Title: TBA

Date: Jul 15, 2014 09:00 AM

URL: http://pirsa.org/14070006

Abstract:

Exceptional service in the national interest

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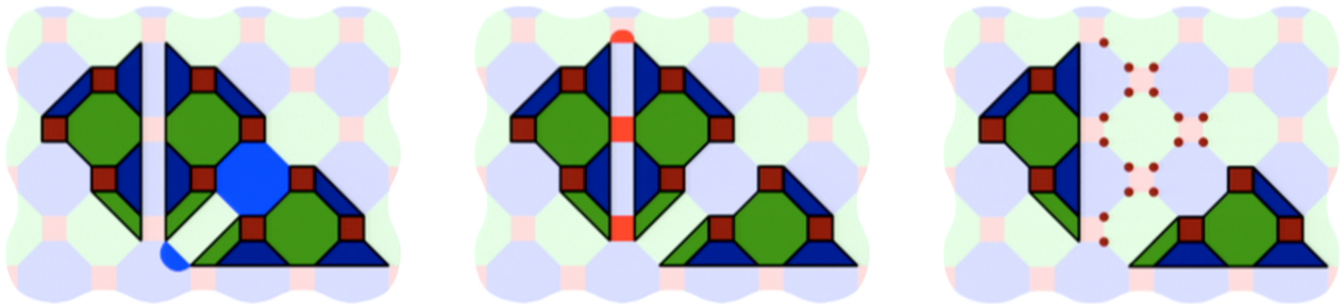

Quantum computing by color-code lattice surgery

Andrew J. Landahl

Joint work with Ciarán Ryan-Anderson

Sandia National Laboratories

15 July 2014

Topological quantum codes

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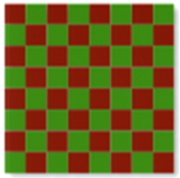
Why they are awesome

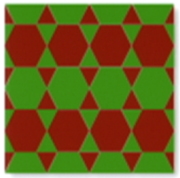
1. High accuracy thresholds.
2. 2D local quantum processing.
3. Low quantum circuit overheads.
4. Efficient decoding algorithms.
5. Smooth interpolation between desired effective error rates.

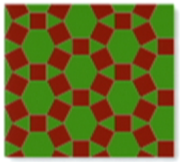
Anderson's Classification Theorem [1]

The only planar topological stabilizer codes with nonlocal logical operators are surface codes or color codes.

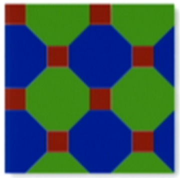
Surface codes

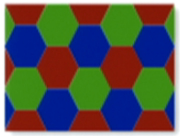

4.4.4.4

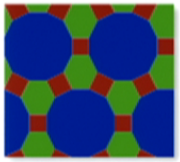

3.6.3.6


3.4.6.4

Color codes


4.8.8


6.6.6


4.6.12

[1] Anderson, arXiv:1107.2502 (2011)

Encoded ("logical") qubits

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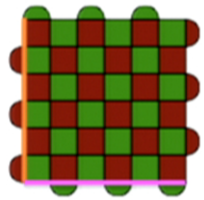
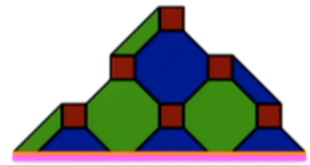
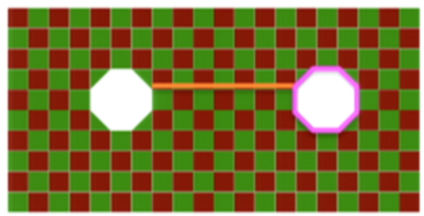
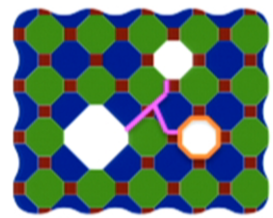
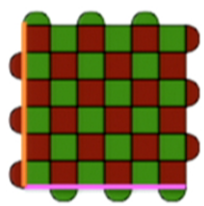
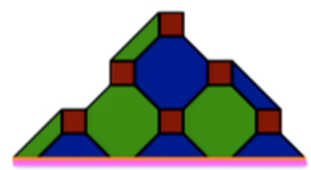
	Surface	Color
Transversal		
Defects		
Lattice surgery		

A nice universal gate basis

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Stabilizer operations: $\{I, |0\rangle, |+\rangle, M_Z, M_X, S, H, CNOT\}$

Non-Clifford operation: $T|+\rangle$

Gottesman-Knill Theorem [1] consequence:

- Can avoid Pauli operators by classically propagating "Pauli frame" to measurement outcomes.
- Does *not* mean we can simulate these circuits efficiently (they act on non-stabilizer states).
- Argument applies to physical-gate syndrome extraction circuits too—they are stabilizer circuits.

We may ***want*** to apply Pauli operators, though, to avoid poly-time classical computation.

[1] Gottesman, arXiv:quant-ph/9807006 (1998).

