Title: Wild and tame scalar-tensor black holes

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Abstract: In scalar-tensor gravity, black holes do not obey the Jebsen-Birkhoff theorem. Non-isolated black holes can be highly dynamical and the teleological concept of event horizon is replaced by the apparent or trapping horizon. Dynamical solutions describing inhomogeneities embedded in cosmological "backgrounds" and the phenomenology of their apparent horizons, which often appear/vanish in pairs, will be described. Isolated black holes, in contrast, have no hair and are the same as in general relativity.

Wild and tame scalar-tensor black holes

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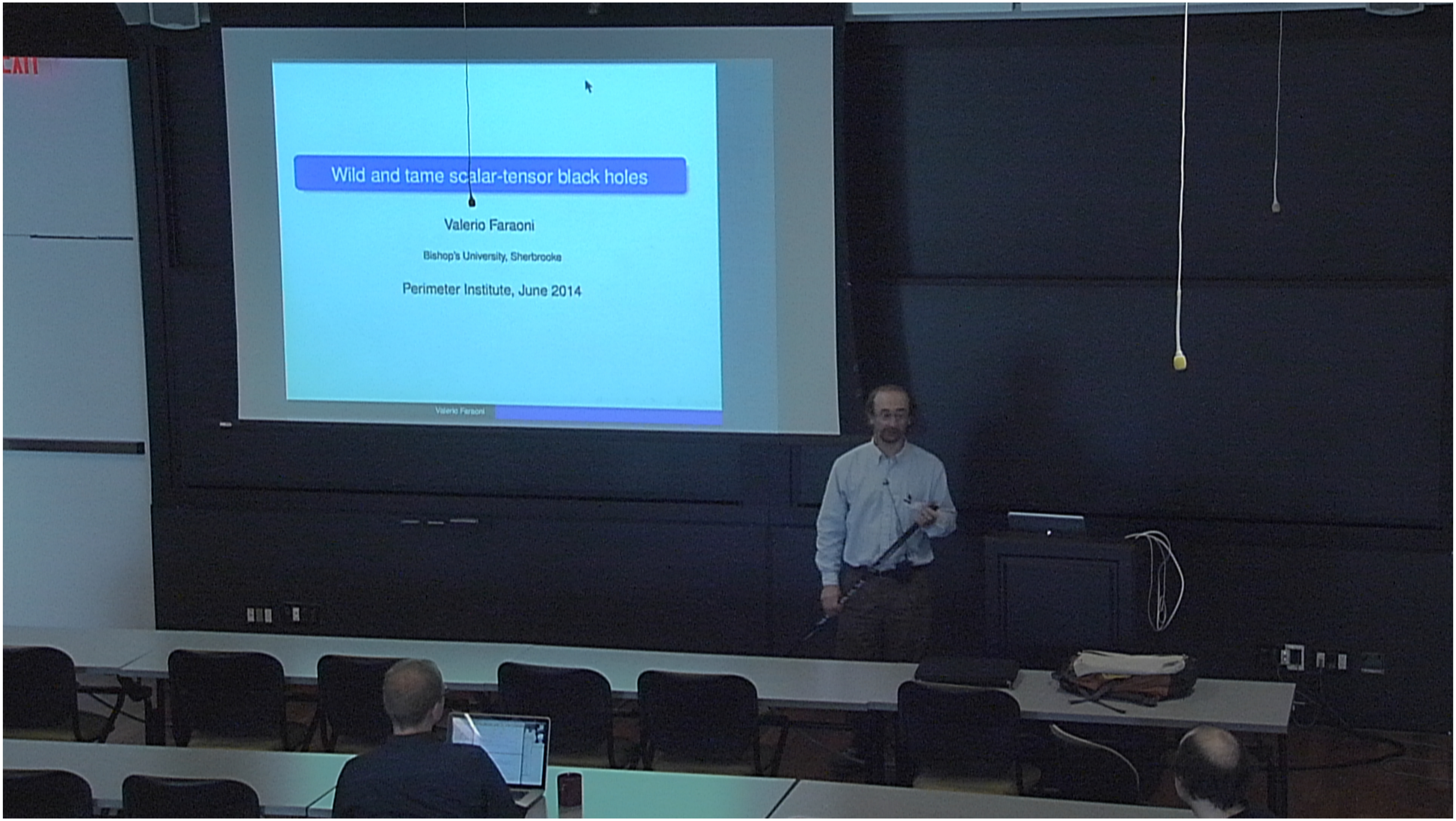
Perimeter Institute, June 2014



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MOTIVATION

Horizon = "a frontier between things observable and things unobservable" (Rindler '56). The horizon, the product of strong gravity, is the most impressive feature of a BH spacetime.
"Textbook" kinds of horizons: Rindler horizons, BH horizons, cosmological horizons; also event, Killing, inner, outer, Cauchy, apparent, trapping, quasi-local, isolated, dynamical, and slowly evolving horizons (Poisson; Wald; Booth; Nielsen; Ashtekar & Krishnan '04; Gourghoulhon & Jaramillo '08). Some horizon notions coincide for stationary BHs.
The (now classic) black hole mechanics and thermodynamics (1970s) focus on *stationary* BHs and *event horizons* but highly dynamical situations are of even greater interest:

- Gravitational collapse.
- Merger BH/compact object.
- Hawking radiation and evaporation of a small BH.
- BHs interacting with non-trivial environments (accretion/emission, backreaction).

