Title: Can Quantum Correlations be Explained Causally?

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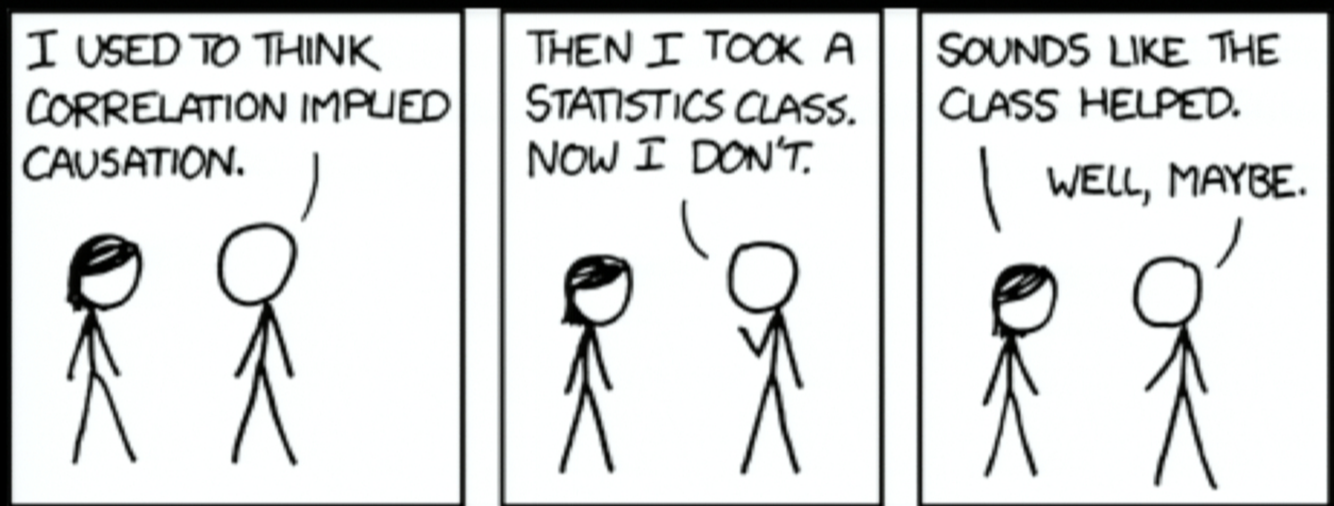
Abstract: There is a strong correlation between the sun rising and the rooster crowing, but to say that the one causes the other is to say more. In particular, it says that making the rooster crow early will not precipitate an early dawn, whereas making the sun rise early (for instance, by moving the rooster eastward) can lead to some early crowing. Intervening upon the natural course of events in this manner is a good way of discovering causal relations. Sometimes, however, we can't intervene, or we'd prefer not to. For instance, in trying to determine whether smoking causes lung cancer, we'd prefer not to force any would-be nonsmokers to smoke. Fortunately, there are some clever tricks that allow us to extract information about what causes what entirely from features of the observed correlations. One of these tricks was discovered by the physicist John Bell in 1964. In a groundbreaking paper, he used it to demonstrate the seeming impossibility of providing a causal explanation of certain quantum correlations. This revealed a fundamental tension between quantum theory and Einstein's theory of relativity --the two central pillars of modern physics. It is a tension that is still with us today.

Can Quantum Correlations Be Explained Causally?

Rob Spekkens
Perimeter Institute



From XKCD comics

ISSYP 2014

Simpson's Paradox

P(recovery | drug) > P(recovery | no drug)

Simpson's Paradox

P(recovery | drug) > P(recovery | no drug)

P(recovery | drug, male) < P(recovery | no drug, male)

