

Title: Exploring N=1 theories of class S through Higgsing, dualizing and twisting

Date: Jul 15, 2014 02:30 PM

URL: <http://pirsa.org/14070024>

Abstract: We study a class of 4d N=1 SCFTs obtained from partial compactifications of 6d N=(2, 0) theory on a Riemann surface with punctures. We identify theories corresponding to curves with general type of punctures through nilpotent Higgsing and Seiberg dualities. The `quiver tails' of N=1 class S theories turn out to differ significantly from N=2 counterpart and have interesting properties. Various dual descriptions for such a theory can be found by using colored pair-of-pants decompositions. Especially, we find N=1 analog of Argyres-Seiberg duality for the SQCD with various gauge groups. We compute anomaly coefficients and superconformal indices to verify our proposal.

Γ $N=1$ theories of class S

- Higgsing, Dualities, Twists

[P. Agarwal, JS] 1311, 2945

[P. Agarwal, I. Behr, K. Moriyoshi, JS]
1407, xxxr

1. $N=1$ class S

2. Nilpotent Higgsing

3. "Fan" (quiver)

4. Fan from $N=1$ d

5. $N=1$ Argyros

6. Conclusion

1. $N=1$ case S

$[BW] [R^3W] [X_\omega] [GUTY] [BB]$

$g+n$

- $T \in ADE$

- Riemann surface $\mathcal{C}_{g,n}$

- $(p, q) \in \mathbb{Z}_{20}$ $p+q = 2g-2+n$

- For each puncture $(\underline{p}_i, \underline{\sigma}_i)$
 $\underline{p}_i: S^4/\mathbb{Z}_2 \hookrightarrow \mathbb{P}^1$, $\underline{\sigma}_i \in \mathbb{Z}_2$

1. $N=1$ case S

$[BW] [R^3W] [X_e] [GUTY] [BB]$

Data

$T \in ADE$ + Twisting
Riemann surface $\mathcal{C}_{g,n}$
 $(p, q) \in \mathbb{Z}_{20}$ $p+q = 2g-2+n$
For each puncture $(\underline{p}_i, \underline{\sigma}_i)$
 $\underline{p}_i \in \mathbb{Z} \hookrightarrow p, \quad \underline{\sigma}_i \in \mathbb{Z}$

$[BW] [B^3W] [X_e] [GMTY] [BB]$

$T \in ADE$

Riemann surface

$\mathbb{C}_{g,n}$

$(p, q) \in \mathbb{Z}_{\geq 0}$

$p+q = 2g-2+n$

For each puncture

$(p_i, \sigma_i) \neq$
 $p_i \in \mathbb{N} \hookrightarrow p, \sigma_i \in \mathbb{Z}$

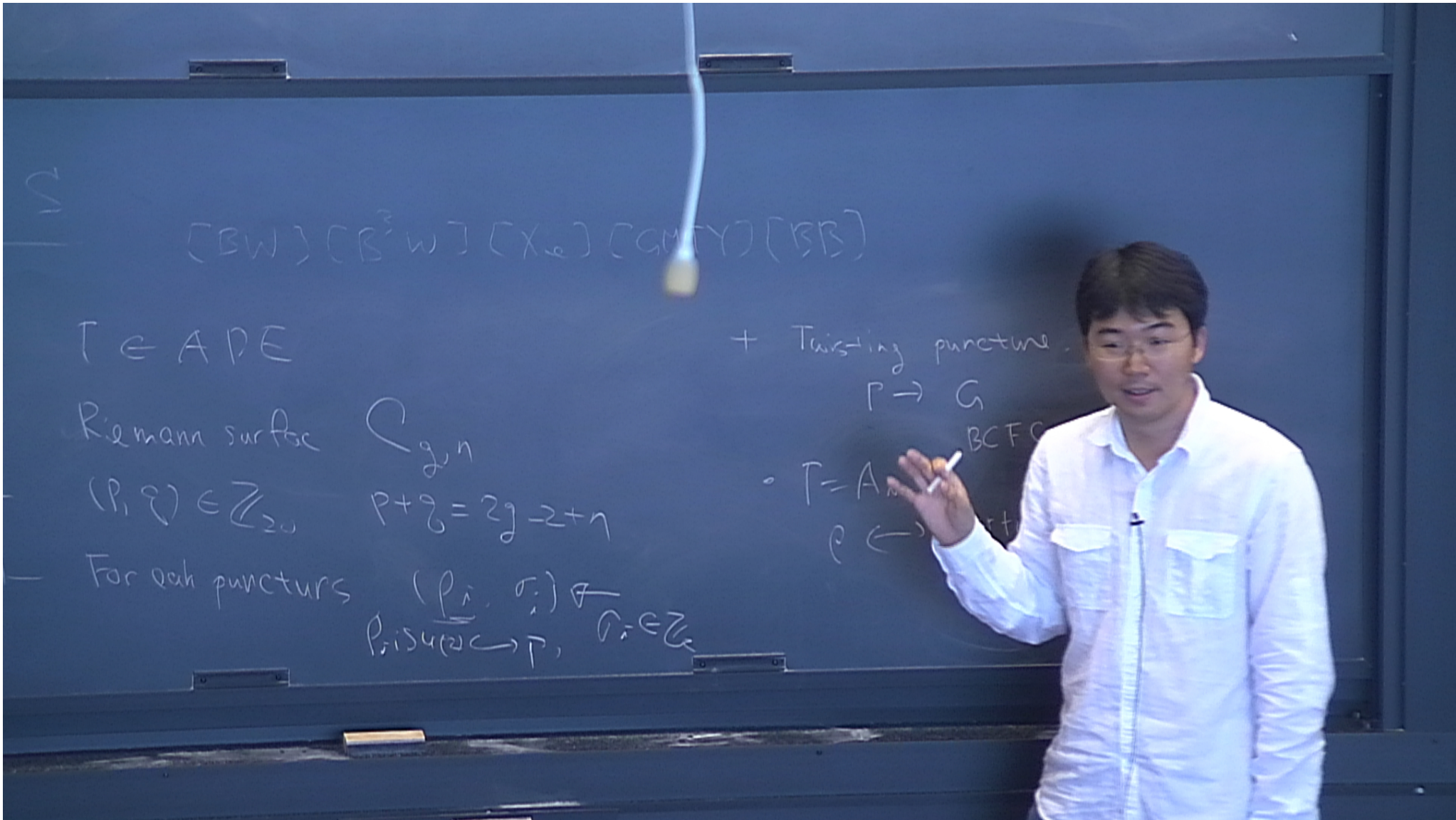
+ Twisting puncture

$P \rightarrow G$

BCFG

$\tau = A_n$

$p \leftarrow$



$[BW] [B^3W] [X_{\infty}] [GMTY] [BB]$

$T \in ADE$

Riemann surface

$\mathcal{C}_{g,n}$

$(p, q) \in \mathbb{Z}_{2n}$

$$p + q = 2g - 2 + n$$

For each puncture

$(p_i, \sigma_i) \neq$
 $p_i \in \mathbb{Z} \hookrightarrow p, \quad \sigma_i \in \mathbb{Z}_k$

+ Twisting puncture

$P \rightarrow G$

$$\bullet T = A_{n-1}$$

$P \in$

$N=1$ class S

$[BW] [B^3W] [X_6] [GUTY] [BB]$

Data

$T \in ADE$

+ Twisting part
 $\Gamma \rightarrow$

Riemann surface $\mathcal{C}_{g,n}$

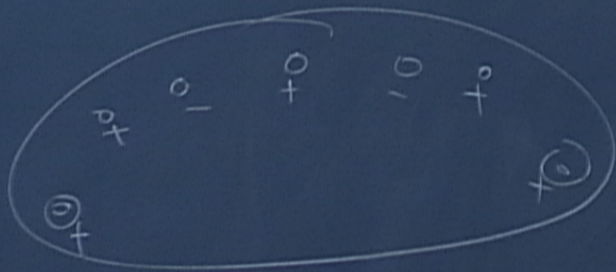
$(p, q) \in \mathbb{Z}_{\geq 0}$ $p+q = 2g - 2 + n$

$\bullet T = A_{n-1}$
 $\Gamma \leftarrow$

For each puncture

$(p_i, \sigma_i) \neq$
 $p_i \in \mathbb{N} \cup \{0\} \hookrightarrow p, \sigma_i \in \mathbb{Z}$

