Title: Space-Time Circuit-to-Hamiltonian construction and Its Applications

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Abstract: The circuit-to-Hamiltonian construction translates a dynamics (a quantum circuit and its output) into statics (the groundstate of a circuit Hamiltonian) by explicitly defining a quantum register for a clock. The standard Feynman-Kitaev construction uses one global clock for all qubits while we consider a different construction in which a clock is assigned to each point in space where a qubit of the quantum circuit resides. We show how one can apply this construction to one-dimensional quantum circuits for which the circuit Hamiltonian realizes the dynamics of a vibrating string. We discuss how the construction can be used (1) in quantum complexity theory to obtain new and stronger results in QMA and (2) how one can realize, based on this construction, universal quantum adiabatic computation and a universal quantum walk using a 2D interacting particle Hamiltonian. See http://arxiv.org/abs/1311.6101

Space-Time Circuit-to-Hamiltonian construction and its applications

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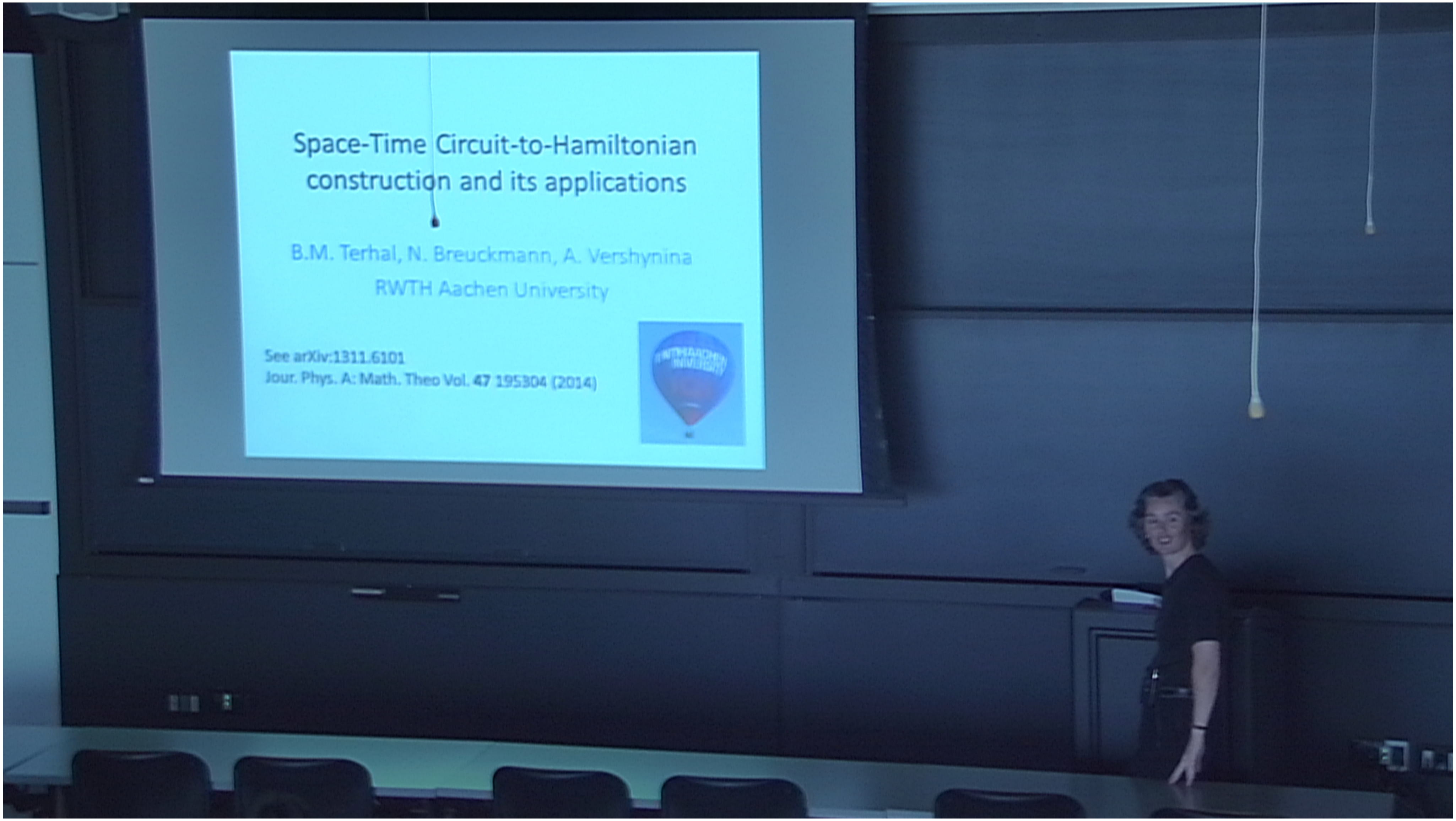
See arXiv:1311.6101
Jour. Phys. A: Math. Theo Vol. **47** 195304 (2014)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Circuit-to-Hamiltonian construction (Feynman-Kitaev)

Mapping from time-dependent circuit to the ground-state of a Hamiltonian H.

Circuit has n qubits and gates U_1, ...U_L. Introduce a clock register |t>: |t=0>....|t=L> and let

$$H_{circuit} = \sum_{t=1}^{L} \left(-U_t \otimes |t\rangle\langle t-1| + h.c. + |t-1\rangle\langle t-1| + |t\rangle\langle t|\right)$$

Ground-state of $H_{circuit}$ is history state of the circuit (for any ξ)

$$|\varphi_{his}\rangle = \frac{1}{\sqrt{L+1}} \sum_{t=0}^{L} U_t \ldots . U_1 \, |\xi\rangle \otimes |t\rangle$$

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Features of Circuit-to-Hamiltonian Mapping

- View as particle hopping on a time-line, gate U_t is executed when particle hops from position t-1 to t.
- Realize clock register using a domain wall clock (e.g. $|111100000\rangle$) or particle clock (e.g. $|0000100000\rangle$ so that $|t-1\rangle\langle t|, |t\rangle\langle t|$ acts on O(1) (3 resp. 2) clock qubits.
- Read out answer of computation from history state
- Spectrum of H, independent of gates: $E_k \propto 1 - \cos(\frac{\pi k}{L+1})$. Gap $\Delta \geq \Theta({}^1/_{L^2})$

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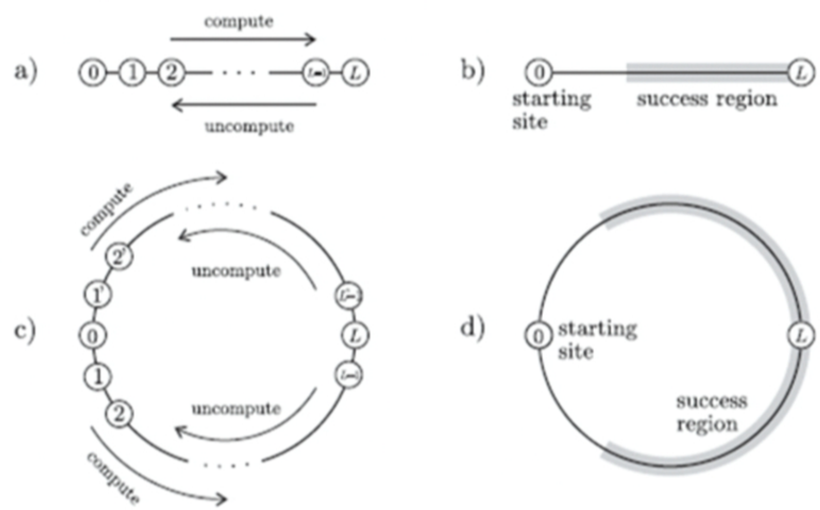
Use of Circuit-to-Hamiltonian Mapping I: universal continuous quantum walk

Quantum Walk (e.g. Nagaj) on a line or circle:
Start walk in t=0 state, evolve with $H_{circuit}$ for random time s$\sim L^2$ (L gates in original quantum circuit) such that one approximately samples a time from the (uniform) distribution.
If we finds a time in the output region, output of entire computation is available.