Encoded CNOT gate

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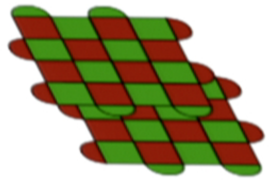
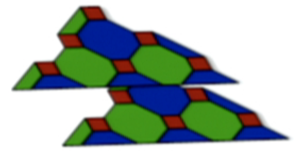
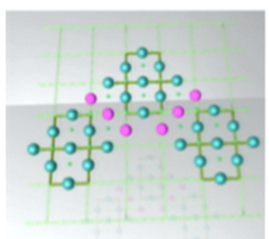
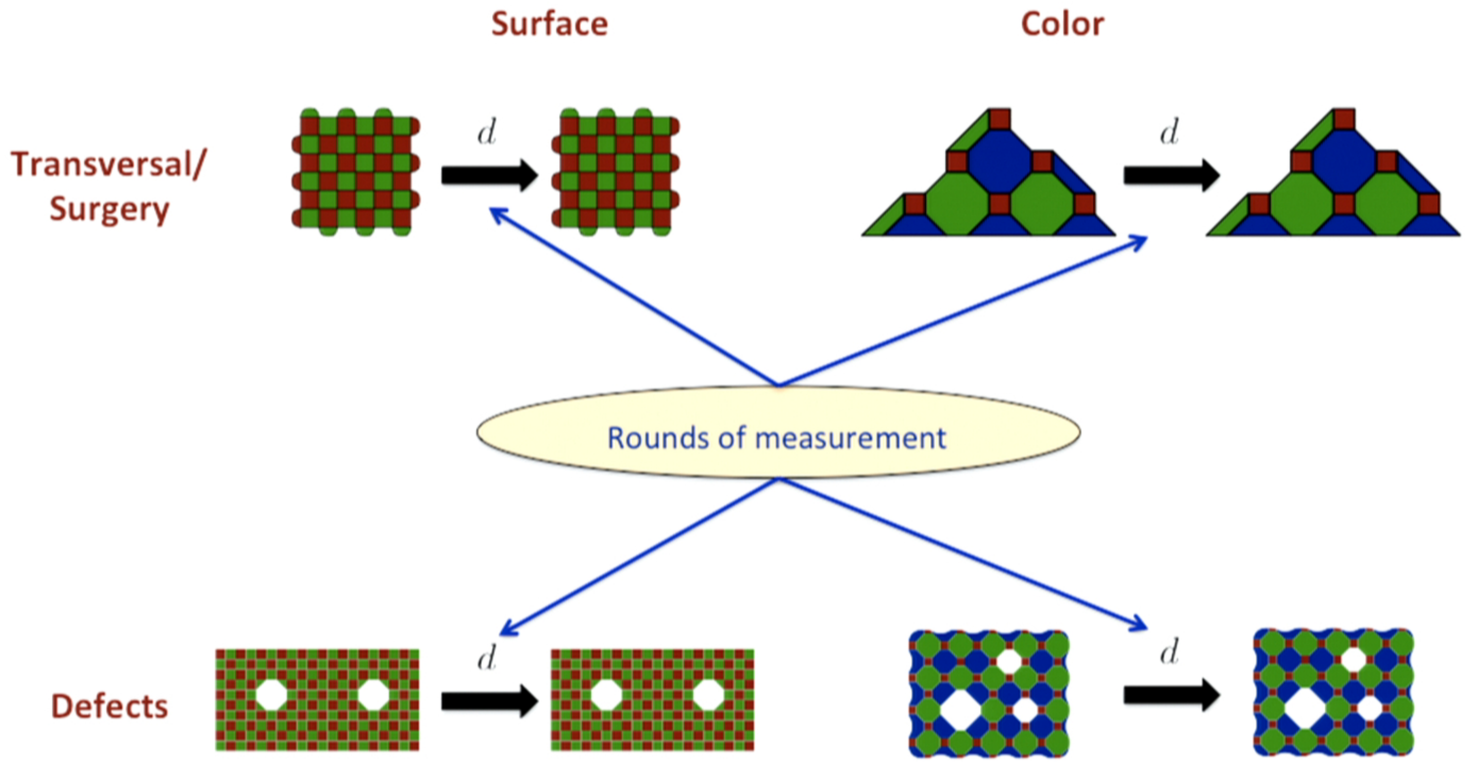
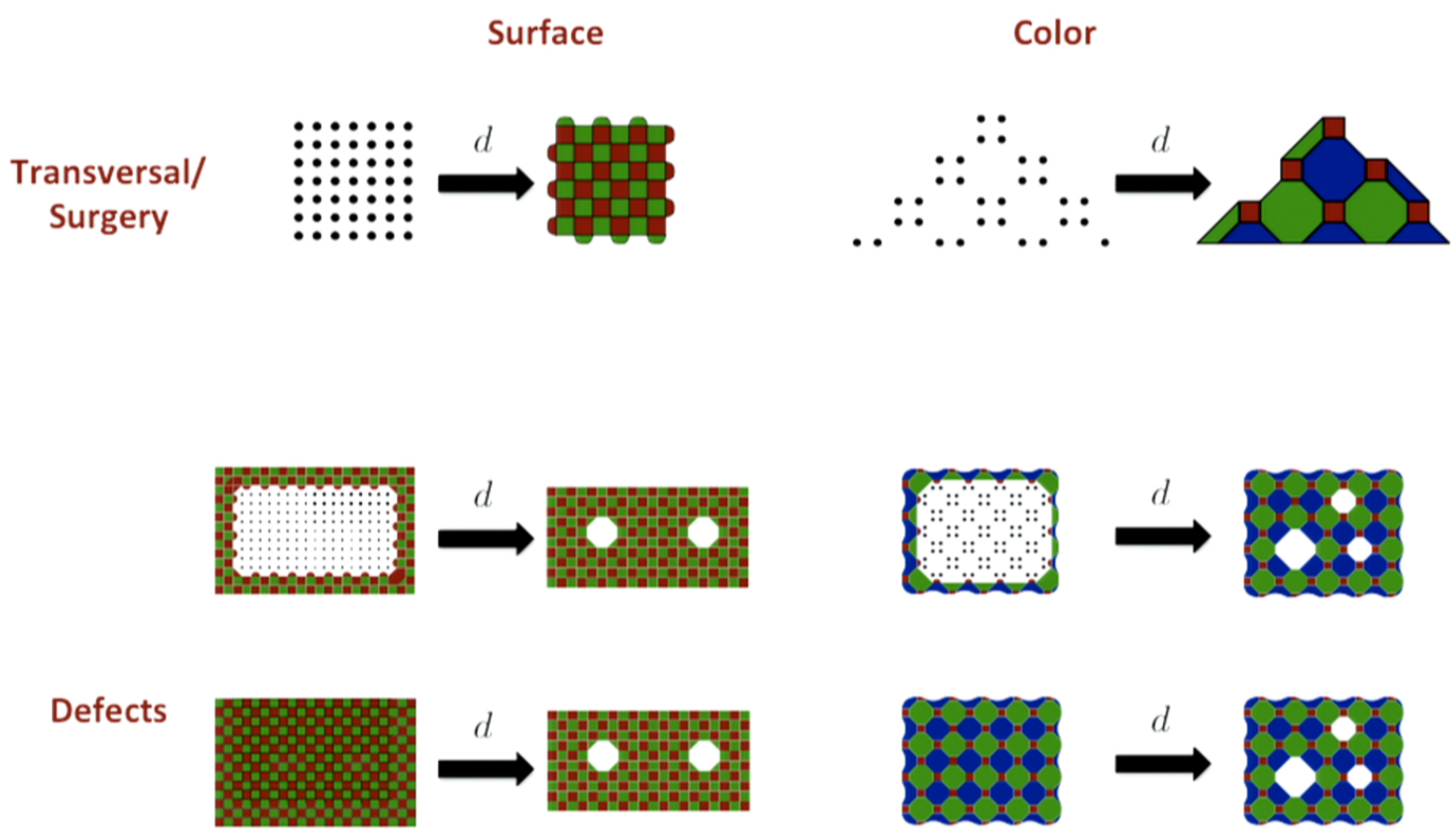
	Surface	Color	
Transversal			Swapping
Defects	[1] 	[2] 	Qubits
Lattice surgery	[3] 	?	Awesome

[1] Fowler & Devitt, arXiv:1209.0510 (2012). [2] Fowler, PRA **83**, 042310 (2011). [3] Horsman *et al.*, NJP **14**, 123011 (2012).

Encoded identity gate



Encoded Pauli preparation gates



Encoded Pauli measurement gates

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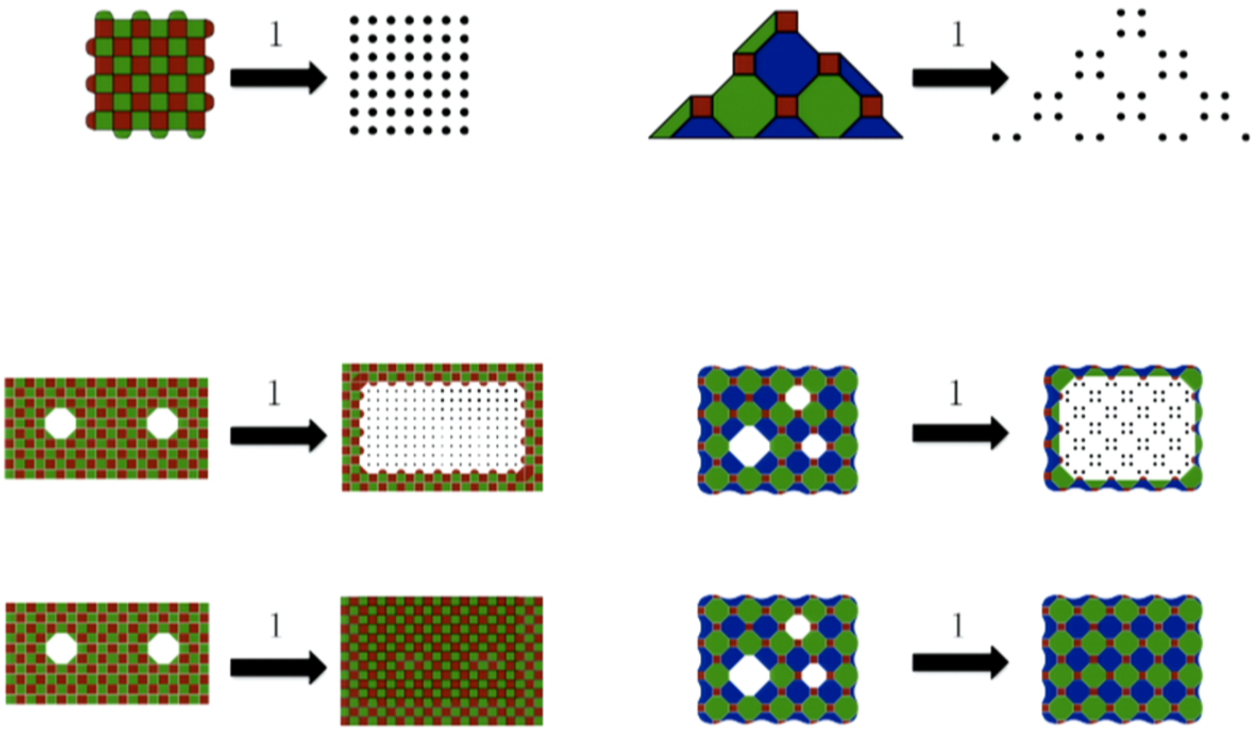
Surface | **Color**

