

Title: An introduction to quantum LDPC code

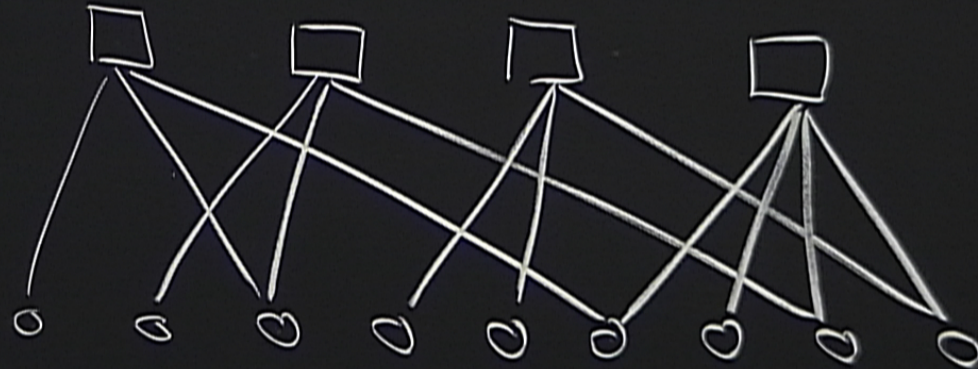
Date: Jul 14, 2014 09:10 AM

URL: <http://pirsa.org/14070000>

Abstract: In this talk, I will cover some basic notions of quantum LDPC codes, focusing on the similarities and distinctions with their classical cousins. Topics will include definitions of stabilizer quantum LDPC codes (CSS and general), iterative decoding algorithms, dual spin models, and obstructions caused by error degeneracy. The talk will be informal and a good occasion to ask questions.

$$\{x: H_{K \times n} x = 0\} = C$$

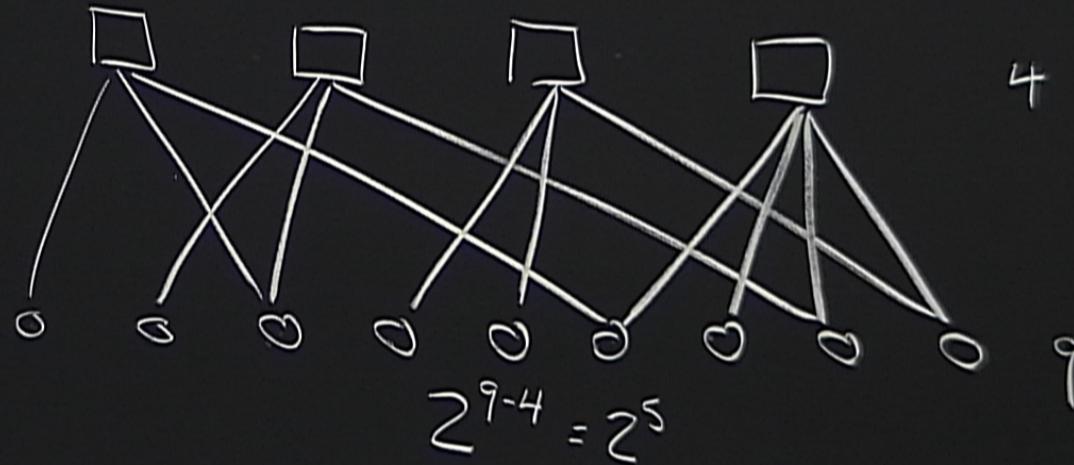
Tanner graph





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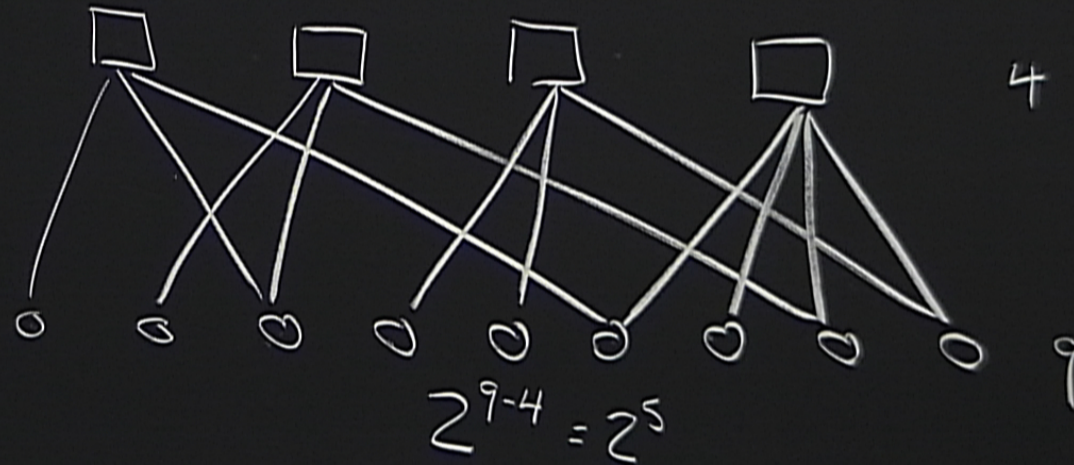
Tanner graph





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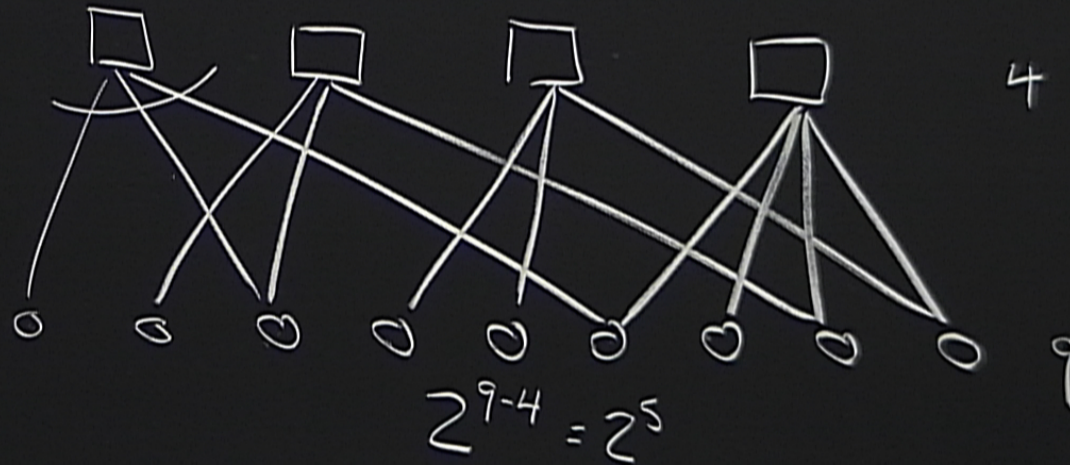
Tanner graph





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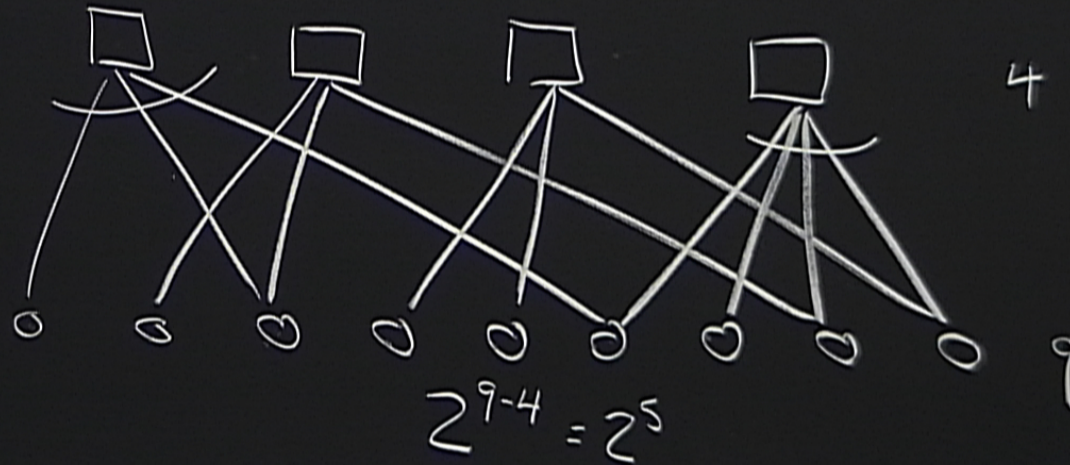
Tanner graph





$$\{x: H_{K \times n} x = 0\} = C$$

Tanner graph





## Quantum codes

$$Q = \{ | \psi \rangle, \overset{\text{Projector}}{\uparrow} \Pi | \psi \rangle = | \psi \rangle \}$$