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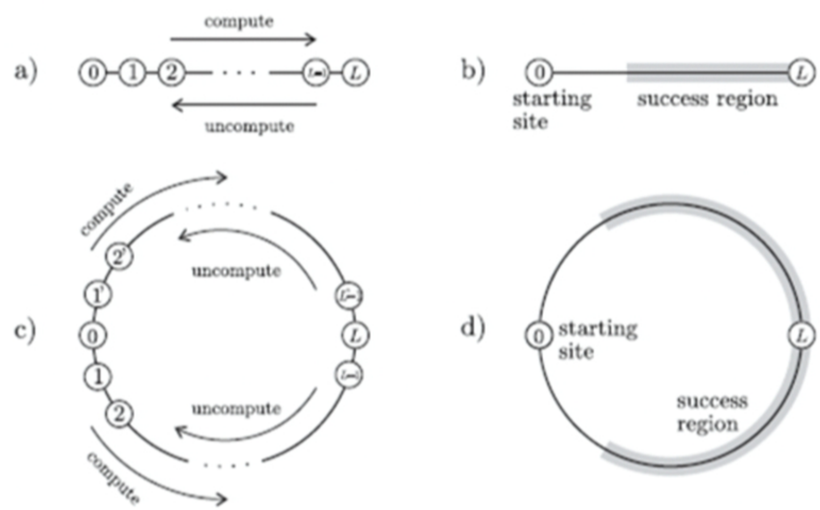
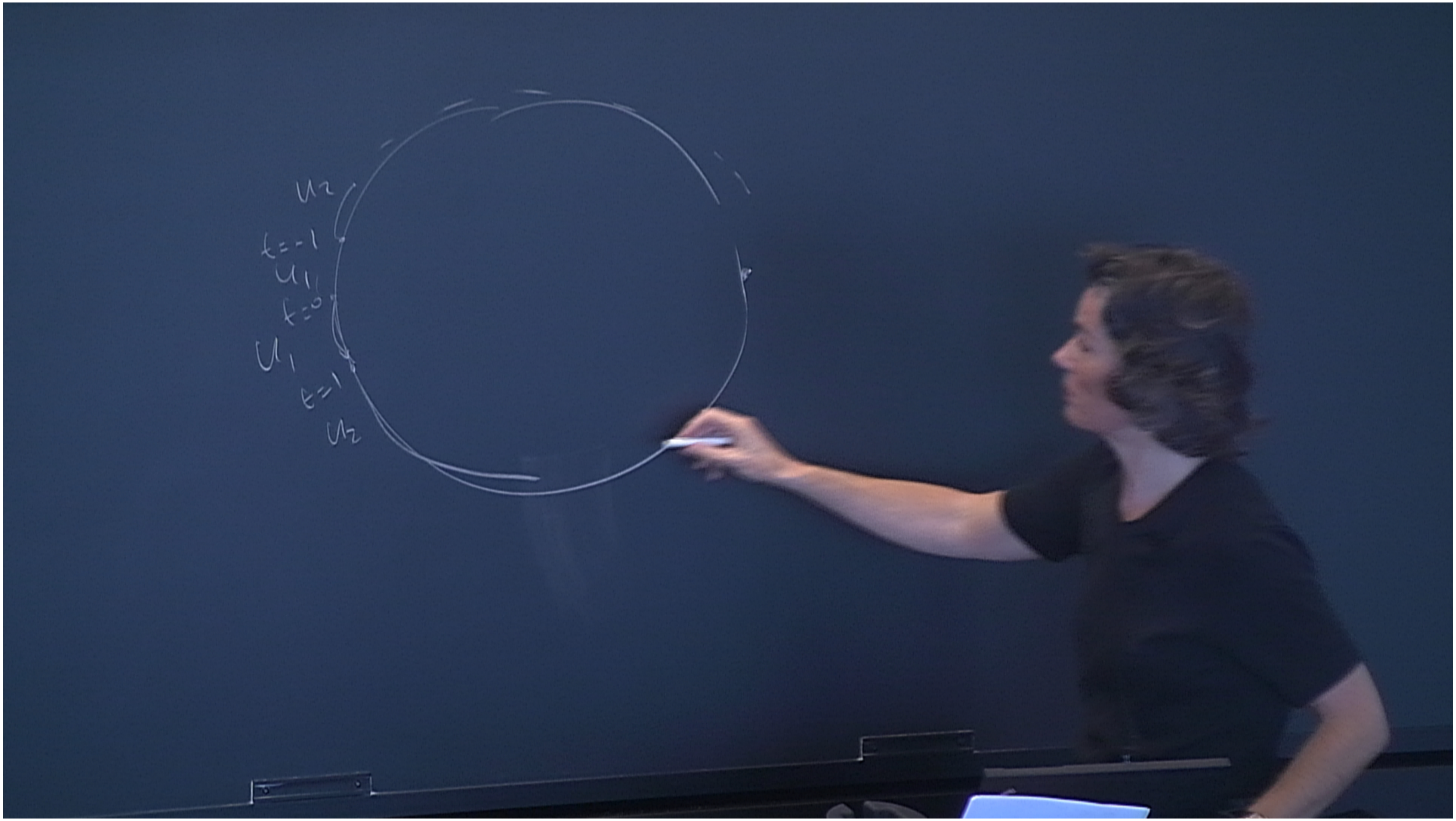
