Title: Atomic Clocks Monitored to 0.2 ns using Satellite Geodesy

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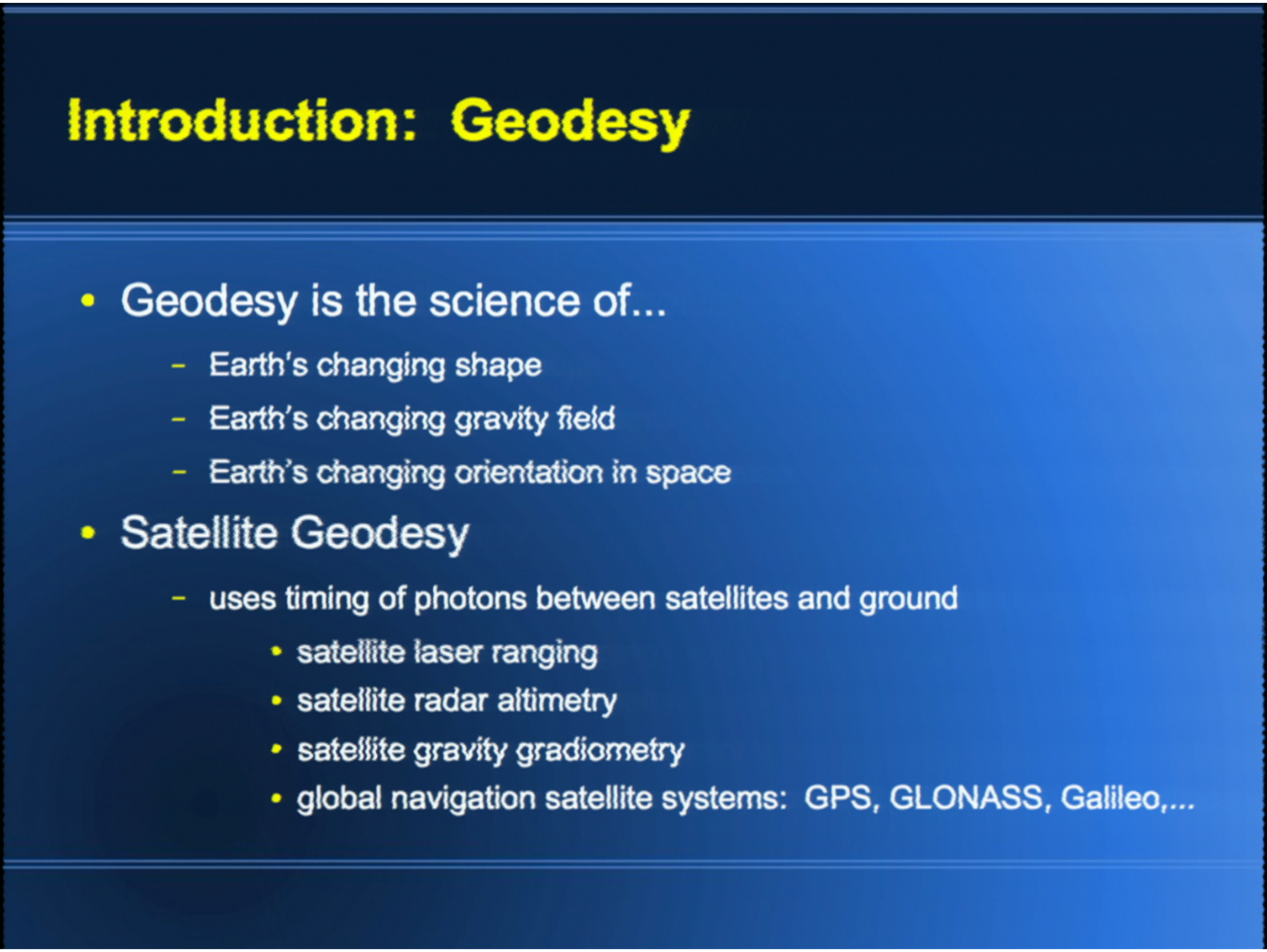
Abstract: <span>Satellite geodesy uses the timing of photons from satellites to determine the Earthâ€™s time varying shape, gravity field, and orientation in space, with accuracies of <1 part per billion, or millimeters at the Earthâ€™s surface, and centimeters at satellite altitude. Implicit in mm-level GPS positioning is the modeling of widely separated atomic clocks with sub-ns precision. The precise monitoring of the relative timing phases between widely separated atomic clocks forms the metrological basis of a recently proposed approach to detect topological dark matter of a type that affects fundamental constants. Relative clock time can be updated as often as every second using the current global network of geodetic GPS stations that record data at that rate, though many more geodetic GPS stations record data every 30 seconds. Thus GPS could be used as the worldâ€™s largest dark matter detector, potentially sensitive to dark matter structures sweeping through the entire system >100 seconds, corresponding to speeds <500 km sÂ¬-1 relative to the solar system. Here it is shown that relative timing phases can be determined to ~0.2 ns between the global network of atomic clocks at many geodetic GPS stations on the Earthâ€™s surface separated as far as ~12,000 km, plus those aboard the 30 GPS satellites separated as far as ~50,000 km. Available atomic clock types include caesium (Cs), rubidium (Rb), and (on the ground) hydrogen maser (Hm). Achieving sub-ns relative timing precision requires (1) dual-frequency carrier phase data measured at the few mm level, (2) rigorous modeling of many aspects of the Earth system and GPS satellite dynamics, and (3) stochastic estimation of biases in the system. For example, solar radiation pressure from momentum exchange with photons hitting the satellites perturbs orbits at the few-meter level. Imperfect modeling, such as knowledge of the satellite attitude, requires us to estimate orbit acceleration biases as they slowly vary in time. For mm-level positioning applications, clock phases are considered to be unknown biases to be estimated as a white noise process, that is, estimated independently at every data epoch without constraint. By virtue of the common view of satellites simultaneously by multiple ground stations, relative clock time can be determined between all clocks in the entire satellite-ground system by estimating all biases in a global inversion. Since the timing phase between Hm clocks can be accurately extrapolated forward in time, they set the standard by which upper limits can be set on the precision of timing at any specific instant. As a feasibility study, a custom analysis of original raw GPS phase data was designed using the GIPSY OASIS II software (from NASA JPL), processing data from ~40 ground stations of various atomic clock type. An analysis of data from GPS stations that are positioned at the few-millimeter level every day indicates that Hm clock time is determined at to ~0.2 ns. Since the smoothness of Hm clocks is not assumed anywhere in the modeling, and that station clock type has no influence on positioning precision, one can infer that timing at the 0.2 ns level is also the case for less predictable atomic clocks such as Rb and Cs, thus providing a window into possibly different coupling of dark matter with different clock types.


Satellite Geodesy
uses timing of photons between sat
satellite laser ranging
satellite radar altimetry
satellite gravity gradiometry
global navigation

