Title: Measuring the overhead of a quantum error correcting code

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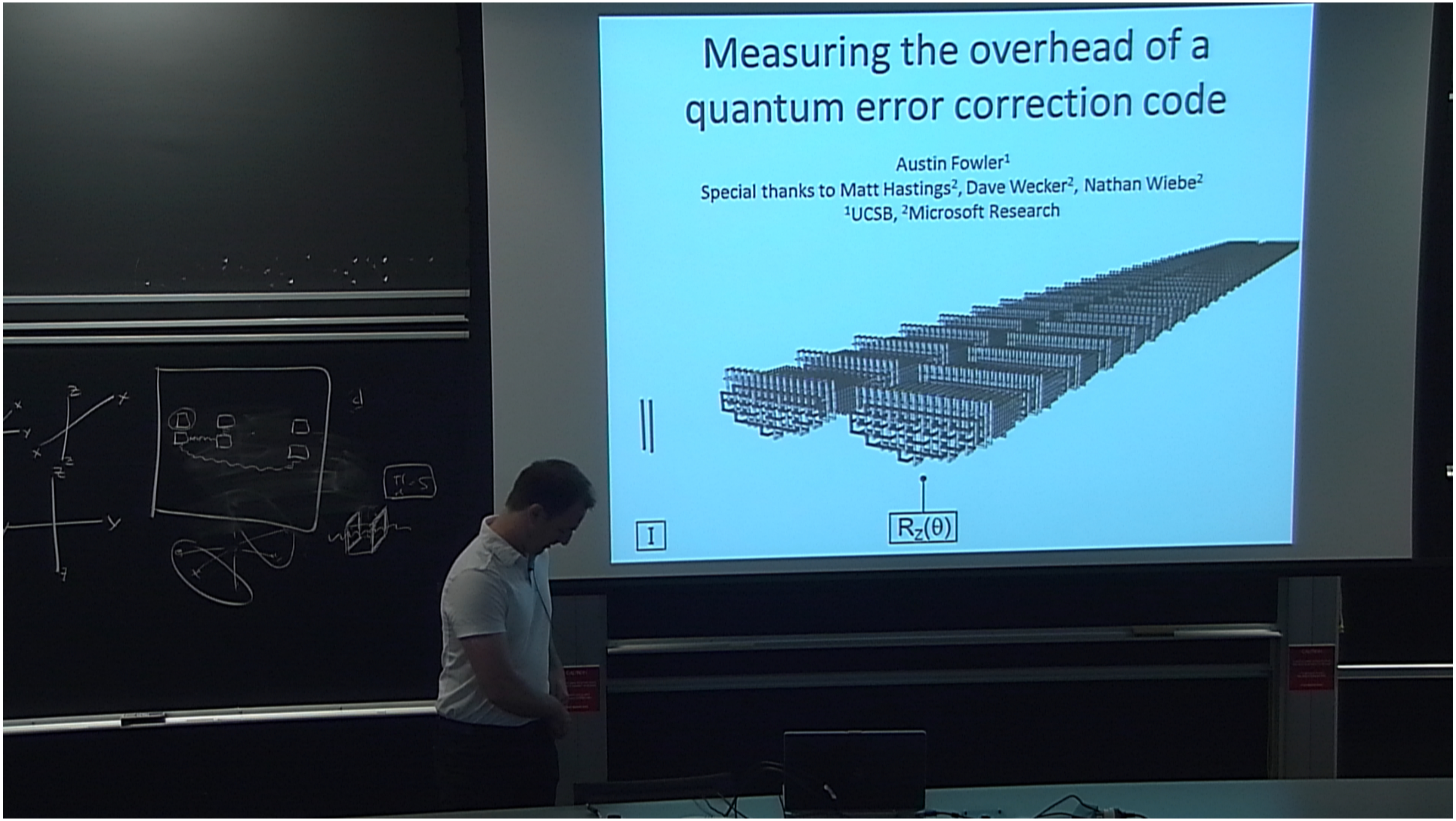
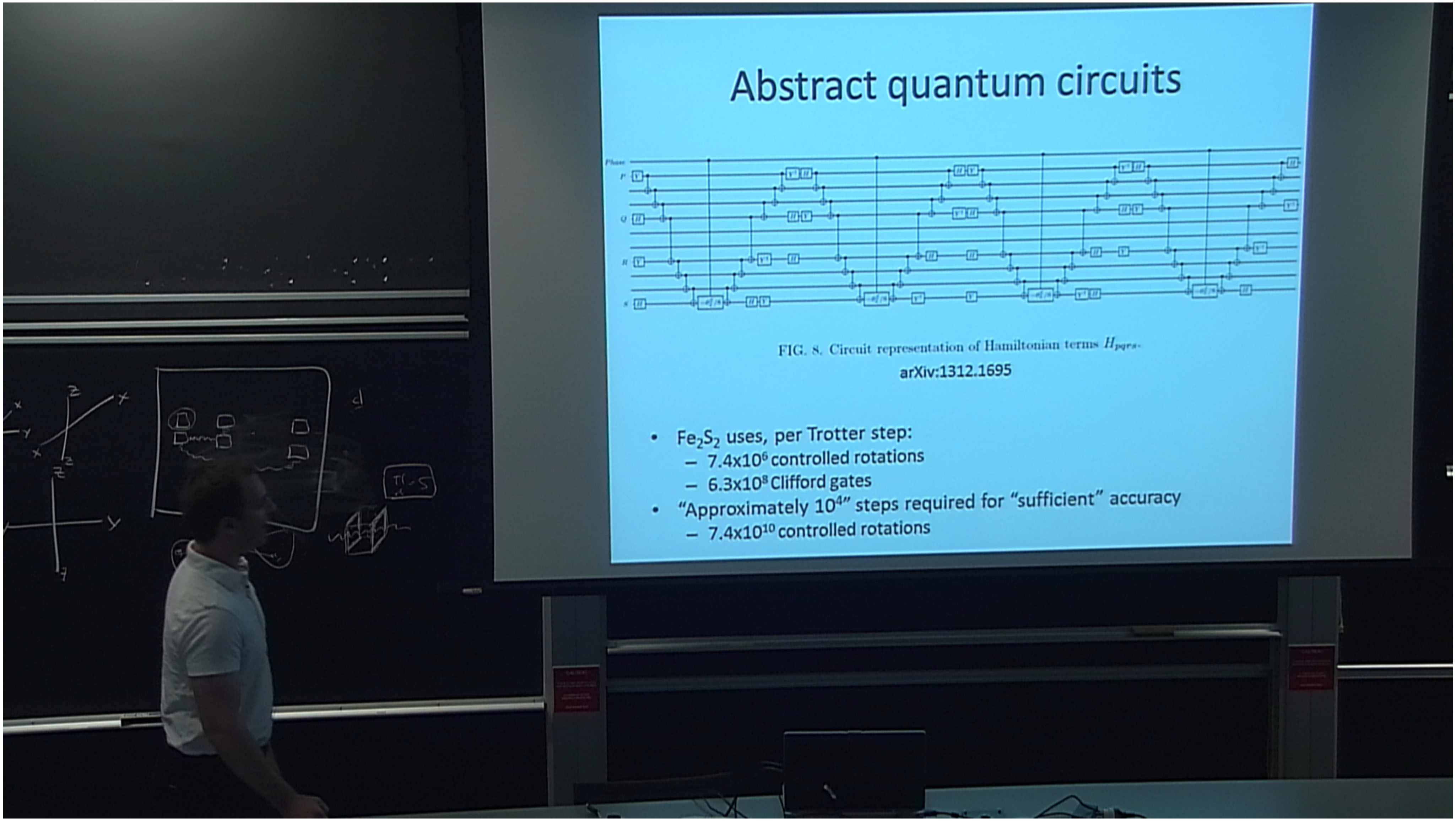
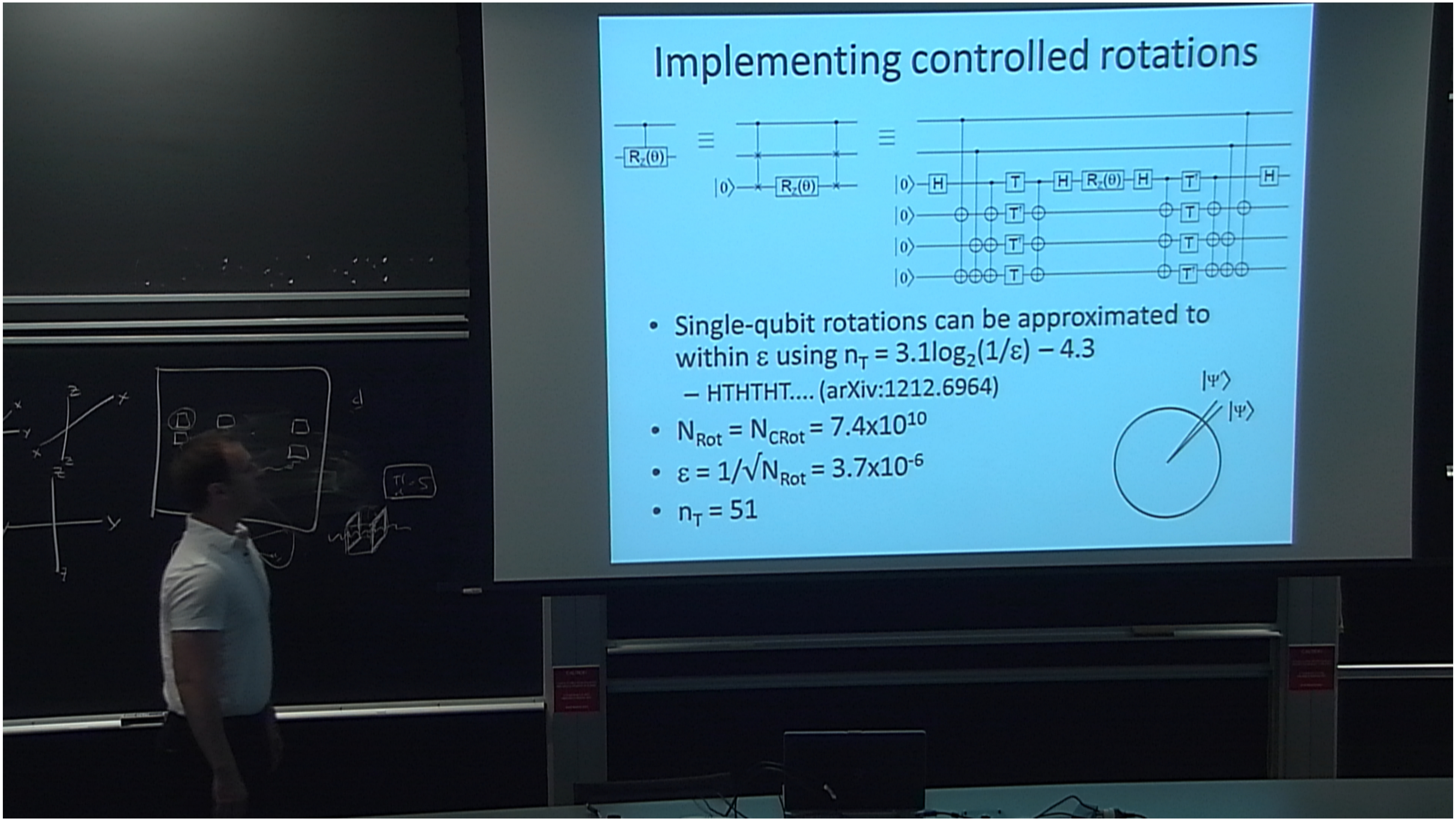
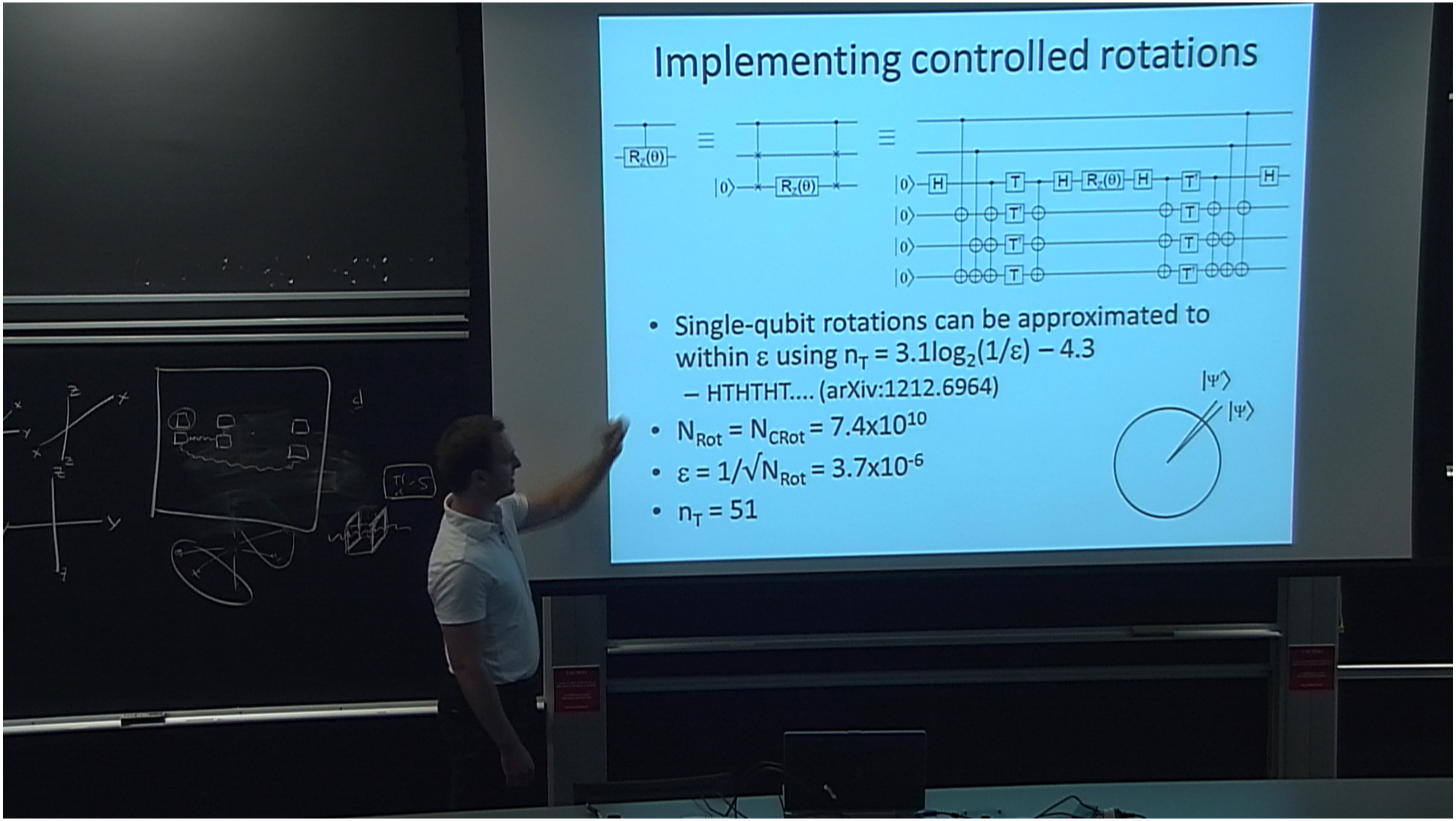
URL: http://pirsa.org/14070014

Abstract: If one's goal is large-scale quantum computation, ultimately one wishes to minimize the amount of time, number of qubits, and qubit connectivity required to outperform a classical system, all while assuming some physically reasonable gate error rate. We present two examples of such an overhead study, focusing on the surface code with and without long-range interactions.


