Title: Gauge color codes

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Abstract: I will describe a new class of topological quantum error correcting codes with surprising features. The constructions is based on color codes: it preserves their unusual transversality properties but removes important drawbacks. In 3D, the new codes allow the effectively transversal implementation of a universal set of gates by gauge fixing, while error-dectecting measurements involve only 4 or 6 qubits. Furthermore, they do not require multiple rounds of error detection to achieve fault-tolerance.


Outline
Introduction
Transversal gates in color codes
Gauge color codes
Single-shot fault-tolerant error correction
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Outline

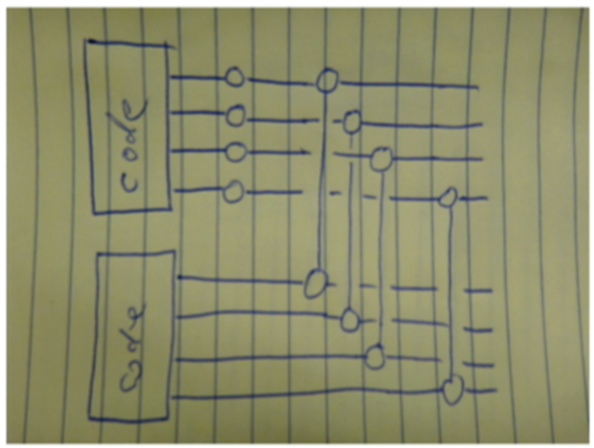
- Introduction
- Transversal gates in color codes
- Gauge color codes
- Single-shot fault-tolerant error correction

Local codes

Local computation

Transversal gates

- Aim: do not spread errors
- How: act on subsystems separately



Transversal gates

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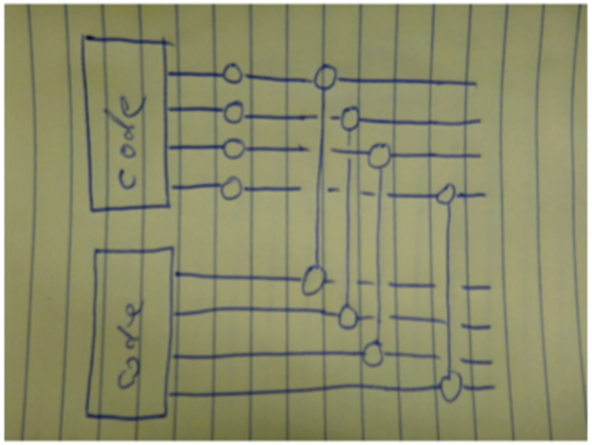
- How: act on subsystems separately



Transversal gates

No code admits a universal transversal set of gates

Eastin & Knill '09

Alternatives:

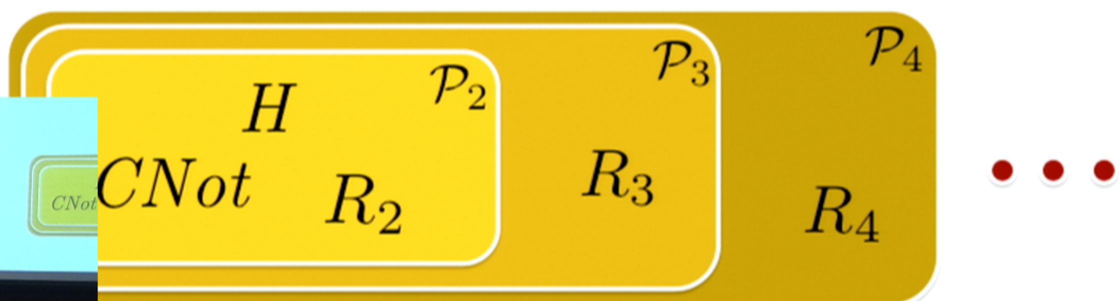
- Magic state distillation Bravyi & Kitaev '03
- Gauge fixing Paetznick & Reichardt '13
- Concatenation Jochym-O'Connor & Laflamme '14

Dimensional restrictions

Bravyi & Koenig '13

$$\mathcal{P}_D := \{U \,|\, U\mathcal{P}U^\dagger \subseteq \mathcal{P}_{D-1}\}, \qquad \mathcal{P}_1 := \mathcal{P}$$

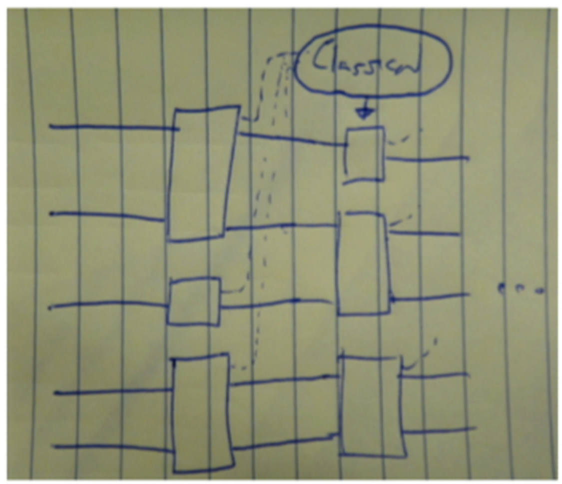
Gottesman & Chuang '99



$$R_D := \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^D} \end{pmatrix}$$

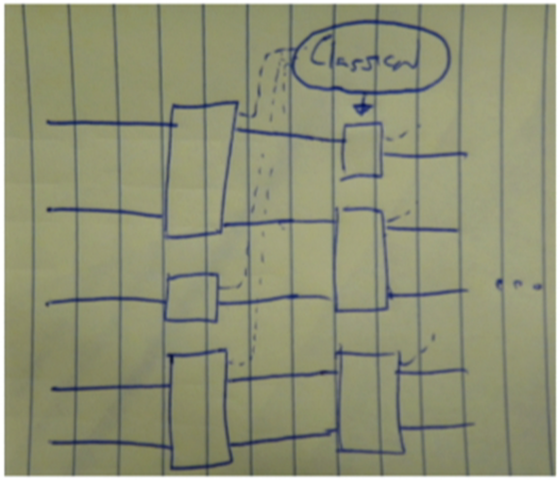
Effectively local operations

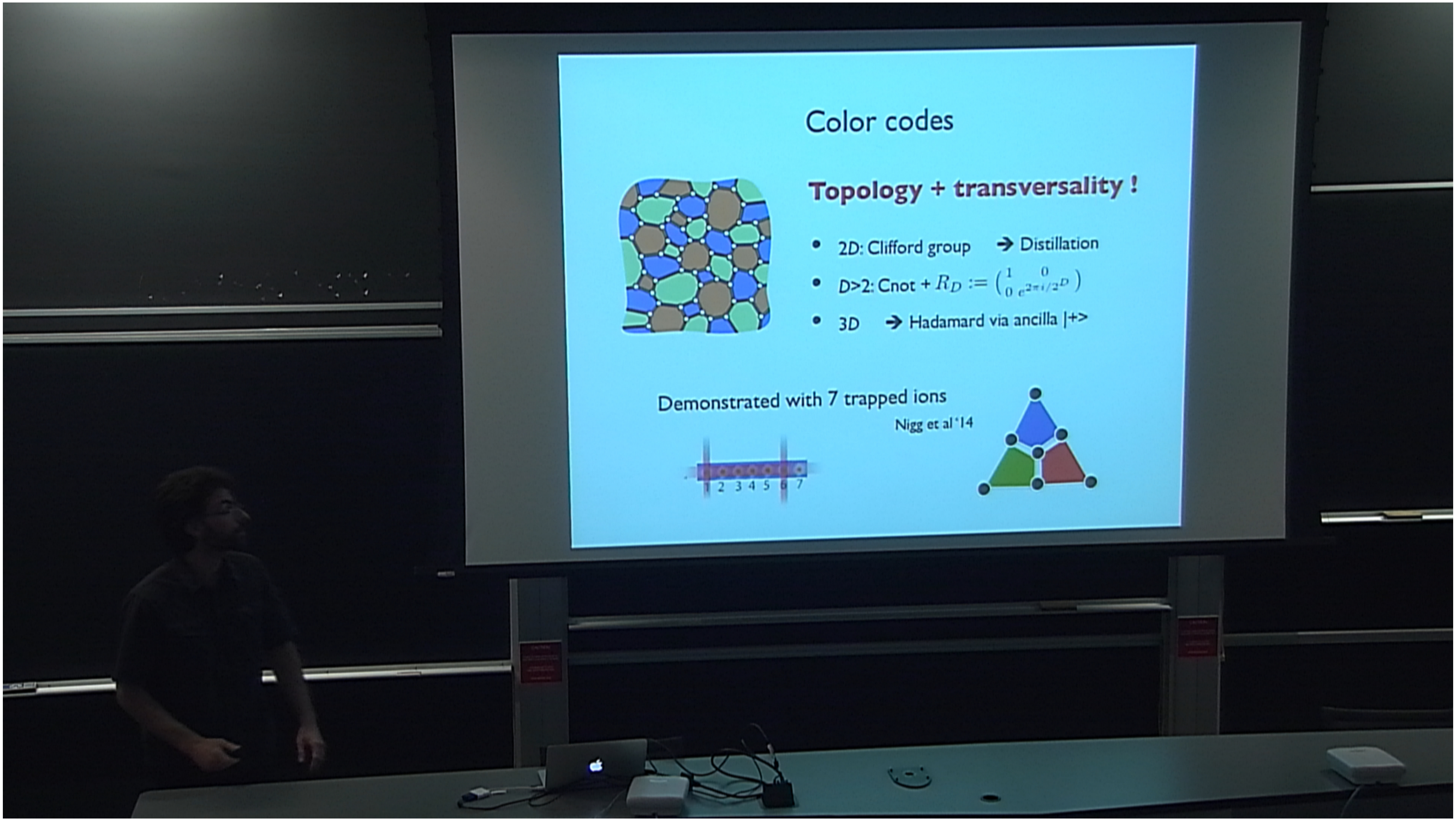
- Local quantum op + global classical computation
- Caution: can yield non-local errors!

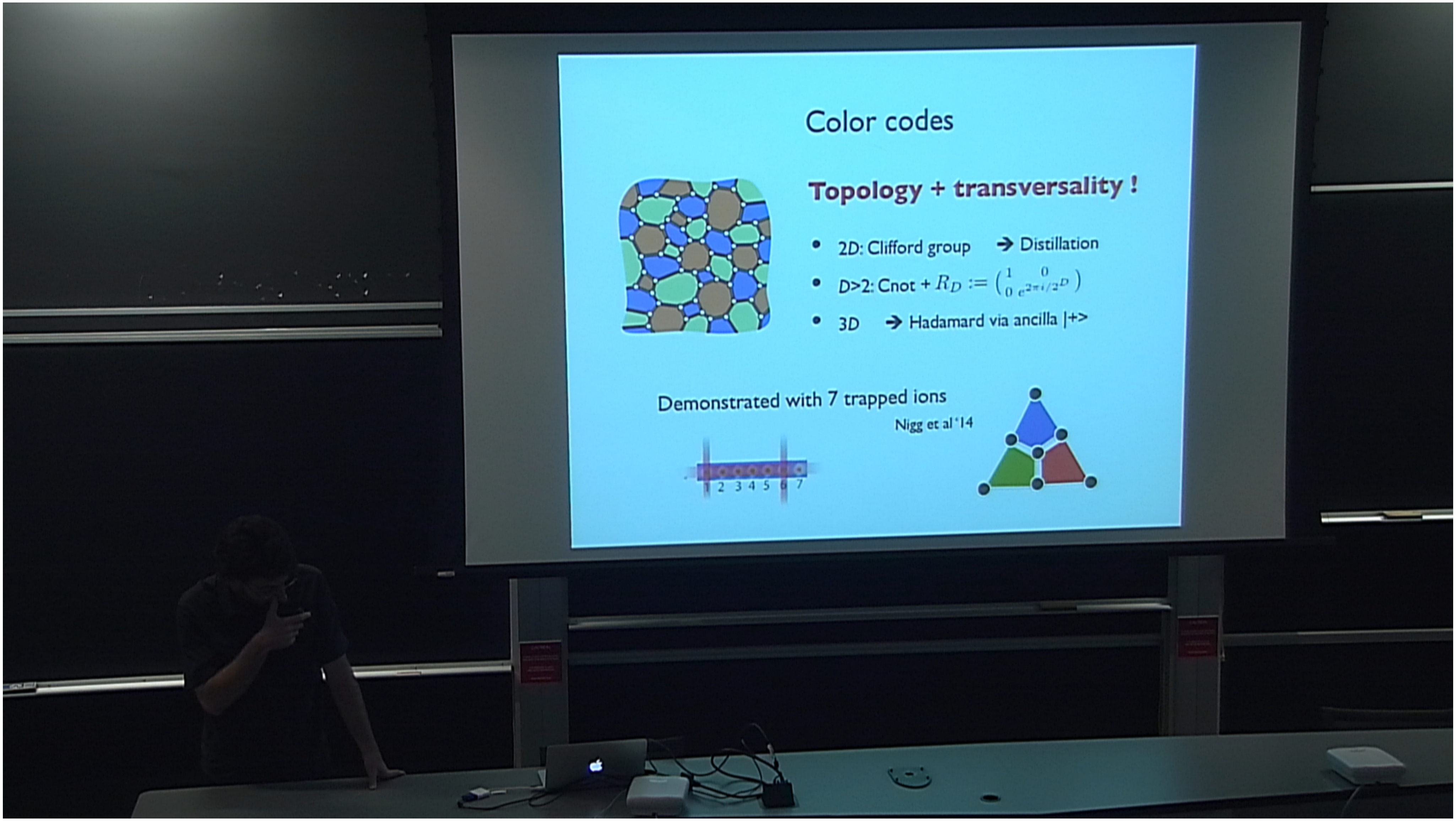


Effectively local operations

- Defines a class of codes with error correction/ initialization local in this sense.
- Contains abelian anyons, but not non-abelian ones!