VARIOUS NOTIONS OF HORIZON

Basic notions: congruence of null geodesics (tangent $l^a = dx^a/d\lambda$, affine parameter λ): $l_a l^a = l^c \nabla_c l^a = 0$. Metric h_{ab} in the 2-space orthogonal to l^a: pick another null vector field n^a such that $l^c n_c = -1$, then

$$h_{ab} \equiv g_{ab} + l_a n_b + l_b n_a \,.$$

h_{ab} purely spatial, $h^a{}_b$ is a projection operator on the 2-space orthogonal to l^a. The choice of n^a is not unique but the geometric quantities of interest do not depend on it once l^a is fixed. Let $\eta^a =$ geodesic deviation, define $B_{ab} \equiv \nabla_b l_a$, orthogonal to the null geodesics. The transverse part of the deviation vector is

$$\tilde{\eta}^a \equiv h^a{}_b \, \eta^b = \eta^a + (n^c \eta_c) l^a$$

and the orthogonal component of $l^c \nabla_c \eta^a$ is

$$(\widetilde{l^c \nabla_c \eta^a}) = h^a{}_b h^c{}_d B^b{}_c \, \tilde{\eta}^d \equiv \tilde{B}^a{}_d \, \tilde{\eta}^d$$

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$$\frac{d\theta}{d\lambda} = \kappa\,\theta - \frac{\theta^2}{2} - \sigma^2 + \omega^2 - R_{ab} l^a l^b \,.$$

A compact and orientable surface has two independent directions orthogonal to it: ingoing and outgoing null geodesics with tangents l^a and n^a. Basic definitions for closed 2-surfaces:

- A *normal surface* corresponds to $\theta_l > 0$ and $\theta_n < 0$.
- A *trapped surface* has $\theta_l < 0$ and $\theta_n < 0$. Both outgoing and ingoing null rays converge here, outward-propagating light is dragged back by strong gravity.
- A *marginally outer trapped* (or *marginal*) *surface (MOTS)* corresponds to $\theta_l = 0$ (where l^a is the outgoing null normal to the surface) and $\theta_n < 0$.
- An *untrapped surface* is one with $\theta_l \theta_n < 0$.
- An *antitrapped surface* corresponds to $\theta_l > 0$ and $\theta_n > 0$.
- A *marginally outer trapped tube (MOTT)* is a 3-D surface which can be foliated entirely by marginally outer trapped (2-D) surfaces.

horizon is a Killing horizon for the Killing vector

$$k^a = (\partial/\partial t)^a + \Omega_H (\partial/\partial\varphi)^a \ ,$$

linear combination of the time and rotational symmetry vectors, $\Omega_H =$ angular velocity at the horizon (this statement requires the assumption that the Einstein-Maxwell equations hold and some assumption on the matter stress-energy tensor). Attempts to use conformal Killing horizons have not been fruitful.

Apparent horizons (AHs)

A *future apparent horizon* is *the closure of a 3-surface which is foliated by marginal surfaces*; defined by the conditions on the time slicings (Hayward '93)

$$\theta_l = 0 \ ,$$
$$\theta_n < 0 \ ,$$

where θ_l and θ_n are the expansions of the future-directed outgoing and ingoing null geodesic congruences, respectively *that on its 2-D "time slicings"* (Hayward '93)

$$\theta_l = 0\,,$$
$$\theta_n < 0\,,$$
$$\mathcal{L}_n\,\theta_l = n^a \nabla_a\,\theta_l < 0\,,$$

Last condition distinguishes between inner and outer Hs and between AHs and trapping Hs (sign distinguishes between future and past horizons).

Past inner trapping horizon (PITH): exchange l^a with n^a and reverse signs in the inequalities,

$$\theta_n = 0\,,$$
$$\theta_l > 0\,,$$
$$\mathcal{L}_l \theta_n = l^a \nabla_a\,\theta_n > 0\,.$$

The PITH identifies a white hole or a cosmological horizon. As one moves just inside an outer trapping horizon, one encounters trapped surfaces, while trapped surfaces are

In spherical symmetry, the Kodama vector mimics the properties of a Killing vector and originates a (miracolously) conserved current and a surface gravity.
Defined only for spherically symmetric spacetimes. Let the metric be

$$ds^2 = h_{ab} dx^a dx^b + R^2 d\Omega^2_{(2)} \,,$$

where $a, b = 0, 1$ and R is the areal radius and $d\Omega^2_{(2)} = d\theta^2 + \sin^2\theta \, d\varphi^2$. Let ϵ_{ab} =volume form of h_{ab}; the *Kodama vector* is

$$K^a \equiv -\epsilon^{ab} \nabla_b R \,.$$

K^a lies in the 2-D (t, R) surface. In a static spacetime the Kodama vector is parallel (not equal) to the timelike Killing vector. When timelike, the Kodama vector defines a class of preferred observers (it is timelike in asymptotically flat regions). Divergence-free, $\nabla_a K^a = 0$, so the Kodama energy current $J^a \equiv G^{ab} K_b$ is covariantly conserved, $\nabla^a J_a = 0$ even if there is no timelike Killing vector ("Kodama miracle").

The Noether charge associated with the Kodama conserved current is the Misner-Sharp-Hernandez energy.

Spherical symmetry

Misner-Sharp-Hernandez mass defined in GR and for spherical symmetry, coincides with the Hawking-Hayward quasi-local mass (Hawking '68; Hayward '94). Use areal radius R, write

$$ds^2 = h_{ab}dx^a dx^b + R^2 d\Omega_{(2)}^2 \qquad (a, b = 1, 2). \qquad (1)$$

then

$$1 - \frac{2M}{R} \equiv \nabla^c R \nabla_c R$$

Formalism of Nielsen and Visser '06, general spherical metric is

$$ds^2 = -e^{-2\phi(t,R)} \left[1 - \frac{2M(t,R)}{R} \right] dt^2 + \frac{dR^2}{1 - \frac{2M(t,R)}{R}} + R^2 d\Omega_{(2)}^2$$

where $M(t, R)$ *a posteriori* is the Misner-Sharp-Hernandez mass. Recast in Painlevé-Gullstrand coordinates as

$$ds^2 = -\frac{e^{-2\phi}}{(\partial\tau/\partial t)^2} \left(1 - \frac{2M}{R} \right) d\tau^2 + \frac{2e^{-\phi}}{\partial\tau/\partial t} \sqrt{\frac{2M}{R}}\, d\tau dR + dR^2 + R^2 d\Omega_{(2)}^2$$