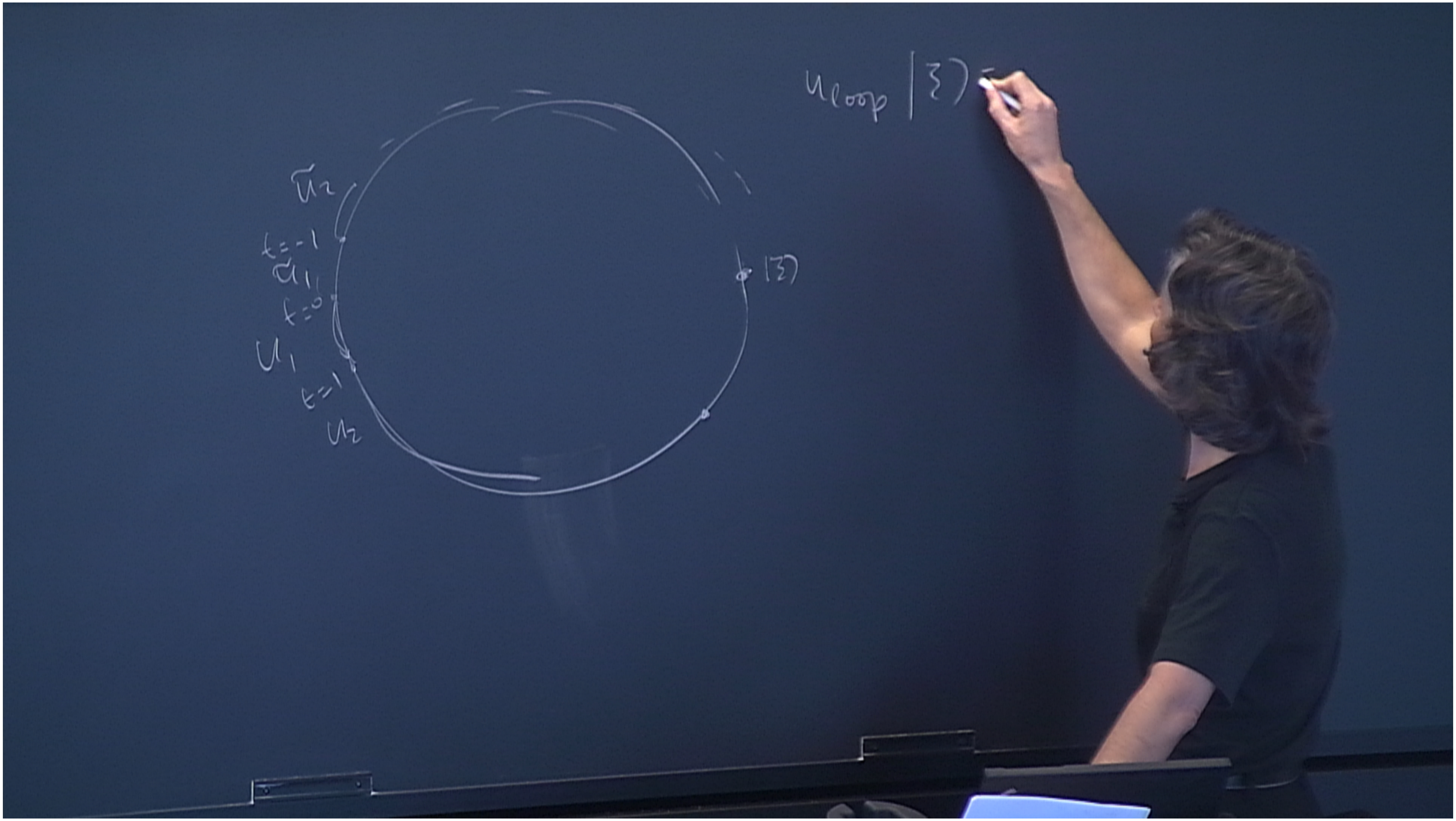
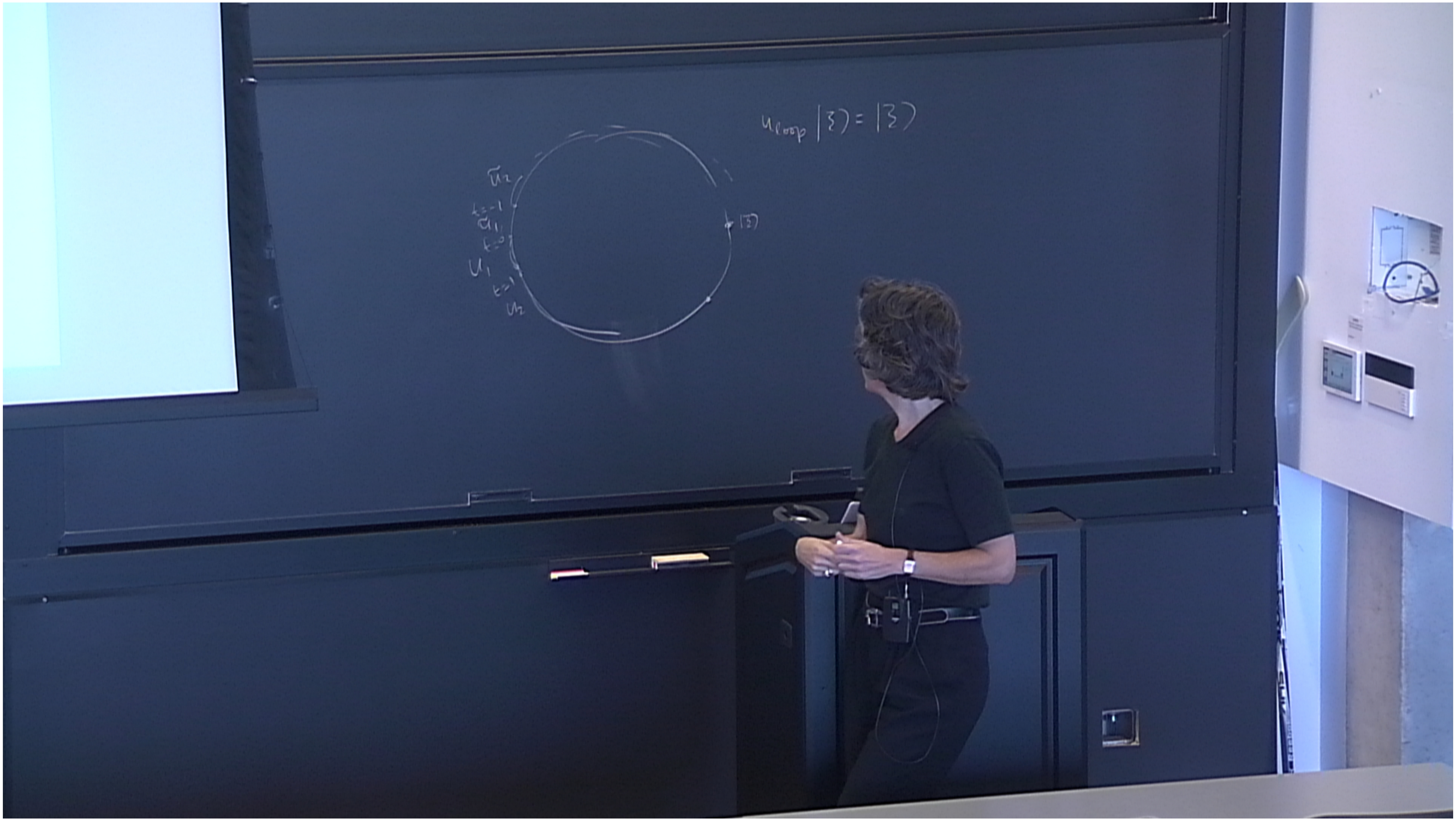
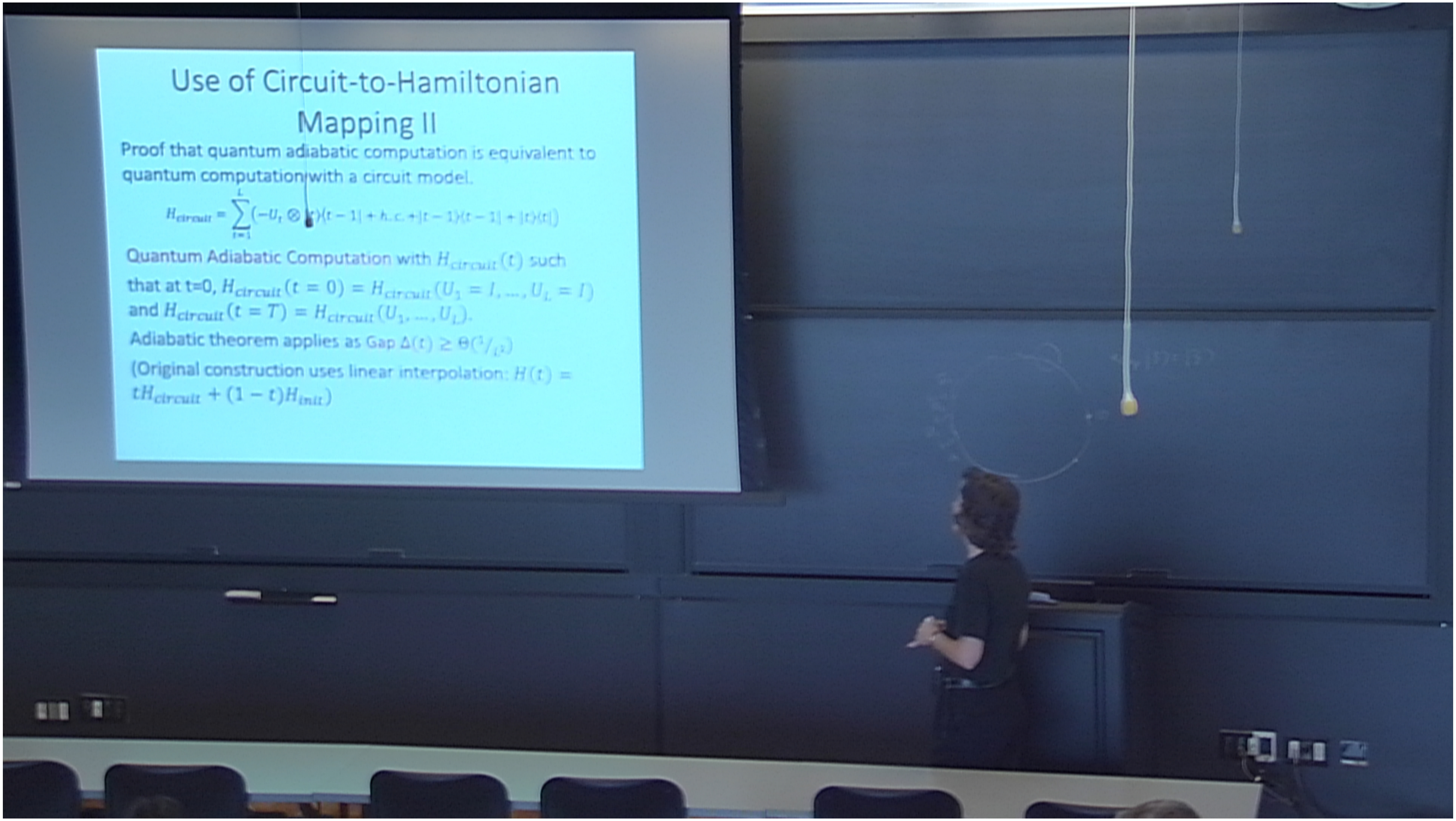
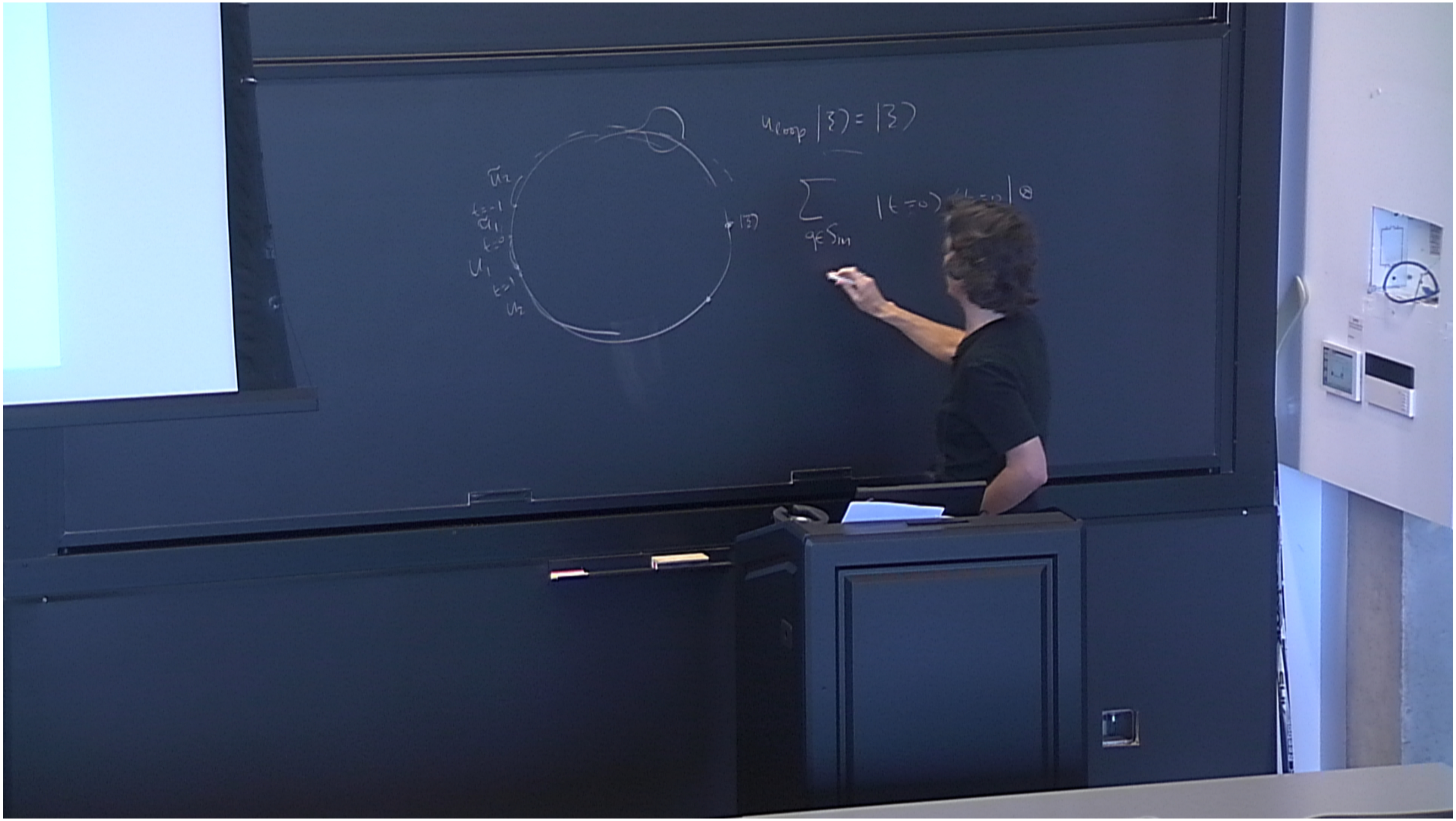