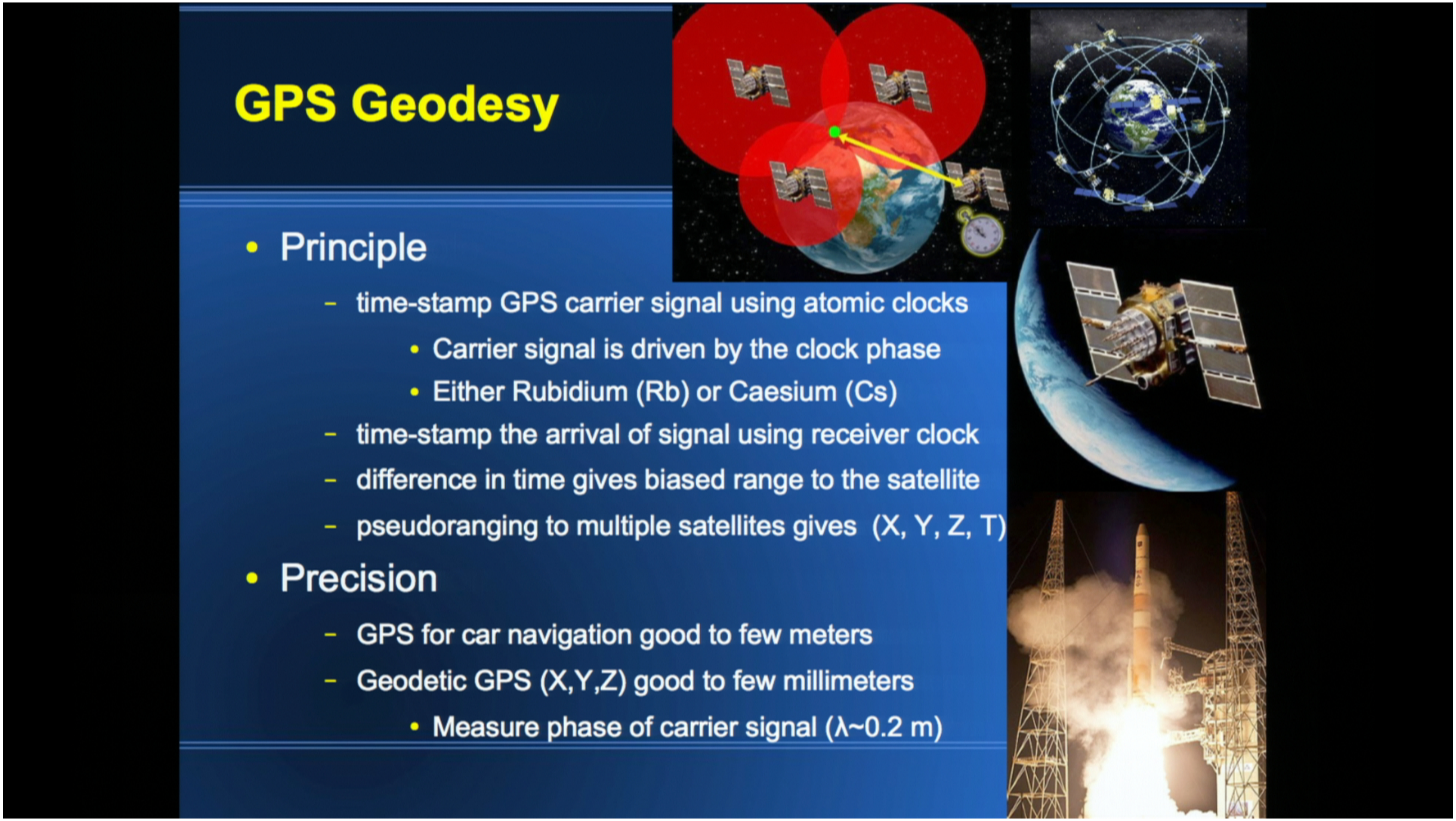
Overview

- What is satellite geodesy?
- GPS geodesy
- GPS clocks and relativity
- Modeling GPS light time equation
- Global inversion for all clock biases
- Data analysis demonstration
- Results
- Conclusions



Introduction: Geodesy

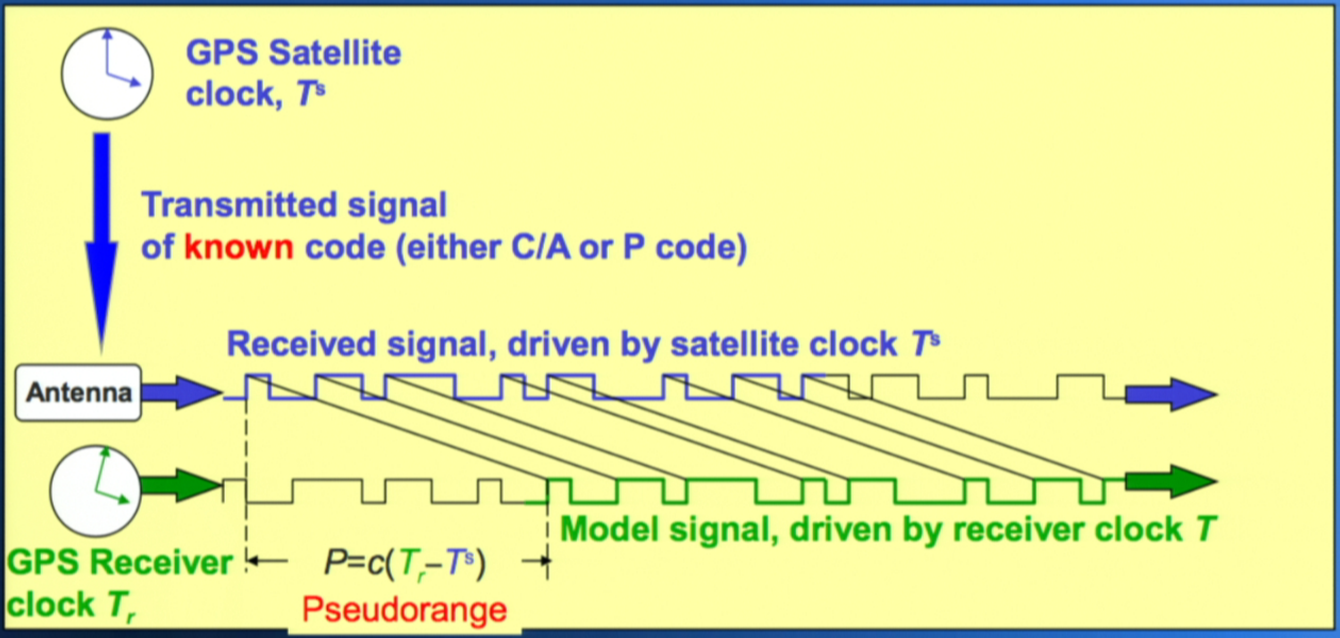
- Geodesy is the science of...
  - Earth's changing shape
  - Earth's changing gravity field
  - Earth's changing orientation in space
- Satellite Geodesy
  - uses timing of photons between satellites and ground
    - satellite laser ranging
    - satellite radar altimetry
    - satellite gravity gradiometry
    - global navigation satellite systems: GPS, GLONASS, Galileo,...



GPS Geodesy

- Principle
  - time-stamp GPS carrier signal using atomic clocks
    - Carrier signal is driven by the clock phase
    - Either Rubidium (Rb) or Caesium (Cs)
  - time-stamp the arrival of signal using receiver clock
  - difference in time gives biased range to the satellite
  - pseudoranging to multiple satellites gives (X, Y, Z, T)
- Precision
  - GPS for car navigation good to few meters
  - Geodetic GPS (X,Y,Z) good to few millimeters
    - Measure phase of carrier signal ($\lambda \sim 0.2$ m)

GPS Pseudorange Measurement (1 m)



GPS Satellite clock, T^s

Transmitted signal of **known** code (either C/A or P code)

Received signal, driven by satellite clock T^s

Antenna

Model signal, driven by receiver clock T

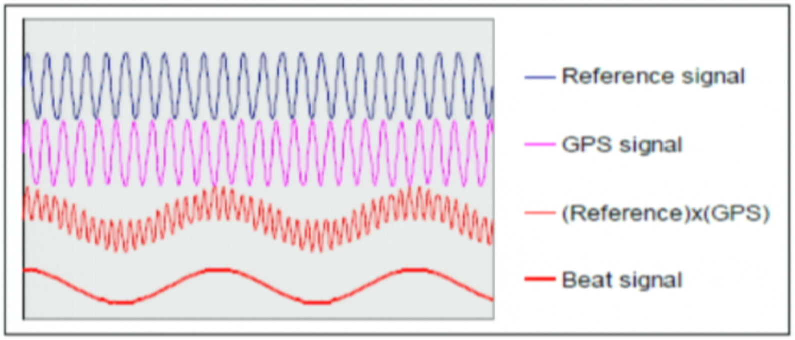
GPS Receiver clock T_r

$P=c(T_r-T^s)$

Pseudorange

GPS Carrier Phase Measurement (1 mm)

Receiver multiplies GPS satellite signal × reference signal from local oscillator



$$S(t) = S_0 \cos\varphi_S(t)$$

$$R(t) = R_0 \cos\varphi_R(t)$$

$$S(t) \times R(t) = \tfrac{1}{2} R_0 S_0 \left[\cos\left(\varphi_R - \varphi_S\right) + \cos\left(\varphi_R + \varphi_S\right)\right]$$

$$S(t) \otimes R(t) = \tfrac{1}{2} R_0 S_0 \cos\left(\varphi_R - \varphi_S\right)$$

