

Title: Is scalable quantum error correction realistic? Some projects, thoughts and open questions.

Date: Jul 15, 2014 03:30 PM

URL: <http://pirsa.org/14070010>

Abstract:

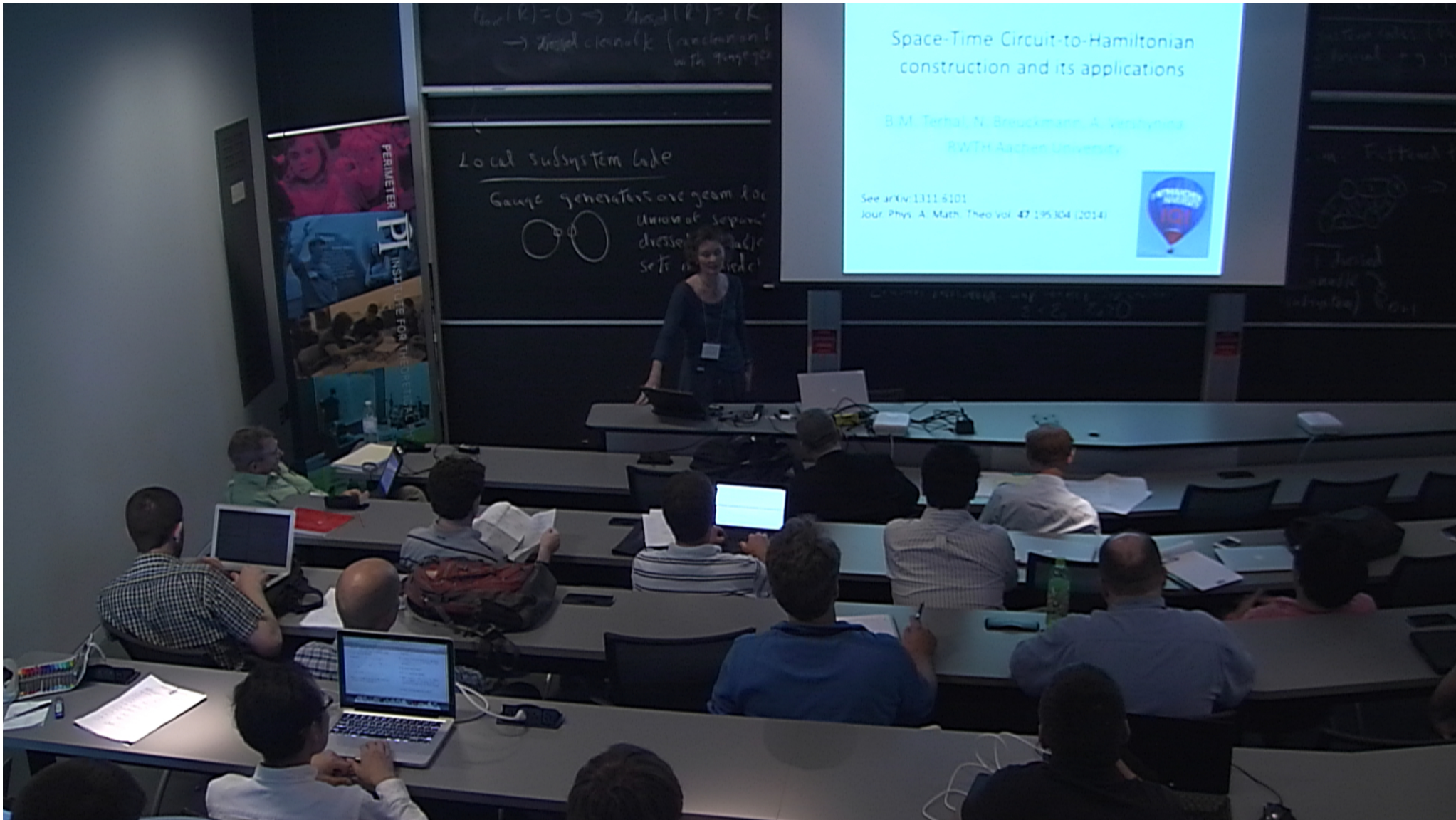
Space-Time Circuit-to-Hamiltonian construction and its applications

B.M. Terhal, N. Breuckmann, A. Vershynina
RWTH Aachen University

See arXiv:1311.6101

Jour. Phys. A: Math. Theo Vol. **47** 195304 (2014)





Circuit-to-Hamiltonian construction (Feynman-Kitaev)

Mapping from time-dependent circuit to the ground-state of a Hamiltonian H .

Circuit has n qubits and gates U_1, \dots, U_L . Introduce a clock register $|t\rangle$: $|t=0\rangle, \dots, |t=L\rangle$ and let

$$H_{circuit} = \sum_{t=1}^L (-U_t \otimes |t\rangle\langle t-1| + h.c. + |t-1\rangle\langle t-1| + |t\rangle\langle t|)$$

Ground-state of $H_{circuit}$ is **history state of the circuit** (for any ξ)

$$|\varphi_{his}\rangle = \frac{1}{\sqrt{L+1}} \sum_{t=0}^L U_t \dots U_1 |\xi\rangle \otimes |t\rangle$$