Transversal/ Surgery

Defects



Key circuits for lattice surgery

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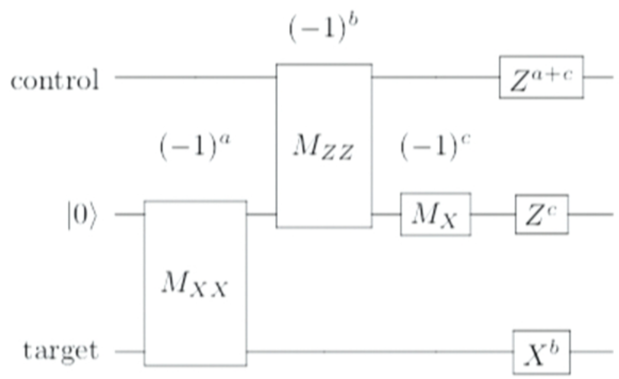
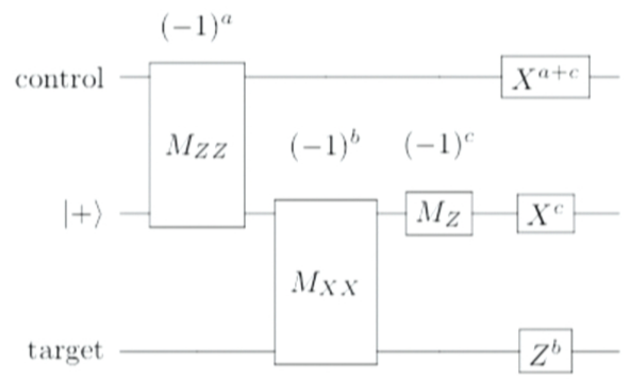
Two circuits equivalent to the CNOT gate





- Used heavily to combat biased noise in [Aliferis & Preskill, PRA **78**, 052331 (2008)].
- Left circuit used implicitly in [Horsman *et al.*, New J. Phys. **14**, 123011 (2012)].

Encoded color-code M_{XX} and M_{ZZ}

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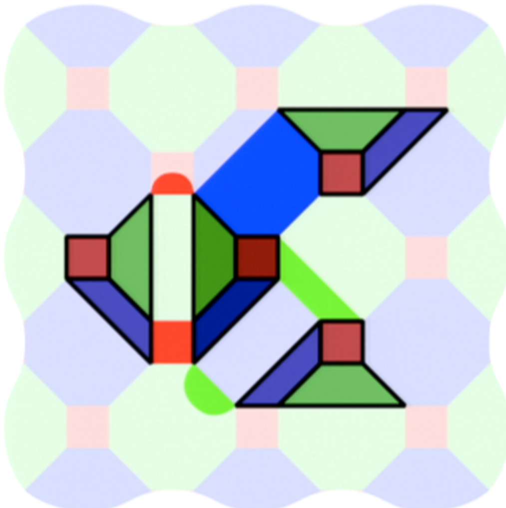
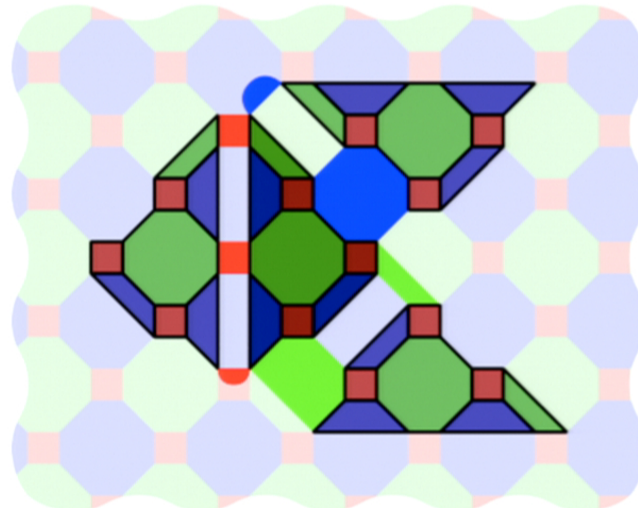
Can make measurements along any of three sides of the central logical qubit





- Measure only *X* (*Z*) on same-colored light checks along the "osculant" interface; their product is *XX* (*ZZ*).
- Half of the "half-octagon" *Z* (*X*) checks along interface vanish during fusion because they anticommute.
- Other half of half-octagon *Z* (*X*) checks deform to full octagons; measure *X* and *Z* checks on these.
- Digon checks can be "folded in" to reuse syndrome qubits in other faces, if desired.

Encoded surface-code M_{XX} and M_{ZZ}

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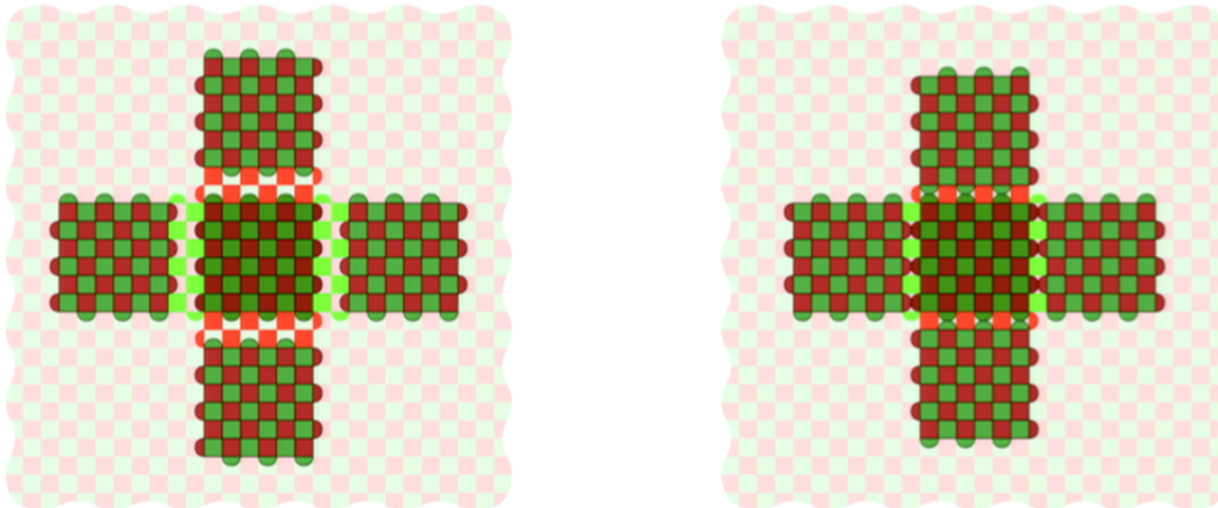
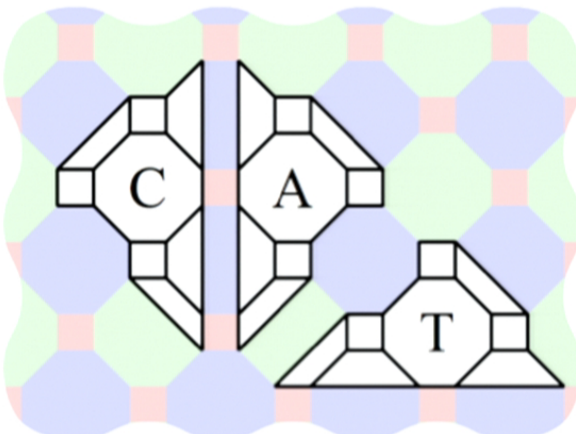
Osculant operations for surface codes

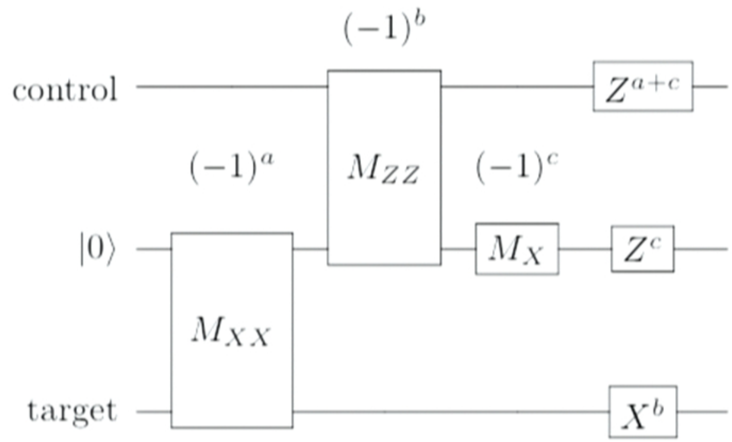


- Surface codes can only measure M_{XX} in one direction and M_{ZZ} in the other.
- Right figure compresses construction from [Horsman *et al.*, New J. Phys. **14**, 123011 (2012)].

