Title: Zen and the art of atomic time-keeping (Tutorial on atomic clocks for particle theorists)

Date: Jun 16, 2014 02:30 PM

URL: http://pirsa.org/14060009

Abstract:

Zen and the art of atomic time-keeping

Tutorial on atomic clocks for particle theorists

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University of Nevada, Reno, USA

What will I cover?

- Basic atomic clock concepts
- Application of these concepts to a dark matter search

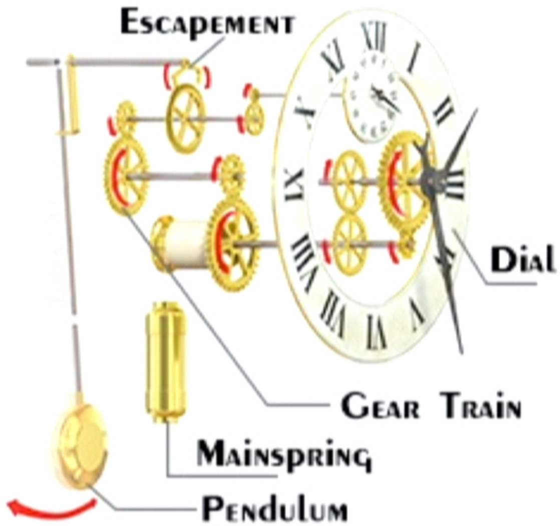
What is time?

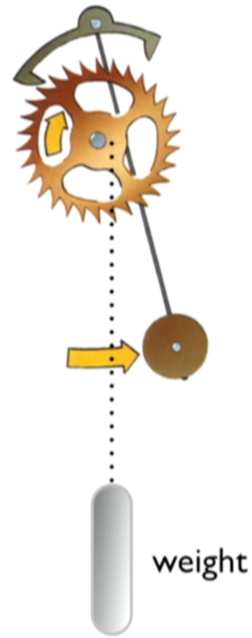
TIME (according to Merriam-Webster)

a : the measured or measurable period during which an action, process, or condition exists or continues

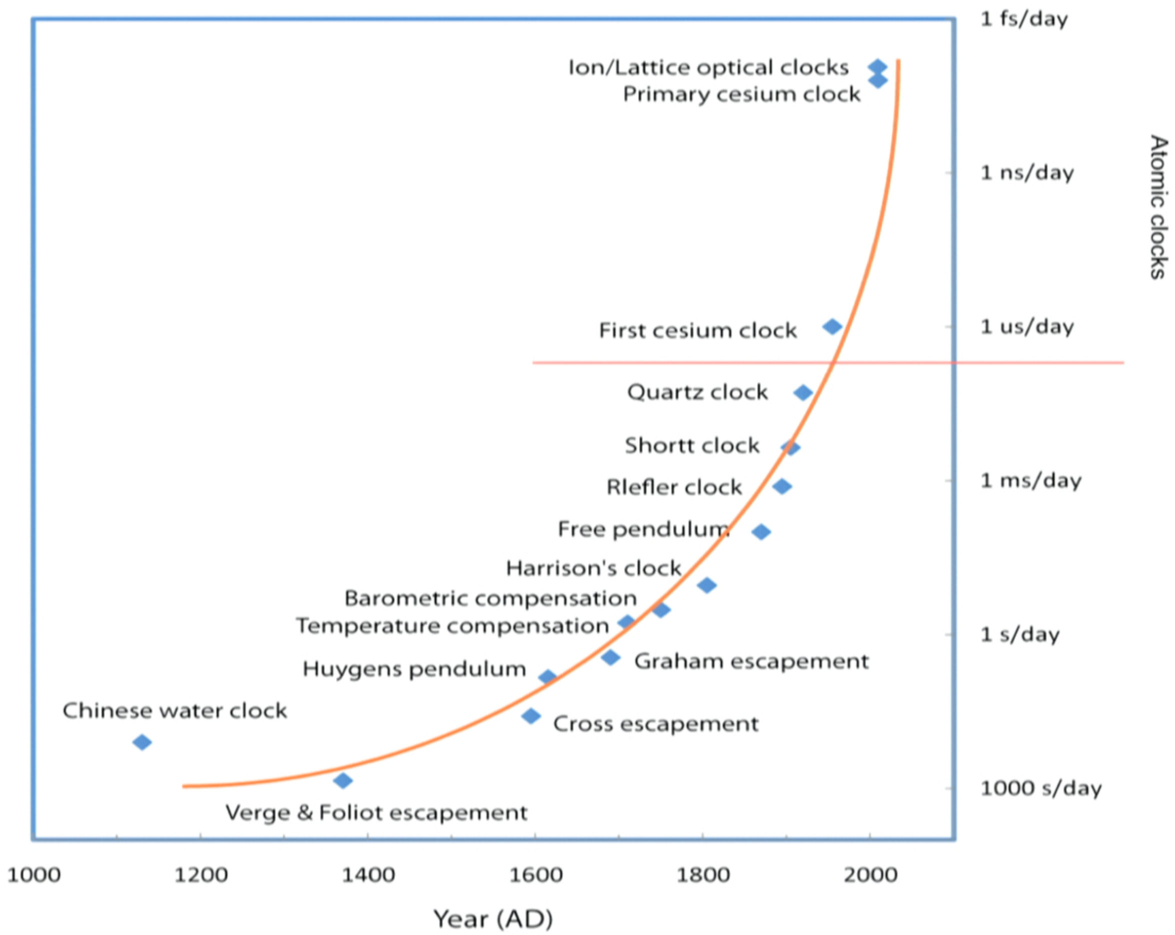
b : a nonspatial continuum that is measured in terms of events which succeed one another from past through present to future

