Title: Inferring causal structure: a quantum advantage

Date: Jul 17, 2014 03:30 PM

URL: http://pirsa.org/14070021

Abstract: A fundamental question in trying to understand the world -- be it classical or quantum -- is why things happen. We seek a causal account of events, and merely noting correlations between them does not provide a satisfactory answer. Classical statistics provides a better alternative: the framework of causal models proved itself a powerful tool for studying causal relations in a range of disciplines. We aim to adapt this formalism to allow for quantum variables and in the process discover a new perspective on how causality is different in the quantum world. Causal inference is a central task in the context of causal models: given observed statistics over a set of variables, one aims to infer how they are causally related. Yet in the seemingly simple case of just two classical variables, this is impossible (unless one makes additional assumptions). I will show how the analogous task for quantum variables can be solved. This quantum advantage is reminiscent of the advantages that quantum mechanics offers in computing and communication, and may lead to similarly rich insights. Our scheme is corroborated by data obtained in collaboration with Kevin Resch's experimental group. Time permitting, I will also address other applications of the quantum causal models. arXiv:1406.5036

Inferring Causal Structure: a Quantum Advantage

and other applications of Quantum Causal Models

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Quantum Foundations Seminar, PI

July 17th 2014

Outline

- Introduction
  - models of causality for the quantum world
- The task
  - causal inference, and why it is hard
- Method
  - how quantum conditionals provide an advantage
- Results
  - experimental verification
- Encore
  - causal tomography
- Other projects

Introduction

causal models
for the quantum world

Clinical trial of [redacted]

Introduction

Rather than merely observing correlations between events, science seeks to explain these correlations in terms of causal influences. In the context of classical variables, the concept of causation has been rigorously defined, and a framework for describing systems in terms of their causal relations has been established [Pearl_book, SpirtesEtAl_book].

Method

Its applications are manifold; a testament to the fact that a causal model captures the essence of "how the system works". In a sense, it describes how information flows from one event to the other. What would a similar account of the relations between a set of quantum variables look like? I will discuss some ways in which classical causal models must be adapted to accommodate quantum variables, highlighting how causation and information processing are different from the classical case.

Results

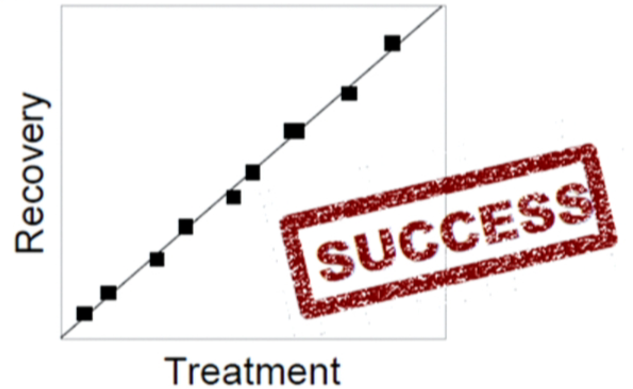
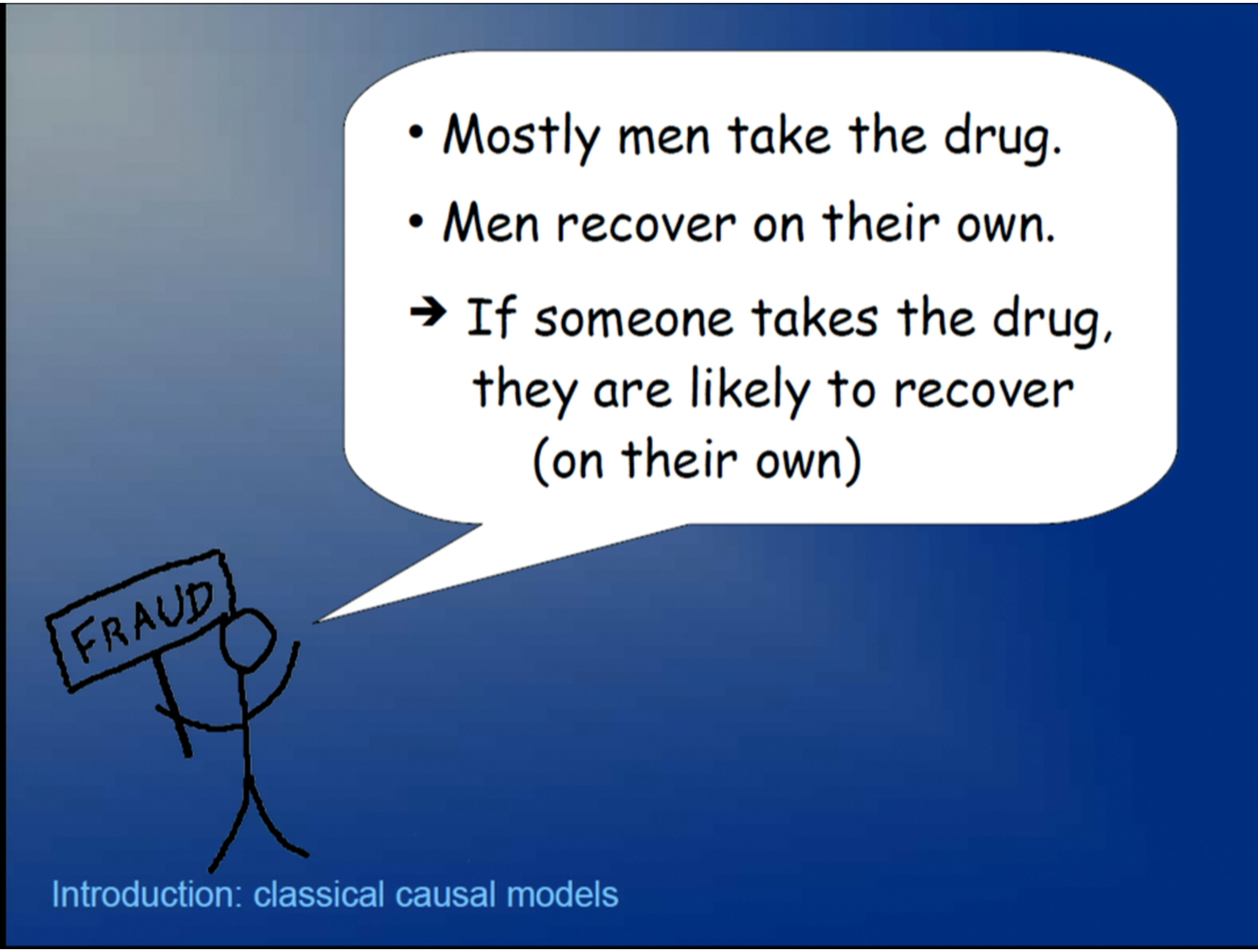


Fig. 1: Recovery correlates with treatment to a statistical significance of 20 standard deviations.

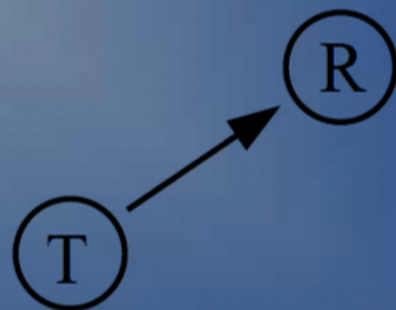
Conclusion

In particular, one such difference allows us to solve a task that is impossible to solve classically. "Causal inference" refers to the problem of determining the causal relations between a set of variables, given observational data. In the case of two classical variables, the correlations that can arise if one variable is a direct cause of the other are precisely the same as those that can arise from a common cause acting on both, so it is impossible to deduce the causal structure from them. Yet for quantum variables, we show that the correlations do encode a signature of the causal structure, which allows us to solve the causal inference problem. We illustrate this with data from a proof-of-concept experiment that corroborates our scheme for quantum causal inference [Agnew_draft].