Measuring the overhead of a quantum error correction code
Austin Fowler1
Special thanks to Matt Hastings2, Dave Wecker2, Nathan Wiebe2
1UCSB, 2Microsoft Research
I
Rz(θ)


Abstract quantum circuits
FIG. 8. Circuit representation of Hamiltonian terms H_{pqrs}.
arXiv:1312.1695
• Fe_2S_2 uses, per Trotter step:
– 7.4×10^6 controlled rotations
– 6.3×10^8 Clifford gates
• "Approximately 10^4" steps required for "sufficient" accuracy
– 7.4×10^{10} controlled rotations


Implementing controlled rotations
$R_z(\theta)$
$|0\rangle$
H
T
$T^\dagger$
• Single-qubit rotations can be approximated to within ε using $n_T = 3.1\log_2(1/\varepsilon) - 4.3$
– HTHTHT.... (arXiv:1212.6964)
• $N_{Rot} = N_{CRot} = 7.4\times10^{10}$
• $\varepsilon = 1/\sqrt{N_{Rot}} = 3.7\times10^{-6}$
• $n_T = 51$
$|\Psi'\rangle$
$|\Psi\rangle$


Implementing controlled rotations
Rz(θ)
|0⟩
H
T
T†
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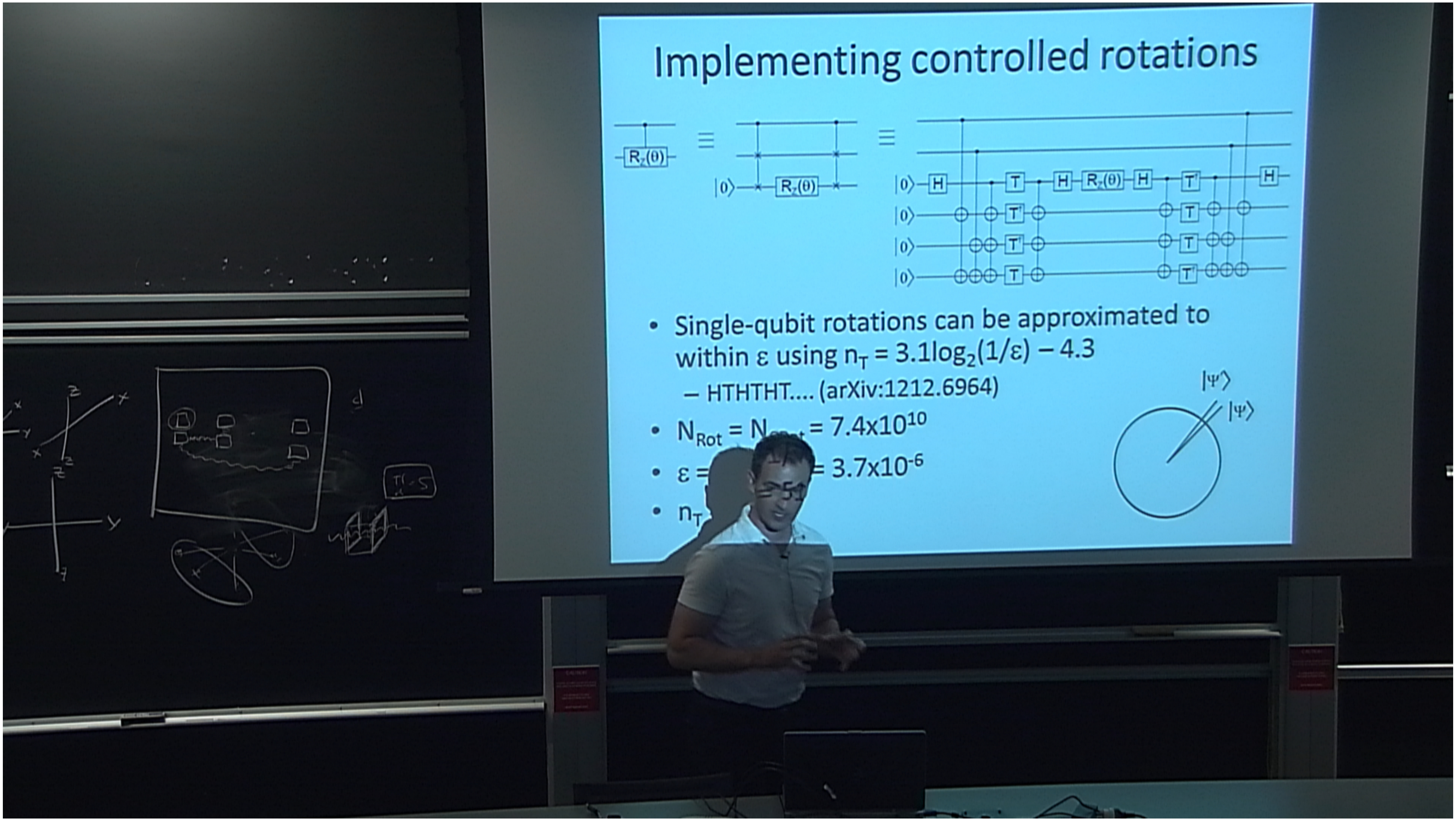
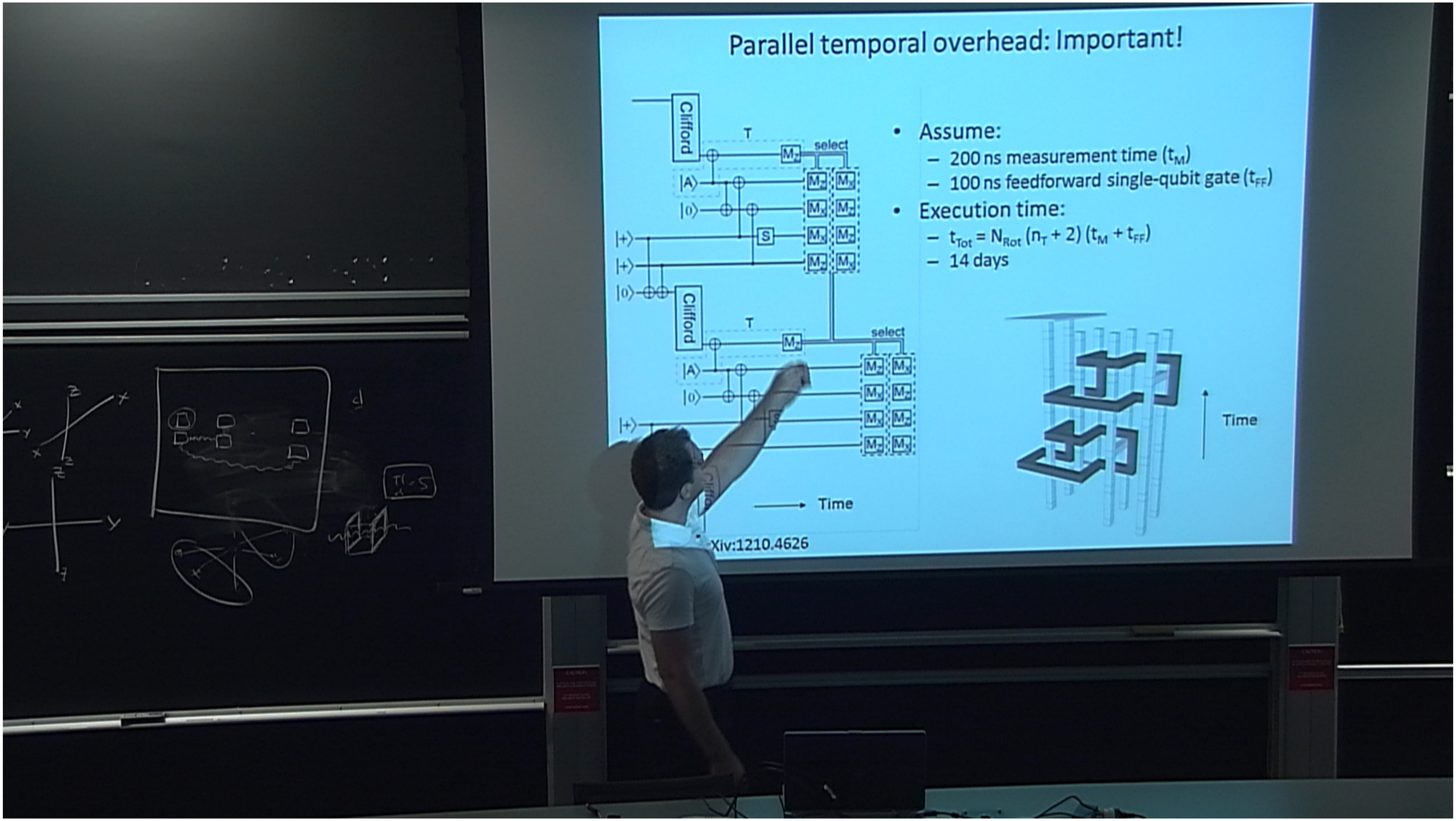
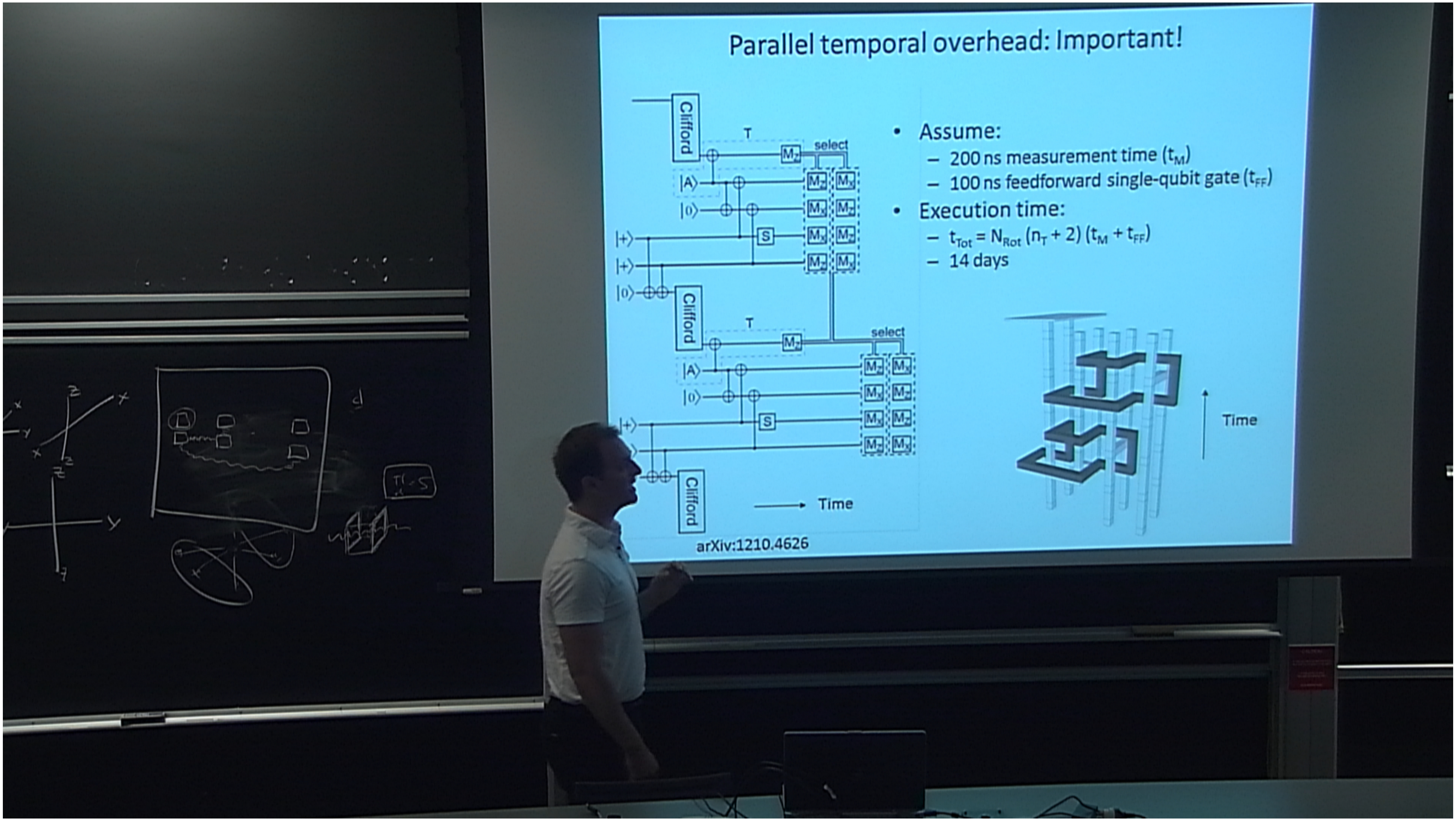
• ε = 1/√N_Rot = 3.7x10^-6
• n_T = 51
|Ψ'⟩
|Ψ⟩


Implementing controlled rotations
R_z(θ)
≡
|0⟩
R_z(θ)
≡
|0⟩
H
T
R_z(θ)
• Single-qubit rotations can be approximated to within ε using n_T = 3.1log_2(1/ε) – 4.3
– HTHTHT…. (arXiv:1212.6964)
• N_Rot = N
= 7.4x10^10
• ε =
= 3.7x10^-6
• n_T
