Title: Quantum Error Correction for Ising Anyon Systems

Date: Jul 14, 2014 05:00 PM

URL: http://pirsa.org/14070005

Abstract: We consider two-dimensional lattice models that support Ising anyonic excitations and are coupled to a thermal bath, and we propose a phenomenological model to describe the resulting short-time dynamics, including pair-creation, hopping, braiding, and fusion of anyons. By explicitly constructing topological quantum error-correcting codes for this class of system, we use our thermalization model to estimate the lifetime of quantum information stored in the code space. To decode and correct errors in these codes, we adapt several existing topological decoders to the non-Abelian setting: one based on Edmond's perfect matching algorithm and one based on the renormalization group. These decoders provably run in polynomial time, and one of them has a provable threshold against a simple iid noise model. Using numerical simulations, we find that the error correction thresholds for these codes/decoders are comparable to similar values for the toric code (an Abelian sub-model consisting of a restricted set of allowed anyons). To our knowledge, these are the first threshold results for quantum codes without explicit Pauli algebraic structure. Joint work with Courtney Brell, Simon Burton, Guillaum Dauphinais, and David Poulin, arXiv:1311.0019.

Error Correction for Ising Anyons

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Seefeld, 2 July 2014

arXiv:1311.0019

Topological Error Correction

- Key problem for a robust quantum memory:
  Nature decoheres fragile superpositions of quantum states
- Quantum error correction was developed to deal with this, but the requirements are quite daunting
- The key idea of topological quantum error correction is:

A. Kitaev, Ann. Phys. 2003

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- The key idea of topological quantum error correction is:

If Nature acts locally, *then encode the information* globally

A. Kitaev, Ann. Phys. 2003

Topological Quantum Computation

- Topological Quantum Codes:
  - Store information robustly in global degrees of freedom

    e.g. Toric Code
- Topological Quantum Computation:
  - Process information stored in global degrees of freedom
  - Quantum gates correspond to topological invariants – robust to deformation
- Ising anyons (and other nonAbelian anyons) can be used for computation, but *they still need error correction*!
  The goal of this project is to demonstrate that this is possible.

Topological Error Correction

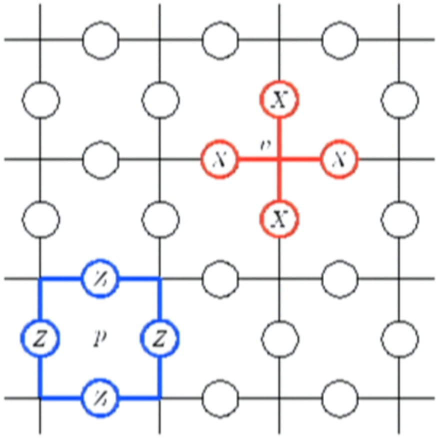
The paradigmatic example is Kitaev's toric code

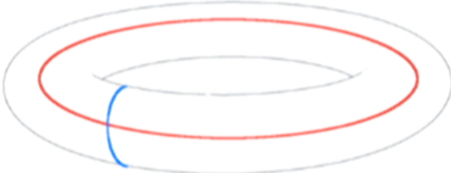
A. Kitaev, Ann. Phys. 2003

Topological Error Correction

The paradigmatic example is Kitaev's toric code

- Qubits on edges of a periodic square lattice
- Local stabilizer operators associated to vertices and plaquettes
- Logical operators are non-contractible loops around the torus
- Can interpret the stabilizer operators as giving rise to localized particles
- Are these particles bosons or fermions?

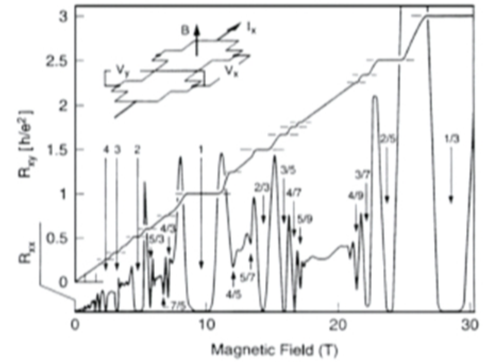


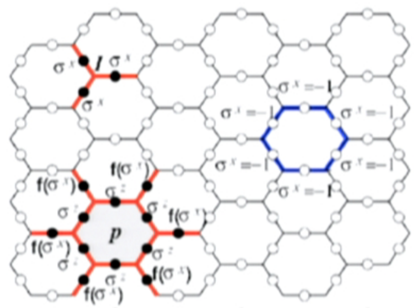


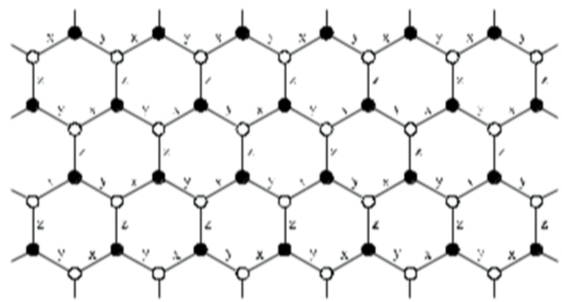
A. Kitaev, Ann. Phys. 2003

Ising anyons?

- Experimentally accessible
  - Fractional quantum Hall systems
  - Kitaev honeycomb model
- String-net (Levin-Wen) models







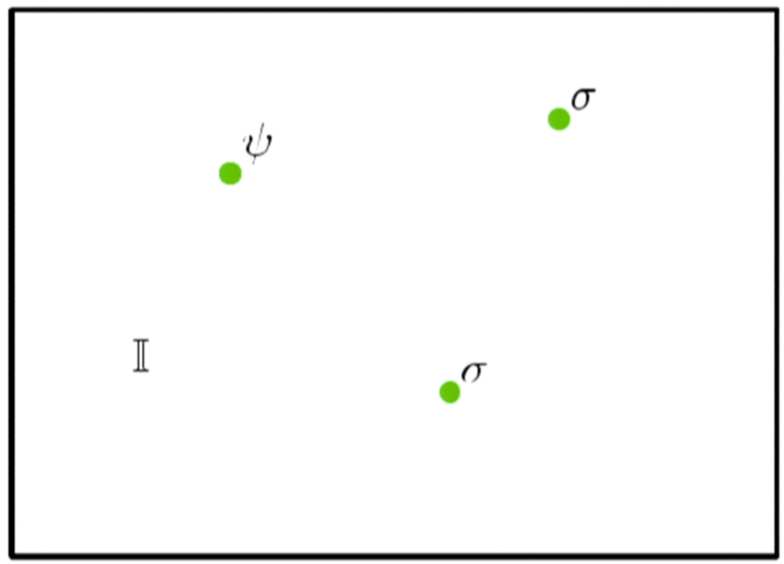
- Nonabelian, but classically simulable
- Ising anyons + magic = universal quantum computation

Bravyi, PRA (2006)

Ising Anyons and their phenomenology

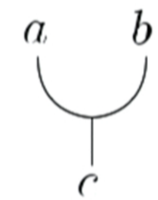
- Consider local excitations—anyons—in a 2D quantum system
  - Anyon types = conserved charges
  - Vacuum $\mathbb{I}$
  - Ising anyons