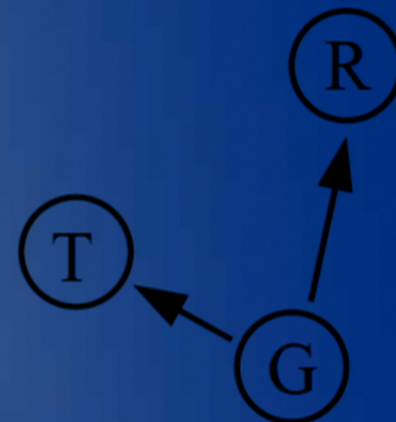
Introduction: classical causal models


• Mostly men take the drug.
• Men recover on their own.
→ If someone takes the drug, they are likely to recover (on their own)
FRAUD
Introduction: classical causal models

More than correlation: Causation



direct cause



common cause

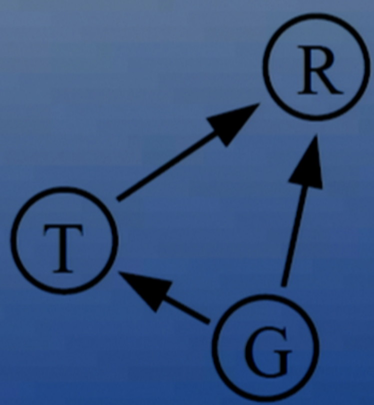
Introduction: classical causal models

Classical causal models

over a set of variables describe causal influences:

Structure

who influences whom



directed acyclic graph

+

Parameters

dependence on parents

$$P(R|TG)$$
$$P(T|G)$$
$$P(G)$$

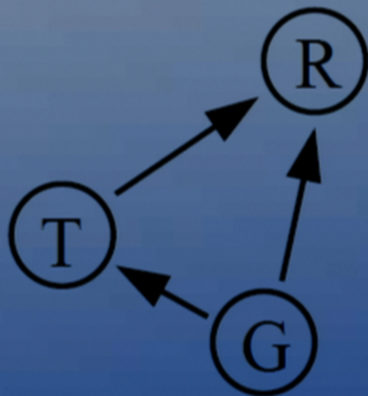
conditional probabilities

Introduction: classical causal models

"Causality – reasoning, models and inference" J. Pearl, Cambridge University Press, 2009.

Quantum causal models

Structure



directed acyclic graph

+

Parameters

?

quantum conditionals

Quantum conditionals

Defining property: rule of inference

classical

$$P(B) = \Sigma_a P(B|A) P(A)$$

quantum

$$\rho_B = Tr_A(\rho_{B|A} \rho_A)$$

Introduction: quantum causal models

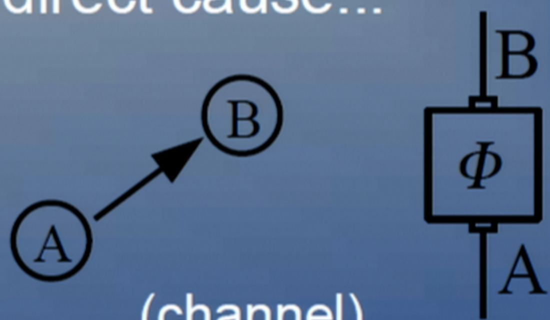
The task:

inferring causal structure

Inferring causal structure

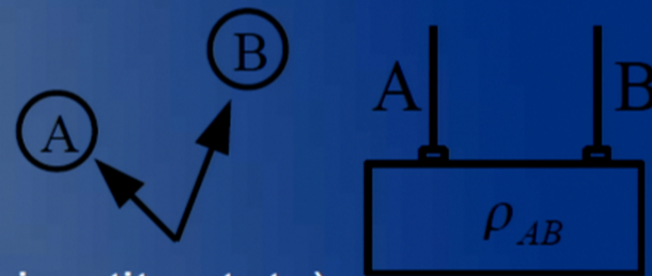
Given statistics *P(A,B)*, ...

direct cause...



(channel)

...or common cause?



(bipartite state)

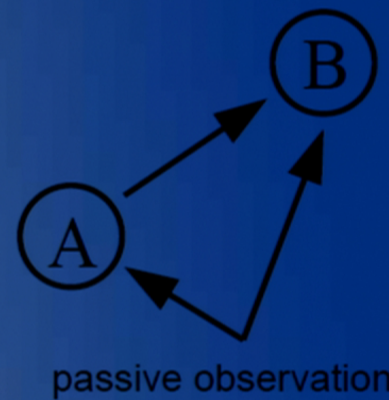
Fundamental task across fields of science:
determine how the system works, so as to control it

The task: classical causal inference

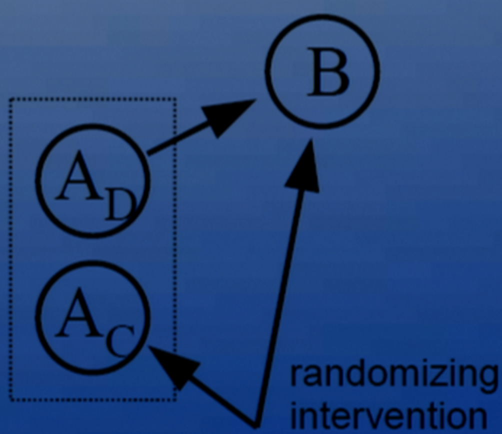
Observation and intervention

For two classical variables,

- all $P(A,B)$ are compatible with both causal structures
  ⇒ Inference is *impossible*.

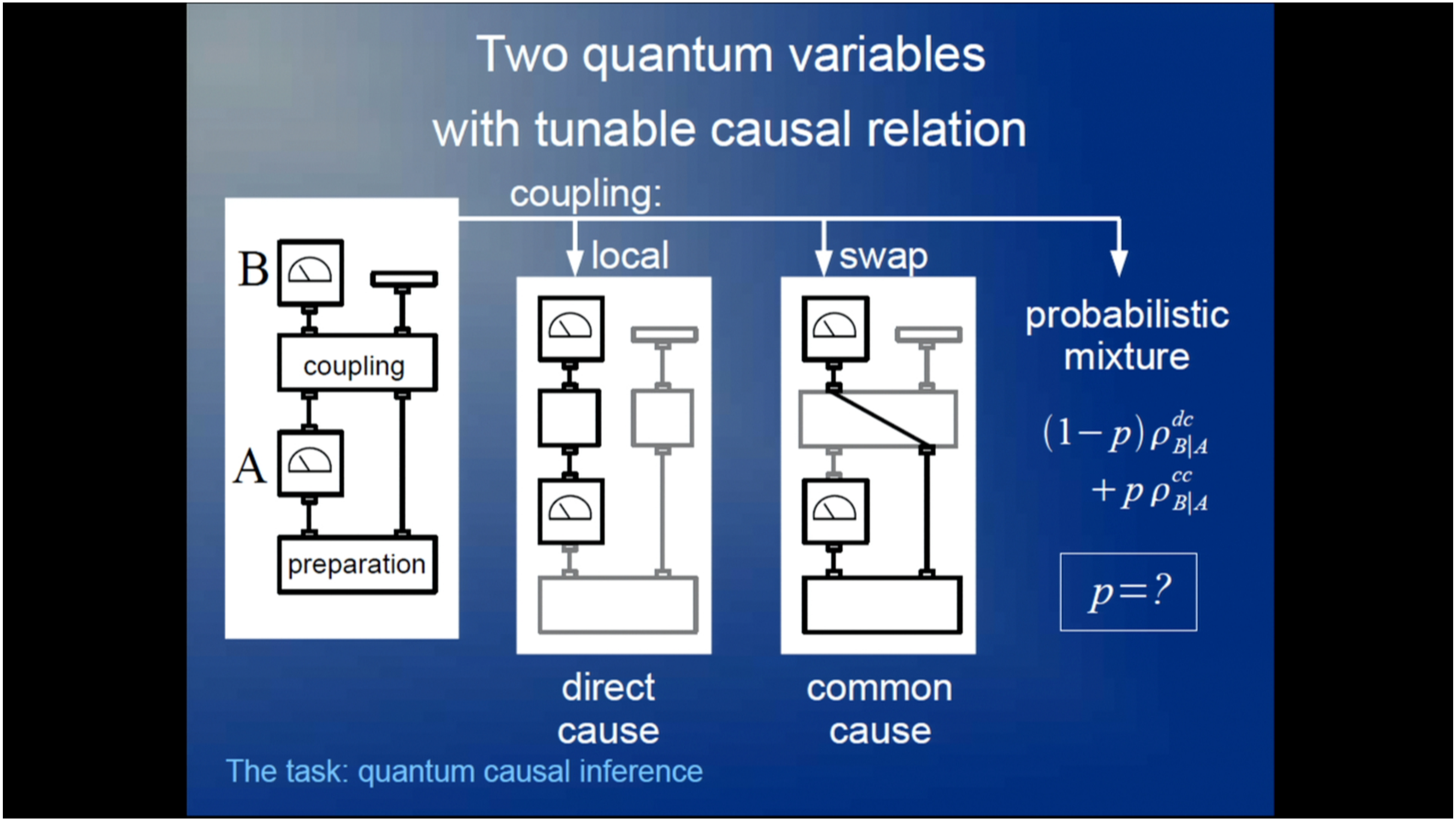


- ... but if we *intervene* (measure and reprepare A),



- correlation B-A_C quantifies common-cause influence;
- B-A_D quantifies direct cause.
  ⇒ Inference is *trivial*.