Color codes
Topology + transversality !
2D: Clifford group → Distillation
D>2: Cnot + $R_D := \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^D} \end{pmatrix}$
3D → Hadamard via ancilla |+>
Demonstrated with 7 trapped ions
Nigg et al '14
1 2 3 4 5 6 7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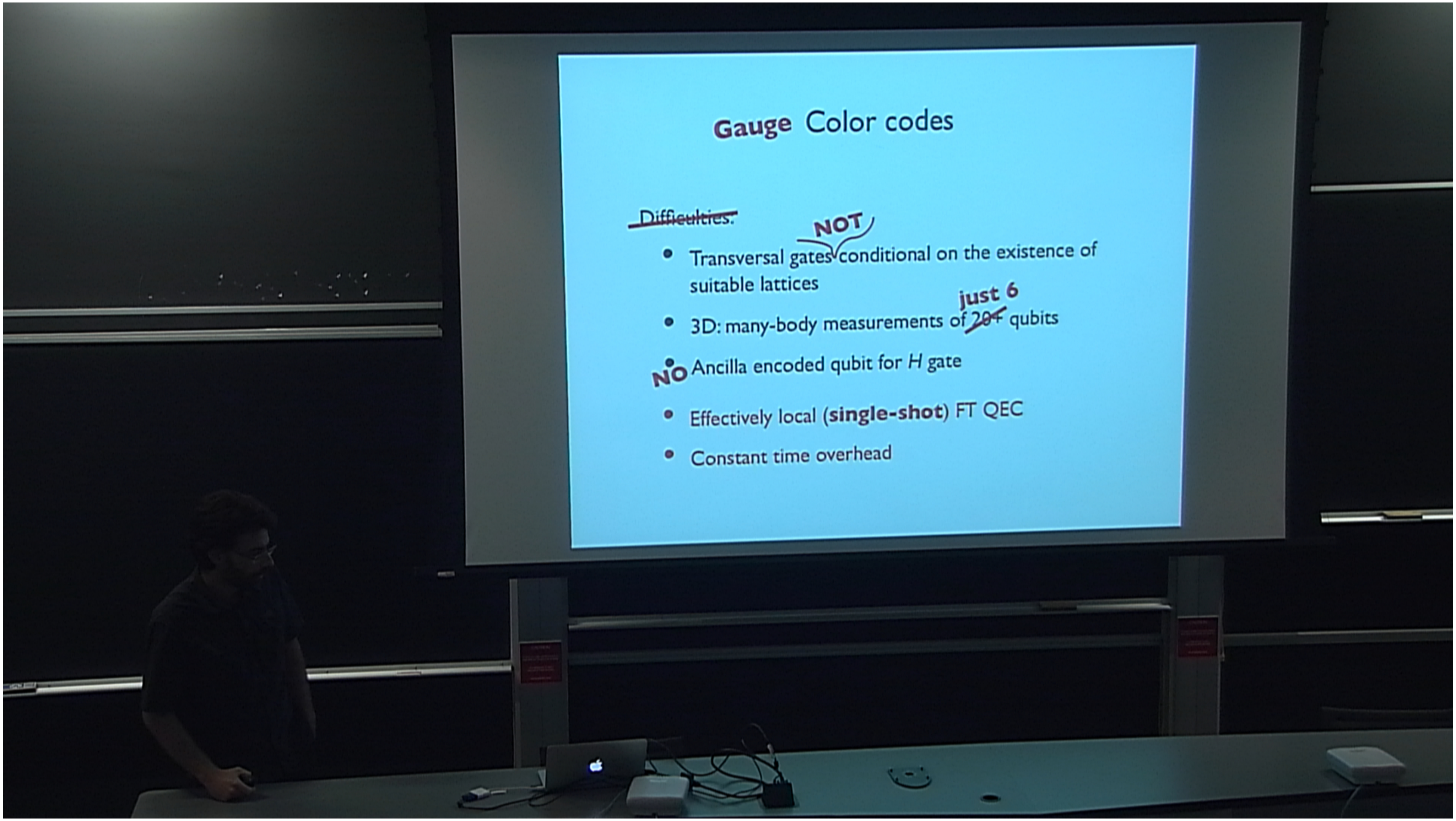
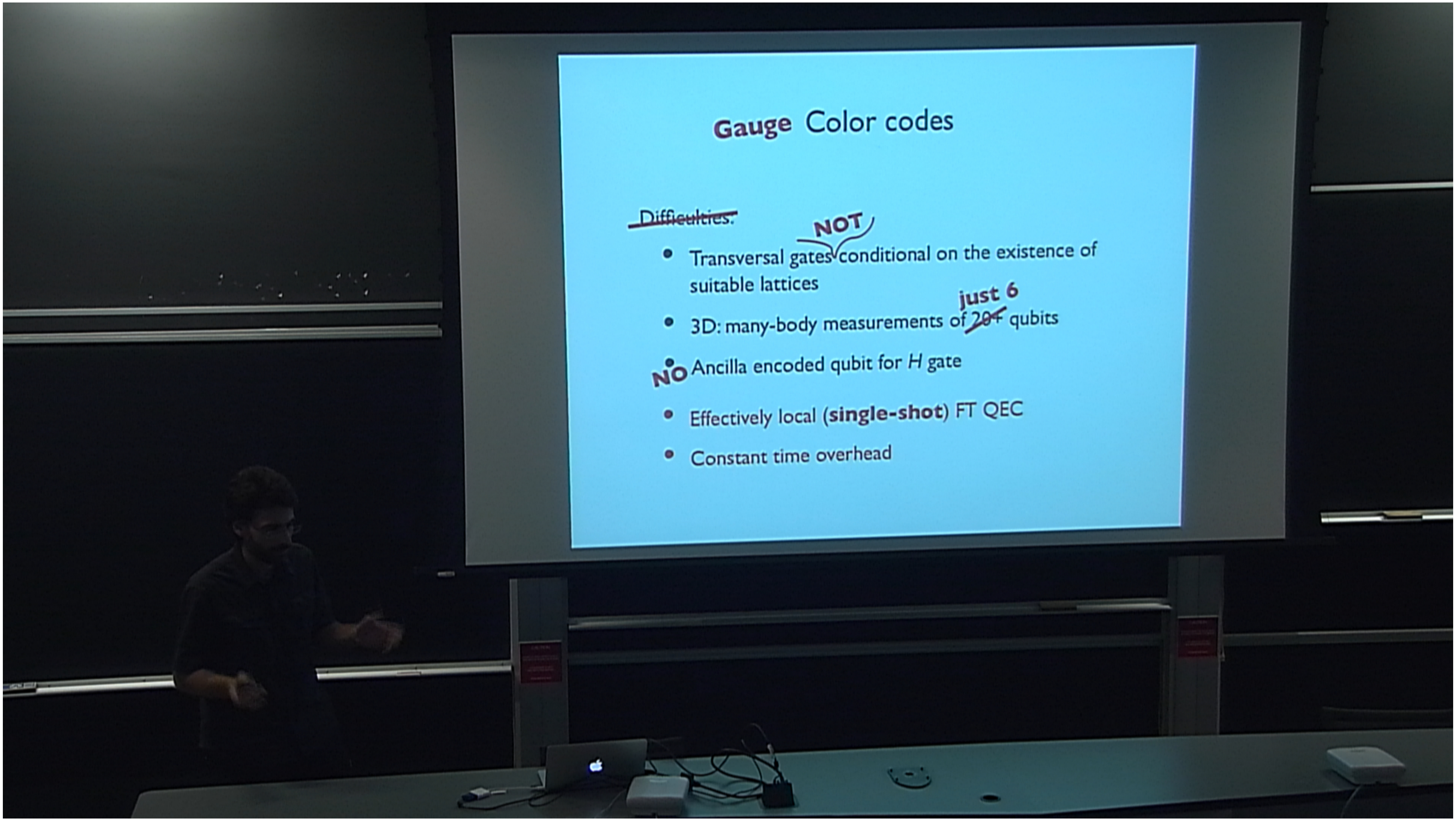
Color codes

Difficulties:

- Transversal gates conditional on the existence of suitable lattices
- 3D: many-body measurements of 20+ qubits
- Ancilla encoded qubit for *H* gate


Gauge Color codes
Difficulties:
NOT
Transversal gates conditional on the existence of suitable lattices
just 6
3D: many-body measurements of 20+ qubits
NO
Ancilla encoded qubit for H gate
Effectively local (single-shot) FT QEC
Constant time overhead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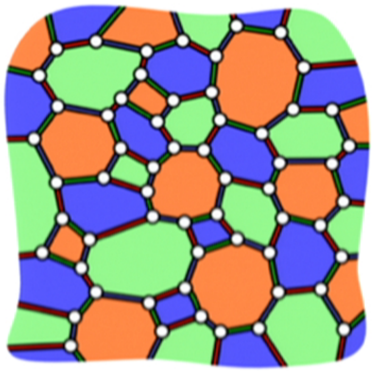
2*D* color codes

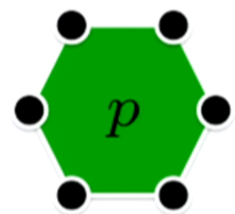


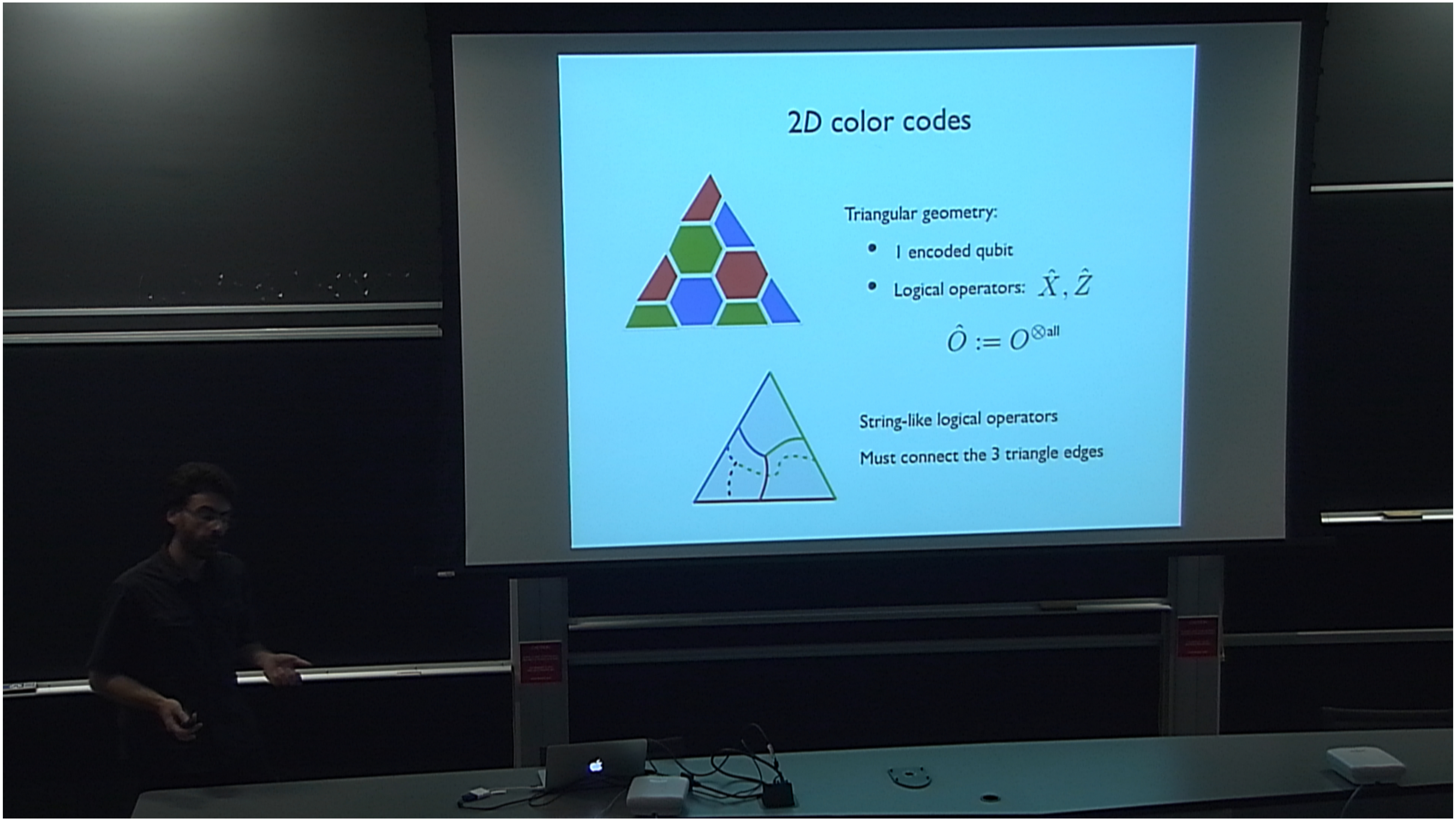
- 2-colexes
- 1 qubit per vertex
- 2 stabilizer generators per plaquette

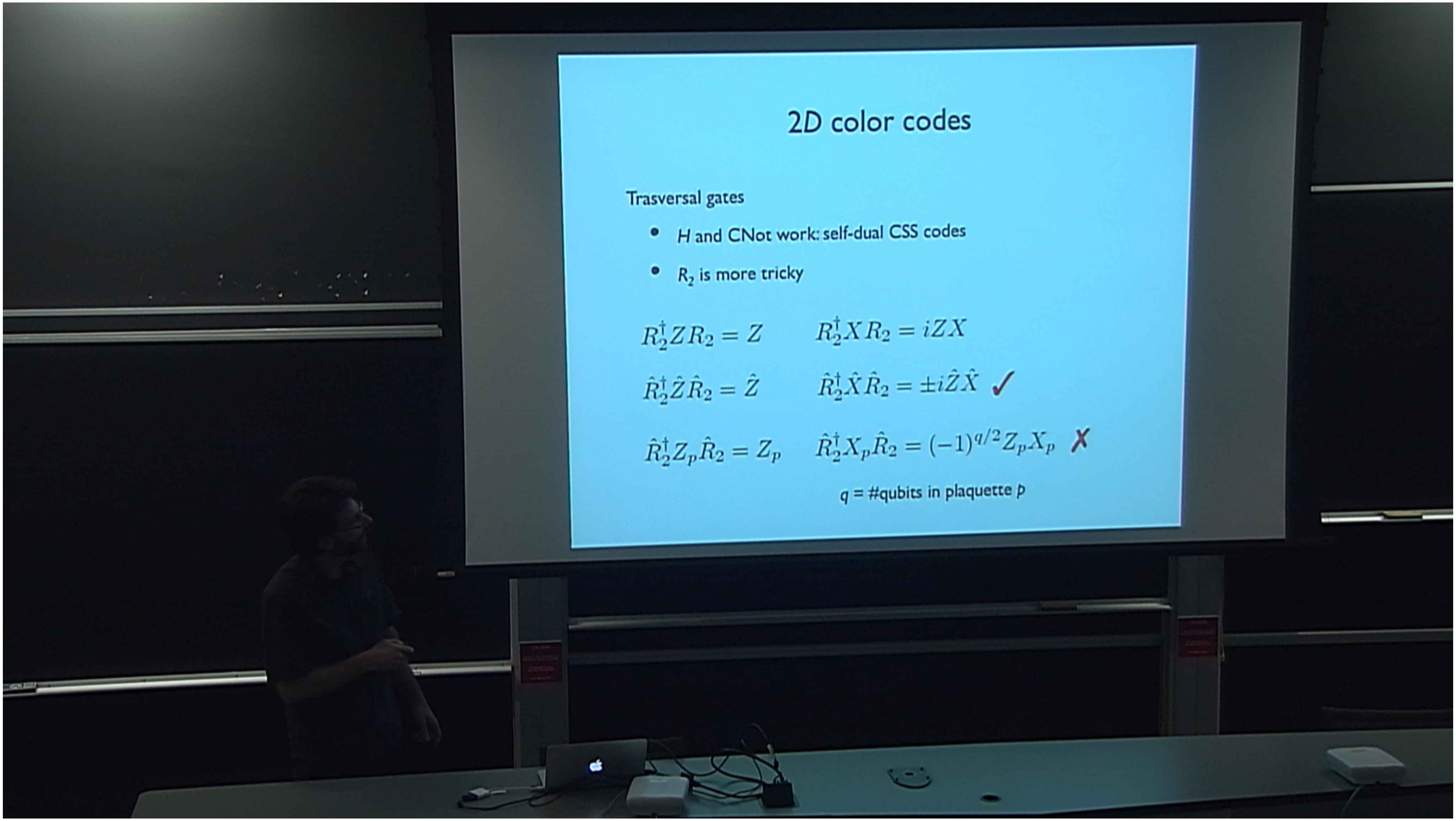
$$\mathcal{S} = \langle X_p, Z_p \rangle_p$$