|Ψ'⟩
|Ψ⟩



Parallel temporal overhead: Important!
Assume:
– 200 ns measurement time (t_M)
– 100 ns feedforward single-qubit gate (t_{FF})
Execution time:
– $t_{Tot} = N_{Rot} (n_T + 2) (t_M + t_{FF})$
– 14 days
Clifford
T
select
Time
Xiv:1210.4626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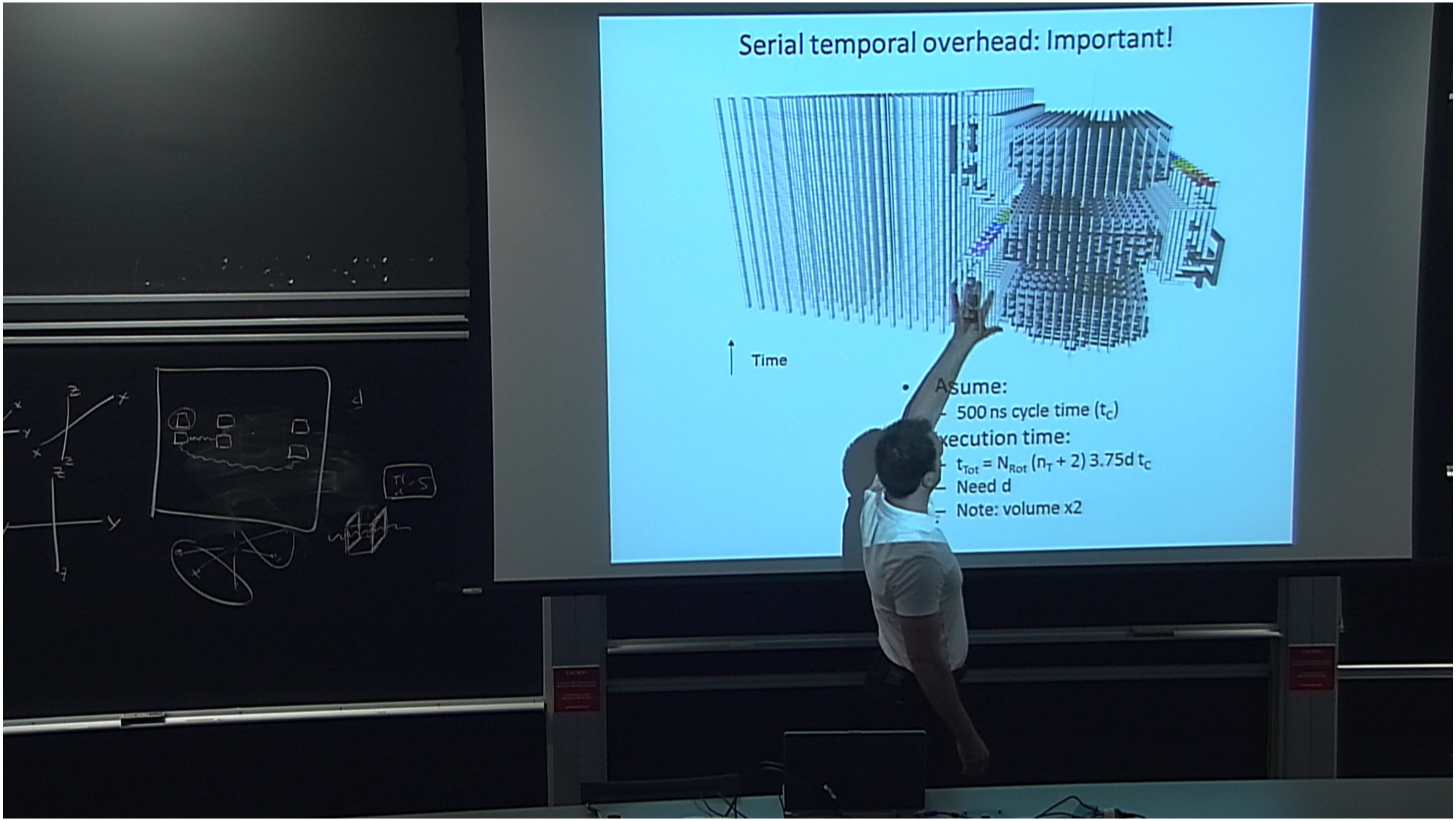
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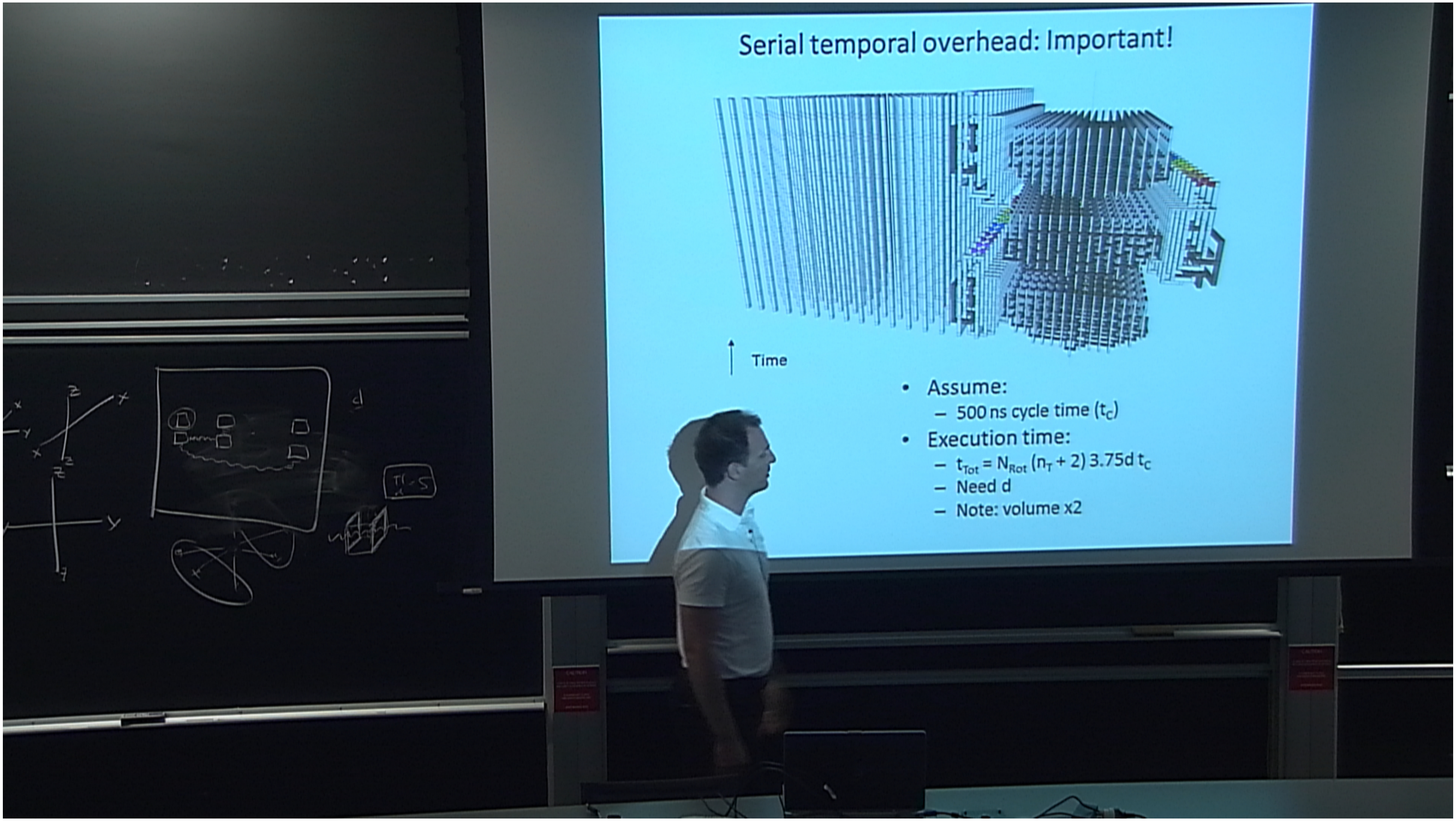
Time
Asume:
500 ns cycle time (t_C)
xecution time:
$t_{Tot} = N_{Rot} (n_T + 2)\ 3.75d\ t_C$
Need d
Note: volume x2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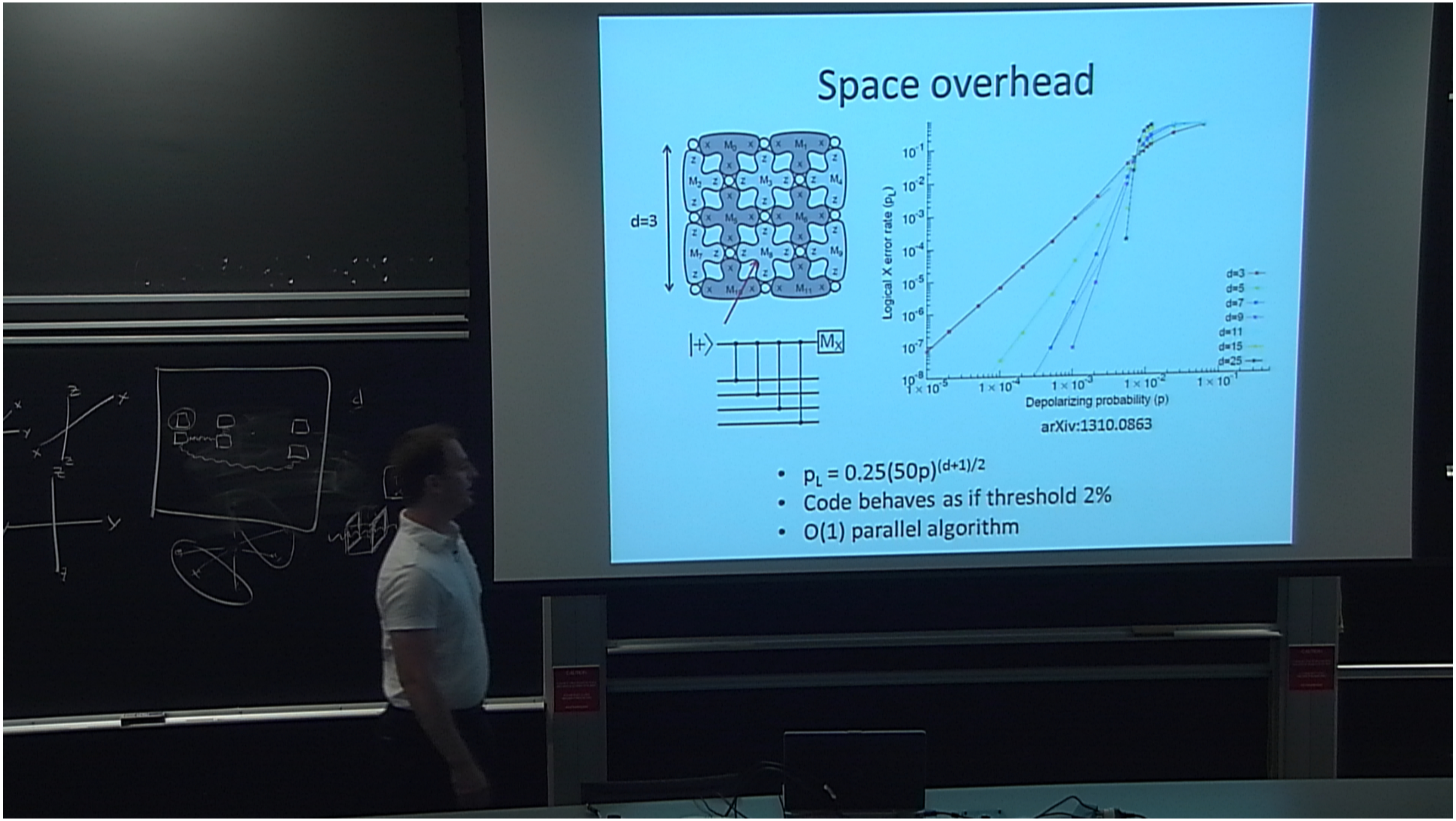
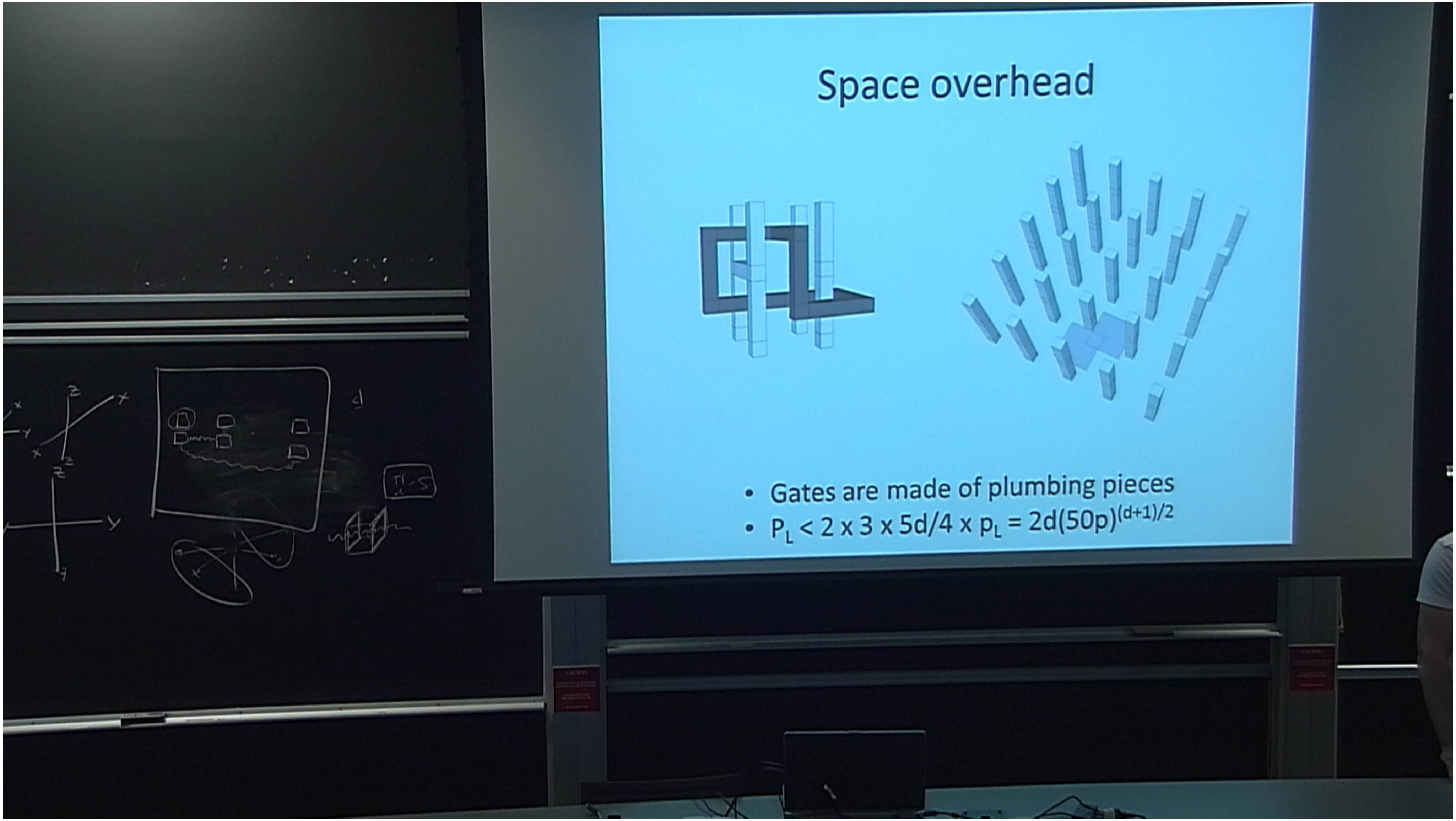