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with $g_{ab}l^a n^b = -2$. Expansions are

$$\theta_{l,n} = \pm \frac{2}{R}\left(1 \mp \sqrt{\frac{2M}{R}}\right)$$

A sphere of radius R is *trapped* if $R < 2M$, *marginal* if $R = 2M$, *untrapped* if $R > 2M$. AHs located by

$$\frac{2M\left(\tau, R_{AH}\right)}{R_{AH}(\tau)} = 1 \quad \Longleftrightarrow \quad \nabla^c R \nabla_c R \mid_{AH} = 0 \Longleftrightarrow \quad g^{RR} \mid_{AH} = 0\,,$$

Inverse metric is

$$\left(g^{\mu\nu}\right) = \frac{1}{c^2}\begin{pmatrix} 1 & -v & 0 & 0 \\ -v & -(c^2 - v^2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

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COSMOLOGICAL BHs IN SCALAR-TENSOR AND $f(R)$ GRAVITY

or A COSMOLOGICAL BESTIARY

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The Husain-Martinez-Nuñez solution of GR

HMN '94 new phenomenology of AHs. This spacetime describes an inhomogeneity in a spatially flat FLRW "background" sourced by a free, minimally coupled, scalar field.

$$
\begin{aligned}
ds^2 &= (A_0\eta + B_0)\left[-\left(1-\frac{2C}{r}\right)^{\alpha} d\eta^2 + \frac{dr^2}{\left(1-\frac{2C}{r}\right)^{\alpha}} \right. \\
&\quad \left. + r^2\left(1-\frac{2C}{r}\right)^{1-\alpha} d\Omega^2_{(2)}\right] , \\
\phi(\eta, r) &= \pm\frac{1}{4\sqrt{\pi}} \ln\left[D\left(1-\frac{2C}{r}\right)^{\alpha/\sqrt{3}} (A_0\eta + B_0)^{\sqrt{3}}\right] ,
\end{aligned}
$$

where $A_0, B_0, C, D \geq 0$ constants, $\alpha = \pm\sqrt{3}/2$, $\eta > 0$. The additive constant B_0 becomes irrelevant and can be dropped whenever $A_0 \neq 0$. When $A_0 = 0$, the HMN metric degenerates

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into the static Fisher spacetime (Fisher '48)

$$ds^2 = -V^{\nu}(r)\, d\eta^2 + \frac{dr^2}{V^{\nu}(r)} + r^2 V^{1-\nu}(r) d\Omega_{(2)}^2 \,,$$

where $V(r) = 1 - 2\mu/r$, μ and ν are parameters, and the Fisher scalar field is

$$\psi(r) = \psi_0 \ln V(r) \,.$$

(a.k.a. Janis-Newman-Winicour-Wyman solution, rediscovered many times, naked singularity at $r = 2C$, asympt. flat). The general HMN metric is conformal to the Fisher metric with conf. factor $\Omega = \sqrt{A_0 \eta + B_0}$ equal to the scale factor of the "background" FLRW space and with only two possible values of the parameter ν. Set $B_0 = 0$. Metric is asympt. FLRW for $r \to +\infty$ and is FLRW if $C = 0$ (in which case the constant A_0 can be eliminated by rescaling η).

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Ricci scalar is

$$R^a{}_a = 8\pi \nabla^c \phi \nabla_c \phi = \frac{2\alpha^2 C^2 \left(1 - \frac{2C}{r}\right)^{\alpha-2}}{3r^4 A_0 \eta} - \frac{3A_0^2}{2\,(A_0\eta)^3 \left(1 - \frac{2C}{r}\right)^{\alpha}} \,,$$

spacetime singularity at $r = 2C$ (for both values of α). ϕ also diverges there, Big Bang singularity at $\eta = 0$. $2C < r < +\infty$ and $r = 2C$ corresponds to zero areal radius

$$R(\eta, r) = \sqrt{A_0 \eta}\, r \left(1 - \frac{2C}{r}\right)^{\frac{1-\alpha}{2}} \,.$$

Use comoving time t, then

$$t = \int d\eta\, a(\eta) = \frac{2\sqrt{A_0}}{3}\,\eta^{3/2}\,, \qquad \eta = \left(\frac{3}{2\sqrt{A_0}}\, t\right)^{2/3}\,, \quad a(t) = a_0\, t^{1/3}\,,$$

HMN solution in comoving time reads

$$ds^2 = -\left(1 - \frac{2C}{r}\right)^{\alpha} dt^2 + a^2 \left[\frac{dr^2}{\left(1 - \frac{2C}{r}\right)^{\alpha}} + r^2 \left(1 - \frac{2C}{r}\right)^{1-\alpha} d\Omega_{(2)}^2\right]$$

with

$$\phi(t,r) = \pm \frac{1}{4\sqrt{\pi}} \ln \left[D \left(1 - \frac{2C}{r} \right)^{\alpha/\sqrt{3}} a^{2\sqrt{3}}(t) \right] .$$

Areal radius $R(t,r)$ increases for $r > 2C$. In terms of R, setting

$$A(r) \equiv 1 - \frac{2C}{r} , \qquad B(r) \equiv 1 - \frac{(\alpha+1)C}{r} ,$$

we have $R(t,r) = a(t) r A^{\frac{1-\alpha}{2}}(r)$ and a time-radius cross-term is eliminated by introducing a new T with $dT = \frac{1}{F} (dt + \beta dR)$,

$$\beta(t,R) = \frac{HRA^{\frac{3(1-\alpha)}{2}}}{B^2(r) - H^2 R^2 A^{2(1-\alpha)}} ;$$

then

$$ds^2 = -A^\alpha(r)\left[1-\frac{H^2R^2A^{2(1-\alpha)}(r)}{B^2(r)}\right]F^2dt^2$$
$$+\frac{H^2R^2A^{2-\alpha}}{B^2(r)}\left[1+\frac{A^{1-\alpha}(r)}{B^2(r)-H^2R^2A^{2(1-\alpha)}(r)}\right]dR^2+R^2d\Omega^2_{(2)}$$