$$X_p = \bigotimes_{i \in p} X_i$$

$$Z_p = \bigotimes_{i \in p} Z_i$$



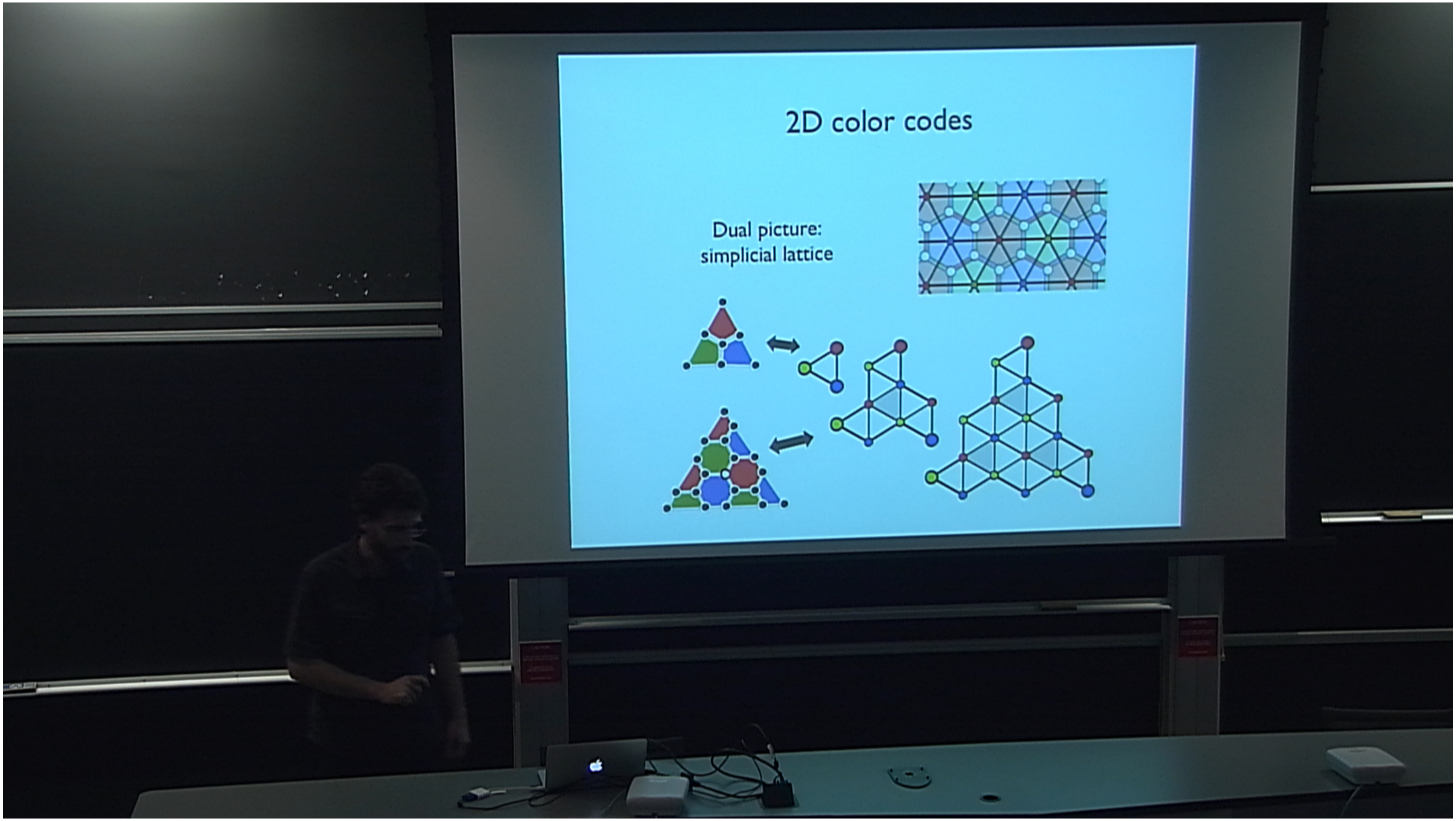

2D color codes
Triangular geometry:
1 encoded qubit
Logical operators: $\hat{X}, \hat{Z}$
$\hat{O} := O^{\otimes \text{all}}$
String-like logical operators
Must connect the 3 triangle edges

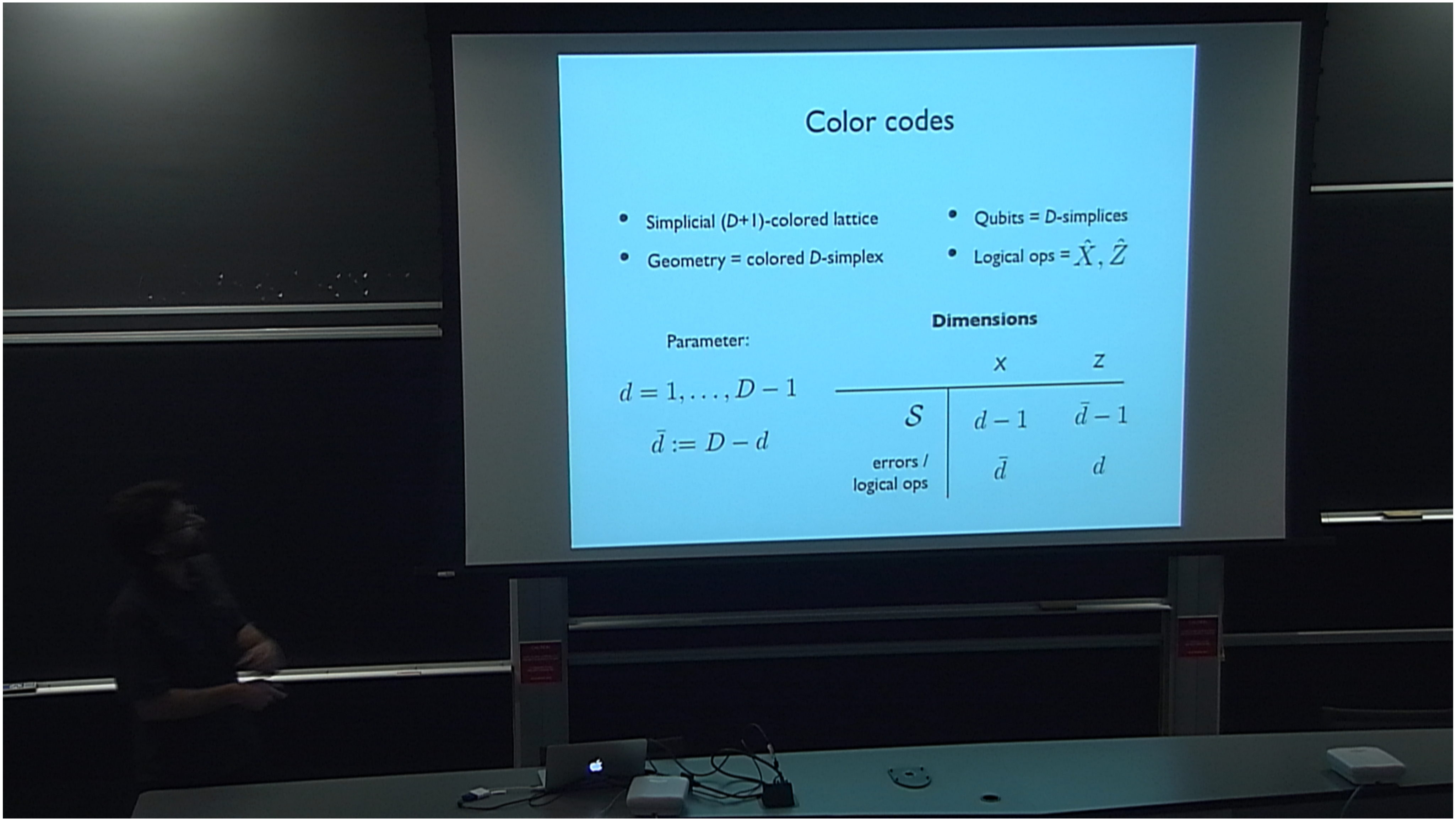

2D color codes
Trasversal gates
H and CNot work: self-dual CSS codes
R_2 is more tricky
$R_2^\dagger Z R_2 = Z$
$R_2^\dagger X R_2 = iZX$
$\hat{R}_2^\dagger \hat{Z} \hat{R}_2 = \hat{Z}$
$\hat{R}_2^\dagger \hat{X} \hat{R}_2 = \pm i \hat{Z} \hat{X}$ ✓
$\hat{R}_2^\dagger Z_p \hat{R}_2 = Z_p$
$\hat{R}_2^\dagger X_p \hat{R}_2 = (-1)^{q/2} Z_p X_p$ ✗
q = #qubits in plaquette p

2D color codes

Instead, inverse gate on some qubits!

R_2 ● $\qquad R_2^\dagger = ZR_2$ ○


2D color codes
Dual picture:
simplicial lattice


Color codes
• Simplicial (D+1)-colored lattice
• Geometry = colored D-simplex
• Qubits = D-simplices
• Logical ops = $\hat{X}, \hat{Z}$
Parameter:
$d = 1, \ldots, D-1$
$\bar{d} := D - d$
Dimensions
x
z
$\mathcal{S}$
$d-1$
$\bar{d}-1$
errors / logical ops
$\bar{d}$
d

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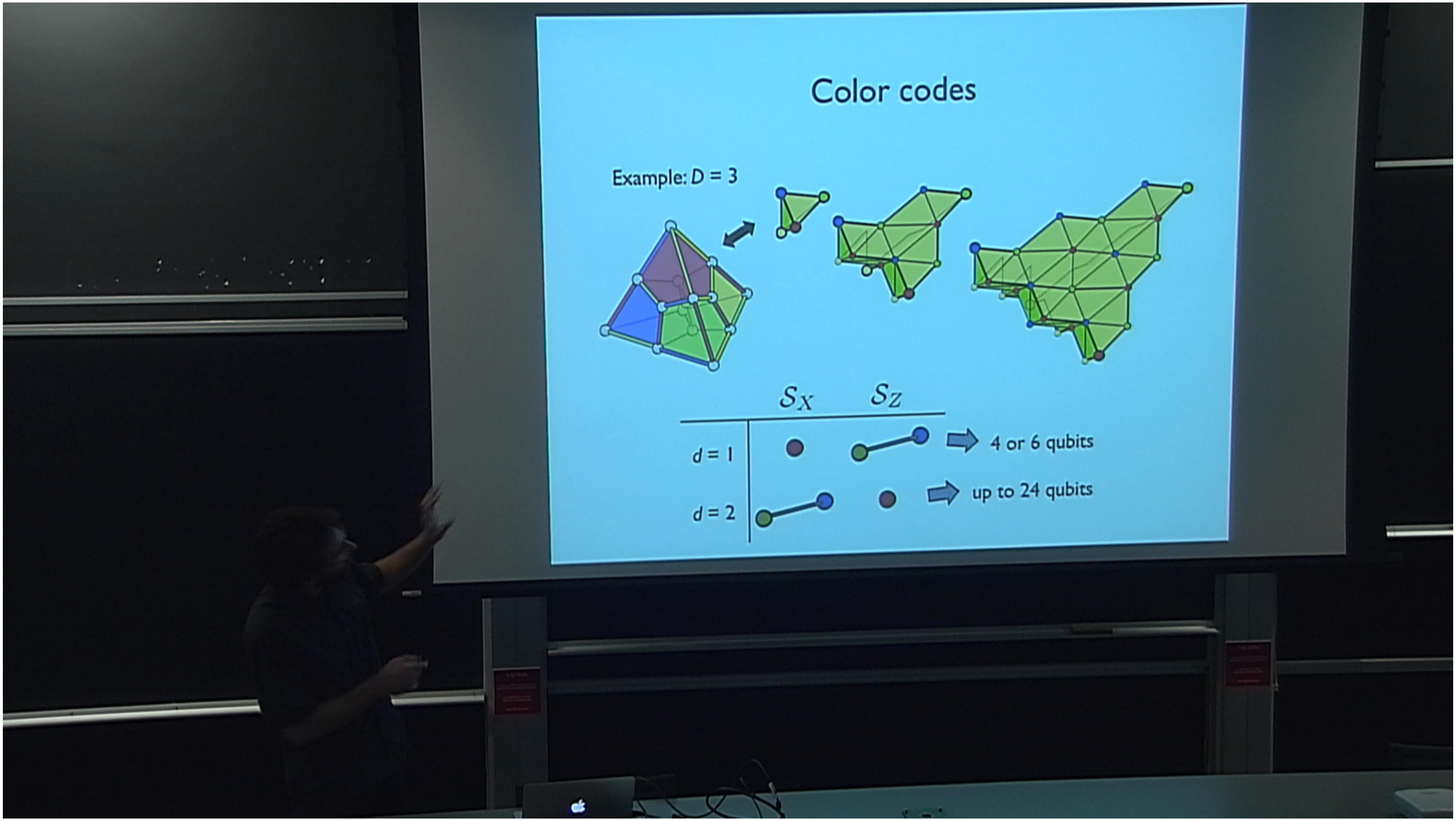
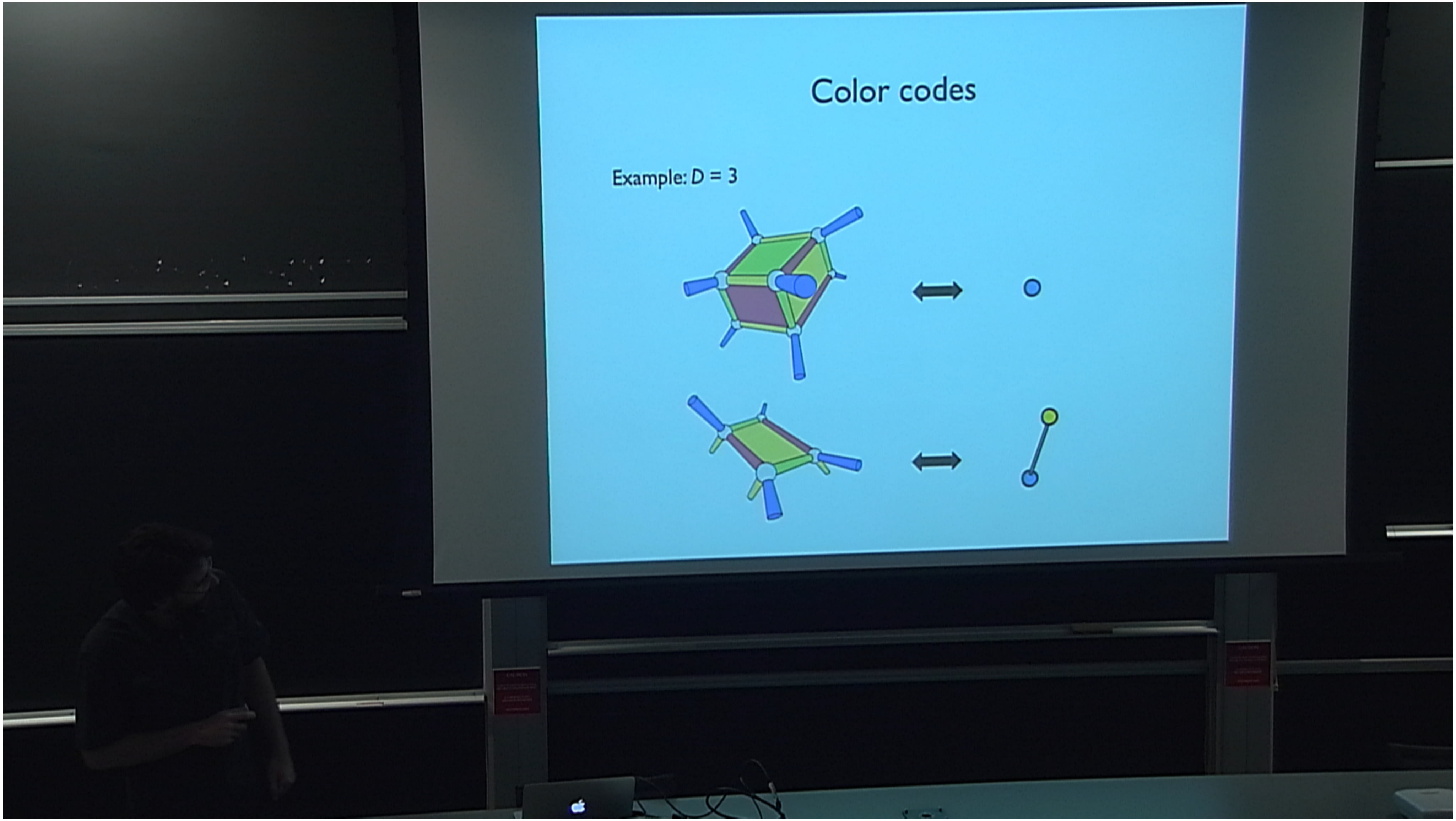
Dimensions