"Baseband filter" removes high frequency

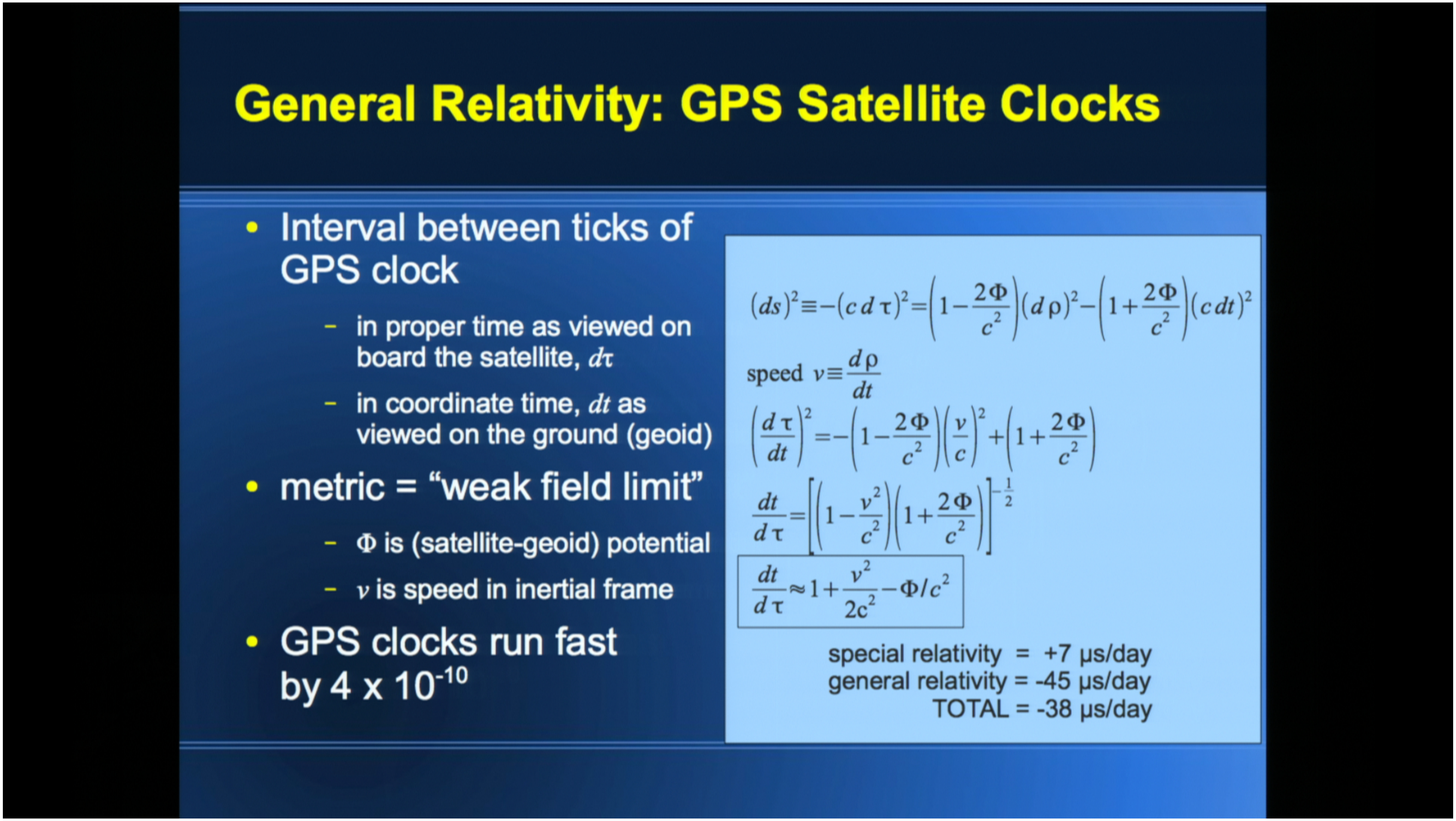
"Carrier phase" = phase of baseband signal (Ambiguity)

Carrier phase measurement: $\Phi = \left(\phi_R - \phi_S\right) + \mathrm{N}$

Model as satellite-receiver clock: $\Phi = f\left(T_R - T_S\right) + \mathrm{N}$

Convert cycles to range $L = \lambda\Phi = c\left(T_R - T_S\right) + N\lambda$

Measurement error < 1 mm



General Relativity: GPS Satellite Clocks

- Interval between ticks of GPS clock
  - in proper time as viewed on board the satellite, $d\tau$
  - in coordinate time, dt as viewed on the ground (geoid)
- metric = “weak field limit”
  - Φ is (satellite-geoid) potential
  - v is speed in inertial frame
- GPS clocks run fast by 4 x 10^{-10}

$$(ds)^2 \equiv -(c\,d\tau)^2 = \left(1-\frac{2\Phi}{c^2}\right)(d\rho)^2 - \left(1+\frac{2\Phi}{c^2}\right)(c\,dt)^2$$

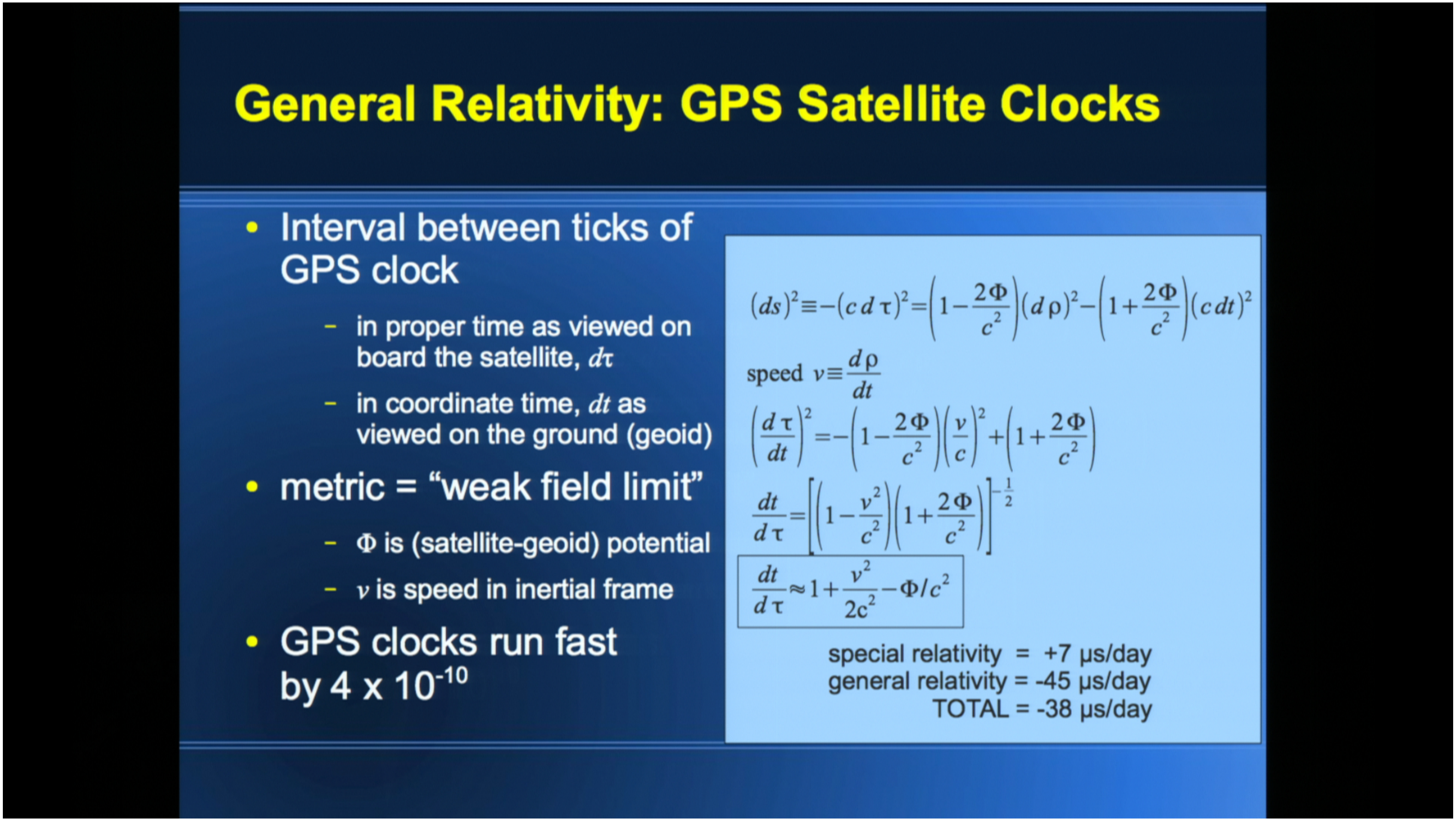
speed $v \equiv \frac{d\rho}{dt}$

$$\left(\frac{d\tau}{dt}\right)^2 = -\left(1-\frac{2\Phi}{c^2}\right)\left(\frac{v}{c}\right)^2 + \left(1+\frac{2\Phi}{c^2}\right)$$