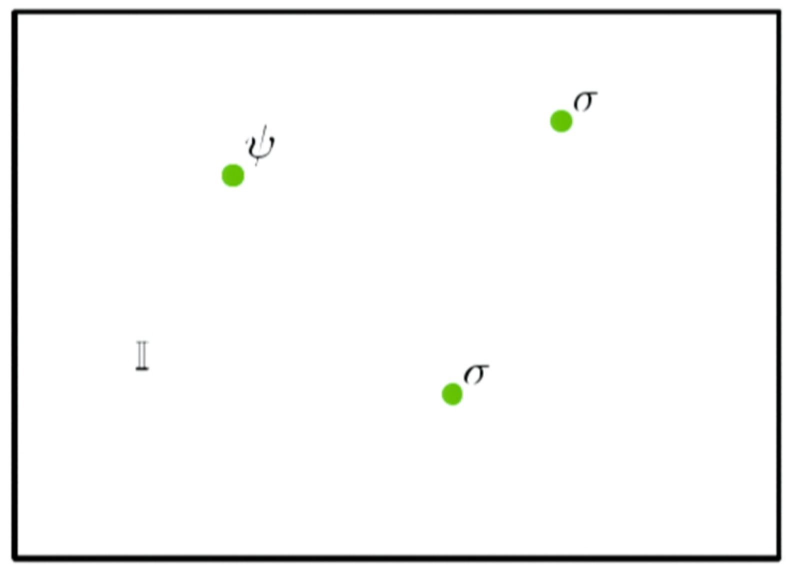
    ψ, σ
- Two processes of interest:
  - Fusion
  - Braiding



Anyon Fusion

- Fusion is commutative, associative, distributive
  - Total charge of pair of anyons
  - $a \times b = c + d + \ldots$





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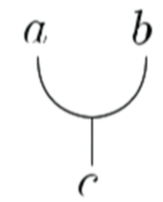
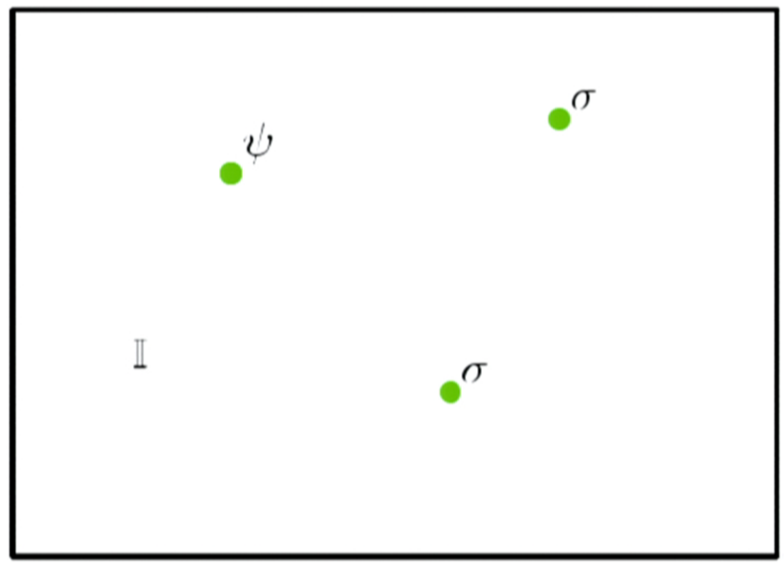


- Ising anyons:
  - $\psi \times \psi = \mathbb{I}$
  - $\psi \times \sigma = \sigma$
  - $\sigma \times \sigma = \mathbb{I} + \psi$



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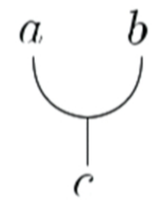

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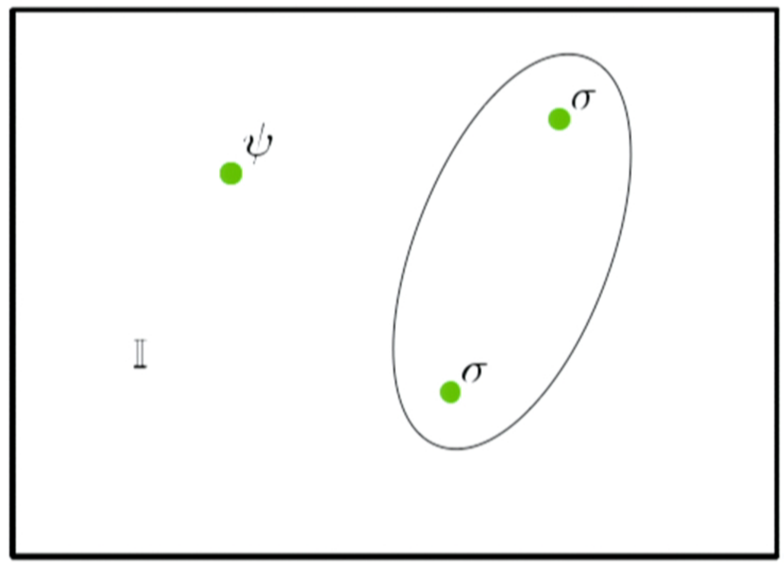
nonlocal Hilbert space
nonAbelian anyon



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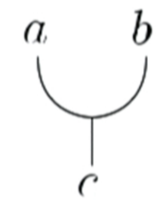

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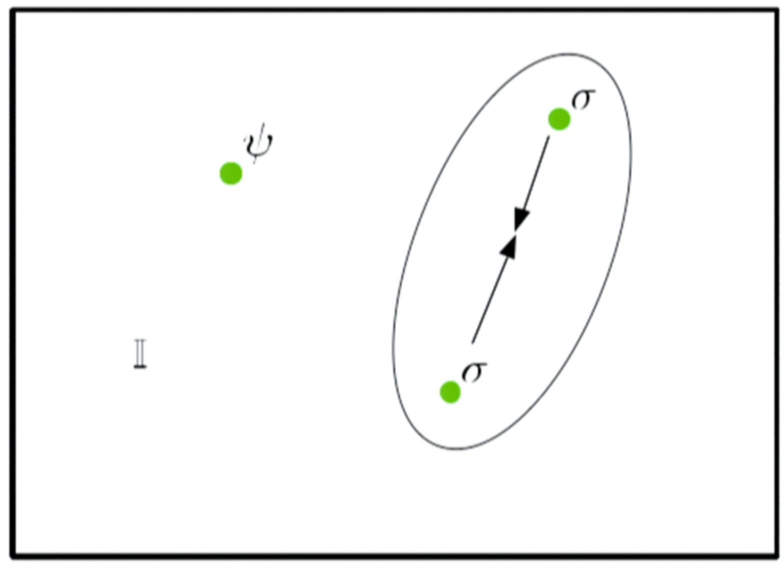

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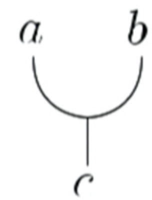

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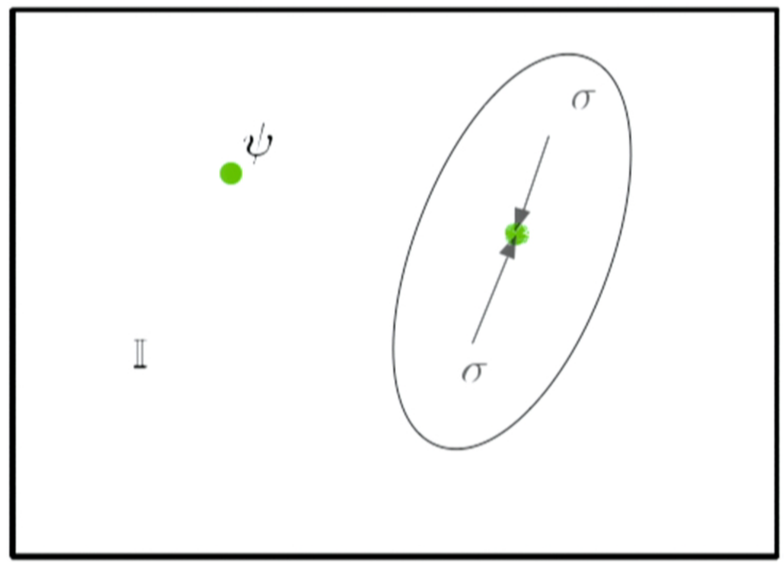
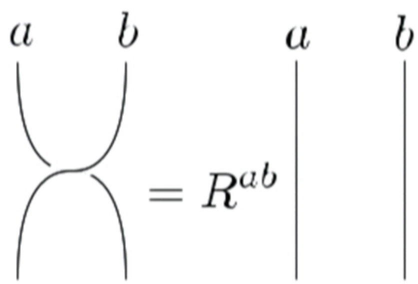