Use of Circuit-to-Hamiltonian Mapping II

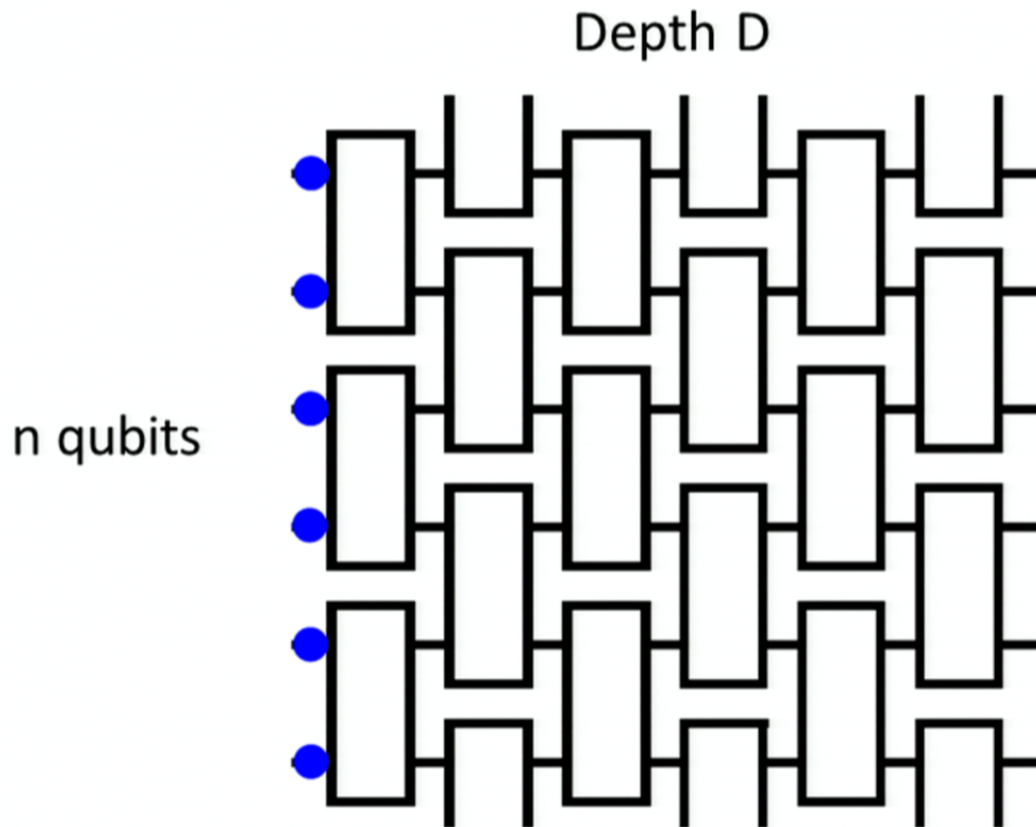
Proof that quantum adiabatic computation is equivalent to quantum computation with a circuit model.

$$H_{circuit} = \sum_{t=1}^L (-U_t \otimes |t\rangle\langle t-1| + h.c. + |t-1\rangle\langle t-1| + |t\rangle\langle t|)$$

Quantum Adiabatic Computation with $H_{circuit}(t)$ such that at $t=0$, $H_{circuit}(t=0) = H_{circuit}(U_1 = I, \dots, U_L = I)$ and $H_{circuit}(t=T) = H_{circuit}(U_1, \dots, U_L)$.

Adiabatic theorem applies as **Gap** $\Delta(t) \geq \Theta(1/L^2)$

(Original construction uses linear interpolation: $H(t) = tH_{circuit} + (1-t)H_{init}$)



For simplicity, we assume we have a 1D quantum circuit with nearest-neighbor interactions (and periodic boundary conditions). Such circuit is universal for computation if $D = \text{poly}(n)$.

A different construction?

Mizel et al. 'Ground State Quantum Computation' in 1999 & PRL 99, 070502 (2007)) consider a (fermionic) Hamiltonian (for adiabatic QC) with the following features:

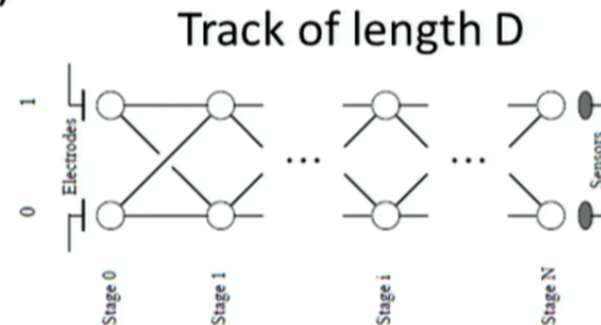
A qubit q in a quantum circuit of depth D is represented by $2(D+1)$ (fermionic) modes, $a_i(q), b_i(q), i = 0 \dots D$

For example: electron in left/right quantum dot or electron spin.

Particles (fermions) can hop on this track of length D .

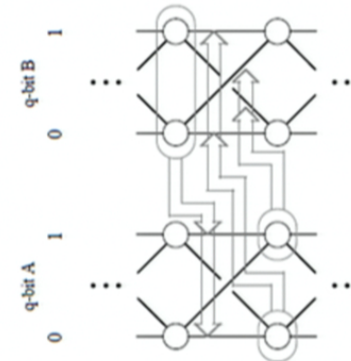
Construct $H_{circuit}$ such that

- Single qubit gate U is represented by a single particle hopping on such track (and changing its internal state).



Mizel et al. construction

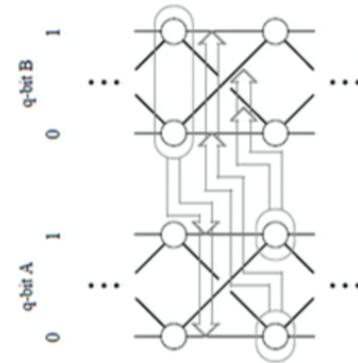
- Two-qubit gate $C(U)$, e.g. CNOT is represented as **pairs of particles hopping together**. They can only move forward or backward when they are both 'at the gate'.



