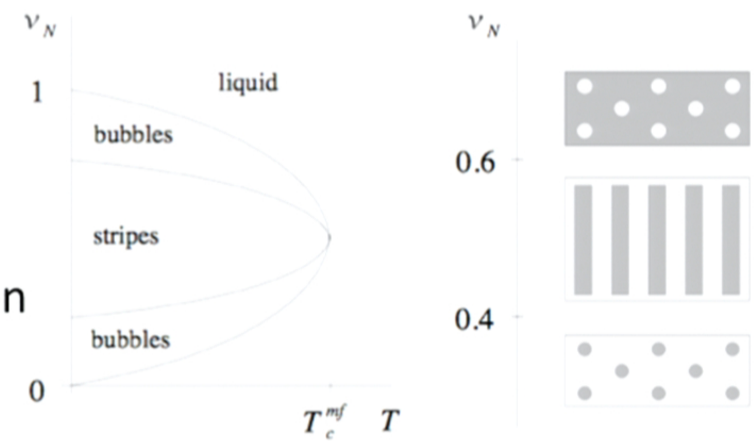
Quantum Hall Stripe and Bubble Phases

- Want $l \sim a$ for measurable effects.
- In the quantum Hall regime (low doping semiconductor 2DEG).
- Investigate stripe and bubble phases with doping

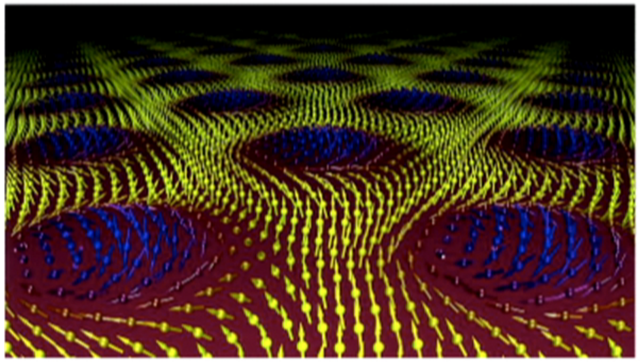
electrons partially filling higher LL

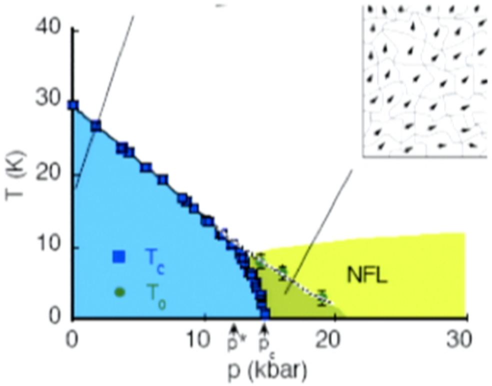
⬇

Spontaneously-breaking translation



Electron-Phonon coupling in Skyrmion crystals





- Breaking of regular translation symmetry $[Q_a, P] = 0$
  So Non Fermi Liquid behavior is *not* expected.

- Can have other mechanisms for unusual transport but is a Fermi liquid with well defined Goldstone modes.

HW-Sid-Sri-AV: arXiv:1309.7047v2
(we corrected error in v1)

Stability of Goldstone Bosons

- In a Lorentz invariant system Goldstone Bosons are always stable.
  - No overdamping (even in the presence of massless Dirac fermions etc.)

- In a metal, the answer depends only on the nature of the broken symmetry. If

$$[Q_a, P_i] = 0$$

weak coupling is irrelevant.

Otherwise, coupling is relevant and could flow to nonFemri liquid, superconductor, …

Conclusion

- A general understanding of coupling between Goldstones and fermions is now available.

- The general criterion for this coupling to be relevant at the lowest energies is $[Q_a, P] \neq 0$, and depends only on the symmetry breaking pattern.

- This allowed us to understand the existing results as well as predict a new (and probably the only other) physical example of nonvanishing coupling – translation symmetry breaking in a field.