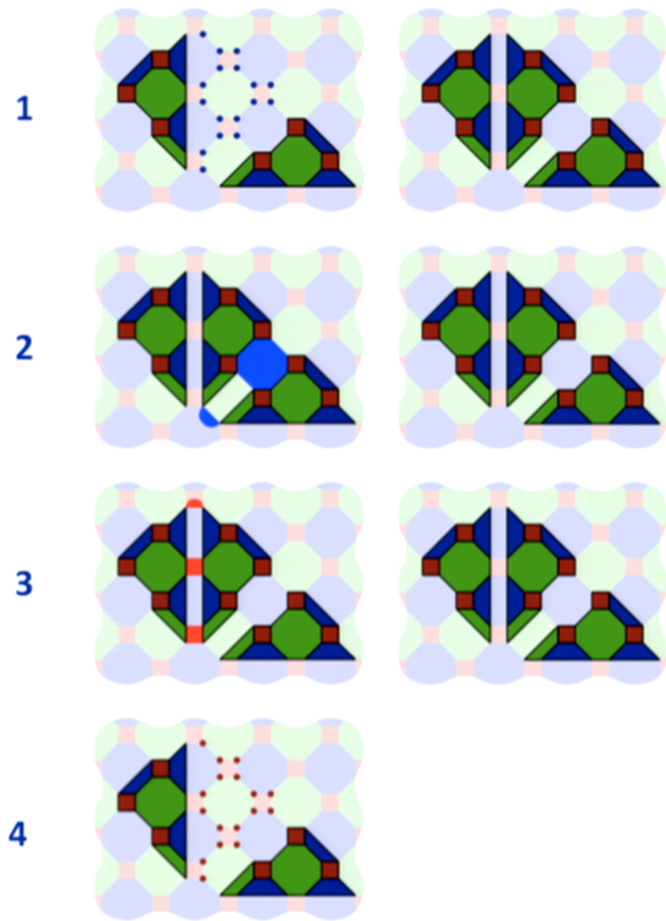
Encoded color-code CNOT gate

The setting:

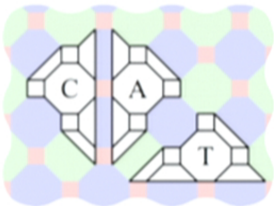


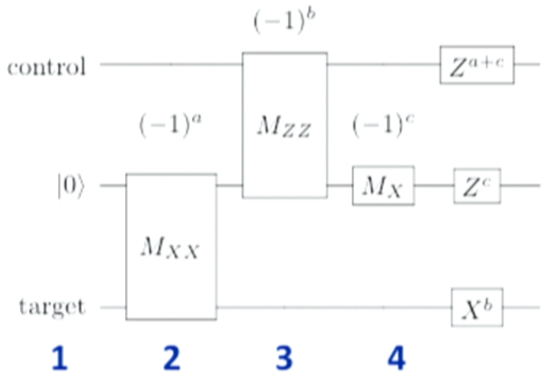


Encoded color-code CNOT gate



The setting:





Encoded color-code CNOT gate

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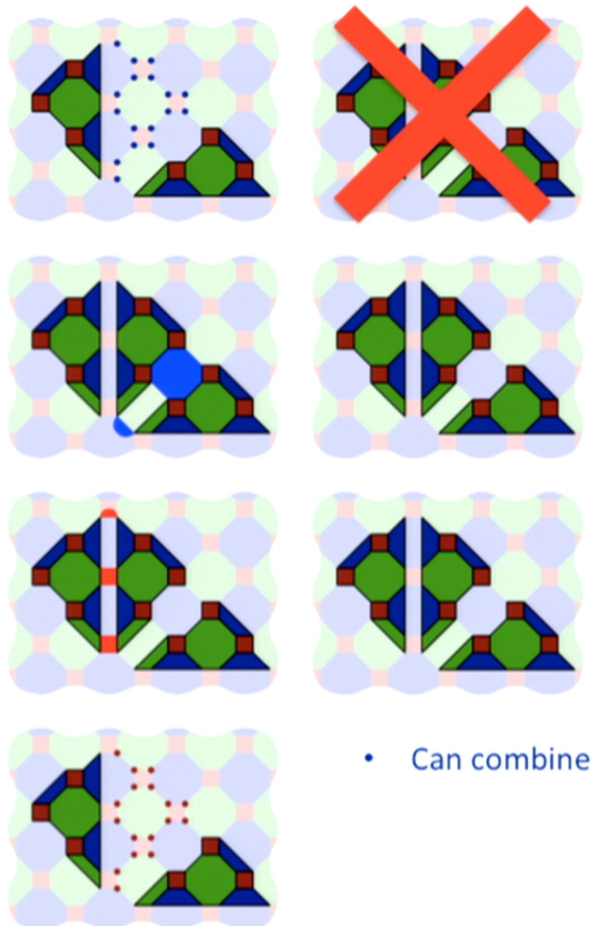
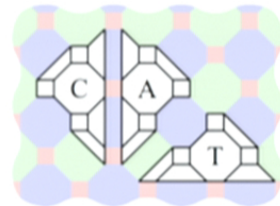
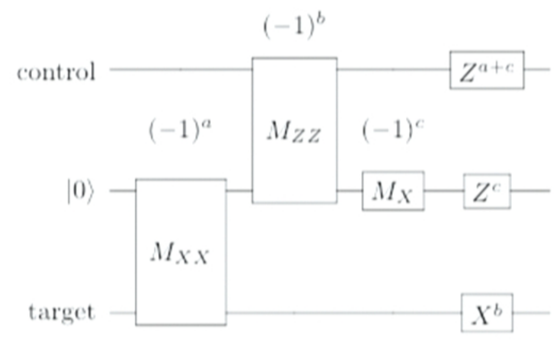
1
2
3
4



The setting:





- Can combine prep with fuse (1b)

Encoded color-code CNOT gate

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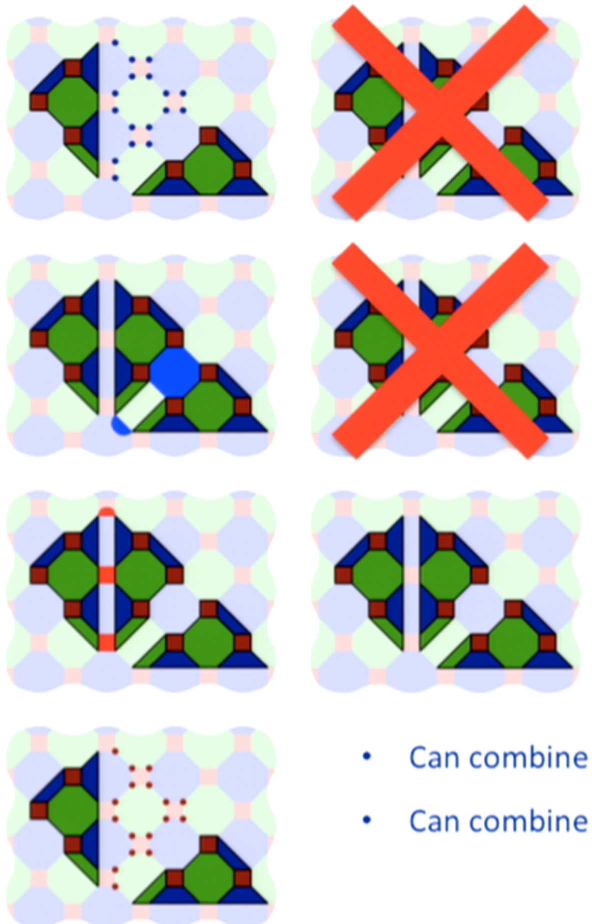
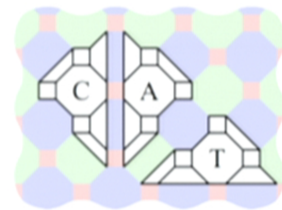
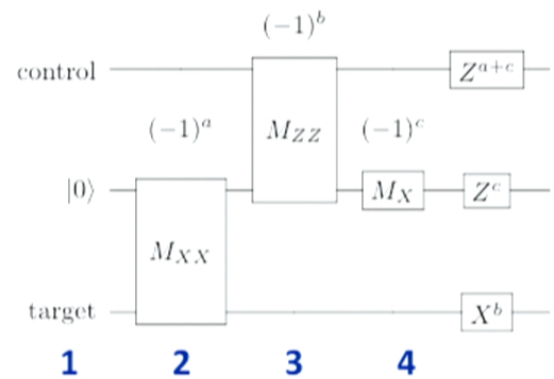
1
2
3
4