- Modes can also be bosonic (fermionic nature not relevant as particles hop on a line).

Mizel et al. construction

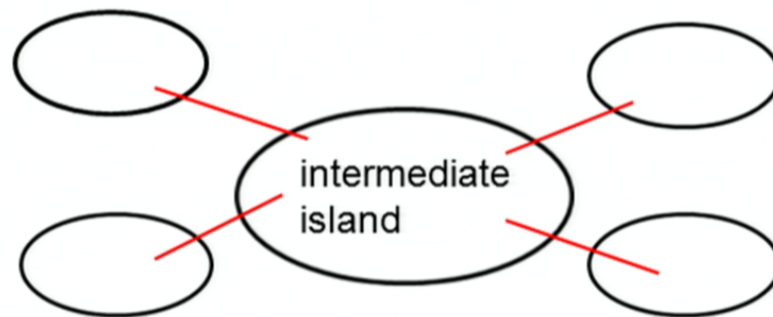
- Two-qubit gate $C(U)$, e.g. CNOT is represented as **pairs of particles hopping together**. They can only move forward or backward when they are both 'at the gate'.



- Modes can also be bosonic (fermionic nature not relevant as particles hop on a line).

How to make particles hop together?

Particles can hop through intermediate island: on this island the joint occupancy of particles is favored (**attractive interaction**), although the state with no particles on the island is the ground-state.



Perturbative analysis with weak hopping terms to and from island in the degenerate 'no-island occupancy' groundspace will give rise to **effective joint hopping terms (and weaker single-particle hopping terms!)**

Interaction (Coulomb for fermions, cross Kerr-nonlinearity for bosons) will always be needed.

New circuit-to-Hamiltonian construction

If quantum circuit is 1D, then Mizel et al. Hamiltonian $H_{circuit}$ is an interacting (fermion) 2D Hamiltonian (or 2D qubit Hamiltonian with 4 qubit interactions).

Properties of this construction are not well understood.

We show that their construction is an example of a general space-time circuit-to-Hamiltonian construction through which we can get new QMA (and quantum adiabatic QC & quantum walk) results.

Previous work by Margolus & Janzing on quantum walks.

Space-Time Circuit-to-Hamiltonian Construction

Define a clock for each qubit q : $|t_q = 0, \dots, D\rangle$.

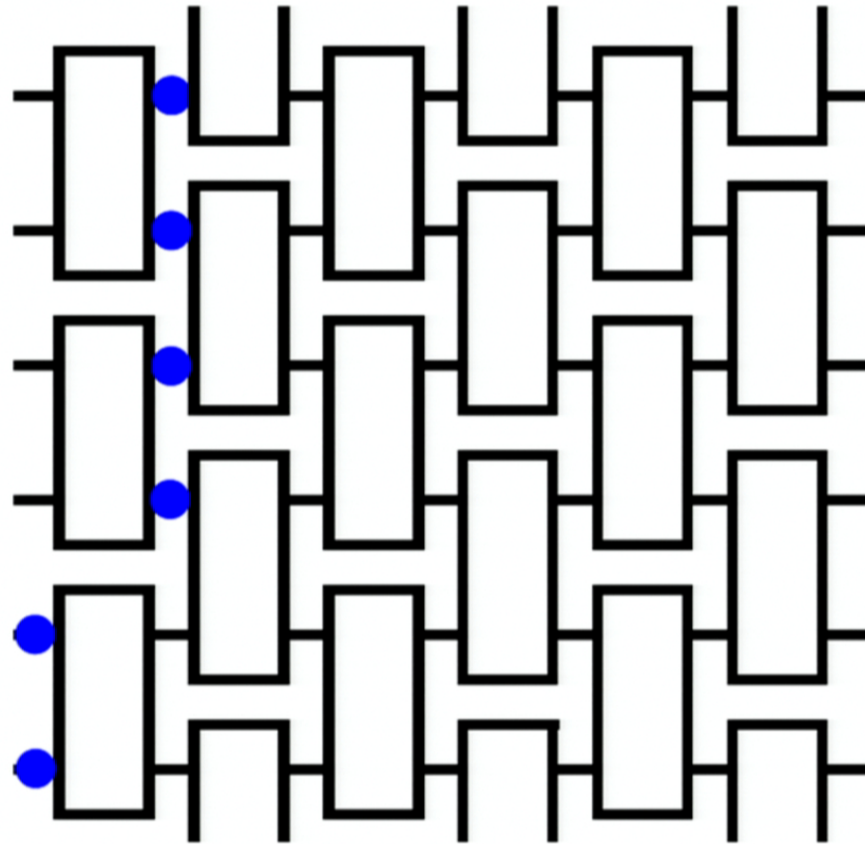
Time configuration $\mathbf{t} = (t_1, t_2, \dots, t_n)$