U2
t=-1
U1
t=0
U1
t=1
U2


$U_{loop}|\xi\rangle =$
$\tilde{U}_2$
$t=-1$
$\tilde{U}_1$
$t=0$
U_1
$t=1$
U_2
$|\xi\rangle$


u_loop |ξ⟩ = |ξ⟩
|ξ⟩
t=-1
t=0
U_1
t=1
u_2


Use of Circuit-to-Hamiltonian Mapping II
Proof that quantum adiabatic computation is equivalent to quantum computation with a circuit model.
$H_{circuit} = \sum_{t=1}^{L} (-U_t \otimes |t\rangle\langle t-1| + h.c. + |t-1\rangle\langle t-1| + |t\rangle\langle t|)$
Quantum Adiabatic Computation with $H_{circuit}(t)$ such that at t=0, $H_{circuit}(t=0) = H_{circuit}(U_1 = I, \dots, U_L = I)$ and $H_{circuit}(t=T) = H_{circuit}(U_1, \dots, U_L)$.
Adiabatic theorem applies as Gap $\Delta(t) \geq \Theta(1/L^2)$
(Original construction uses linear interpolation: $H(t) = tH_{circuit} + (1-t)H_{init}$)



Use of Circuit-to-Hamiltonian Mapping III: quantum complexity theory

Quantum Cook-Levin Theorem (Kitaev): proof that determining the lowest energy of a n-qubit Hamiltonian with 1/poly(n) accuracy is QMA-complete, i.e. hard for quantum computers.

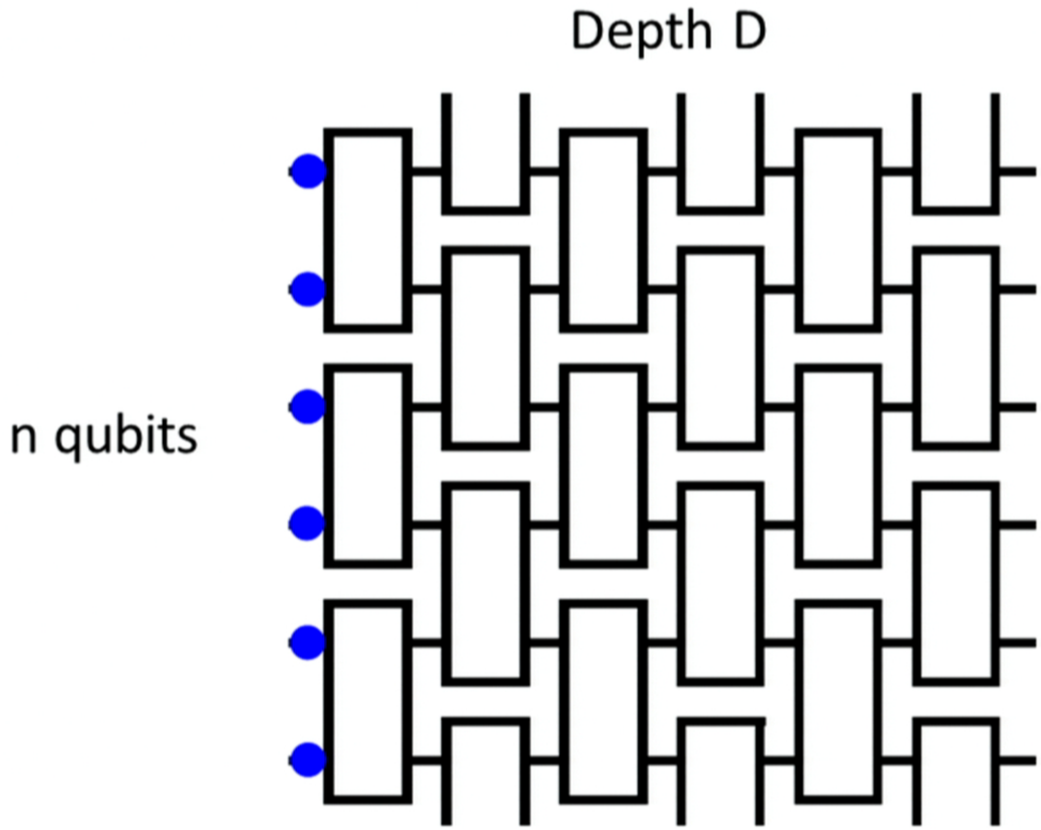
Idea: any problem in QMA (quantum NP) has a verification circuit which (approximately) outputs 0 or 1 depending on input being a valid proof.

Construct $H = H_{circuit} + H_{input} + H_{output}$ which has a low energy state iff there exists an input (proof) such that circuit outputs 1 and only high-energy states when circuit outputs 0.

Some previous QMA results

- Two-qubit Hamiltonians on a planar graph (via perturbation gadgets)
- Hamiltonians on a line with nearest-neighbor qudit interactions (strongest result: 8-dimensional qudits)
- 2D Hubbard model with magnetic field

Tools are limited (perturbation gadgets, modifications of Cook-Levin Theorem), so we need new tricks.



For simplicity, we assume we have a 1D quantum circuit with nearest-neighbor interactions (and periodic boundary conditions). Such circuit is universal for computation if D=poly(n).

A different construction?

