Title: Numerical investigations of singularities in general relativity

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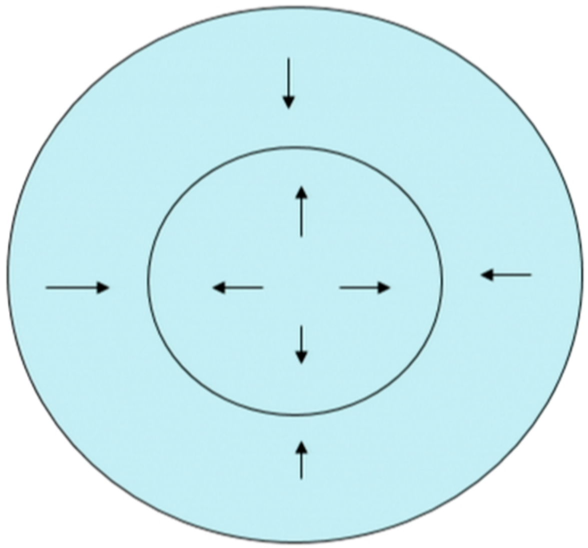
Abstract: The singularity theorems of general relativity tell us that spacetime singularities form in gravitational collapse, but tell us very little about the precise nature of these singularities. More information can be found using analytic approximations and numerical simulations. It is conjectured that inside black holes are two types of singularities: one that is spacelike, local, and oscillatory, and the other that is null and weak. This talk will review what numerical simulations of singularities have been done and the extent to which the above conjecture has been verified by the simulations.

- Gravitational collapse
- Singularity theorems
- BKL conjecture
- Gowdy spacetimes
- General spacetimes

Star in equilibrium between gravity and pressure. Gravity can win and the star can collapse



Singularity theorems

Once a trapped surface forms
(given energy and causality conditions)
Some observer or light ray ends in a
Finite amount of time

Very general circumstances for
Singularity formation

Very little information about
The nature of singularities

Approach to the singularity

As the singularity is approached, some terms in the equations are blowing up

Other terms might be negligible in comparison

Therefore the approach to the singularity might be simple

BKL Conjecture

As the singularity is approached time derivatives become more important than spatial derivatives. At each spatial point the dynamics approaches that of a homogeneous solution.

Is the BKL conjecture correct? Perform numerical simulations and see

Gowdy spacetimes

$ds^2 = e^{(\lambda+t)/2} (- e^{-2t} dt^2 + dx^2) +$

$e^{-t} [e^{P} (dy+Qdz)^2 + e^{-P} dz^2]$

P, Q and λ depend only on t and x

The singularity is approached as t goes to infinity

x,y and z are periodic, space is a 3-torus

Einstein field equations

$$P_{tt} - e^{2P} Q_t^2 - e^{-2t} P_{xx} + e^{2(P-t)} Q_x^2 = 0$$

$$Q_{tt} + 2 P_t Q_t - e^{-2t} (Q_{xx} + 2 P_x Q_x) = 0$$

(subscript means coordinate derivative)

Note that spatial derivatives are multiplied by decaying exponentials

Numerical results (Berger, Moncrief, DG, Isenberg, Weaver)

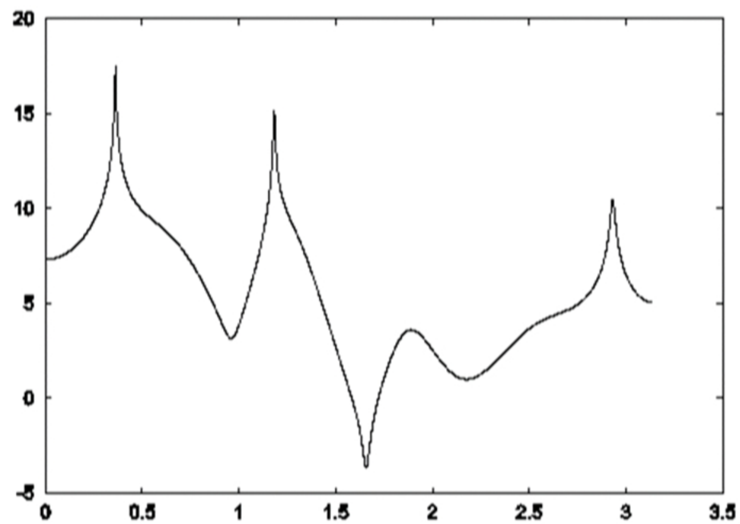
As t goes to infinity

$P = P_0(x)+tv_0(x)$

$Q = Q_0(x)$

But there are spikes

P



General case

Variables are scale invariant commutators of tetrad (Uggla et al, DG, Gundlach)

$e_0 = N^{-1}\, d_t \qquad e_\alpha = e_\alpha{}^i\, d_i$

$[e_0, e_\alpha] = u_\alpha\, e_0 - (H\delta_\alpha{}^\beta + \sigma_\alpha{}^\beta)\, e_\beta$

$[e_\alpha, e_\beta] = (2\, a_{[\alpha}\delta_{\beta]}{}^\gamma + \varepsilon_{\alpha\beta\delta}\, n^{\delta\gamma})\, e_\gamma$

The vacuum Einstein equations become evolution equations and constraint equations for the scale invariant variables.