Simpson's Paradox

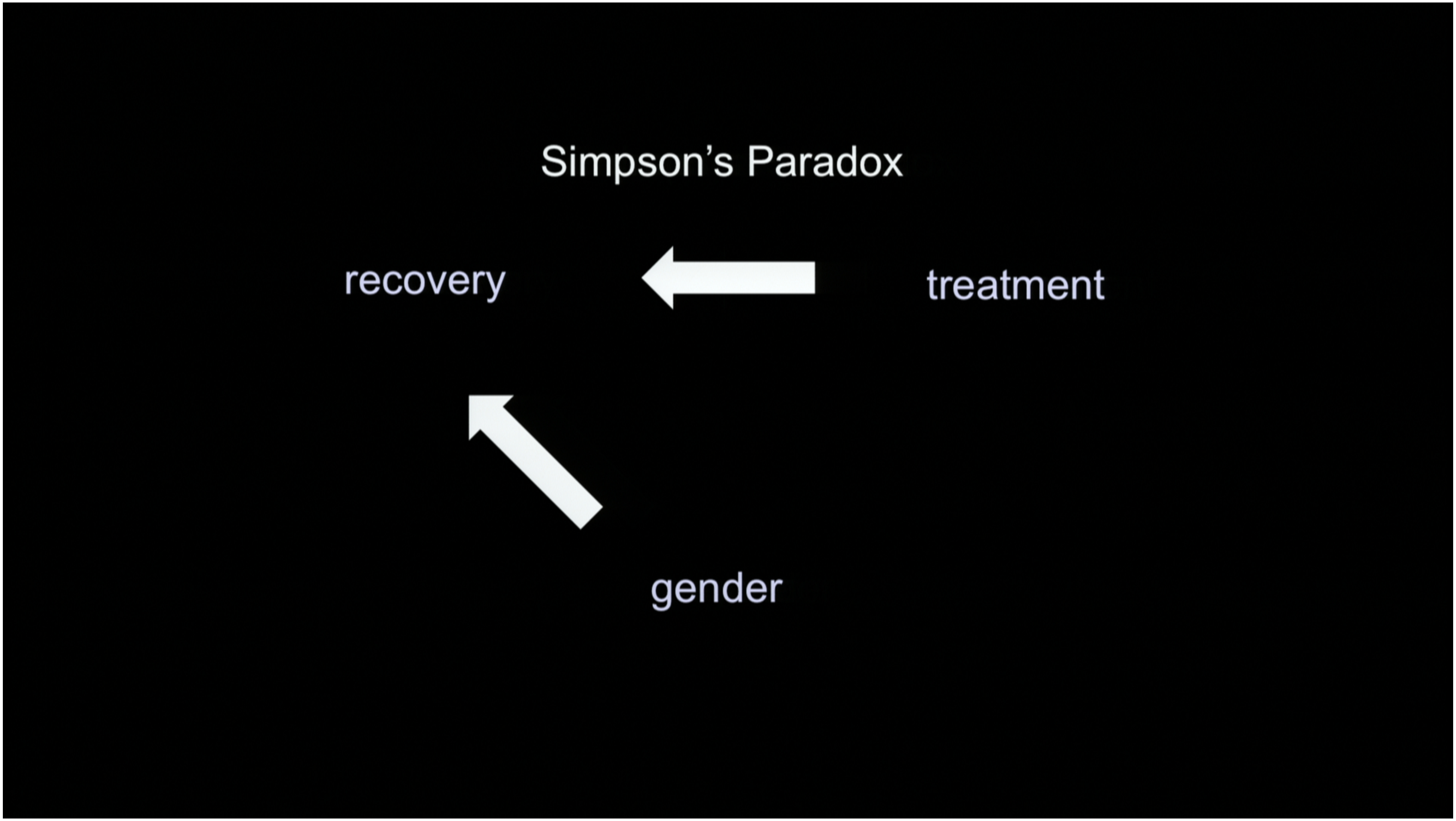
P(recovery | drug) > P(recovery | no drug)

P(recovery | drug, male) < P(recovery | no drug, male)

P(recovery | drug, female) < P(recovery | no drug, female)

Recovery probability

	drug	no drug
male	180/300 = 60%	70/100 = 70%
female	20/100 = 20%	90/300 = 30%
combined	200/400 = 50%	160/400 = 40%


Simpson's Paradox
recovery
treatment
gender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Simpson's Paradox

$$P(\text{recovery} \mid do(\text{drug})) \neq P(\text{recovery} \mid observe(\text{drug}))$$

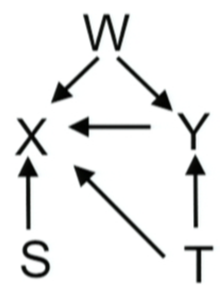
causation | correlation

How can we infer
causal relations from correlations?

J. Pearl, Causality: Models, Reasoning and Inference
P. Spirtes, C. Glymour, R. Scheines, Causation, Prediction and Search

Causal Model

Causal Structure



Causal-Statistical Parameters

$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

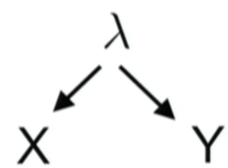
$P(Y|T,W)$

Reichenbach's principle

Statistical correlations must be explained causally

If X and Y are correlated, then

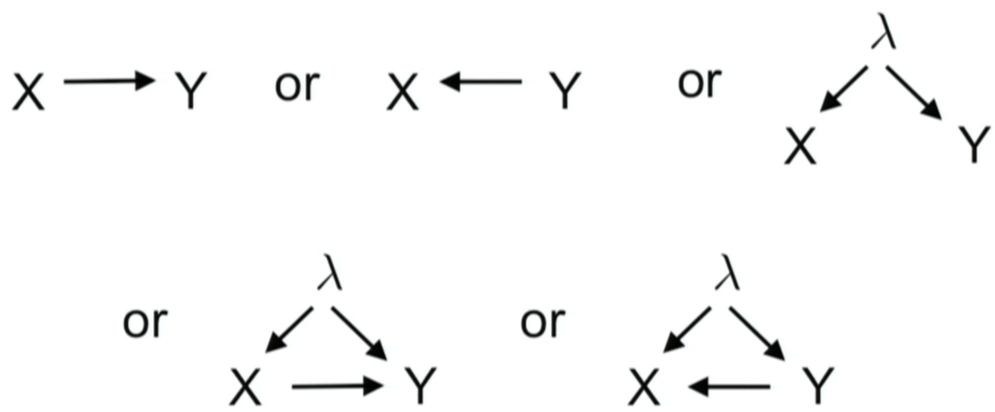
$X \longrightarrow Y$ or $X \longleftarrow Y$ or



Reichenbach's principle

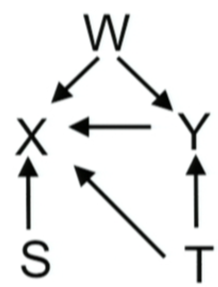
Statistical correlations must be explained causally