The task: classical causal inference



Two quantum variables
with tunable causal relation
coupling:
B
A
coupling
preparation
local
swap
probabilistic
mixture
$(1-p)\,\rho^{dc}_{B|A}$
$+\,p\,\rho^{cc}_{B|A}$
$p=?$
direct
cause
common
cause
The task: quantum causal inference

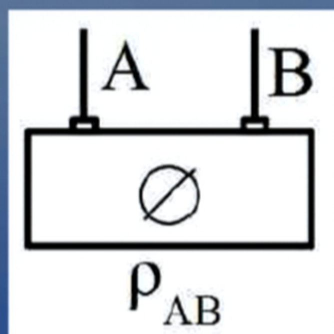
Method

the quantum advantage

Peculiarities of quantum conditionals I

Classical $P(B|A)$ can characterize any causal relation, but...

Common-cause quantum conditionals



$$\rho_{B|A} \equiv \rho_A^{-1/2} \rho_{AB} \rho_A^{-1/2}$$

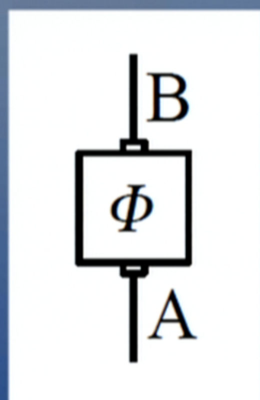
property: $\rho^{cc}_{B|A} \geq 0$

Method – the quantum advantage

M Leifer and RW Spekkens, "Formulating quantum theory as a causally neutral theory of Bayesian inference", arXiv:1107.5849

Peculiarities of quantum conditionals I

Direct-cause quantum conditionals



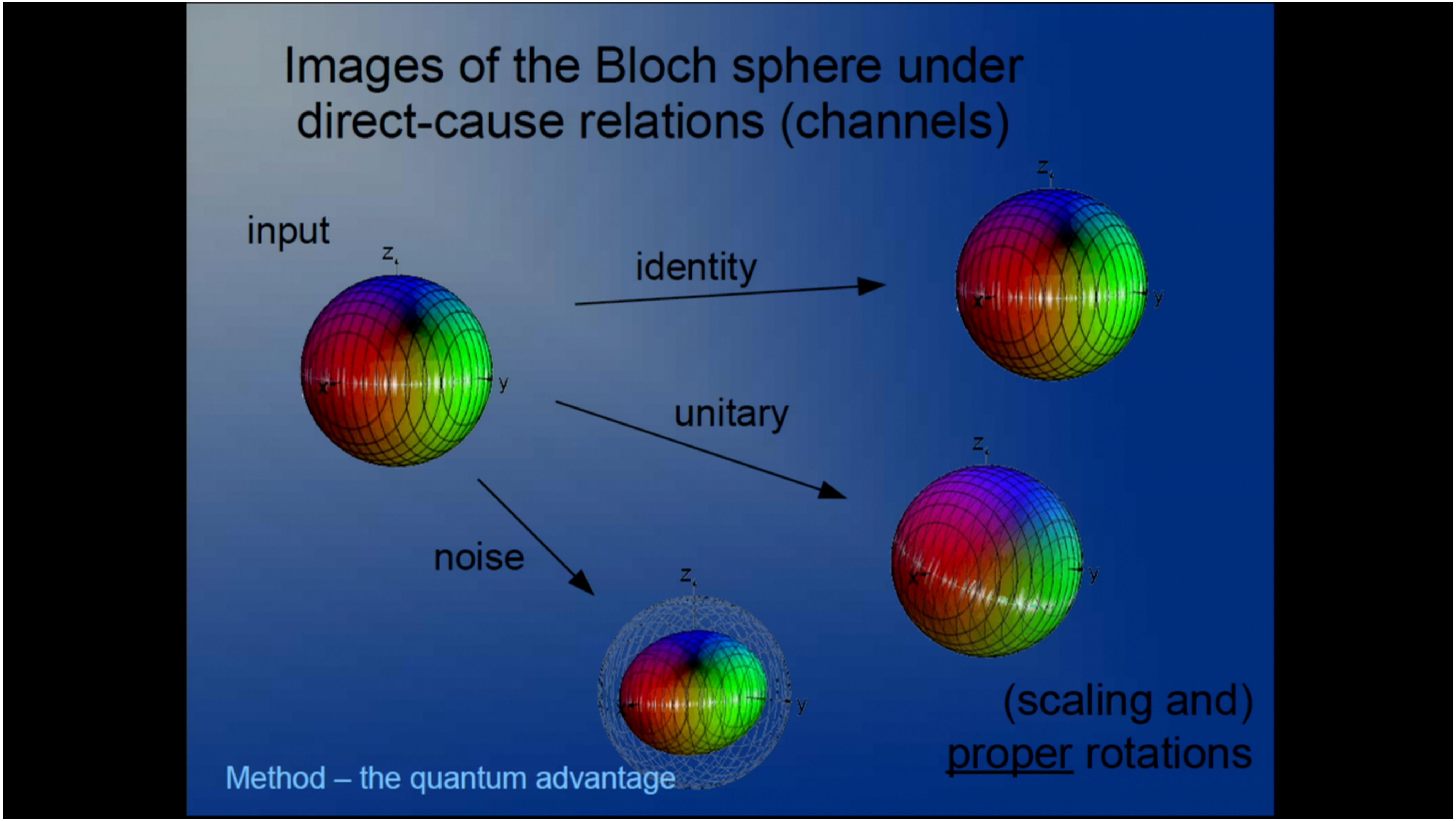
Choi-Jamiolkowski isomorphism:

$$\rho_B = \Phi(\rho_A) = Tr_A(\rho^{dc}_{B|A}\rho_A)$$

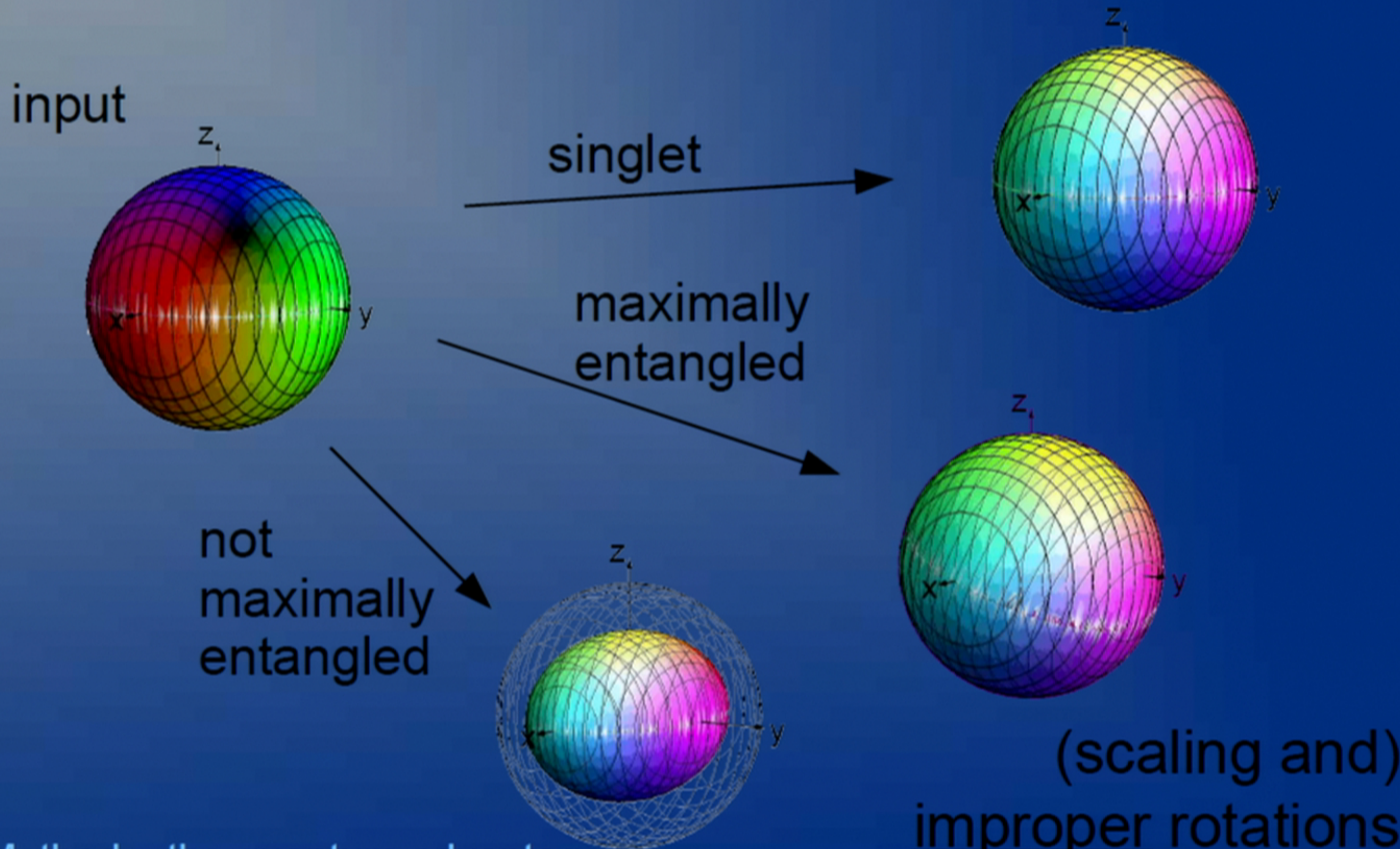
$$\rho^{dc}_{B|A} = \sum_{jk} \Phi(|j\rangle\langle k|) \otimes |k\rangle\langle j|$$

property: $\left(\rho^{dc}_{B|A}\right)^{T_A} \geqslant 0$

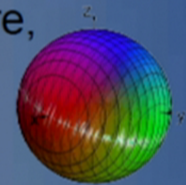
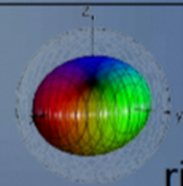
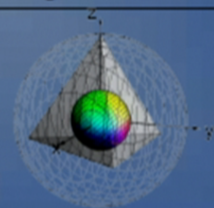
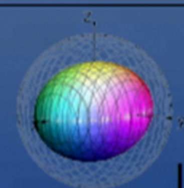
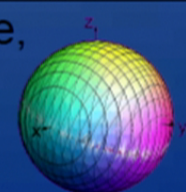
Method – the quantum advantage


Images of the Bloch sphere under direct-cause relations (channels)
input
z
x
y
identity
z
x
y
unitary
z
x
y
noise
z
x
y
(scaling and)
proper rotations
Method – the quantum advantage

Images of the Bloch sphere under common-cause relations (steering)



Method – the quantum advantage

direct-cause, noiseless	unit sphere, right-handed 	$(\rho_{B\|A})^{T_A} \geqslant 0$ rank-one	unitary channel
direct-cause, noisy	 ellipsoid, right-handed	$(\rho_{B\|A})^{T_A} \geqslant 0$ $\rho_{B\|A} \ngeqslant 0$	non-unitary channel
undecidable (classical)	ellipsoid inside tetra-hedron* 	$(\rho_{B\|A})^{T_A} \geqslant 0$ $\rho_{B\|A} \geqslant 0$	entanglement-breaking channel, separable state,
common-cause, noisy	 ellipsoid, left-handed	$\rho_{B\|A} \geqslant 0$ $(\rho_{B\|A})^{T_A} \ngeqslant 0$	not maximally entangled state
common-cause, noiseless	unit sphere, left-handed 	$\rho_{B\|A} \geqslant 0$ rank-one	pure, maximally entangled bipartite state

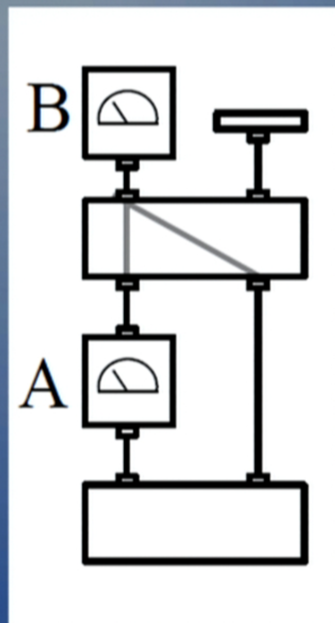
Method – the quantum advantage

*S Jevtic, MF Pusey, D Jennings, T Rudolph, "The quantum steering ellipsoid". arXiv:1303.4724

Results

probabilistic mixture

direct cause $1-p$



common cause p

- implement p
- collect data
- fit to

$$(1-p)\,\rho^{dc}_{B|A} + p\,\rho^{cc}_{B|A}$$

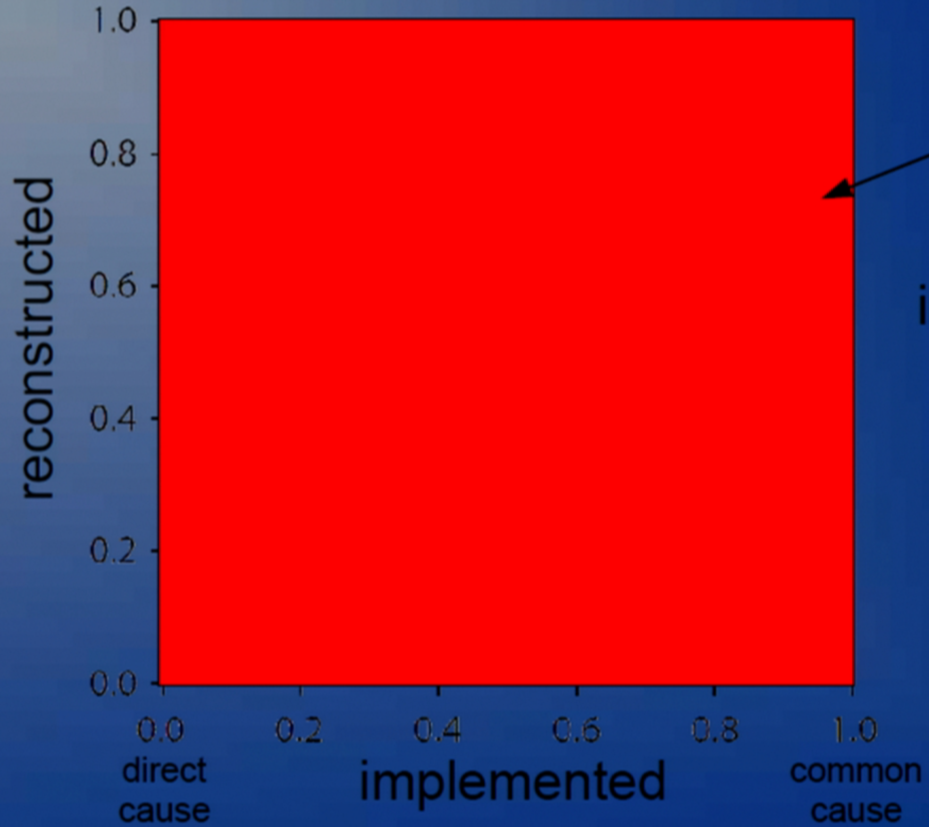
(minimize residue χ^2)

