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Abstract:

Nemátic order in fractional quantum Hall liquids

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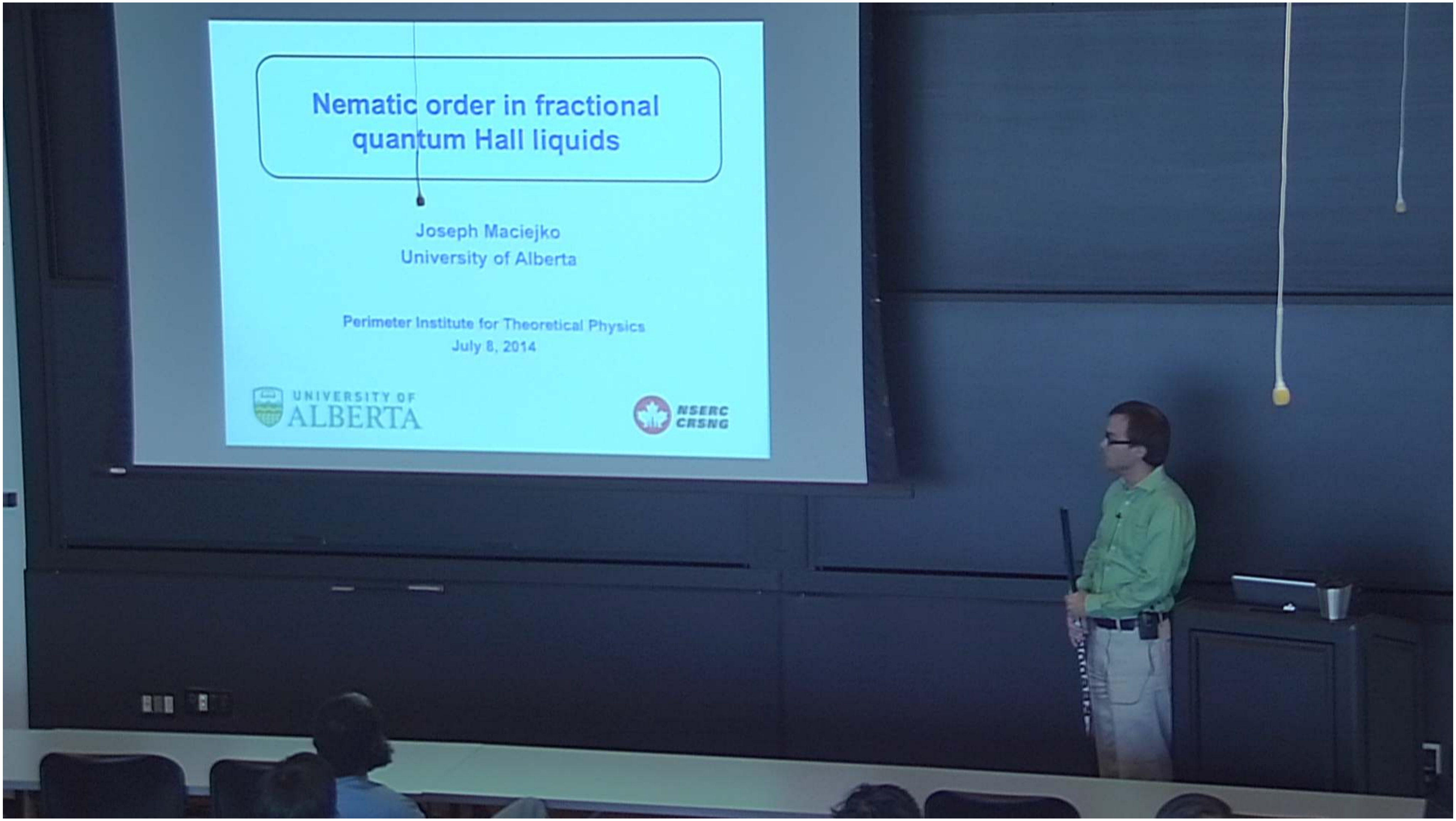
NSERC
CRSNG


Nematic order in fractional quantum Hall liquids
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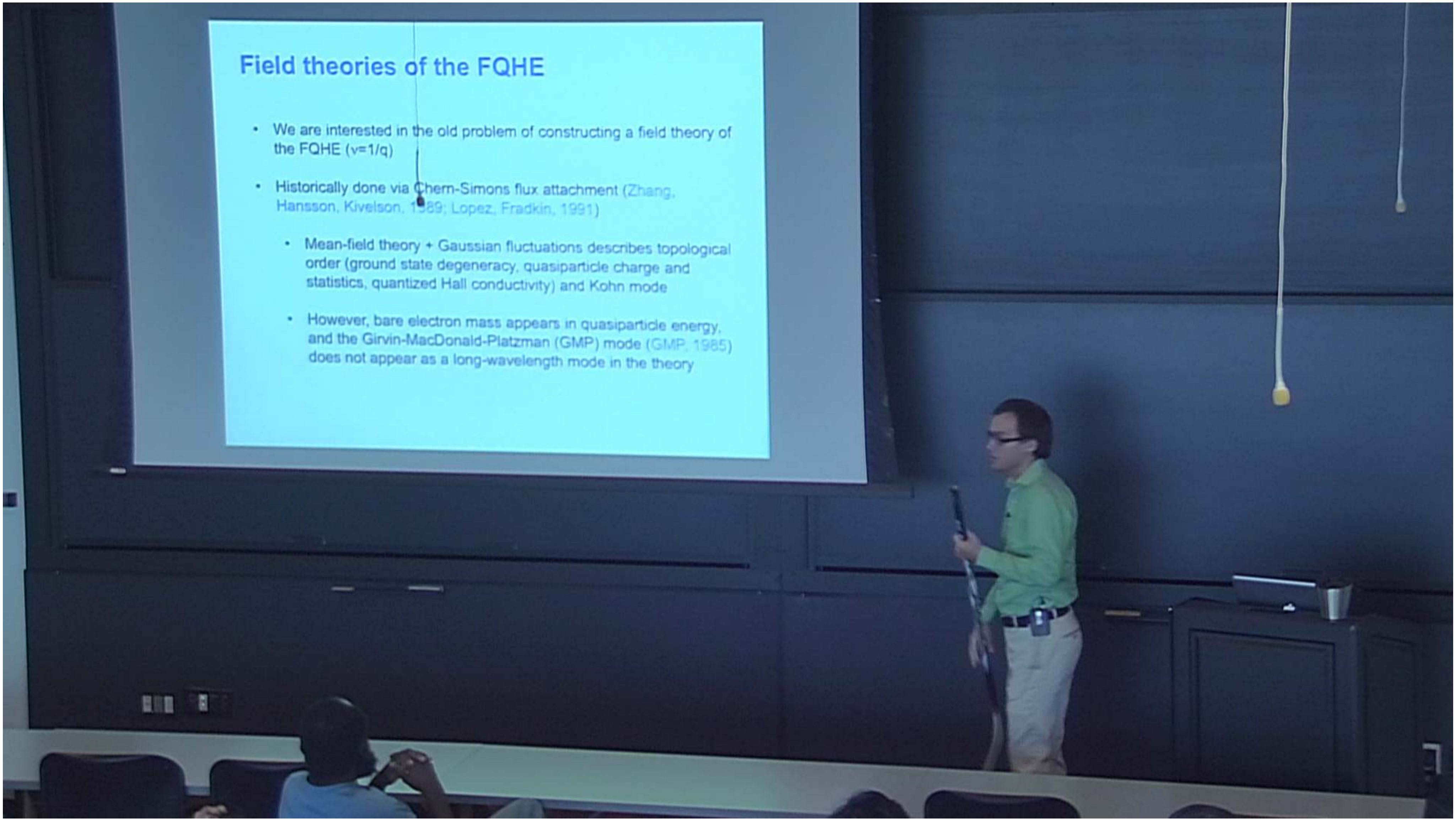
Collaborators