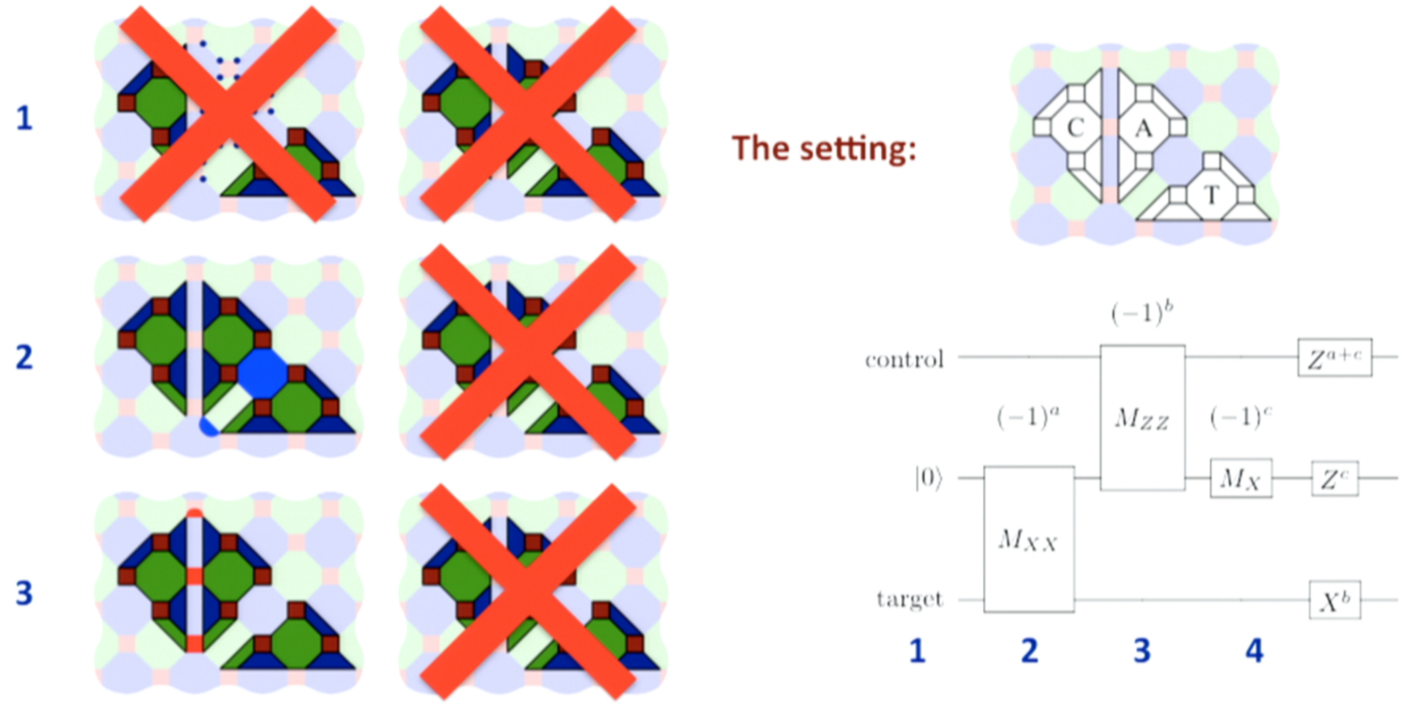
The setting:





- Can combine prep with fuse (1b)
- Can combine split on one side with fuse on another (2b)

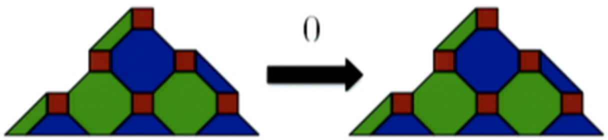
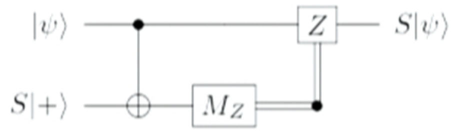
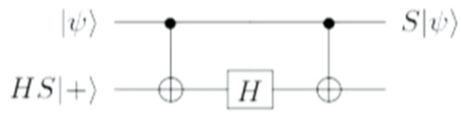
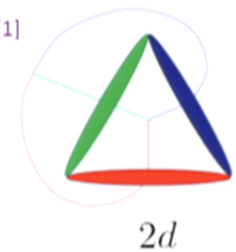
Encoded color-code CNOT gate



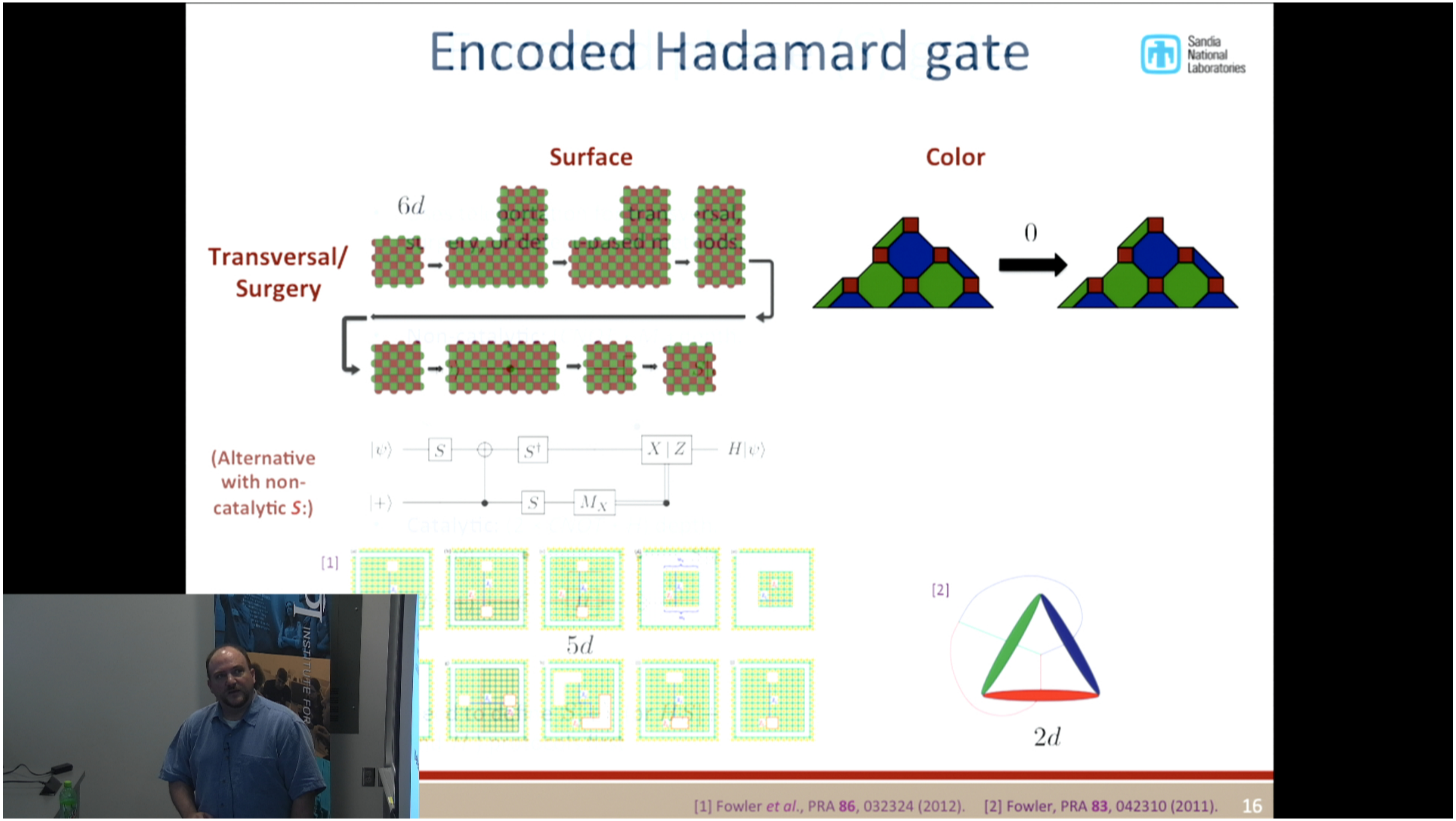
- Can combine prep with fuse (1b)
- Can combine split on one side with fuse on another (2b)
- Can combine split with measure (3b)
- Can combine measure with prep (1a)