If X and Y are correlated, then



Causal Model

Causal Structure	Causal-Statistical Parameters



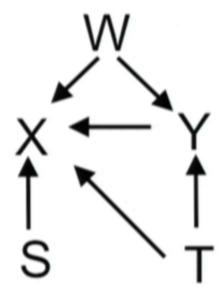
$P(W)$
$P(S)$
$P(T)$
$P(X|S,T,W,Y)$
$P(Y|T,W)$

- Parentless variables are independently distributed

Causal Model

Causal Structure

Causal-Statistical Parameters



$P(W)$

$P(S)$

$P(T)$

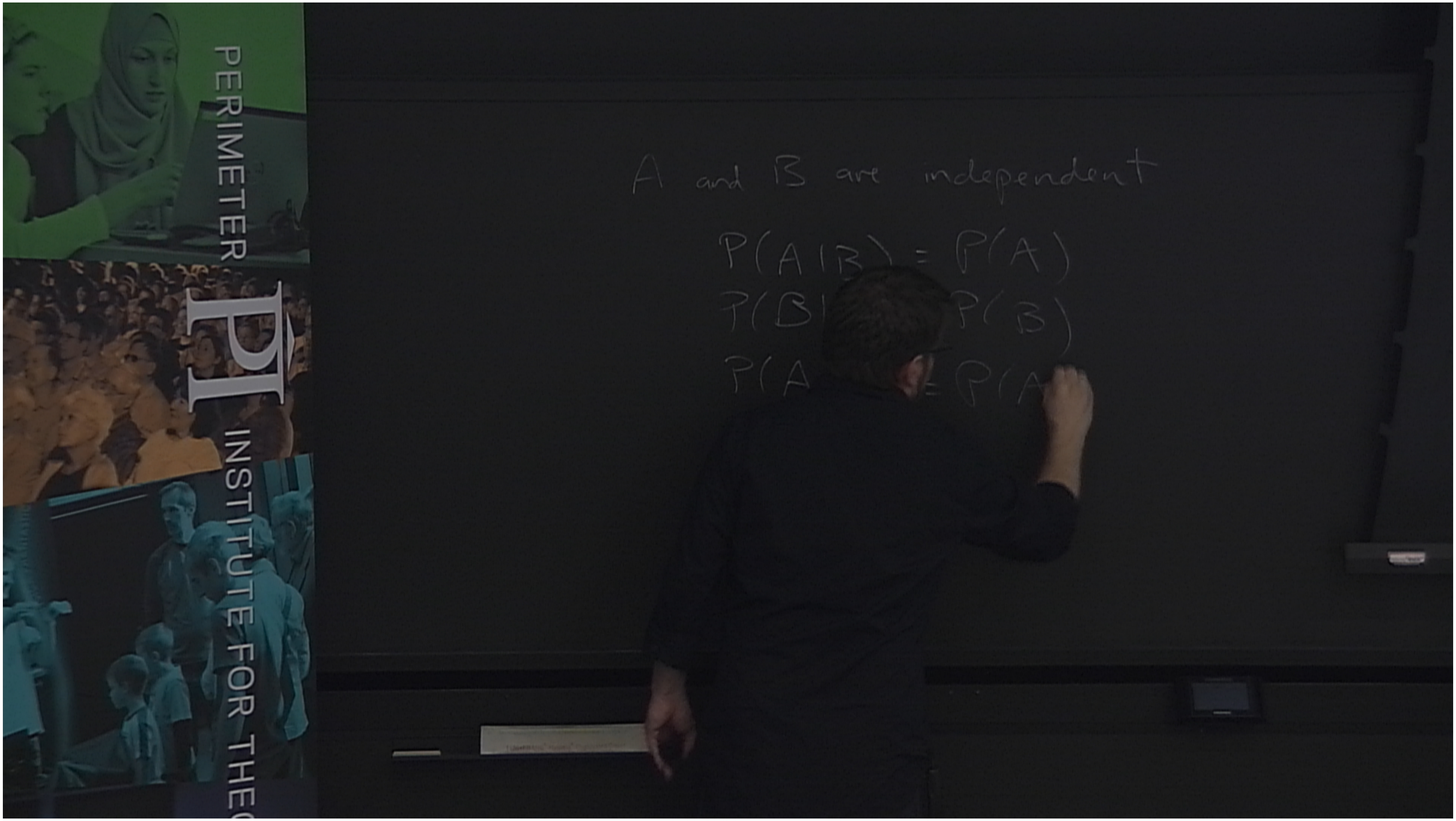
$P(X|S,T,W,Y)$

$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal inference algorithms seek to solve the inverse problem

Inferring facts about the causal structure from statistical independences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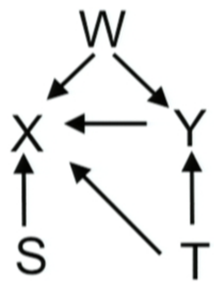
A and B are independent
P(A|B) = P(A)