- Shivaji Sondhi (Princeton)
- Steve Kivelson (Stanford)
- Nicolas Regnault (ENS/CNRS/Princeton)
- Benjamin Hsu (Princeton)
- YeJe Park (Princeton/KAIST)

Field theories of the FQHE

- We are interested in the old problem of constructing a field theory of the FQHE ($\nu=1/q$)
- Historically done via Chern-Simons flux attachment (Zhang, Hansson, Kivelson, 1989; Lopez, Fradkin, 1991)
  - Mean-field theory + Gaussian fluctuations describes topological order (ground state degeneracy, quasiparticle charge and statistics, quantized Hall conductivity) and Kohn mode
  - However, bare electron mass appears in quasiparticle energy, and the Girvin-MacDonald-Platzman (GMP) mode (GMP, 1985) does not appear as a long-wavelength mode in the theory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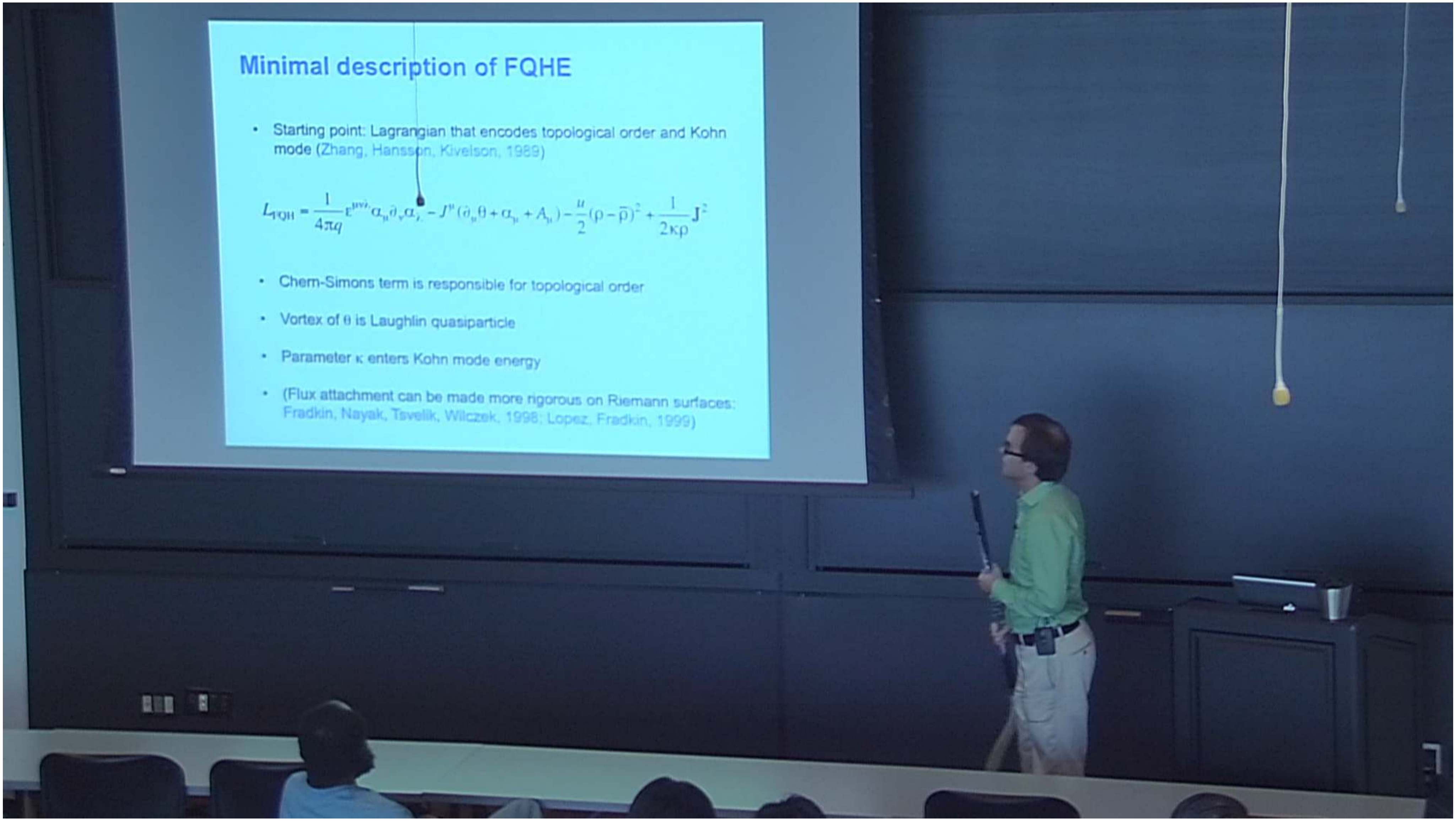
Minimal description of FQHE
• Starting point: Lagrangian that encodes topological order and Kohn mode (Zhang, Hansson, Kivelson, 1989)
$L_{FQH} = \frac{1}{4\pi q}\varepsilon^{\mu\nu\lambda}\alpha_\mu\partial_\nu\alpha_\lambda - J^\mu(\partial_\mu\theta + \alpha_\mu + A_\mu) - \frac{u}{2}(\rho-\bar{\rho})^2 + \frac{1}{2\kappa\rho}\mathbf{J}^2$
• Chern-Simons term is responsible for topological order
• Vortex of θ is Laughlin quasiparticle
• Parameter κ enters Kohn mode energy
• (Flux attachment can be made more rigorous on Riemann surfaces: Fradkin, Nayak, Tsvelik, Wilczek, 1998; Lopez, Fradkin, 1999)