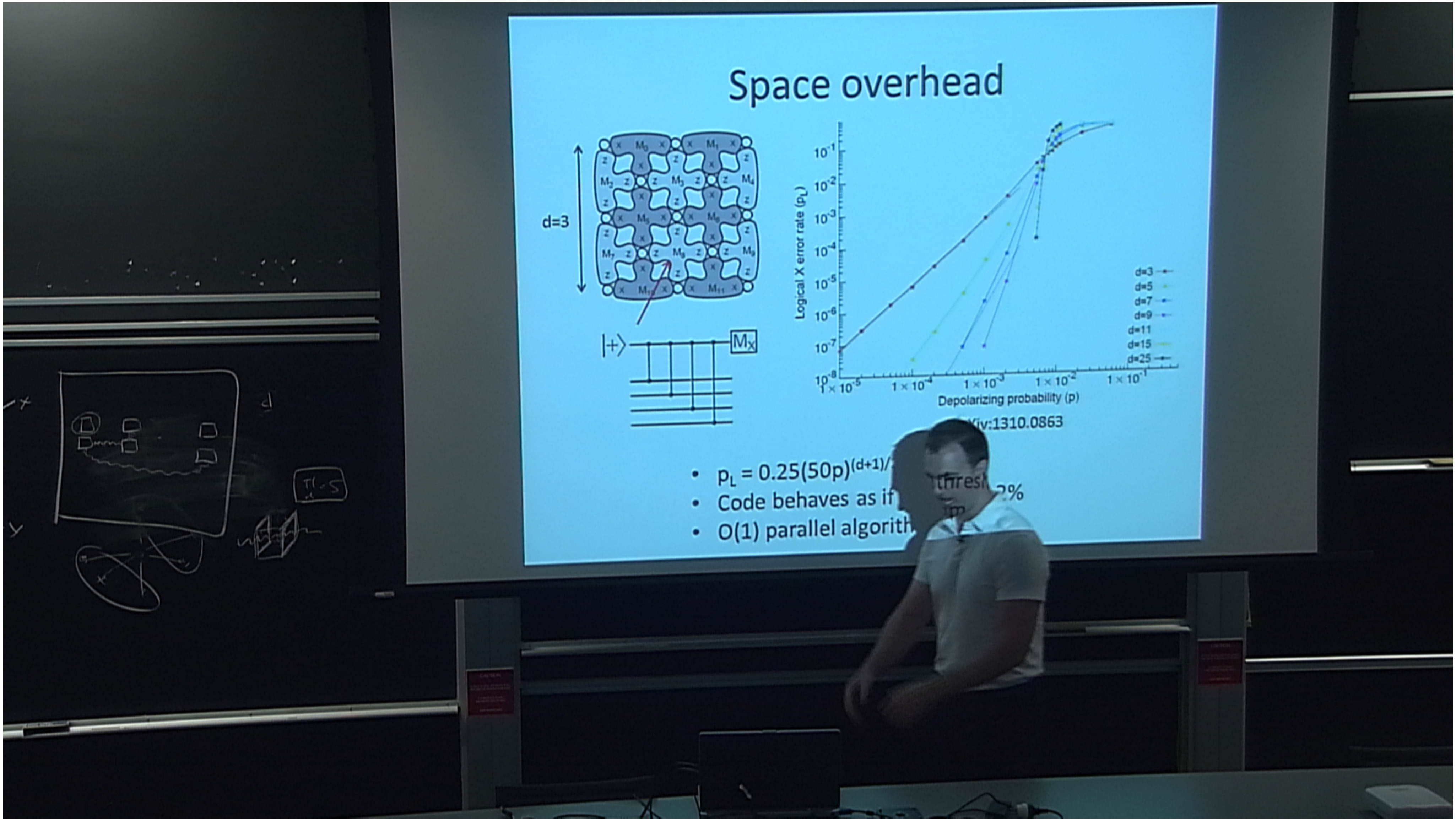


Space overhead
d=3
Logical X error rate (p_L)
Depolarizing probability (p)
d=3
d=5
d=7
d=9
d=11
d=15
d=25
arXiv:1310.0863
• p_L = 0.25(50p)^{(d+1)/2}
• Code behaves as if threshold 2%
• O(1) parallel algorithm



Space overhead
• Gates are made of plumbing pieces
• $P_L < 2 \times 3 \times 5d/4 \times p_L = 2d(50p)^{(d+1)/2}$



Space overhead
d=3
M0
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
M11
|+⟩
Mx
Logical X error rate (pL)
Depolarizing probability (p)
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iv:1310.0863
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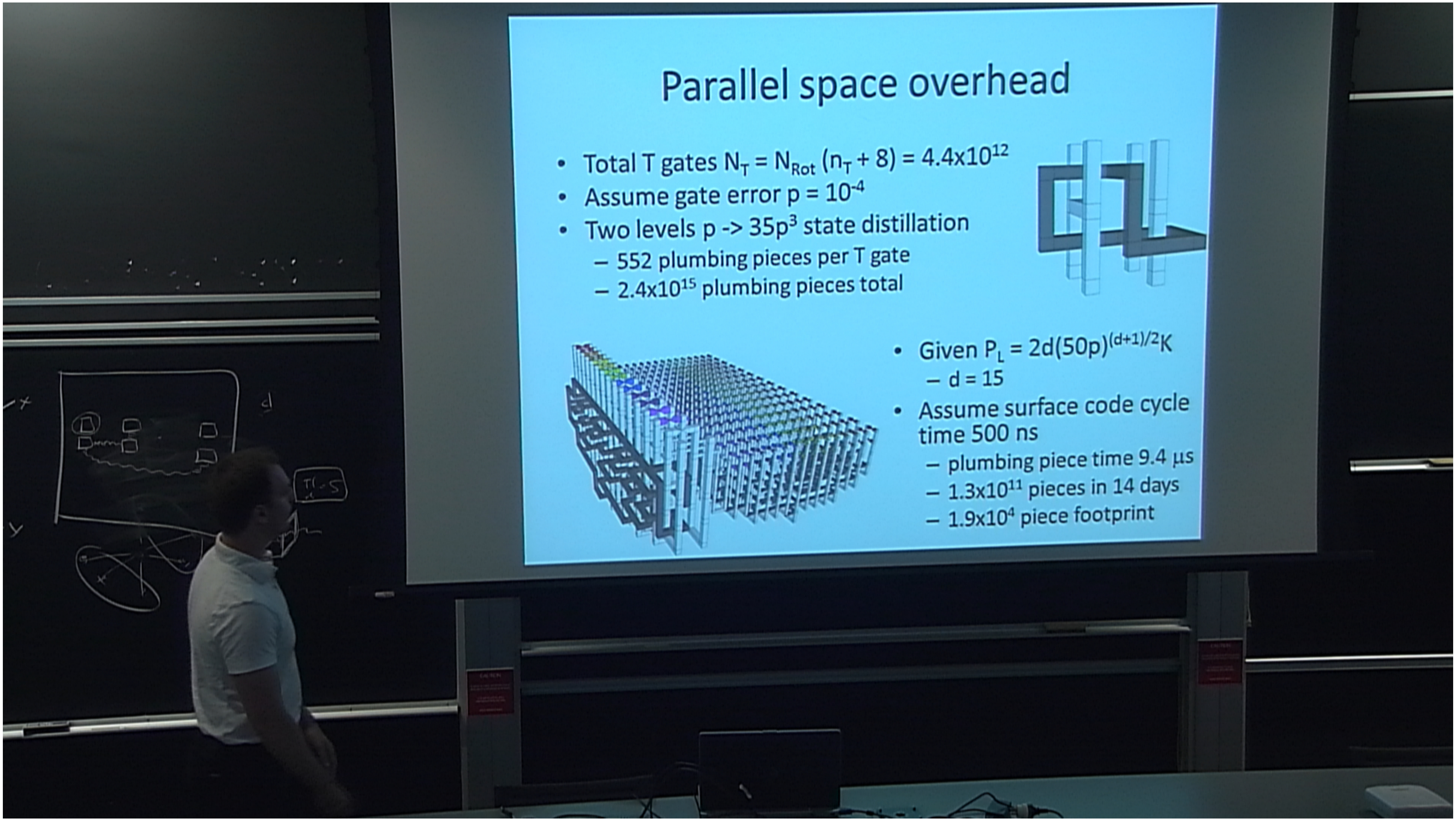