$$\frac{dt}{d\tau} = \left[\left(1-\frac{v^2}{c^2}\right)\left(1+\frac{2\Phi}{c^2}\right)\right]^{-\frac{1}{2}}$$

$$\boxed{\frac{dt}{d\tau} \approx 1+\frac{v^2}{2c^2}-\Phi/c^2}$$

special relativity = +7 μs/day
general relativity = -45 μs/day
TOTAL = -38 μs/day



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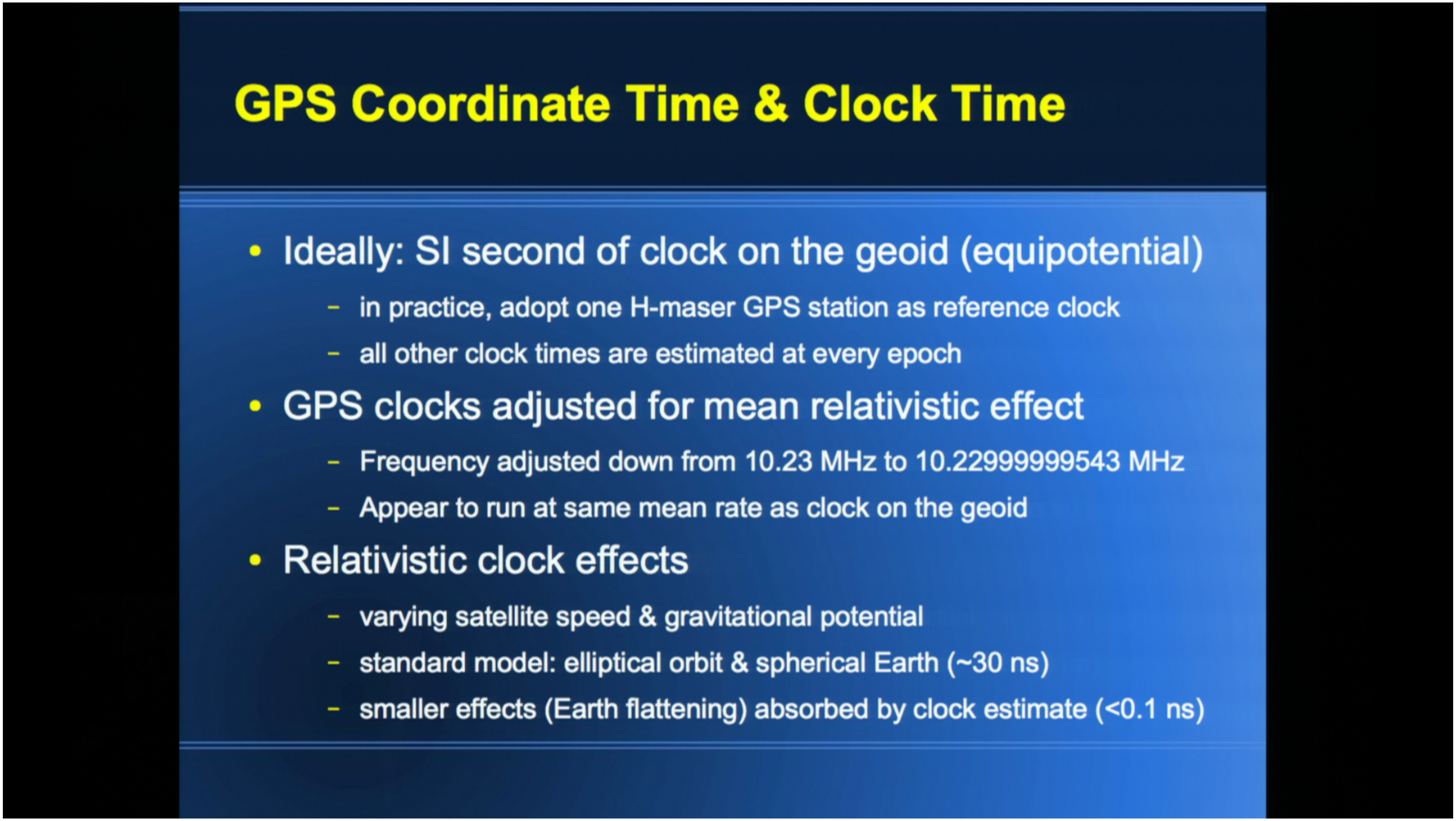
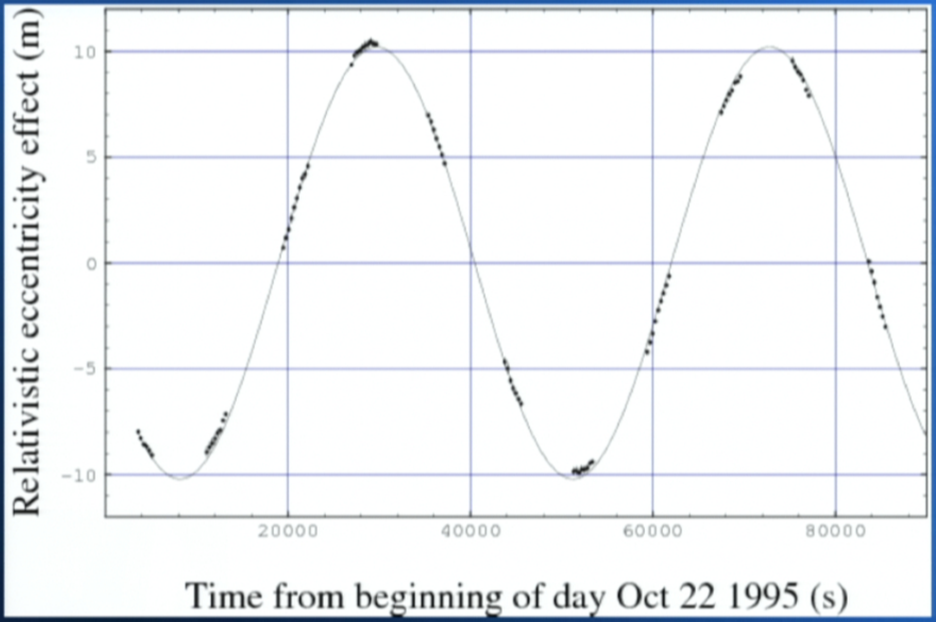


GPS Coordinate Time & Clock Time

- Ideally: SI second of clock on the geoid (equipotential)
  - in practice, adopt one H-maser GPS station as reference clock
  - all other clock times are estimated at every epoch
- GPS clocks adjusted for mean relativistic effect
  - Frequency adjusted down from 10.23 MHz to 10.22999999543 MHz
  - Appear to run at same mean rate as clock on the geoid
- Relativistic clock effects
  - varying satellite speed & gravitational potential
  - standard model: elliptical orbit & spherical Earth (~30 ns)
  - smaller effects (Earth flattening) absorbed by clock estimate (<0.1 ns)