③ Colored pair-of-pants decomposition



$$(p, q) = (3, 2)$$

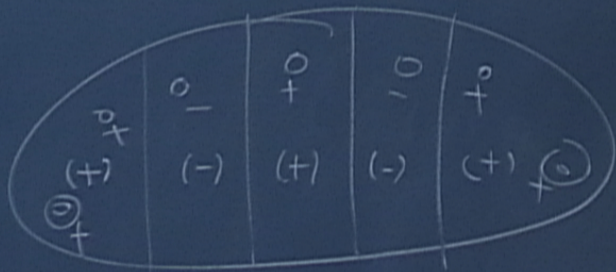
$$n_+ = 3 \quad n_- = 2$$

$$n_+^{\circ} = 2$$

$$p + q = 2g - 2 + n$$



⊙ Colored pair-of-pants decomposition



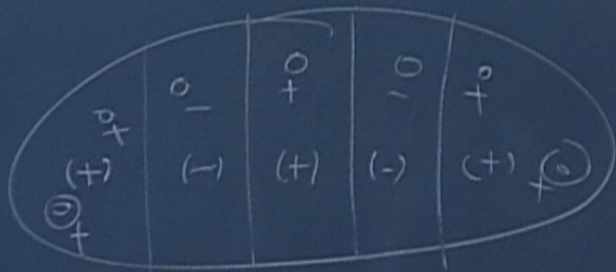
$$n_+ = 3 \quad n_- = 2$$

$$n_+^{\ominus} = 2$$

$$p+q =$$

$$(p, q) = (3, 2)$$

① Colored pair-of-pants decomposition



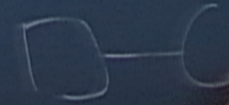
$$(p, q) = (3, 2)$$

$$2 \quad 3$$

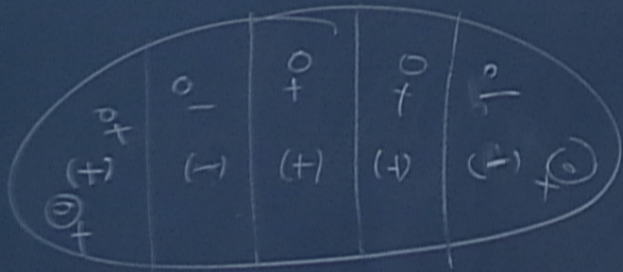
$$n_+ = 3 \quad n_- = 2$$

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$$p + q = 2g - 2 + n$$



① Colored pair-of-pants decomposition

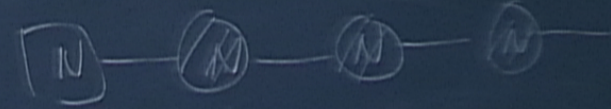


$$n_+ = 3 \quad n_- = 2$$

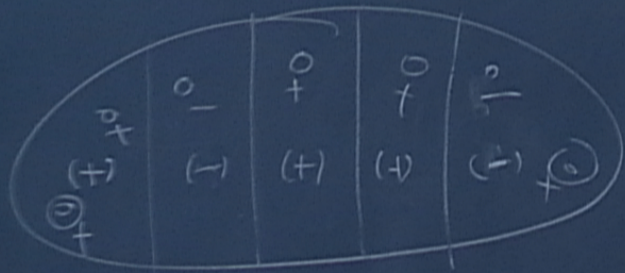
$$n_{\pm}^{\circ} = 2$$

$$p+q = 2+2 = 4$$

$$(p, q) = (3, 2)$$



③ Colored pair-of-pants decomposition



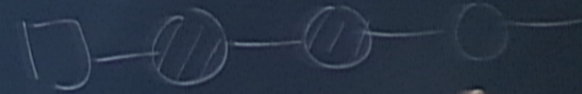
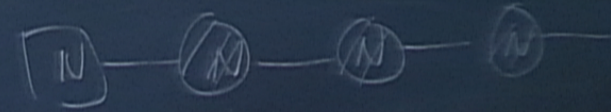
$$(p, q) = (3, 2)$$

$$2 \quad 3$$

$$n_+ = 3 \quad n_- = 2$$

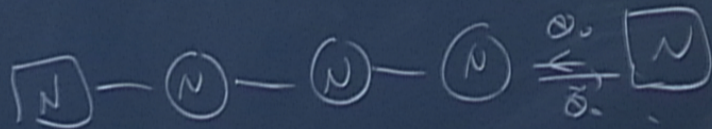
$$n_{\pm}^{\circ} = 2$$

$$p + q = 2g - 2 + n$$



2. Nilpotent Higgsing

• $N=2$



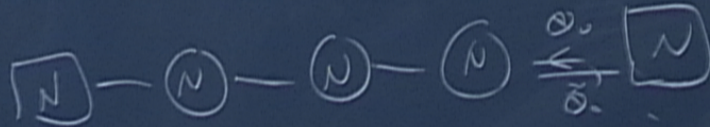
$$M_0 = \tilde{Q}_1 Q_2$$
$$\langle M_0 \rangle = \rho(\sigma^+)$$

$$N = \sum k N_{ik}$$

$$\rightarrow S[U]$$

2. Nilpotent Higgsing

• $N=2$



$$\mu_0 = \tilde{Q}_+ Q_-$$
$$\langle \mu_0 \rangle = \rho(\sigma^+)$$

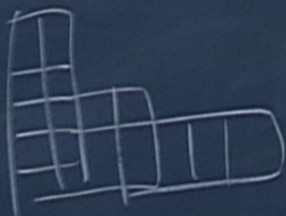
$$N = \sum_k k N_{1k}$$

$$\rightarrow S \left[\prod_k U(N_{1k}) \right]$$

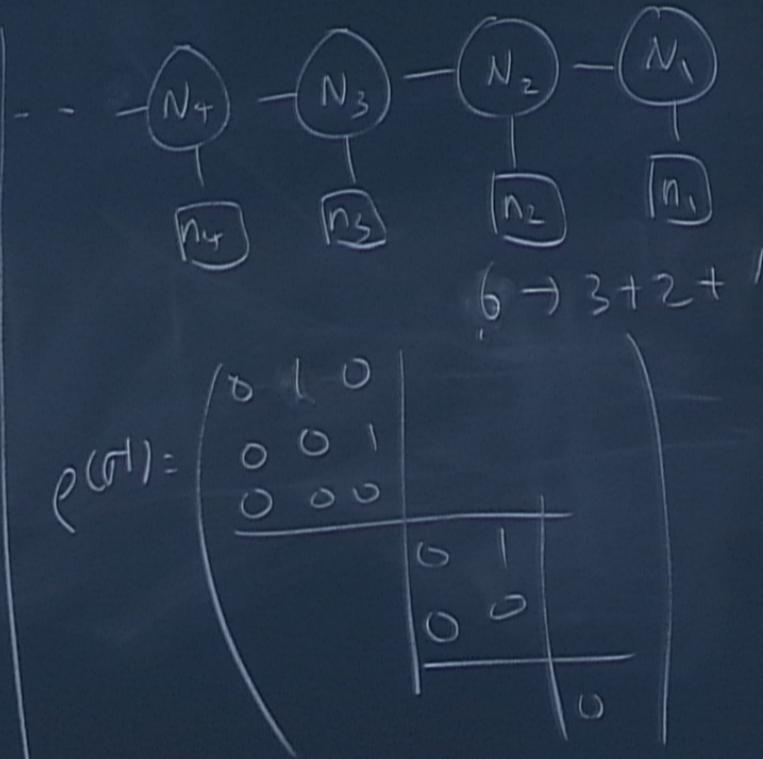


$$N = \sum_k k n_k$$

$$\rightarrow S \left[\prod_k U(n_k) \right]$$



$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$



$$W = \sum_{\tilde{n}} \phi_{\tilde{n}} (\tilde{Q}_{\tilde{n}} \tilde{\Theta}_{\tilde{n}} - \tilde{Q}_{\tilde{n}+1} \tilde{\Theta}_{\tilde{n}+1})$$

$$F_{\phi_{\tilde{n}}} = \tilde{Q}_{\tilde{n}} \tilde{\Theta}_{\tilde{n}} - \tilde{Q}_{\tilde{n}+1} \tilde{\Theta}_{\tilde{n}+1} = 0$$

$$\langle \tilde{Q}_0 \tilde{\Theta}_0 \rangle \rightarrow \langle \tilde{Q}_0 \tilde{\Theta}_0 \rangle = \langle \tilde{\Theta}_1 \tilde{\Theta}_1 \rangle$$