$3d$ rounds!

Encoded phase (S) gate

	Surface	Color
Transversal/ Surgery	• Uses teleportation for transversal, surgery, or defect-based methods.	
	• **Non-catalytic:** ($CNOT + M_Z$) depth. 	
	• **Catalytic:** ($2 \times CNOT + H$) depth. 	[1] 
Defects	• Need to define $S\|+\rangle$ (or $HS\|+\rangle$ and H) protocols first	

[1] Fowler, PRA **83**, 042310 (2011).


Encoded Hadamard gate
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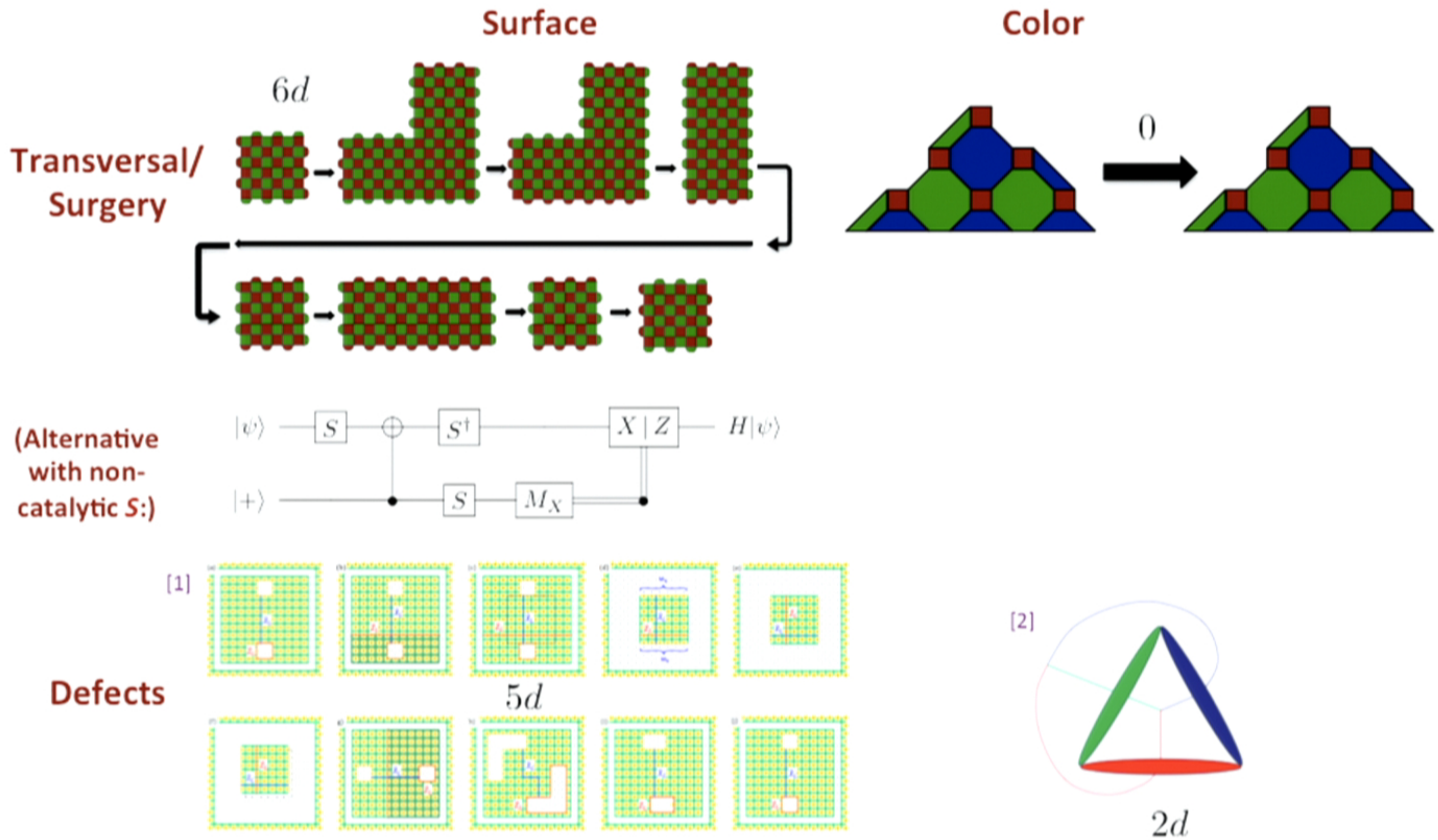
Surface
Color
Transversal/ Surgery
6d
0
(Alternative with non-catalytic S:)
|ψ⟩
S
S†
X | Z
H|ψ⟩
|+⟩
S
M_X
[1]
5d
[2]
2d
[1] Fowler et al., PRA 86, 032324 (2012). [2] Fowler, PRA 83, 042310 (2011).
16

Encoded Hadamard gate



[1] Fowler *et al.*, PRA **86**, 032324 (2012). [2] Fowler, PRA **83**, 042310 (2011).