AHs located by $g^{RR}=0$, or

$$\frac{1}{\eta}=\frac{2}{r^2}\left[r-(\alpha+1)C\right]\left(1-\frac{2C}{r}\right)^{\alpha-1}.$$

For $R\to+\infty$, reduces to $R\simeq H^{-1}$, cosmological AH in FLRW.
Let $x\equiv C/r$, then the AH eq. is

$$HR=\left[1-\frac{(\alpha+1)C}{r}\right]\left(1-\frac{2C}{r}\right)^{\alpha-1}.$$

lhs is

$$HR=\frac{a_0}{3\,t^{2/3}}\,\frac{2C}{x}\,(1-2x)^{\frac{1-\alpha}{2}},$$

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then

$$ds^2 = -A^\alpha(r)\left[1 - \frac{H^2R^2A^{2(1-\alpha)}(r)}{B^2(r)}\right]F^2dt^2 + \frac{H^2R^2A^{2-\alpha}}{B^2(r)}\left[1 + \frac{A^{1-\alpha}(r)}{B^2(r) - H^2R^2A^{2(1-\alpha)}(r)}\right]dR^2 + R^2d\Omega^2_{(2)}$$

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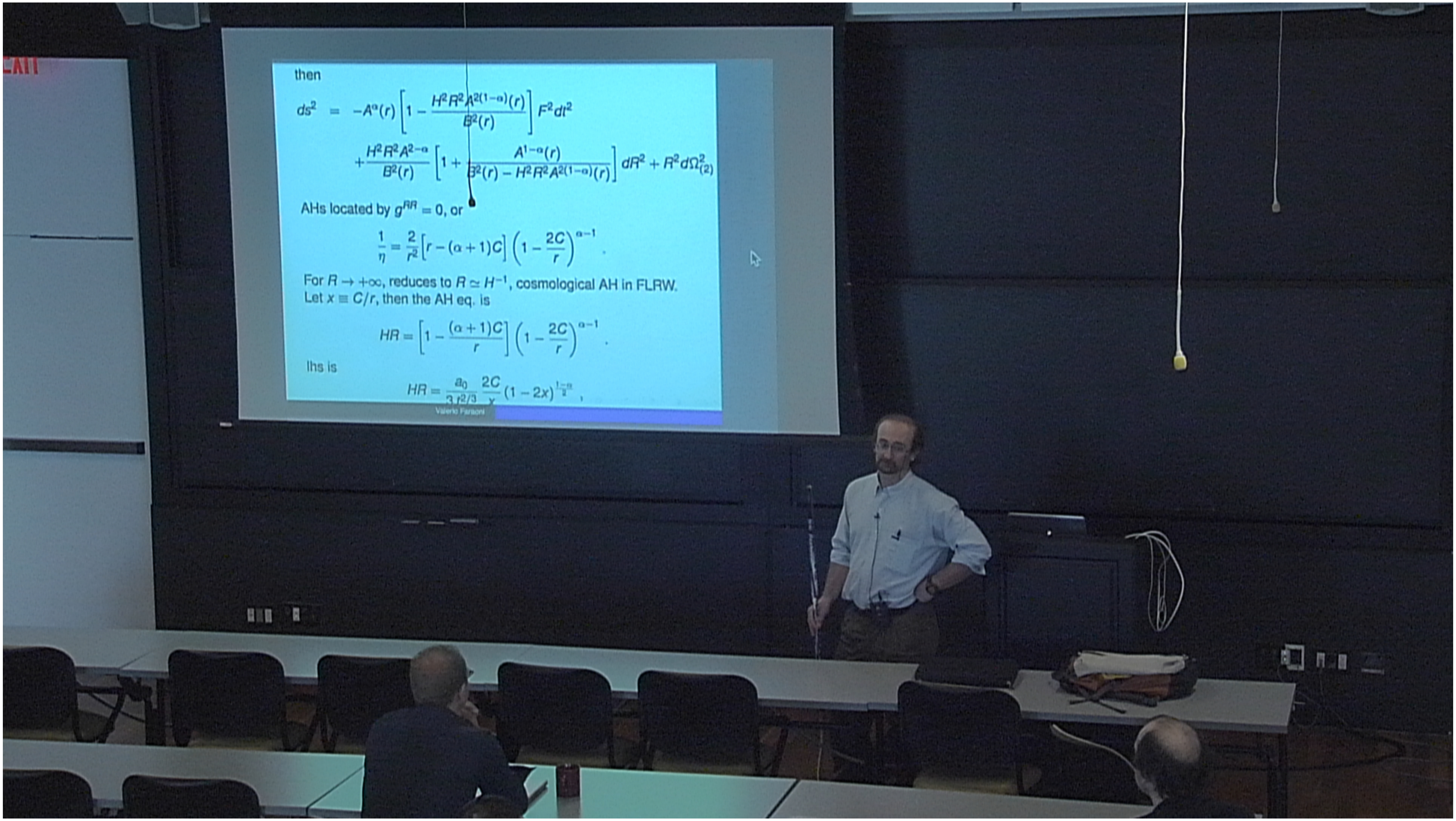
then
$ds^2 = -A^{\alpha}(r)\left[1 - \frac{H^2R^2A^{2(1-\alpha)}(r)}{B^2(r)}\right]F^2dt^2$
$+\frac{H^2R^2A^{2-\alpha}}{B^2(r)}\left[1 + \frac{A^{1-\alpha}(r)}{B^2(r) - H^2R^2A^{2(1-\alpha)}(r)}\right]dR^2 + R^2d\Omega^2_{(2)}$
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rhs is $[1-(\alpha+1)x]\,(1-2x)^{\alpha-1}$ and