1. $N=1$ class
2. Nilpotent
3. "Fan" ()
4. Fan from
5. $N=1$ Arg
6. Conclusion

$$P_{154} \subset \mathbb{P}^3, \quad v_i \in \mathbb{C}$$

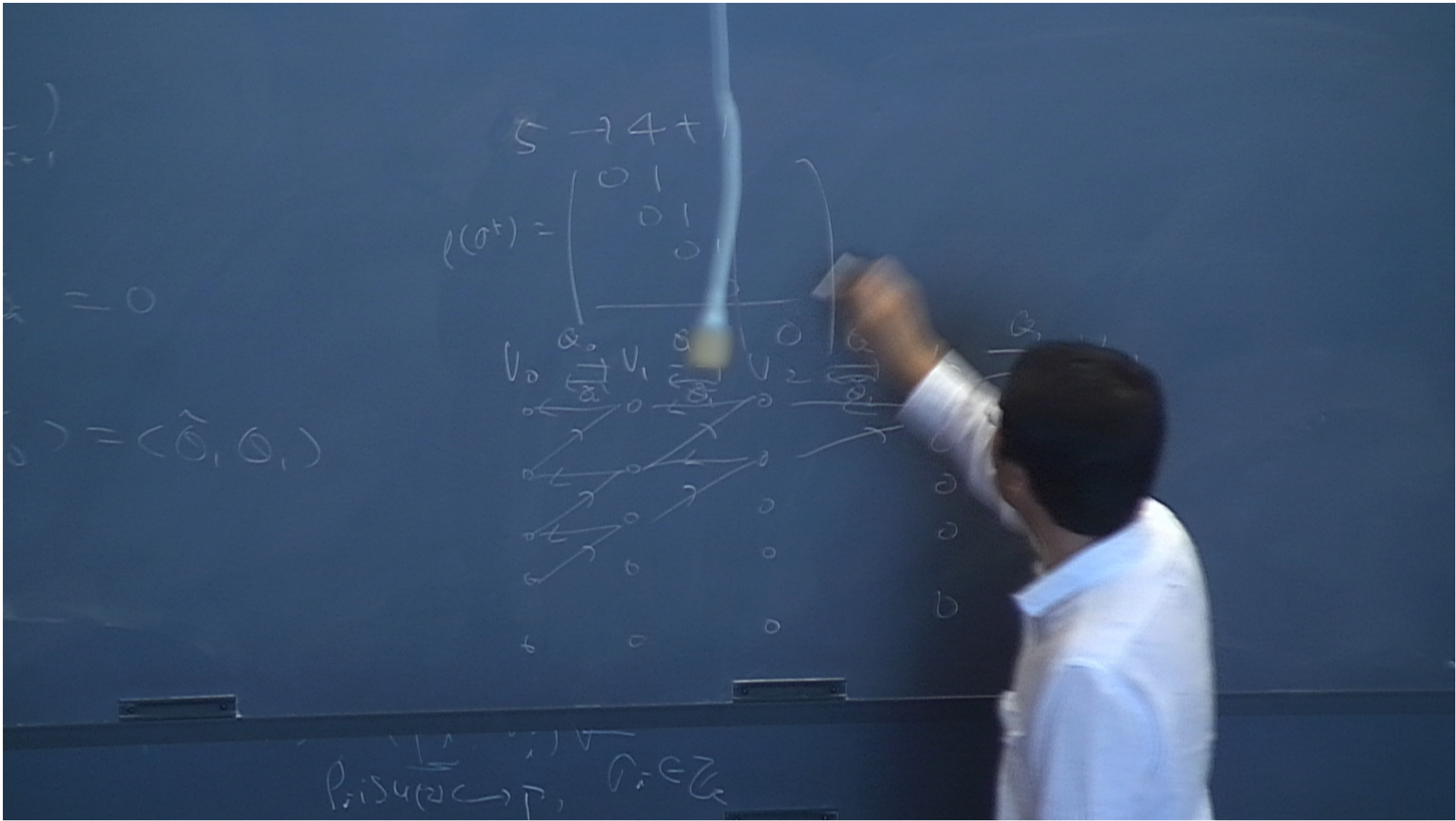
$$W = \sum_{\tilde{n}} \phi_{\tilde{n}} (\tilde{Q}_{\tilde{n}} \tilde{\theta}_{\tilde{n}} - \tilde{Q}_{\tilde{n}+1} \tilde{\theta}_{\tilde{n}+1})$$

$$F_{\phi_{\tilde{n}}} = \tilde{Q}_{\tilde{n}} \tilde{\theta}_{\tilde{n}} - \tilde{Q}_{\tilde{n}+1} \tilde{\theta}_{\tilde{n}+1} = 0$$

$$\langle \tilde{\theta}_0, \tilde{\theta}_0 \rangle \rightarrow \langle \tilde{Q}_0 \tilde{\theta}_0 \rangle = \langle \tilde{\theta}_1, \tilde{\theta}_1 \rangle$$

$$5 \rightarrow 4+$$

$$P(\sigma^+) = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & 1 & \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}$$



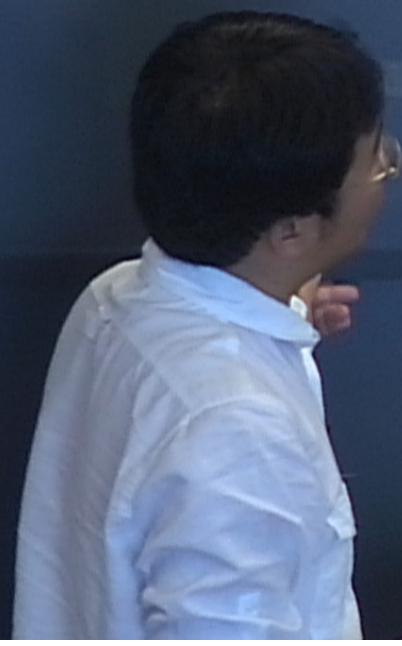
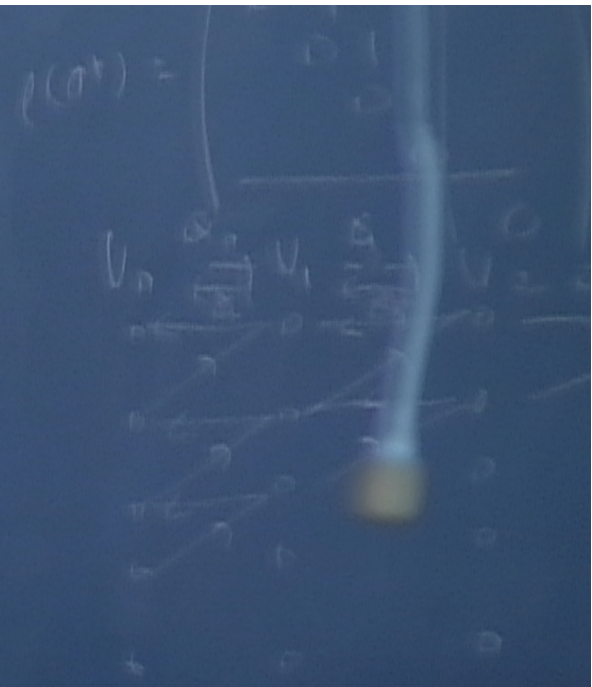
Handwritten mathematical notes on a chalkboard:

- Top left: $5 \rightarrow 4 + 1$
- Top center: $P(\sigma^+) = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$
- Top right: $\langle \hat{0}, \hat{0} \rangle$ and a sequence of nodes $(5) - (5) - \dots - (5)$ with a double arrow \Downarrow pointing to $(1) - (2) - (3) - (4) - (5)$.
- Middle: A diagram showing nodes V_0, V_1, V_2, V_3, V_4 with arrows and a matrix of zeros. The matrix is:

V_0	0	0	0	0	0
V_1	0	0	0	0	0
V_2	0	0	0	0	0
V_3	0	0	0	0	0
V_4	0	0	0	0	0
- Bottom left: $\hat{0} = (\hat{0}, \hat{0}, \dots)$
- Bottom center: $P_i = 54(2) \rightarrow P, P_i \in \mathbb{Z}$

$$F_{\mathbb{R}^n} = \partial_r \hat{\sigma}_r - \hat{\sigma}_{r+1} \partial_r = 0$$

$$\langle \hat{\sigma}_0, \hat{\sigma}_0 \rangle \rightarrow \langle \partial_0 \hat{\sigma}_0 \rangle = \langle \hat{\sigma}_1, \hat{\sigma}_1 \rangle$$



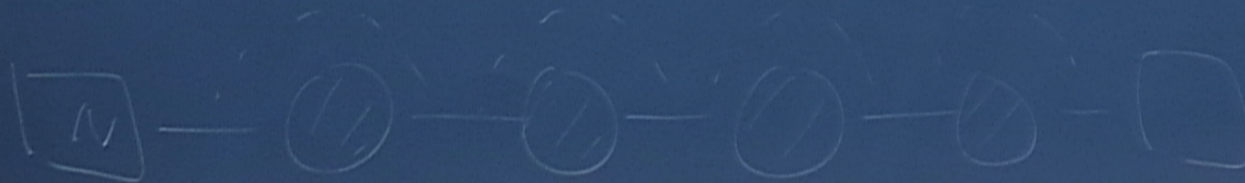
$$(p, q) \in \mathbb{Z}_{\geq 0}^2$$

$$p+q = 2g-2+n$$

For each puncture

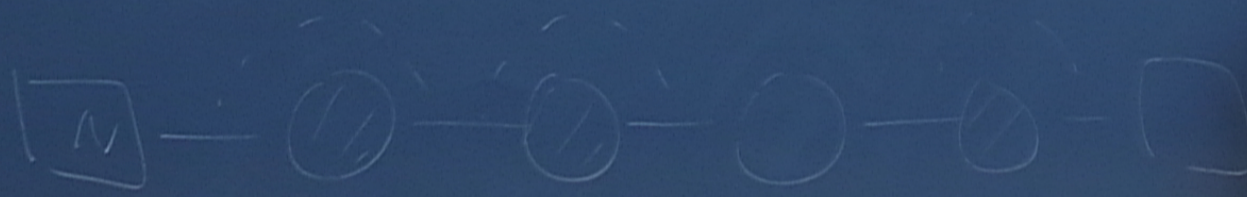
$$(p_i, \sigma_i) \neq$$

$$p_i \in \mathbb{N} \cup \{2\} \hookrightarrow \mathbb{P}^1, \sigma_i \in \mathbb{Z}_g$$



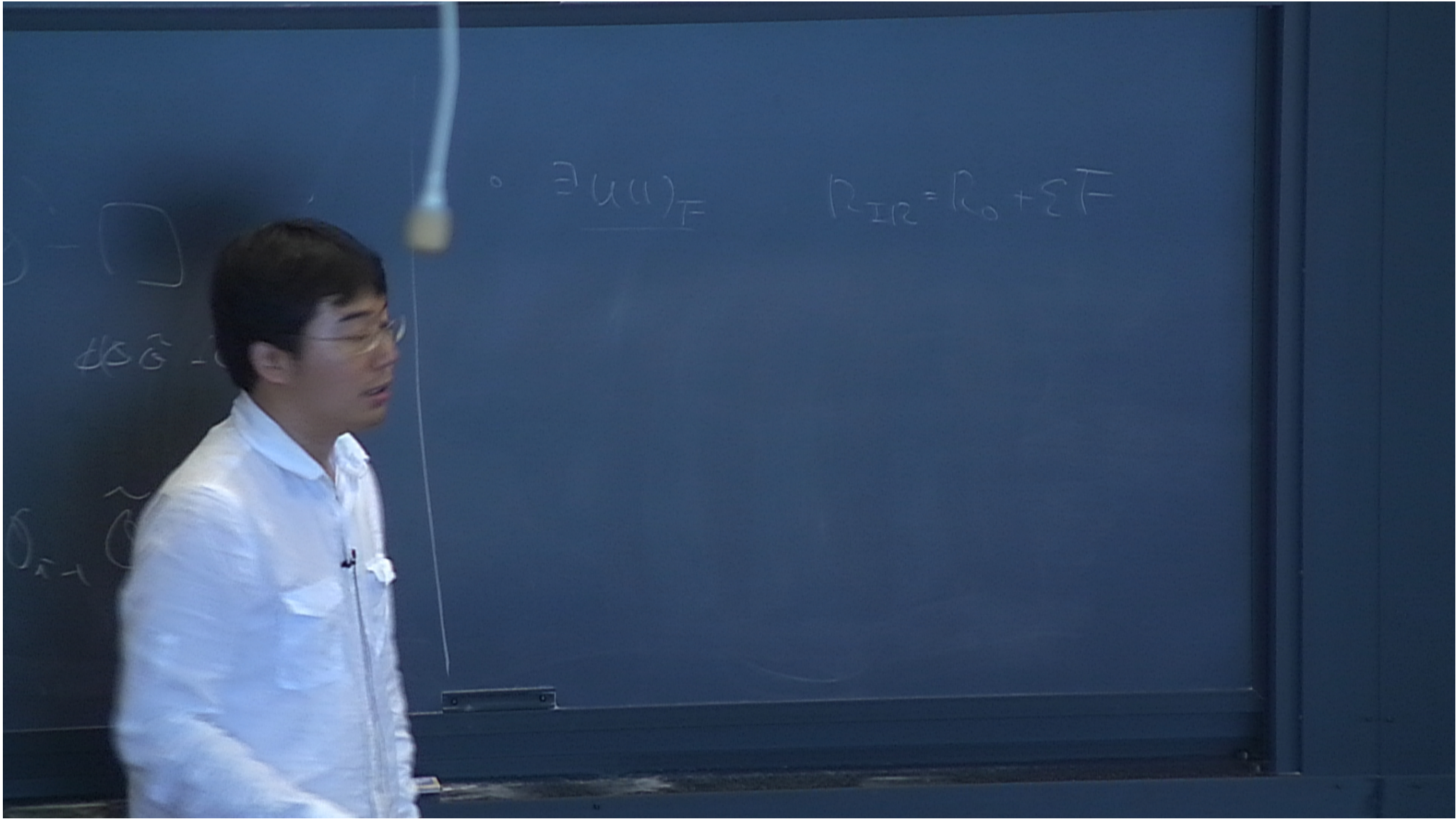
$$W = \sum Q_n \hat{E}_n \hat{E}_{n+1} Q_{n+1}$$