## Quantum codes

$$Q = \{ | \psi \rangle, \overset{\substack{\uparrow \\ \text{Projector}}}{\Pi} | \psi \rangle = | \psi \rangle \} \quad \text{subspace of } \mathcal{H}$$

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 subspace of  $\mathcal{H}$

Stabilizer codes

$$Q = \{ | \psi \rangle, S_j | \psi \rangle = | \psi \rangle \forall j \}$$

Stab. generators  
 $\{ S_j \}$



## Quantum codes

$$Q = \{ |\psi\rangle, \overset{\text{Projector}}{\uparrow} \Pi |\psi\rangle = |\psi\rangle \} \quad \text{subspace of } \mathcal{H}$$

## Stabilizer codes

$$Q = \{ |\psi\rangle, S_j |\psi\rangle = |\psi\rangle \forall j \} \quad \left\{ S_j \right\} \leftarrow \text{stab. generators}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



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$$Q = \{ | \psi \rangle, S_j | \psi \rangle = | \psi \rangle \forall j \}$$
  $\{ S_j \}$  stab. generators  $X \otimes I \otimes Y \dots$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$



# Quantum codes

$$Q = \{ | \psi \rangle, \pi | \psi \rangle = | \psi \rangle \}$$

Subspace of  $\mathcal{H}$

Projector

$$\begin{pmatrix} X & Z & Z & I & X \\ Z & I & I & Z & X \end{pmatrix} \begin{pmatrix} Y \\ X \end{pmatrix}$$

Stabilizer codes

$$Q = \{ | \psi \rangle, S_j | \psi \rangle = | \psi \rangle \}$$

Stab. generators  $\{ S_j \}$   $X \otimes I \otimes Y$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$



$n$  qubits and  $n-k$  independent  $S_j$

$$\dim Q = 2^k$$



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$\in$  Pauli group

$\text{wt}(E) = \#$  of non-identity  
tensor components



$$\dim Q = 2^k$$

$E \in \text{Pauli group}$   
 $\text{wt}(E) = \# \text{ of non-identity tensor components}$

$$|\psi\rangle \in Q \longrightarrow |\phi\rangle = E|\psi\rangle$$

$$S_j |\phi\rangle = S_j E |\psi\rangle = \begin{cases} + E S_j |\psi\rangle & \text{if } [E, S_j] = 0 \\ - E S_j |\psi\rangle & \text{if } \{E, S_j\} = 0 \end{cases} = \begin{cases} |\phi\rangle \\ -|\phi\rangle \end{cases}$$



$$| \psi \rangle \in Q \longrightarrow |\phi\rangle = E | \psi \rangle$$

$$S_j |\phi\rangle = S_j E | \psi \rangle = \begin{cases} + E S_j | \psi \rangle & \text{if } [E, S_j] = 0 \\ - E S_j | \psi \rangle & \text{if } \{E, S_j\} = 0 \end{cases} = |\phi\rangle$$

$$E | \psi \rangle = | \psi' \rangle \in Q$$

$$[E, S_j] = 0 \quad \forall j$$

$wt(E) = \#$  of tensor components



$$| \gamma \rangle \in Q \longrightarrow |\phi\rangle = E|\gamma\rangle$$

$$S_j |\phi\rangle = S_j E|\gamma\rangle = \begin{cases} + E S_j |\gamma\rangle & \text{if } [E, S_j] = 0 \\ - E S_j |\gamma\rangle & \text{if } \{E, S_j\} = 0 \end{cases} = |\phi\rangle$$

undetected error

$$\begin{cases} E|\gamma\rangle = |\gamma'\rangle \in Q \\ [E, S_j] = 0 \quad \forall j \end{cases}$$



$$\dim Q = 2$$

$wt(E) = \#$  of non-identity tensor components

$$|7\rangle \in Q \longrightarrow |\phi\rangle = E|7\rangle$$

$$S_j |\phi\rangle = S_j E|7\rangle = \begin{cases} +E S_j |7\rangle & \text{if } [E, S_j] = 0 \\ -E S_j |7\rangle & \text{if } \{E, S_j\} = 0 \end{cases} = |\phi\rangle$$

undetected error

$$\begin{cases} E|7\rangle = |7'\rangle \in Q \\ [E, S_j] = 0 \quad \forall j \end{cases}$$

Undetected harmful error  
 $[E, S_j] = 0$  and  $E \in \langle S_j \rangle$



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Undetected harmful error

$$[E, S_j] = 0 \text{ and } E \notin \langle S_j \rangle$$

Minimum distance  $d = \min c$



Minimum distance  $d = \min$  weight of undetected harmful error.



Minimum distance  $d = \text{min weight of undetected harmful error.}$

$$[[n, k, d]]$$



# Quantum LDPC codes

$$Q = \{ | \psi \rangle, \Pi_j | \psi \rangle = | \psi \rangle \quad \forall j \}$$

↑ projectors acting on at most  
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↑  
Projector

$$\begin{pmatrix} X & Z \\ Z & 1 \end{pmatrix} \begin{pmatrix} Z & 1 \\ 1 & Z \end{pmatrix} \begin{pmatrix} X & Y \\ Y & X \end{pmatrix}$$

Stabilizer codes

$$\pi_j = \frac{1 + S_j}{2}$$

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Stab. generators  
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$$Q = \{ |\psi\rangle, \pi |\psi\rangle = |\psi\rangle \}$$

↑  
Projector

Subspace of  $\mathcal{H}$

$$\begin{pmatrix} X & Z & Z & I & X \\ Z & I & I & Z & X \end{pmatrix} \begin{pmatrix} Y \\ X \end{pmatrix}$$

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Stabilizer LDPC codes

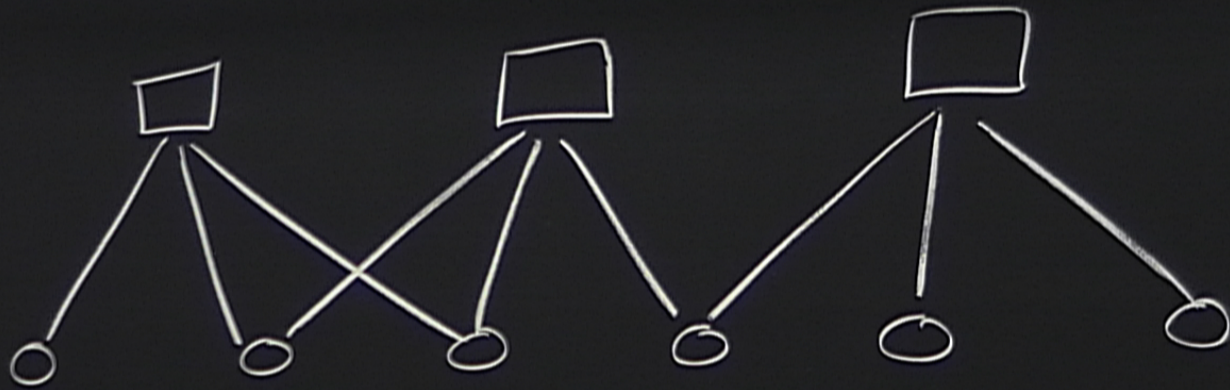


$$Q = \{ | \psi \rangle, \Pi_j | \psi \rangle = | \psi \rangle \quad \forall j \}$$

