Title: f(R) Gravity and Cosmology

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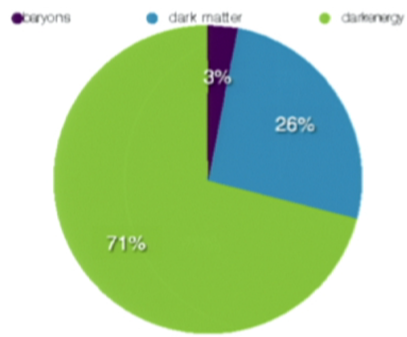
Abstract: A popular alternative to dark energy in explaining the current acceleration of the universe discovered with type Ia supernovae is modifying gravity at cosmological scales. But this is risky: even when everything is well for cosmology, other fundamental and experimental aspects of gravity must be checked in order for the theory to be viable. The successes of modified gravity and its challenges, which have generated a large body of literature in the past ten years, will be reviewed.


CAUTION

INTRODUCTION

Acceleration of the universe discovered 1998 with type Ia supernovae

WMAP, PLANCK, ...: if GR is correct, $\sim 71\%$ of the energy content of the universe is not luminous/dark matter



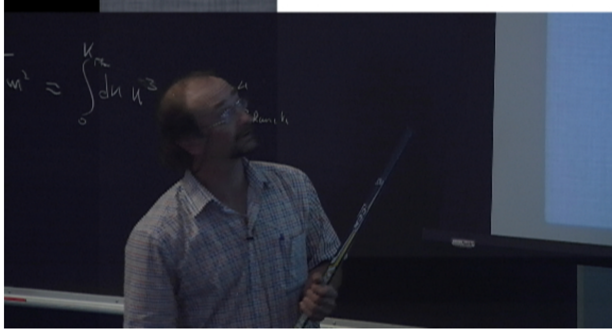
Implications not only for cosmology but also for fundamental physics

Popular explanations:

- $\Lambda \rightarrow$ cosmological constant problem(s) (landscape of string theory?)

$$\rho = \int_0^{k_0} d^3\underline{k}\, \sqrt{k^2 + m^2} \simeq \int_0^{k_{Planck}} dk\, k^3 \sim k^4_{Planck}$$

- **dark energy** - fluid with $P \simeq -\rho$ which comes to dominate late in the matter era. Possibly phantom energy with $P < -\rho$ (violates null energy condition, causes Big Rip)
- **backreaction of inhomogeneities** - lots of problems here
- **inhomogeneous universe** - serious fine-tuning
- **modified gravity**



MODIFIED ("$f(R)$") GRAVITY

IR modification of GR which kicks in only at low curvatures, late in the matter era

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\, f(R) + S^{(matter)}$$

R =Ricci scalar, $\kappa = 8\pi G$, $c = 1$ (Capozziello *et al.* 03, Carroll *et al.* 03)

In principle, the metric g_{ab} contains several degrees of freedom: tensor, vector, scalar; massless/massive

- GR: only massless spin 2 graviton propagates;
- modify Einstein-Hilbert action, then also scalar/vector modes appear;
- $R - 2\Lambda \longrightarrow f(R)$: massless graviton+massive scalar which can drive the cosmic acceleration;
- $R - 2\Lambda \longrightarrow f\left(R, R_{ab}R^{ab}, R_{abcd}R^{abcd}, ...\right)$: also massive gravitons and vector degrees of freedom.

Reviews in T.P. Sotiriou & VF, *Rev. Mod. Phys.* 2010; A. De Felice & S. Tsujikawa, *Living Rev. Relat.* 2010

$\geq$ 1500 articles in 2004-2014

$f(R)$ gravity taken very seriously by many authors. For now, regard as a toy model.

Modifying gravity is risky! Consequences can be:

- deviations from GR on terrestrial/Solar System scales which violate experimental constraints on PPN parameters;
- instabilities, ghosts, superluminal propagation;
- Cauchy problem not well-posed (loss of predictability).

(Long) history of $f(R)$ gravity

- Weyl '19; Eddington '22; Bach, Lanczos, Schrödinger, ...; historical review H.-J. Schmidt gr-qc/0602017
- quadratic corrections $f(R) = R + aR^2$ from first loop renormalization of GR (Utyama & de Witt '62, Stelle '77, ...);
- Starobinsky inflation without scalar fields '80;
- motivations from M-theory claimed for non-quadratic corrections (Nojiri & Odintsov '04); $f(R)$ found in asymptotic safety (Saueressig, Percacci, ...)

$$\text{Prototype: } f(R) = R - \mu^4/R\,, \quad \mu \sim H_0 \sim 10^{-33}\text{ eV}$$

(Capozziello *et al.* 03, Carroll *et al.* 03)
correction is negligible at large R, kicks in as $R \to 0$. Soon ruled out by Dolgov-Kawasaki instability+ weak field limit.