Term in $H_{circuit}$ for a two-qubit gate U on *qubit* q, p at time $s+1$

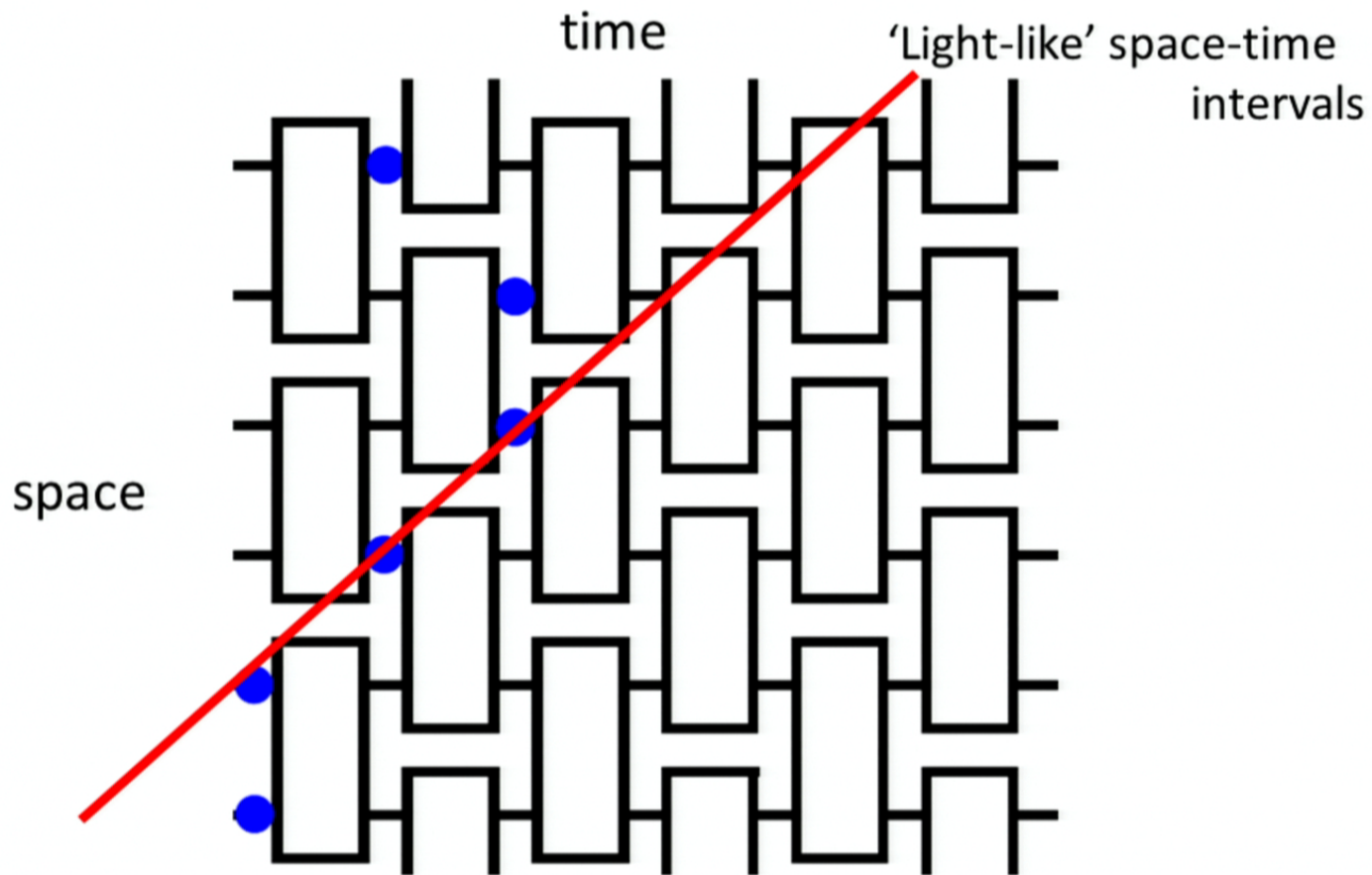
$$\begin{aligned}
 & -U \otimes |t_p = s + 1, t_q = s + 1\rangle\langle t_c = s, t_q = s| + h.c. \\
 & \quad + |t_p = s, t_q = s\rangle\langle t_p = s, t_q = s| \\
 & \quad + |t_p = s + 1, t_q = s + 1\rangle\langle t_p = s + 1, t_q = s + 1|
 \end{aligned}$$

“Times of interacting qubits are moved ahead/backward if they are synchronized”.

The previous fermionic model effectively corresponds to a certain clock realization (is thus unitarily equivalent).



Possible time-configuration



A valid time-configuration has only space-like or light-like space-time intervals while for an invalid time-configuration some intervals are time-like (with respect to a 2D Minkowski metric $g = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$).

Causal Structure of the Circuit

A valid time-configuration has only space-like or light-like space-time intervals while for an invalid time-configuration some intervals are time-like with respect to a 2D Minkowski metric $g = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $c=1$.

Two qubits p and q: $c|t_p - t_q| = |x_p - x_q|$: **light-like** (equivalent to $\Delta x_\mu \Delta x^\mu = 0$ with $\Delta x_1 = c(t_p - t_q)$, $\Delta x_2 = x_p - x_q$)

Two qubits p and q: $c|t_p - t_q| < |x_p - x_q|$: **space-like** (equivalent to $\Delta x_\mu \Delta x^\mu > 0$)

Similar for time-like intervals

Causal Structure of the Circuit

A valid time-configuration has only space-like or light-like space-time intervals while for an invalid time-configuration some intervals are time-like with respect to a 2D Minkowski metric $g = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $c=1$.