$$t(x) = \left\{\frac{2Ca_0}{3}\,\frac{(1-2x)^{3(1-\alpha)}}{x\,[1-(\alpha+1)x]}\right\}^{3/2},$$

$$R(x) = a_0\,t^{1/3}(x)\,\frac{2C}{x}\,(1-2x)^{\frac{1-\alpha}{2}}.$$

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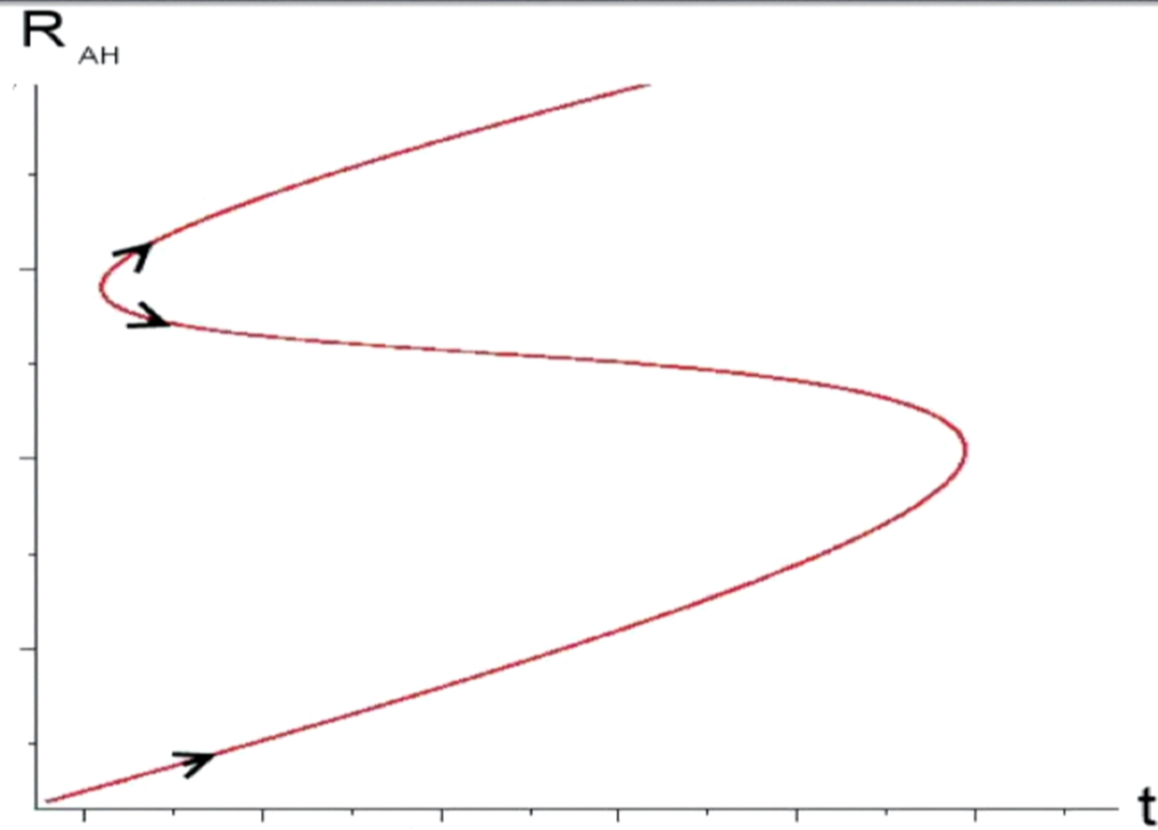

If $\alpha = \sqrt{3}/2$, between the Big Bang and a critical time t_* there

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The conformal cousin of the HMN solution

Solution of BD gravity found by Clifton, Mota, and Barrow '05 by conformally transforming the HMN solution,

$$ds^2 = -A^{\alpha\left(1-\frac{1}{\sqrt{3}\,\beta}\right)}(r)\,dt^2 + A^{-\alpha\left(1+\frac{1}{\sqrt{3}\,\beta}\right)}(r)\,t^{\frac{2(\beta-\sqrt{3})}{3\beta-\sqrt{3}}}\left[dr^2 + r^2 A(r)d\Omega_{(2)}^2\right],$$

$$\phi(t,r) = A^{\frac{\pm 1}{2\beta}}(r)\,t^{\frac{2}{\sqrt{3}\,\beta-1}},$$

where

$$A(r) = 1 - \frac{2C}{r}, \quad \beta = \sqrt{2\omega+3}, \quad \omega > -3/2.$$

Singularities at $r = 2C$ and $t = 0$ ($2C < r < +\infty$ and $t > 0$). The scale factor is

$$a(t) = t^{\frac{\beta-\sqrt{3}}{3\beta-\sqrt{3}}} \equiv t^{\gamma}. \quad (5)$$

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Interpreted in VF & A. Zambrano Moreno '12. Rewrite as

$$ds^2 = -A^{\sigma}(r)\,dt^2 + A^{\Theta}(r)\,a^2(t)dr^2 + R^2(t,r)d\Omega^2_{(2)}\,,$$

where

$$\sigma = \alpha\left(1 - \frac{1}{\sqrt{3}\,\beta}\right)\,, \quad \Theta = -\alpha\left(1 + \frac{1}{\sqrt{3}\,\beta}\right)\,,$$

and

$$R(t,r) = A^{\frac{\Theta+1}{2}}(r)\,a(t)\,r$$

Study the area of the 2-spheres of symmetry:
$\partial R/\partial r = a(t)A^{\frac{\Theta-1}{2}}(r)\,(1 - r_0/r)$ where

$$r_0 = (1-\Theta)C\,, \quad R_0(t) = \left(\frac{\Theta+1}{\Theta-1}\right)^{\frac{\Theta+1}{2}}(1-\Theta)a(t)\,C\,.$$