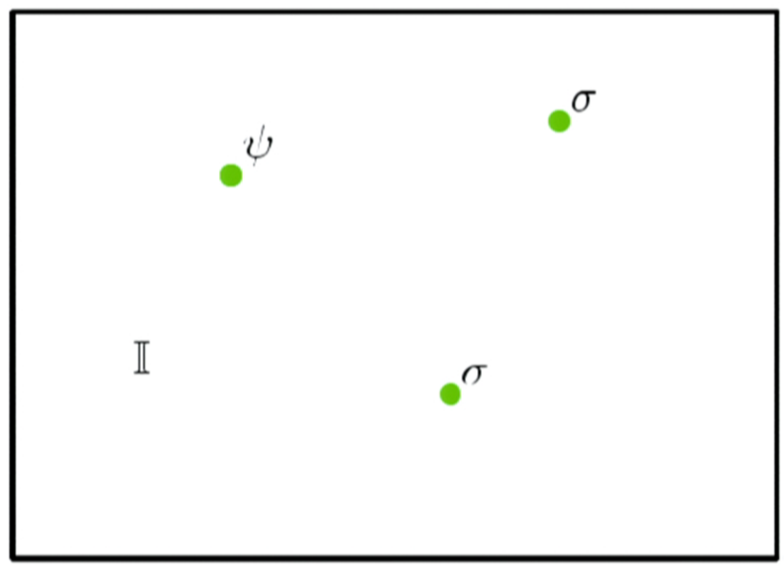
Error Correction for Ising Anyons

Noncommutivity makes things challenging...

Anyon Dynamics

- Fusion outcomes depend on the history of the anyons
- Can be changed by braiding anyons around one another



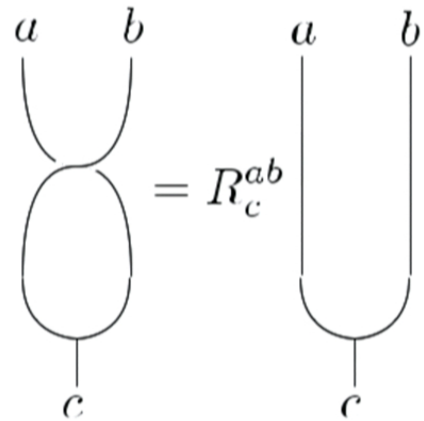


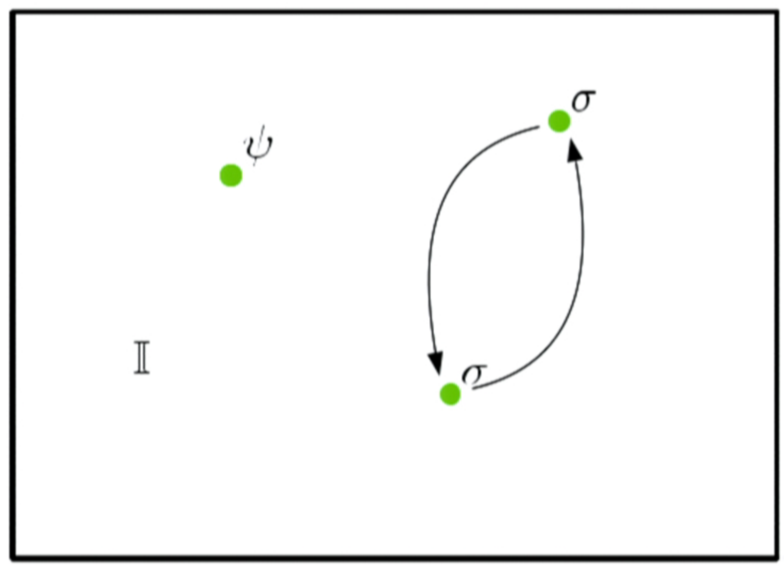
Anyon Dynamics

- Ising anyons:

$$R_{\mathbb{I}}^{\psi,\psi} = -1 \qquad R_{\mathbb{I}}^{\sigma,\sigma} = e^{-i\frac{\pi}{8}}$$

$$R_{\sigma}^{\psi,\sigma} = R_{\sigma}^{\sigma,\psi} = -i \qquad R_{\psi}^{\sigma,\sigma} = e^{3i\frac{\pi}{8}}$$

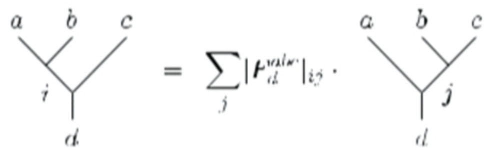


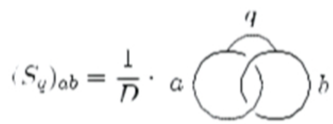


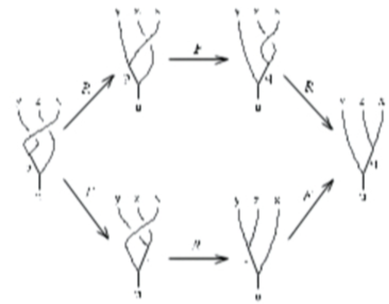
Error Correction for Ising Anyons

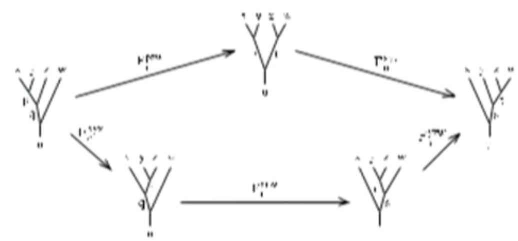
Anyon Dynamics - Math

- Anyons described by "unitary braided fusion category"
  - R-matrices
  - Fusion rules
  - F-symbols (associators)
  - (Modular) S-matrix
- Consistency relations
  - Hexagon identity
  - Pentagon identity





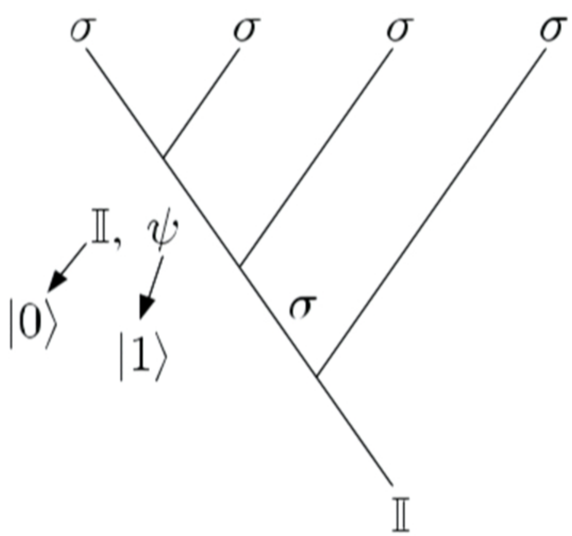




Kitaev, Ann. Phys. (2006)