	X	Z
$\mathcal{S}$	$d-1$	$\bar{d}-1$
errors / logical ops	$\bar{d}$	d


Color codes
Example: D = 3
$\mathcal{S}_X$
$\mathcal{S}_Z$
d = 1
4 or 6 qubits
d = 2
up to 24 qubits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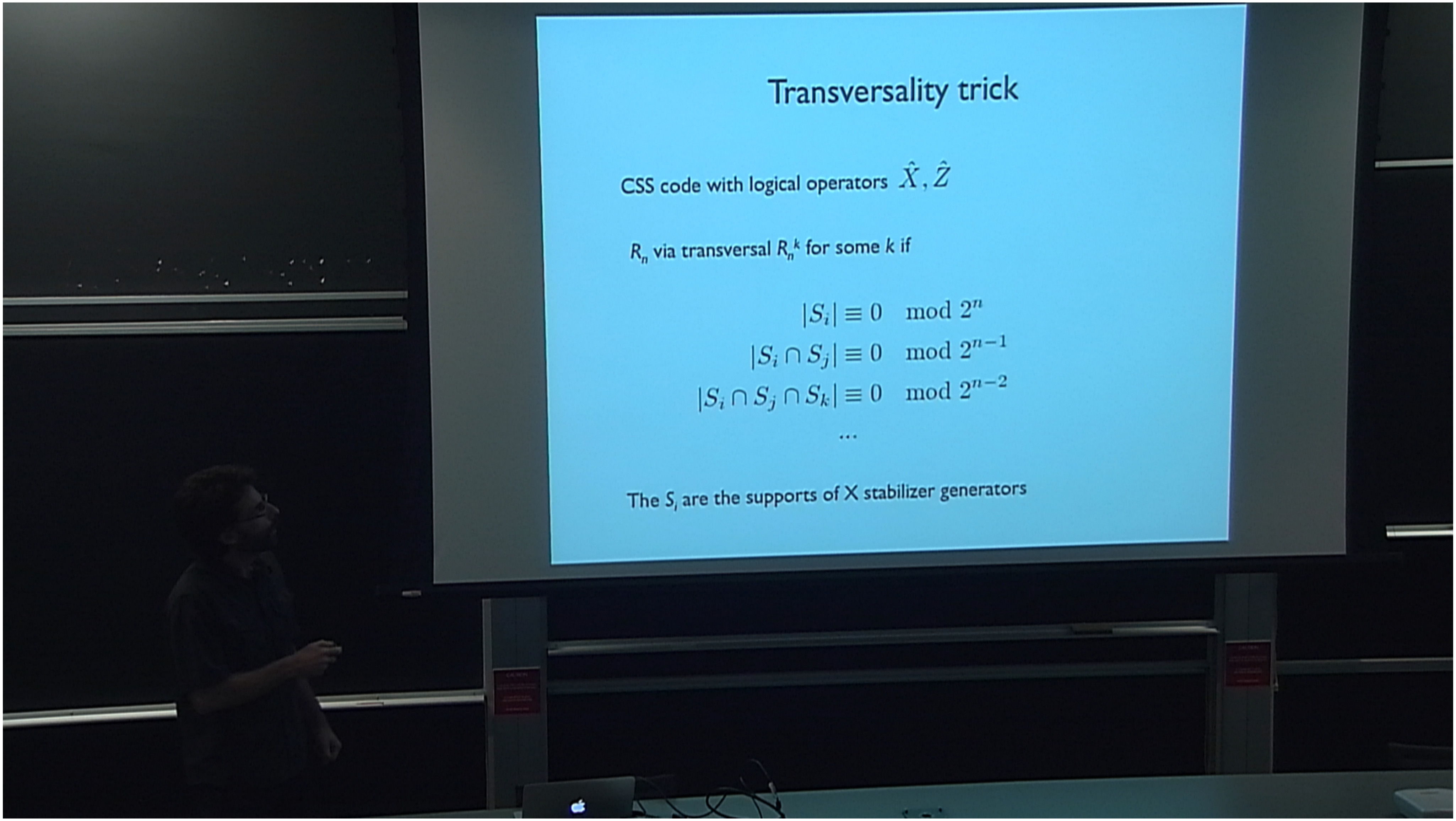
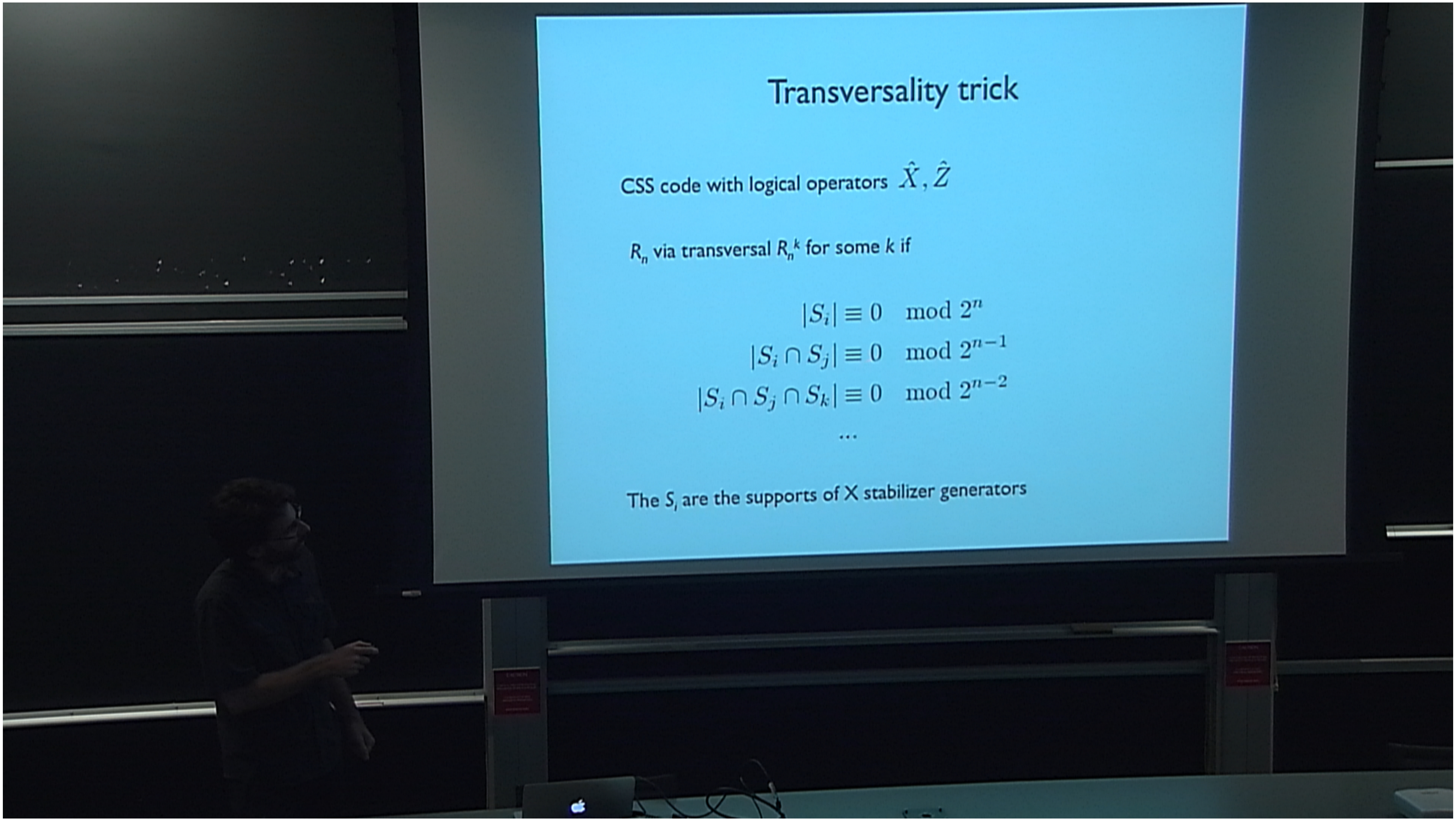