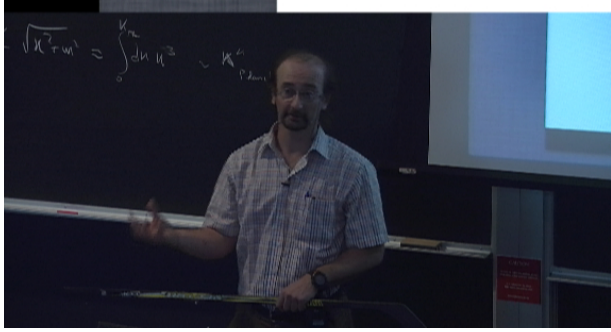
VERSIONS OF $f(R)$ COSMOLOGY/GRAVITY

- Metric (2nd order) formalism (Capozziello *et al.* '03, Carroll *et al.* '03) ← focus on this
- Palatini (1st order) formalism (Vollick '03)
- metric-affine gravity (Sotiriou & Liberati '07)

Metric formalism

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, f(R) + S^{(matter)}$$

varying w.r.t. g^{ab} yields



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$$f'(R)R_{ab} - \frac{f(R)}{2}\, g_{ab} = \underbrace{\nabla_a \nabla_b f'(R) - g_{ab}\Box f'(R)}_{\text{4th order}} + \kappa T_{ab}$$

where $' \equiv d/dR$.
Trace equation is

$$3\Box f'(R) + Rf'(R) - 2f(R) = \kappa T$$

for dynamical variable $f'(R)$. 2nd order eq., not the algebraic equation $R = -\kappa T$ of GR.
The effective gravitational coupling $G/f'(R)$ requires $f'(R) > 0$ (graviton carries positive kinetic energy).

Formally,

$$G_{ab} = \frac{\kappa}{f'(R)} \left(T^{(m)}_{ab} + T^{(eff)}_{ab} \right) ,$$

$$T^{(eff)}_{ab} = \frac{1}{\kappa} \left[\frac{f(R) - Rf'(R)}{2} g_{ab} + \nabla_a \nabla_b f'(R) - g_{ab} \Box f'(R) \right]$$

effective stress-energy tensor (ρ not ≥ 0).
In a spatially flat FLRW space with metric

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$

field eqs. reduce to

$$H^2 = \frac{\kappa}{3f'} \left[\rho^{(m)} + \frac{Rf' - f}{2} - 3H\dot{R}f'' \right] ,$$

$$2\dot{H} + 3H^2 = -\frac{\kappa}{f'} \left[P^{(m)} + f'''(\dot{R})^2 + 2H\dot{R}f'' + \ddot{R}f'' + \frac{f - Rf'}{2} \right] .$$

where

$$\phi(\chi) = f'(\chi)\,, \qquad V(\phi) = \chi f'(\chi) - f(\chi)$$

reduces to the previous action if $\chi = R$. Vice-versa, variation w.r.t. g^{ab} yields

$$G_{ab} = \frac{1}{\phi}\left(\nabla_a \nabla_b \phi - g_{ab}\Box\phi - \frac{V}{2}\, g_{ab}\right) + \frac{\kappa}{\phi}\, T^{(m)}_{ab}\,,$$

variation w.r.t. χ yields

$$R\,\frac{d\phi}{d\chi} - \frac{dV}{d\chi} = (R-\chi)\, f''(\chi) = 0 \;\Rightarrow \chi = R \;\text{ if }\; f'' \neq 0\,.$$

$\phi = f'(R)$ is a dynamical field and satisfies the **dynamical** eq.