The 1 Qubit Ising Anyon Computer

- Encode a qubit in 4 anyons



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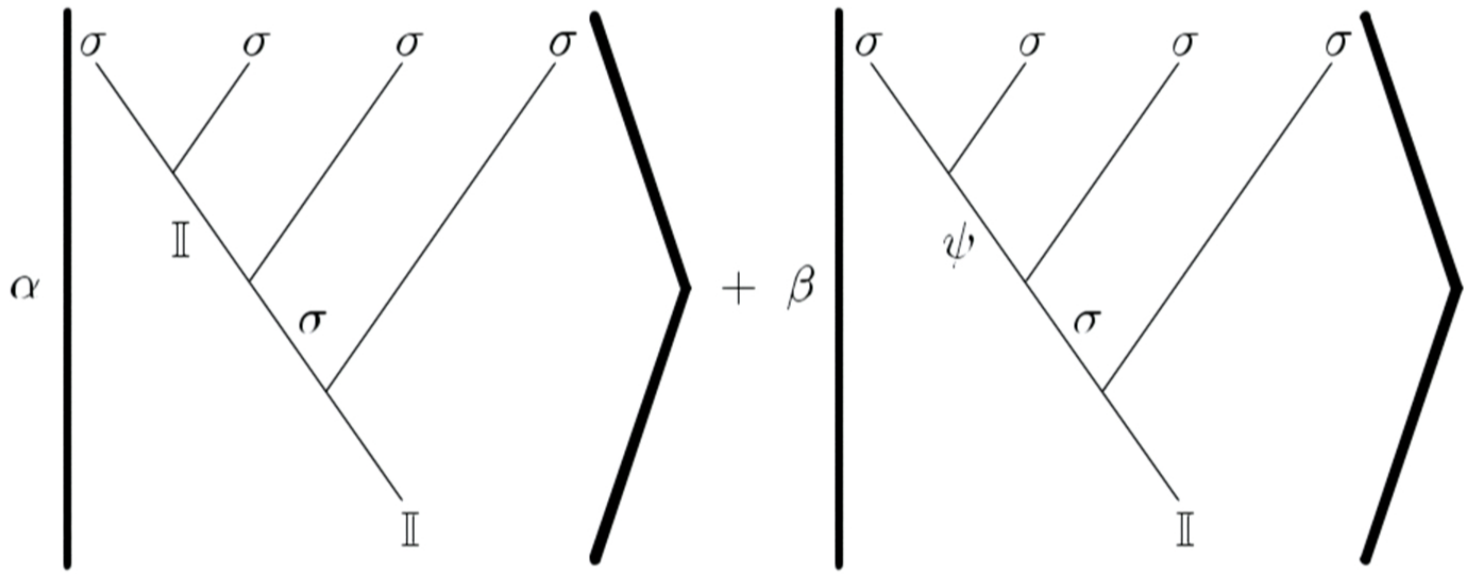

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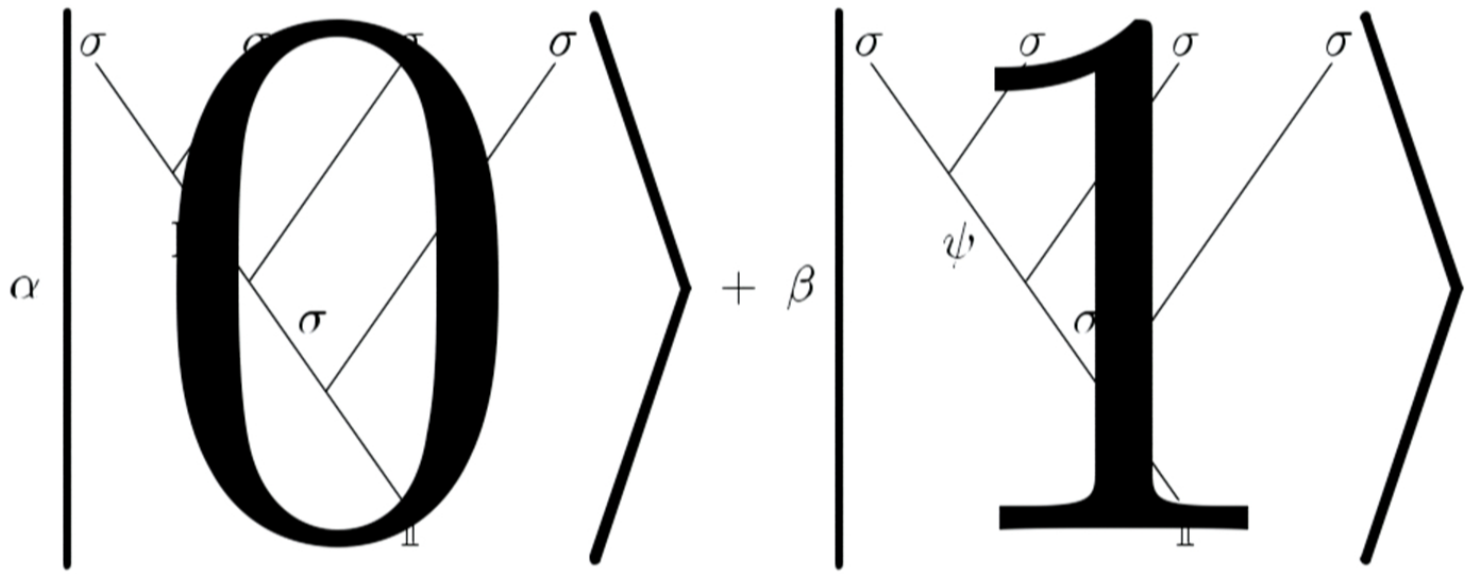
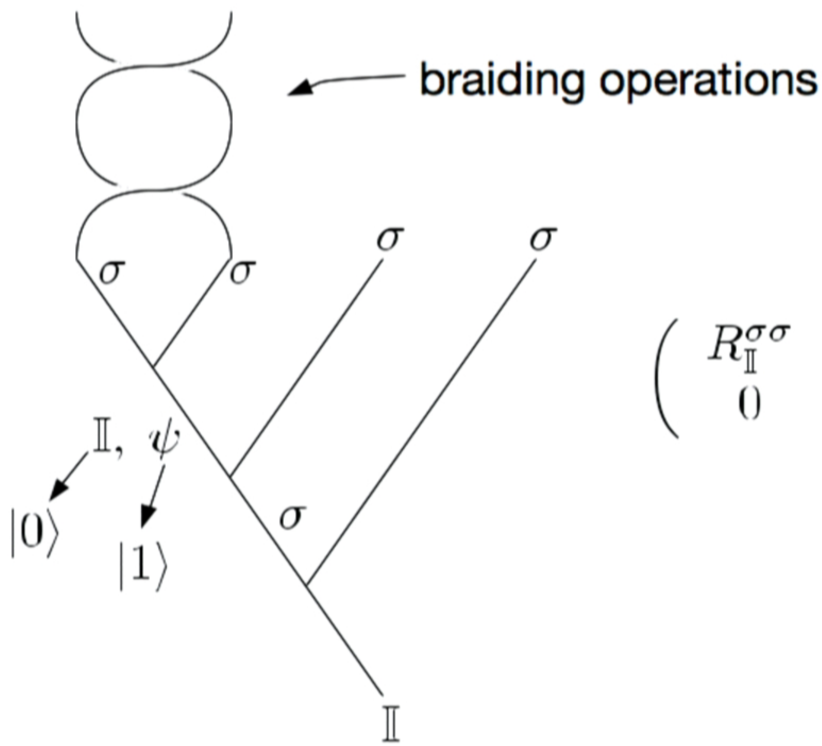