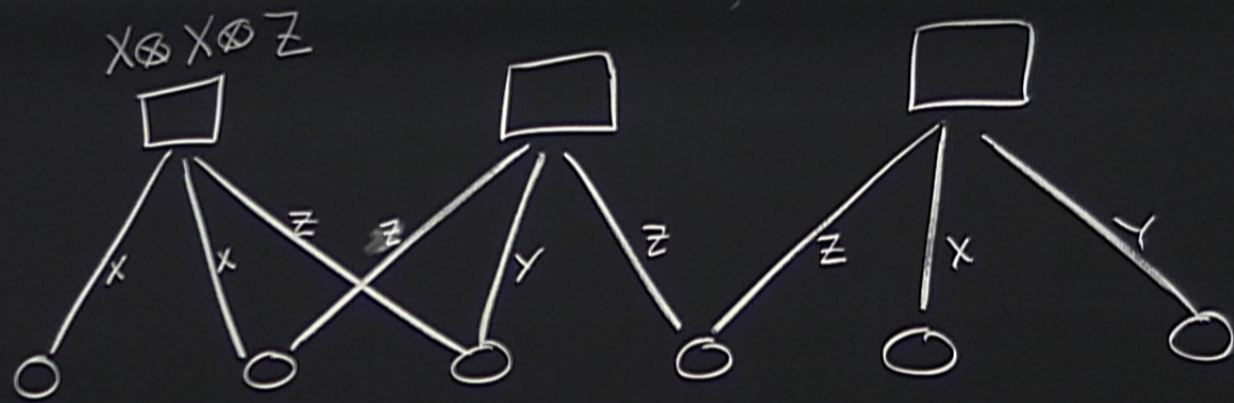
↑ projectors acting on at most  
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Stabilizer LDPC codes  $\Pi_j = \frac{1 + S_j}{2}$  ← commuting Pauli operators

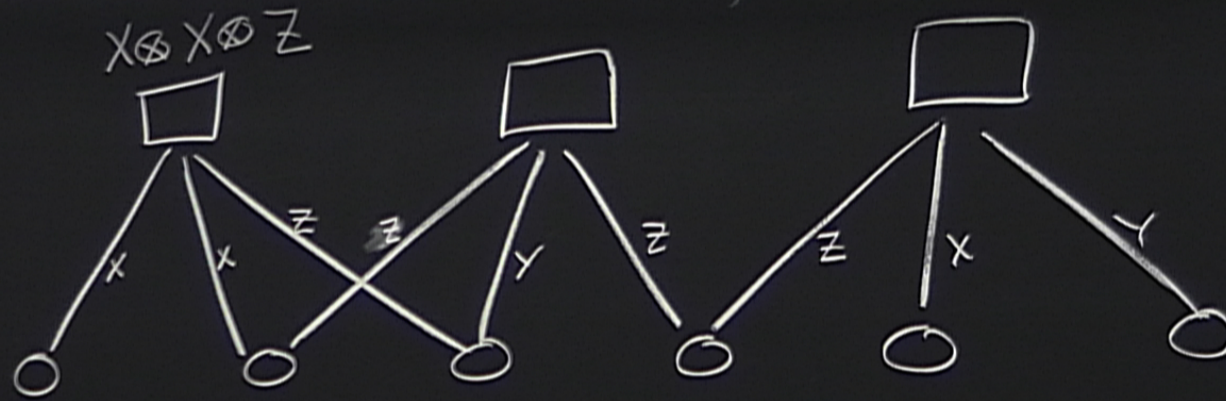








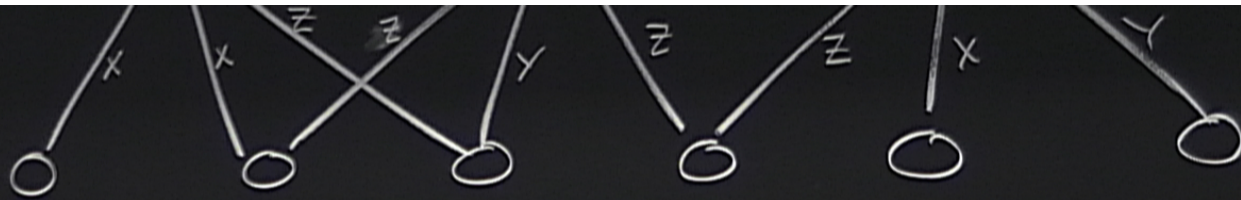




CSS Quantum LDPC

Stab generators are all  $X$  and  $Z$ .





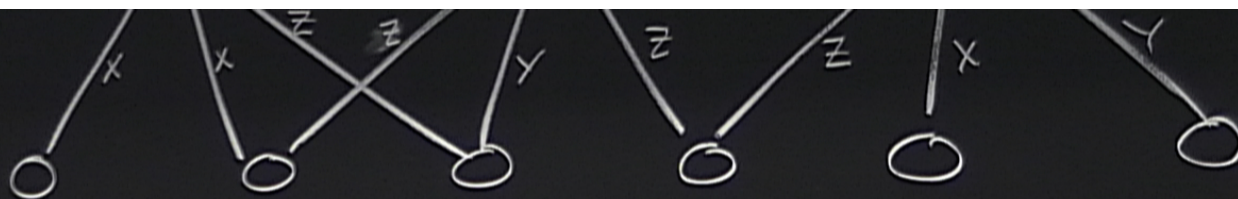
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$$S_j^x = X^{v_j}$$

$$S_k^z = Z^{u_k}$$





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$$v_j \cdot u_k = 0 \pmod{2} \quad \forall j, k.$$

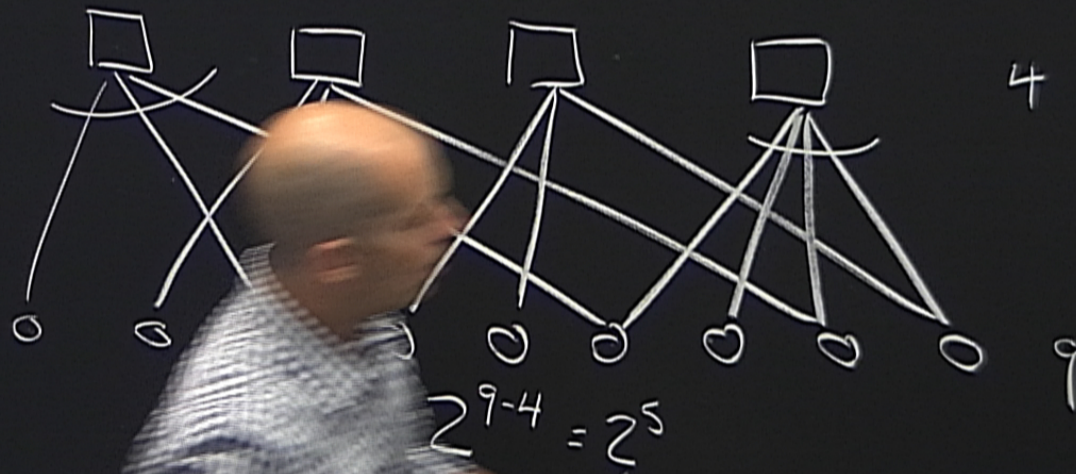


$$\{x: Hx = 0\} = C$$

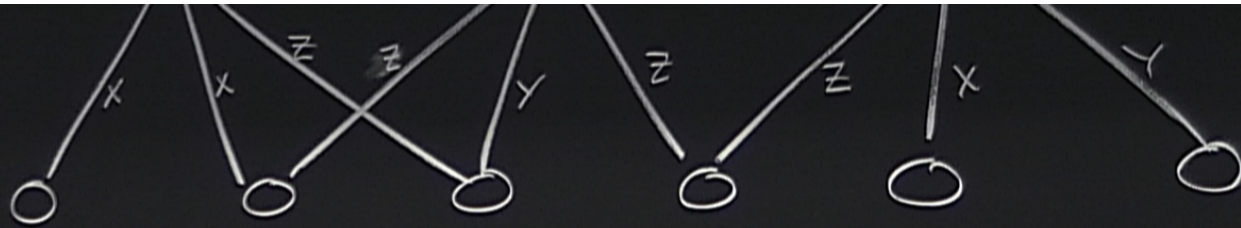
$K \times n$

Tanner graph

$$H = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_s \end{pmatrix}$$







CSS Quantum LDPC

Stab generators are all  $X$  and  $Z$ .