Critical value r_0 lies in the physical region $r_0 > 2C$ if $\Theta < -1$. R has the limit

$$R(t,r) = \frac{r\,a(t)}{\left(1 - \frac{2C}{r}\right)^{\left|\frac{\Theta+1}{2}\right|}} \rightarrow +\infty \quad \text{as } r \rightarrow 2C^{+} \qquad (6)$$

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AHs: use $dr = \frac{dR - A^{\frac{\Theta+1}{2}}(r)\,\dot{a}(t) r dt}{A^{\frac{\Theta-1}{2}}\,a(t)\,\frac{C(\Theta+1)}{r} + A^{\frac{\Theta+1}{2}}(r)\,a(t)}$, turn line element into

$$ds^2 = -\frac{(D_1 A^\sigma - H^2 R^2)}{D_1}\,dt^2 - \frac{2HR}{D_1}\,dtdR + \frac{dR^2}{D_1} + R^2 d\Omega^2_{(2)}$$

Inverse metric is

$$(g^{\mu\nu}) = \begin{pmatrix} -\frac{1}{A^\sigma} & -\frac{HR}{A^\sigma} & 0 & 0 \\ -\frac{HR}{A^\sigma} & \frac{(D_1 A^\sigma - H^2 R^2)}{A^\sigma} & 0 & 0 \\ 0 & 0 & R^{-2} & 0 \\ 0 & 0 & 0 & R^{-2}\sin^{-2}\theta \end{pmatrix}.$$

AHs located by $g^{RR} = 0$, or $D_1(r)A(r) = H^2(t)R^2(t,r)$. There are solutions which describe apparent horizons with the "S-curve" phenomenology of the HMN solution of GR. AH eq. satisfied also if the rhs is time-independent, $H = \gamma/t = 0$,

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$$ds^2 = -A^{b+1}(r)dt^2 + \frac{dr^2}{A^{a+1}(r)} + \frac{r^2 d\Omega^2_{(2)}}{A^a(r)},$$

$$\phi(r) = \phi_0 A^{\frac{a-b}{2}}(r),$$

with $\phi_0 > 0$, a, b constants,

$$\omega(a,b) = -2\left(a^2+b^2-ab+a+b\right)(a-b)^{-2}.$$

Reproduced by setting $(a,b) = \left(\frac{4\alpha}{3}-1, \frac{2\alpha}{3}-1\right)$, then $\omega\left(\frac{4\alpha}{3}-1, \frac{2\alpha}{3}-1\right) = 0$ for $\alpha = \pm\sqrt{3}/2$. The nature of the CL spacetime depends on sign(a) (choice $\alpha = \pm\sqrt{3}/2$ (Vanzo, Zerbini, VF '12). For $a \geq 0 \leftrightarrow \alpha = +\sqrt{3}/2$, $a \simeq 0.1547$, and $\Theta = -\frac{4\alpha}{3} \simeq -1.1547 < -1$ CL contains a wormhole throat which coincides with an AH at $r_0 = 2C\left(\frac{1-\Theta}{2}\right) > 2C$.
For $a < 0 \leftrightarrow \alpha = -\sqrt{3}/2$, $a \simeq -2.1547$, and $\Theta \simeq 1.1547 > 0$, there are no AHs and CL contains a naked singularity.

$$\rho^{(m)}(t, \varrho) = \rho_0^{(m)} \left(\frac{a_0}{a(t)} \right)^{3\gamma} A^{-2\alpha}$$

Matter source is a perfect fluid with $P^{(m)} = (\gamma - 1)\,\rho^{(m)}$ with $\gamma =$const. $m, \alpha, \phi_0, a_0, \rho_0^{(m)}, t_0$ are > 0. Areal radius is

$$r = a(t)\varrho \left(1 + \frac{m}{2\alpha\varrho} \right)^2 A^{\frac{1}{\alpha}(\alpha-1)(\alpha+2)} = a(t)\tilde{r} A^{\frac{1}{\alpha}(\alpha-1)(\alpha+2)}$$

Require that $\omega_0 > -3/2$ and $\beta \geq 0$. Interpreted in VF, Vitagliano, Sotiriou, Liberati '12, solve $g^{RR} = 0$ numerically. According to the parameter values, several behaviours are possible. The "S-curve" familiar from the HMN solution is reproduced in a certain region of the parameter space, but different behaviours appear for other combinations of the parameters. In certain regions of the parameter space, CMB contains a naked singularity created with the universe. In other regions of the parameter space, pairs of black hole and cosmological apparent horizons appear and bifurcate, or merge and disappear. Larger parameter space involved, CMB class exhibits most varied and richer phenomenology of AHs seen (some new one).

Other solutions

Few other solutions known in BD theory Sakai & Barrow, Einstein-Gauss-Bonnet gravity Nozawa & Maeda '08, higher order gravity Charmousis, Lovelock gravity Maeda, Willison, Ray '11 (Einstein-Gauss-Bonnet and Lovelock appropriate in $D > 4$, bestiary then includes Myers-Perry BHs, black strings, black rings, black Saturns, *etc.*). Add stringy/supergravity BHs.
Misner-Sharp-Hernandez mass and Kodama vector defined in GR and Einstein-Gauss-Bonnet gravity (perhaps in FLRW in $f(R)$ gravity)

[T.P. Sotiriou & VF PRL 2012]

- In GR spacetime singularities are generic (Hawking & Penrose) and they are usually cloacked by horizons (Cosmic Censorship).
- GR: **stationary** black holes (endpoint of grav. collapse) must be **axisymmetric** (Hawking '72). Asympt. flat black holes in GR are simple.
- Non-asympt. flat black holes can be very complicated: "cosmological" black holes have appearing/disappearing apparent horizons (McVittie, generalized McVittie, LTB, Husain-Martinez-Nuñez, Fonarev, ...). Interaction between black hole and cosmic "background".
- Scalar-tensor, $f(R)$ gravity, higher order gravity, low-energy effective actions for quantum gravity, etc.: Birkhoff's theorem is lost.