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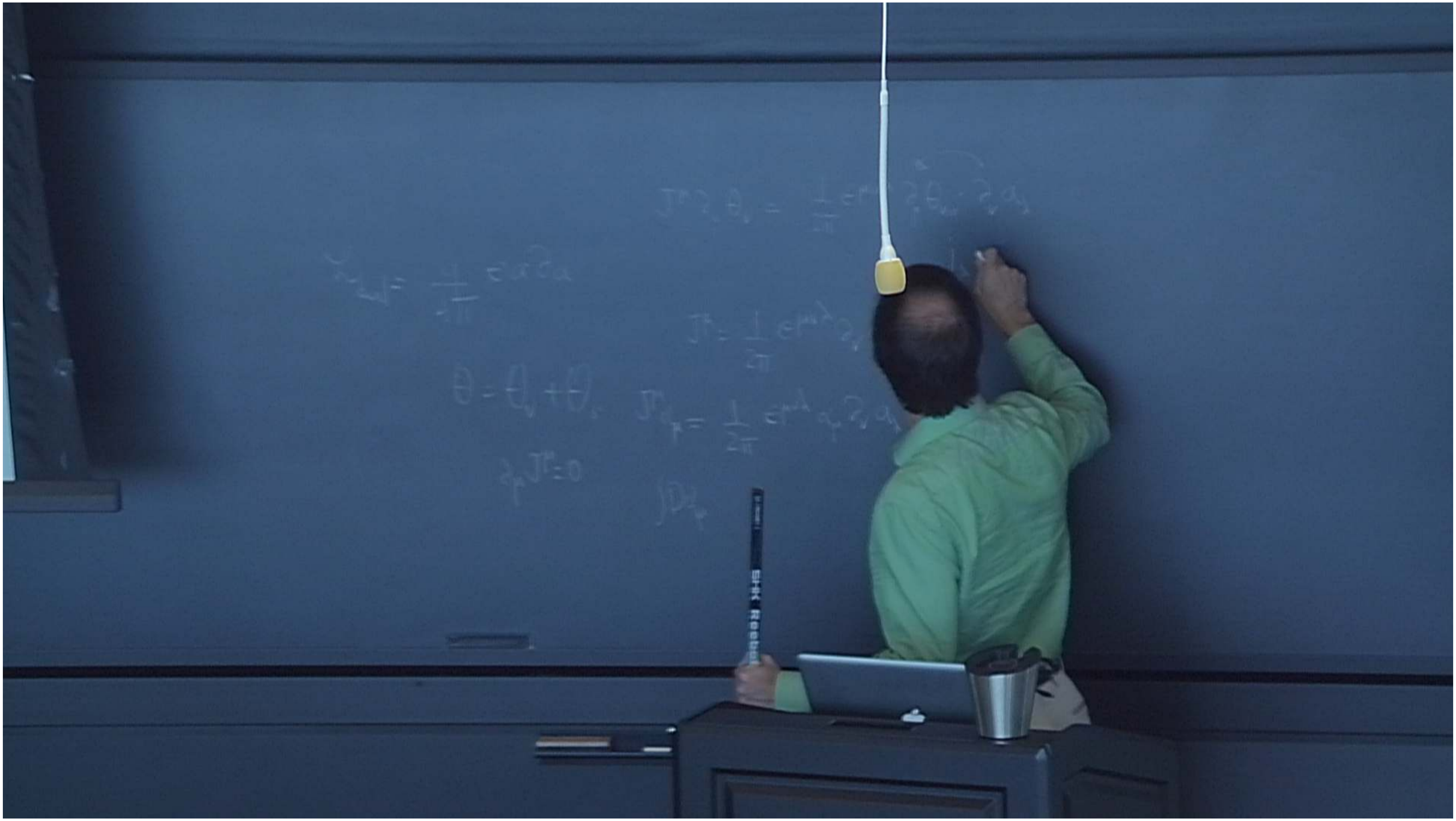
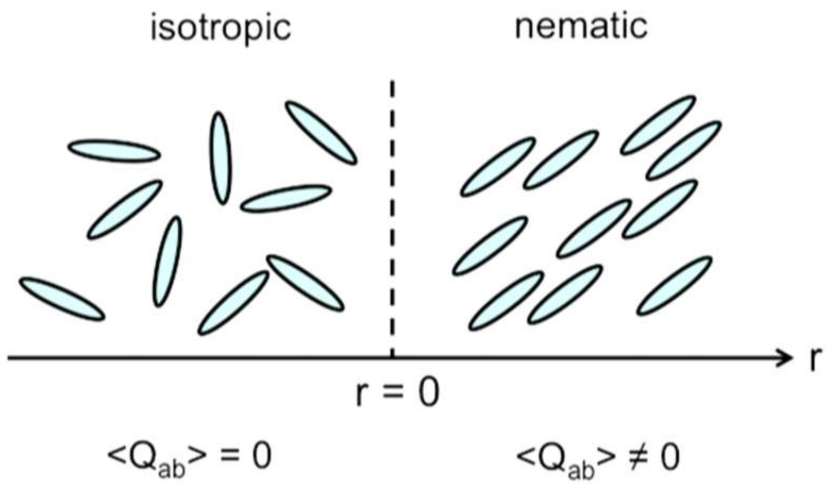
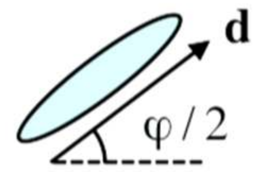


Nematic order

- Spontaneous breaking of SO(2) rotation invariance is described by a real **symmetric traceless** nematic order parameter Q_{ab}





$$\mathbf{d} \sim -\mathbf{d}$$

$$Q_{ab} = Q(2d_a d_b - \delta_{ab})$$

- The GMP mode is a fluctuating quadrupole at long wavelengths (Lee and Zhang, 1991; Yang et al., 2012)

- We identify the GMP mode with fluctuations of Q_{ab} in the isotropic phase

Dynamics of the order parameter

- General form of the Lagrangian: $L = L_{\text{FQH}} + L_{\text{OP}} + L_{\text{FQH-OP}}$
- Work near the transition, $Q_{ab} << 1$

$$L_{\text{OP}} = \lambda\varepsilon^{bc} Q_{ab}\partial_t Q_{ca} - K(\partial_a Q_{bc})^2 - rQ_{ab}Q_{ba} - u(Q_{ab}Q_{ba})^2$$

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quantum dynamics: Berry phase term as in QH ferromagnet (Sondhi et al., 1993)

dictated by Landau theory: SO(2) rotation symmetry

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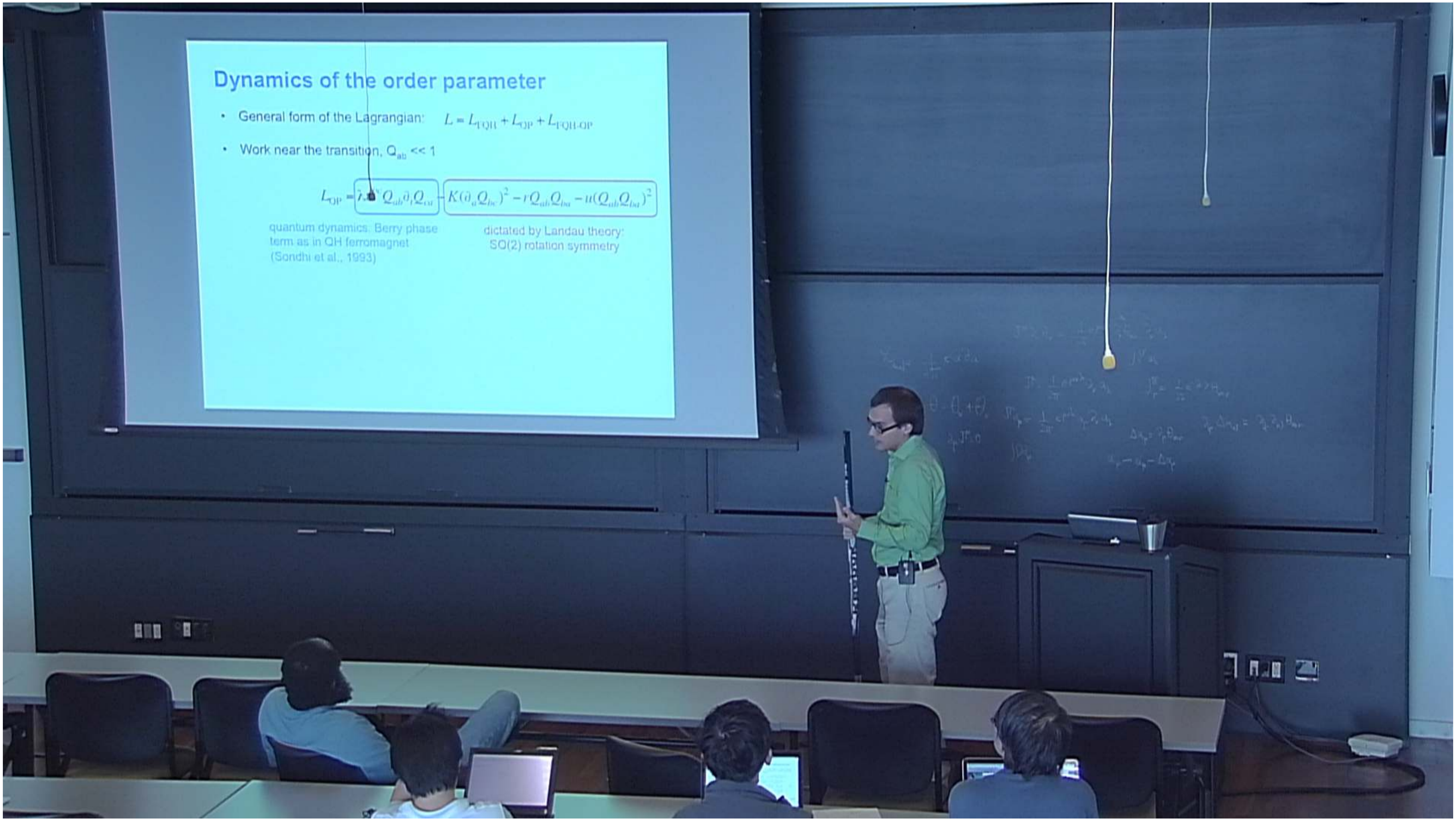