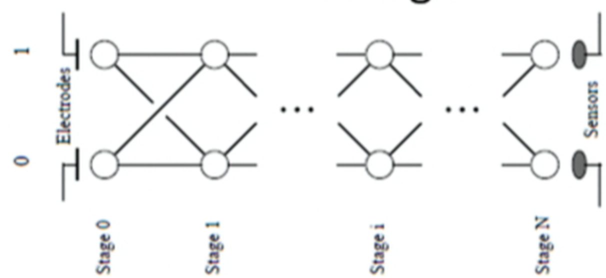
Mizel et al. 'Ground State Quantum Computation' in 1999 & PRL 99, 070502 (2007)) consider a (fermionic) Hamiltonian (for adiabatic QC) with the following features:

A qubit q in a quantum circuit of depth D is represented by 2(D+1) (fermionic) modes, $a_i(q), b_i(q), i = 0 \dots D$

For example: electron in left/right quantum dot or electron spin.

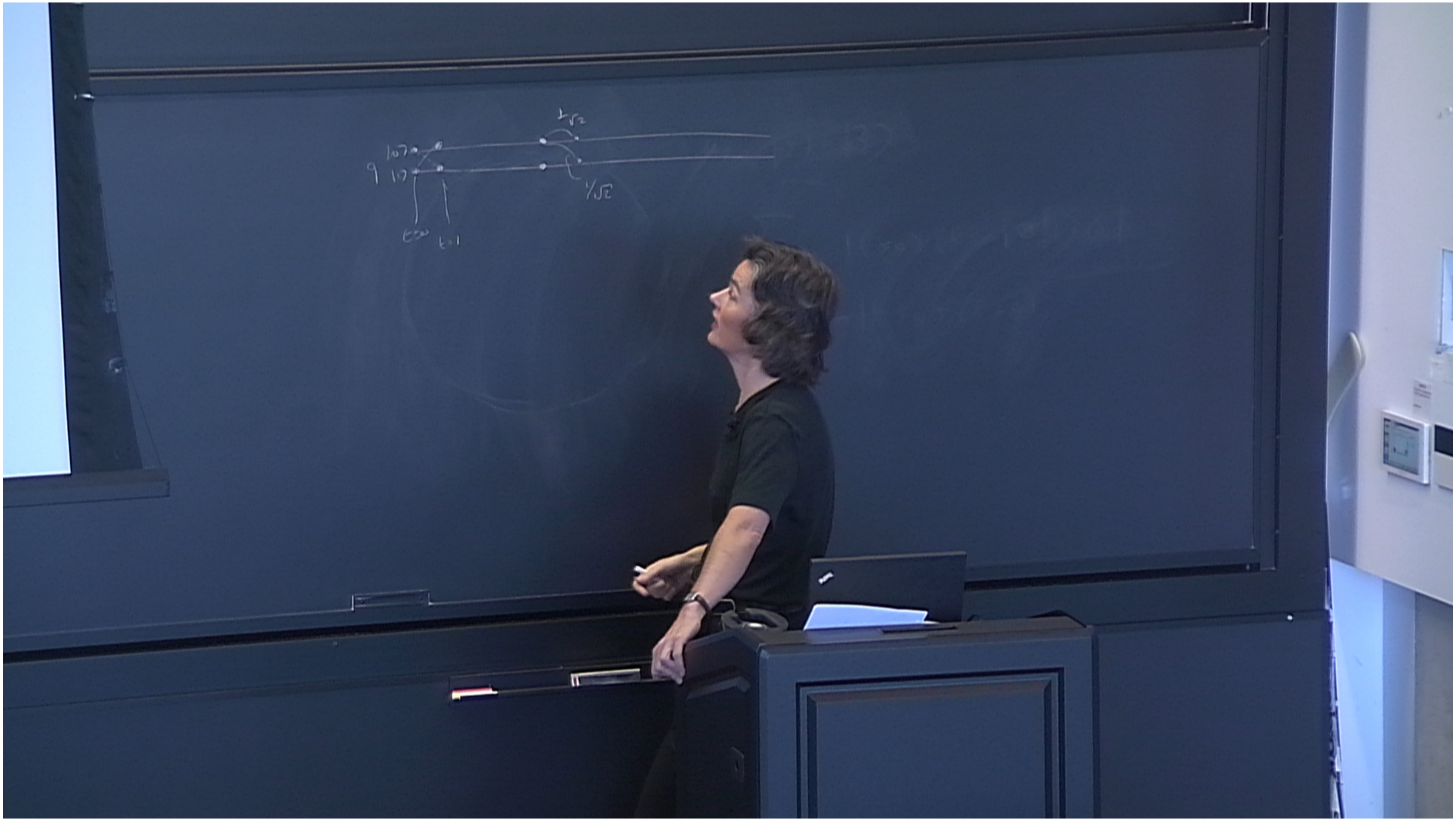
Particles (fermions) can hop on this track of length D.

Track of length D



Construct $H_{circuit}$ such that

- Single qubit gate U is represented by a single particle hopping on such track (and changing its internal state).



Space-Time Circuit-to-Hamiltonian Construction

Define a clock for each qubit q: $|t_q = 0, \ldots, D\rangle$.

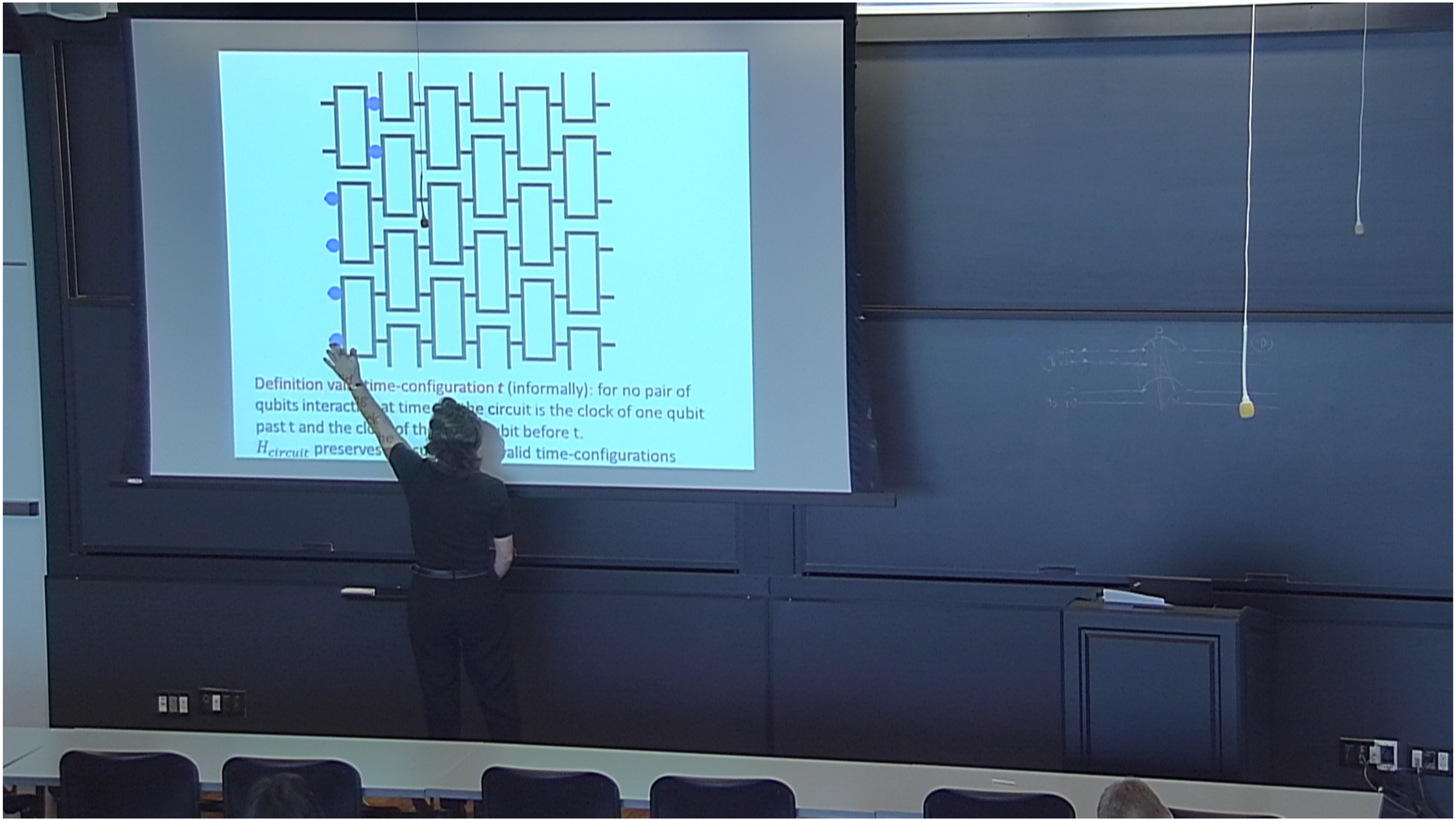
Time configuration $\boldsymbol{t} = (t_1, t_2 \ldots . t_n)$

Term in $H_{circuit}$ for a two-qubit gate U on *qubit q, p* at time s+1

$$-U \otimes \left|t_p = s+1, t_q = s+1\right\rangle\!\left\langle t_c = s, t_q = s\right| + h.c.$$
$$+|t_p = s, t_q = s\rangle\langle t_p = s, t_q = s|$$
$$+|t_p = s+1, t_q = s+1\rangle\langle t_p = s+1, t_q = s+1|$$

"Times of interacting qubits are moved ahead/backward if they are synchronized".