Causal Model



$P(W)$
$P(S)$
$P(T)$
$P(X|S,T,W,Y)$
$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Def'n: A and B are conditionally independent given C

$$P(A|B,C) = P(A|C)$$
$$P(B|A,C) = P(B|C)$$
$$P(A,B|C) = P(A|C)P(B|C)$$

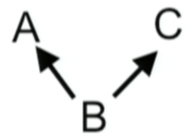
Denote this
$(A \perp B|C)$

fork

$A \not\perp C$

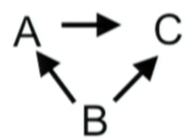
$(A \perp C|B)$

fork



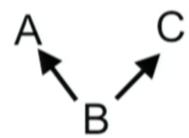
$$\Rightarrow \quad A \not\perp C \qquad (A \perp C|B)$$

confounded cause



$$\Rightarrow \quad A \not\perp C \qquad (A \not\perp C|B)$$

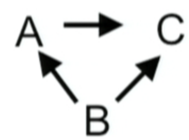
fork



$A \not\perp C$

$(A \perp C|B)$

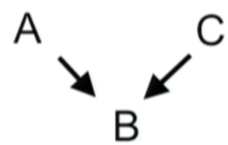
confounded cause



$A \not\perp C$

$(A \not\perp C|B)$

collider

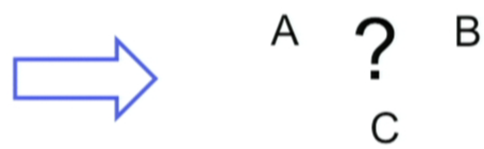


$A \perp C$