Choose CMC slicing to asymptotically approach the singularity.

Results of simulations

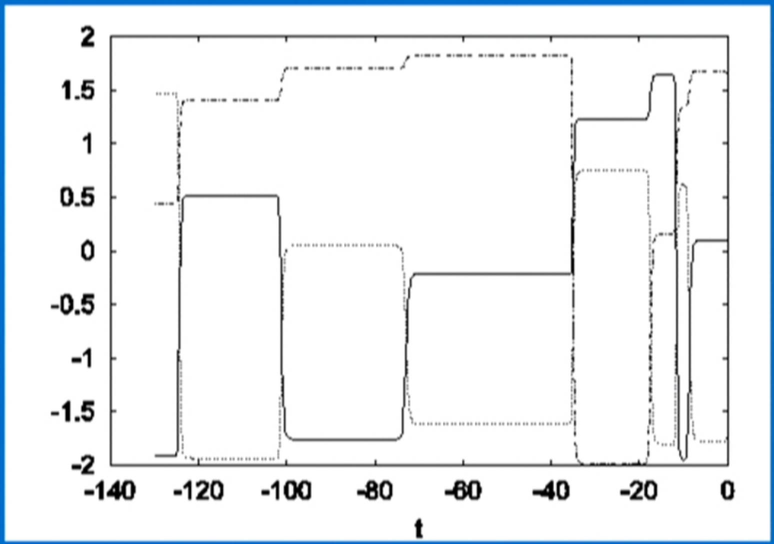
Spatial derivatives become negligible

At each spatial point the dynamics of the scale invariant variables becomes a sequence of "epochs" where the variables are constant, punctuated by short "bounces" where the variables change rapidly