$$3\Box\phi + 2V(\phi) - \phi\,\frac{dV}{d\phi} = \kappa\, T$$

(trace equation). In terms of the dynamical ϕ,

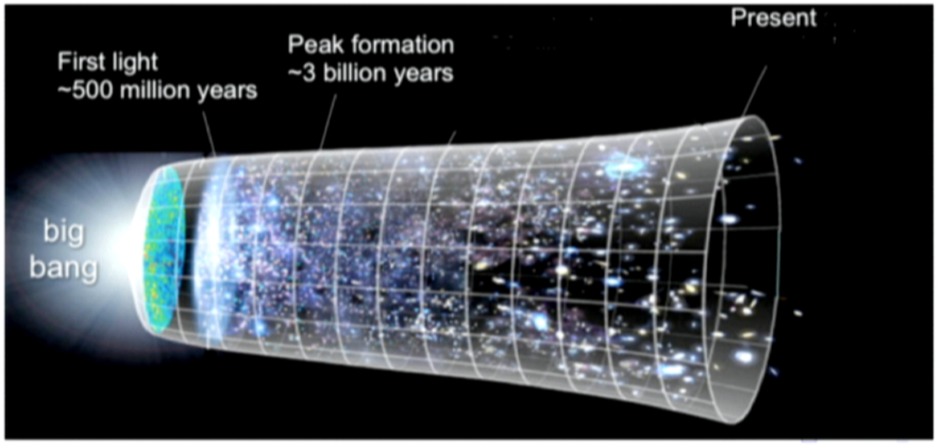
$$S = \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}\,\left[\phi R - V(\phi)\right] + S^{(m)}\,,$$

$$V(\phi) = \chi(\phi)\phi - f(\chi(\phi));$$

VIABILITY CRITERIA FOR $f(R)$ GRAVITY

- correct cosmological dynamics
- no instabilities
- no ghosts
- correct Newtonian/post-Newtonian limit
- well-posed Cauchy problem
- cosmological perturbations compatible with CMB, LSS

1) Correct cosmological dynamics



- early inflation (or substitute)
- radiation era (well constrained by Big Bang nucleosynthesis)
- matter era
- present accelerated epoch
- the future: de Sitter? Big Rip?

smooth transitions between consecutive eras required. Exit from radiation era could have problems (Amendola, Polarski, Tsujikawa, Gannouiji, ... but "designer $f(R)$ gravity" is possible: prescribe desired $a(t)$ and integrate an ODE for $f(R)$ that produces it (Nojiri & Odintsov '06, Song, Hu & Sawicky '06, Faulkner *et al.* '06).
$f(R)$ not determined uniquely, rather contrived forms.

2) Instabilities

Prototype: $f(R) = R - \mu^4/R\,, \quad \mu \sim H_0 \sim 10^{-33}$ eV suffers from the Dolgov-Kawasaki ('03) instability. Generalize to any metric $f(R)$ gravity (VF '06):

parametrize deviations from GR as

$$f(R) = R + \epsilon\varphi(R)\,, \qquad [\epsilon] = [m^2]\,, [\varphi] = [0]\,, \qquad f'' \neq 0$$

trace equation gives

$$\Box R + \frac{\varphi'''}{\varphi''}\nabla^c R \nabla_c R + \left(\frac{\epsilon\varphi' - 1}{3\epsilon\varphi''}\right) R = \frac{\kappa T}{3\epsilon\varphi''} + \frac{2\varphi}{3\varphi''}$$

locally, $g_{ab} = \eta_{ab} + h_{ab}\,, \quad R = -\kappa T + R_1$. To 1st order,

$$\ddot{R}_1 - \nabla^2 R_1 - \frac{2\kappa\varphi'''}{\varphi''}\dot{T}\dot{R}_1 + \frac{2\kappa\varphi'''}{\varphi''}\vec{\nabla}T \cdot \vec{\nabla}R_1$$

$$+ \underbrace{\frac{1}{3\varphi''}\left(\frac{1}{\epsilon} - \varphi'\right)}_{\text{effective mass squared}} R_1 = \kappa\ddot{T} - \kappa\nabla^2 T - \frac{(\kappa T\varphi' + 2\varphi)}{3\varphi''}$$

Effective mass squared is $m^2 \simeq \frac{1}{3\varphi''}$:

stability for $f''(R) \geq 0$

Example: $f(R) = R - \mu^4/R$ has $f'' < 0$ and instability manifests in $t \sim 10^{-26}$ s.

Physical interpretation: effective grav. coupling is $G_{eff} = \frac{G}{f'(R)} > 0$.
If $\frac{dG_{eff}}{dR} = \frac{-f''G}{(f')^2} > 0$, G_{eff} increases with R and large curvature$\longrightarrow$effect of gravity becomes stronger$\longrightarrow$larger $G_{eff}(R)$, positive feedback.
GR, with $f'' = 0$, corresponds to neutral stability.