$\Rightarrow$ reconstruct p

Results

Probability of common cause by passive observation

Classical variables



small χ^2

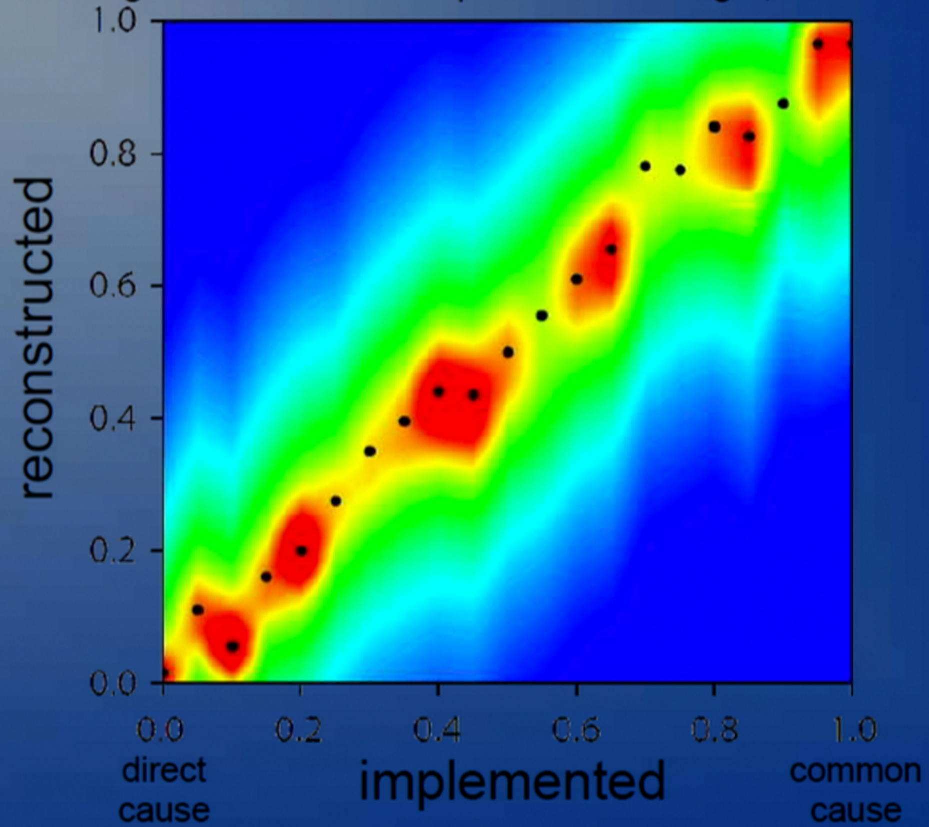
$\Rightarrow$ good fit

inconclusive

Results

Probability of common cause – experimental results

KR, M Agnew, L Vermeyden, D Janzing, RW Spekkens and KJ Resch,
"Inferring causal structure: a quantum advantage", arXiv:1406.5036

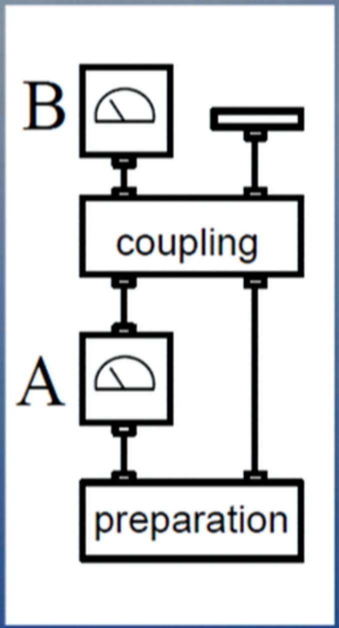


Results

Encore

Causal Tomography

Peculiarities of quantum conditionals II



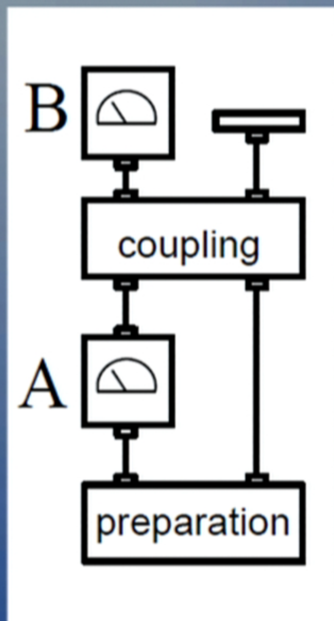
coupling: $U = \sqrt{1-p}\,\mathbf{1} + i\sqrt{p}\,U_{swap}$

Common-cause and direct-cause simultaneously

Map of inferences from A to B:

Encore: causal tomography

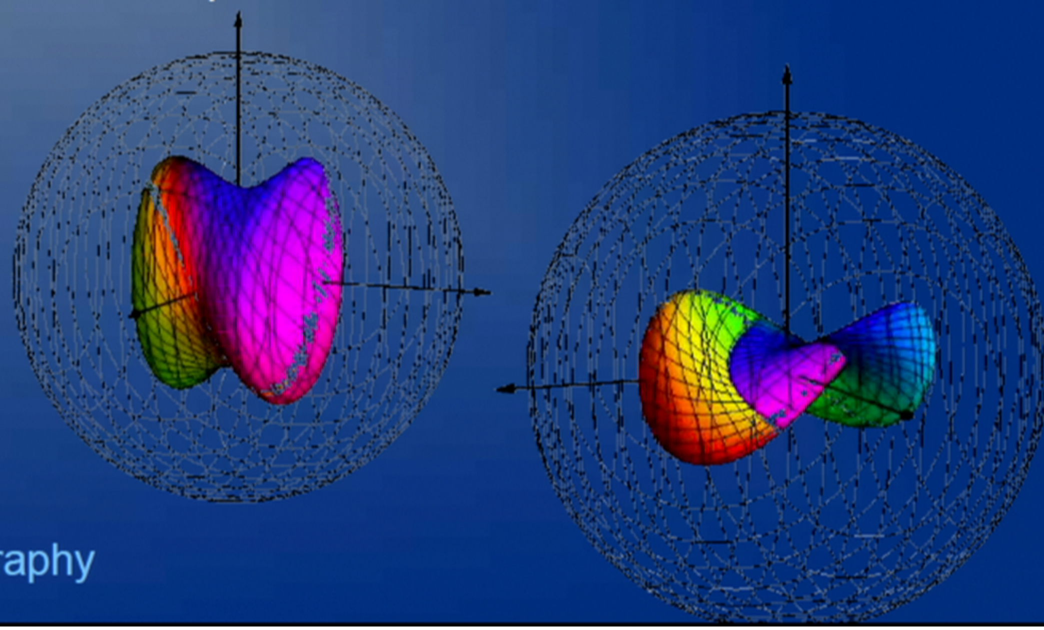
Peculiarities of quantum conditionals II



coupling: $U = \sqrt{1-p}\,\mathbf{1} + i\sqrt{p}\,U_{swap}$

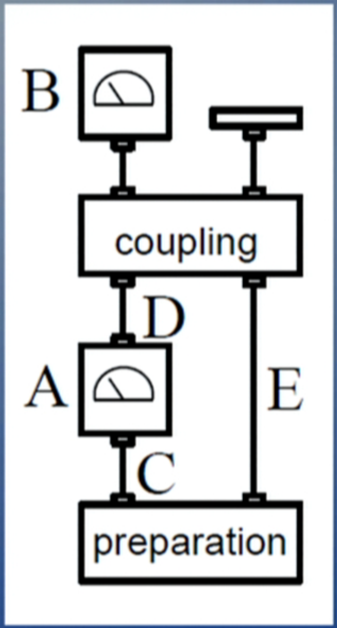
Common-cause and direct-cause simultaneously

Map of inferences from A to B:



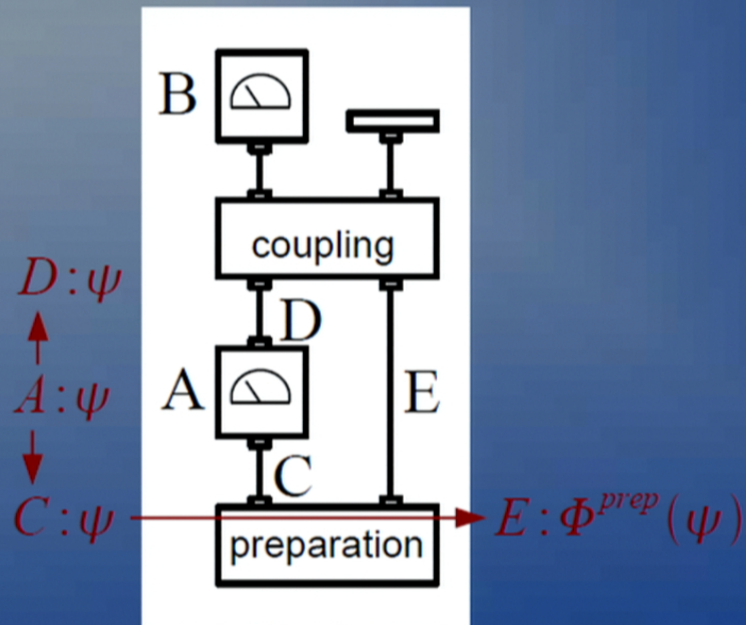
Encore: causal tomography

Paths of inference



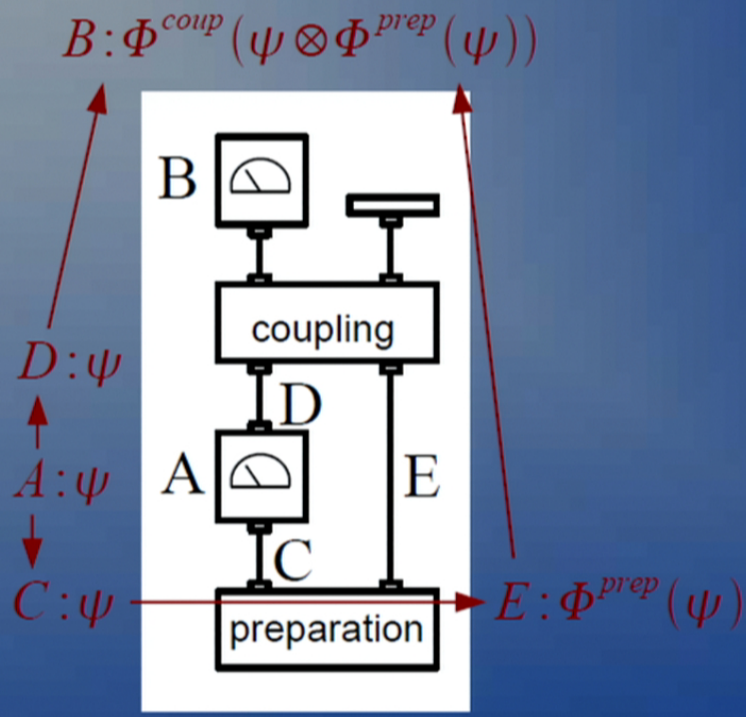
Encore: causal tomography

Paths of inference



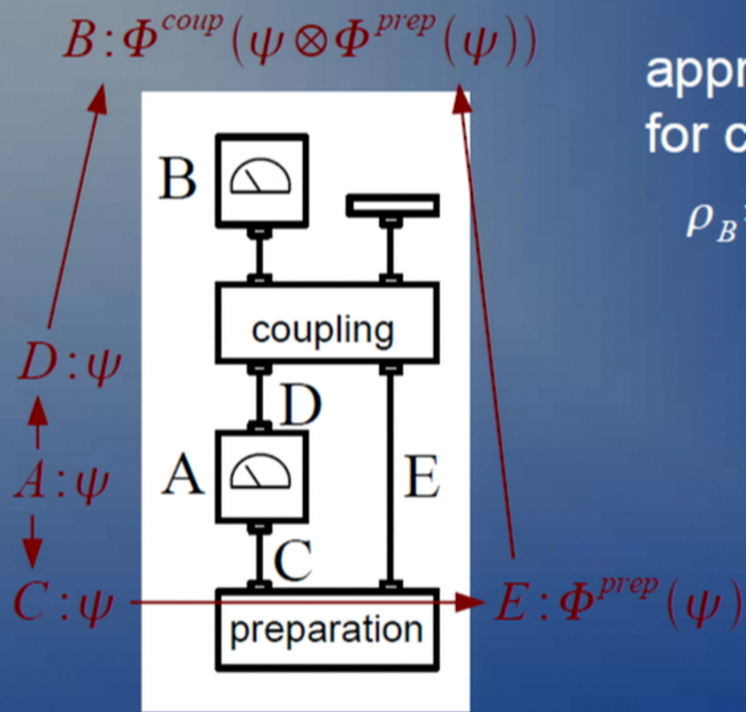
Encore: causal tomography

Paths of inference



Encore: causal tomography

Paths of inference

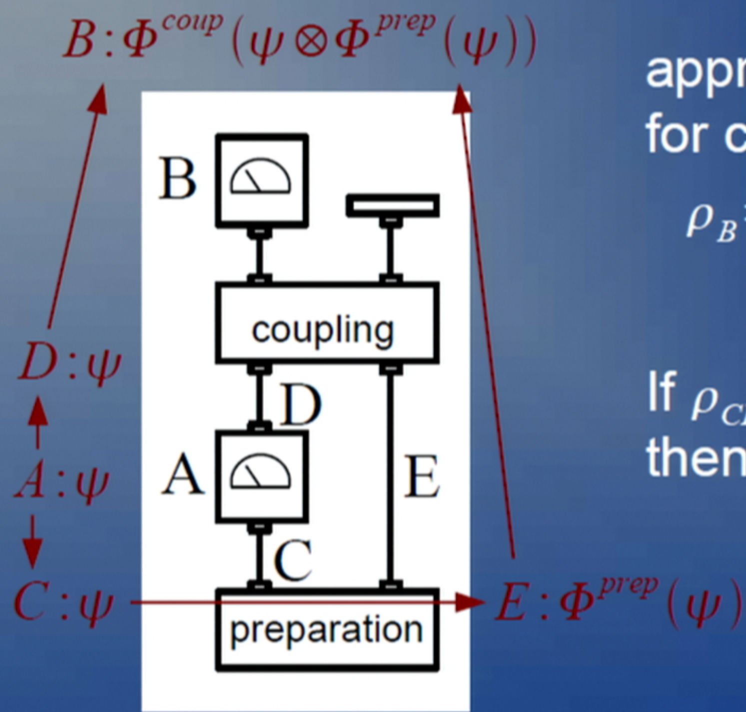


appropriate conditional
for coherent superpositions:

$$\rho_B = Tr_{CD}(\rho_{B|CD}\,\rho_{CD})$$

Encore: causal tomography

Paths of inference



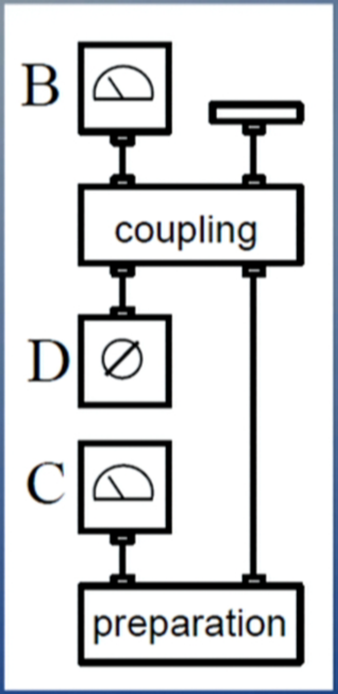
appropriate conditional for coherent superpositions:

$$\rho_B = Tr_{CD}(\rho_{B|CD}\,\rho_{CD})$$

If $\rho_{CD} = |\psi\rangle\langle\psi|_C \otimes |\psi\rangle\langle\psi|_D$ then ρ_B is not linear in ψ

Encore: causal tomography

Causal tomography

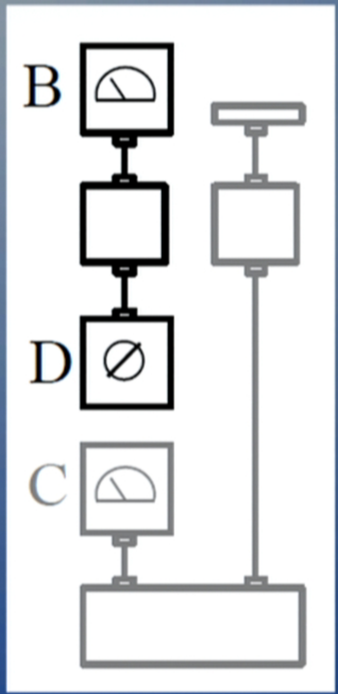


- characterizes
what A tells us about B
if their relation is both
common-cause and direct-cause

$$\rho_{BC|D} = \Sigma_{stu} C_{stu} \sigma_u^B \otimes \sigma_s^C \otimes \sigma_t^D$$