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[T.P. Sotiriou & VF PRL 2012]

- In GR spacetime singularities are generic (Hawking & Penrose) and they are usually cloacked by horizons (Cosmic Censorship).
- GR: **stationary** black holes (endpoint of grav. collapse) must be **axisymmetric** (Hawking '72). Asympt. flat black holes in GR are simple.
- Non-asympt. flat black holes can be very complicated: "cosmological" black holes have appearing/disappearing apparent horizons (McVittie, generalized McVittie, LTB, Husain-Martinez-Nuñez, Fonarev, ...). Interaction between black hole and cosmic "background".
- Scalar-tensor, $f(R)$ gravity, higher order gravity, low-energy effective actions for quantum gravity, etc.: Birkhoff's theorem is lost.

Prototype: Brans-Dicke theory (Jordan frame)

$$S_{BD} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega_0}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

- Hawking '72: endpoint of axisymmetric collapse in this theory must be GR black holes. Result generalized *for spherical symmetry only* by Bekenstein + Mayo '96, Bekenstein '96, + bits and pieces of proofs.
- What about more general theories?

A SIMPLE PROOF

This work (T.P. Sotiriou & VF 2012, *Phys. Rev. Lett.* 108, 081103): extend result to *general* scalar-tensor theory

$$S_{ST} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega(\varphi)}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi - V(\varphi) + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

(this action includes metric and Palatini $f(R)$ gravity). We require

- **asymptotic flatness** (collapse on scales $\ll H_0^{-1}$): $\varphi \to \varphi_0$ as $r \to +\infty$, $V(\varphi_0) = 0$, $\varphi_0 V'(\varphi_0) = 2V(\varphi_0)$
- **stationarity** (endpoint of collapse).

Use Einstein frame $\hat{g}_{\mu\nu} \to g_{\mu\nu} = \varphi \hat{g}_{\mu\nu}$, $\varphi \to \phi$ with

$$d\phi = \sqrt{\frac{2\omega(\varphi) + 3}{16\pi}} \frac{d\varphi}{\varphi} \qquad (\omega \neq -3/2)$$

brings the action to

$$S_{ST} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - U(\phi) + L_m(\hat{g}_{\mu\nu},\psi)\right]$$

where $U(\phi) = V(\varphi)/\varphi^2$. Field eqs. are

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} = \frac{\omega(\varphi)}{\varphi^2}\left(\hat{\nabla}_\mu\varphi\hat{\nabla}_\nu\varphi - \frac{1}{2}\hat{g}_{\mu\nu}\,\hat{\nabla}^\lambda\varphi\hat{\nabla}_\lambda\varphi\right) + \frac{1}{\varphi}\left(\hat{\nabla}_\mu\hat{\nabla}_\nu\varphi - \hat{g}_{\mu\nu}\hat{\Box}\varphi\right) - \frac{V(\varphi)}{2\varphi}\hat{g}_{\mu\nu},$$

$$(2\omega+3)\,\hat{\Box}\varphi = -\omega'\,\hat{\nabla}^\lambda\varphi\hat{\nabla}_\lambda\varphi + \varphi\,V' - 2V,$$

$\Omega = \Omega(\varphi) \longrightarrow$ same symmetries as in the J. frame:

- ξ^μ timelike Killing vector (stationarity)
- ζ^μ spacelike at spatial infinity (axial symmetry).

multiply $\Box\phi = U'(\phi)$ by U', integrate over $\mathcal{V} \longrightarrow$

$$\int_{\mathcal{V}} d^4x\sqrt{-g}\, U'(\phi)\Box\phi = \int_{\mathcal{V}} d^4x\sqrt{-g}\, U'^2(\phi)$$