Two qubits p and q : $c|t_p - t_q| = |x_p - x_q|$: **light-like** (equivalent to $\Delta x_\mu \Delta x^\mu = 0$ with $\Delta x_1 = c(t_p - t_q)$, $\Delta x_2 = x_p - x_q$)

Two qubits p and q : $c|t_p - t_q| < |x_p - x_q|$: **space-like** (equivalent to $\Delta x_\mu \Delta x^\mu > 0$)

Similar for time-like intervals



PERIMETER

PI INSTITUTE FOR THE

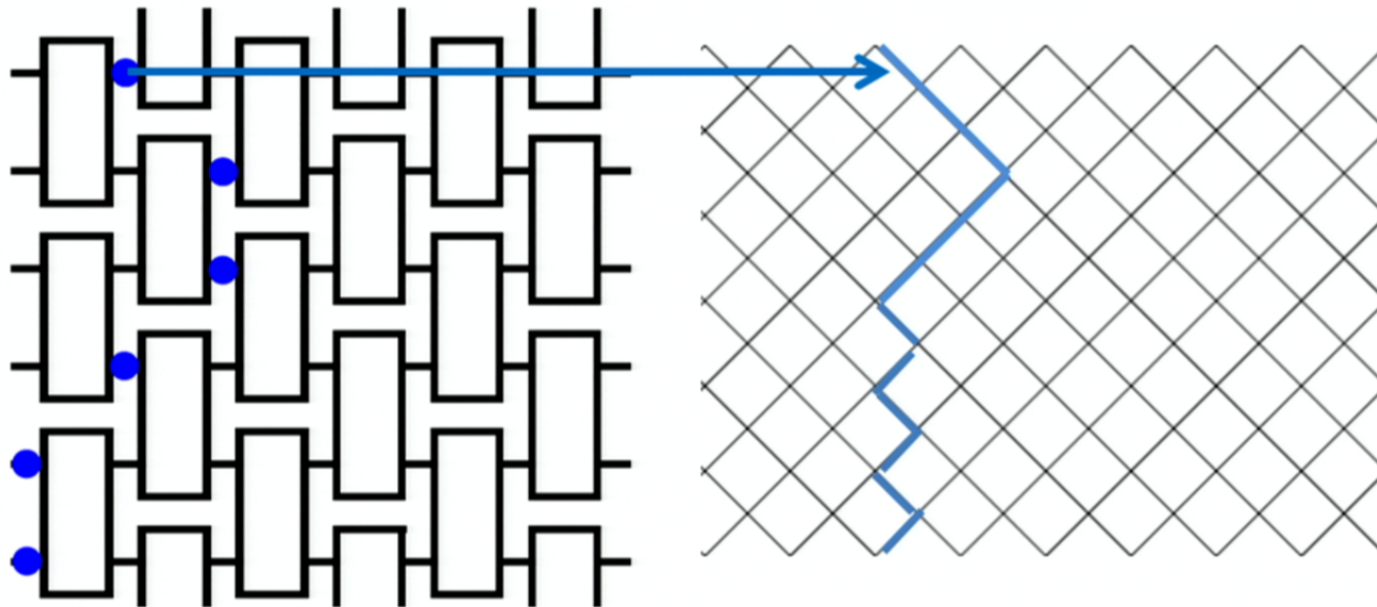
Outer Space
 every second of our life, you are being bombarded by cosmic rays from space. They are protons, ions and electrons like muons.

Local subsystem code

Sample error correction code is a quantum local subsystem code of separate dressed cleanable sets is dressed c!

A very useful representation

Valid time-configurations are **closed strings on a cylinder** if we have a one-dimensional circuit on a circle.



Picture on the right: each gate is represented as square and blue string is vertex in the graph. Points in time are on the edges. Strings (vertices) are connected by transitions $\langle \leftrightarrow \rangle$.

Application for QMA

Using this lowerbound on the gap, we can prove that (informally)

“determining the lowest eigenvalue of a two-dimensional interacting fermion model (periodic boundary conditions in both directions) in the sector where there is one fermion per line is QMA-complete.”

To prove this one needs a.o. to add spatially-local terms to Hamiltonian in order to penalize improper time-configuration (realized by blue quartic operators in the picture)

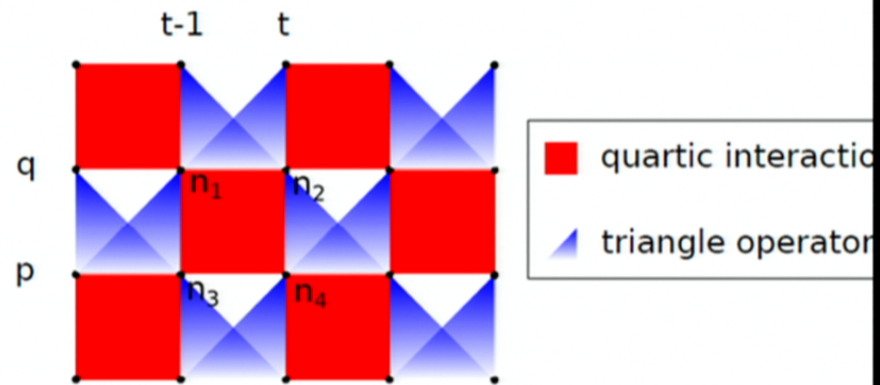


Figure 5: The black dots are fermionic sites, each with two modes (an \uparrow or \downarrow spin, say). The (red) squares represent the quartic gate interactions and the (blue) triangle operators penalize improper fermionic configurations (improper time-configuration). A (blue) triangle operator with top corner a and bottom corners b and c enforces the constraint $n_a(1 - n_b - n_c)$. The lattice has periodic boundary conditions in both directions.