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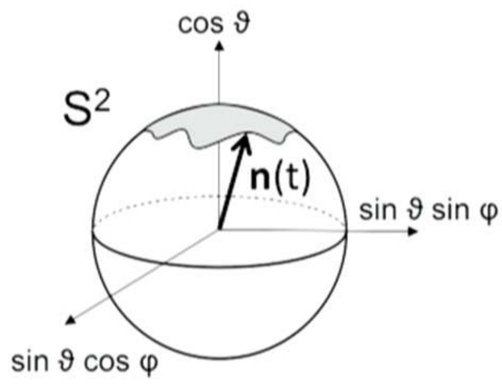

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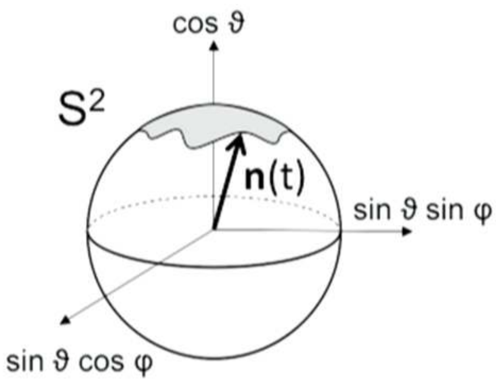

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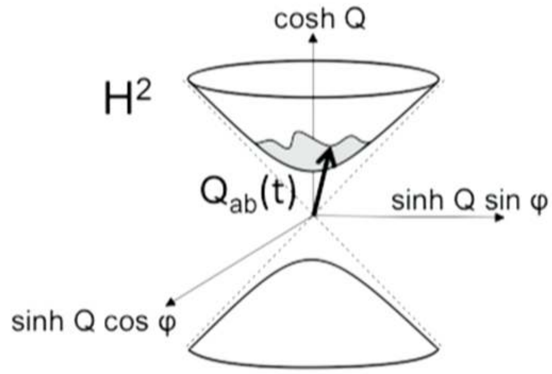

Field theory of the transition

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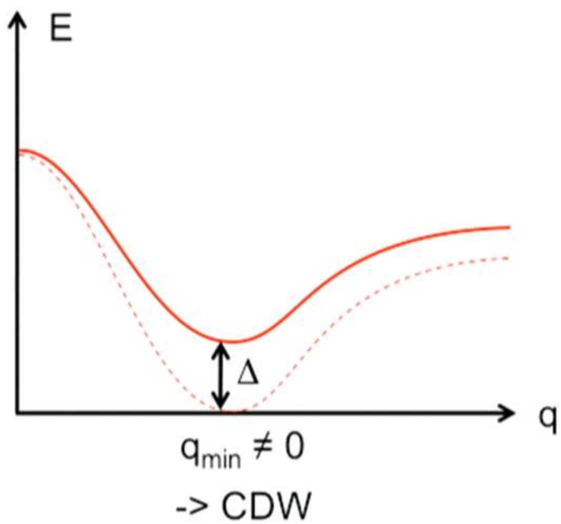
JM, Hsu, Kivelson, Park, Sondhi, PRB 88, 125137 (2013)

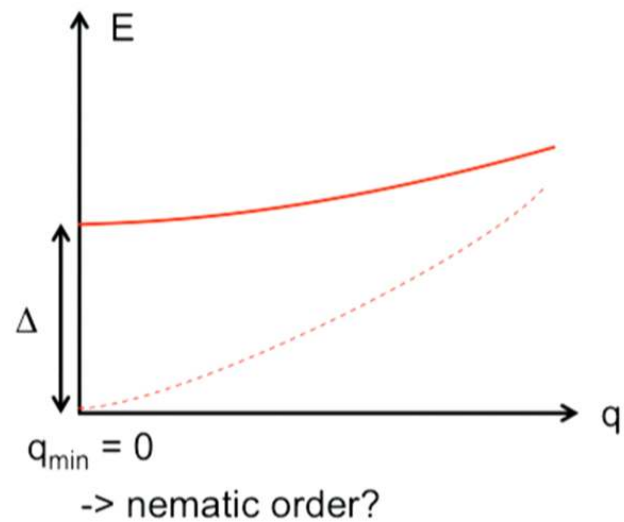
- Resulting Lagrangian parallels closely Haldane's geometric field theory (Haldane, 2011) for a dynamical metric $g_{ab}(\mathbf{r},t)$ upon identifying

$$g_{ab} = (\exp Q)_{ab} \approx \delta_{ab} + Q_{ab}$$

Isotropic phase

- Supports gapped Laughlin quasiparticles and two long-wavelength gapped modes: GMP and Kohn modes
- We conjecture that if the GMP gap can be made to collapse at q=0, there should be an instability towards nematic order





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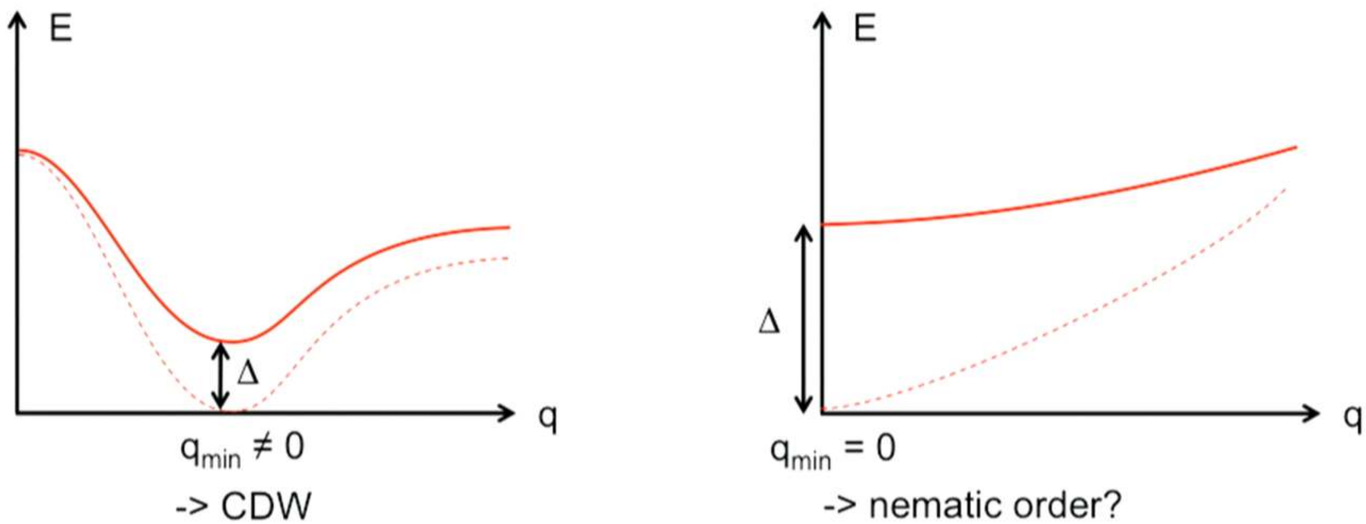