$(A \not\perp C|B)$

$$A \perp B$$

and no other independence relations



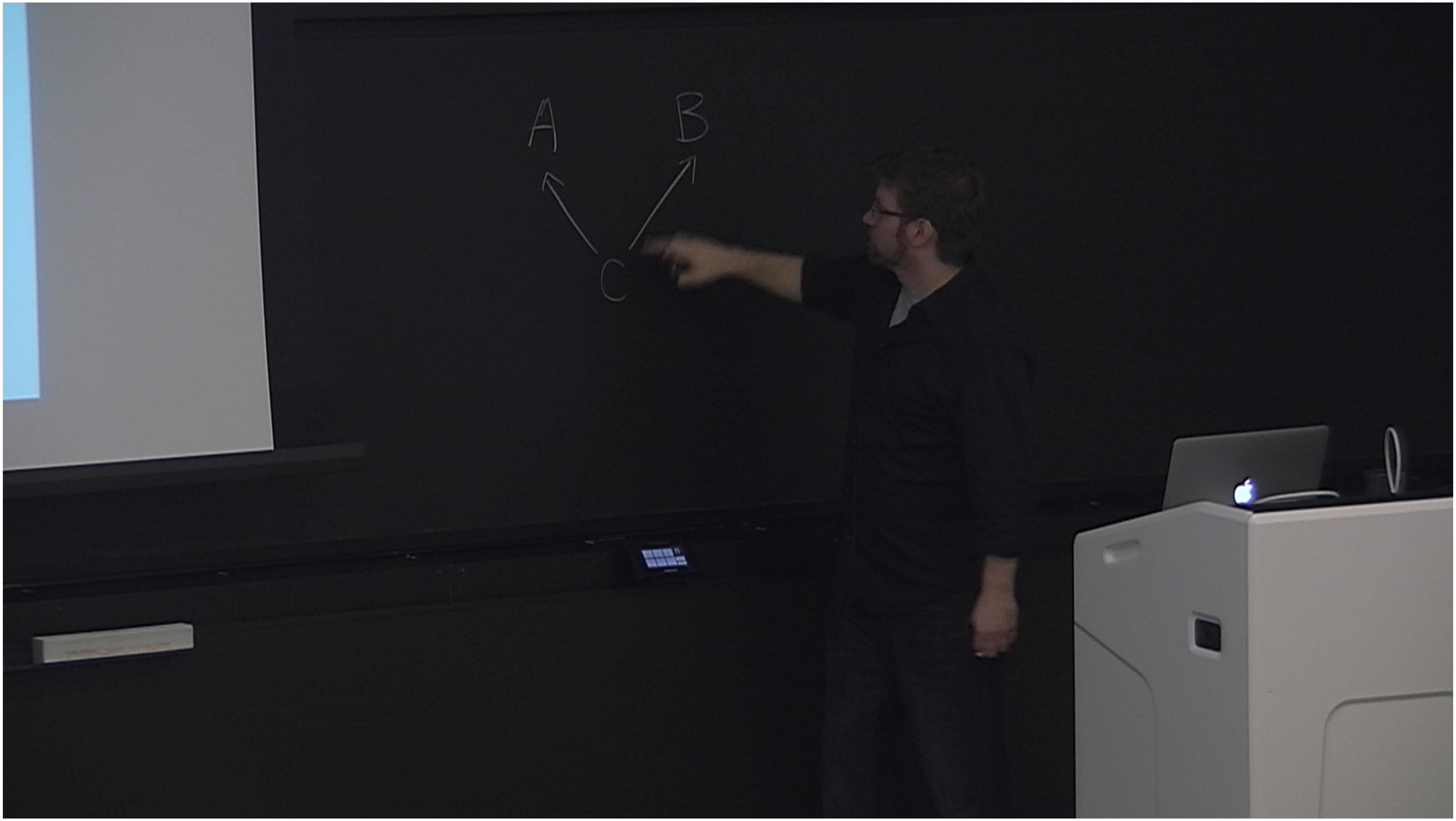
$A \perp B$

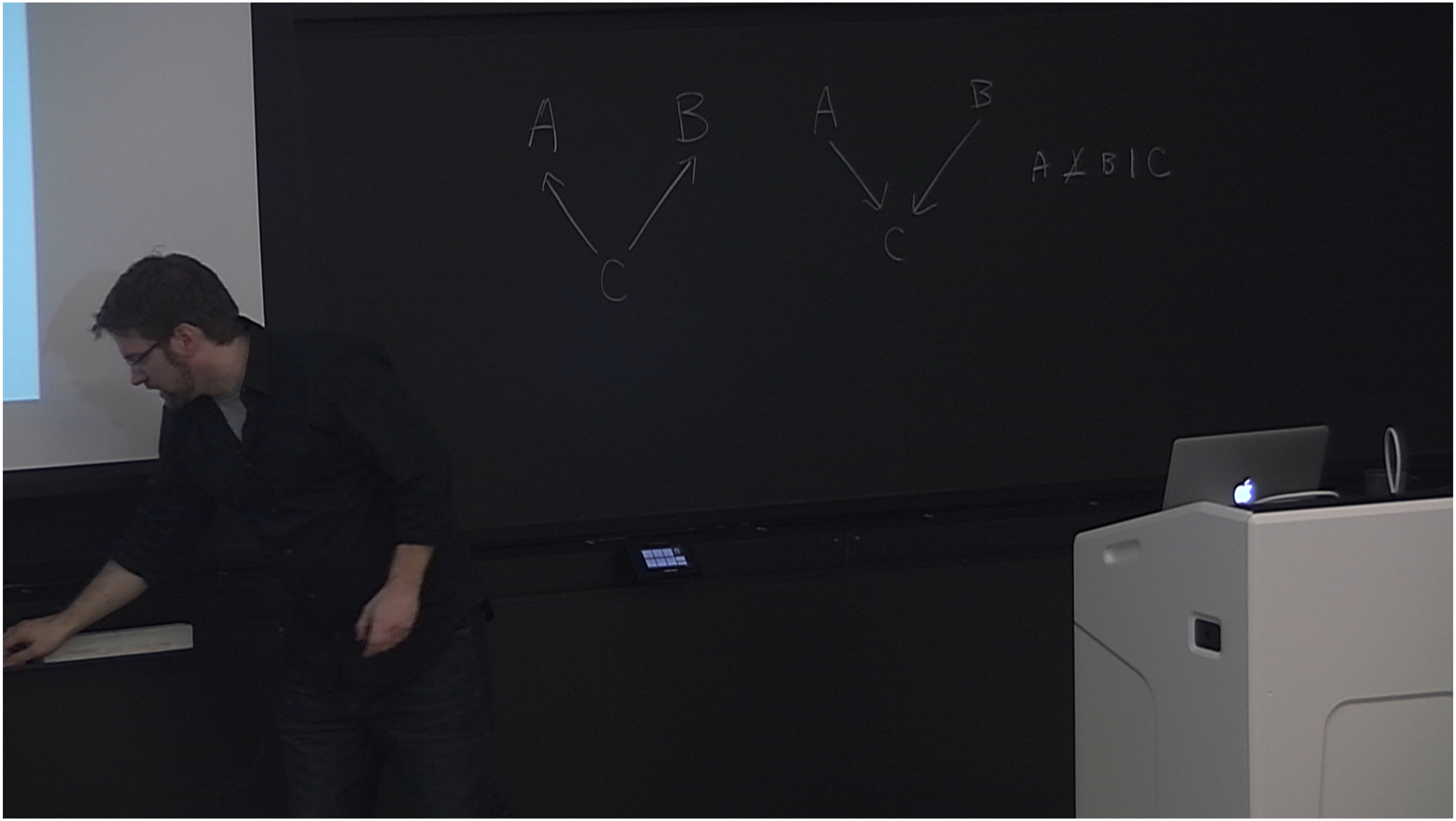
and no other independence relations

⇒

A ? B

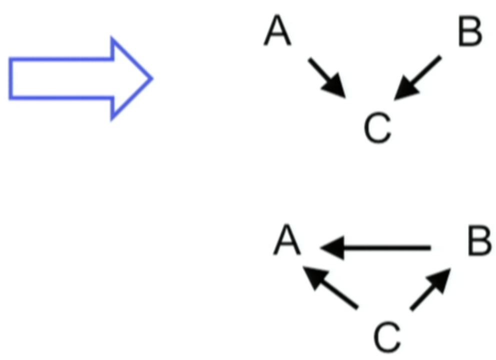
C


A
B
C


A B
C
A B
C
A ⊥̸ B | C

$A \perp B$

and no other independence relations



$P(A)$
$P(B)$
$P(C|A,B)$ ✓

$P(C)$
$P(B|C)$
$P(A|B,C)$ ✗

No Fine-tuning!

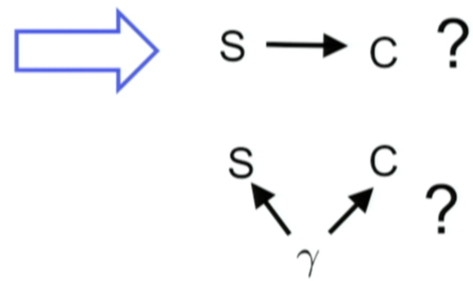
A key assumption of causal discovery algorithms

No fine-tuning

A causal model of an observed distribution is fine-tuned if the conditional independences in the distribution only hold for a set of measure zero of the values of the causal-statistical parameters in the model

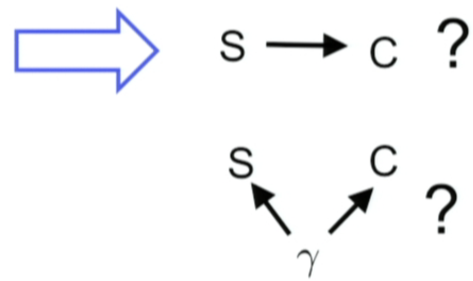
Does smoking cause lung cancer?