The previous fermionic model effectively corresponds to a certain clock realization (is thus unitarily equivalent).


Definition val time-configuration t (informally): for no pair of
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past t and the clo of th bit before t.
$H_{circuit}$ preserves
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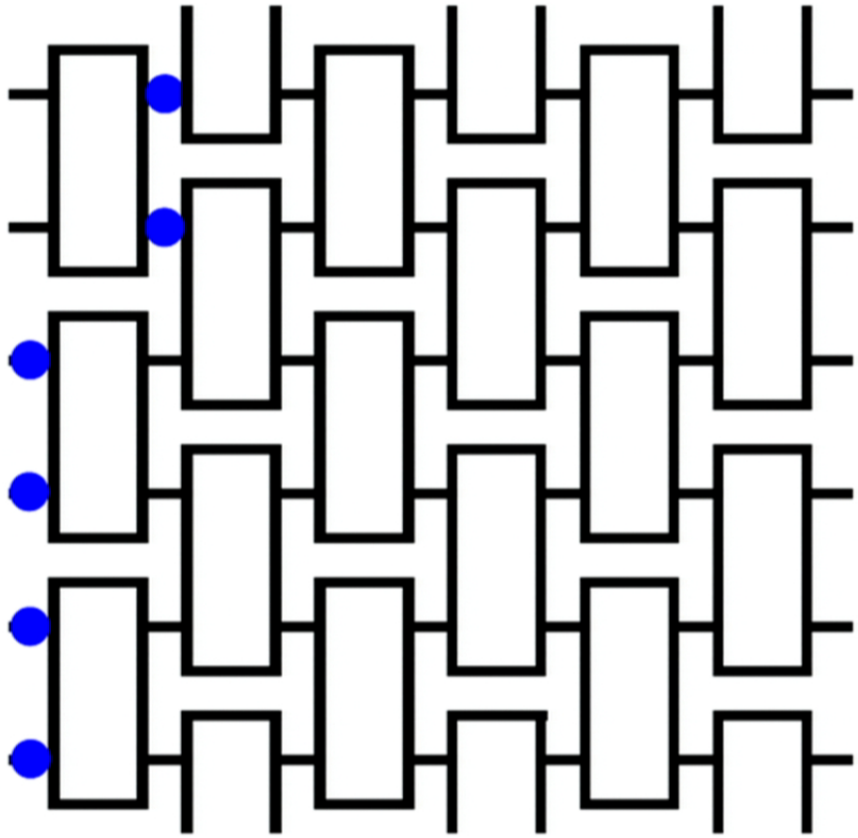
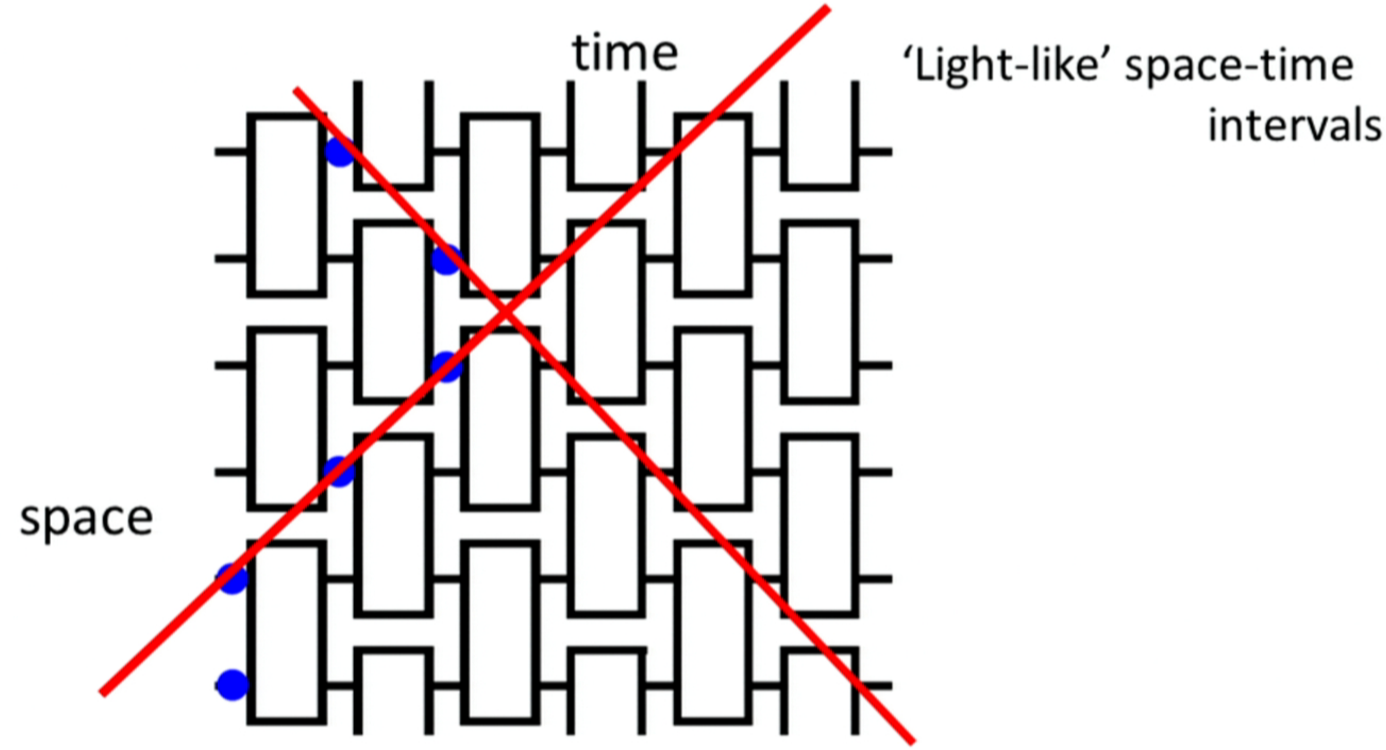


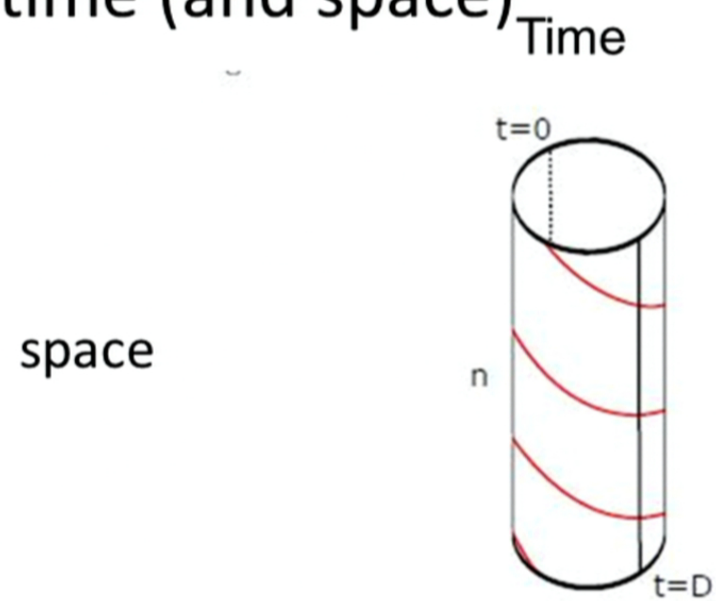
Definition valid time-configuration ***t*** (informally): for no pair of qubits interacting at time t in the circuit is the clock of one qubit past t and the clock of the other qubit before t.