Encore: causal tomography

Causal tomography



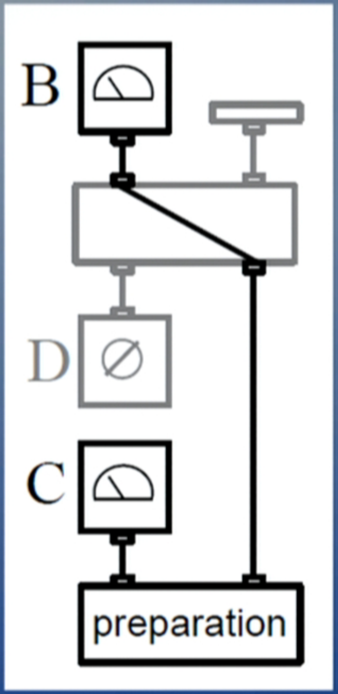
$$\rho_{BC|D} = \rho_{B|D} \otimes \rho_C$$

- characterizes what A tells us about B if their relation is both common-cause and direct-cause

- purely direct-cause
  $\Rightarrow$ tomography of process

Encore: causal tomography

Causal tomography



$$\rho_{BC|D} = \rho_{BC} \otimes \mathbf{1}_D$$

- characterizes
what A tells us about B
if their relation is both
common-cause and direct-cause

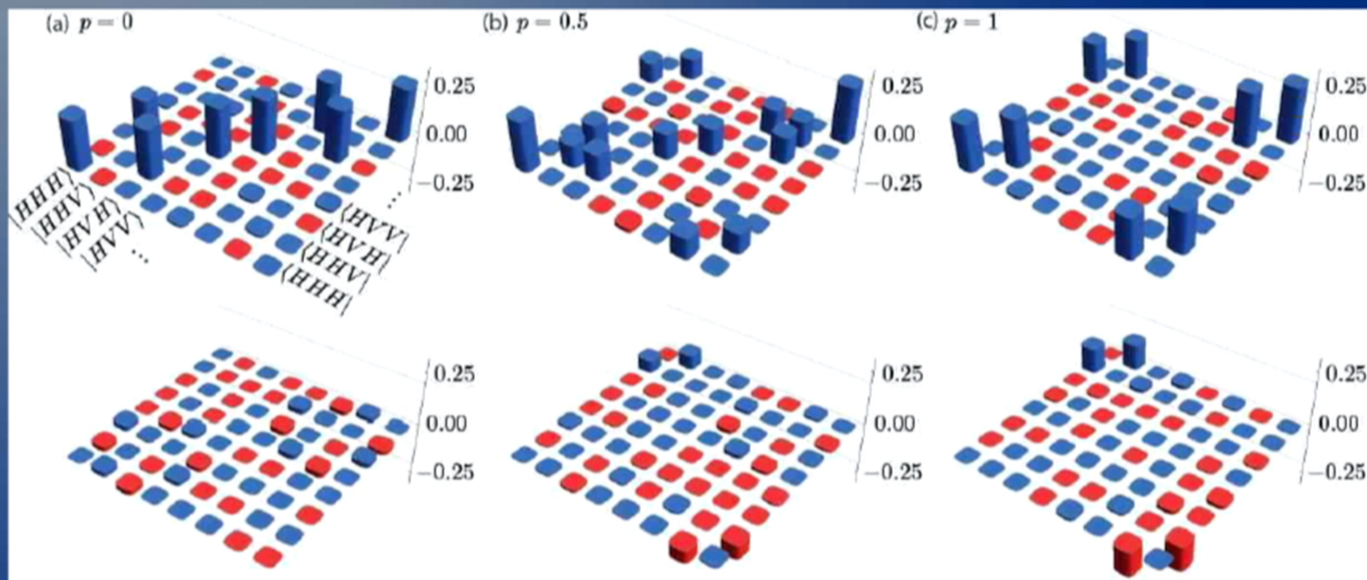
- purely direct-cause
$\Rightarrow$ tomography of process

- purely common-cause
$\Rightarrow$ tomography of bipartite state

Encore: causal tomography

$\rho_{BC|D}$ reconstructed from experimental statistics

KR, M Agnew, L Vermeyden, D Janzing, RW Spekkens and KJ Resch,
"Inferring causal structure: a quantum advantage", arXiv:1406.5036

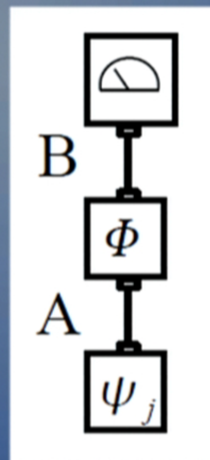


Encore: causal tomography

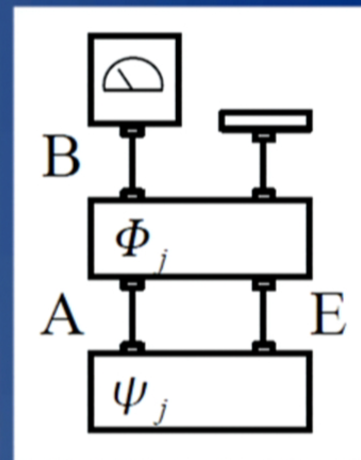
Other projects

Quantum process tomography in the presence of initial correlations

ideal:



real:



naive process tomography fails (eg not completely positive map)

⇒ want

- test for environmental back-action (detect non-Markovianity)
- characterize process despite these effects

Other projects: process tomography with initial correlations

Quantum process tomography in the presence of initial correlations

known:

marginal input $\rho_A^{(j)} = Tr_E(\rho_{AE}^{(j)})$

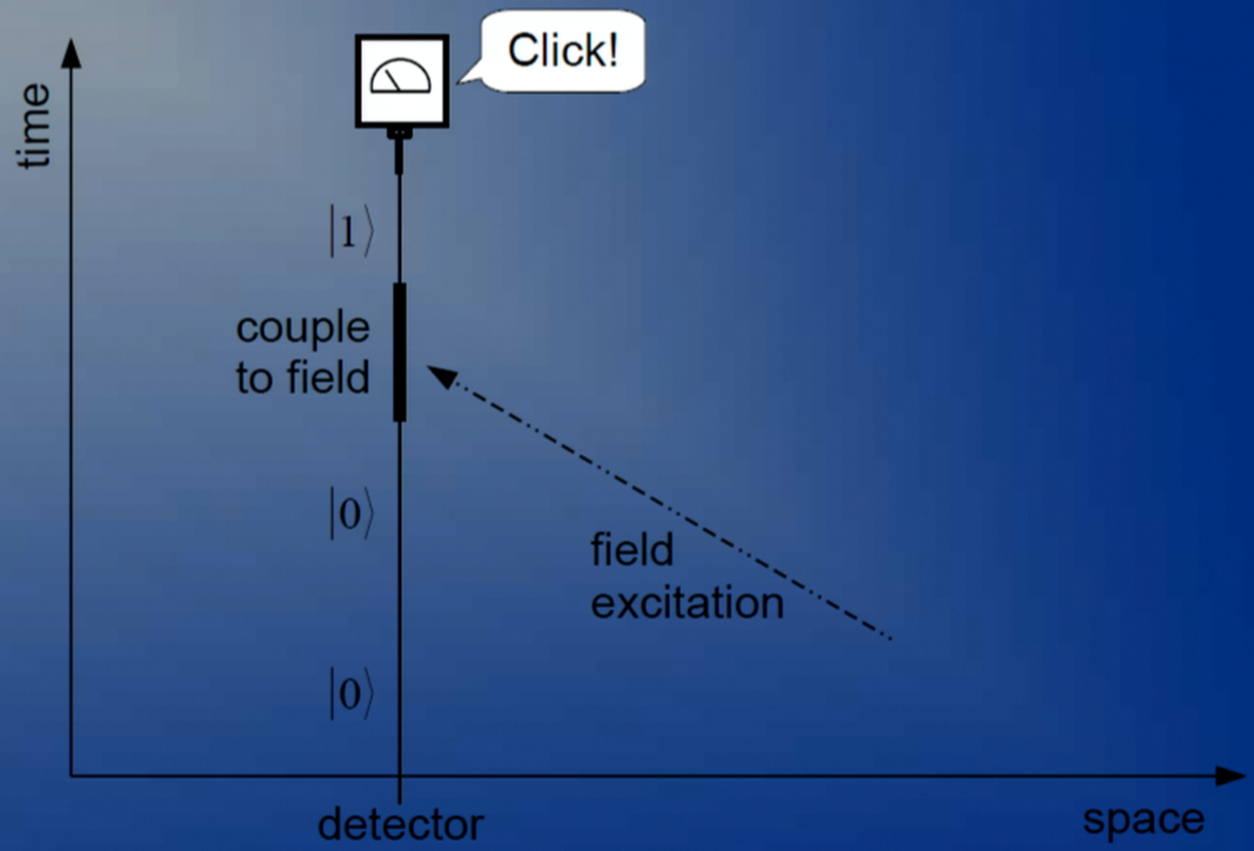
output $\rho_B^{(j)}$

wanted:

minimal dimension of E required to explain data

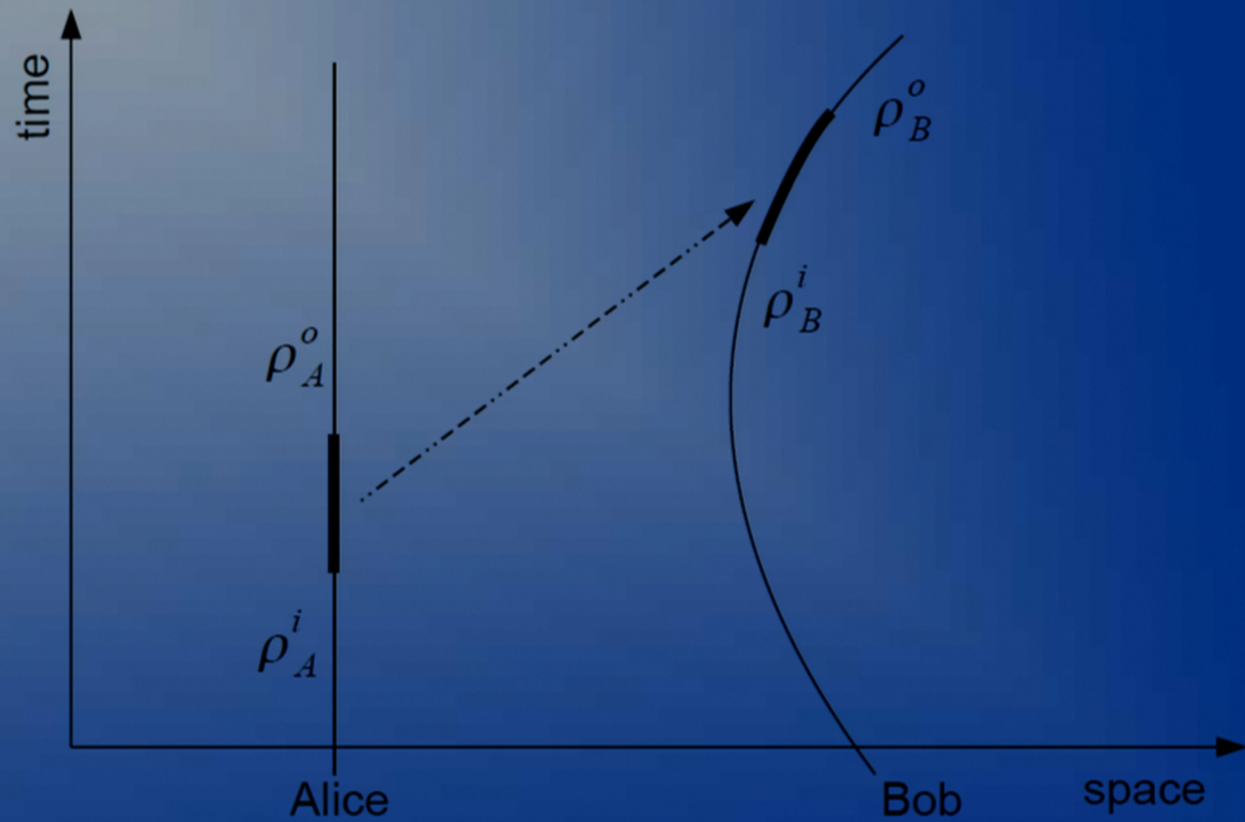
Other projects: process tomography with initial correlations

Probing causality in quantum field theory



Other projects: causality in quantum fields

Setup: two detectors in space-time



Other projects: causality in quantum fields

Study causal influences:

- structure
  - straightforward
- qualify and quantify:
  - How much classical information is transmitted?
  - How much quantum information is transmitted?
  - ...

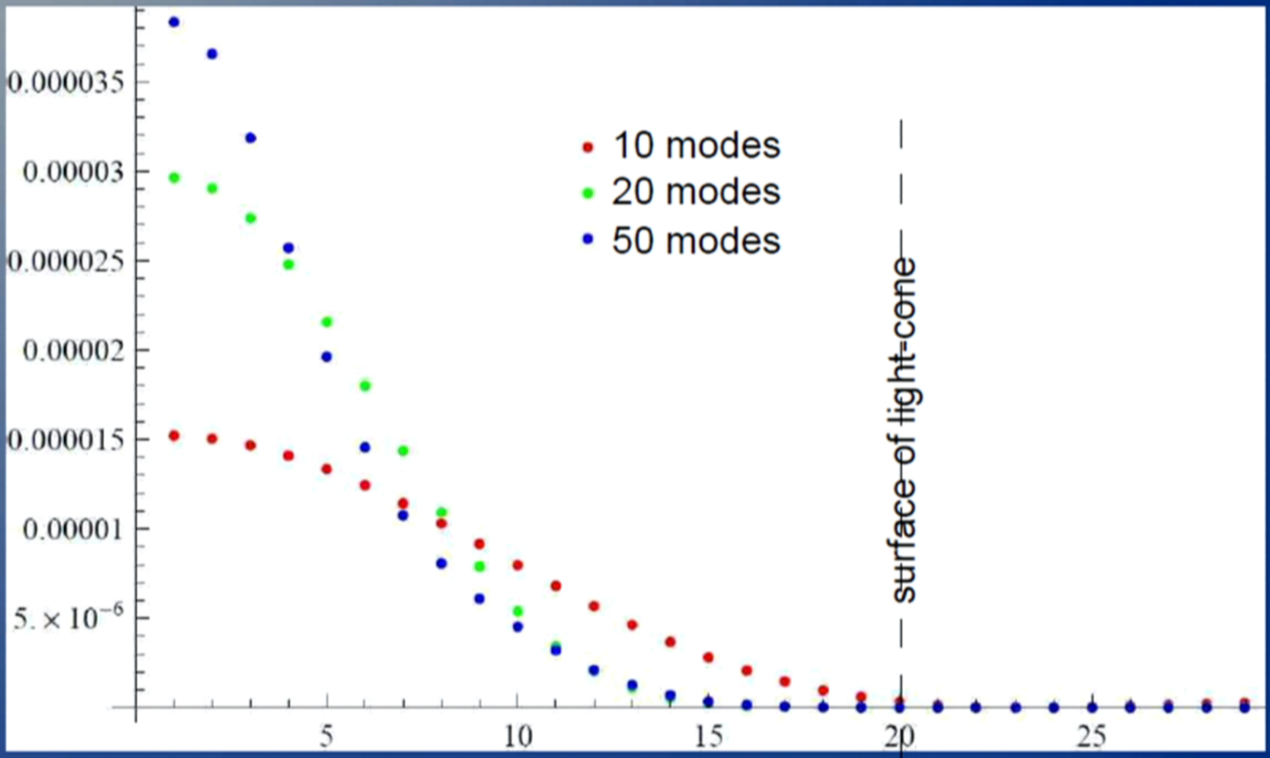
... as a function of

- space-time
- acceleration
- UV cut-off
- switching function
- ...

joint work with Eduardo Martín-Martínez, Robert Jonsson and Achim Kempf

Other projects: causality in quantum fields

Mutual information vs spatial separation



see also RH Jonsson, E Martín-Martínez, A Kempf, "Quantum signaling in cavity QED", PRA 89, 022330

Other projects: causality in quantum fields


The End