The 1 Qubit Ising Anyon Computer

- Perform gates by braiding

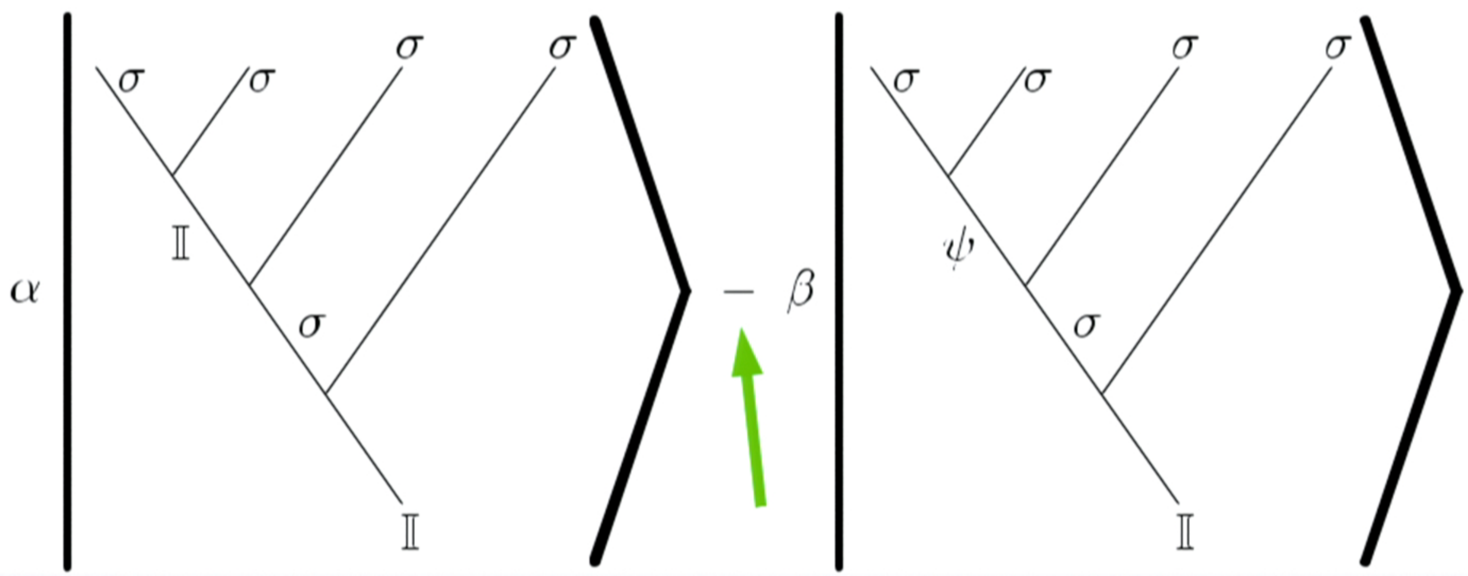


$$\begin{pmatrix} R^{\sigma\sigma}_{\mathbb{I}} & 0 \\ 0 & R^{\sigma\sigma}_{\psi} \end{pmatrix}^2 = e^{-i\frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The 1 Qubit Ising Anyon Computer

- Perform gates by braiding (alternatively tunneling)

$=$ up to global phase of $e^{-i\frac{\pi}{4}}$



Simulating Ising Anyon Dynamics

- Assign each σ anyon a Majorana mode c
- Eigenvalues of $M = -ic_1c_2$ correspond to fusion outcomes
- Braiding two neighboring σ anyons clockwise gives

$$c_1 \to c_2 \qquad\qquad c_2 \to -c_1$$

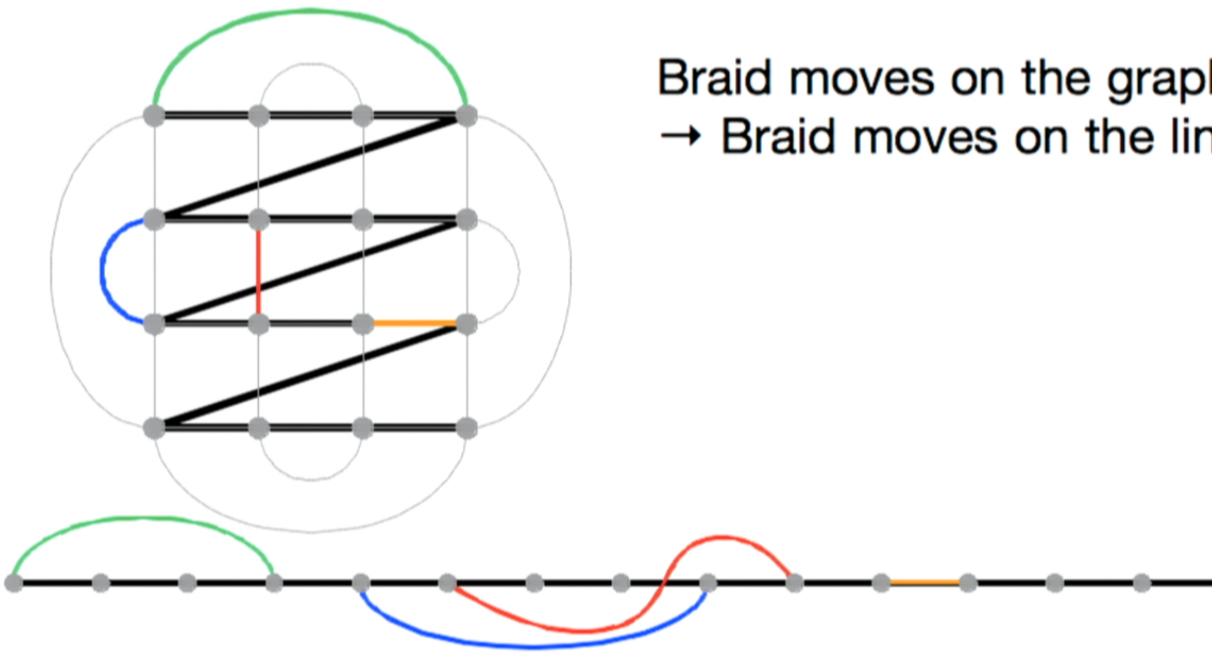
- Keep track of 'stabilizers' of Majorana operators
- Could map via Jordan-Wigner to Clifford operations

- ψ particles only introduce global phases, can largely be ignored

Bravyi PRA (2006)

Simulating Ising Anyon Dynamics

- Linearized braid sequences

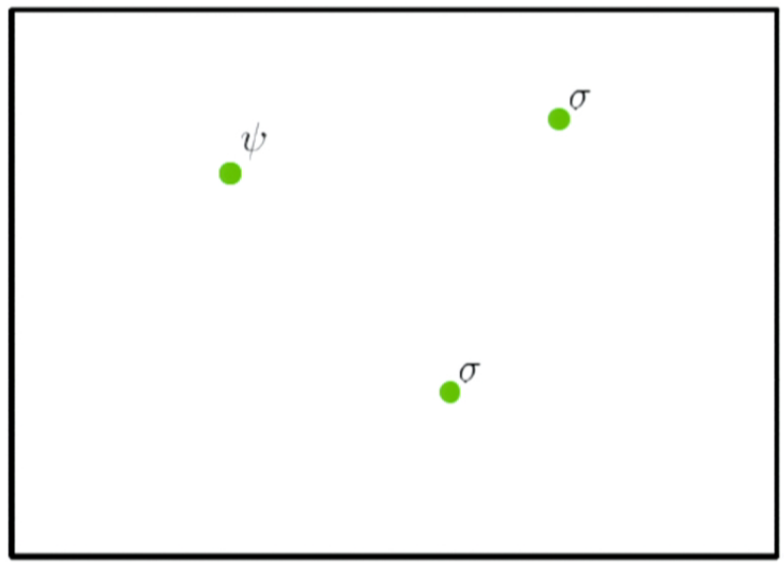


Braid moves on the graph
→ Braid moves on the line

Phenomenological Model of Anyon Dynamics

- Discretize space
- At each point, 3-dimensional Hilbert space
  - Basis $|\mathbb{I}\rangle$, $|\psi\rangle$, $|\sigma\rangle$

- Pair Creation
- Hopping
- Exchange
- Decoherence



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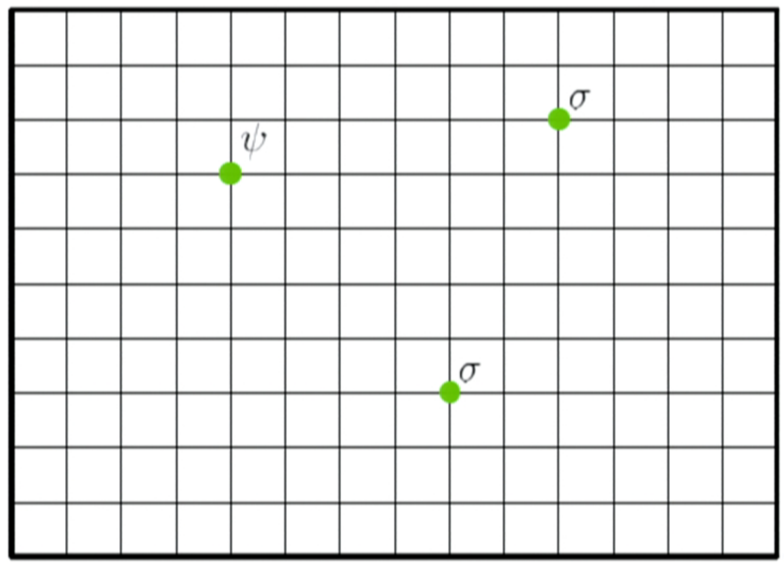