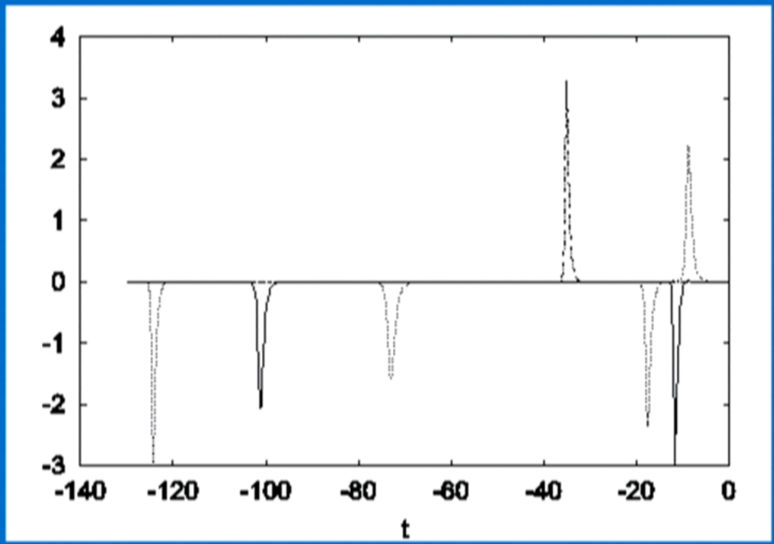
In CMC slicing $H=(1/3)e^{-t}$

$C_{abcd}C^{abcd}=e^{-4t}\ F[\Sigma_{\alpha\beta},N_{\alpha\beta},\ldots]$

Σ

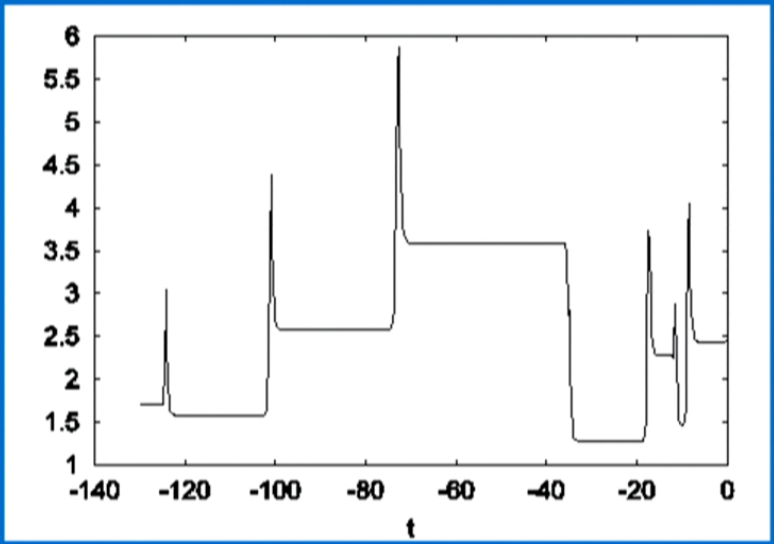


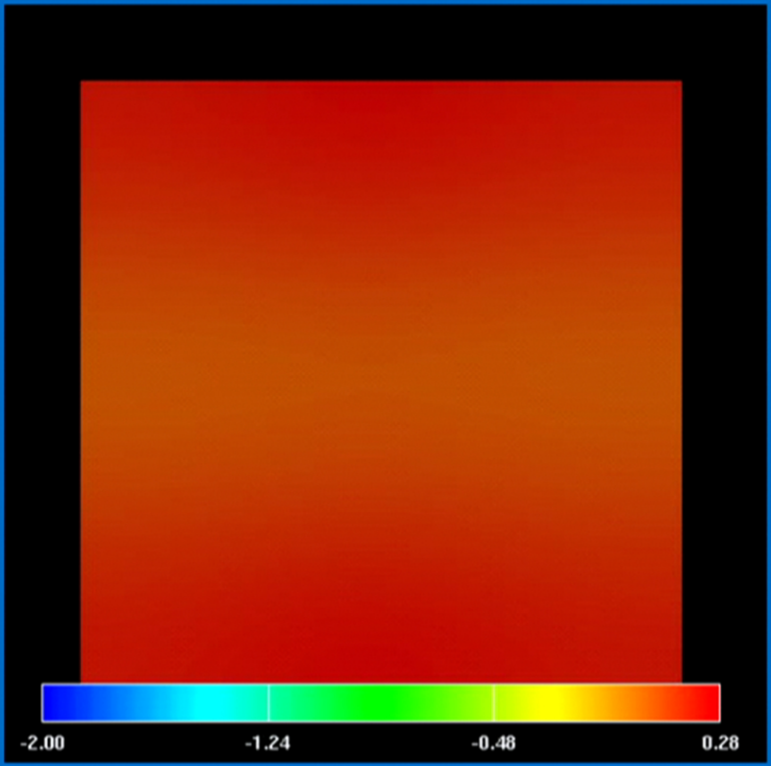
N

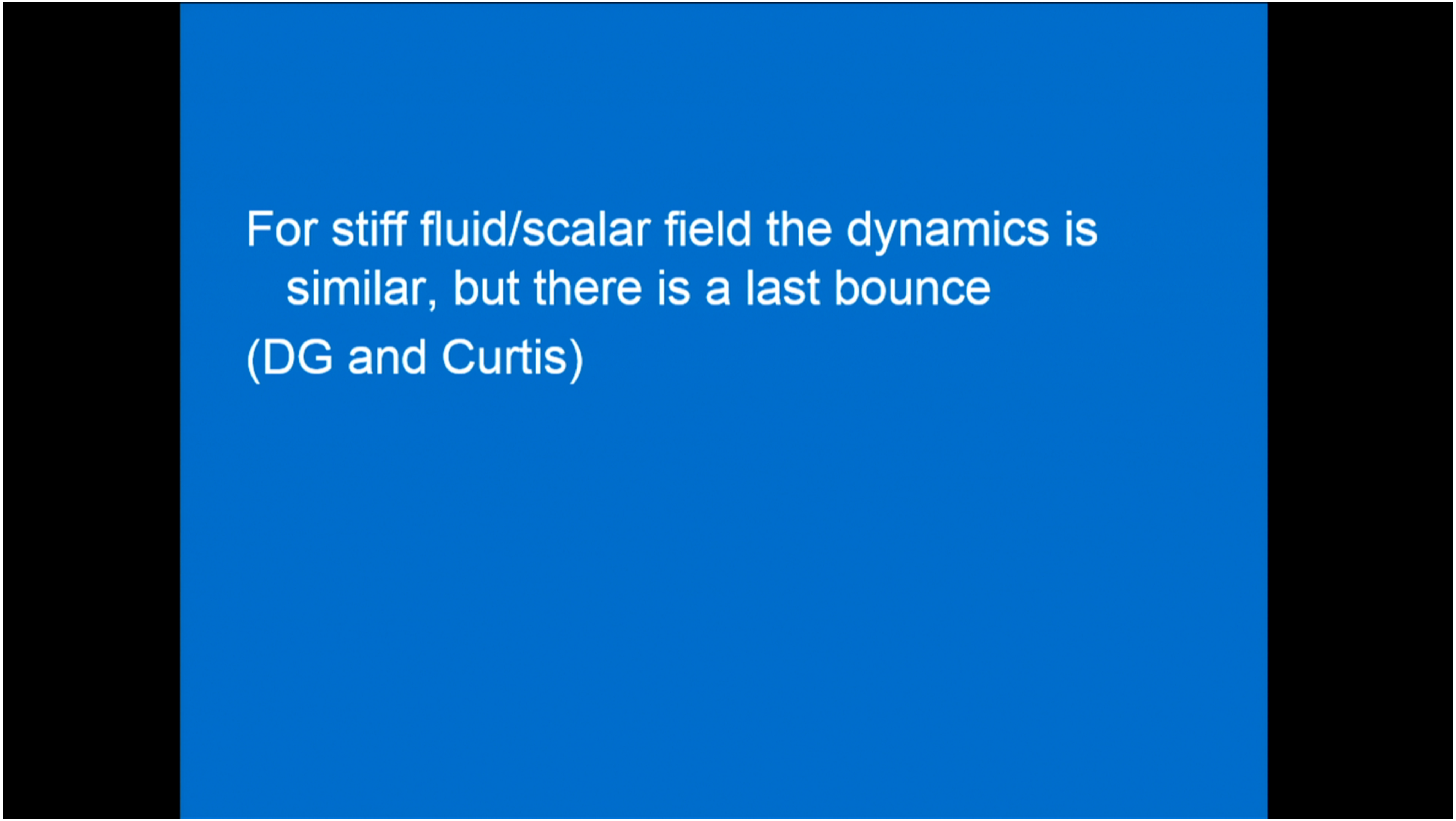


$\Sigma_{\alpha\beta}$ between bounces is characterized by a single number u. BKL conjecture that at a bounce u goes to u-1 if u>2 and to 1/(u-1) if u<2.

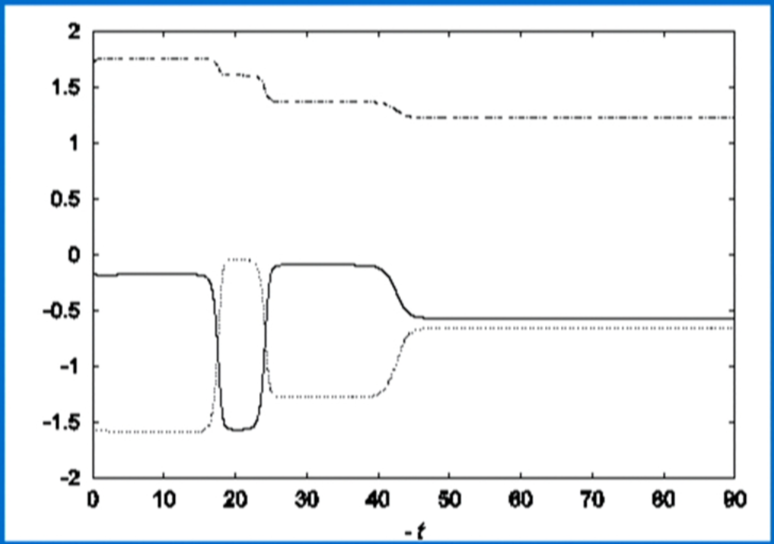
u




-2.00
-1.24
-0.48
0.28


For stiff fluid/scalar field the dynamics is similar, but there is a last bounce
(DG and Curtis)

Σ for stiff fluid



Resolution of spikes

- Use AMR

(DG and Pretorius)

So far only in 1d and ekpyrotic potential

Vacuum 2d work in progress

Null Singularities

- Inner horizon of Kerr and RN are unstable
- Conjecture (Israel and Poisson) the inner horizon becomes a null singularity
- Mathematical results establish that null singularities exist
- Numerical simulations (Brady and Smith…) for spherically symmetric case with a scalar field.

To Do List

- Resolve spikes in more than 1d
- Simulate BKL behavior in the asymptotically flat case
- Simulate null singularities beyond spherical symmetry

Conclusions

- Numerical simulations give us some understanding of singularities
- But there is still a lot left to do