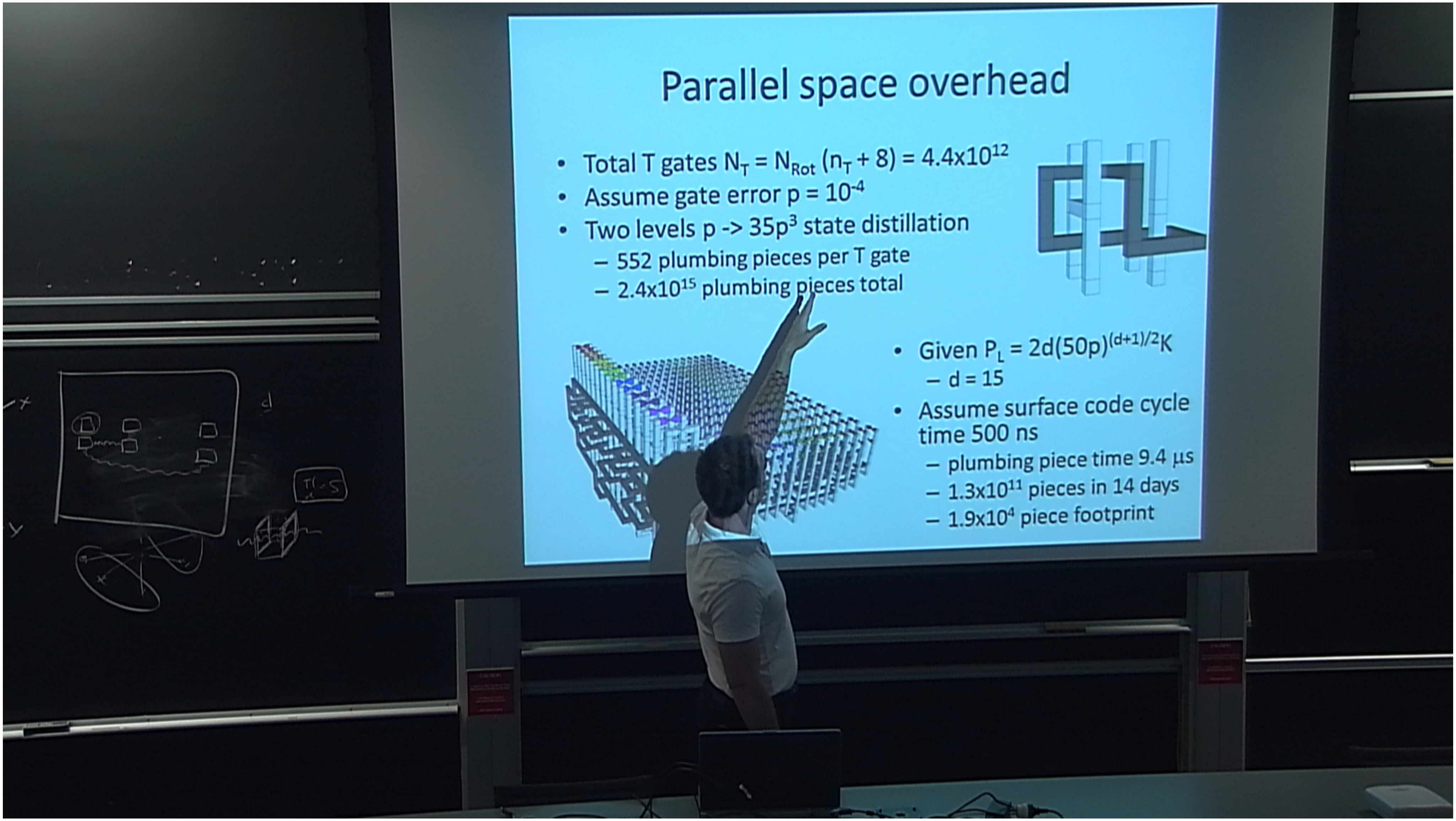


Parallel space overhead
• Total T gates $N_T = N_{Rot} (n_T + 8) = 4.4x10^{12}$
• Assume gate error $p = 10^{-4}$
• Two levels $p \rightarrow 35p^3$ state distillation
– 552 plumbing pieces per T gate
– $2.4x10^{15}$ plumbing pieces total
• Given $P_L = 2d(50p)^{(d+1)/2}K$
– d = 15
• Assume surface code cycle time 500 ns
– plumbing piece time 9.4 μs
– $1.3x10^{11}$ pieces in 14 days
– $1.9x10^4$ piece footprint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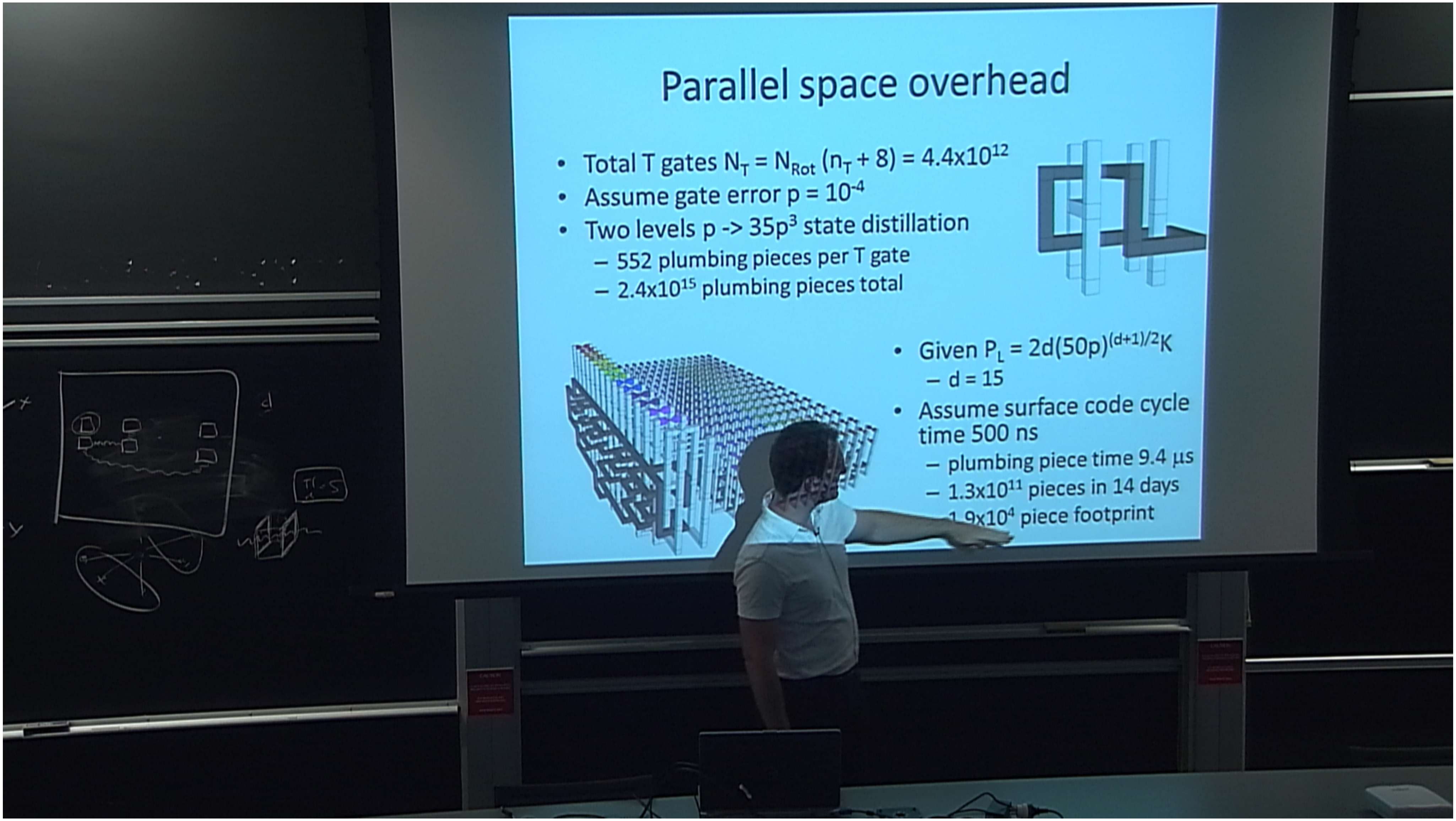
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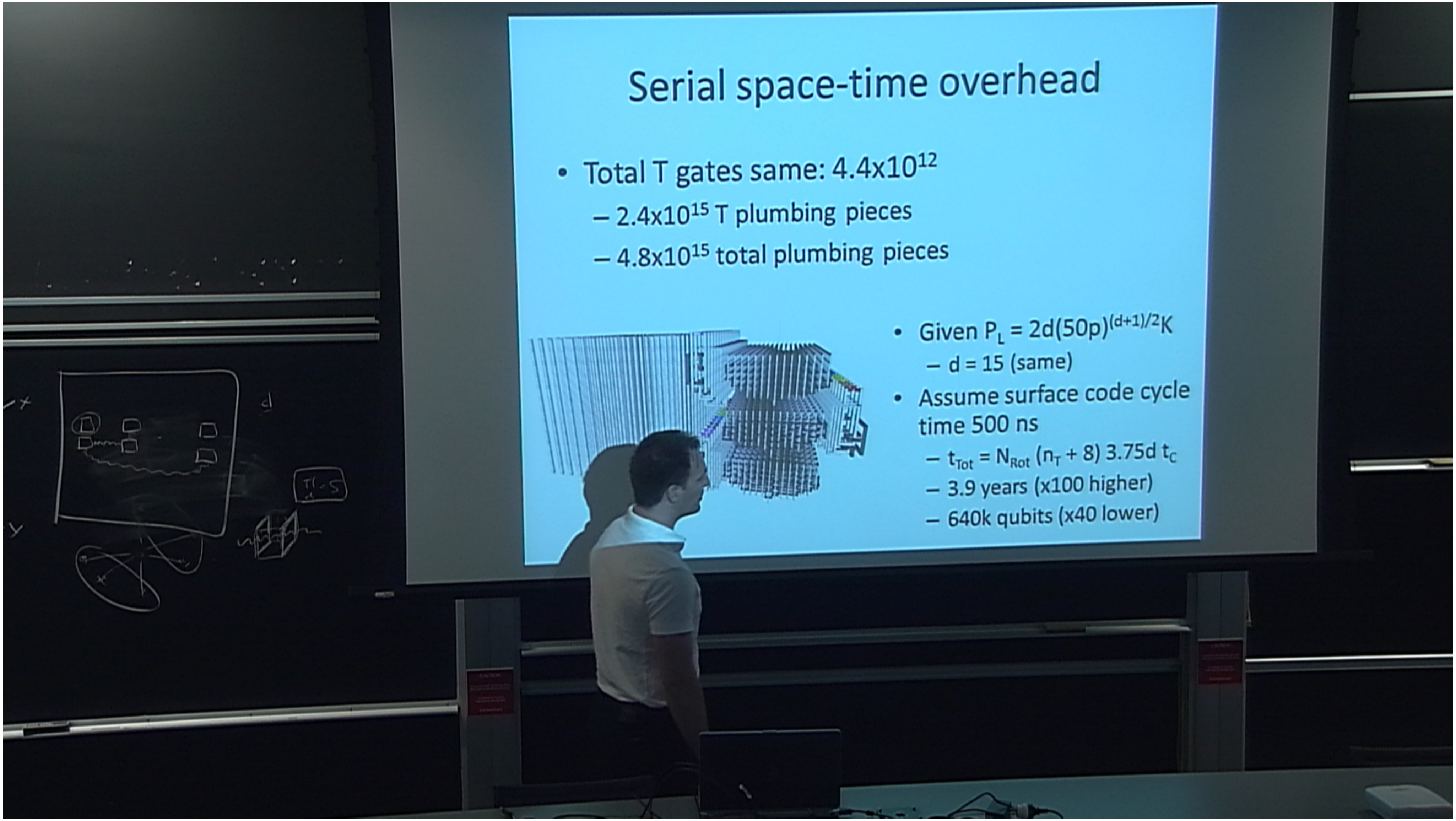
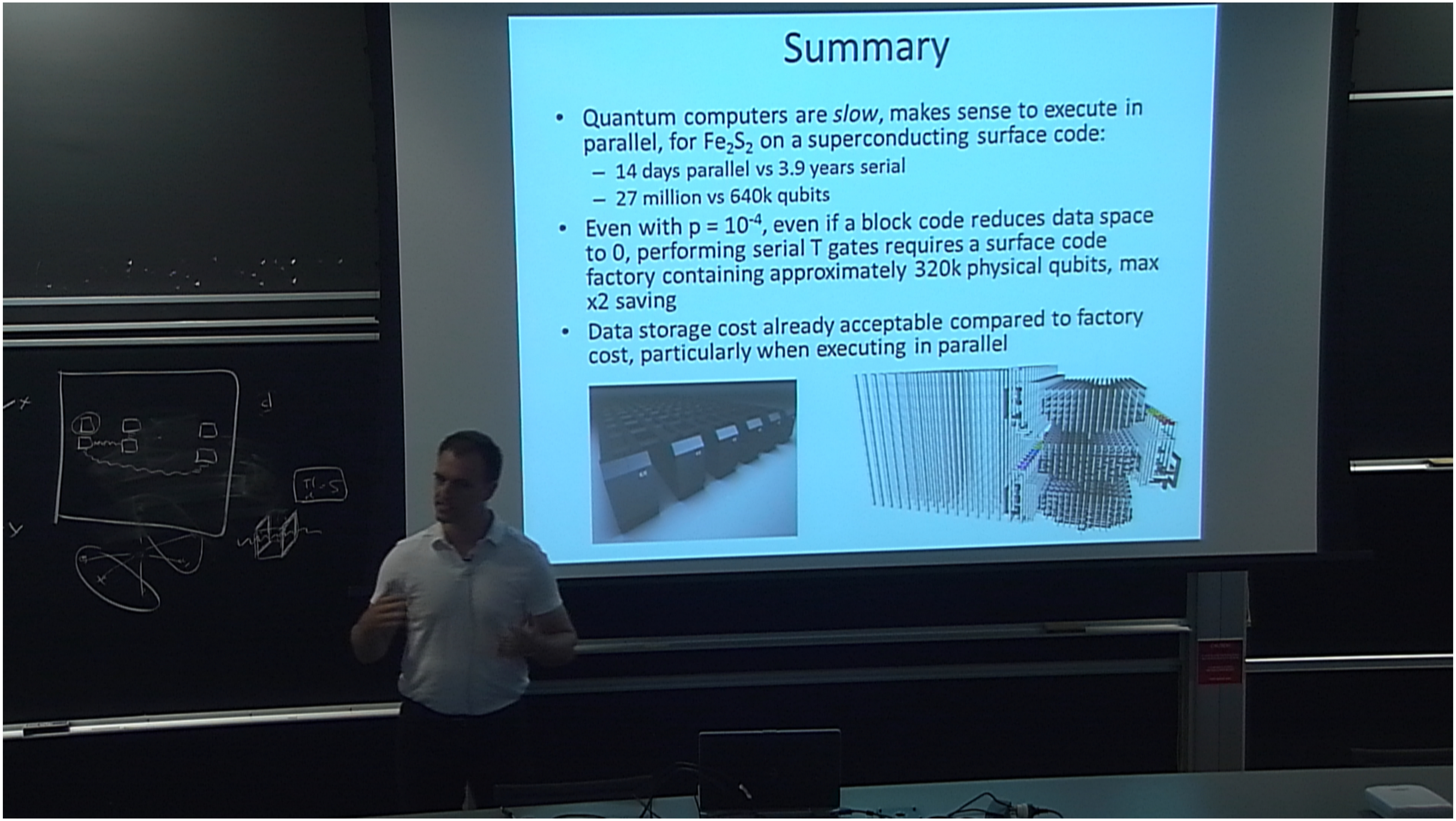