Relativistic Effect due to Elliptical Orbit

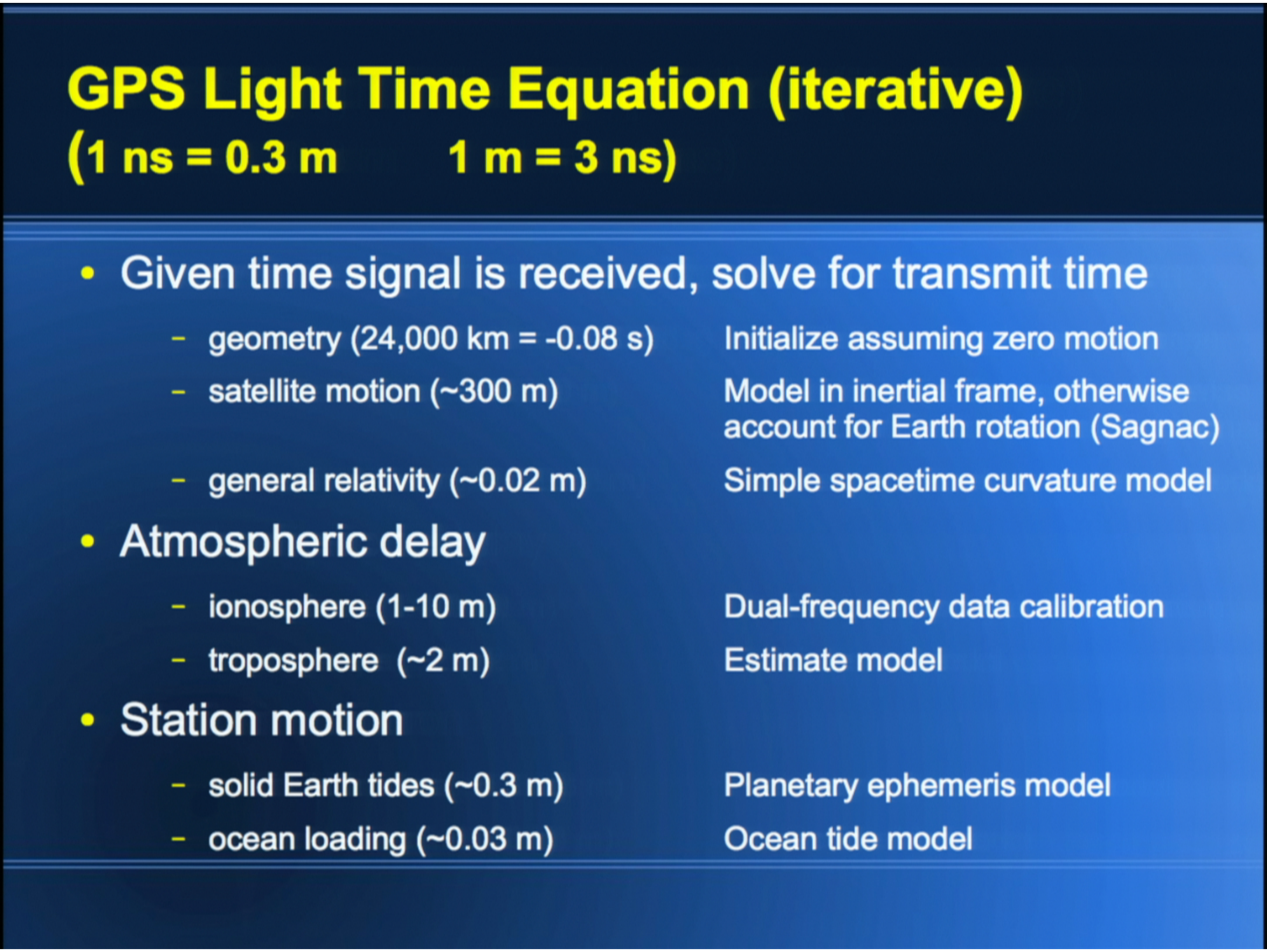


Sinusoidal Effect
~30 ns amplitude
~12 hour period

Conventional model assumes spherical Earth, elliptical orbit

Function of satellite position & velocity:

$$\boxed{-\frac{2}{c^2}\mathbf{X}\cdot\mathbf{V}}$$



GPS Light Time Equation (iterative)

(1 ns = 0.3 m   1 m = 3 ns)

- Given time signal is received, solve for transmit time

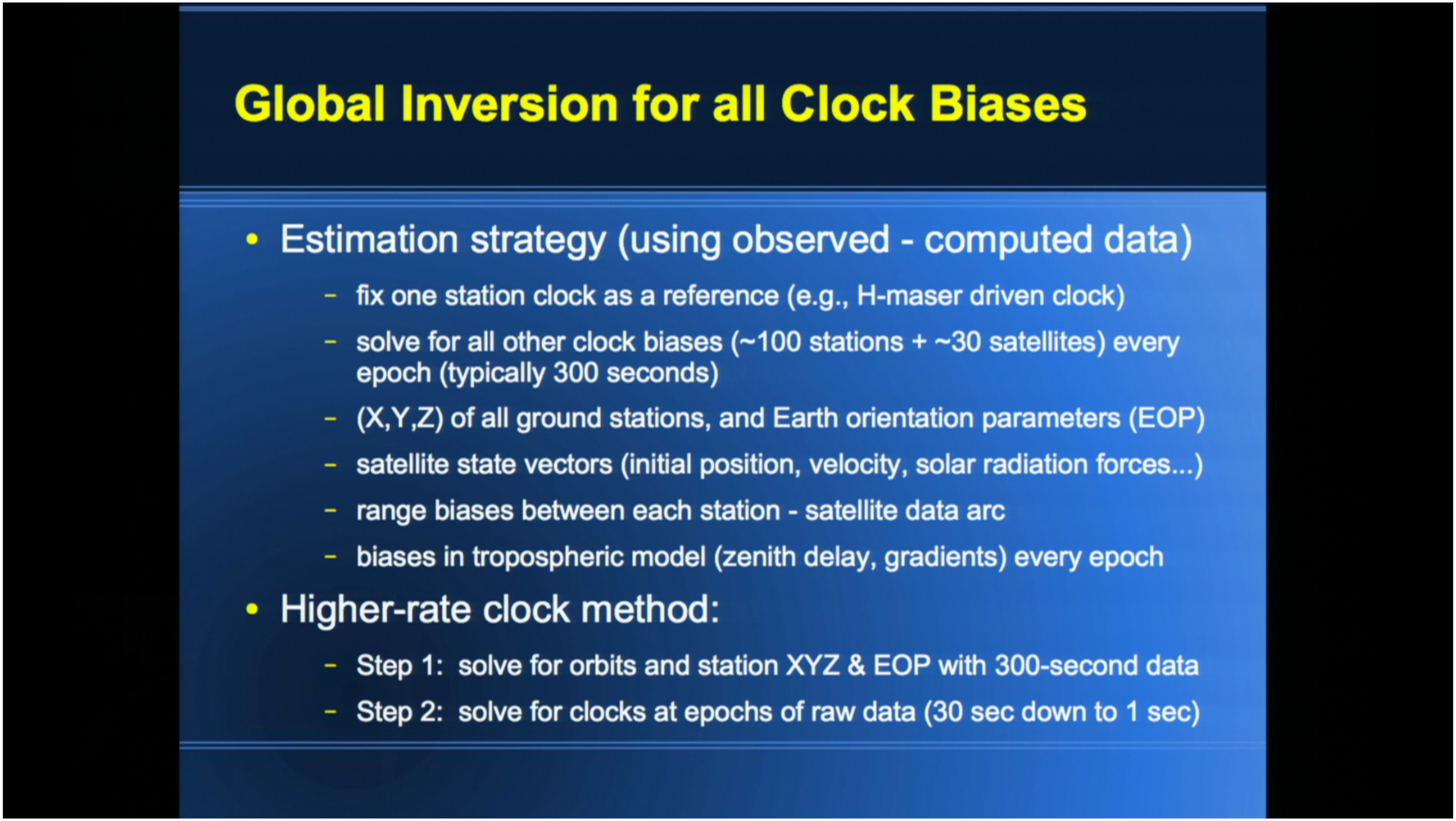
– geometry (24,000 km = -0.08 s)	Initialize assuming zero motion
– satellite motion (~300 m)	Model in inertial frame, otherwise account for Earth rotation (Sagnac)
– general relativity (~0.02 m)	Simple spacetime curvature model