rewrite as

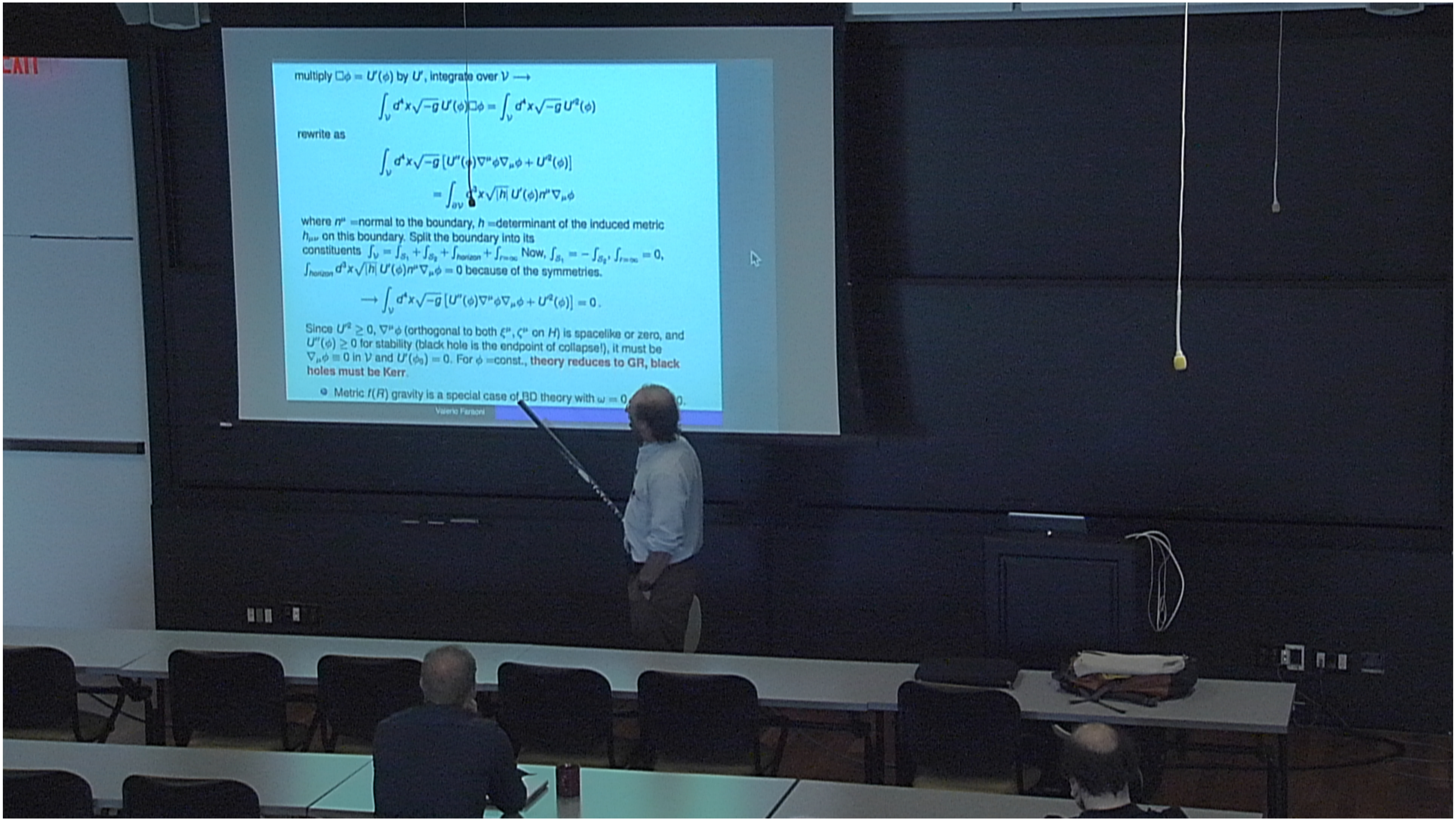
$$\int_{\mathcal{V}} d^4x\sqrt{-g}\,\left[U''(\phi)\nabla^\mu\phi\nabla_\mu\phi + U'^2(\phi)\right] = \int_{\partial\mathcal{V}} d^3x\sqrt{|h|}\, U'(\phi)n^\mu\nabla_\mu\phi$$

where n^μ =normal to the boundary, h =determinant of the induced metric $h_{\mu\nu}$ on this boundary. Split the boundary into its constituents $\int_{\mathcal{V}} = \int_{\mathcal{S}_1} + \int_{\mathcal{S}_2} + \int_{horizon} + \int_{r=\infty}$ Now, $\int_{\mathcal{S}_1} = -\int_{\mathcal{S}_2}$, $\int_{r=\infty} = 0$, $\int_{horizon} d^3x\sqrt{|h|}\, U'(\phi)n^\mu\nabla_\mu\phi = 0$ because of the symmetries.

$$\longrightarrow \int_{\mathcal{V}} d^4x\sqrt{-g}\,\left[U''(\phi)\nabla^\mu\phi\nabla_\mu\phi + U'^2(\phi)\right] = 0\,.$$

Since $U'^2 \geq 0$, $\nabla^\mu\phi$ (orthogonal to both ξ^μ, ζ^μ on H) is spacelike or zero, and $U''(\phi) \geq 0$ for stability (black hole is the endpoint of collapse!), it must be $\nabla_\mu\phi \equiv 0$ in $\mathcal{V}$ and $U'(\phi_0) = 0$. For ϕ =const., **theory reduces to GR, black holes must be Kerr**.

- Metric $f(R)$ gravity is a special case of BD theory with $\omega = 0$ and $V \neq 0$.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$\longrightarrow \int_{\mathcal{V}} d^4x\sqrt{-g}\left[U''(\phi)\nabla^{\mu}\phi\nabla_{\mu}\phi + U'^2(\phi)\right] = 0\,.$
Since $U'^2 \geq 0$, $\nabla^{\mu}\phi$ (orthogonal to both ξ^{μ}, ζ^{μ} on H) is spacelike or zero, and $U''(\phi) \geq 0$ for stability (black hole is the endpoint of collapse!), it must be $\nabla_{\mu}\phi = 0$ in $\mathcal{V}$ and $U'(\phi_0) = 0$. For ϕ =const., theory reduces to GR, black holes must be Kerr.
Metric $f(R)$ gravity is a special case of BD theory with $\omega = 0$,
Valerio Faraoni

- for $\omega = -3/2$, vacuum theory reduces to GR, Hawking's theorem applies (Palatini $f(R)$ gravity is a special BD theory with $\omega = -3/2$ and $V \neq 0$).

Exceptions not covered by our proof:

- theories in which $\omega \to \infty$ somewhere
- theories in which φ diverges (at ∞ or on the horizon)
  ex: maverick solution of Bocharova *et al.* '80 (unstable).
- Proof extends immediately to electrovacuum/conformal matter ($T = 0$).

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CONCLUSIONS AND OPEN PROBLEMS

- Rich bestiary (phenomenology and dynamics) of evolving horizons.
- Are AHs/trapping horizons the "right' quantities for thermodynamics? Is their thermodynamics meaningful? Is the Kodama prescription correct? (conflicting views)
- An adiabatic approximation should be meaningful. Do fast-evolving horizons require non-equilibrium thermodynamics?
- Even though Birkhoff's theorem is lost, black holes which are the endpoint of axisymmetric gravitational collapse (and asympt. flat) in *general* scalar-tensor gravity are the same as in GR (*i.e.*, Kerr-Newman). Proof extends to electrovacuum.
- Exceptions (exact solutions) are unphysical or unstable solutions which cannot be the endpoint of collapse, or do not satisfy the Weak/Null Energy Condition.
- Asymptotic flatness is a technical assumption, but can't eliminate it at the moment. Excludes "large" primordial black holes in a "small" universe.
- What about more general theories with other degrees of freedom?