$S \not\perp C$



Does smoking cause lung cancer?

$S \not\perp C$



Suppose you also observe

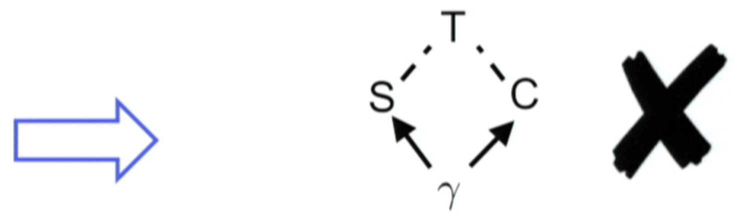
$$S \perp C \mid T$$

and no other independences

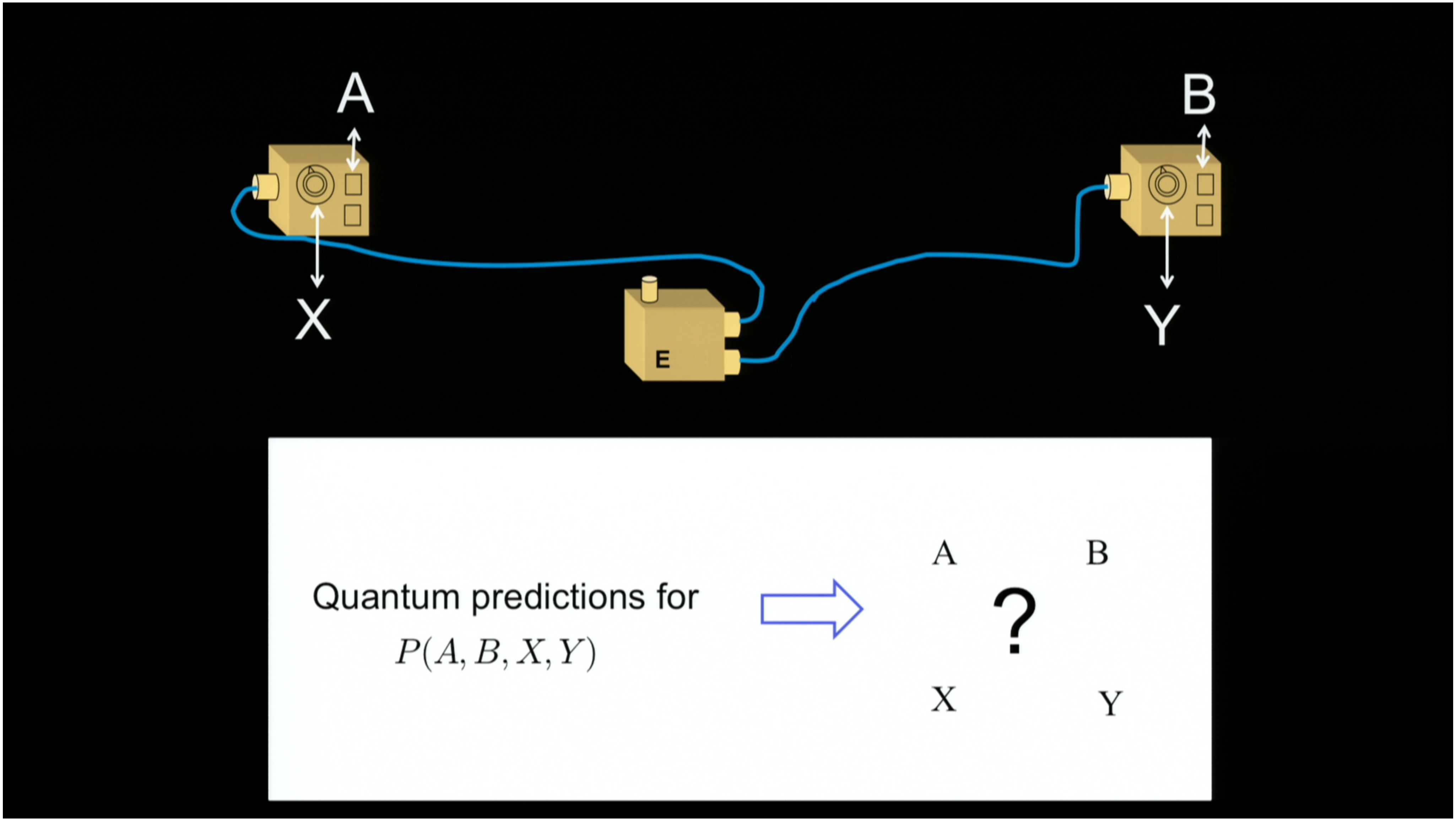
Does smoking cause lung cancer?