Transversality trick
CSS code with logical operators $\hat{X}, \hat{Z}$
R_n via transversal R_n^k for some k if
$|S_i| \equiv 0 \mod 2^n$
$|S_i \cap S_j| \equiv 0 \mod 2^{n-1}$
$|S_i \cap S_j \cap S_k| \equiv 0 \mod 2^{n-2}$
...
The S_i are the supports of X stabilizer generators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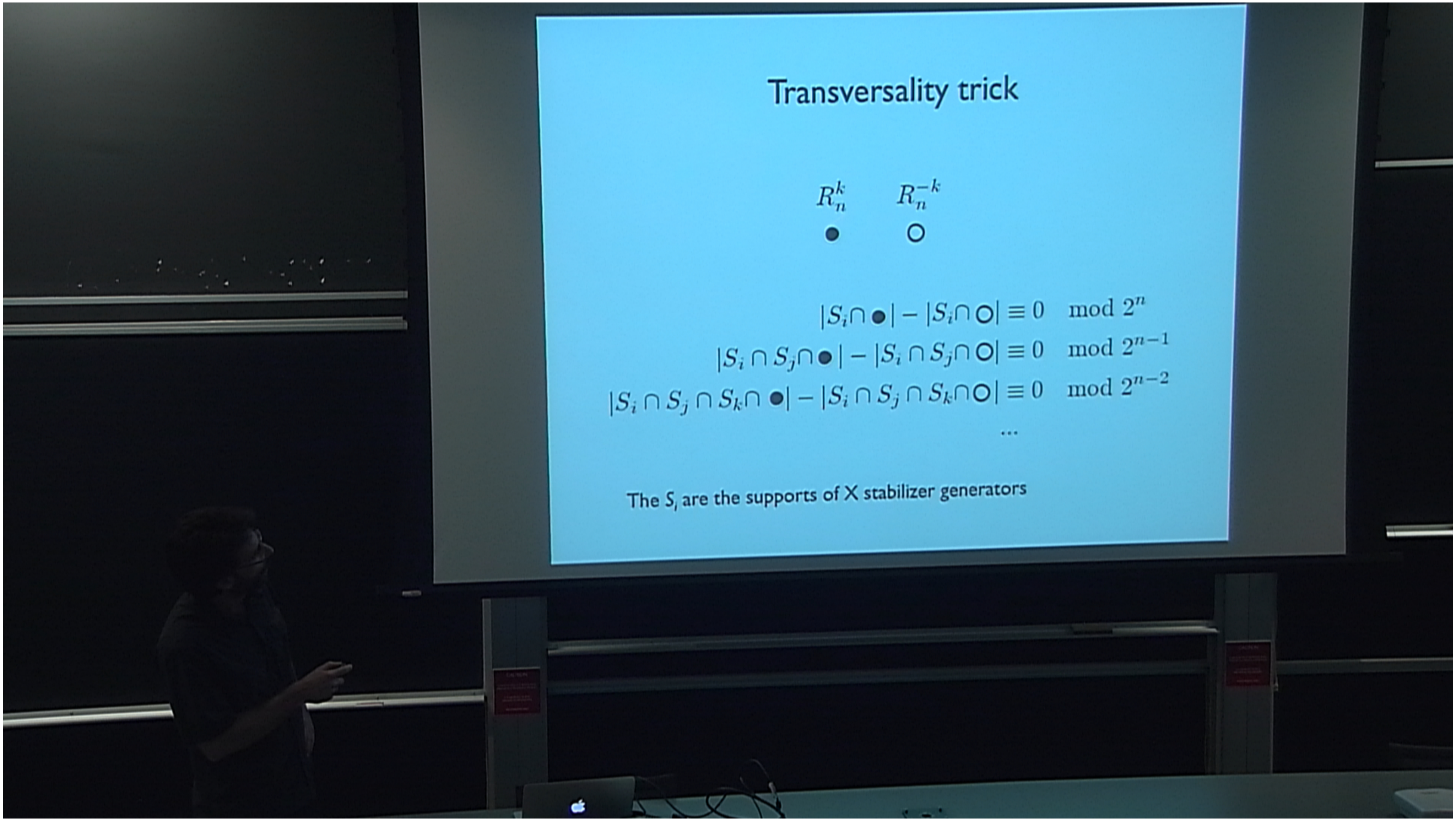
Transversality trick
R_n^k ●
R_n^{-k} ○
$|S_i \cap \bullet| - |S_i \cap \circ| \equiv 0 \mod 2^n$
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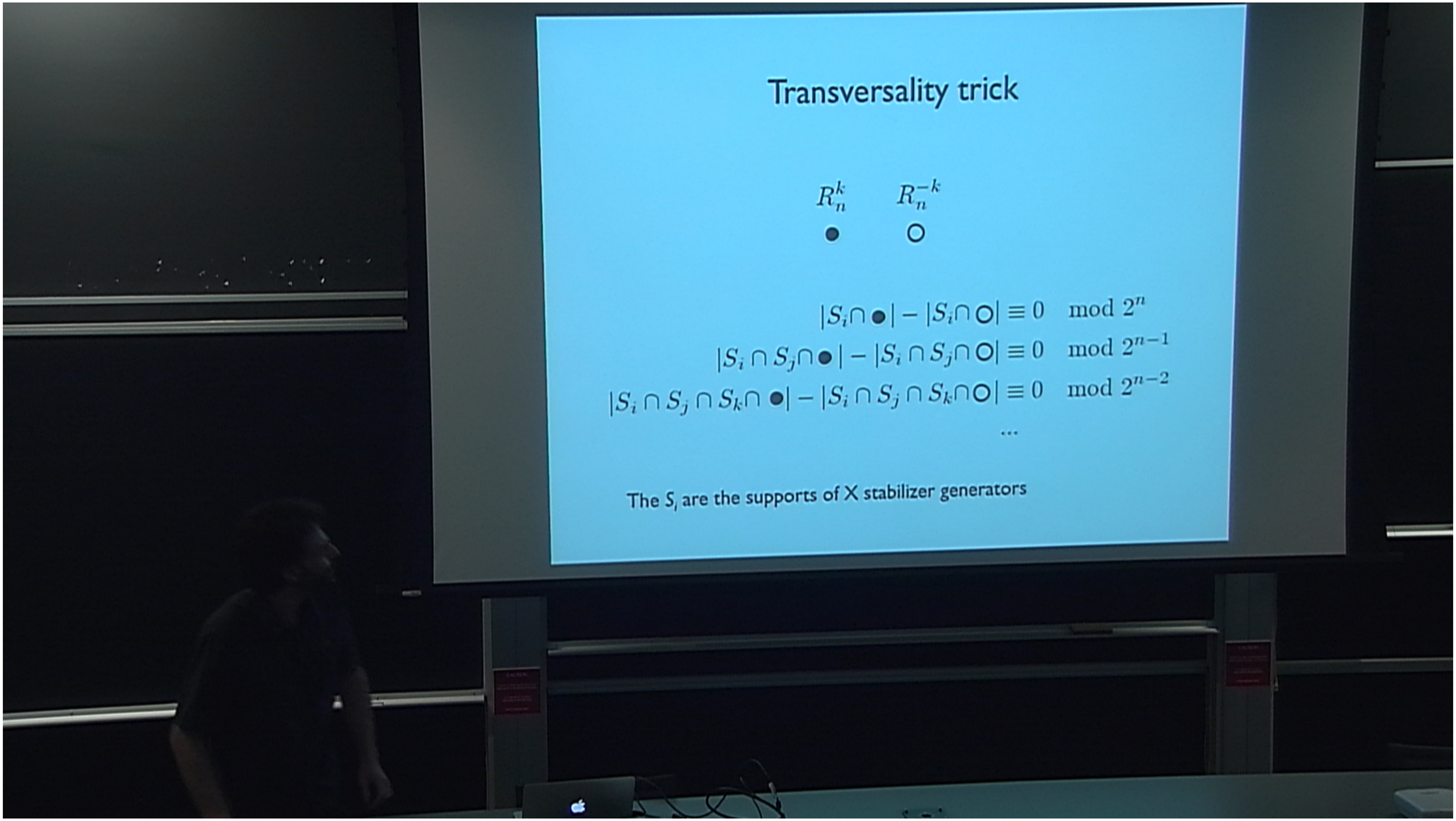
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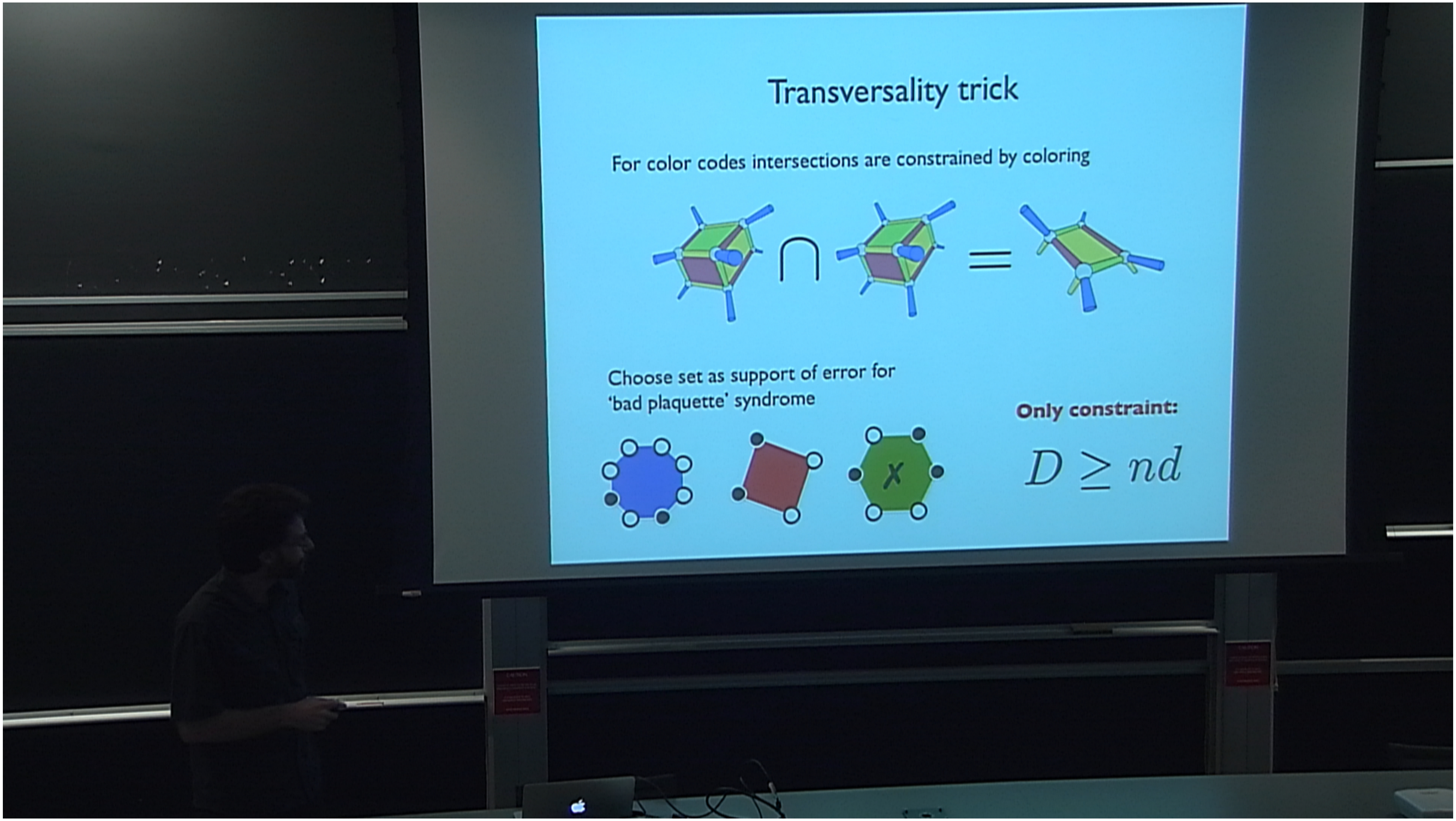
Transversality trick
For color codes intersections are constrained by coloring
Choose set as support of error for
'bad plaquette' syndrome
Only constraint:
$D \geq nd$

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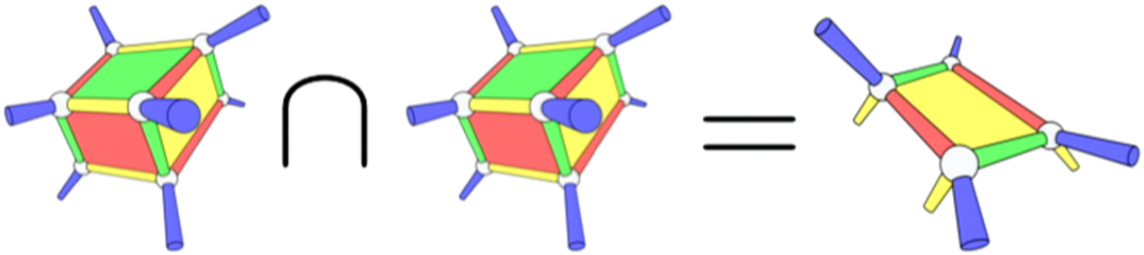