Quantum Walk or How fast can kindergartners holding hands move forward

Question:

Imagine we start the particles on a line (all times synchronized) and let the system evolve with $H_{circuit}$ for a random, but sufficiently long time t . Then we check where the particles are, has the computation been executed? Can this be done in a time scaling with the depth D of the circuit?

Take a modified circuit Hamiltonian where for a 2-qubit gate we have

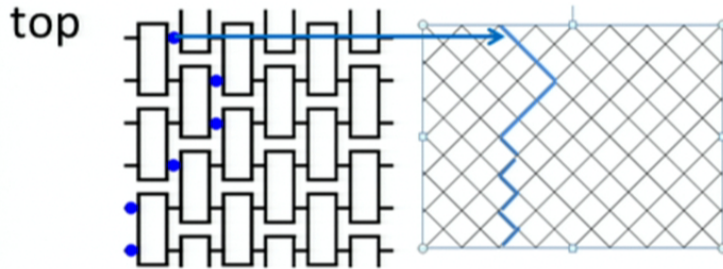
$$-U \otimes |t_p = s + 1, t_q = s + 1\rangle\langle t_c = s, t_q = s| + h.c.$$

Hence no diagonal terms: this simplifies the dynamics!

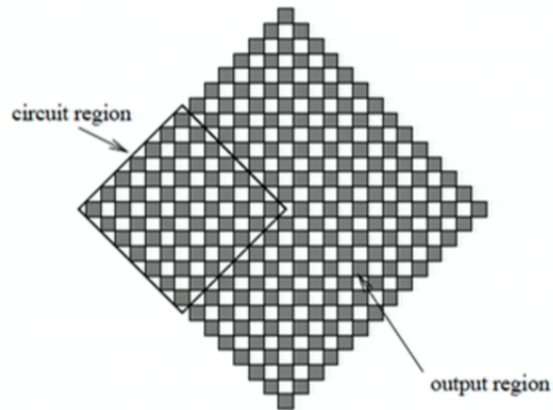
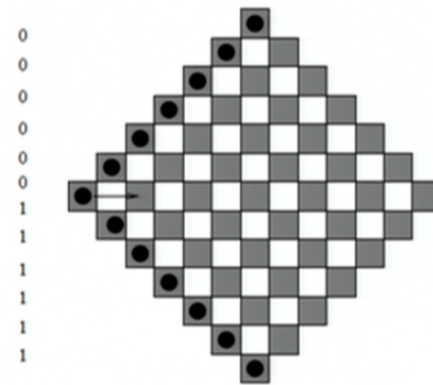
Do not consider a torus but a different 2D geometry so that boundary is fixed (no dynamics of a boundary point).

Dynamics of the string

The dynamics of the string is that of $n/2$ particles (the position of $n/2$ segments going, say, left) moving around on a line, all starting at the top



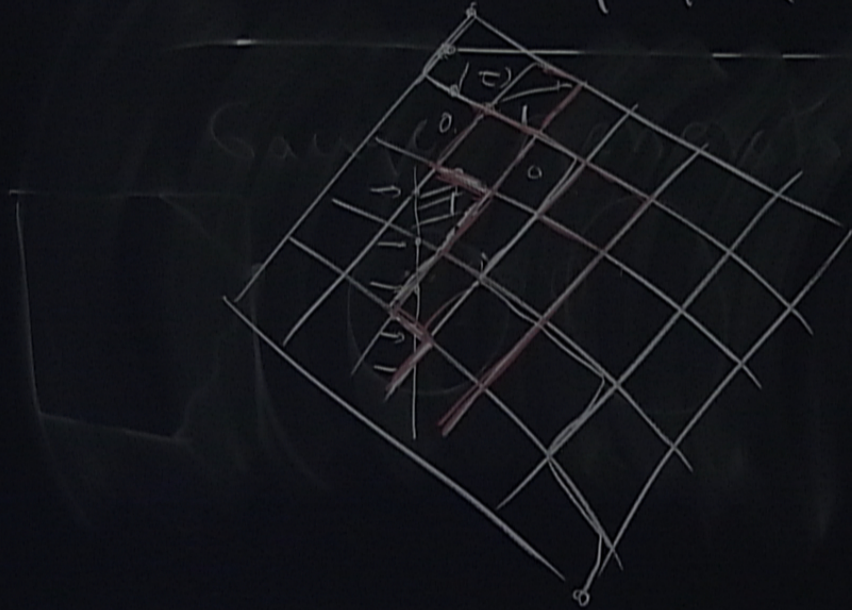
Particles with spins on edges!



String runs through center of grey squares. Black dots are input configuration.

D. Janzing, *Phys Rev. A* 75, 012307 (2007)
In Janzing's model, there are particles with spin on grey squares which can only hop if there are nearest-neighbor particles

Local subsystem code



Same as in previous management loc
of separate
dressed cleanable
set is dressed c!

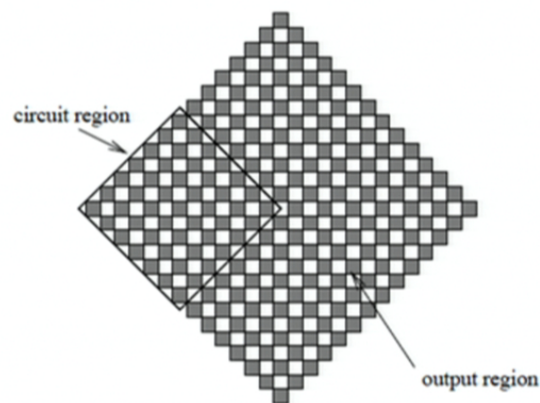
Entropy pushes the time forward...