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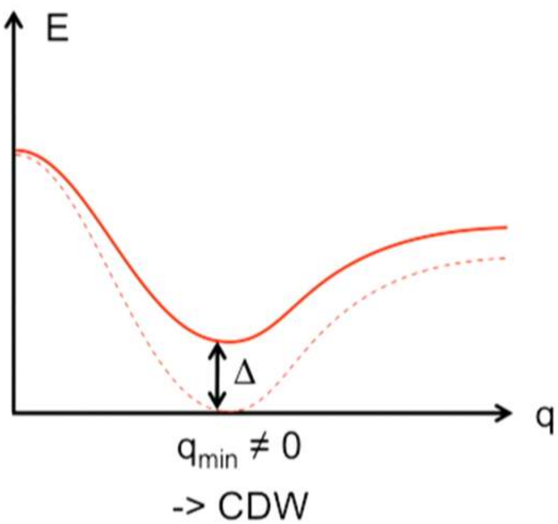
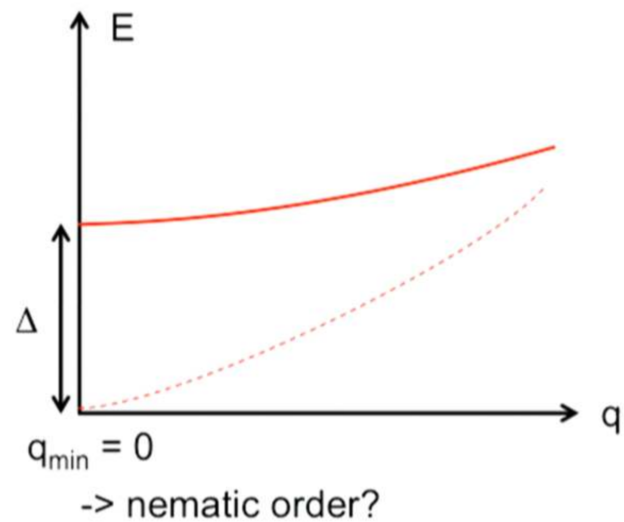




z=2 critical point

- At long wavelengths, the nematic and gauge sectors decouple and the nematic sector undergoes a z=2 transition (~2D dilute Bose gas)
- z=2 scaling for FQH nematics first discussed by Mulligan, Nayak, Kachru (2010, 2011) in a pure gauge theory approach
- Laughlin quasiparticle and Kohn mode remain gapped through the transition

Nematic phase

- Linearly dispersing gapless Goldstone mode
- Because Q_{ab} is charge neutral, dc Hall conductivity remains quantized in the nematic phase, as in the experiment by Xia et al., 2011 (see also Mulligan, Nayak, Kachru, 2010, 2011)
- Optical Hall conductivity receives corrections in the nematic phase:

$$\sigma_{xy}(\mathbf{q},\omega) = \frac{1}{2\pi m} + \underbrace{W}_{\propto Q^2}\hat{q}_x\hat{q}_y i\omega + \mathcal{O}(\omega^2) \qquad (\nu=1/m)$$

- Nematic vortices (disclinations):
  - have logarithmically divergent energy
  - carry (unquantized) electric charge
  - have anyonic statistics

Numerical study on the torus

- Can this transition occur in a concrete model for the FQHE?
- Numerically study interacting electrons in the LLL at filling $\nu=1/3$

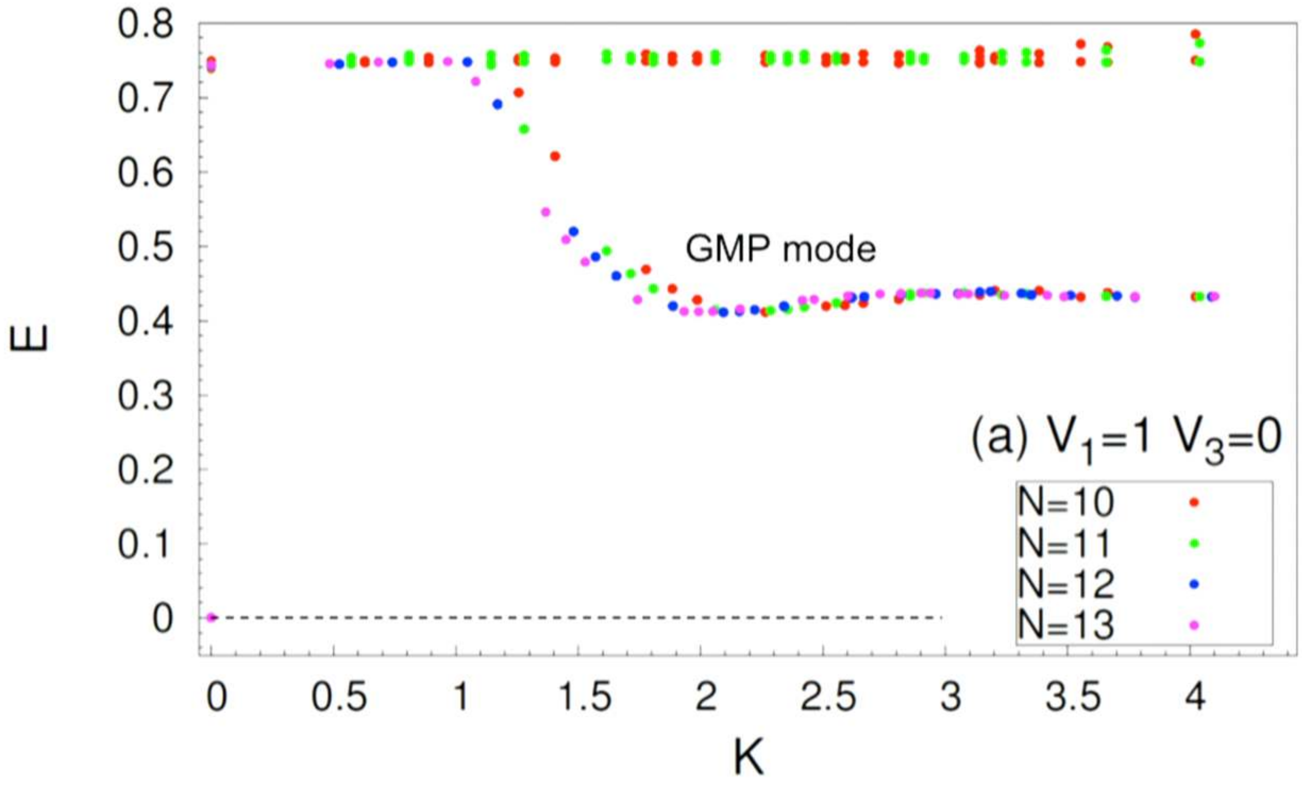
$$\mathcal{H} = \sum_{\mathbf{q}} V(\mathbf{q}) : \rho_{\mathbf{q}} \rho_{-\mathbf{q}} :$$

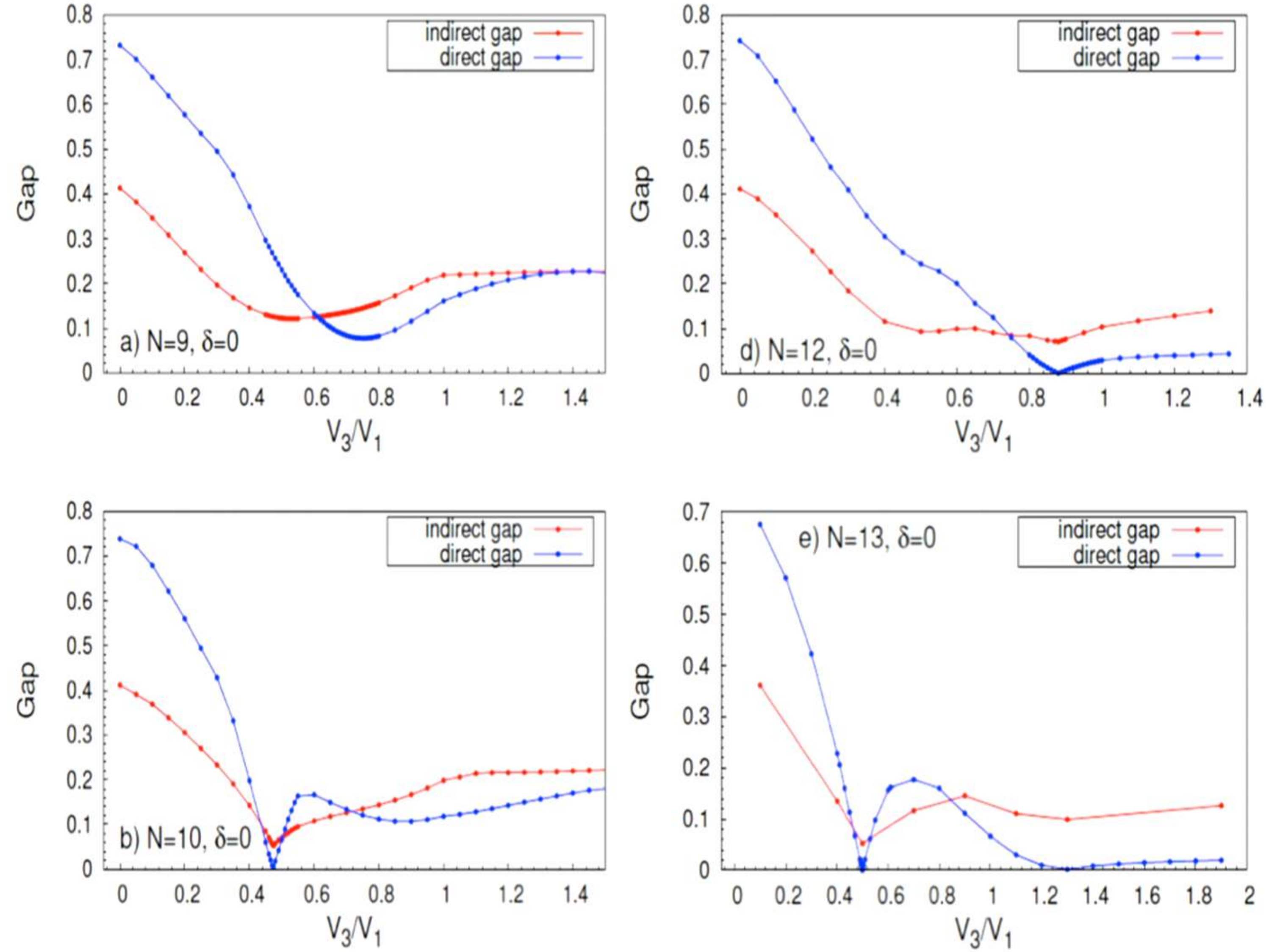
- Decompose into Haldane pseudopotentials (Haldane, 1983):