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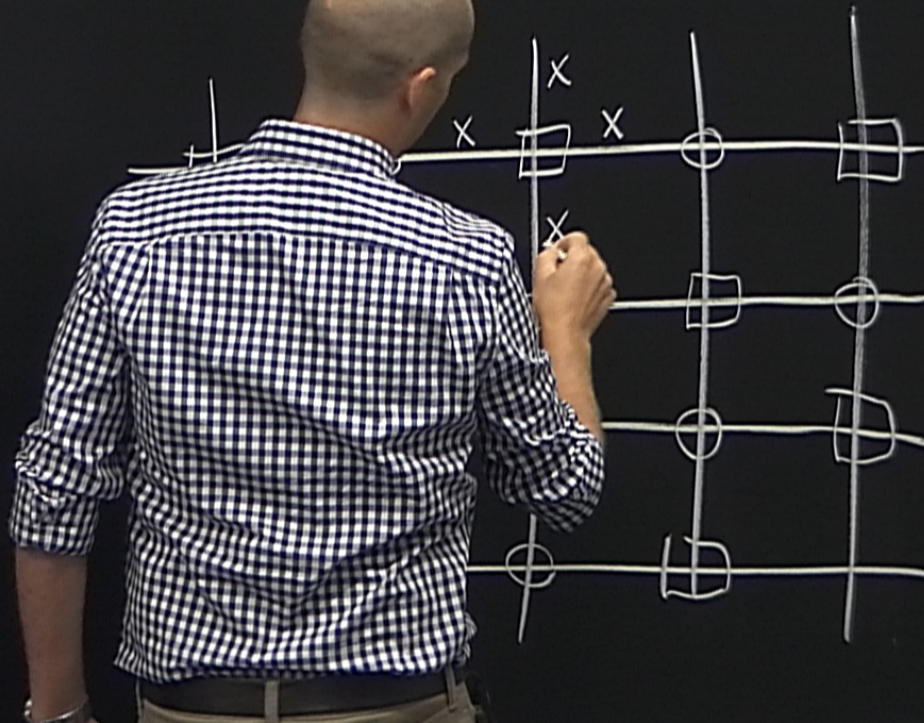
$$v_j \cdot u_k = 0 \pmod{2} \quad \forall j, k.$$

$$Hu_k = 0$$



Minimum distance  $d = \text{min weight of undetected harmful error.}$

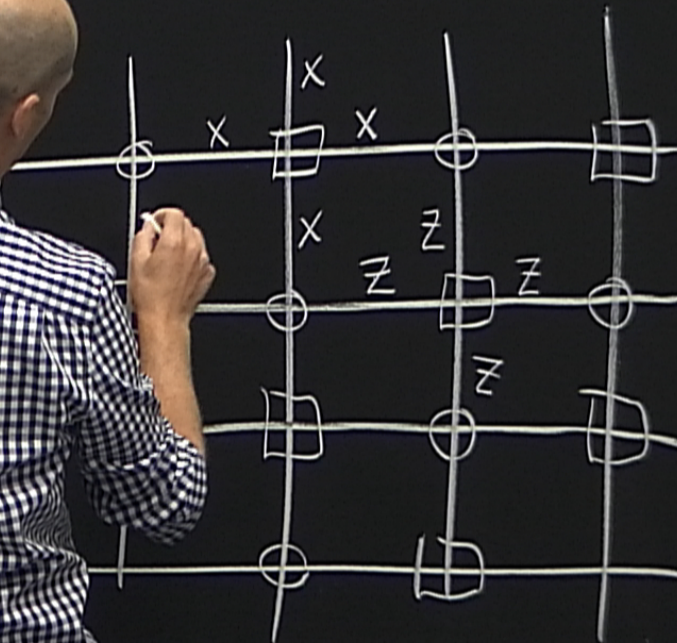
$$[[n, k, d]]$$





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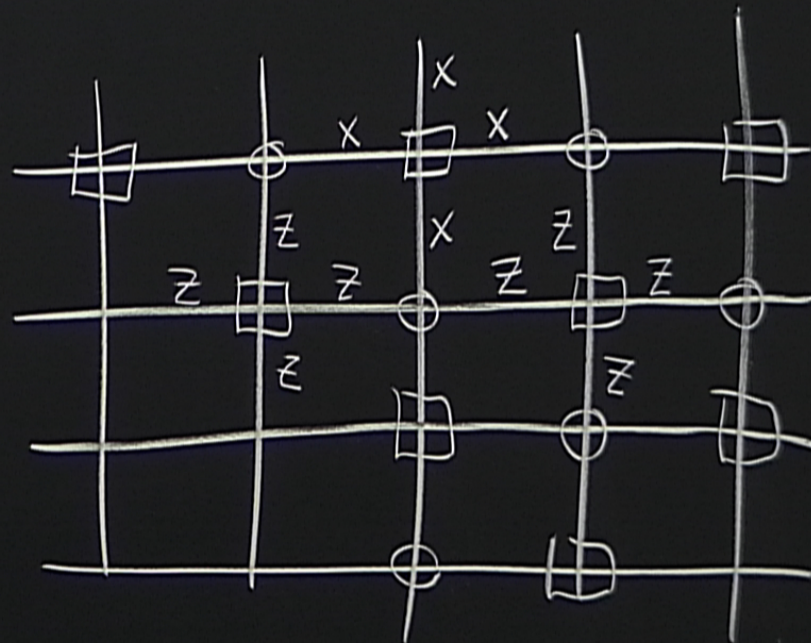
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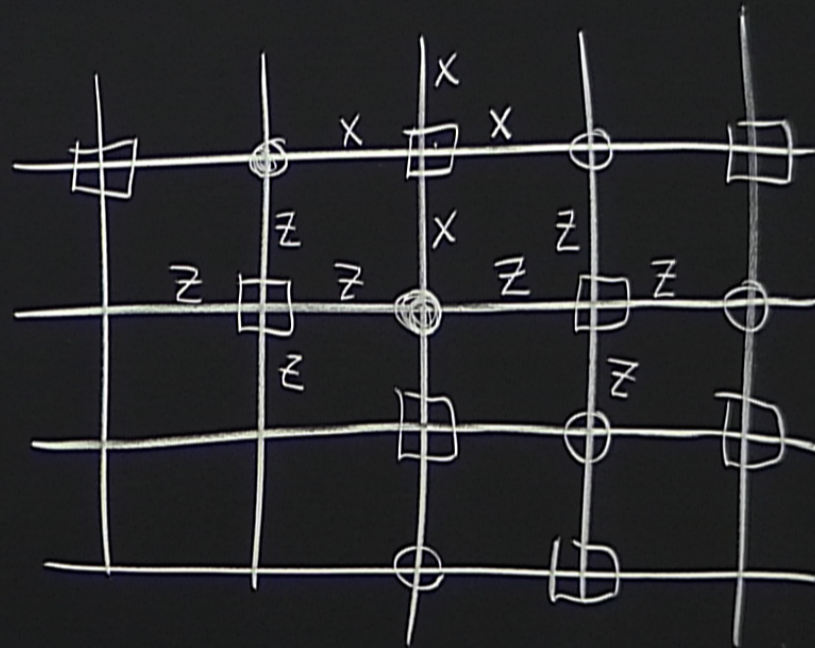
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$$E \quad F = ES \quad \swarrow \quad E \langle S_j \rangle$$

Impossible to tell  $E$  and  $F$  apart by their syndrome.