Let circuit region be $k \times k$ and the whole region is $n \times n$.

One can prove (Janzing, PRA, 2007)

Thm: That there is a time $O(k)$ such that the probability of finding the whole string in the output region is at least $1 - 12/k$ (if $n \gg k$)

Also for $k = (n-1)/4$ (or less): the probability of finding the string in the output region for a random time goes to 1 as $n \rightarrow \infty$ (there are many more strings in the bulk of the lattice!)

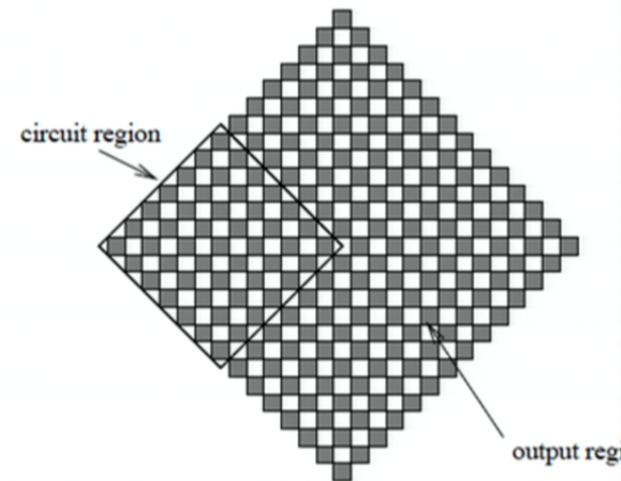


Analysis for dynamics on the torus is more involved due to moving boundary and absence of entropy argument.

Quantum Adiabatic Computation

Quantum walk is not very noise robust.

Quantum Adiabatic Computation?
And minimal control? Turn a knob and particles move down hand-in-hand so that at the end of the quantum adiabatic computation, the computation has been executed.



Idea: **Turning Hopping Terms Off and On or Turn Layer Off and On.**
Constant Gap.

See Flammia+Bacon, Adiabatic Gate Teleportation

Flammia+Bacon+Crosswhite, Adiabatic Quantum Transistors

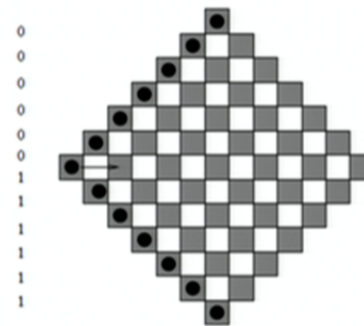
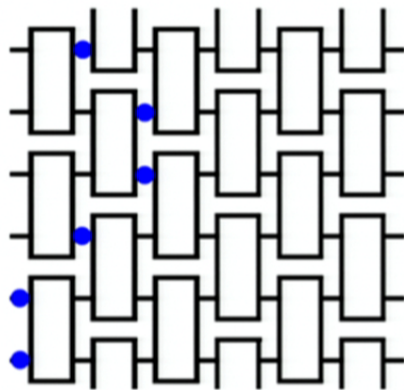
Quantum Adiabatic Computation

Consider the quantum adiabatic Hamiltonian

$$H(\lambda) = \lambda H_{circuit} + (1 - \lambda)H_{init}$$

in the state space of strings, i.e. bit strings of length n with $n/2$ 1s.

H_{init} penalizes clocks of nearest-neighbor qubits to be synchronized and breaks symmetry between beginning and end of the circuit by forcing times of top & bottom qubit to be at the beginning.



Exact map onto q-deformed Heisenberg ferromagnet!

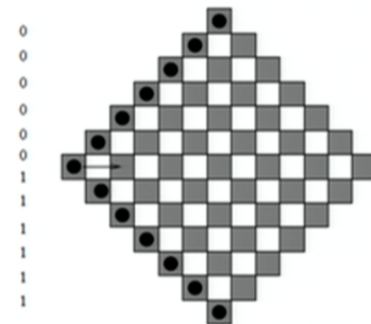
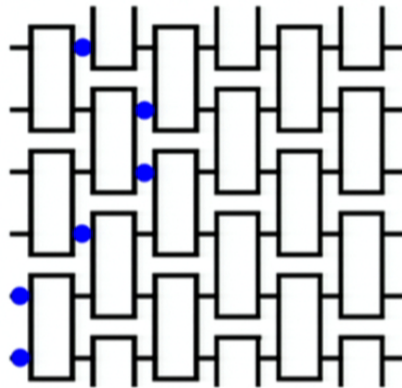
$$H(\lambda) \propto c(\lambda)(1 - \lambda)(Z_n - Z_1) - \frac{1}{2} \sum_{i=1}^{n-1} (\lambda (X_i X_{i+1} + Y_i Y_{i+1}) + Z_i Z_{i+1})$$

Well-studied model (Bethe ansatz etc.): $\frac{q+q^{-1}}{2} = \frac{1}{\lambda}$ (for some $c(\lambda)$)

For $\lambda = 1$: Heisenberg ferromagnet with $SU(2)$ symmetry.

For $\lambda < 1$: q-deformed Heisenberg ferromagnet with $SU_q(2)$ symmetry.

Gap (Koma/Nachtergaele): $2(1 - \lambda \cos(\frac{\pi}{n}))$. **Constant gap for $\lambda < 1$.**



Questions

Continuum Limit of Space-Time Model in 1+1 dim. and 3+1 dim?