$S \perp C \mid T$

and no other independence relations



Inferring facts about the causal structure from the strength of correlations

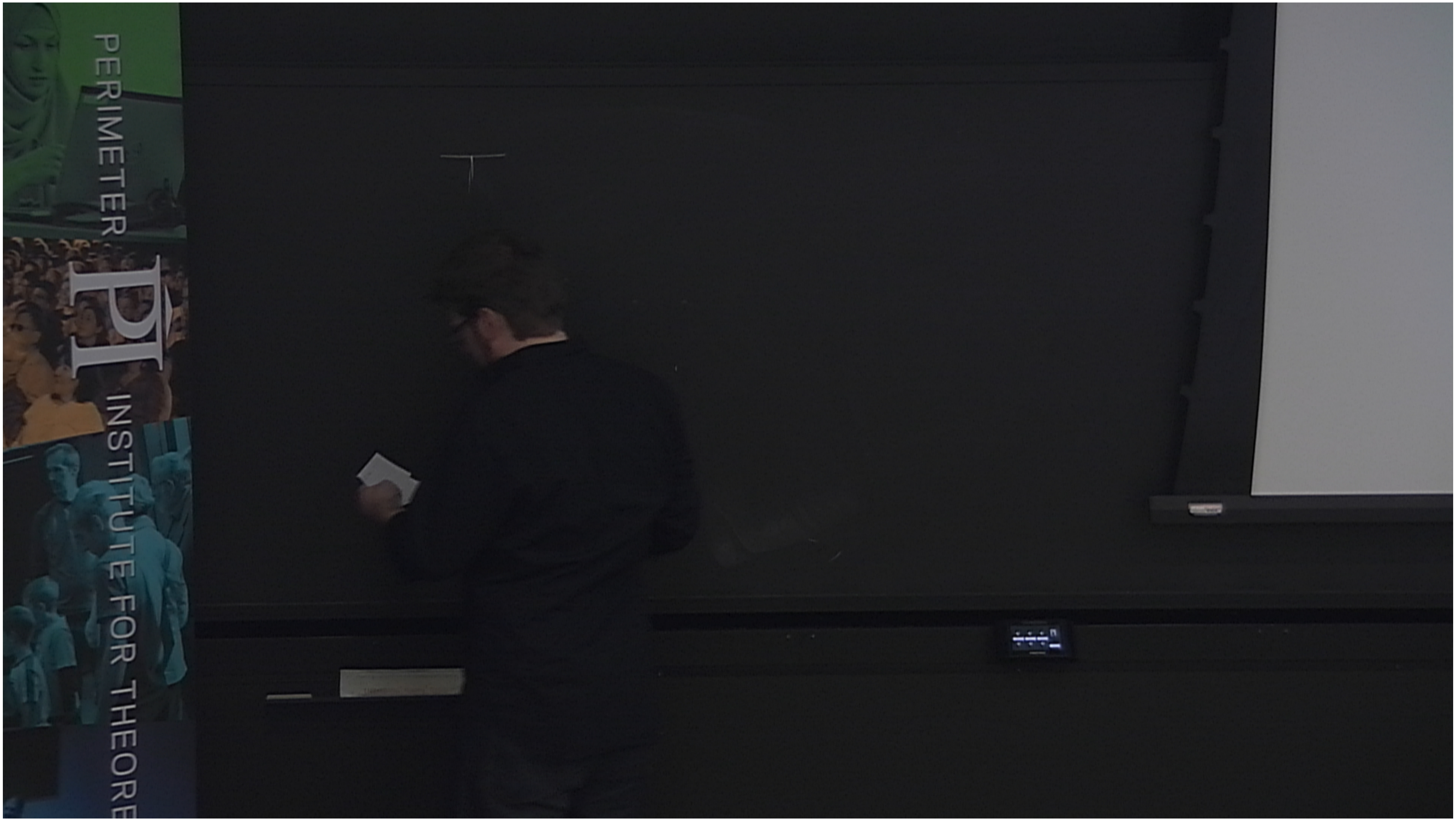

A
B
X
E
Y
Quantum predictions for
$P(A, B, X, Y)$
A
B
?
X
Y

There are two possible measurements, H and T, with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 1

1. Whenever the same measurement is made on A and B, the outcomes always agree	H and H or T and T
2. Whenever different measurements are made on A and B, the outcomes always disagree	H and T or T and H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There are two possible measurements, H and T, with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 2