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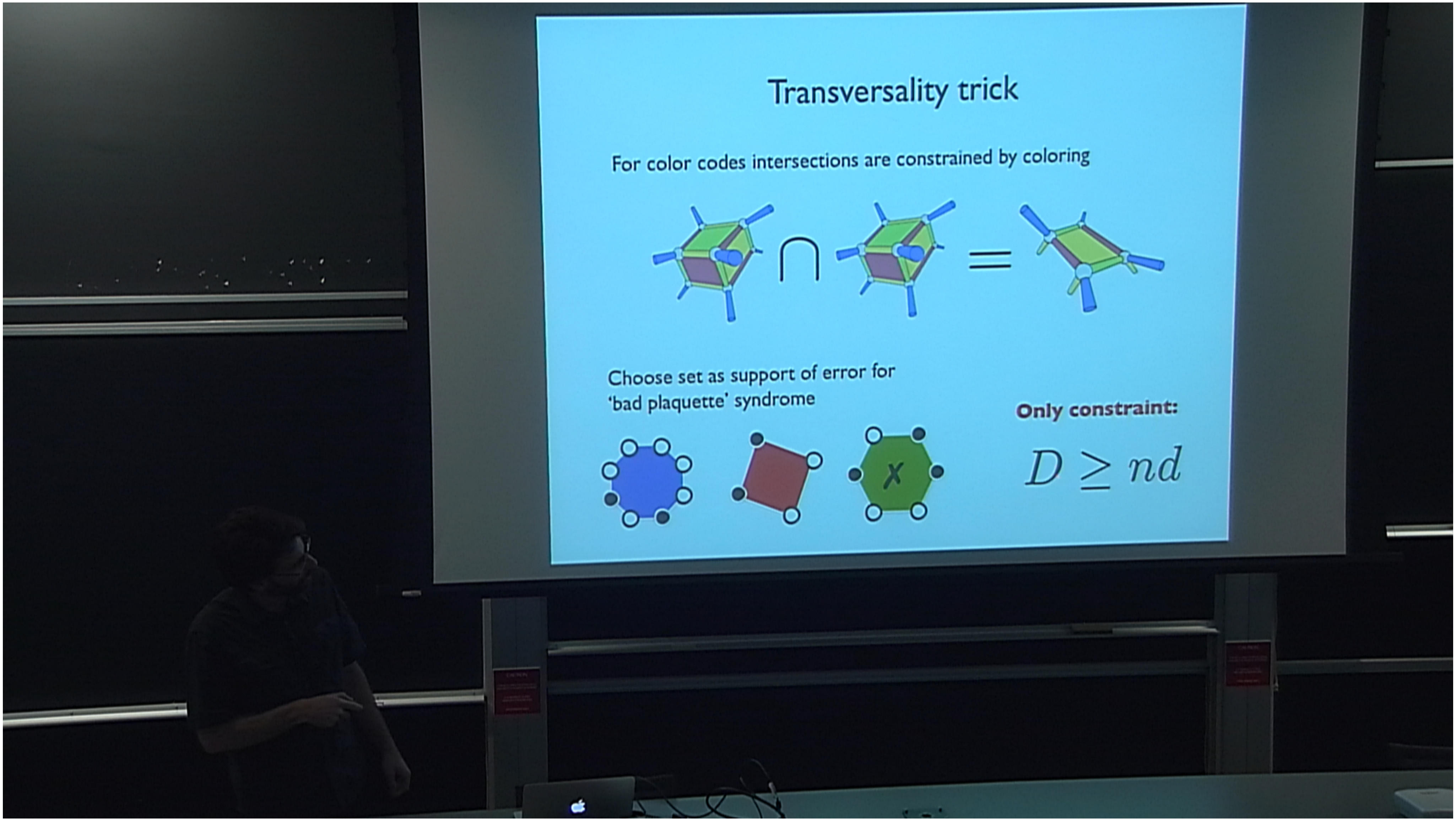
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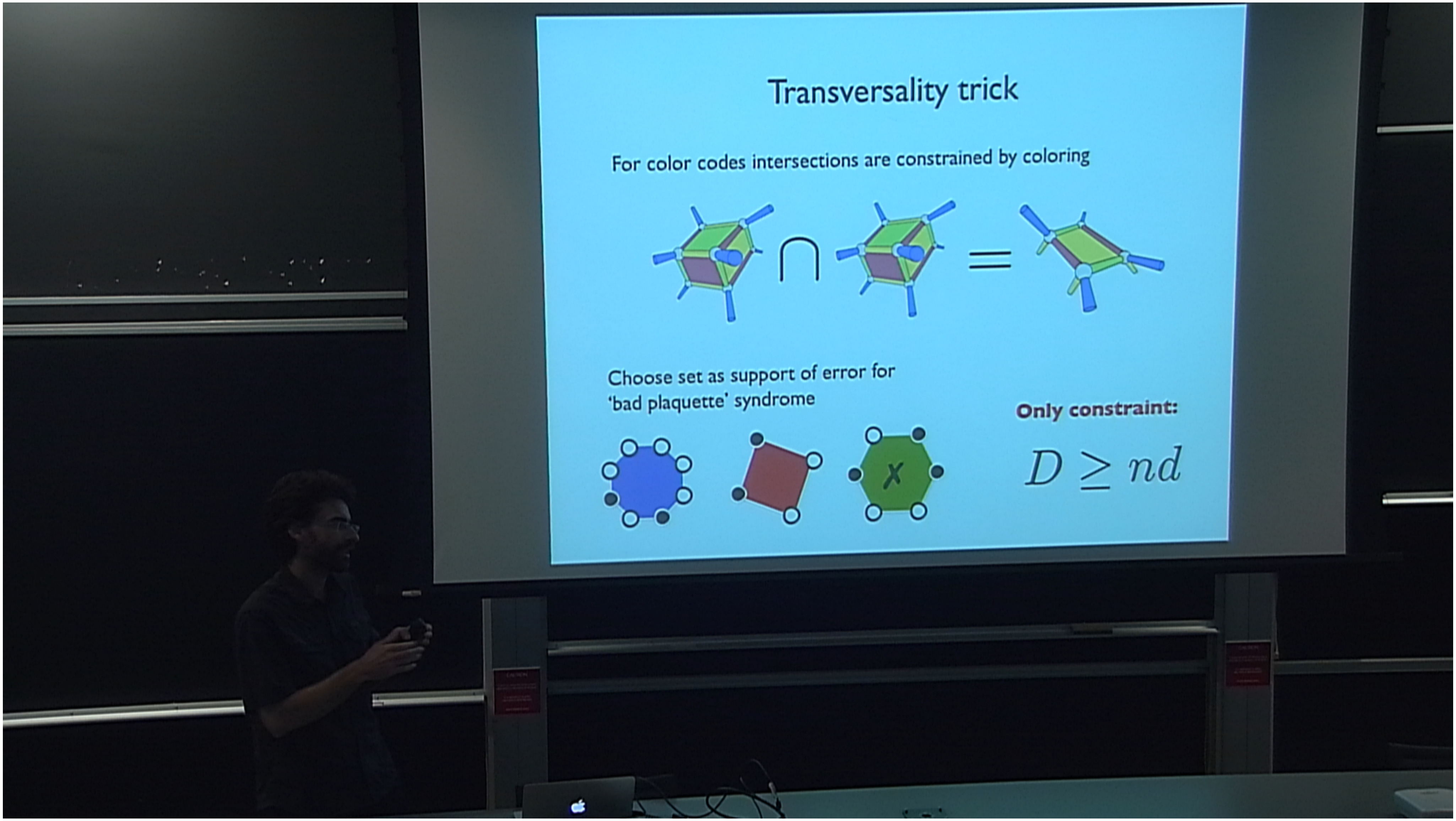
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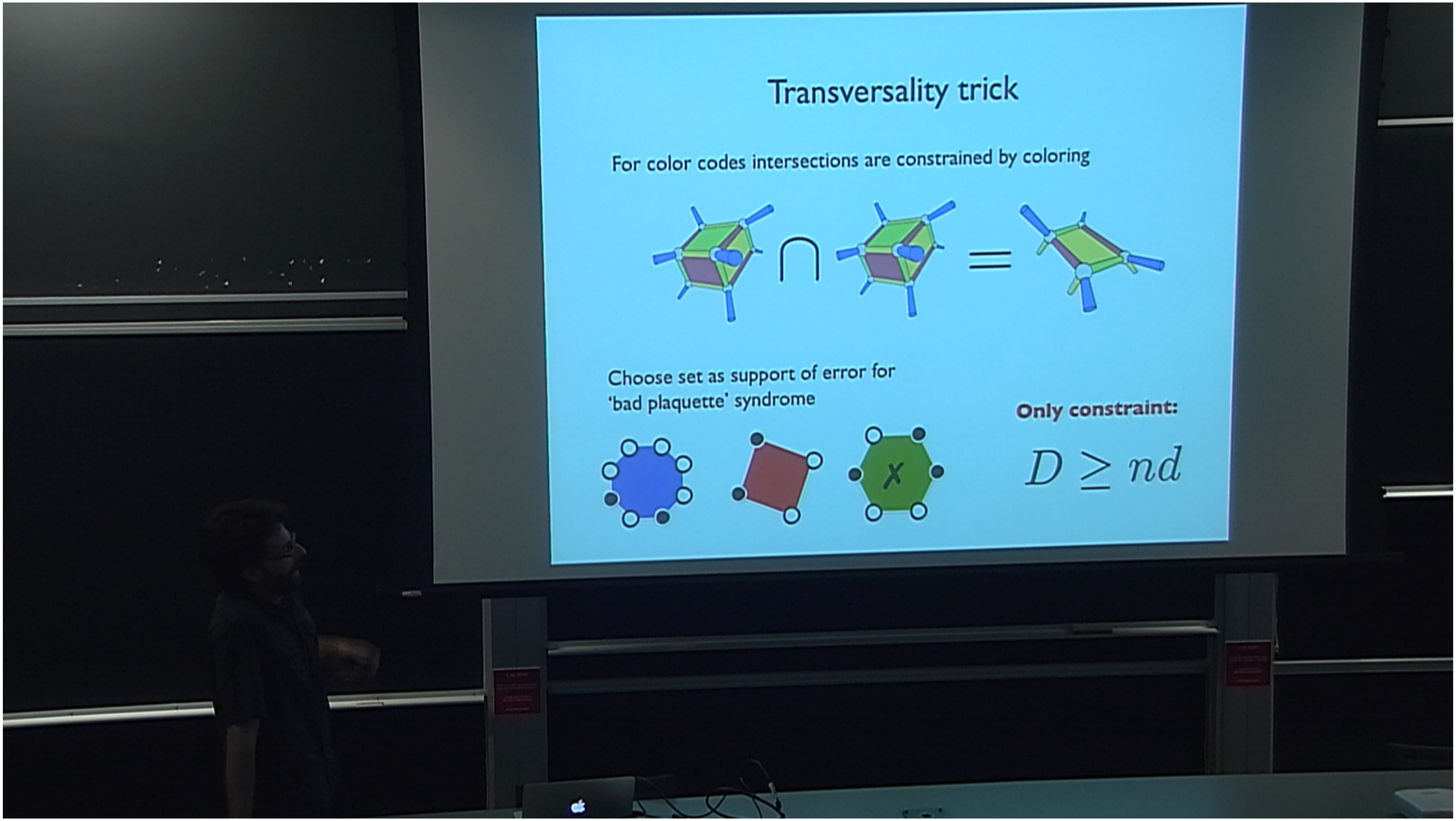
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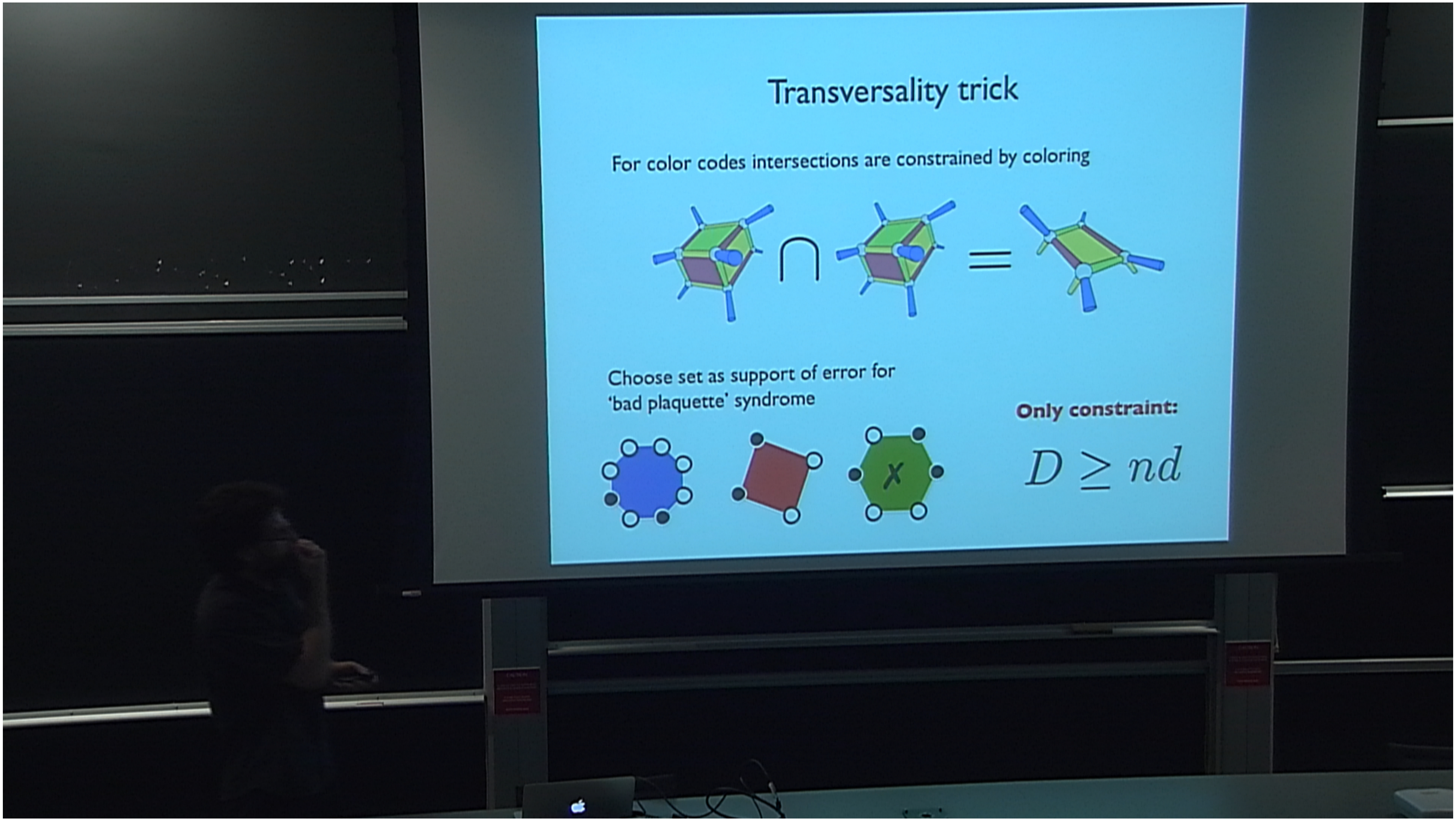
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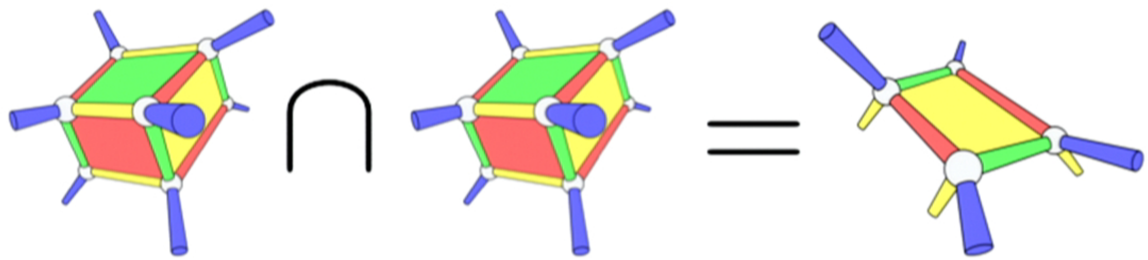