- Atmospheric delay

– ionosphere (1-10 m)	Dual-frequency data calibration
– troposphere (~2 m)	Estimate model

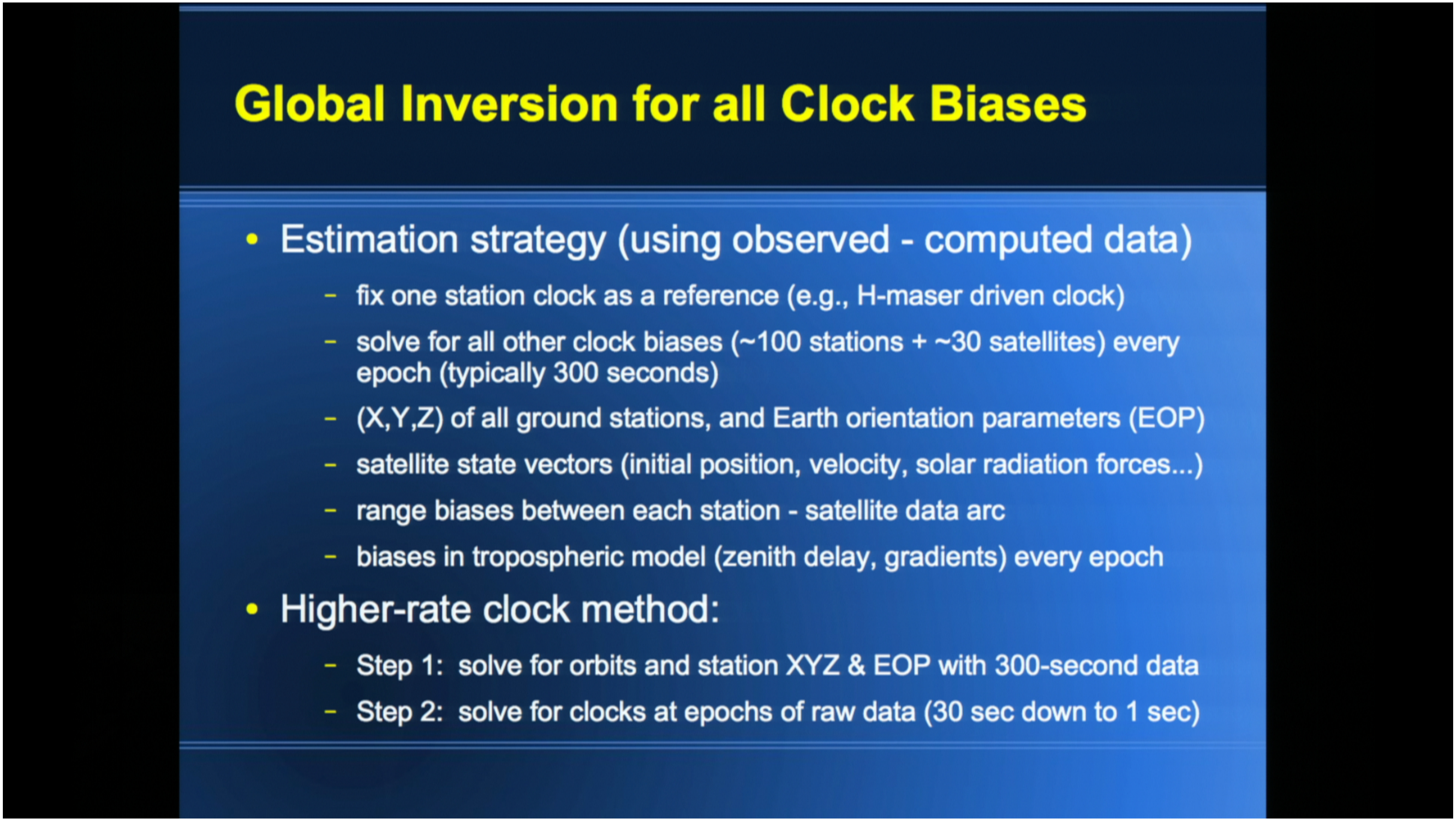
- Station motion

– solid Earth tides (~0.3 m)	Planetary ephemeris model
– ocean loading (~0.03 m)	Ocean tide model



Global Inversion for all Clock Biases

- Estimation strategy (using observed - computed data)
  - fix one station clock as a reference (e.g., H-maser driven clock)
  - solve for all other clock biases (~100 stations + ~30 satellites) every epoch (typically 300 seconds)
  - (X,Y,Z) of all ground stations, and Earth orientation parameters (EOP)
  - satellite state vectors (initial position, velocity, solar radiation forces...)
  - range biases between each station - satellite data arc
  - biases in tropospheric model (zenith delay, gradients) every epoch
- Higher-rate clock method:
  - Step 1: solve for orbits and station XYZ & EOP with 300-second data
  - Step 2: solve for clocks at epochs of raw data (30 sec down to 1 sec)



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Data Analysis Demonstration

Using GIPSY OASIS II software from JPL

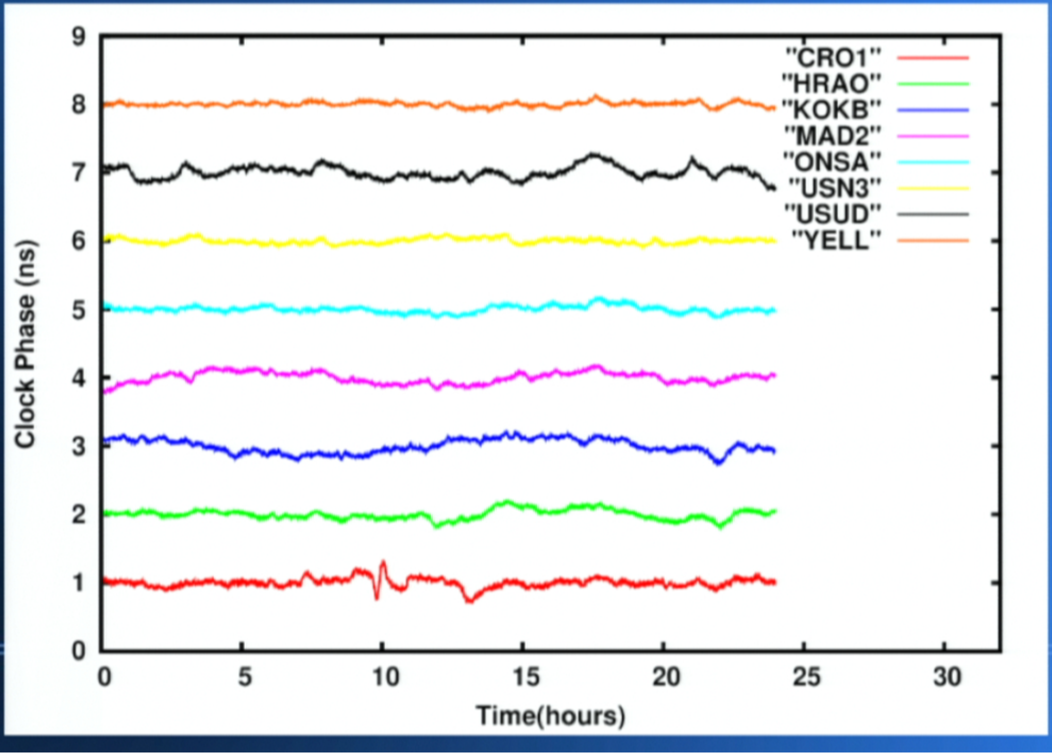
- 40 geodetic GPS stations
  - Fix satellite orbits to those published by JPL
  - Clock bias estimated every 30 sec