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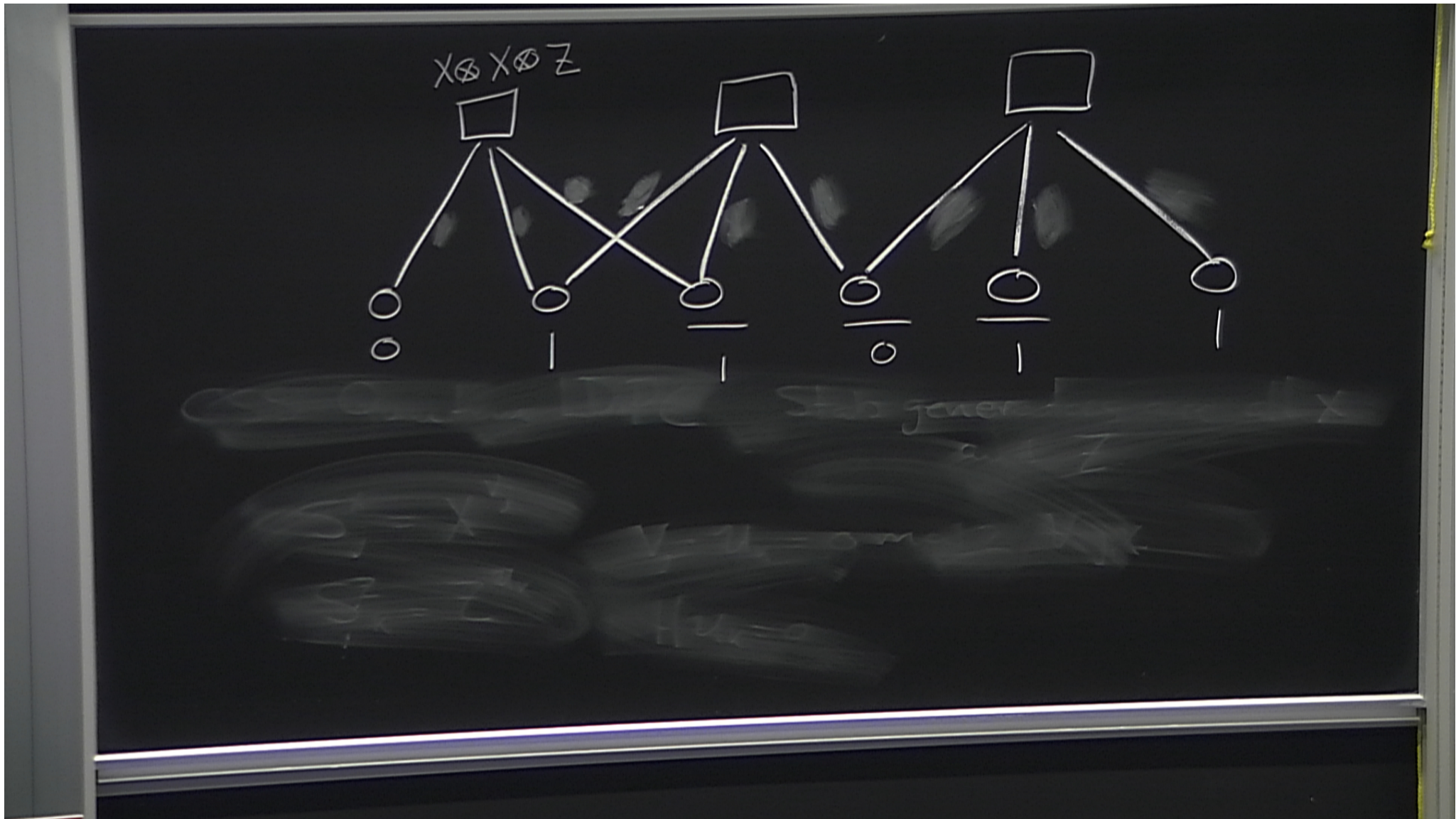
# Degeneracy

$$E \quad F = ES \quad \leftarrow E \langle S_j \rangle$$

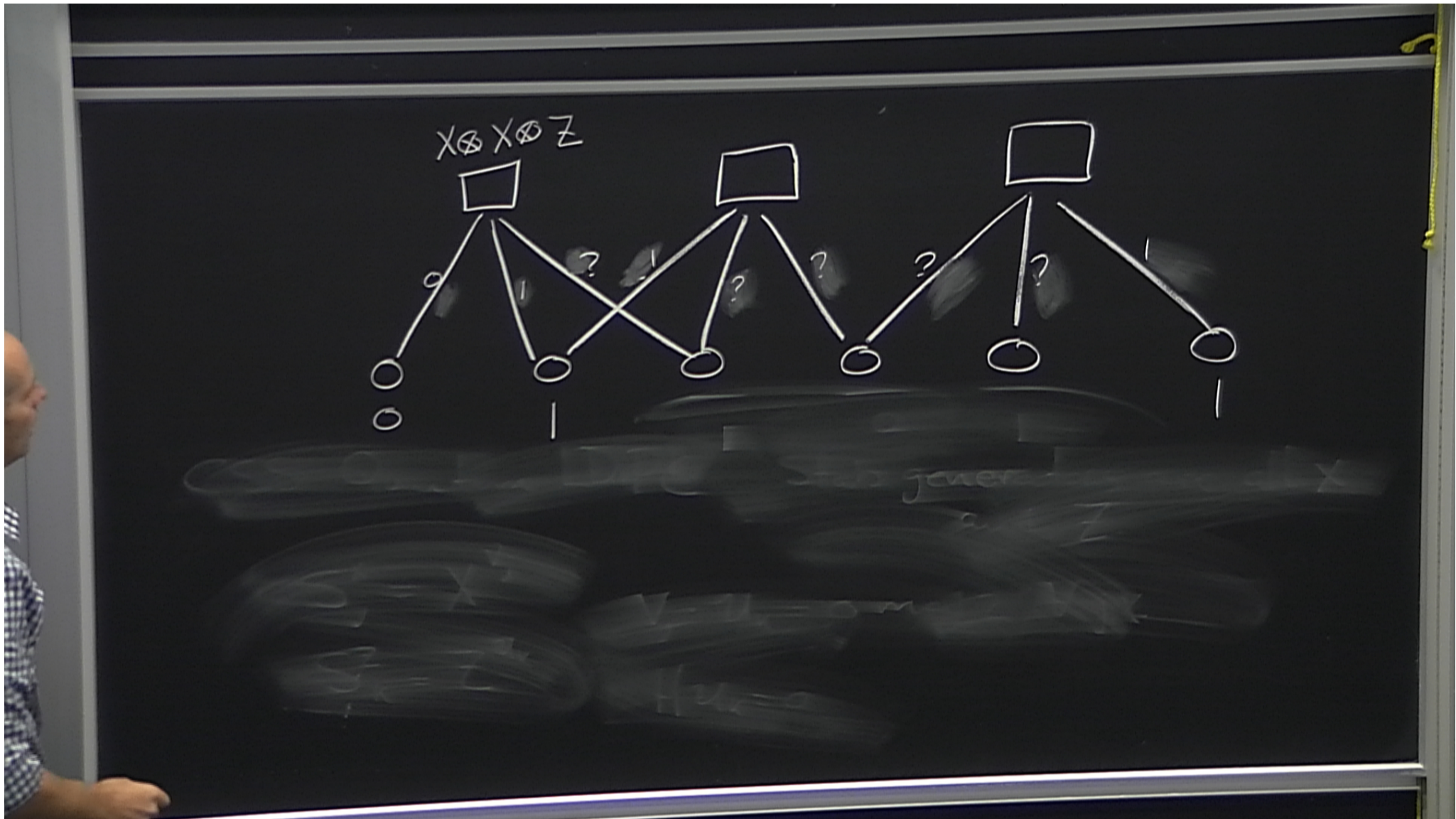
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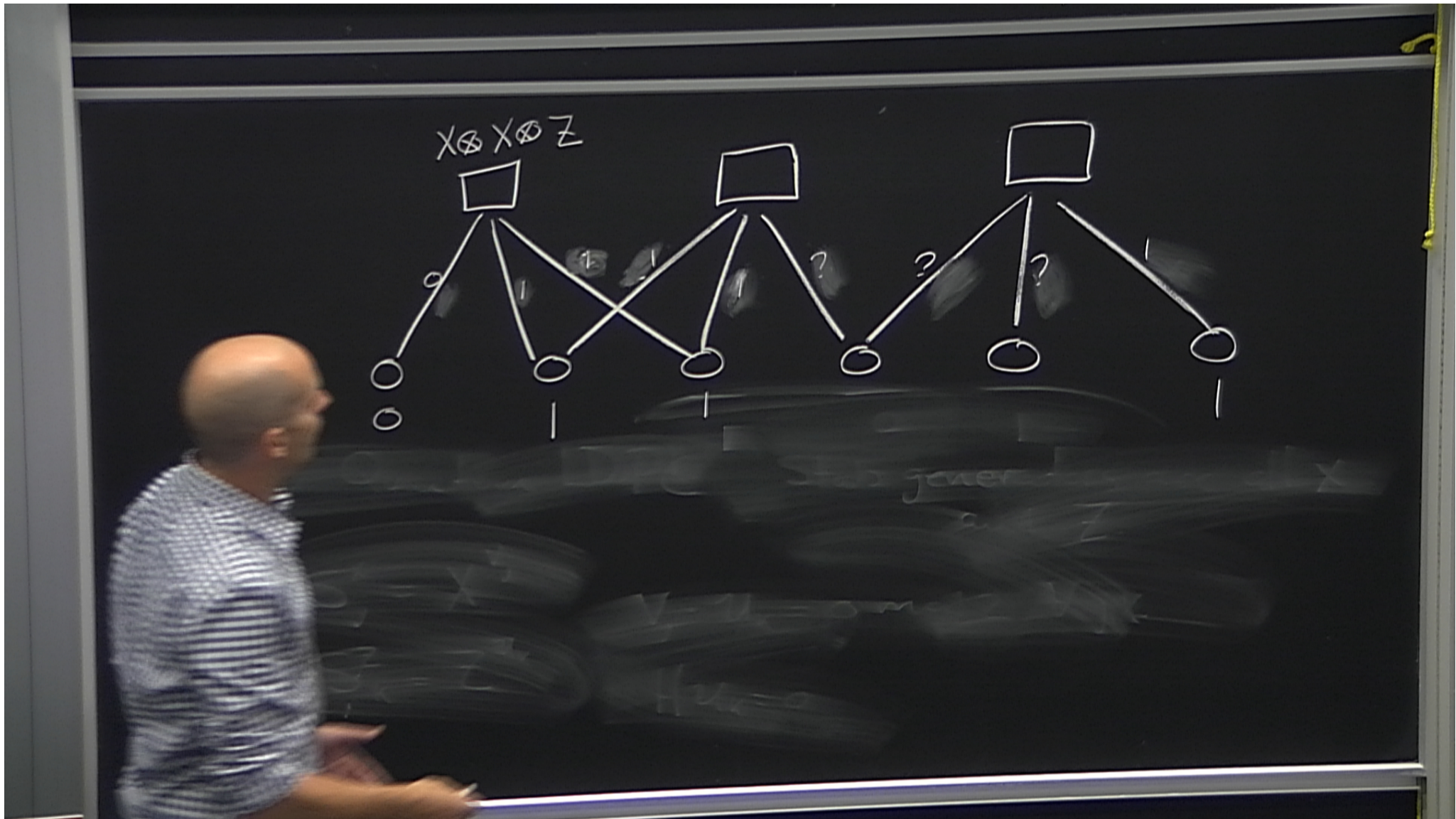