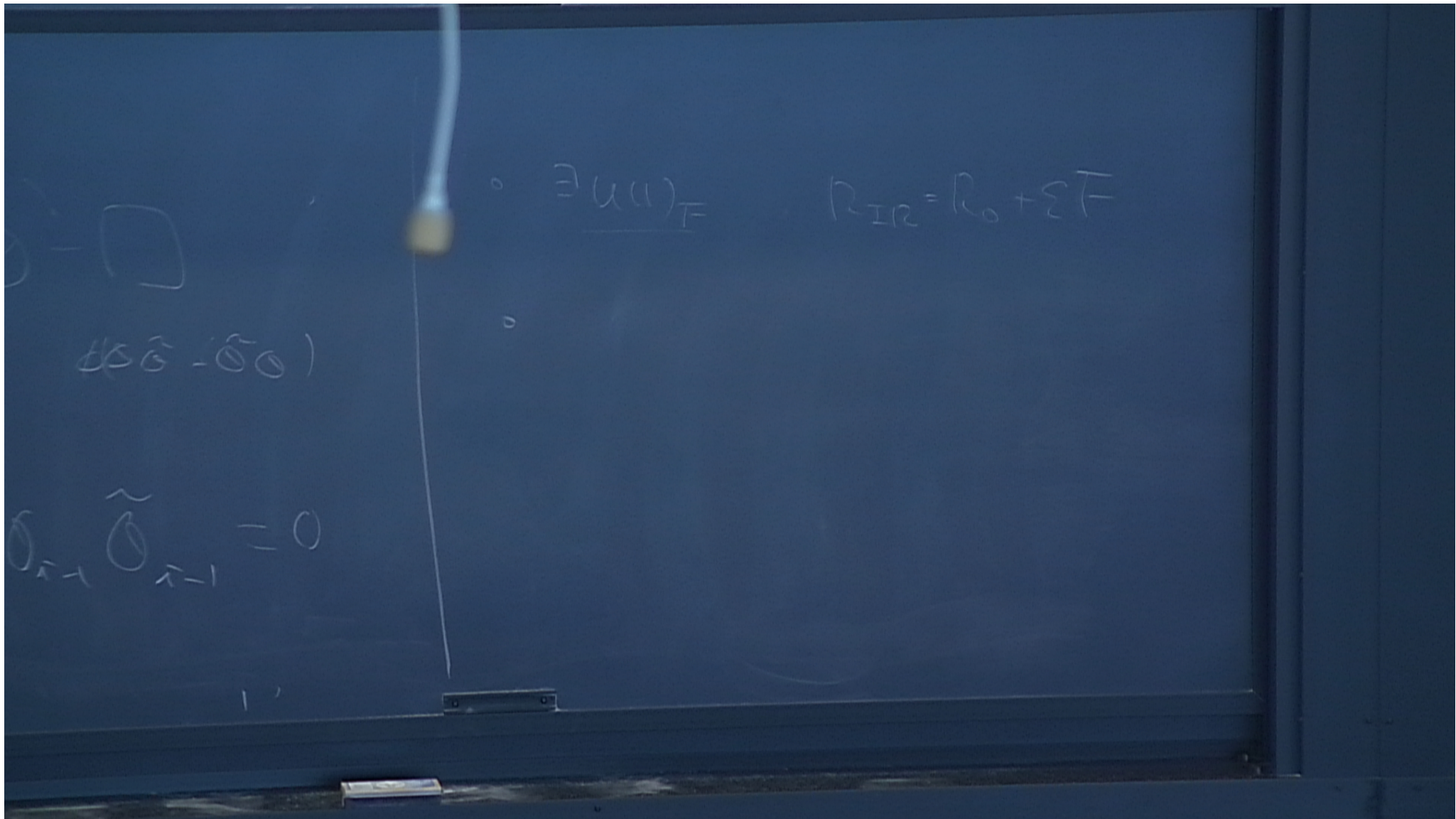
$$F_Q \sim \hat{E}_n \hat{E}_{n+1} Q_{n+1} - Q_n \hat{E}_{n-1} \hat{E}_n = 0$$

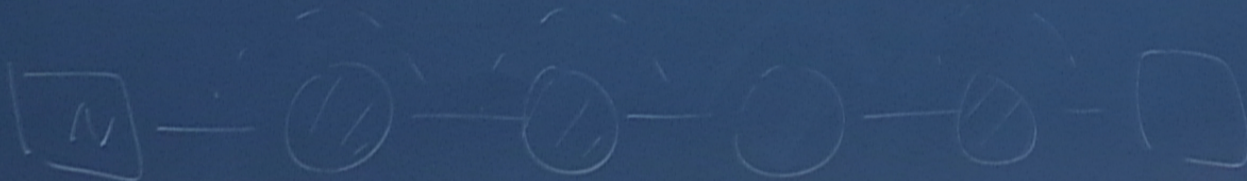


$$W = \sum Q_n \hat{E}_n \hat{E}_{n+1} Q_{n+1} \quad HSE = \hat{E}$$

$$F_Q \sim \hat{E}_n \hat{E}_{n+1} Q_{n+1} - Q_n \hat{E}_{n-1} \hat{E}_n$$

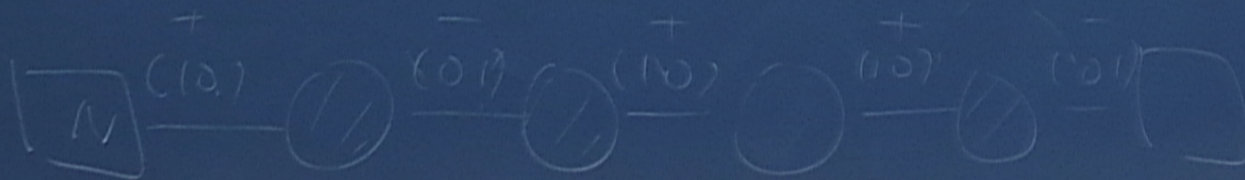






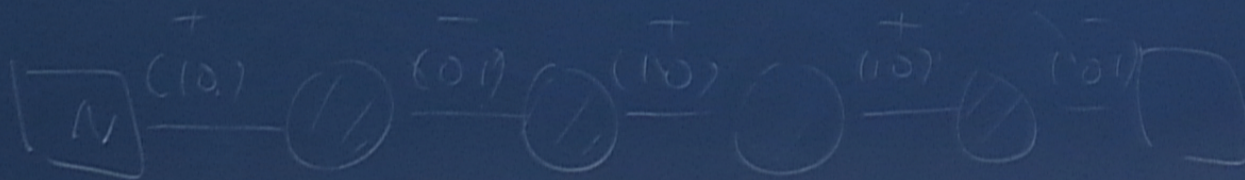
$$W = \sum Q_n \hat{E}_n \hat{E}_{n+1} Q_{n+1} \quad (\delta \hat{E} = \hat{E} \delta)$$

$$F_Q \sim \hat{E}_n \hat{E}_{n+1} Q_{n+1} - Q_n \hat{E}_{n-1} \hat{E}_n = 0$$



$$L = \sum \Theta_n \tilde{\Theta}_n \tilde{\Theta}_{n+1} \Theta_{n+1} \quad (\Delta \tilde{\Theta} = \tilde{\Theta}_0)$$

$$\tilde{\Theta}_n \tilde{\Theta}_{n+1} \Theta_{n+1} - \Theta_n \tilde{\Theta}_{n-1} \tilde{\Theta}_{n-1} = 0$$



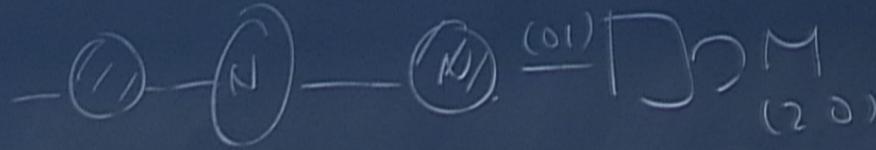
$$W = \sum \theta_{\bar{n}} \tilde{\theta}_{\bar{n}} \tilde{\theta}_{\bar{n}+1} \theta_{\bar{n}+1} \quad (\delta \tilde{\theta} = \tilde{\theta} \ominus \theta)$$

$$F_Q \sim \tilde{\theta}_{\bar{n}} \tilde{\theta}_{\bar{n}+1} \theta_{\bar{n}+1} - \theta_{\bar{n}} \theta_{\bar{n}-1} \tilde{\theta}_{\bar{n}-1} = 0$$

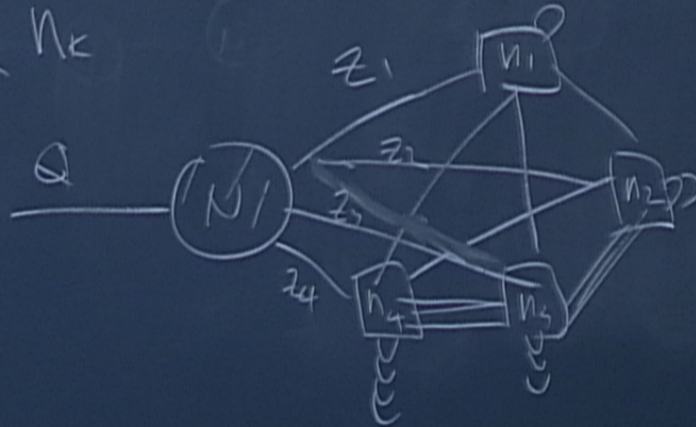
3. Fans

3. Fans

$$W = M \delta \tilde{z}$$



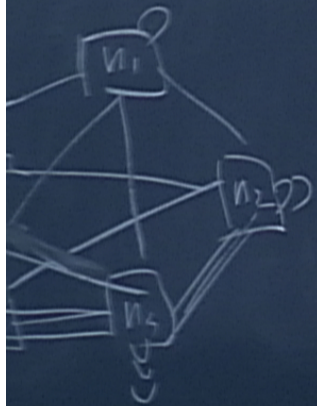
$$N = \sum k n_k$$



	(J_+, J_-)
z, \tilde{z}	$(1, 1-\kappa)$
$M_{ij}^{(p)}$	$(0, \kappa + j - 2p)$
\odot	$(0, 1)$