$H_{circuit}$ preserves the subspace of valid time-configurations

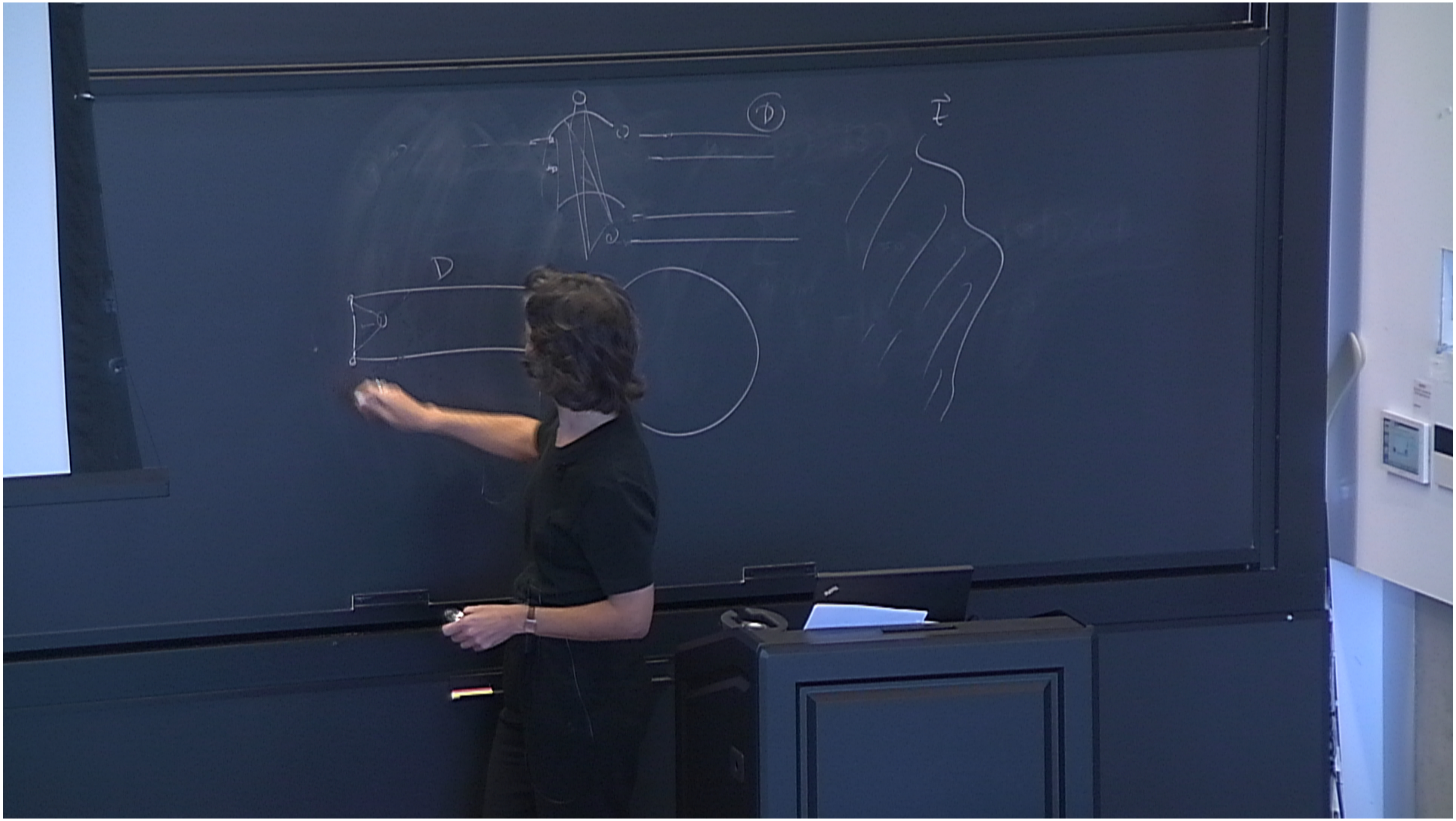


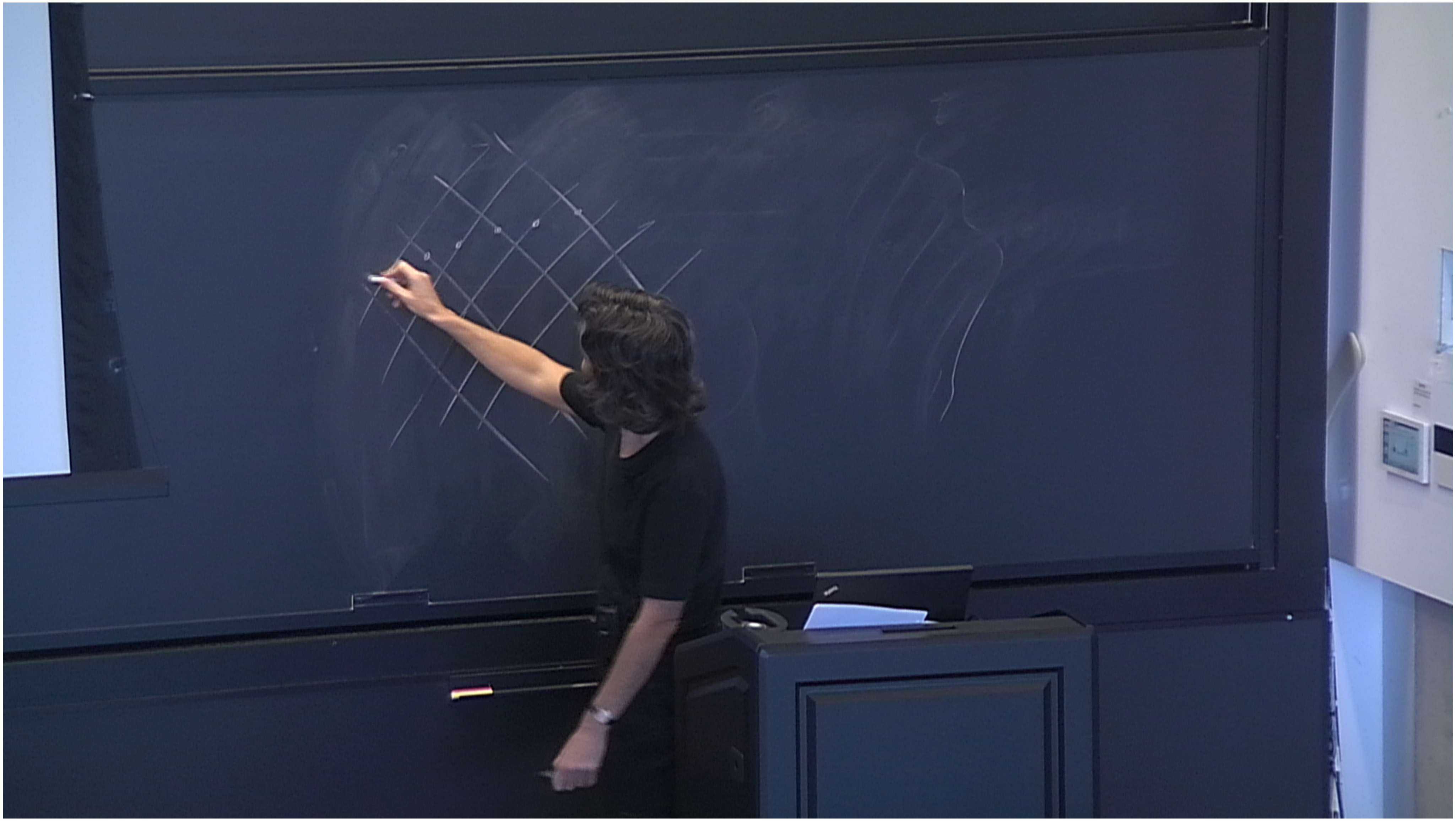
Zero-energy pure light-like **t** can be avoided. The ground-state of the circuit Hamiltonian equals history state
$\sum_{proper\ \boldsymbol{t}} V(\boldsymbol{t} \leftarrow \boldsymbol{0})|\xi\rangle|\boldsymbol{t} = t_1 \dots t_n\rangle$.
with $V(\boldsymbol{t} \leftarrow \boldsymbol{0})$ those unitaries which are applied to go from **0** (all clocks reading t=0) to time-string **t**.