Is it of experimental interest/use/advantage to consider a model where the quantum information carrying degrees are moving instead of stationary? Photons/electrons...

Movement needs to be robust: no back-scattering, no particle staying behind before a gate.

Can we show that quantum adiabatic computation can be made fault-tolerant (including fault-tolerance against movement errors)?

Is Quantum Error Correction Realistic?

B.M. Terhal

JARA Institute for Quantum Information
RWTH Aachen University

Review *Quantum Error Correction for Quantum Memories*
(under revision at Rev. Mod. Phys.)
Email gmemoryreview@gmail.com for corrections,
comments in the next few weeks!



Some Thoughts

What physics is a QC going to be built from?

Quantized electromagnetic fields interacting with matter (electrons).
Only quantized EM fields (optical, RF, microwave) gives just linear optics, not strong enough for universality.

But interaction with matter is ruled by a constant

the dimension-less **fine-structure constant**

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \approx 7.297 \times 10^{-3}$$

A threshold given by physics?

Some Thoughts

What physics is a QC going to be built from?

Quantized electromagnetic fields interacting with matter (electrons).
Only quantized EM fields (optical, RF, microwave) gives just linear optics, not strong enough for universality.

But interaction with matter is ruled by a constant

the dimension-less **fine-structure constant**

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \approx 7.297 \times 10^{-3}$$

A threshold given by physics?

Weak Coupling determined by α

Workhorse of quantum optics: Jaynes-Cummings Model

$$H = -\frac{\omega_q}{2}Z + \omega_r a^\dagger a + g(\sigma_- a^\dagger + \sigma_+ a)$$

In cavity QED $\frac{g}{\omega} = O(10^{-6})$ (3D cavity, small dipole moment of atomic levels).

In 1D circuit QED $\frac{g}{\omega} = O(10^{-2})$ (1D cavity, large dipole moment)

(Devoret, Girvin, Schoelkopf, Ann. Physics 2007):

Rydberg atom in 3D cavity $\frac{g}{\omega} \leq O(\alpha^{\frac{3}{2}})$

Analysis for 1D circuit cavity with similar dipolar coupling: $\frac{g}{\omega} \leq O(\sqrt{\alpha})$

Circuit QED: $\omega_q, \omega_r = O(1)\text{GHz}, g = O(10)\text{MHz}$

Engineering: different coupling in circuit QED can give ultra-strong coupling regime $\frac{g}{\omega} \sim \alpha^{-1/2}$

Consequences of weak interaction

- Photon detection and single photon generation non-trivial (through conversion of photon into electric current). Not yet existing/under development in microwave domain (low energy photons..), hence low-temperature amplifiers needed.
- Effective non-linearities (which can generate squeezing, amplification, codewords of qubit-into-oscillator code) are **weak**

In dispersive regime $\Delta = \omega_q - \omega_r \gg g$, one derives from a Jaynes-Cummings Hamiltonian:

$$H_{eff} = (\omega_r - \chi Z)a^\dagger a - \frac{1}{2}(\omega_q + \chi)Z + \frac{5g^4}{\Delta^3}Z(a^\dagger a)^2, \chi = \frac{g^2}{\Delta}$$

Is linear optics with very small g non-universal for QC (classically simulatable?)

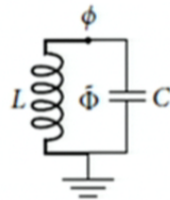
Asymmetry between strength of magnetic and electric field

$$\alpha = \frac{Z_{vacuum}}{2R_K}$$

where $Z_{vacuum} = \frac{1}{\epsilon_0 c}$ is vacuum impedance (strength of electric field over strength of magnetic field in vacuum) in units of Ohm Ω .

Resistance quantum $R_K = \frac{h}{e^2}$ (Hall conductance in integers/fractions of $1/R_K$.)

Consider a LC
Circuit with
impedance



$$H = \frac{Q^2}{2C} + \frac{\phi^2}{2L}, \quad [\phi, Q] = i\hbar.$$

Harmonic oscillator with variables charge Q and magnetic flux ϕ . Capacitance C plays the role of mass of the particle.

If $Z_0 = \sqrt{L/C} \leq Z_{vacuum} = 2R_K \alpha$, **it is hard to get a large inductance L .**

➤ phase variable only small fluctuations (charge: large fluctuations)

Engineering: towards superinductance with multiple **Josephson junctions** (Manachuryan et al.)

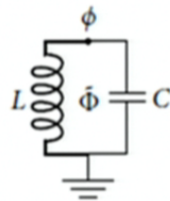
Asymmetry between strength of magnetic and electric field

$$\alpha = \frac{Z_{vacuum}}{2R_K}$$

where $Z_{vacuum} = \frac{1}{\epsilon_0 c}$ is vacuum impedance (strength of electric field over strength of magnetic field in vacuum) in units of Ohm Ω .

Resistance quantum $R_K = \frac{h}{e^2}$ (Hall conductance in integers/fractions of $1/R_K$.)

Consider a LC
Circuit with
impedance



$$H = \frac{Q^2}{2C} + \frac{\phi^2}{2L}, \quad [\phi, Q] = i\hbar.$$

Harmonic oscillator with variables charge Q and magnetic flux ϕ . Capacitance C plays the role of mass of the particle.

If $Z_0 = \sqrt{L/C} \leq Z_{vacuum} = 2R_K \alpha$, it is hard to get a large inductance L .

➤ phase variable only small fluctuations (charge: large fluctuations)

Engineering: towards superinductance with multiple **Josephson junctions** (Manachuryan et al.)