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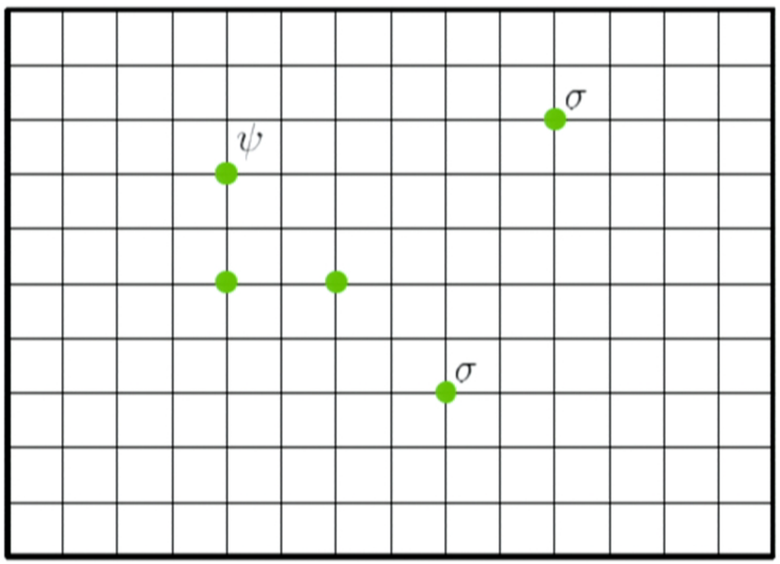
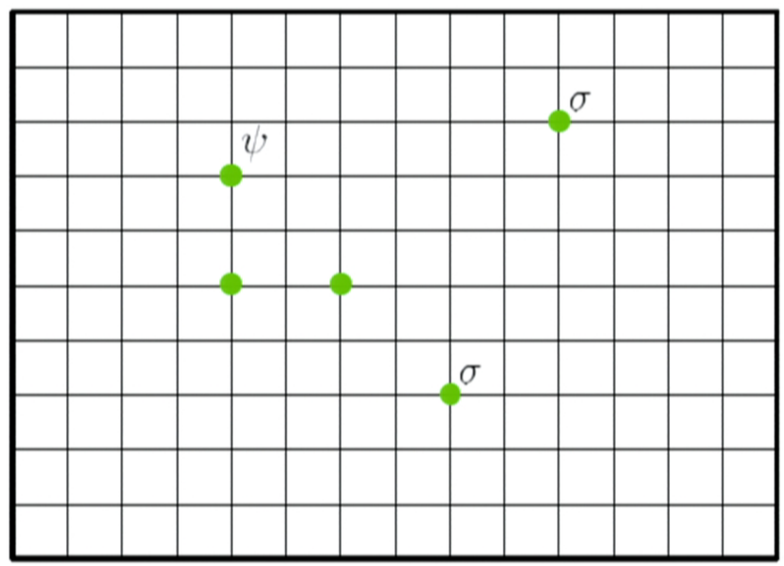


Phenomenological Model of Anyon Dynamics

- Discretize space
- At each point, 3-dimensional Hilbert space
  - Basis $|\mathbb{I}\rangle$, $|\psi\rangle$, $|\sigma\rangle$

- Pair Creation γ_p^σ; γ_p^ψ
- Hopping γ_h
- Exchange γ_e
- Decoherence γ_d



Monte-carlo sampling

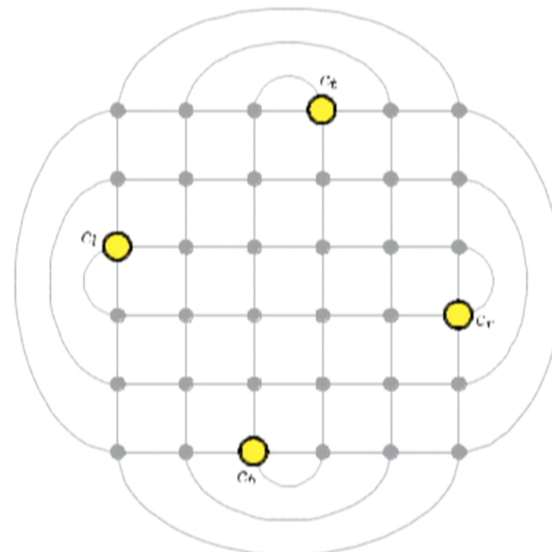
- For each of T timesteps, choose an edge (i,j) uniformly
- Given the charge configuration, determine allowed processes

	Charge configuration at (i,j)			
Process	$\mathbb{I}, \mathbb{I}$	$\mathbb{I},\ q$	$q,\ \mathbb{I}$	$q,\ q'$
Pair creation	✓	✓	✓	✓
Hopping			✓	✓
Exchange		✓	✓	✓
Decoherence			✓	✓

- Sample from the allowed processes on (i,j) according to their relative rates
- We expect this sampling mechanism to be appropriate for high temperatures, far from equilibrium. One could also vary the relative rates with charge density, for example.

Ising Anyon Codes

- Ising Fusion Code
  - 1 qubit memory (2d codespace)
- Spherical boundary conditions
- Pinned σ particles at four equidistant sites
- Anyons always *globally* fuse to vacuum



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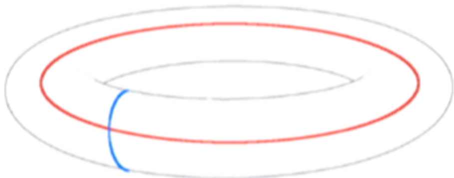
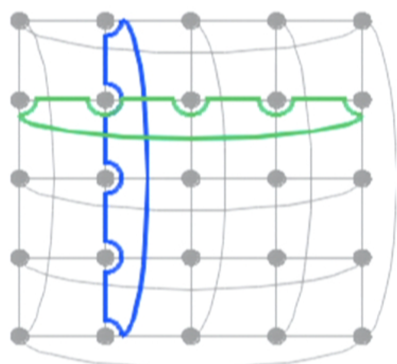
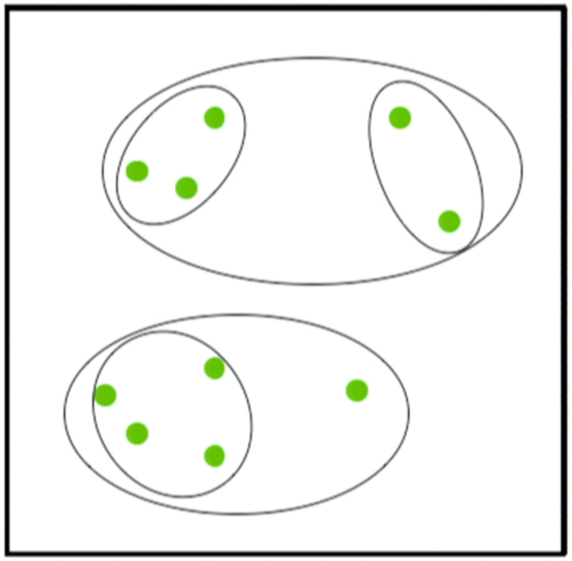




- Ising Topological Code
  - 1 *qutrit* memory (3d codespace)
  - More subtle than fusion code
- Toroidal boundary conditions
- Braid moves are a representation of the Fox braid group
- "Singleton" charge configurations are possible (and herald a logical error)
- Modular symmetry couples the nontrivial cycles on the torus

Decoders

RG decoder
(Bravyi-Haah version)