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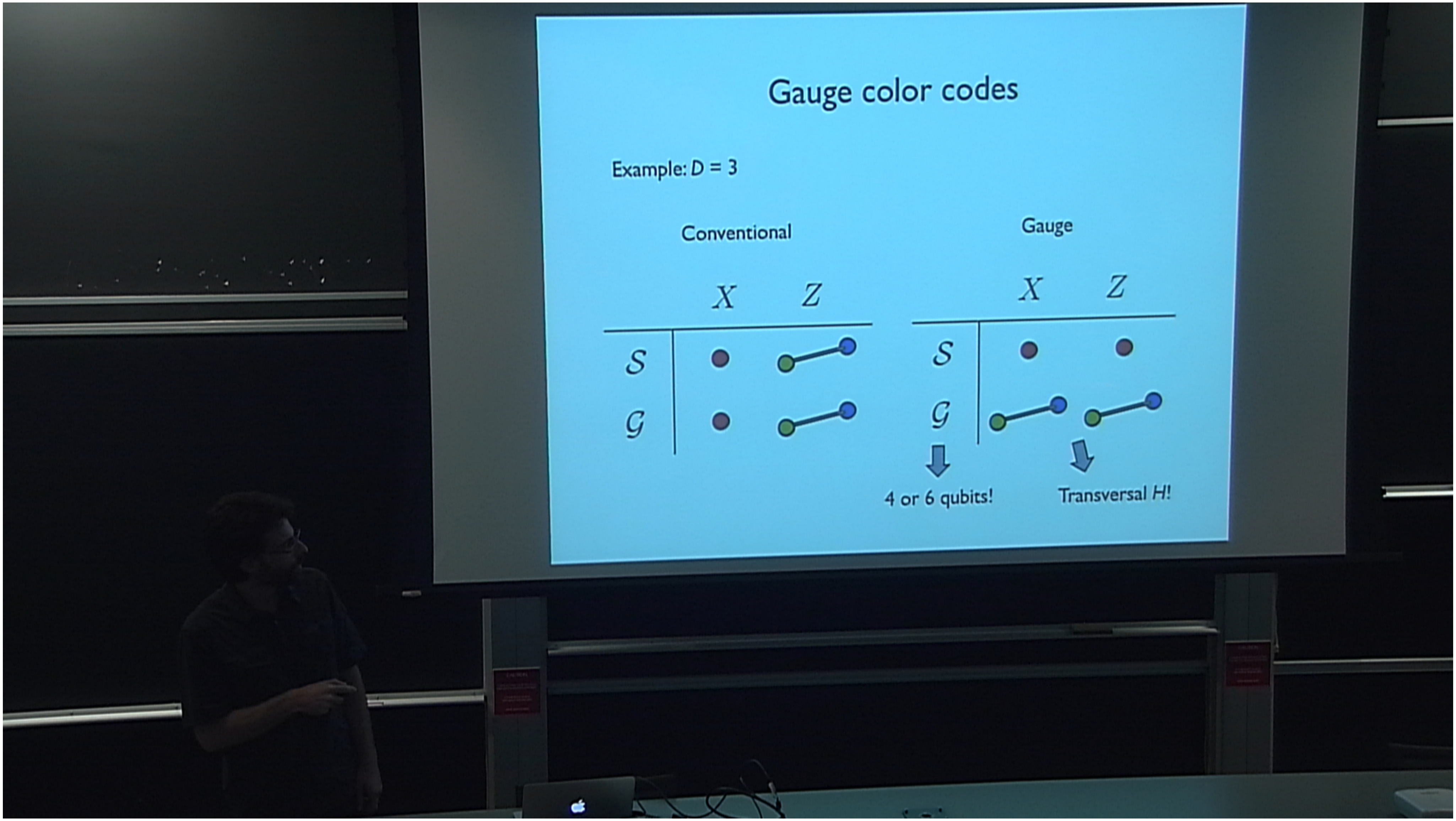
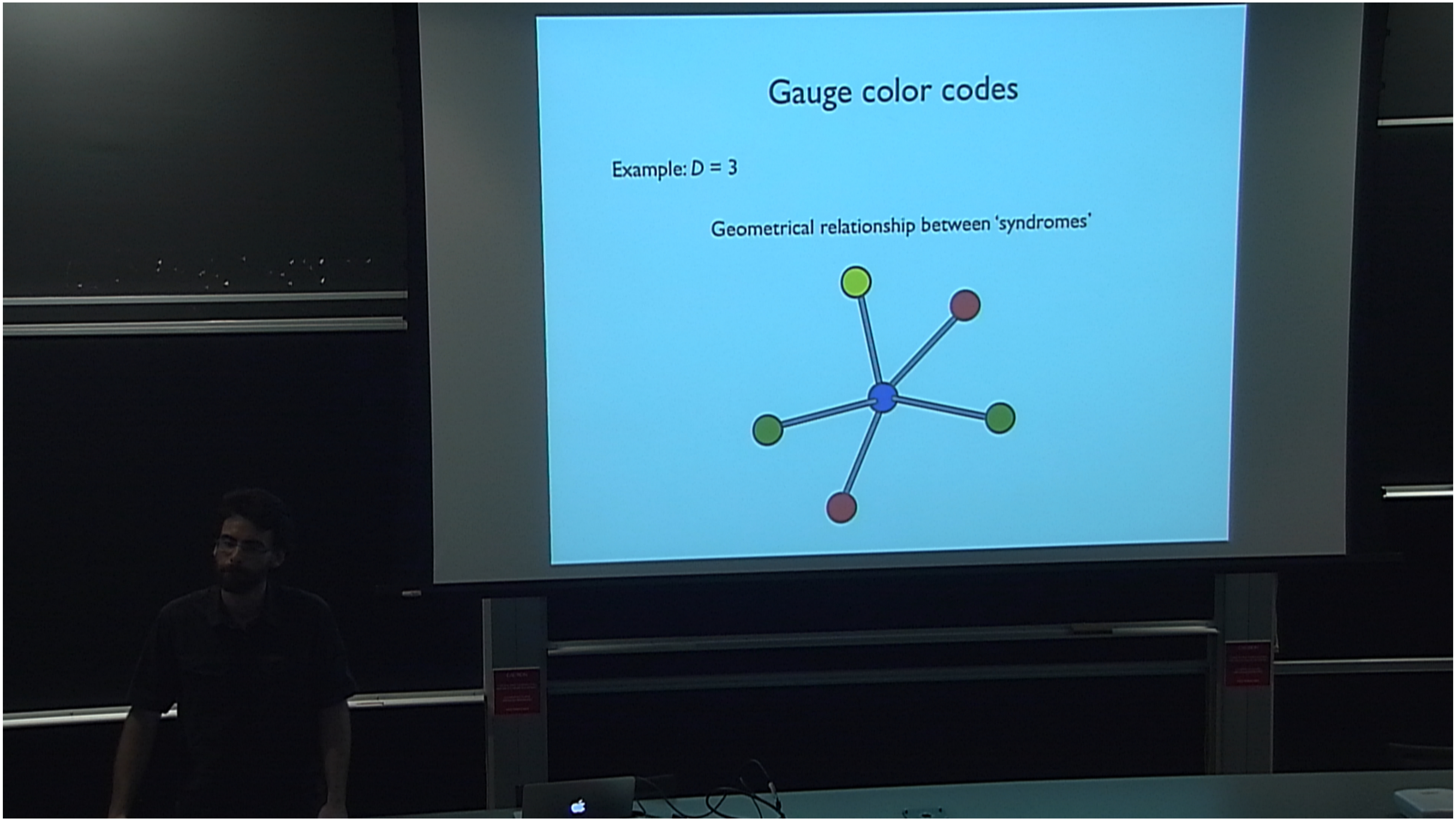
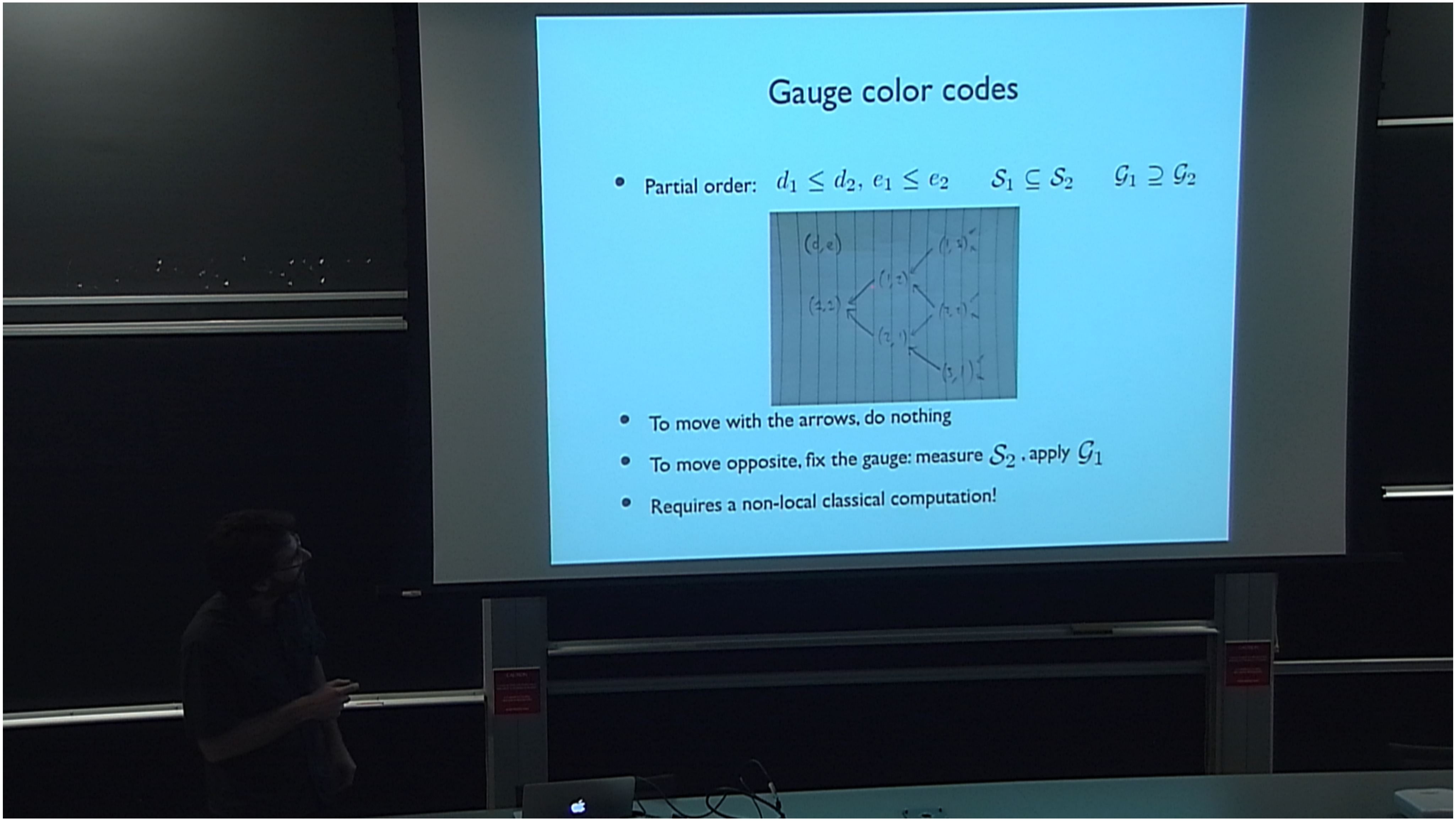
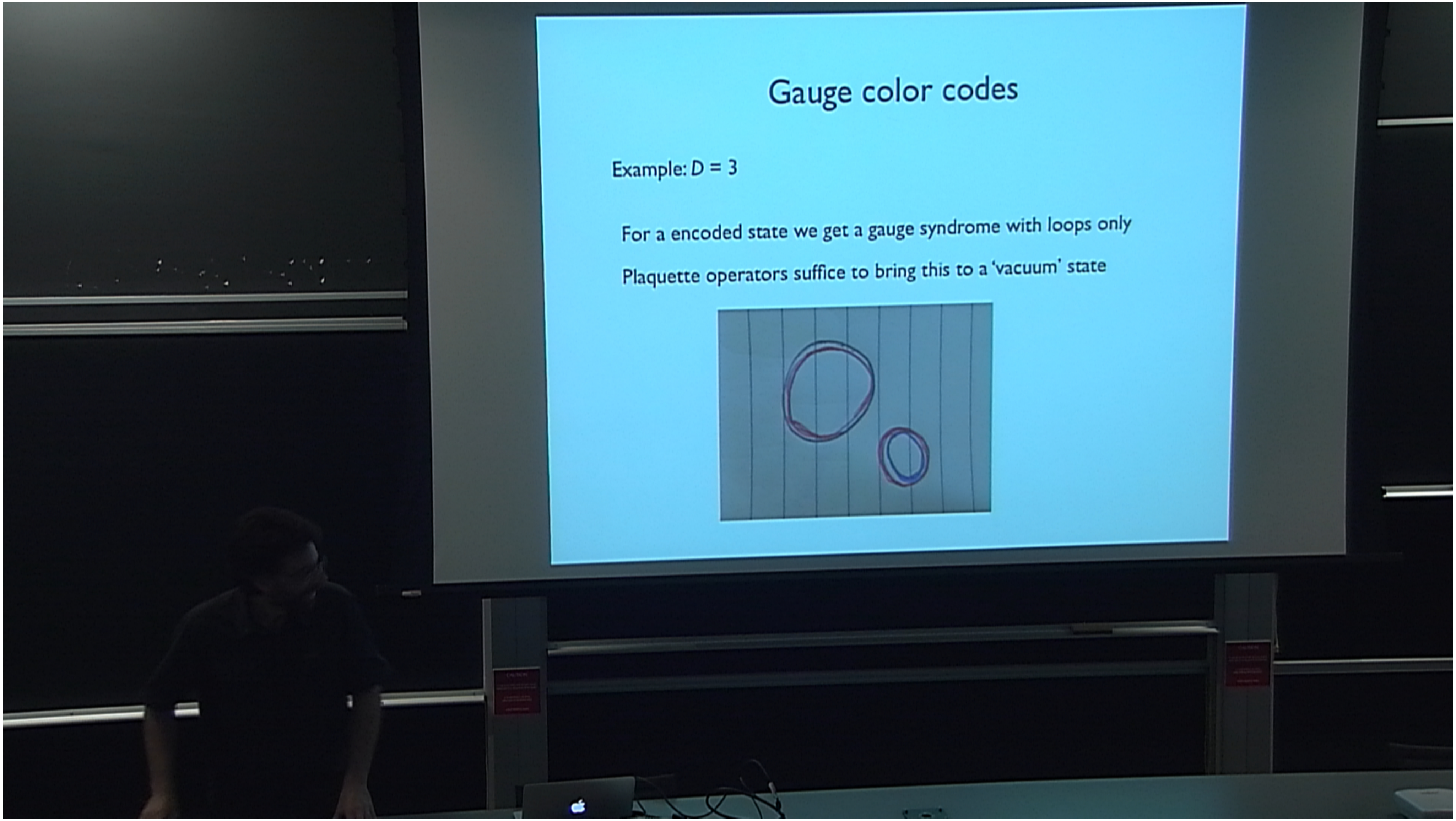
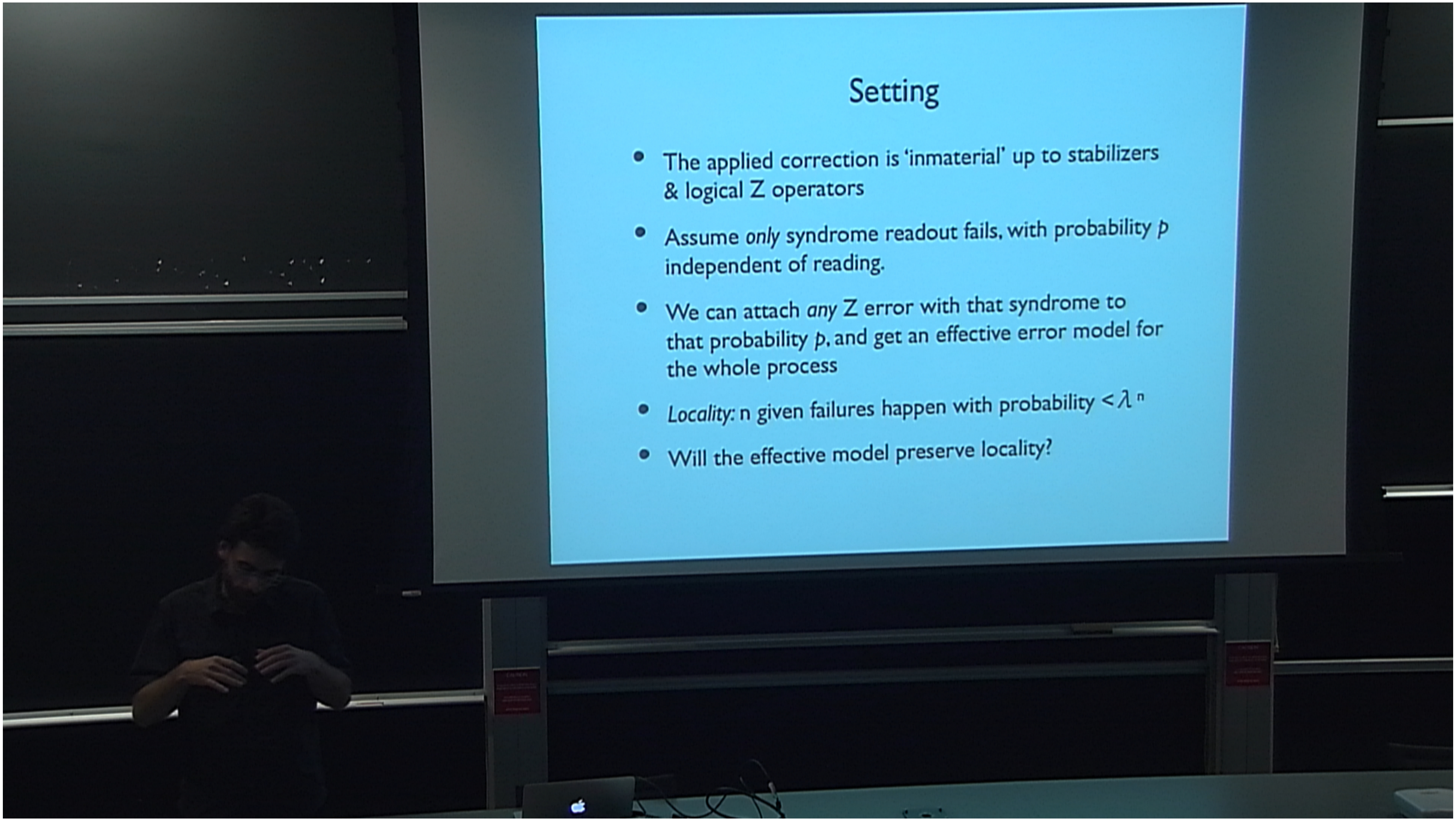

Gauge color codes
Example: D = 3
Conventional
Gauge
X
Z
S
G
4 or 6 qubits!
Transversal H!