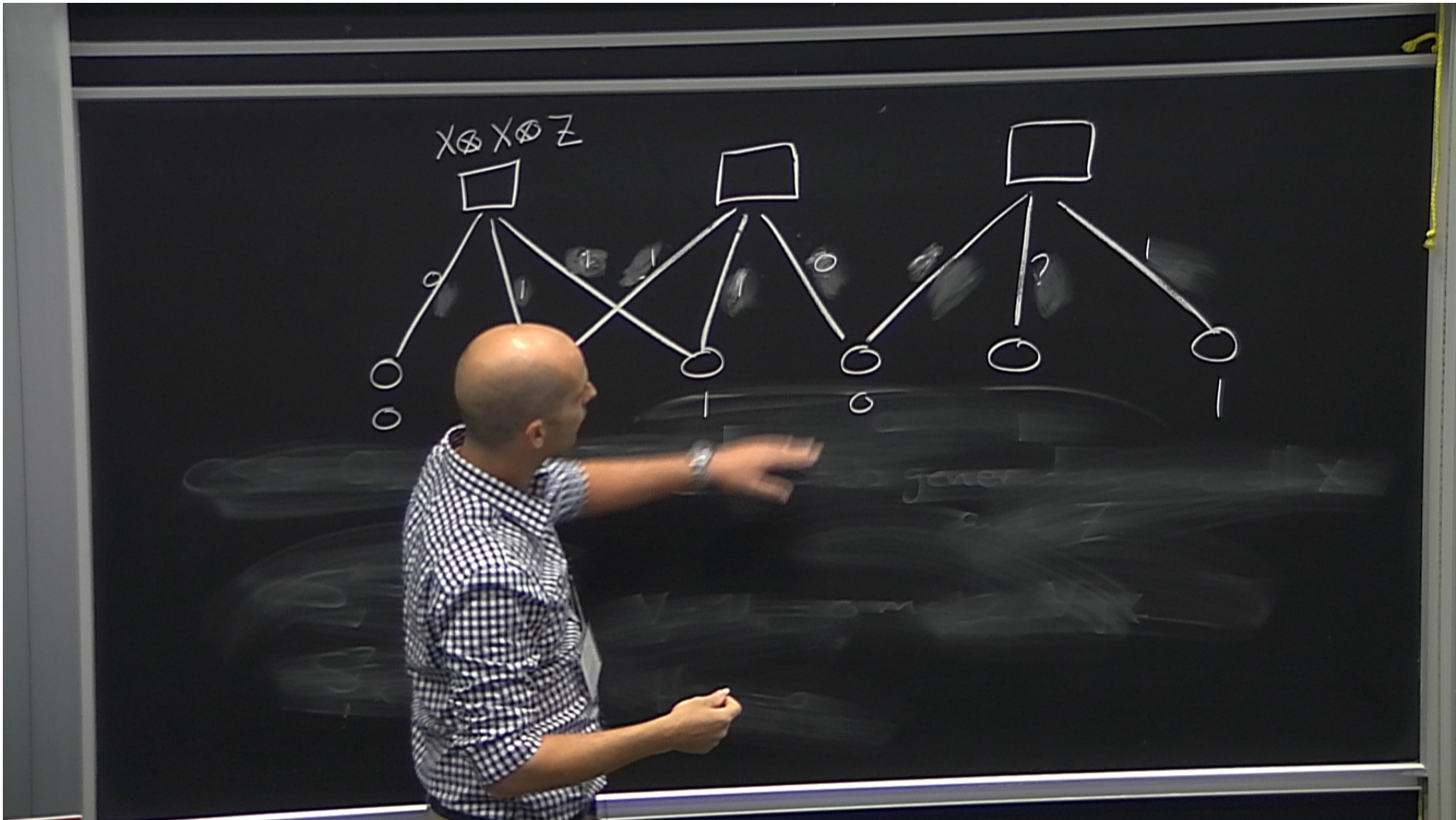




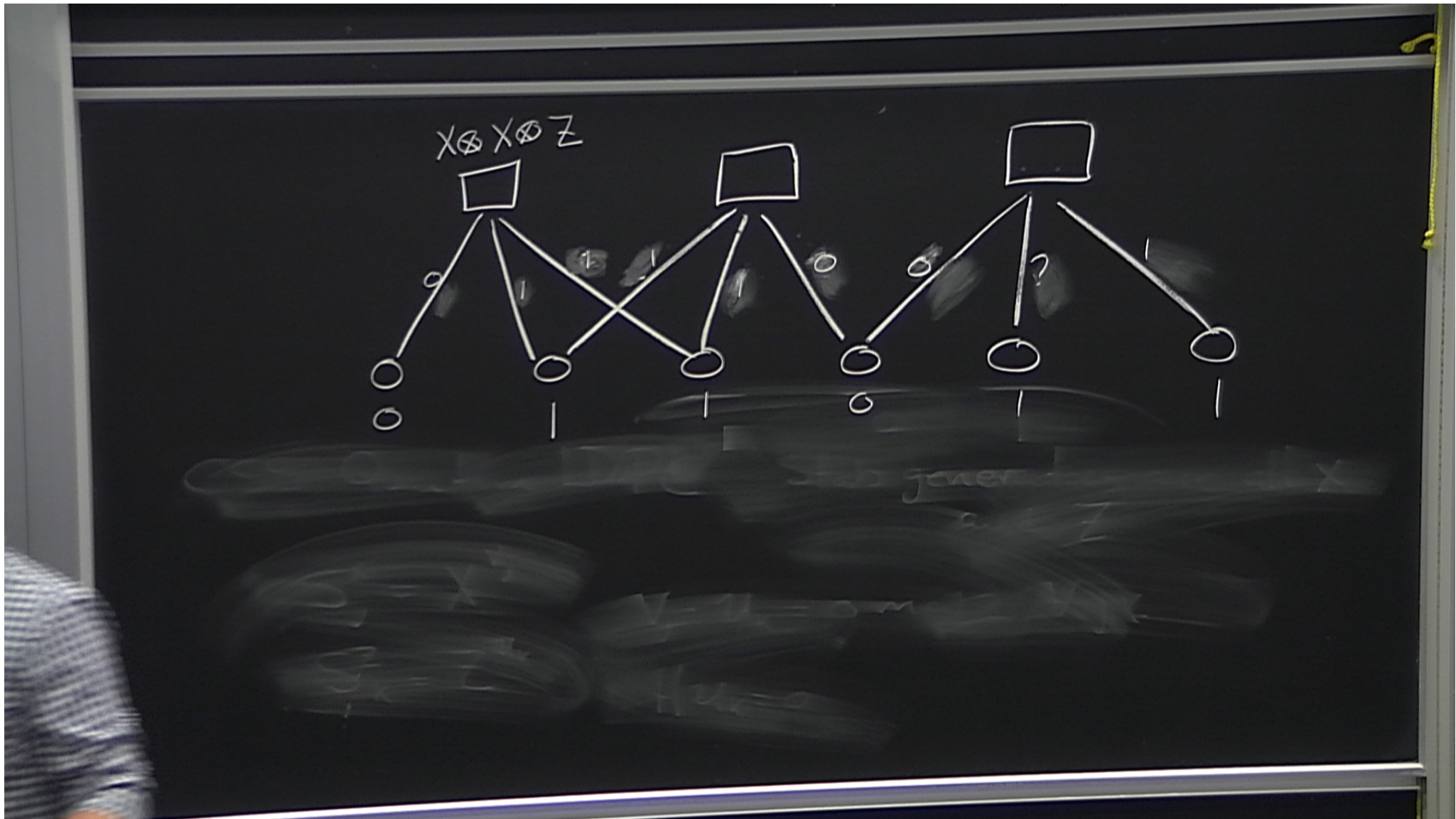




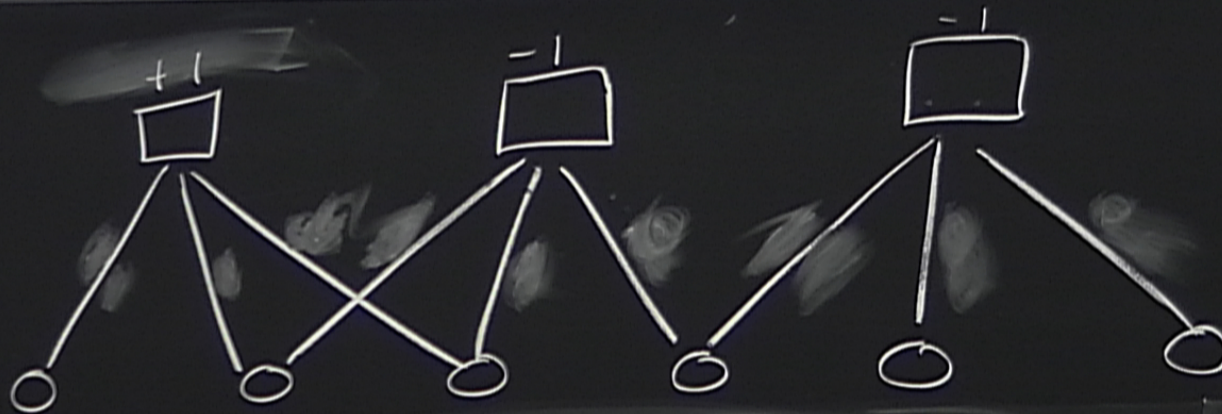




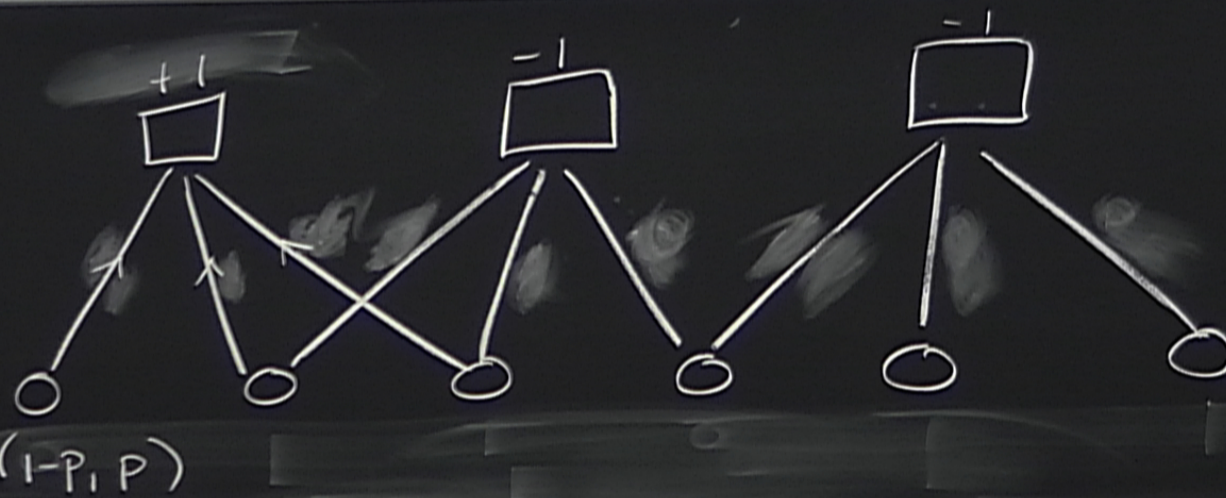






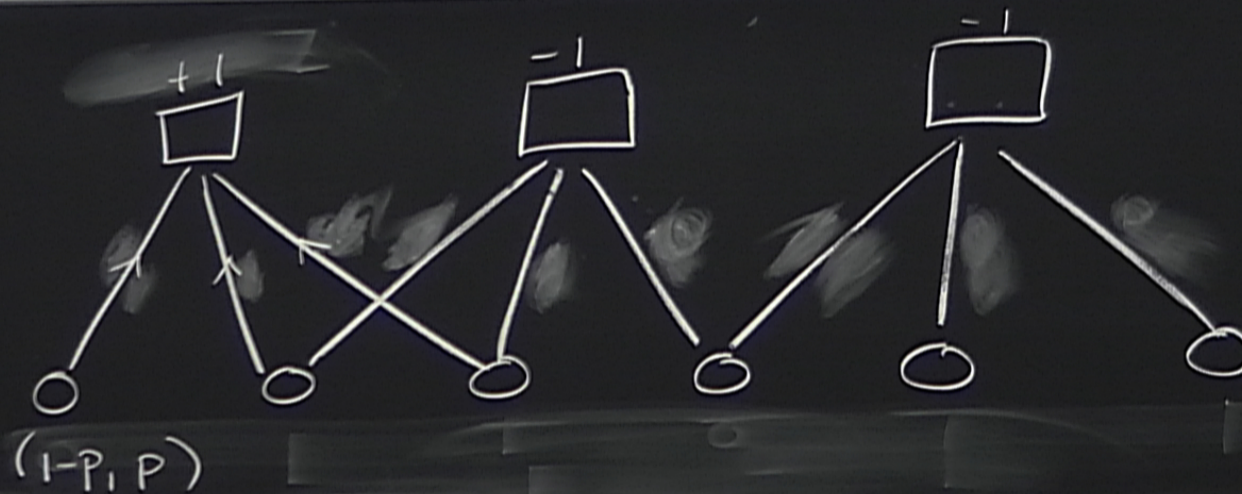






$$M_{b \rightarrow c}^o = \bar{P}_{\text{prior}}$$