Gauge-invariant perturbations of de Sitter space

(VF '05, '07) consider action

$$S = \int d^4x \sqrt{-g} \left[\frac{f(\phi, R)}{2} - \frac{\omega(\phi)}{2} \nabla^c\phi\nabla_c\phi - V(\phi) \right] ,$$

3) Ghosts

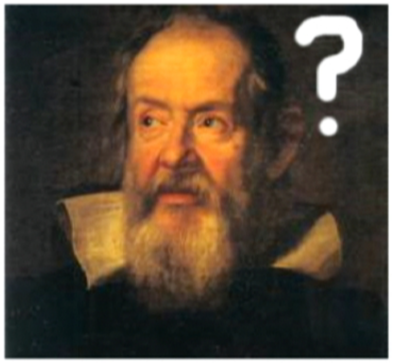
massive states of negative norm, violate unitarity

$f(R)$ gravity is ghost-free

In general $f\left(R, R_{ab}R^{ab}, R_{abcd}R^{abcd}, ...\right)$ contains ghosts (Stelle '77)

Exception: $f\left(R, \underbrace{R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}}_{\text{Gauss-Bonnet combo}}\right)$ gives 2nd order field equations, ghost-free under certain conditions (Nunez & Solganik '04, Comelli '05, Navarro & Van Acoleyen '06)

4) Weak-field limit



early work on $f(R) = R - \mu^4/R$ (Chiba *et al.* '06, Olmo '07)
Find weak-field solution, compute PPN parameter γ. Static, spherically symmetric, non-compact body in a background de Sitter universe described by

$$ds^2 = -\left[1 + 2\Psi(r) - H_0^2 r^2\right] dt^2 + \left[1 + 2\Phi(r) + H_0^2 r^2\right] dr^2 + r^2 d\Omega_{(2)}^2$$

in Schwarzschild coordinates, where

$$|\Psi(r)|\,, |\Phi(r)| \ll 1\,, rH_0 \ll 1\,, R(r) = R_0[dS] + R_1\,, \gamma = -\frac{\Phi(r)}{\Psi(r)}\,.$$

Assumptions:

- $f(R)$ analytical at R_0;
- $mr \ll 1$, where m is the effective mass of the scalar degree of freedom $f'(R)$ (no constraints if $m^{-1} \leq 0.2$ mm);
- $P \simeq 0, \; T = T_0 + T_1, \; T \approx -\rho$.

The trace eq.

$$3f_0'' \Box R_1 + \left(f_0'' R_0 - f_0'\right) R_1 = \kappa T_1$$

becomes

$$\nabla^2 R_1 - m^2 R_1 = \frac{-\kappa \rho}{3f_0''}$$

where $m^2 = \frac{(f_0')^2 - 2f_0 f_0''}{3f_0' f_0''}$, same as in GI perturbation formalism or in propagator calculations. If $mr \ll 1$, solution is

$$\Psi(r) = -\frac{\kappa M}{6\pi f_0'} \frac{1}{r}, \qquad \Phi(r) = \frac{\kappa M}{12\pi f_0'} \frac{1}{r},$$

PPN parameter is

$$\gamma = -\frac{\Phi(r)}{\Psi(r)} = \frac{1}{2}$$

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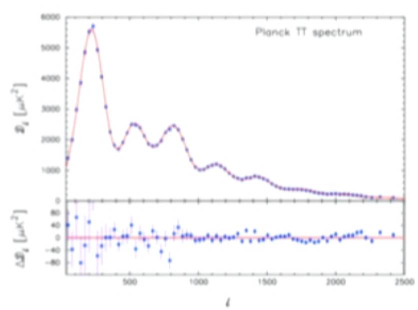
5) Correct dynamics of cosmological perturbations

Assuming evolution of scale factor $a(t)$ typical of ΛCDM model,

- vector/tensor modes are unaffected by $f(R)$ corrections (to 1st order);
- $f'' > 0$ required for stability of scalar perturbations;
- $f(R)$ corrections affect scalar modes: lower large angle anisotropies of CMB (help explain low 4-pole?);
- produce different correlations between CMB and galaxy surveys than in ΛCDM.



- cosmological perturbations (need further study of history growth, comparison with LSS surveys);
- theories $f\left(R, R_{ab}R^{ab}, R_{abcd}R^{abcd}\right)$ need more study;
- Ok, we understand better GR and scalar-tensor gravity better, but where does GR fail?