$$V(\mathbf{q}) = \sum_{n} V_n \mathcal{L}_n \left(\frac{1}{2} \mathbf{q}^2 \right)$$

- Model with V_1 only: Laughlin state is exact ground state
- Consider model with V_1 and V_3 (no exact ground state)

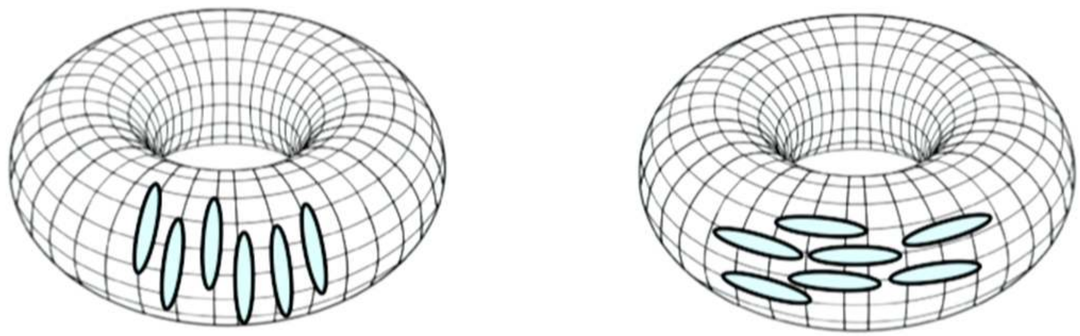
Numerical study on the torus




indirect gap
direct gap
a) N=9, δ=0
d) N=12, δ=0
b) N=10, δ=0
e) N=13, δ=0
Gap
V3/V1

A possible nematic phase?

- Significant finite-size effects, difficult to pinpoint nature of large V_3 phase
- Signature of nematicity? Aspect ratio dependence
- For finite-size torus, O(2) nematic becomes Ising nematic (C_4) ()



$$|\uparrow\rangle \qquad |\downarrow\rangle$$

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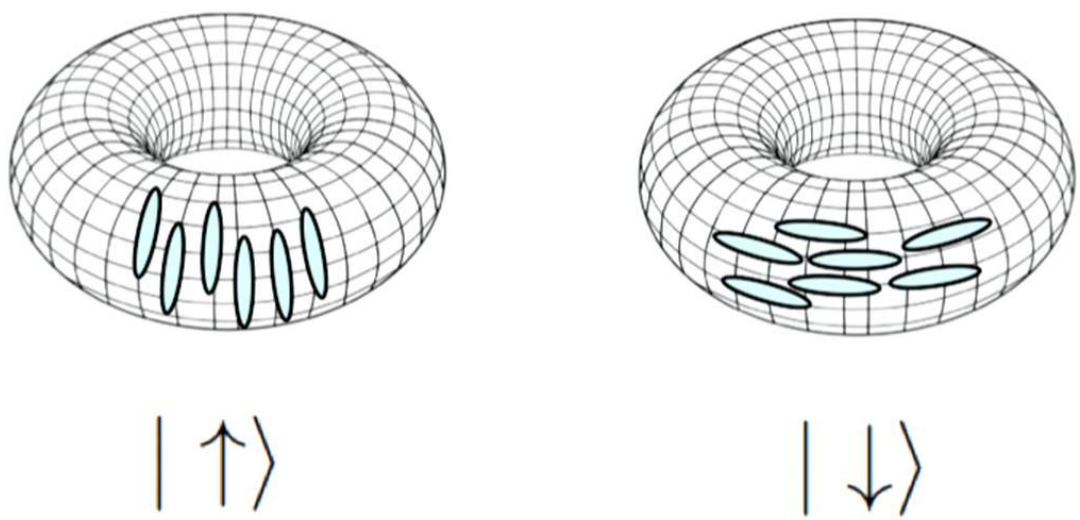
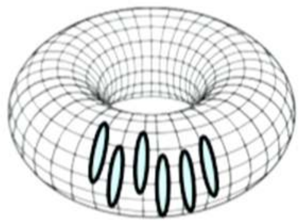


Effective Ising nematic

- Quantum dynamics of **q**=0 Goldstone mode leads to tunneling Δ between the two Ising states (Anderson, 1952)
- Torus aspect ratio $L_y/L_x=1+\delta$ acts as "Zeeman field"
- Effective Hamiltonian for 2-dimensional low-energy subspace in the nematic phase



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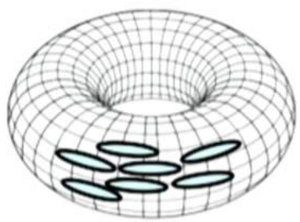


$|\downarrow\rangle$

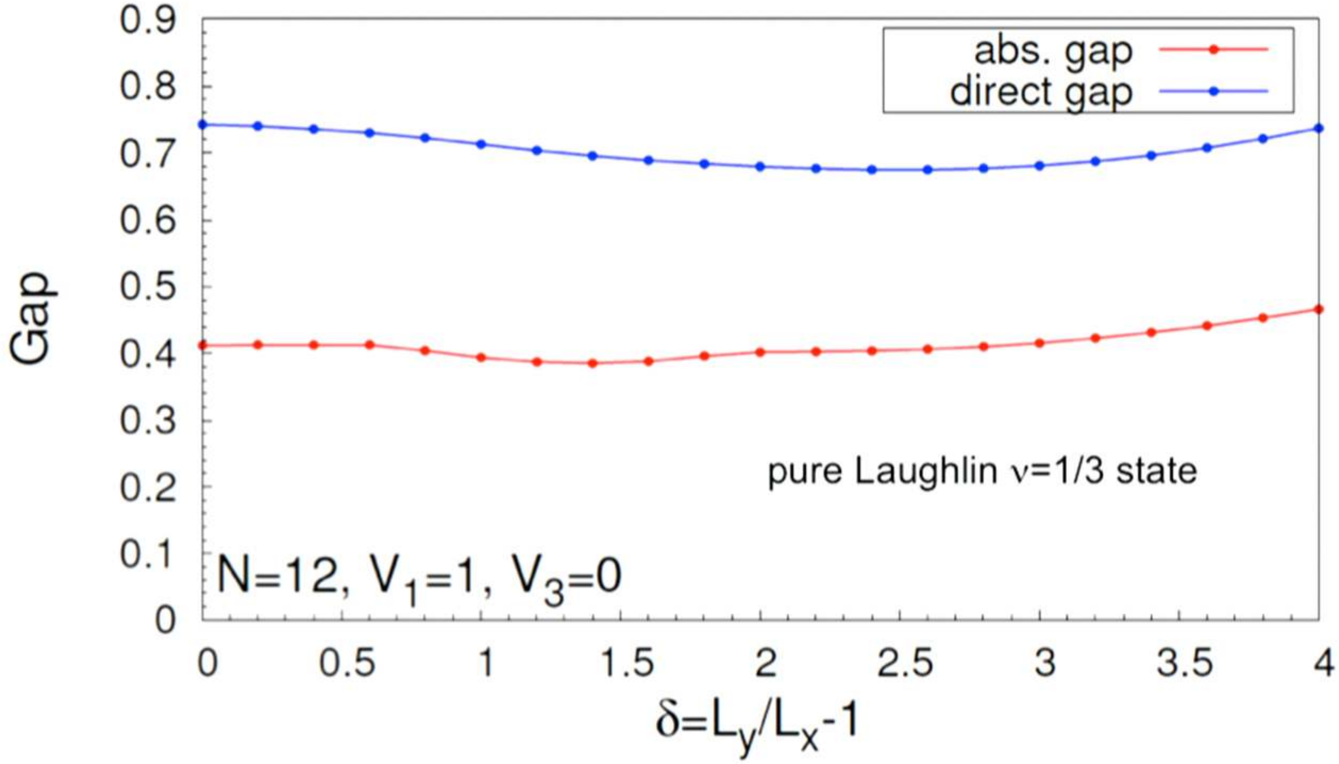
$$H_{\mathrm{eff}} = \Delta\tau_x + E_Z\tau_z$$