$$W = M \delta \tilde{z}$$

$$M_{(2,0)}$$



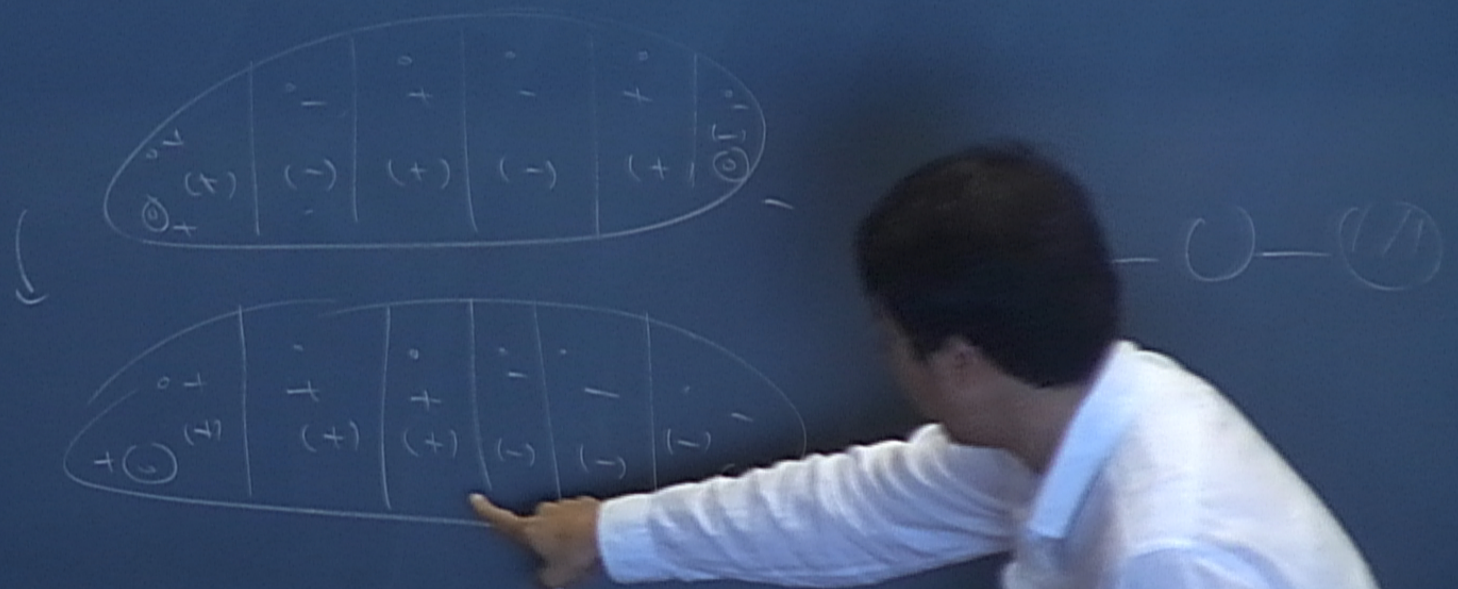
	(J_+, J_-)
z, \tilde{z}	$(1, 1-\kappa)$
$M_{ij}^{(p)}$	$(0, \kappa + j - 2p)$
\odot	$(0, 1)$

$$(J_+, J_-) = (2, 2) \quad R = R_0 + \epsilon F$$

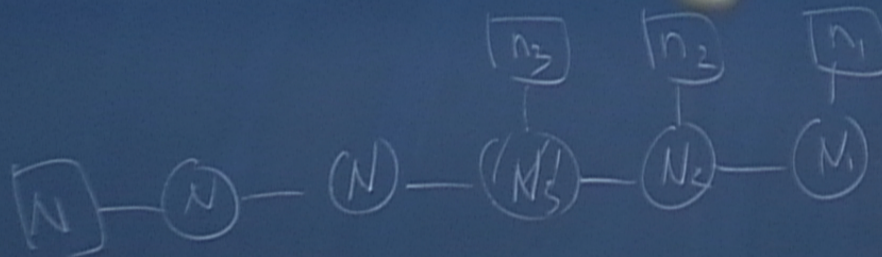
$$W = \sum (\mathbb{Q} \tilde{\mathbb{Q}})^{\hat{i}} (\tilde{z}_+ - z_-)$$

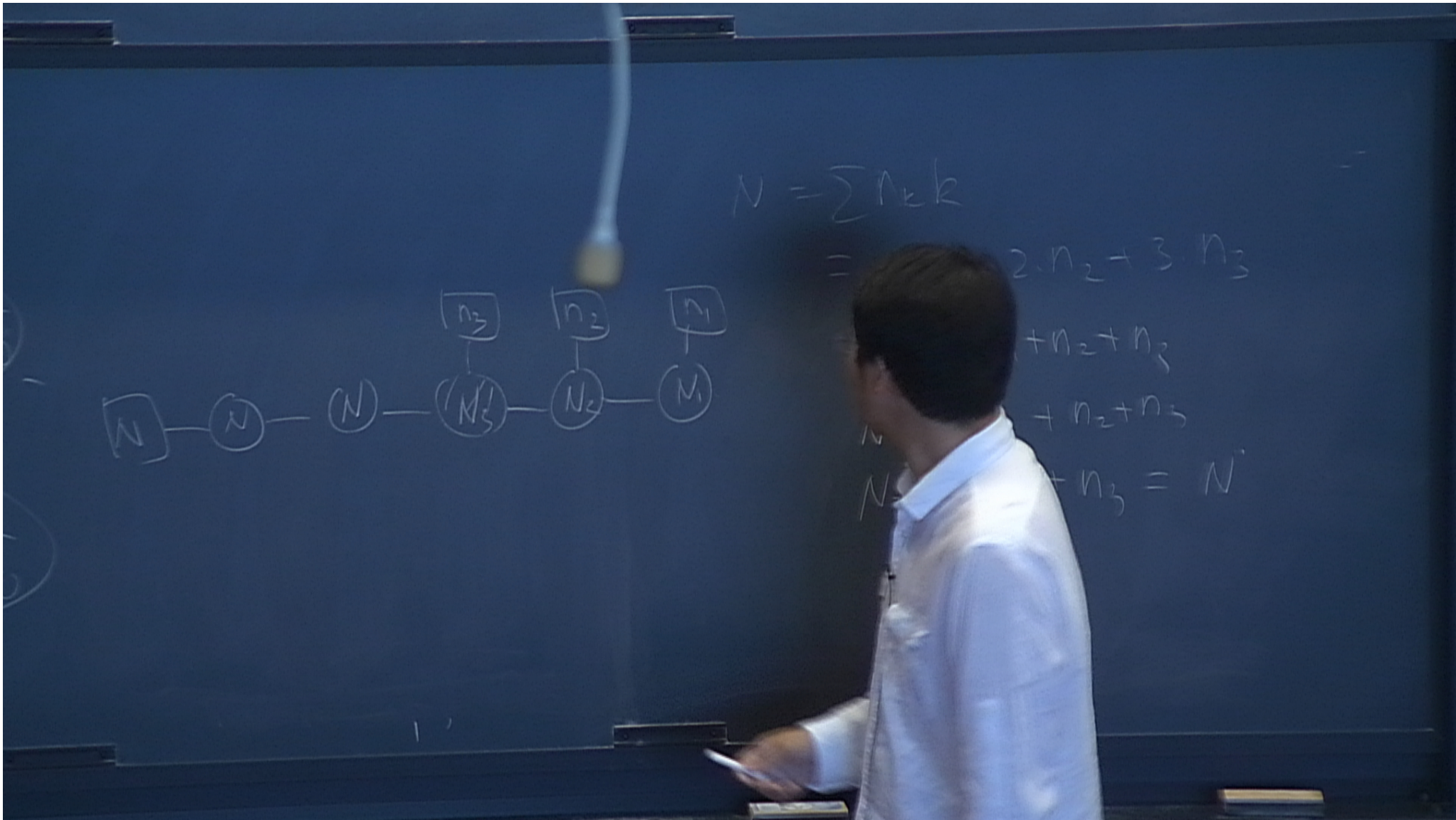
$$+ \sum_{\hat{i}, \hat{j}} \sum_{p=0}^{\min(\hat{i}, \hat{j})-1} \sum_{\tilde{z}_+} M_{ij}^{(p)} z_{\tilde{z}_+} (\mathbb{Q} \tilde{\mathbb{Q}})^p$$