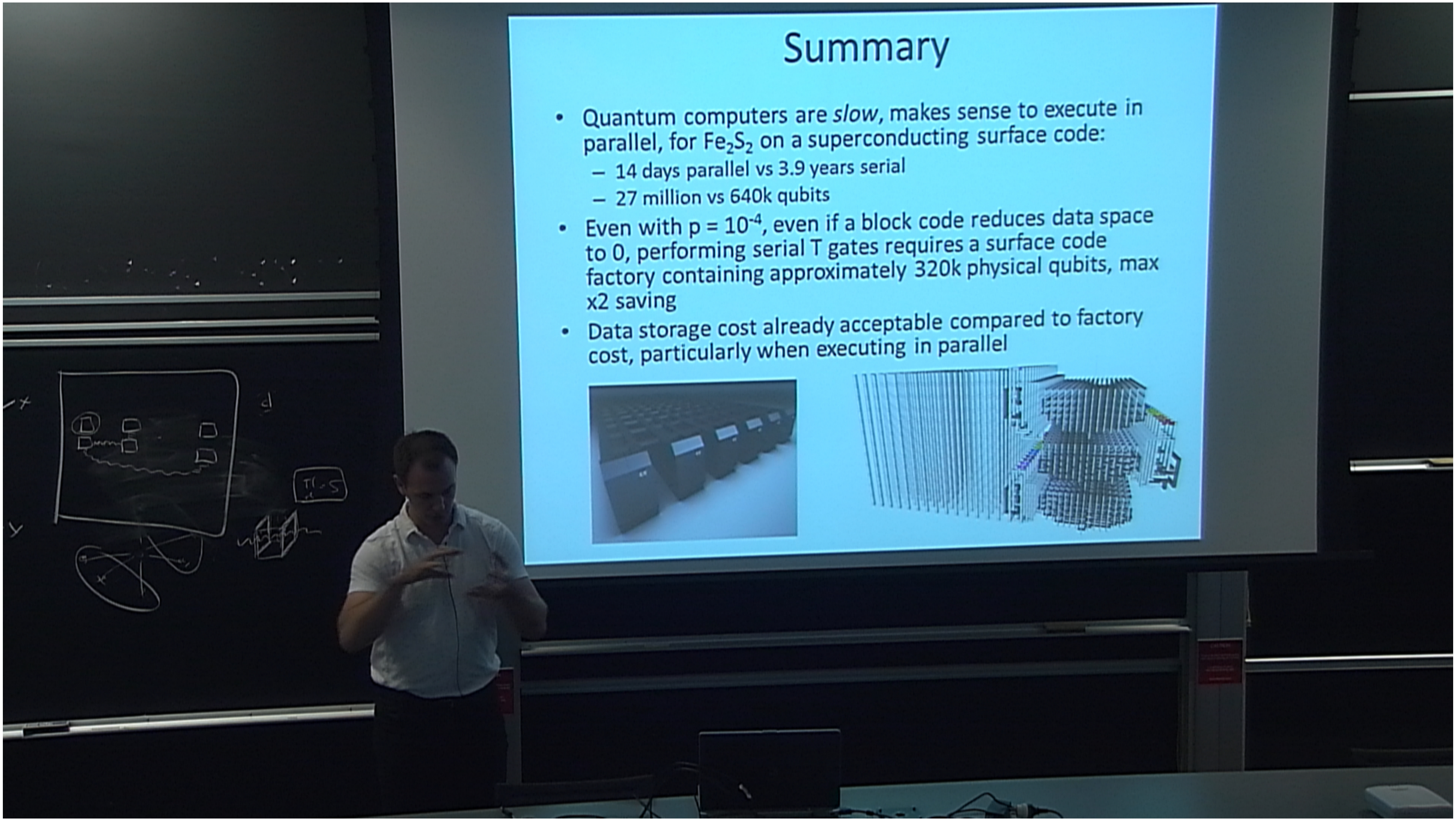

Serial space-time overhead
Total T gates same: 4.4x10^12
– 2.4x10^15 T plumbing pieces
– 4.8x10^15 total plumbing pieces
Given P_L = 2d(50p)^((d+1)/2)K
– d = 15 (same)
Assume surface code cycle time 500 ns
– t_Tot = N_Rot (n_T + 8) 3.75d t_C
– 3.9 years (x100 higher)
– 640k qubits (x40 lower)


Summary
Quantum computers are *slow*, makes sense to execute in parallel, for Fe_2S_2 on a superconducting surface code:
– 14 days parallel vs 3.9 years serial
– 27 million vs 640k qubits
Even with $p = 10^{-4}$, even if a block code reduces data space to 0, performing serial T gates requires a surface code factory containing approximately 320k physical qubits, max x2 saving