Color-code state injection

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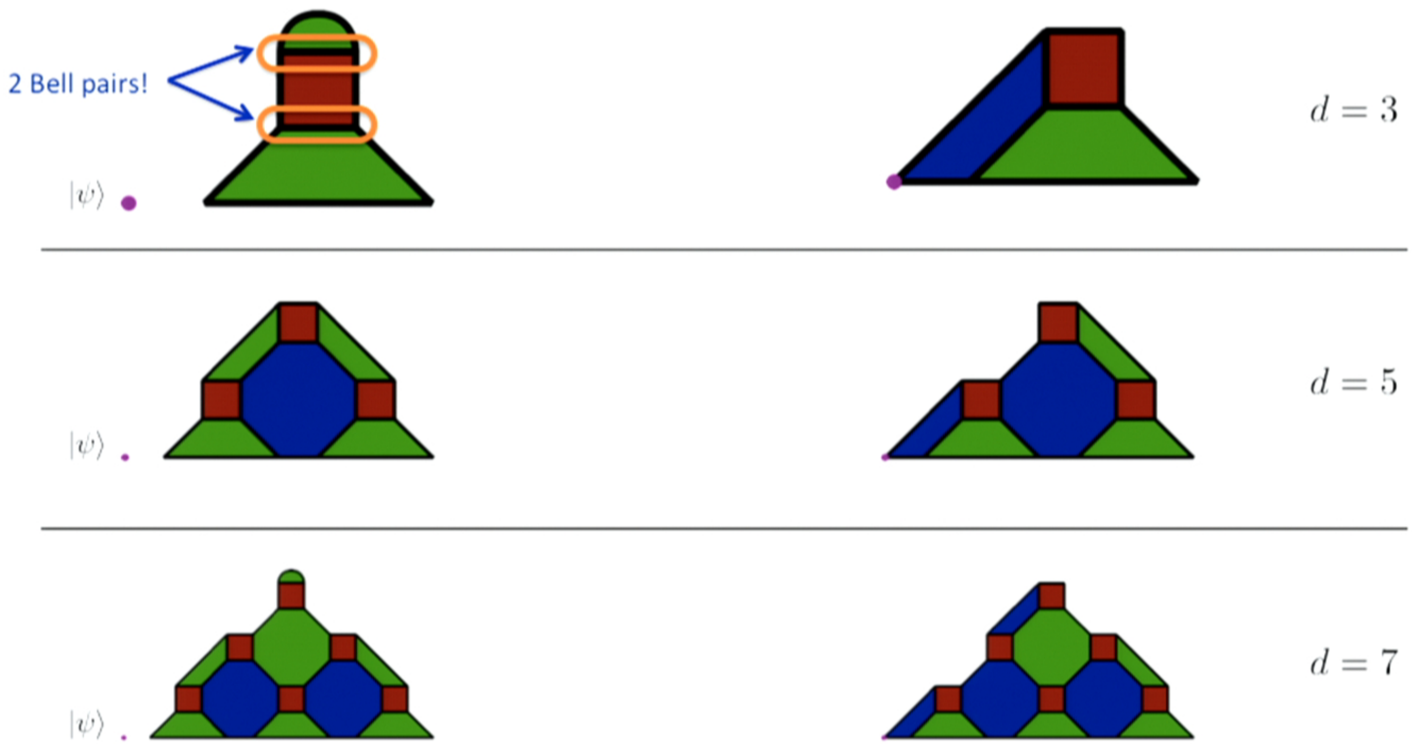
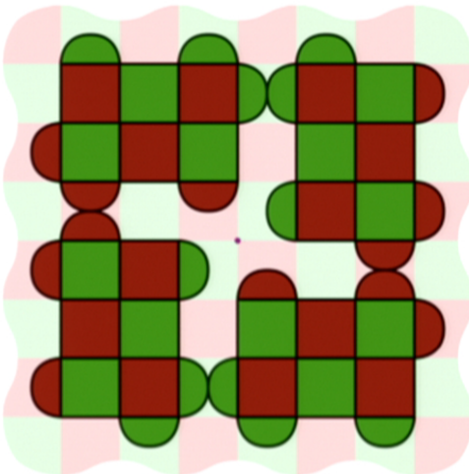
Step 1: 3 rounds

Step2: 3 rounds

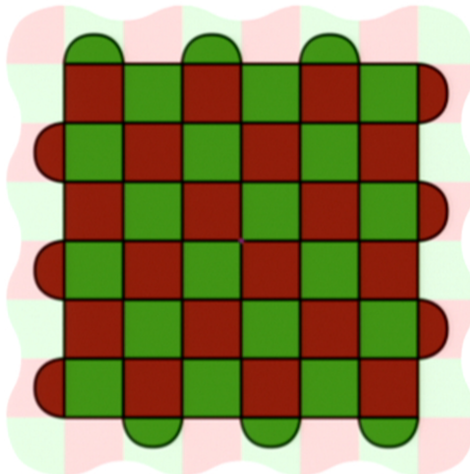


Surface-code state injection

Step 1: 3 rounds



Step2: 3 rounds



Resource recap

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Compared to surface codes, color codes…

- **…use half the qubits for the same distance**
- **…use less than or equal to the measurement rounds per distance**

Color-code lattice surgery (1 syndrome qubit per face)

Gate	$T\|+\rangle$	I	$\|0\rangle$	$\|+\rangle$	M_Z	M_X	H	S	$CNOT$
Depth	6	d			1		0		$3d$
Qubits	d^2+2d-2								$3d^2+6d-6$
Error	$\mathcal{O}(p)$	$\mathcal{O}(p^{(d+1)/2})$							

Surface-code lattice surgery (1 syndrome qubit per face)

Gate	$T\|+\rangle$	I	$\|0\rangle$	$\|+\rangle$	M_Z	M_X	H	S	$CNOT$
Depth	6	d			1		$6d$	$12d$	$3d$
Qubits	$2d^2-2d+1$						$6d^2-6d+3$		
Error	$\mathcal{O}(p)$	$\mathcal{O}(p^{(d+1)/2})$							

But what's the overhead to achieve a fixed level of error suppression?

Need error model and syndrome decoding algorithm to answer…

Syndrome decoding

Surface codes

1. **Optimal decoder:** #P hard? [1]
2. **PEPS decoder:** approximates optimal decoder in linear time! [2]
3. **Matching-based decoders:**
   1. Perfect matching [3]
   2. Renormalization group [4, 5]
   3. Clustering [6, 7]
   4. Global attractive force CA [8]

Color codes

1. **Optimal decoder:** #P hard? [1]
2. **PEPS decoder:** Exists?
3. **IP decoder:** NP hard? [9]
4. **Matching-based decoders:**
   1. Perfect matching [10, 11]
   2. Renormalization group [5, 12, 13]
   3. Clustering?
   4. Global attractive force?
5. **Surface-code decoders** [14, 15]

Not clear how to extend all of these decoders to circuit-level models

[1] Iyer & Poulin, arXiv:1310.3235 (2013).

[2] Bravyi, Suchara, & Vargo, arXiv:1405.4883 (2014).

[3] Dennis JMP 43, 4452 (2002).

[4] Duclos-Cianci & Poulin, PRL **104**, 050504 (2010).

[5] Bravyi & Haah, PRL **111**, 200501 (2013).

[6] Dennis, UCSB PhD thesis (2003).

[7] Harrington, Caltech PhD thesis (2004).

[8] Herold et al., arXiv:1406.2338 (2014).

[9] Landahl, Anderson, & Rice, arXiv:1108.5738 (2011).

[10] Wang et al., QIC **10**, 780 (2010).

[11] Stephens, arXiv:1402.3037 (2014).

[12] Sarvepalli & Raussendorf, arXiv:1111.0831 (2011).

[13] Duclos-Cianci & Poulin, QIC **14**, 0721 (2014).

[14] Bombin, Duclos-Cianci & Poulin, NJP **14**, 073048 (2012).

[15] Delfosse, PRA **89**, 012317 (2014).

Qubit overhead

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Failure probability of encoded data

- Expression depends on *p*, *d*. ("Low *d*" is d < 1/4*p*, "High *d*" is *d* > 1/4*p*.)
- Beautiful formulas only for code-capacity, phenomenological models.
- Surface-code circuit-level Monte Carlo data for non-asymptotic *d* and low *p* fits the following function well [1]:

$$p_{\text{fail}} = A(d)\left(\frac{p}{p_{\text{th}}}\right)^{d/2}$$

Color-code increase to achieve surface-code error suppression

$$d_c = d_s\left(\frac{\log p/p_{\text{th}}^{(s)}}{\log p/p_{\text{th}}^{(c)}}\right) + 2\left(\frac{\log A_s(d)/A_c(d)}{\log p/p_{\text{th}}^{(c)}}\right).$$

≈0

Accuracy threshold estimates

- **Color codes:** 0.082(3)% [1] to 0.143% [2]
- **Surface codes:** 0.502(1)% to 1.140(1)% [3]

Not awesome?

Qubit overhead

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Failure probability of encoded data

- Expression depends on *p*, *d*. ("Low *d*" is d < 1/4*p*, "High *d*" is *d* > 1/4*p*.)
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≈0

Accuracy threshold estimates

- **Color codes:** 0.082(3)% [1] to 0.143% [2]
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Not awesome?