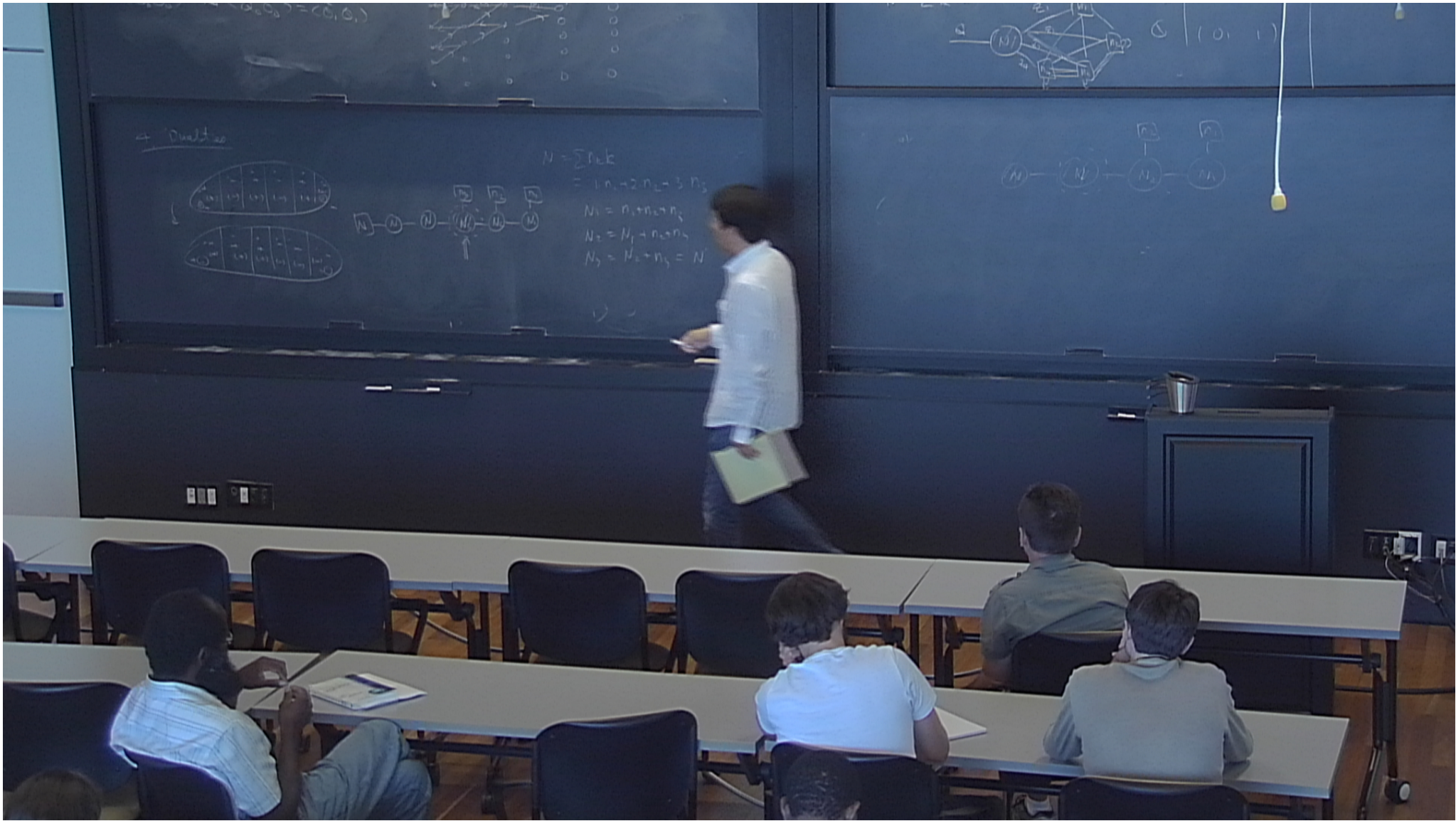
4 Dualities

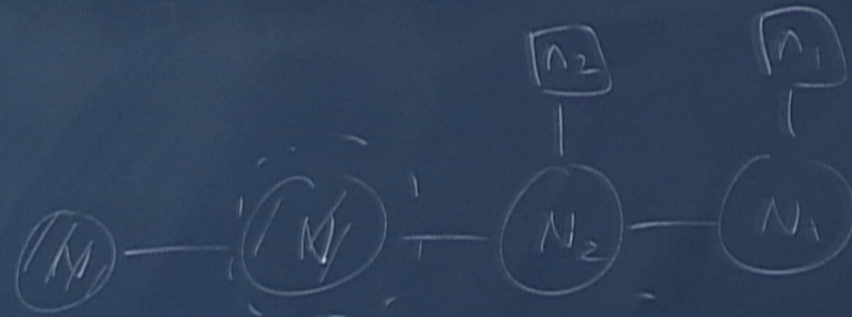


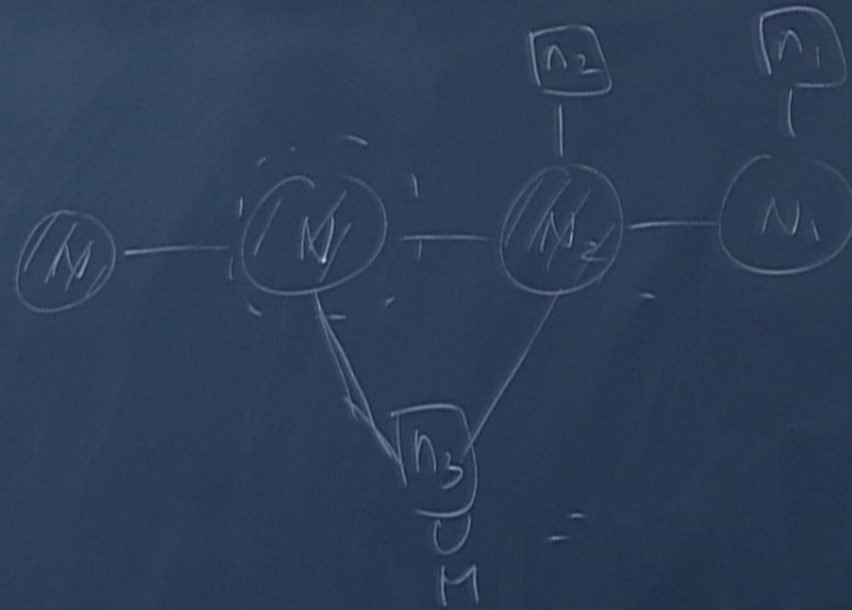
$$N = \sum n_k k$$
$$= 1 \cdot n_1 + 2 \cdot n_2 + 3 \cdot n_3$$