Data storage cost already acceptable compared to factory cost, particularly when executing in parallel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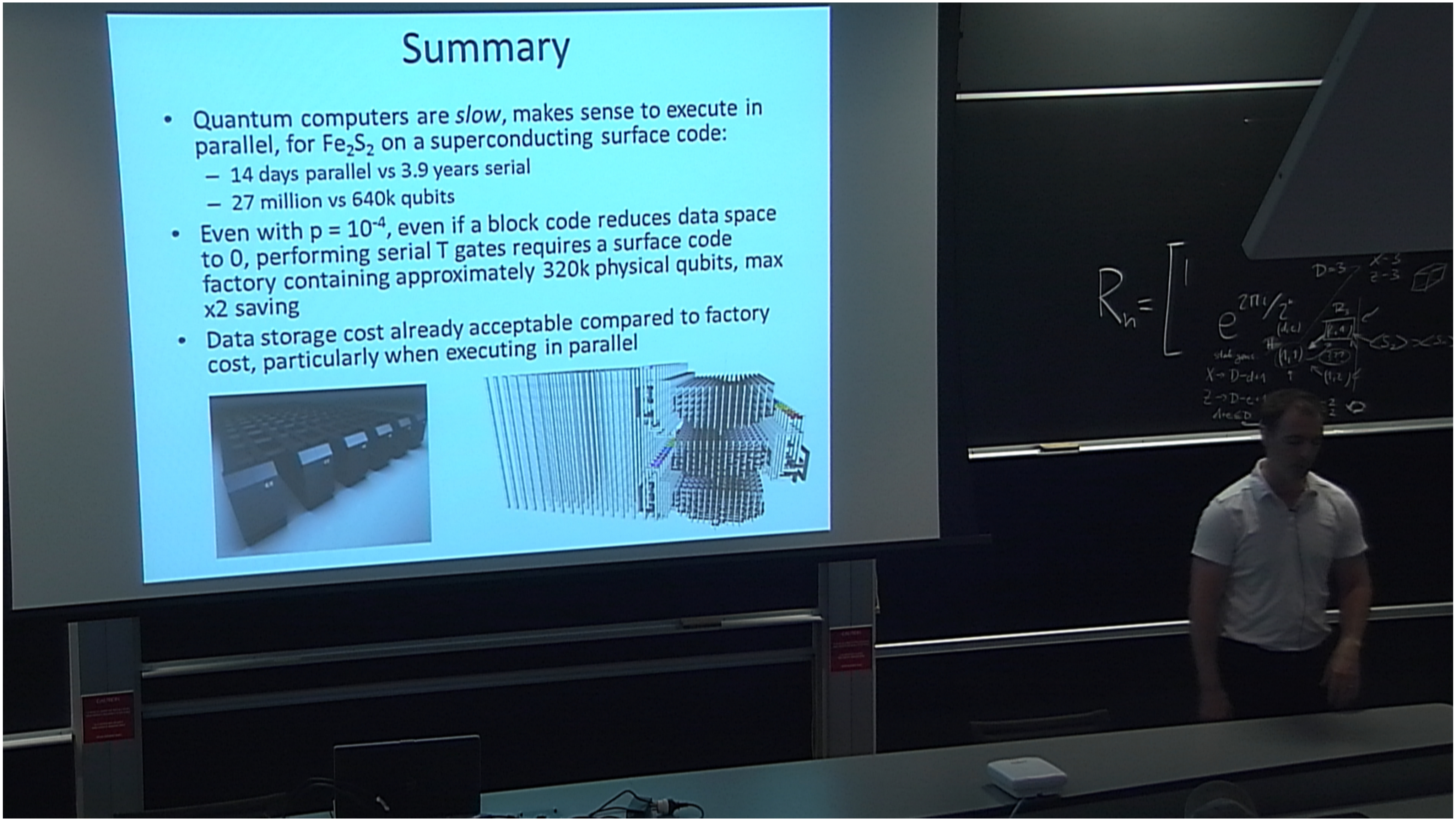
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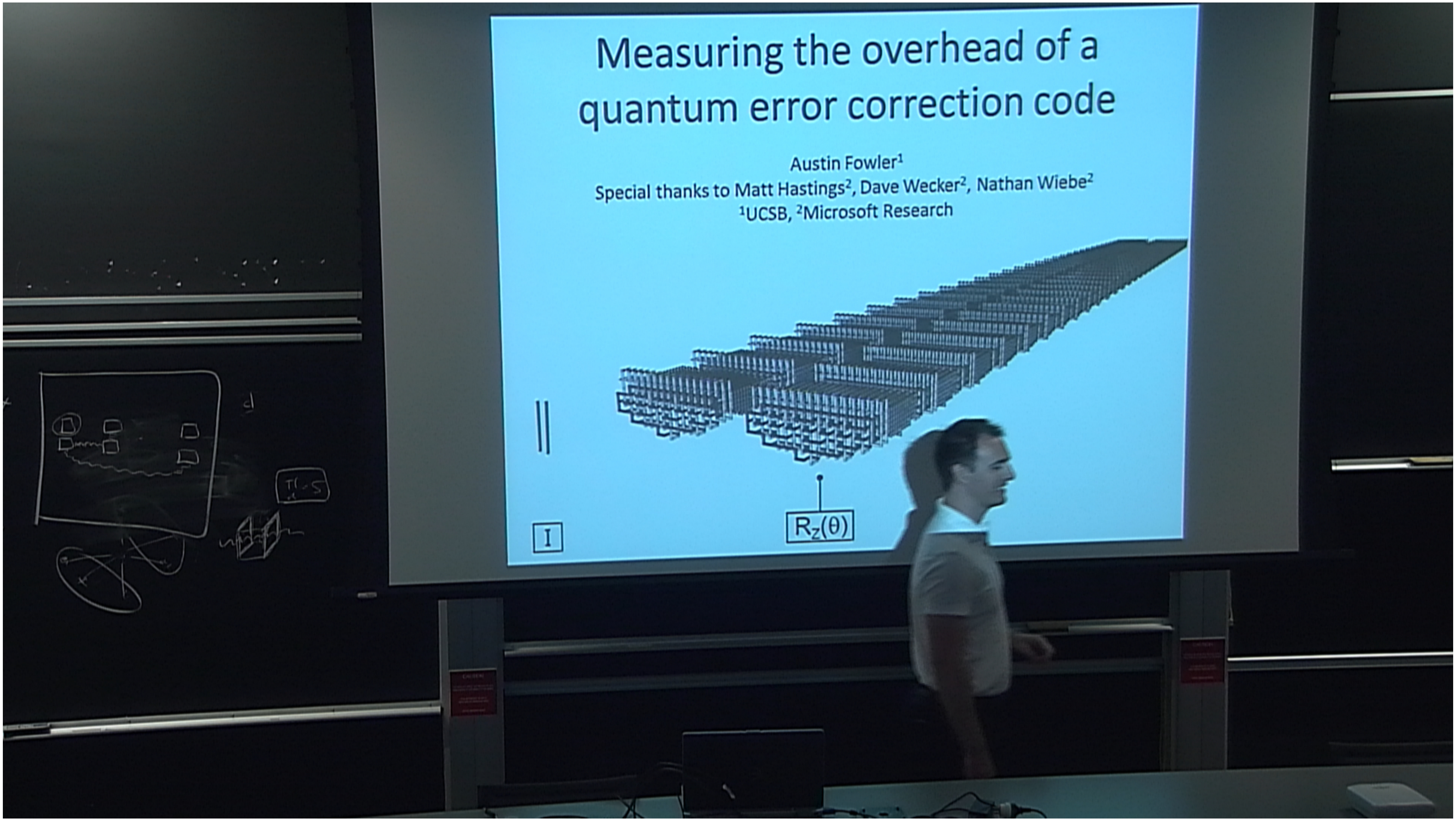
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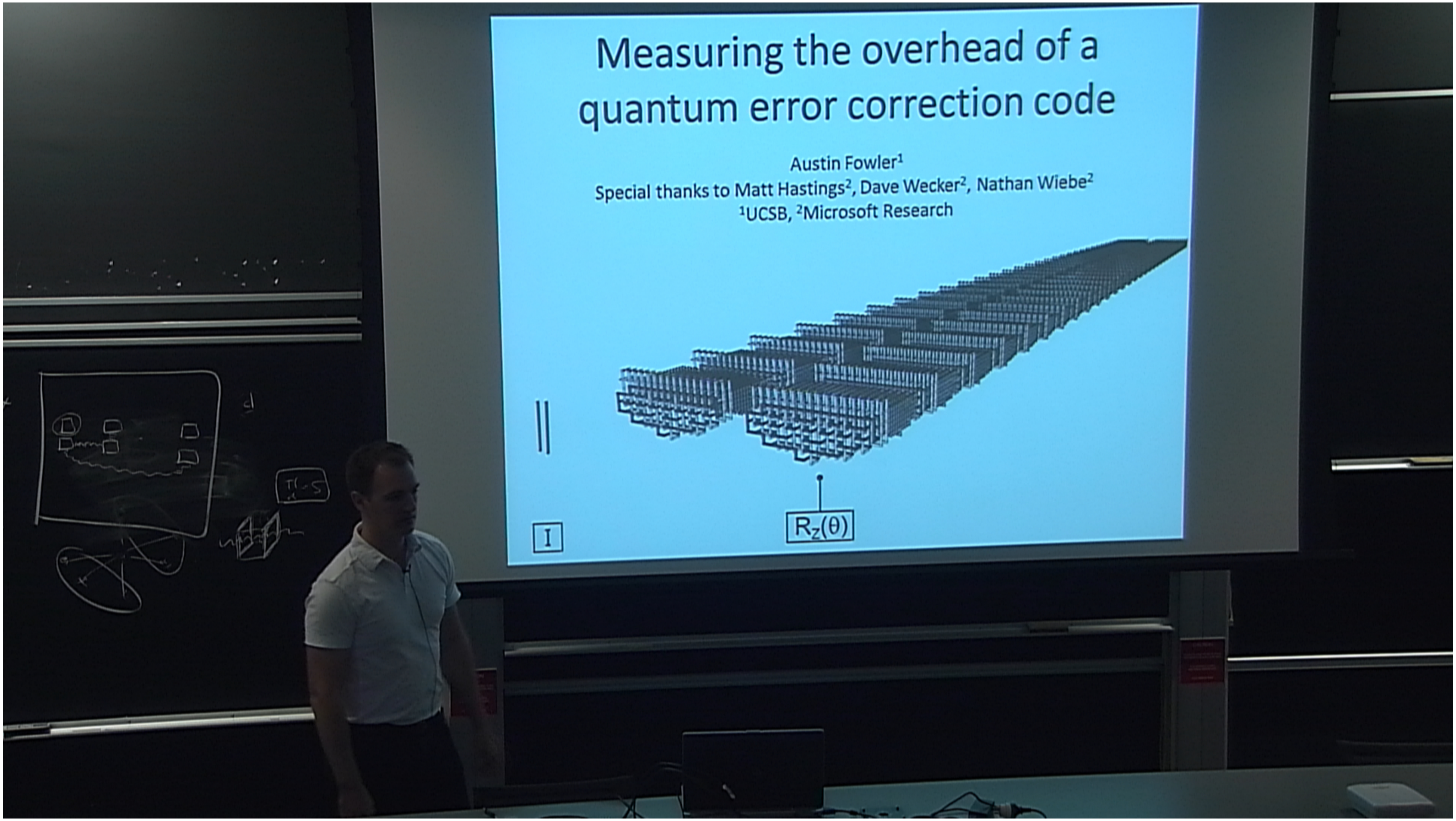
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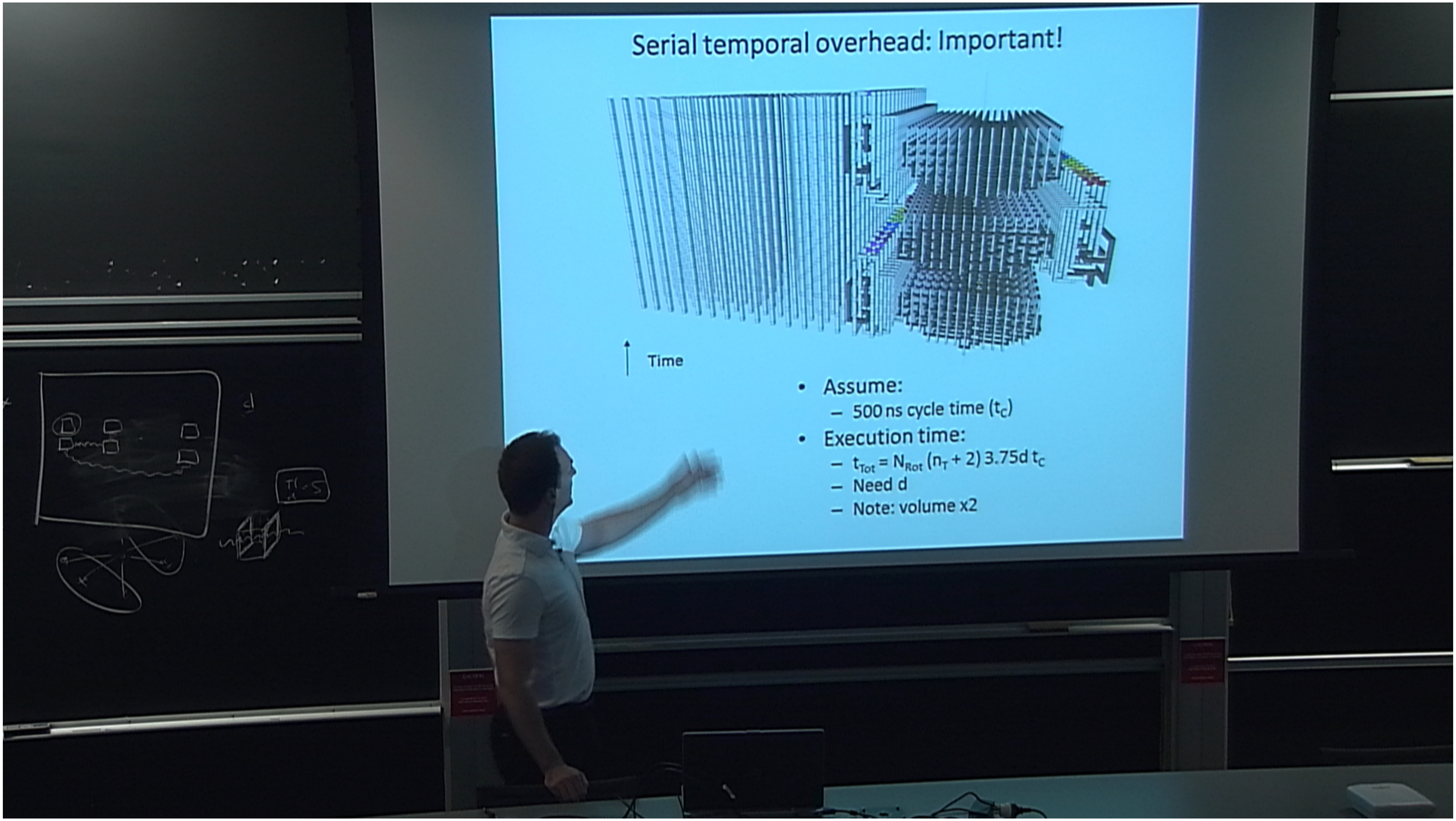
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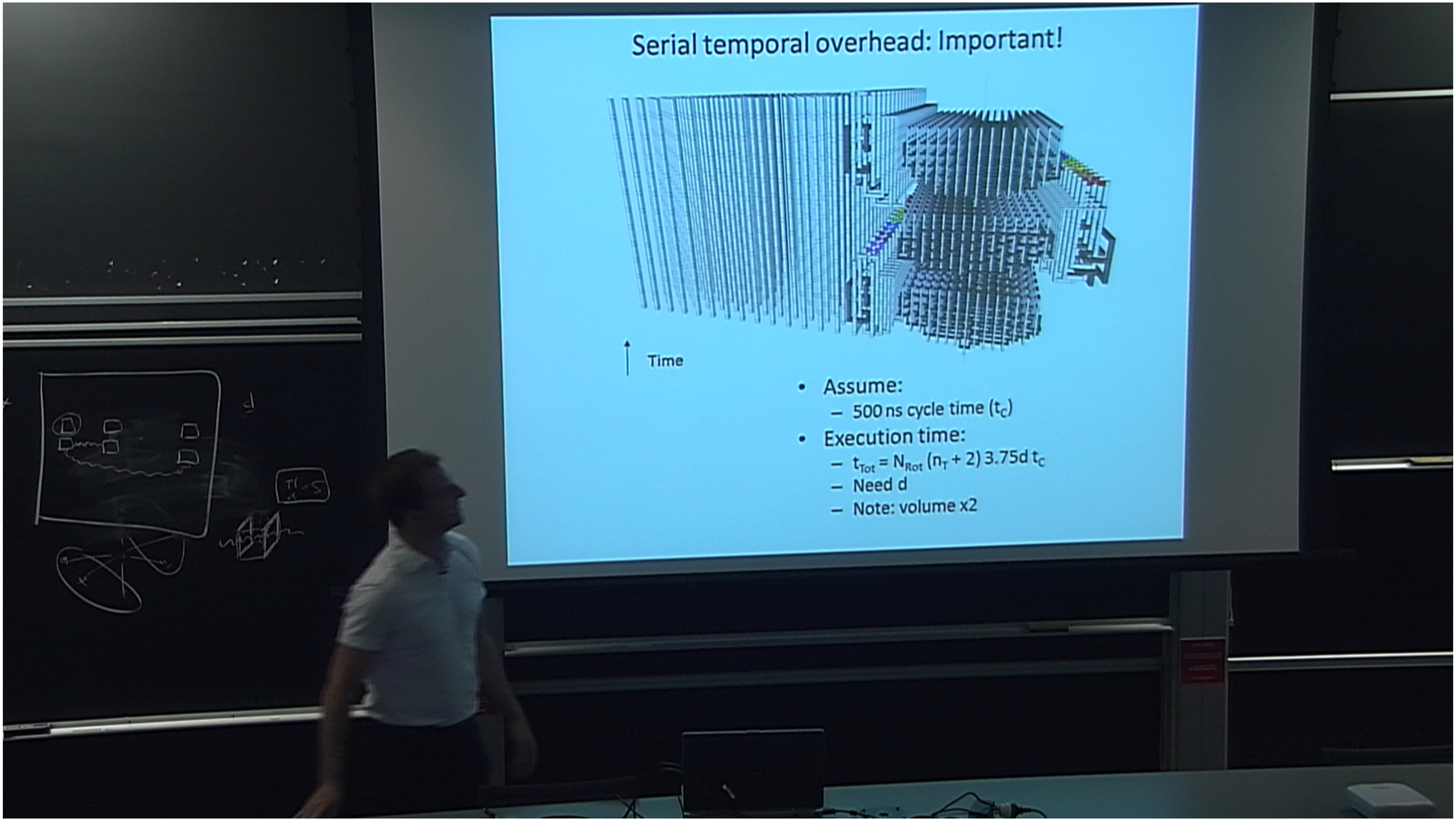
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