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Qubit overhead

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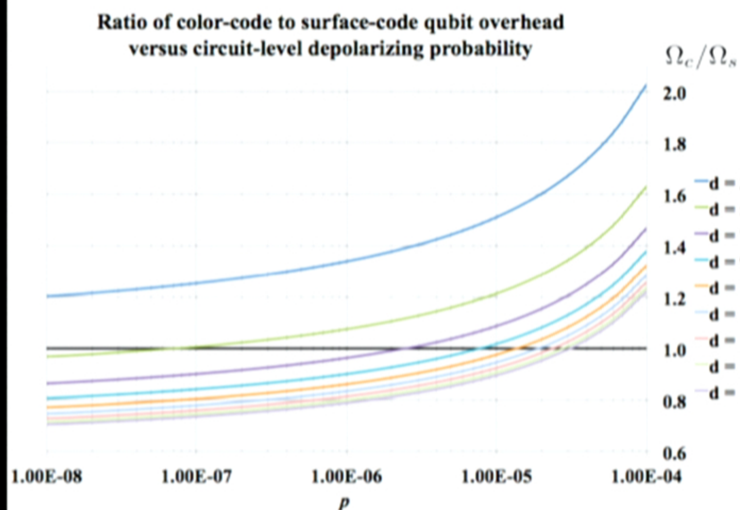
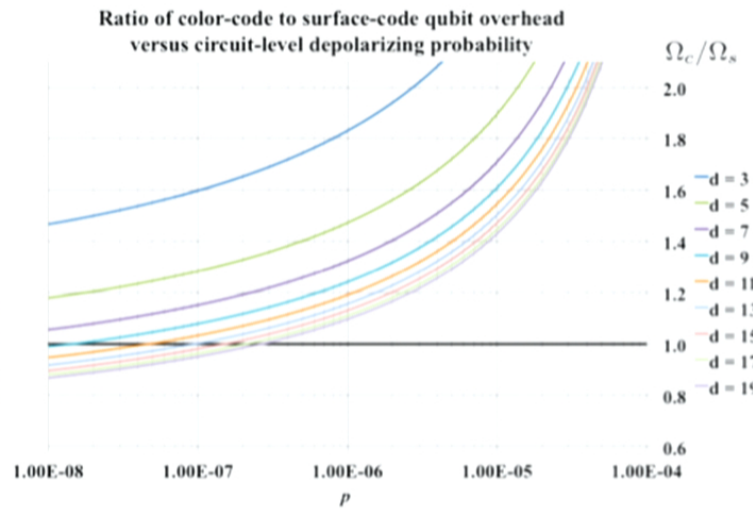
$p_c = 0.143\%, p_s = 0.502\%$ $\qquad$ $p_c = 0.082\%, p_s = 1.140\%$





Color codes **DO** use fewer resources…at low enough *p* and high enough *d*.

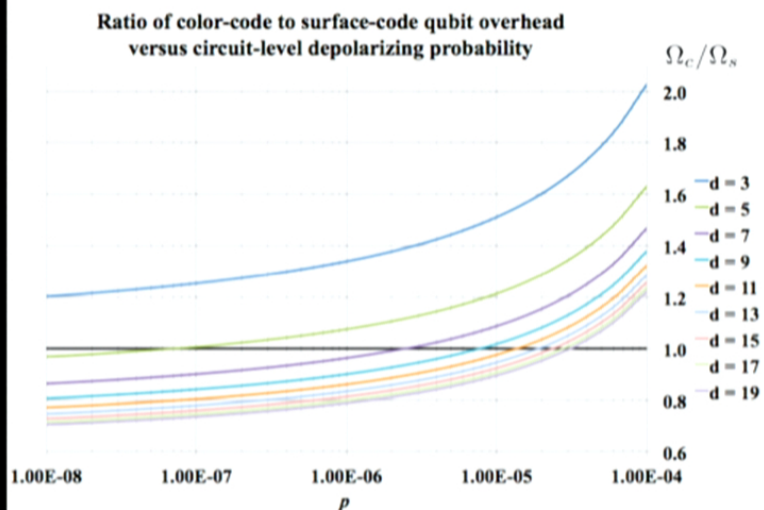
Room for more improvement

- Numerical estimates of overhead for each *p* (removes need for fit $p_{\text{fail}} = A(d)\left(\frac{p}{p_{\text{th}}}\right)^{d/2}$).
- Develop better color-code decoders. (PEPS?)
- Subsystem color codes?

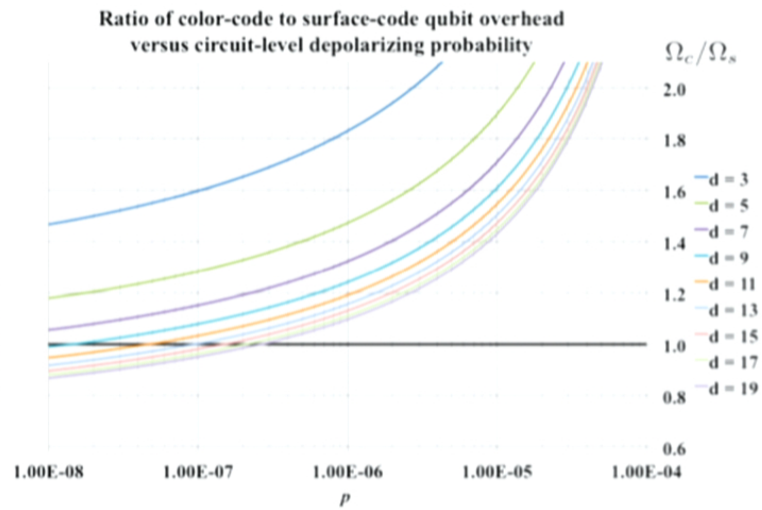
Qubit overhead

Sandia National Laboratories

$p_c = 0.143\%, p_s = 0.502\%$



$p_c = 0.082\%, p_s = 1.140\%$

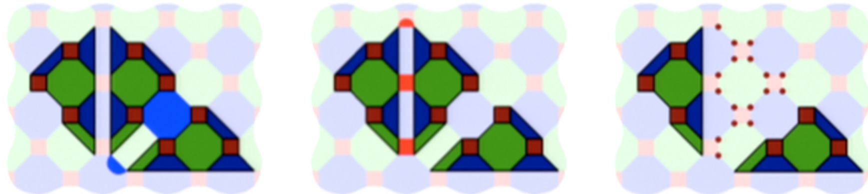


Color codes **DO** use fewer resources…at low enough *p* and high enough *d*.

Room for more improvement

- Numerical estimates of overhead for each *p* (removes need for fit $p_{\text{fail}} = A(d)\left(\frac{p}{p_{\text{th}}}\right)^{d/2}$).
- Develop better color-code decoders. (PEPS?)
- Subsystem color codes?

Summary



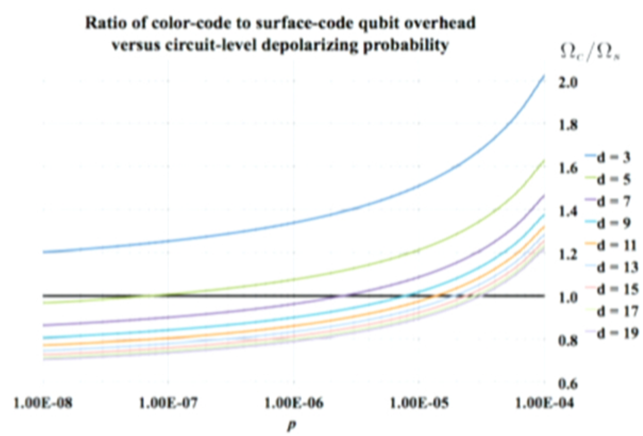
- Half the qubits per code distance
- Fewer qubits per logical failure probability at low enough *p* and high enough *d*
- MUCH faster *H* and *S* gates
- We improved surface-code lattice surgery along the way

Color-code lattice surgery (1 syndrome qubit per face)

Gate	$T\|+\rangle$	I	$\|0\rangle$	$\|+\rangle$	M_Z	M_X	H	S	$CNOT$
Depth	6	d			1		0		$3d$
Qubits	d^2+2d-2								$3d^2+6d-6$
Error	$\mathcal{O}(p)$	$\mathcal{O}(p^{(d+1)/2})$							

Surface-code lattice surgery (1 syndrome qubit per face)

Gate	$T\|+\rangle$	I	$\|0\rangle$	$\|+\rangle$	M_Z	M_X	H	S	$CNOT$
Depth	6	d			1		$6d$	$12d$	$3d$
Qubits	$2d^2-2d+1$						$6d^2-6d+3$		
Error	$\mathcal{O}(p)$	$\mathcal{O}(p^{(d+1)/2})$							




Summary
Half the qubits per code distance
Fewer qubits per logical failure probability at low enough p and high enough d
MUCH faster H and S gates
We improved surface-code lattice surgery along the way
Andrew J. Landahl
PERIMETER