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$$M_{c \rightarrow b}^t$$



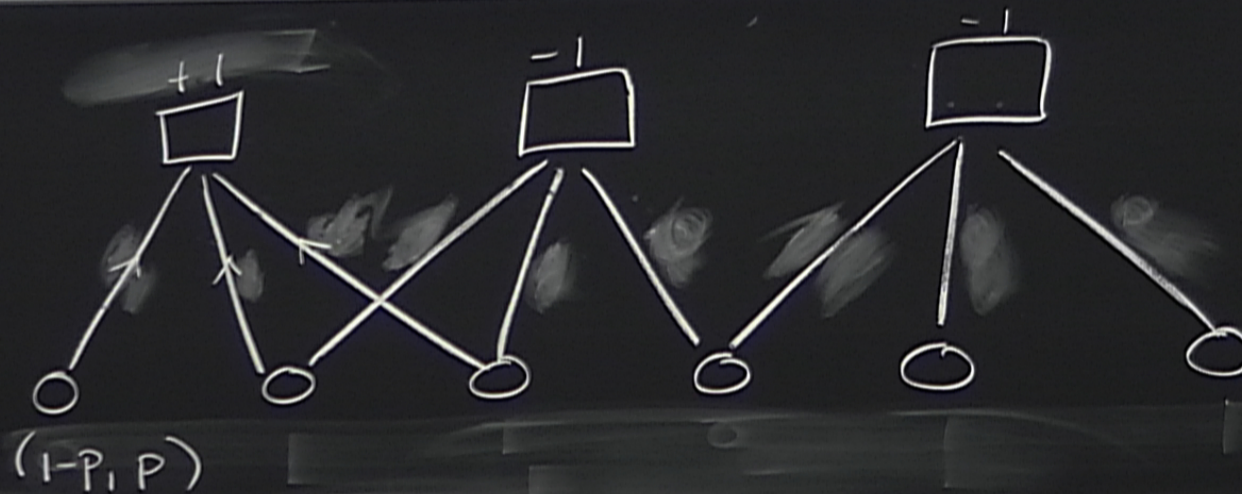


$(1-p, p)$

$\rightarrow_c = \bar{p}_{prior}$

$$m_{c \rightarrow b}^t(x) = \sum_y \prod_{b' \in c \setminus b} m_{b' \rightarrow c}(y) \delta(y \cdot x \text{ "have right Parity"})$$