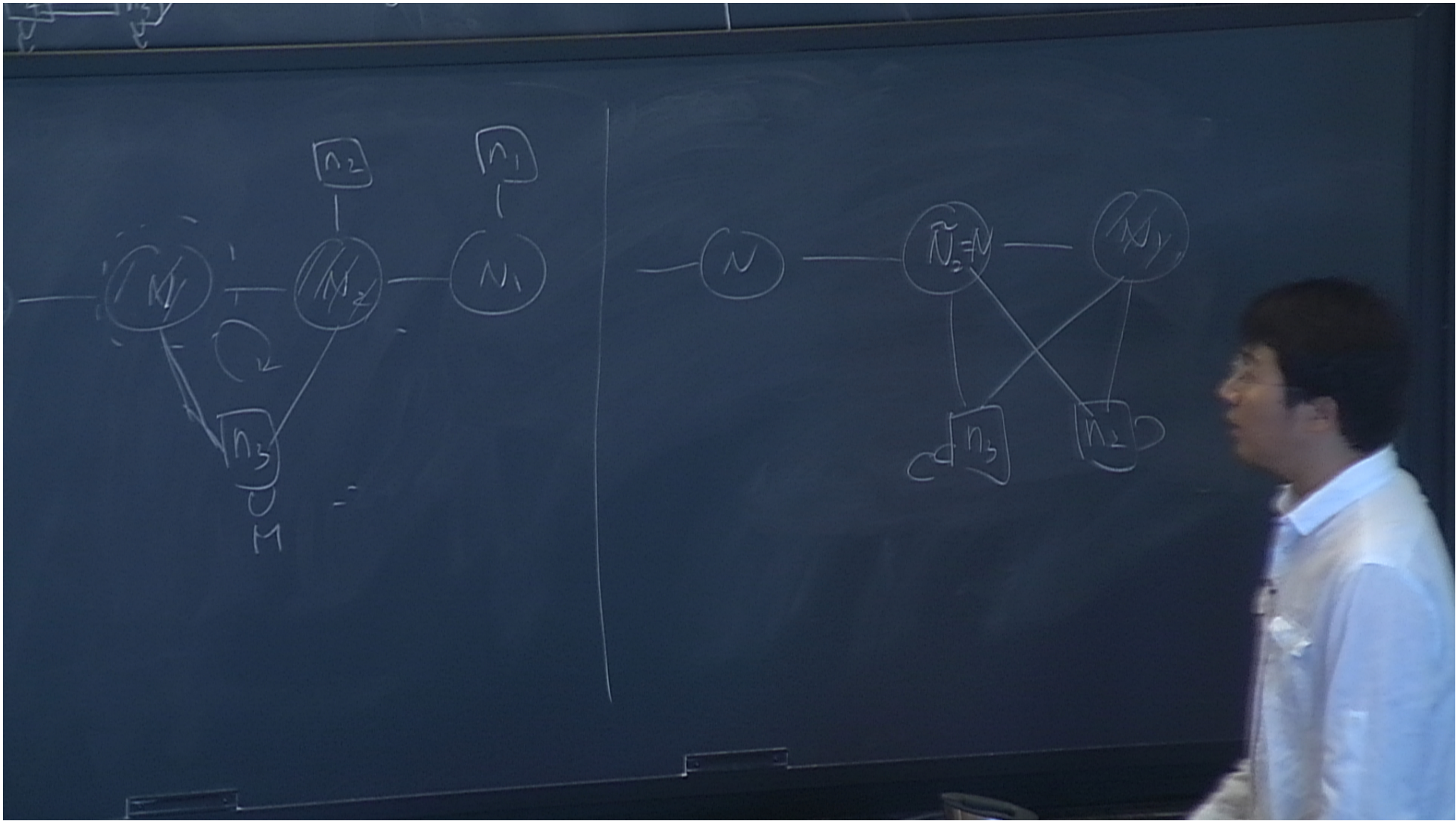


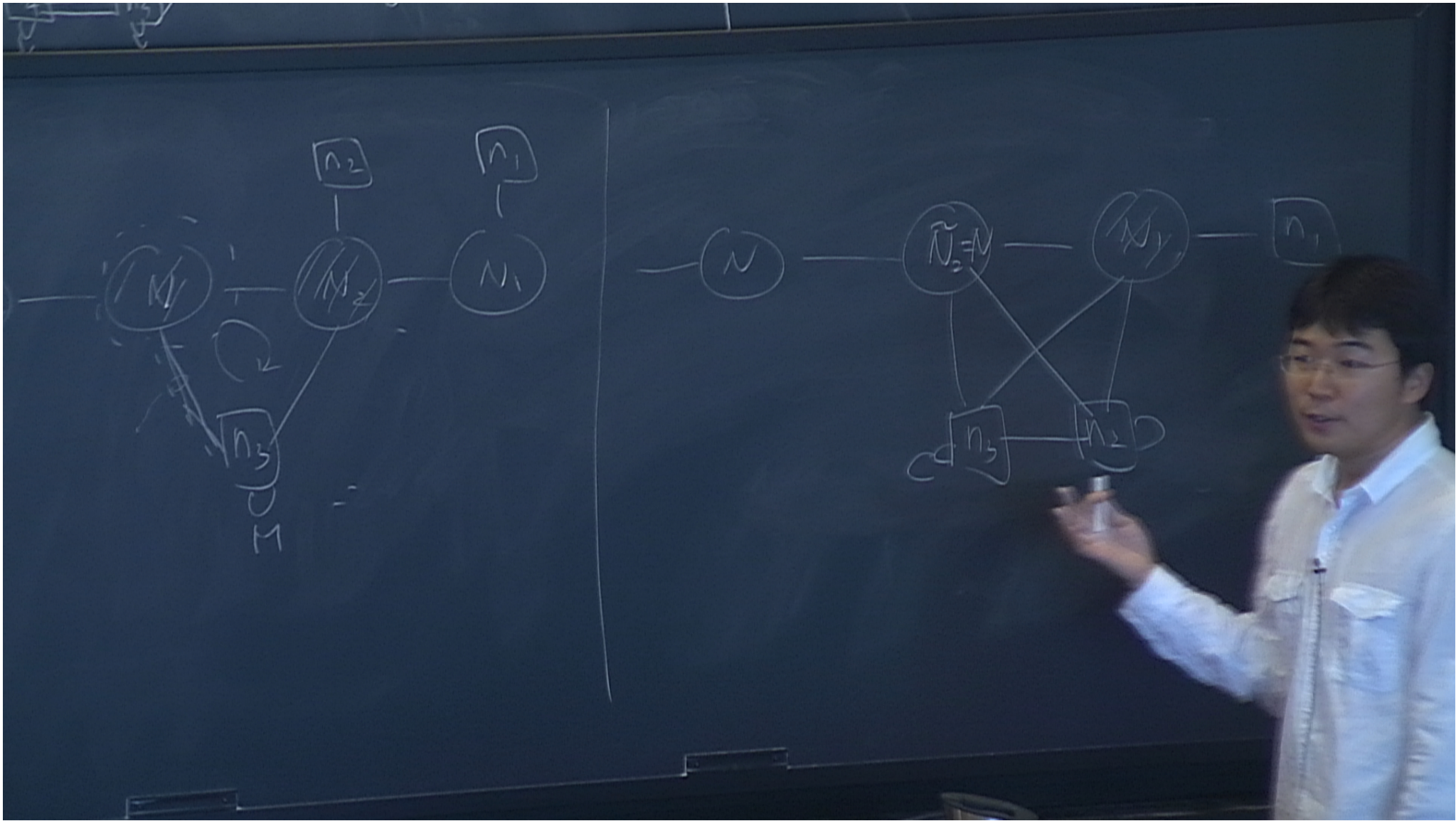






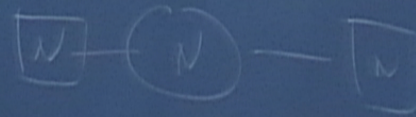
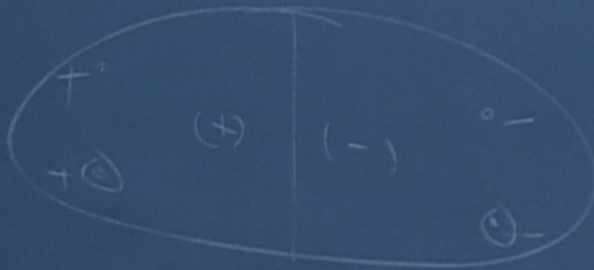






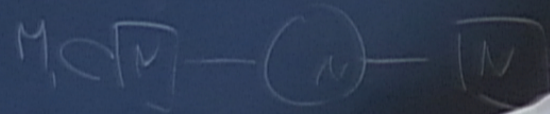
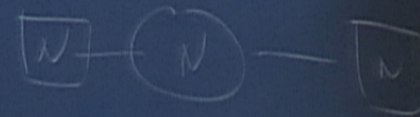
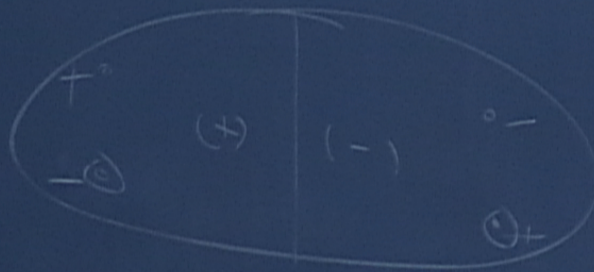
5. $N=1$ Analog of Argyres-Seiberg

SQCD $G = SU(N)$, $N_f = 2N$



s. $N=1$ Analog of Argyres-Seiberg

SQCD $G = SU(N)$, $N_f = 2N$



res-Seiberg

$$\langle M \rangle = R(0^+)$$