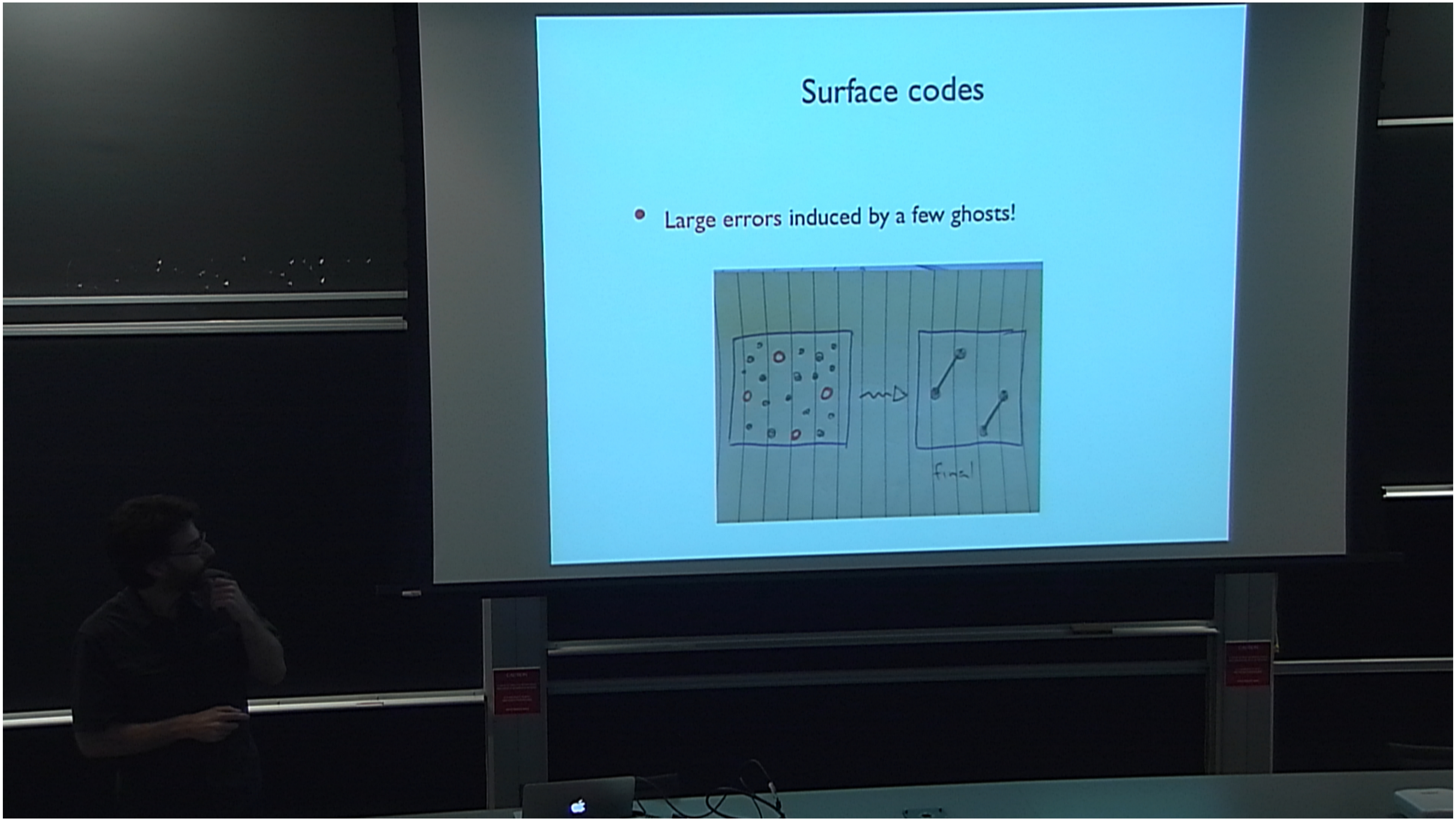


Gauge color codes
Example: D = 3
Geometrical relationship between 'syndromes'


Gauge color codes
Partial order: $d_1 \leq d_2,\ e_1 \leq e_2$ $\quad \mathcal{S}_1 \subseteq \mathcal{S}_2$ $\quad \mathcal{G}_1 \supseteq \mathcal{G}_2$
(d,e)
To move with the arrows, do nothing
To move opposite, fix the gauge: measure $\mathcal{S}_2$, apply $\mathcal{G}_1$
Requires a non-local classical computation!

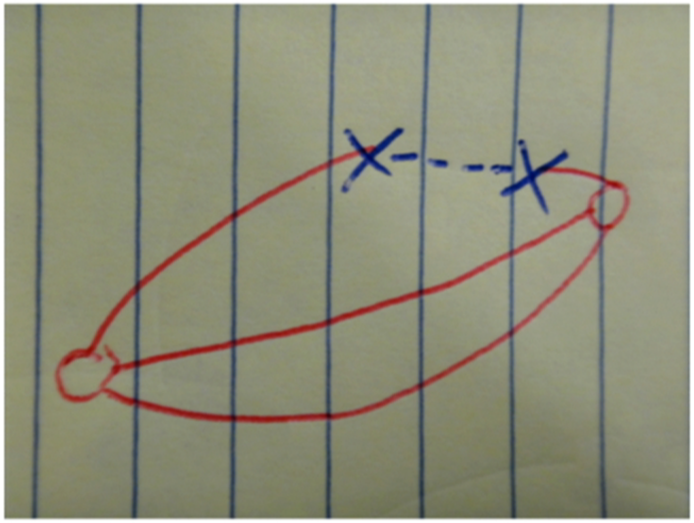

Gauge color codes
Example: D = 3
For a encoded state we get a gauge syndrome with loops only
Plaquette operators suffice to bring this to a 'vacuum' state

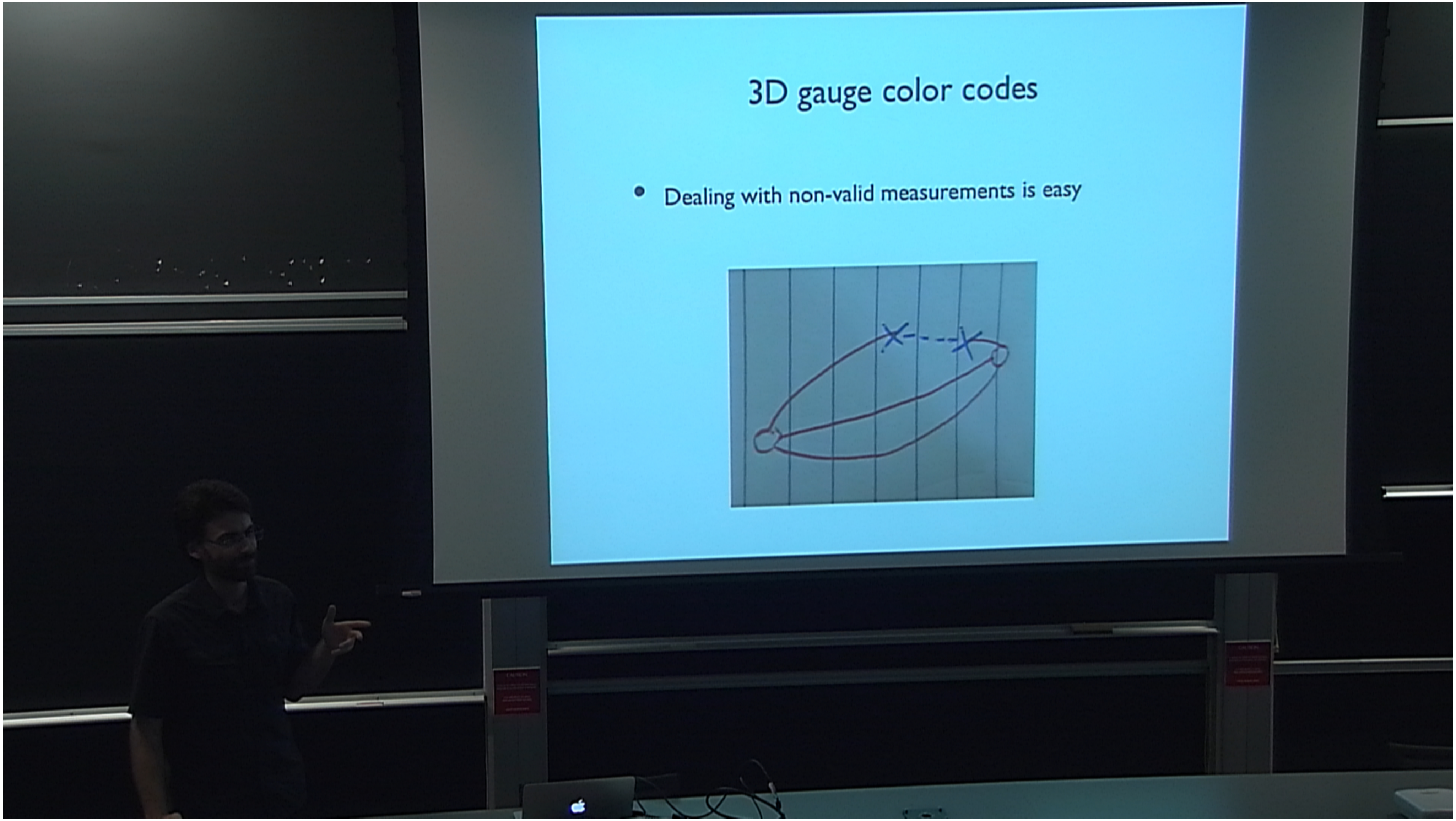

Setting
The applied correction is 'inmaterial' up to stabilizers & logical Z operators
Assume only syndrome readout fails, with probability p independent of reading.
We can attach any Z error with that syndrome to that probability p, and get an effective error model for the whole process
Locality: n given failures happen with probability < λ n
Will the effective model preserve locality?


Surface codes
Large errors induced by a few ghosts!
final

3D gauge color codes

- Dealing with non-valid measurements is easy

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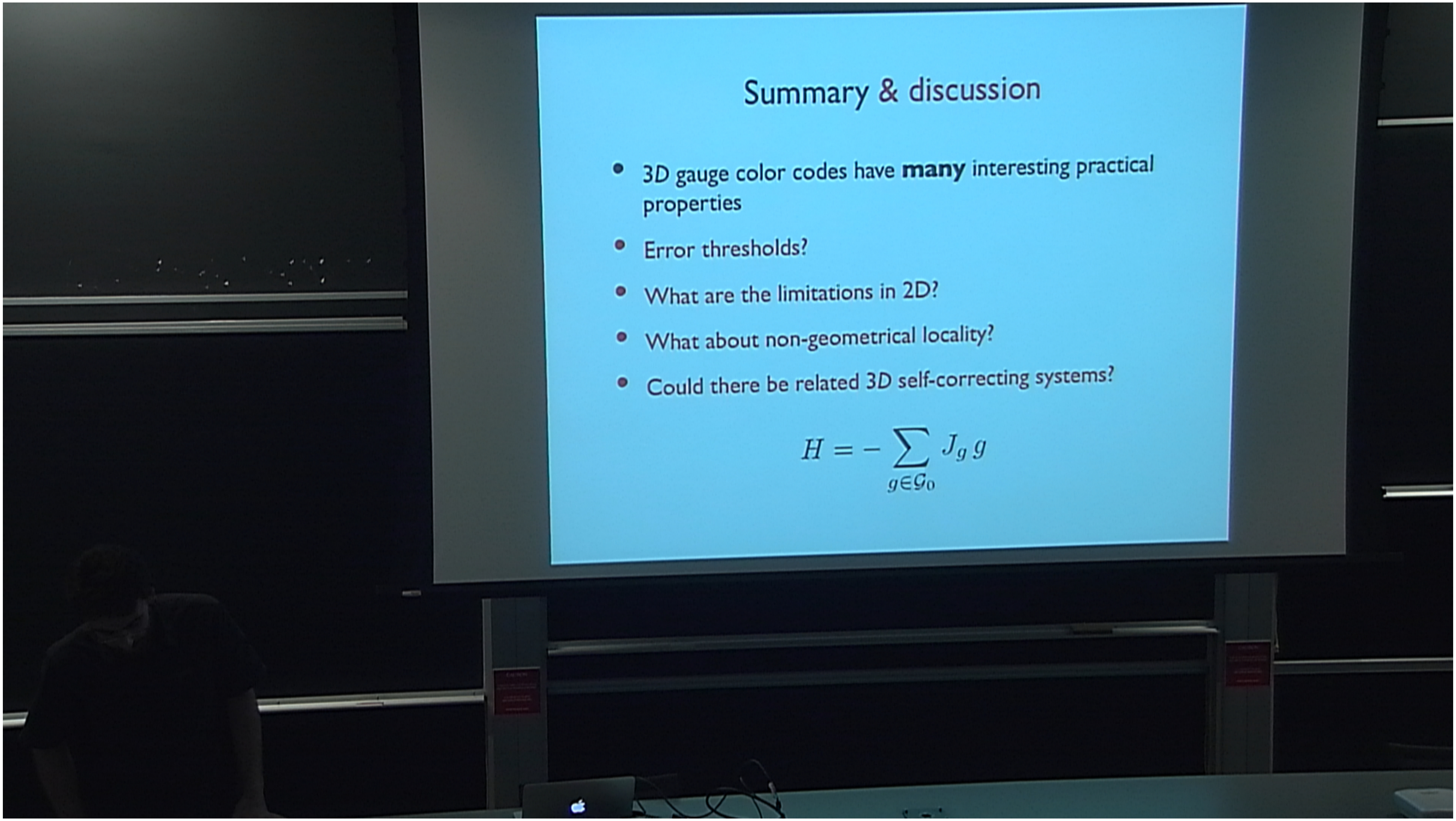
Summary & discussion
3D gauge color codes have many interesting practical properties
Error thresholds?
What are the limitations in 2D?
What about non-geometrical locality?
Could there be related 3D self-correcting systems?
$H = -\sum_{g \in \mathcal{G}_0} J_g \, g$