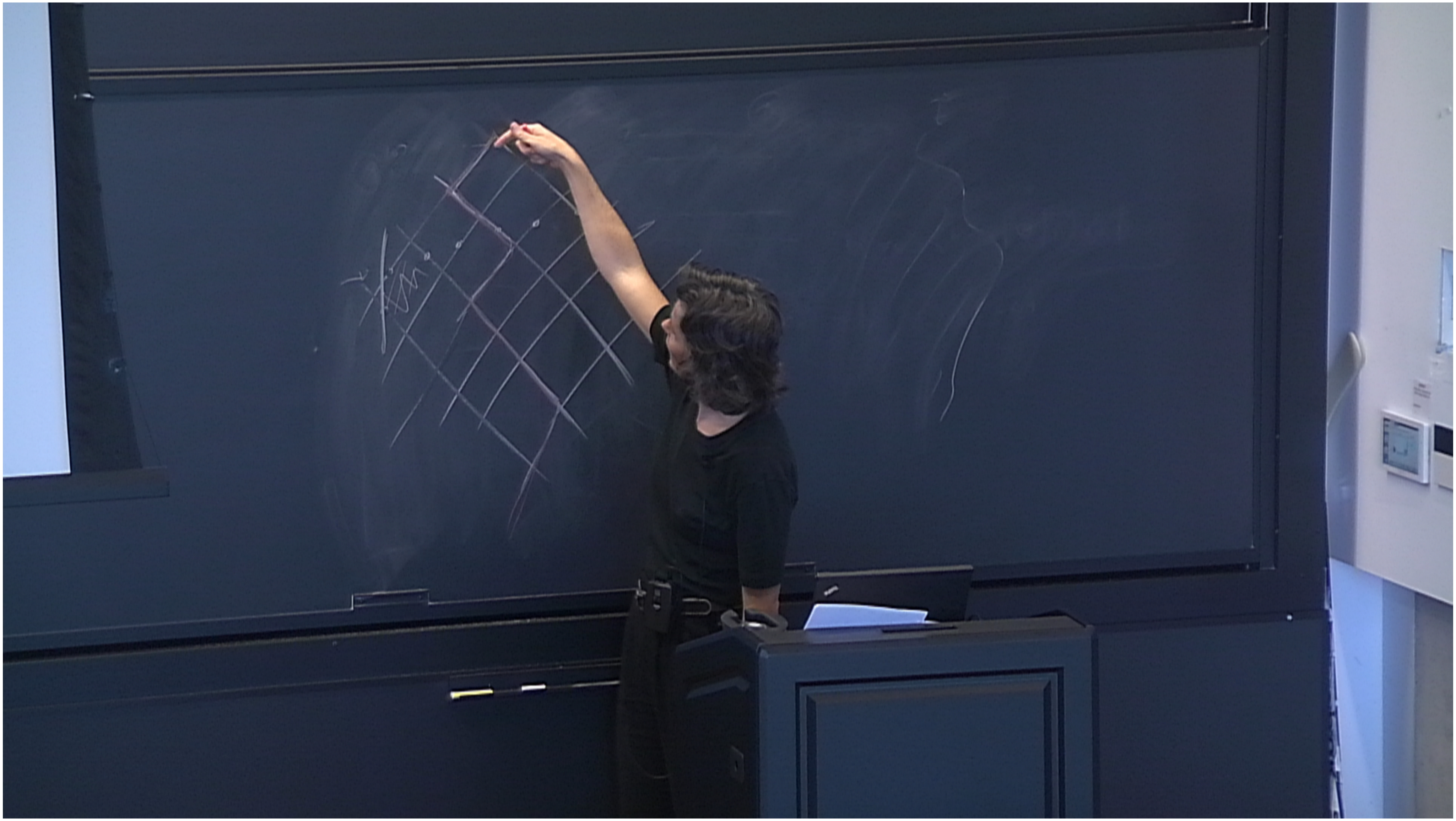
Consequence of periodic boundary conditions in time (and space)



We can construct a closed loop in space-time if n=2 k D for integer k that has zero-energy. When one travels at the speed of light along this loop, one gets back to the same position (in space-time).







Application for QMA

Using this lower bound on the gap, we can prove that (informally)

"determining the lowest eigenvalue of a two-dimensional interacting fermion model (periodic boundary conditions in both directions) in the sector where there is one fermion per line is QMA-complete."

To prove this one needs a.o. to add spatially-local terms to Hamiltonian in order to penalize invalid time-configuration (realized by blue quartic operators in the picture)

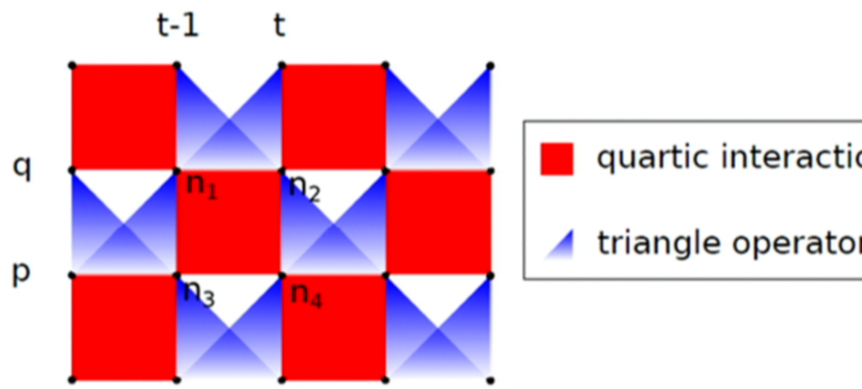


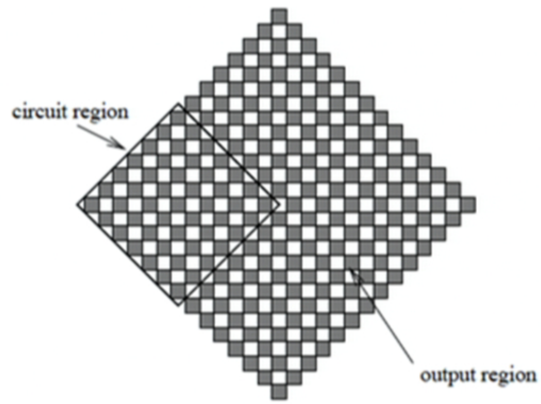
Figure 5: The black dots are fermionic sites, each with two modes (an ↑ or ↓ s say). The (red) squares represent the quartic gate interactions and the (blue) tria operators penalize improper fermionic configurations (improper time-configuratic A (blue) triangle operator with top corner a and bottom corners b and c eq $n_a(1 - n_b - n_c)$. The lattice has periodic boundary conditions in both directions.

Entropy pushes the time forward…

Let circuit region be k x k and the whole region is n x n.

One can prove (Janzing, PRA, 2007)

Thm: That there is a time O(k) such that the probability of finding the whole string in the output region is at least 1-12/k (if n>> k)



Choose $k = \frac{n}{4} - c\sqrt{n}$: the probability of finding the string in the output region for a random time goes to 1 as n→∞ (there are many more strings in the bulk of the lattice!)

Analysis for dynamics on the torus is more involved due to moving boundary and absence of entropy argument.

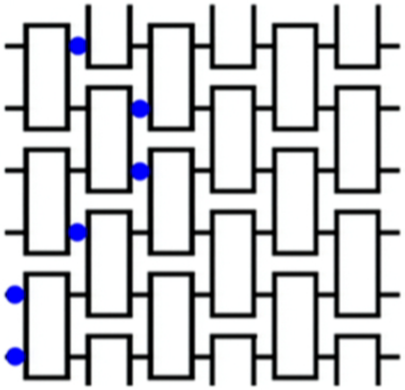
Quantum Adiabatic Computation

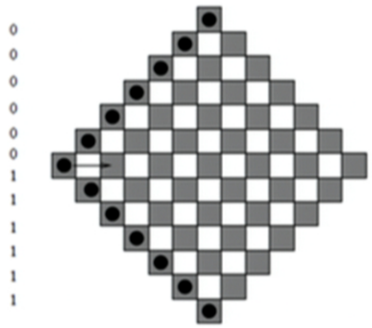
Consider the quantum adiabatic Hamiltonian

$$H(\lambda) = \lambda\, H_{circuit} + (1-\lambda)H_{init} + \sqrt{1-\lambda^2}H_{past}$$

in the state space of strings, i.e. bit strings of length n with n/2 1s.

H_{init} penalizes clocks of nearest-neighbor qubits to be synchronized and H_{past} breaks the symmetry between beginning and end of the circuit by forcing times of top & bottom qubit to be at the beginning.





Questions

- Continuum Limit of Space-Time Model in 1+1 dim. and 3+1 dim: connection with discretized field theories.

- Construction of perturbation gadgets which map joint-hopping interactions unto three types of terms: (1) $a^\dagger b + b^\dagger a$, (2) $n_a = a^\dagger a$, and (attractive or repulsive) interaction (3) $n_a n_b$ for two modes *a* and *b*. This would show the universality of such physical interactions.