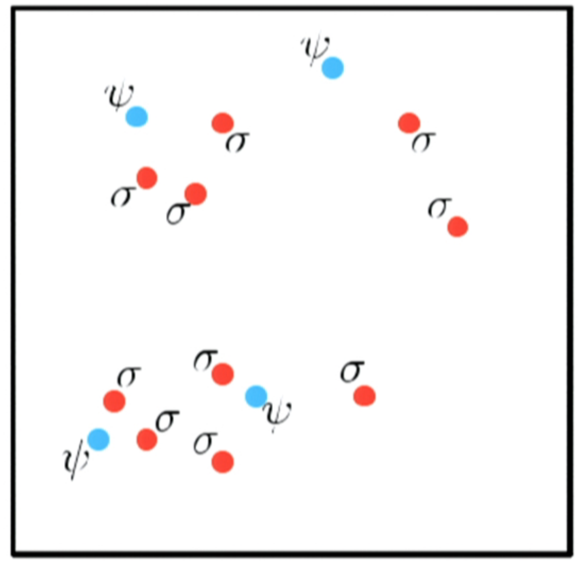
J. Harrington, PhD thesis, Caltech (2004);
Duclos-Cianci & Poulin PRL (2010);
Bravyi & Haah, PRL (2013)

- Cluster defects together at length scale *k* and try to annihilate disjoint clusters to vacuum
- Double length scale *k*
- Repeat until you reach the scale of the lattice.
- Two important differences from the toric code case:
  1) measure at each iteration
  2) do σ first, then ψ

Decoders

- Based on Edmonds' perfect matching algorithm
- Similar to the toric code case, but with important differences
- We iterate over the charge types: first σ, then ψ
- This is guaranteed to fuse to vacuum because of global charge conservation (on a sphere)

PMA



Dennis *et al.*, J. Math. Phys. (2002)

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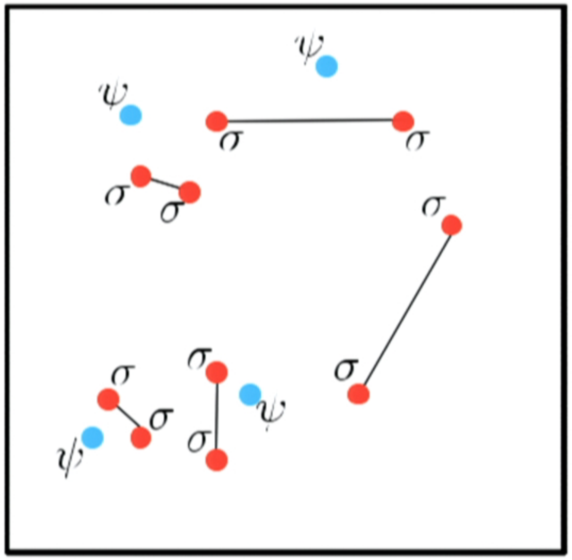

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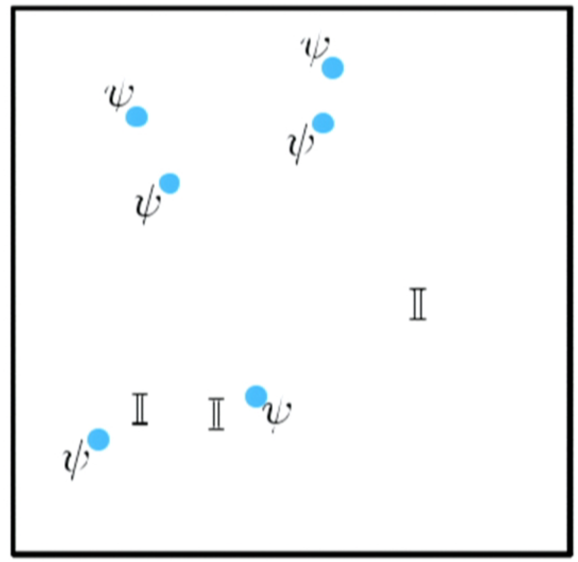

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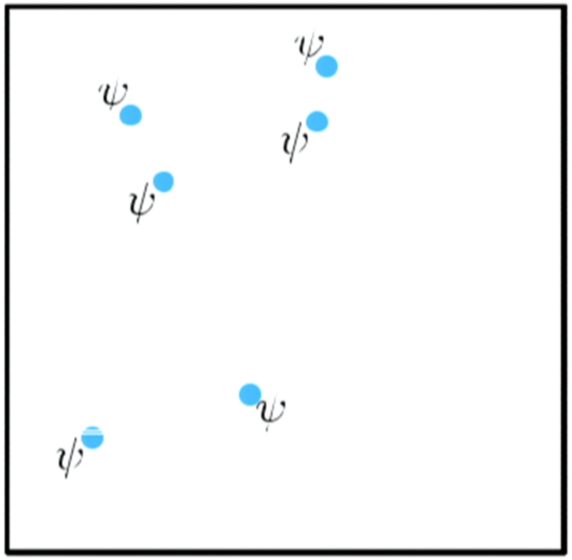

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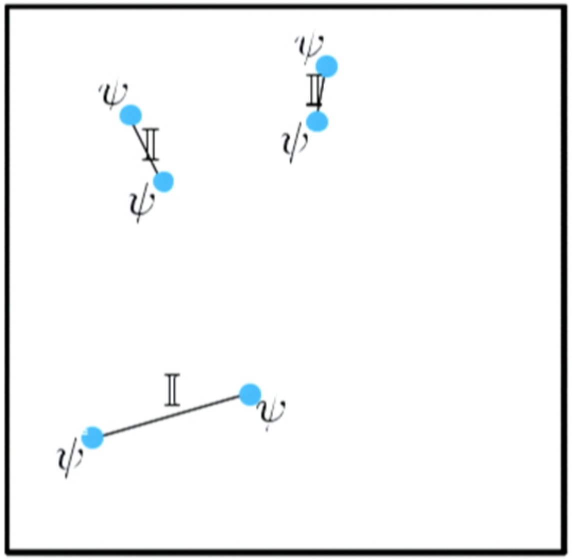
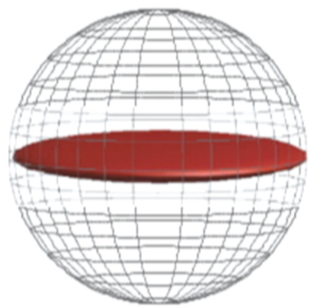


Dennis *et al.*, J. Math. Phys. (2002)

How can we determine if an error has occurred?

- Our codes lack the natural symmetry of a CSS code.
- To test if our logical info is preserved, we use the following theorem:
  A code space is perfectly preserved by the action of a channel iff the channel preserves the generators of the matrix algebra for the code space.
- By inputting any spanning set of non-orthogonal pure states, we can test for a threshold.