$$\Delta \propto e^{-\mathrm{const.}\times Q_0^{3/2}V^{1/4}}$$

$$E_Z \propto \frac{Q_0^2\delta}{\sqrt{V}} \qquad \delta \ll 1$$

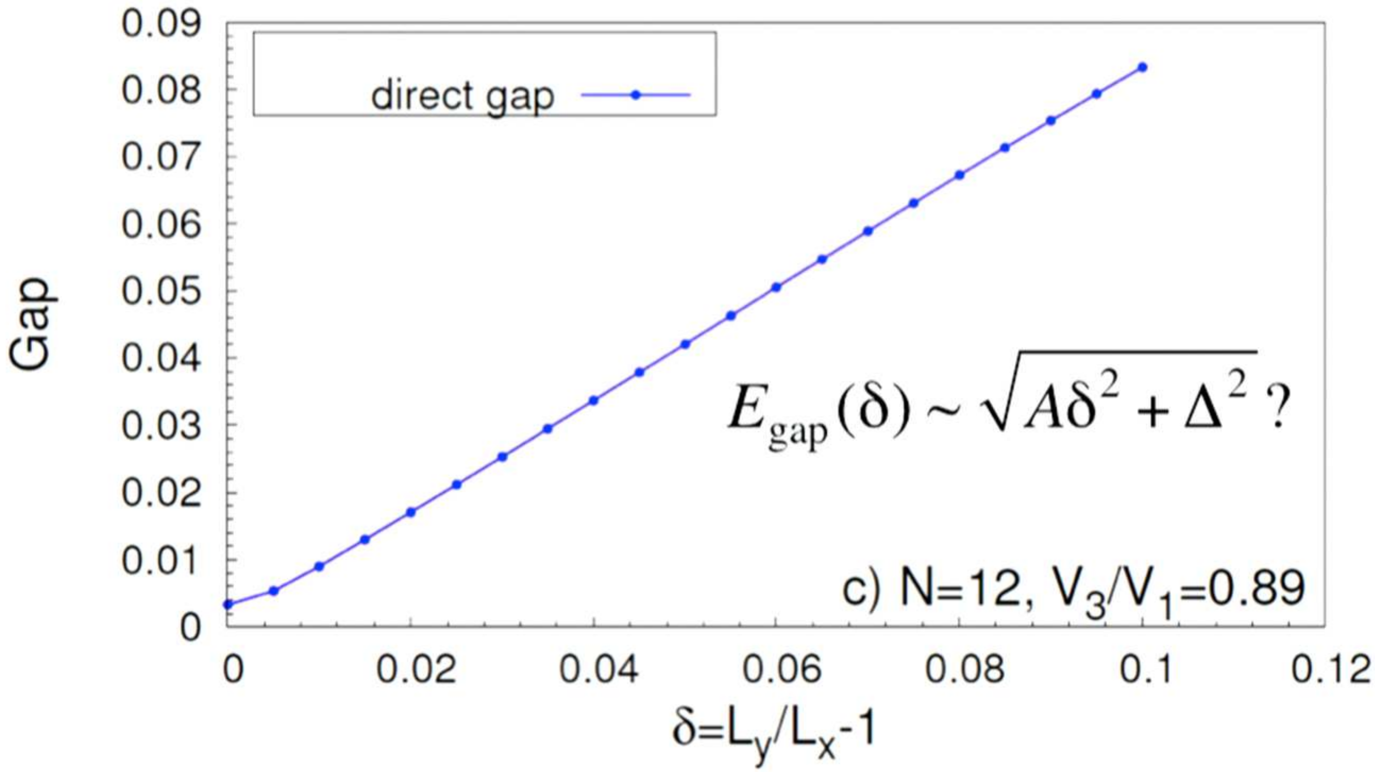
$$E_{\mathrm{gap}}(\delta) \sim \sqrt{A\delta^2 + \Delta^2}$$

Effect of aspect ratio: isotropic phase



- Very little dependence of gap on aspect ratio

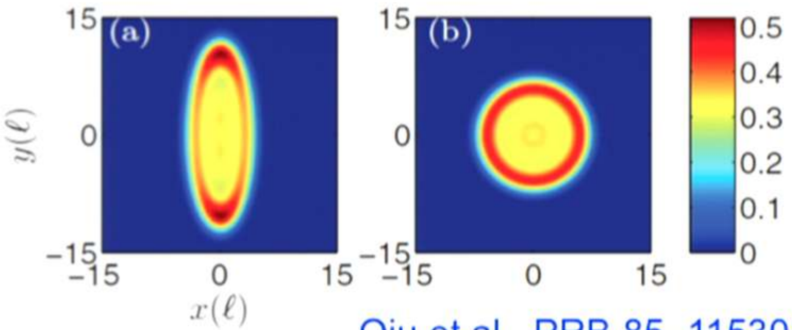
Effect of aspect ratio: nematic phase



Variational calculation

- Use Haldane's family of anisotropic Laughlin states (Qiu et al., 2012; Yang et al., 2012) as variational states

$$|\Psi_L^3(g)\rangle$$



ν=1/3

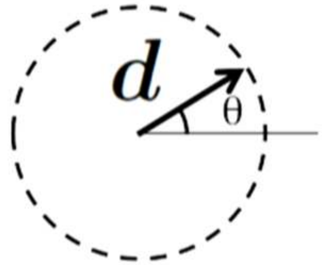
Qiu et al., PRB 85, 115308 (2012)

$$g_{ab} = \begin{pmatrix} \cosh Q + \sinh Q \cos\varphi & \sinh Q \sin\varphi \\ \sinh Q \sin\varphi & \cosh Q - \sinh Q \cos\varphi \end{pmatrix}$$

$$E(Q, \varphi, V_3/V_1) = \frac{\langle \Psi_L^3(Q,\varphi)|H_{V_3/V_1}|\Psi_L^3(Q,\varphi)\rangle}{\langle \Psi_L^3(Q,\varphi)|\Psi_L^3(Q,\varphi)\rangle}$$

Nematic order parameter

$$f(\boldsymbol{d}) = \langle \rho(\boldsymbol{r} + \boldsymbol{d}/2)\rho(\boldsymbol{r} - \boldsymbol{d}/2) \rangle$$