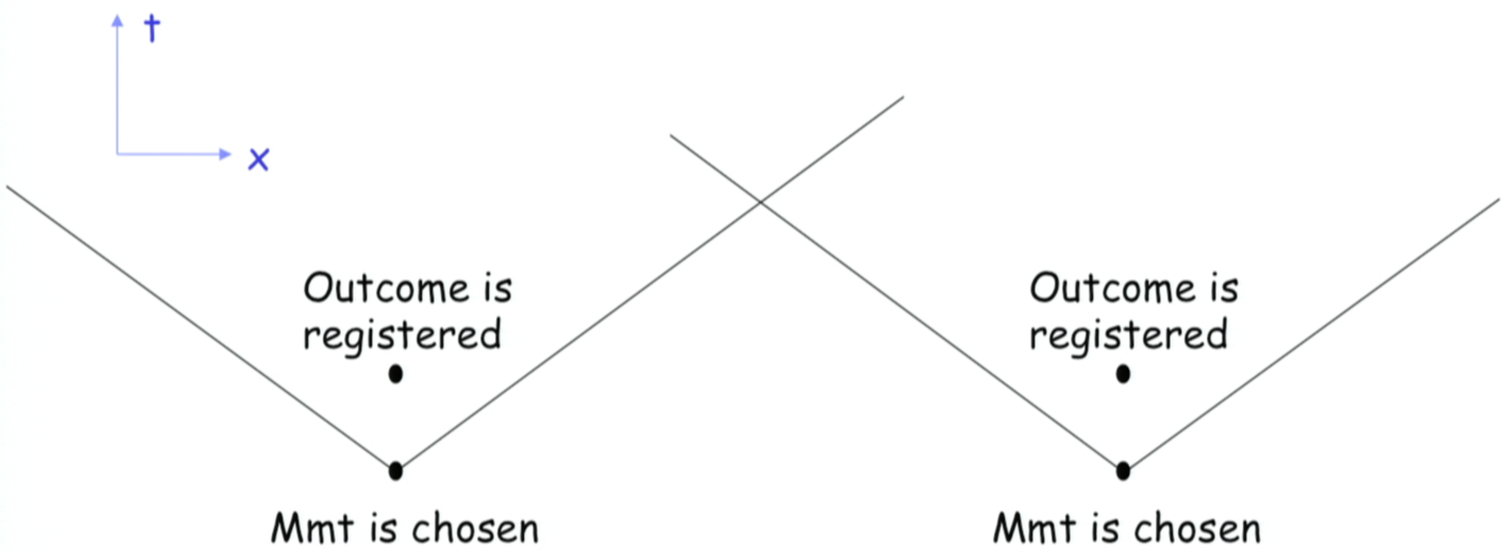
1. Whenever the same measurement is made on A and B, the outcomes always disagree	H and H or T and T
2. Whenever different measurements are made on A and B, the outcomes always agree	H and T or T and H

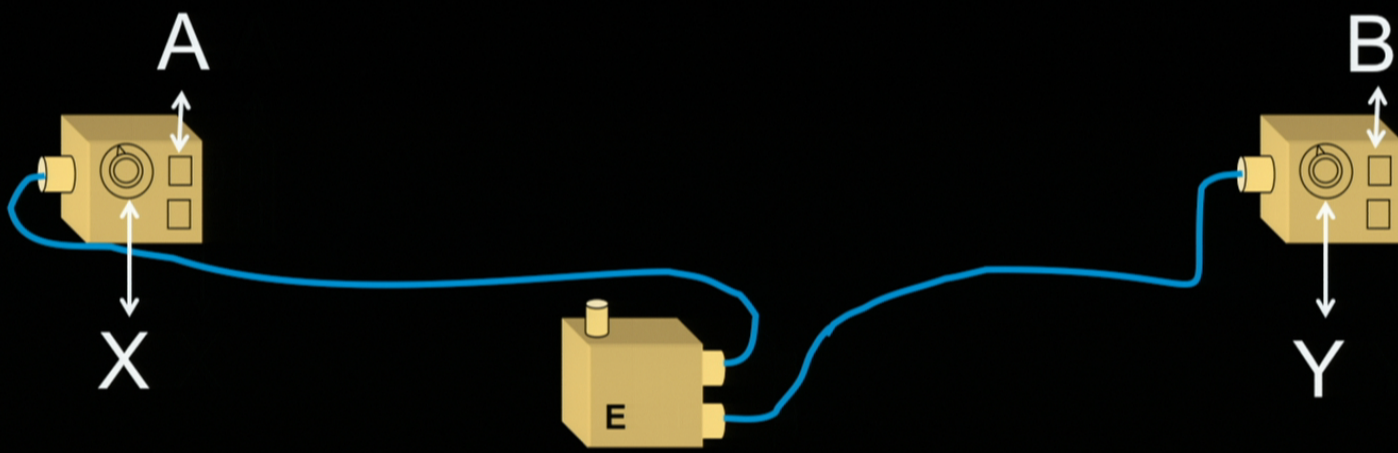

H
G
H
G
T
T
G
R

A: Rig the game so that the choices of settings are not random but instead are correlated with the local strategies

But surely nature isn't so conspiratorial...

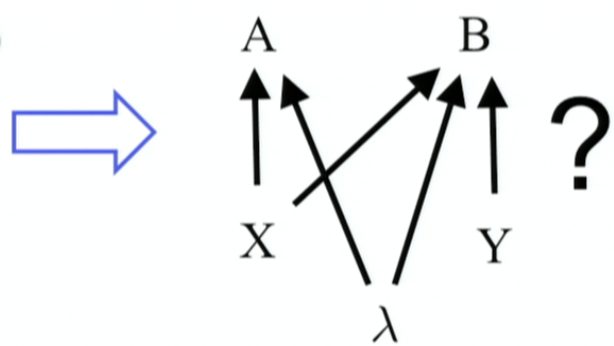
Tension with the theory of relativity





$$P(X,Y)$$
$$= (\tfrac{1}{2}[0] + \tfrac{1}{2}[1])(\tfrac{1}{2}[0] + \tfrac{1}{2}[1])$$

$$P(A,B|X,Y)$$
$$= \tfrac{1}{2}[00] + \tfrac{1}{2}[11] \quad \text{if } XY = 0$$
$$= \tfrac{1}{2}[01] + \tfrac{1}{2}[10] \quad \text{if } XY = 1$$



- Reichenbach's principle

- No fine-tuning

⇩

Contradiction with quantum theory and experiment