R. Blume-Kohout, *et al.* PRL 2008 & PRA 2010

Putting these ingredients together

- So far, we have:
  - Defined an error correcting code
  - Within the context of a phenomenological noise model, we added some noise
  - We simulated the Ising anyons, then measured the syndromes
  - Then we applied our decoder
  - Finally, we test for a logical error
  - Then we repeated many times to estimate the threshold value of the noise strength

Numerical Threshold Calculations

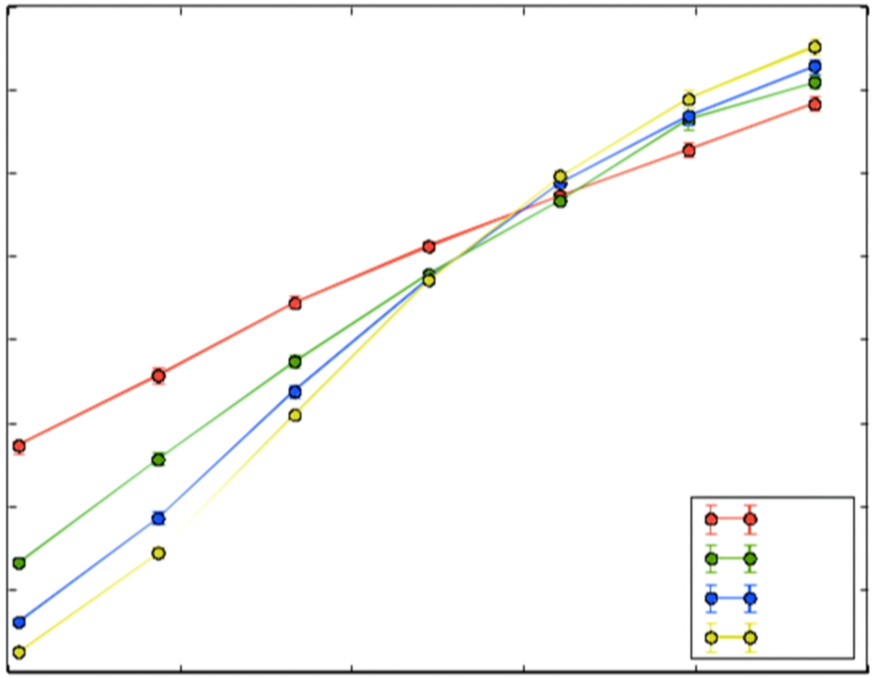
- We use the phenomenological noise model for the 4 pinned anyons code
- To benchmark our decoders, we set the σ creation rate to zero (c.f. the toric code with only one type of error)
- Previous threshold results mostly deal with iid noise; to make this comparable, we use the following (exact) mapping between the error rates:

$$p_{\text{iid}} = \frac{1}{2}\left(1 - e^{-2p_{\text{poi}}}\right) \qquad p_{\text{poi}} = \frac{T_0}{|E|}$$

T_0 = Expected number of physical errors
$|E|$ = Number of edges in the lattice

Numerical Threshold Calculations

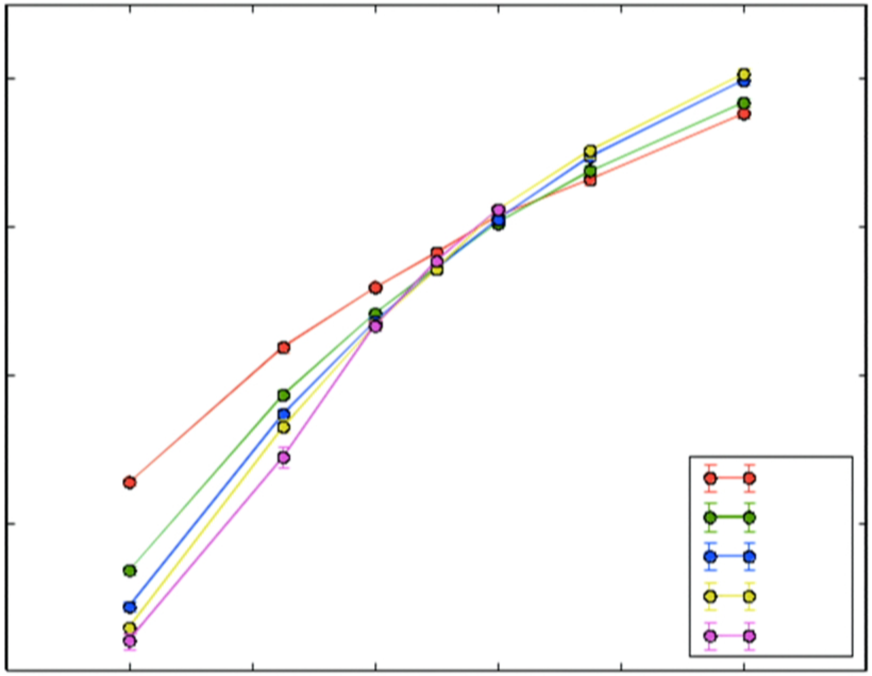
- Benchmark against the case with no σ creation
- Should give comparable performance to the toric code against iid bit-flip errors
- Threshold* of 11.6%
- Discrepancy due to finite size effects and pinned anyons



*This is a coarse estimate which we expect will decrease.

Numerical Threshold Calculations

- Now let's make it nonAbelian, with equal rates of σ and ψ creation
- Hopping, braiding, and decoherence initially turned off
- Threshold of 24% (thermal model)
- Result is insensitive to adding additional error processes



Error Correction for Ising Anyons

Numerical Threshold Calculations

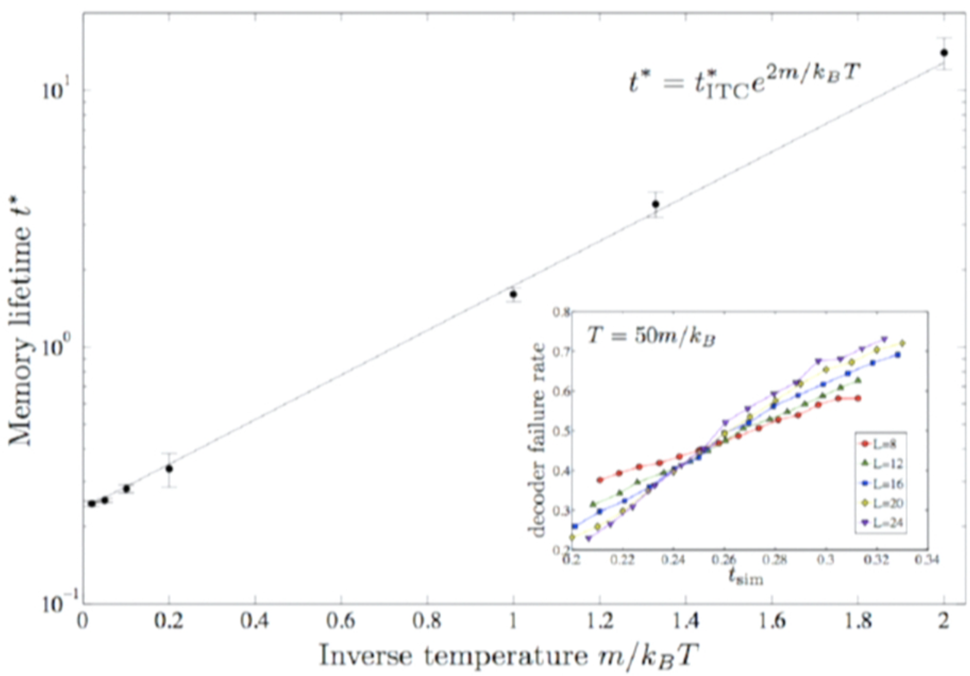
- For the Ising topological code (on a torus) we also did a Metropolis-type algorithm to simulate a coupling to a heat bath
- Define a Hamiltonian that penalizes creation of charges on-site and do standard Metropolis

$$H = \sum_{i \in V} \left(m_\psi P_i^\psi + m_\sigma P_i^\sigma \right)$$

  where the P operators project onto charge eigenstates
- To satisfy the detailed balance condition, the rates of the elementary noise processes must satisfy some simple conditions
- Elementary moves don't map charge eigenstates to charge eigenstates, so we use a semiclassical picture where we decohere (or measure) total charge on each site after each Metropolis step.

Numerical Threshold Calculations

- Because the Metropolis rule (and our chosen rates) obey detailed balance, the resulting state is a Gibbs thermal state
- Memory lifetime increases exponentially with inverse temperature



Error Correction for Ising Anyons

Open Questions

- Decoding general anyon models
  - We can decode... but how can we compute a threshold?
  - Simulating general anyon models is BQP hard, so this seems unlikely to be possible
- Other nonabelian models (Wootton *et al.*, arXiv:1310.3846)
- Fault-tolerant decoding
  - Is this possible? (*Sherbrooke*)
- Other decoders
  - Are "soft" decoders possible for nonAbelian anyons?
- Better and more accurate thresholds
- Microscopic models (e.g. the honeycomb model) (*Sydney*)

Thank you for your attention!