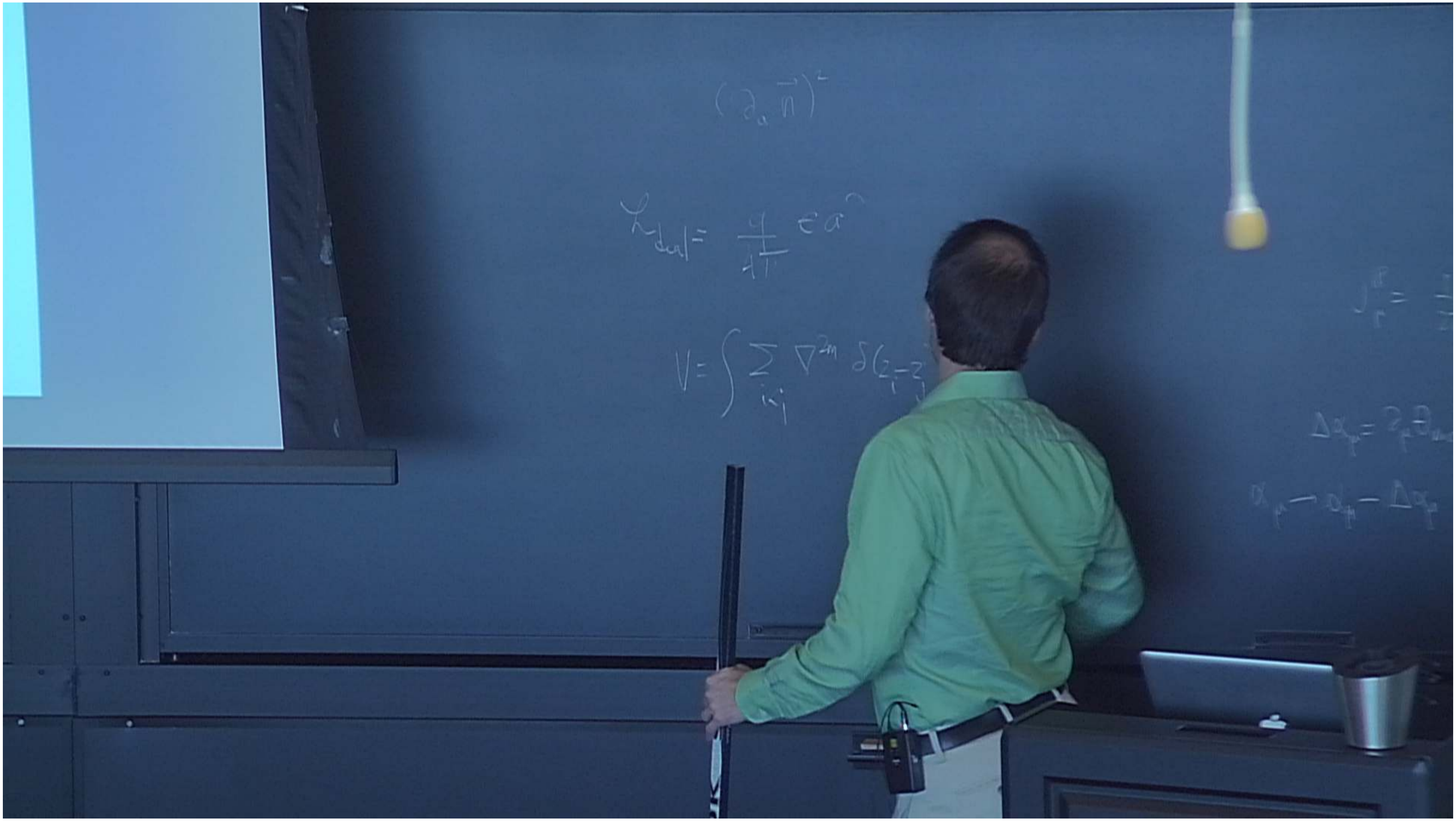


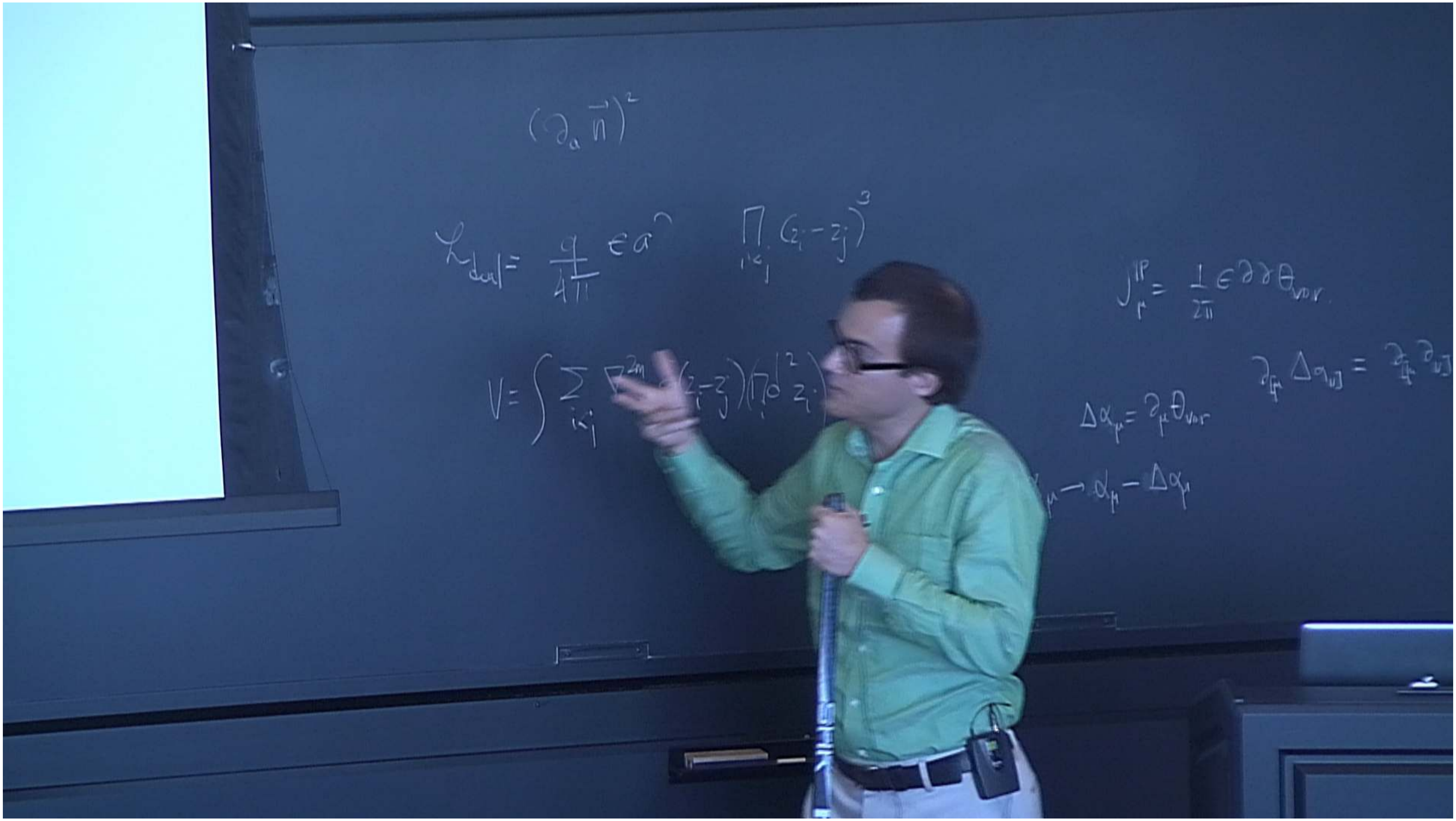
$$|\boldsymbol{d}| \sim \ell_B$$

$$f(\boldsymbol{d}) = f(-\boldsymbol{d}) \iff \mathbf{d} \sim -\mathbf{d}$$

"headless vector"

- Isotropic ground state has $f = 0$
- Odd part of f comes from the commutator of densities at r+d/2 and r-d/2, which vanishes trivially for unprojected densities and vanishes as a consequence of the GMP algebra for LLL projected densities





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