Station AMC2
reference clock

blue = H-maser
red = Rb
green = Cs
black = Quartz

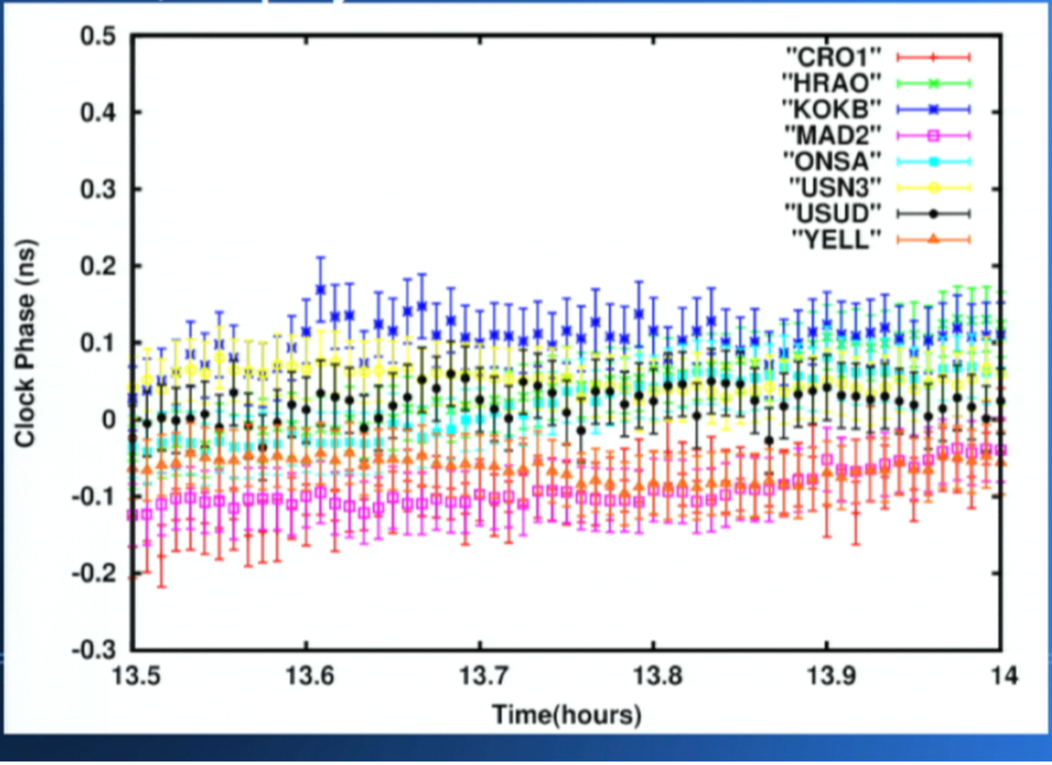
Results:
Top Performing H-Maser Stations

- 24 hours, 2nd order polynomial removed



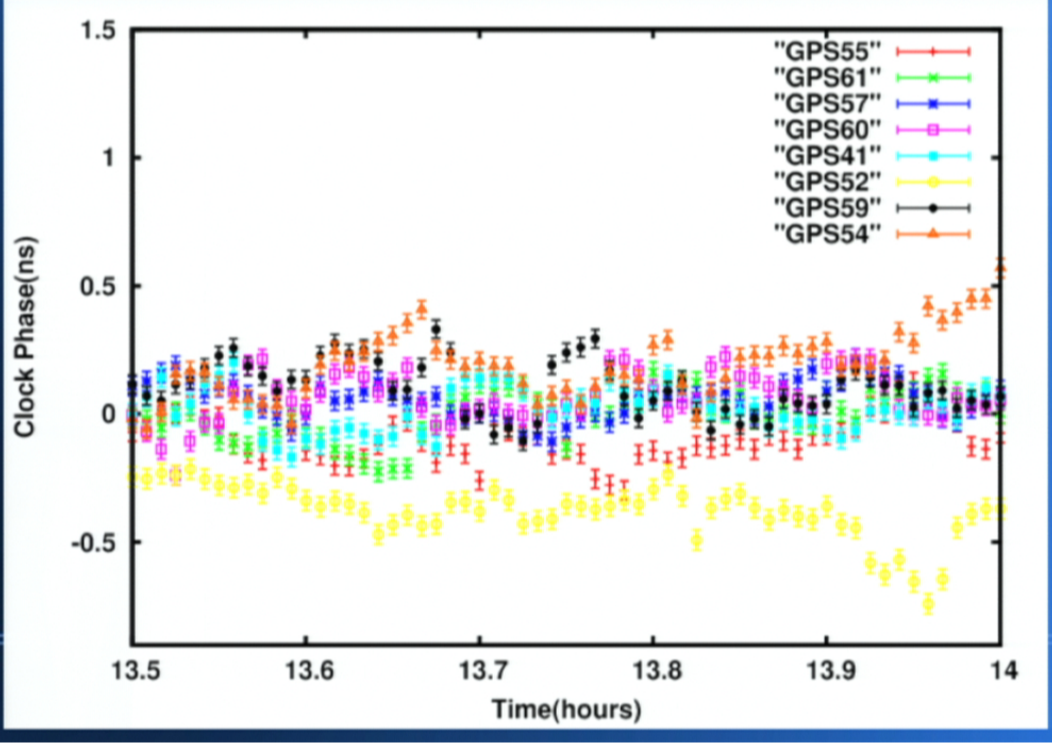
Results:
Top Performing H-Maser Stations