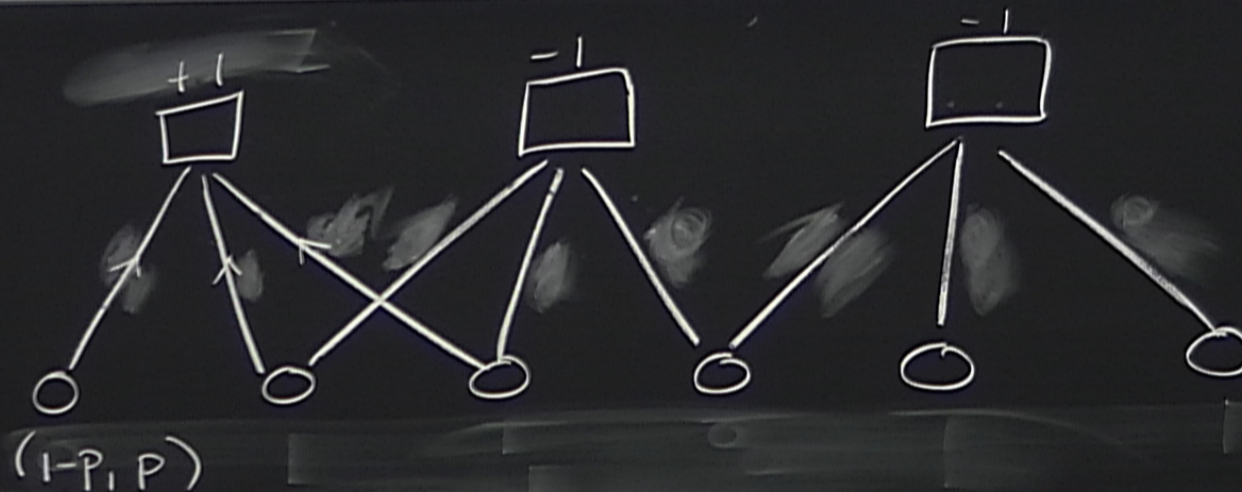




$$m_{b \rightarrow c}^0 = \vec{p}_{\text{prior}}$$

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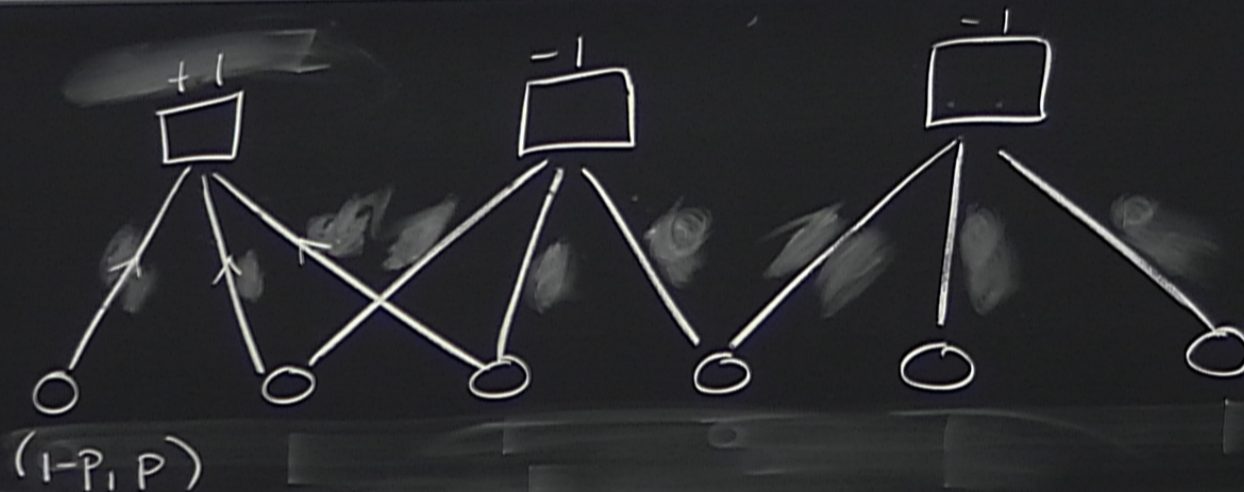


$$m_{b \rightarrow c}^0 = \bar{p}_{\text{prior}}$$

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$$m_{b \rightarrow c}$$



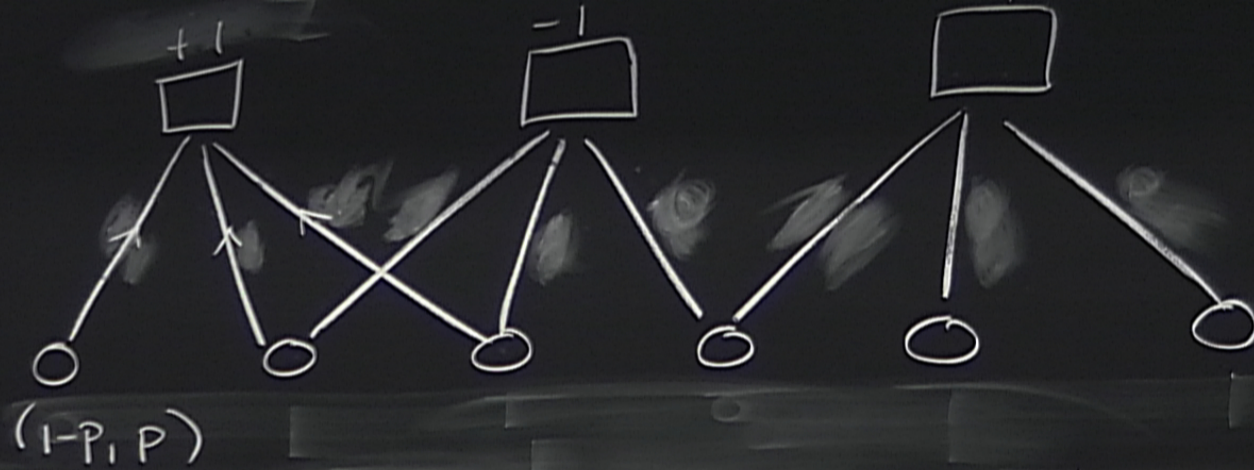


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$$m_{b \rightarrow c}(x) = \bar{P}_{\text{prior}} \cdot \prod_{c' \in b \setminus c} m_{c' \rightarrow b}^{t-1}(x)$$

$$\text{belief}_b(x) = \text{Prior}(x) \times \prod_{c \in b} m_{c \rightarrow b}^t(x)$$

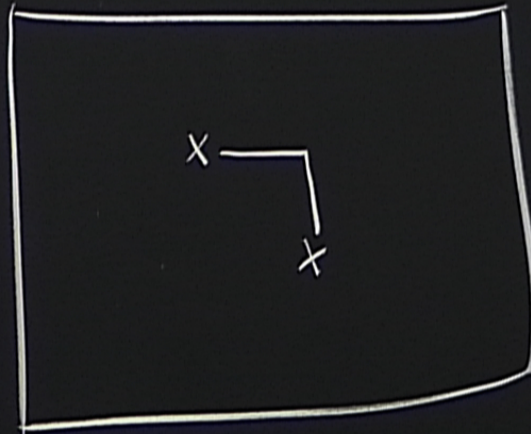


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These are not peaked in a sparse  
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