- 30 minutes, no polynomial removed

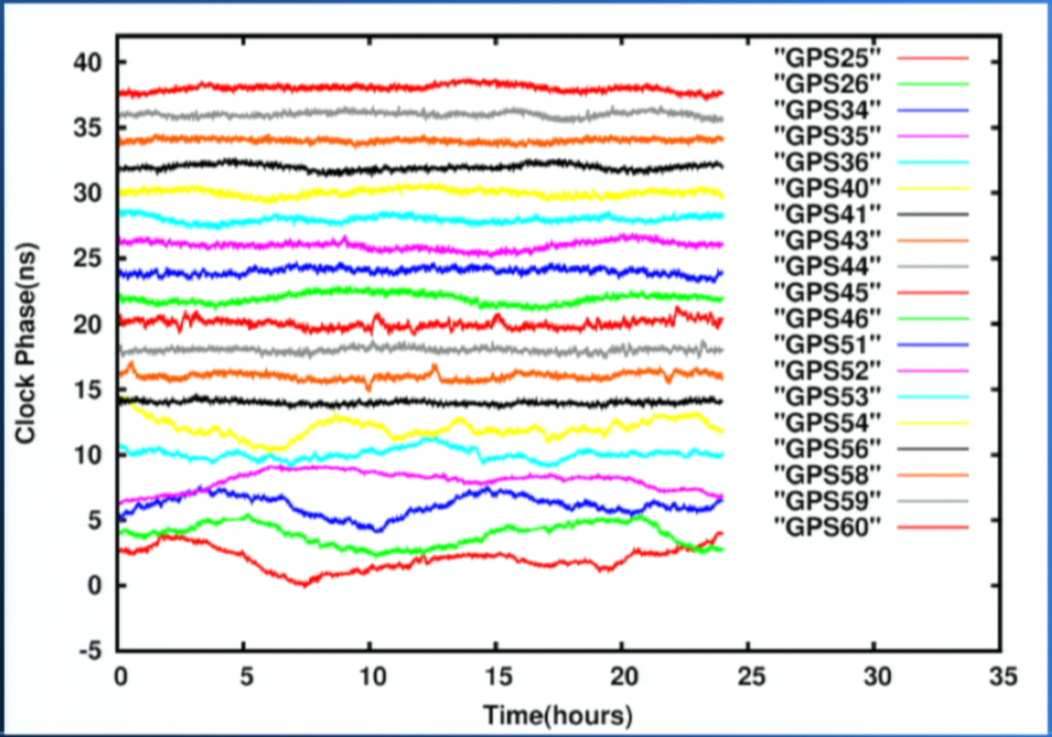


Results:
Top-Performing Satellite Rb Clocks

- 30 minutes, no polynomial removed

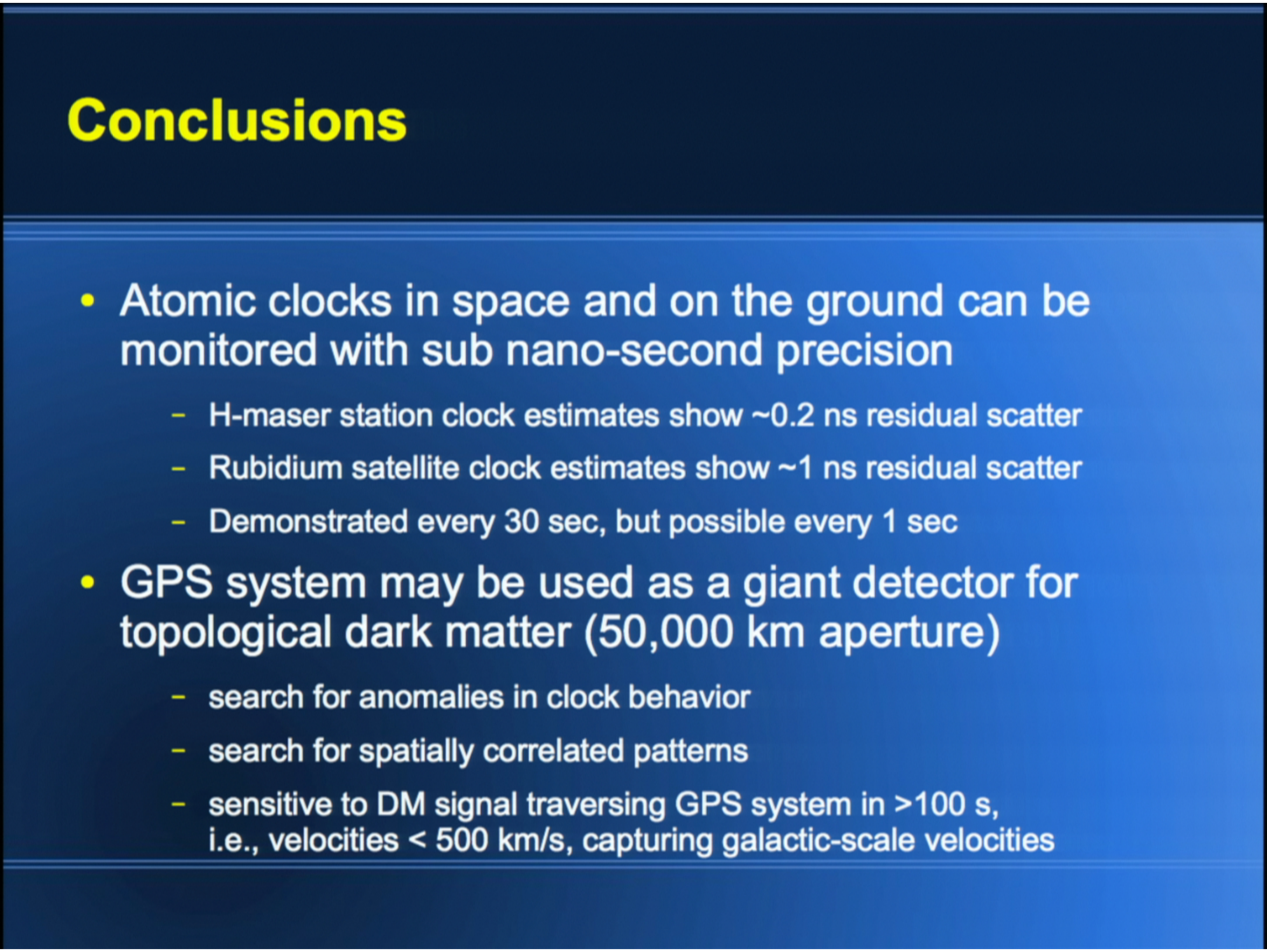


Results:
Satellite Rb clocks “good” & “poor”



Conclusions

- Atomic clocks in space and on the ground can be monitored with sub nano-second precision
    - H-maser station clock estimates show ~0.2 ns residual scatter
    - Rubidium satellite clock estimates show ~1 ns residual scatter
    - Demonstrated every 30 sec, but possible every 1 sec
- GPS system may be used as a giant detector for topological dark matter (50,000 km aperture)
    - search for anomalies in clock behavior
    - search for spatially correlated patterns
    - sensitive to DM signal traversing GPS system in >100 s, i.e., velocities < 500 km/s, capturing galactic-scale velocities



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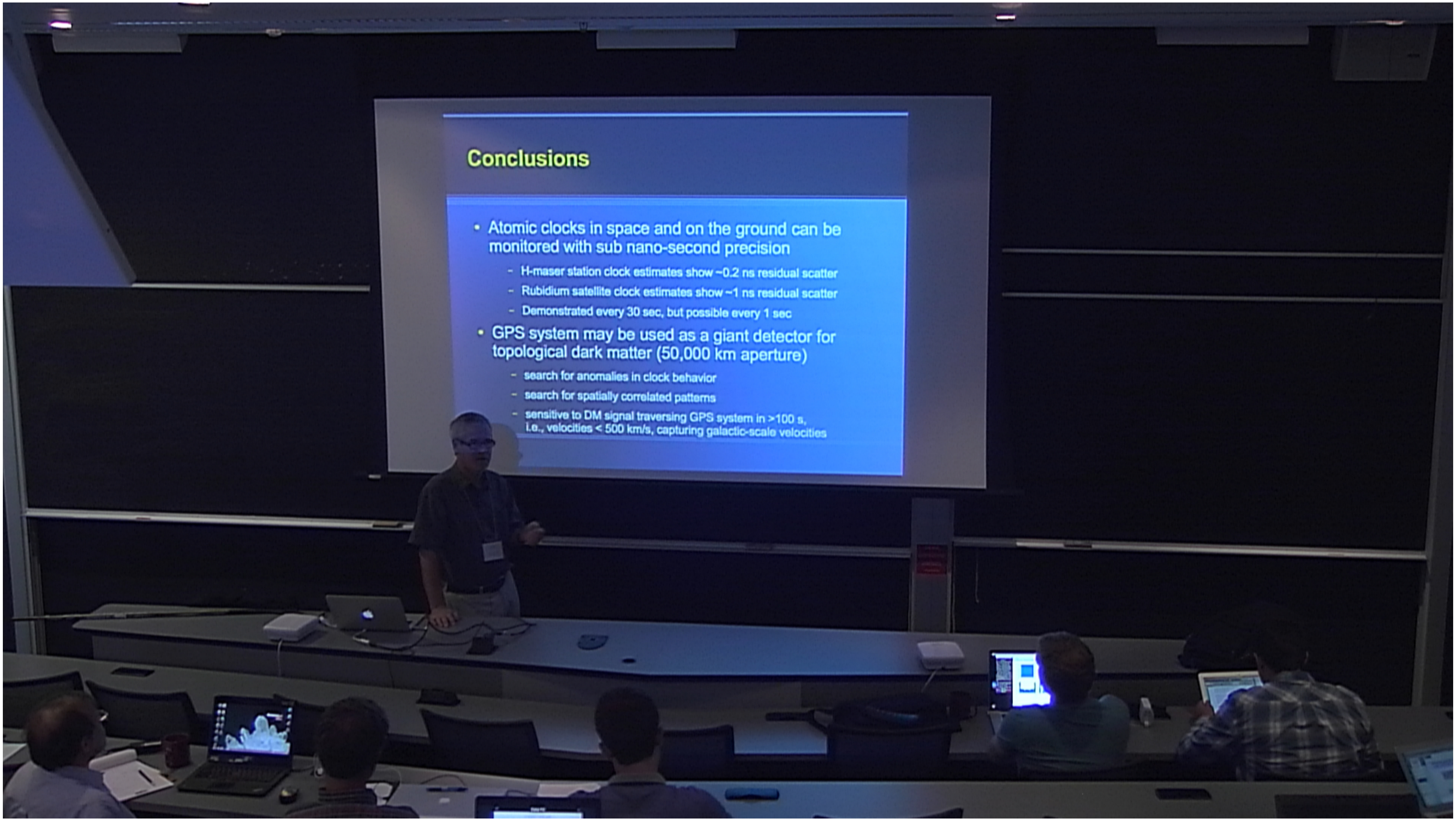

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