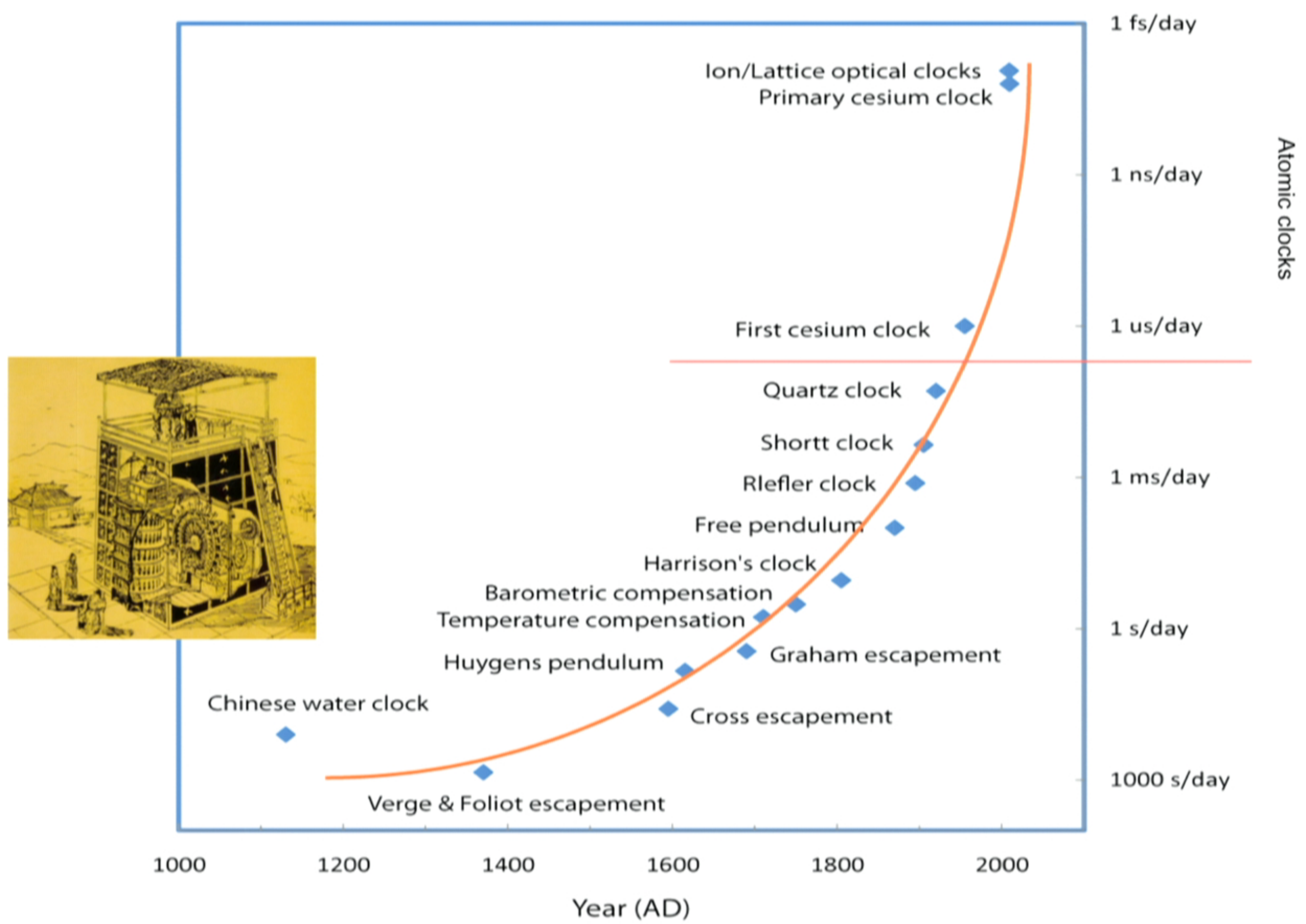
How do we tell time?





Time = (number of oscillations) x (known period)


1 fs/day
Ion/Lattice optical clocks
Primary cesium clock
1 ns/day
Atomic clocks
First cesium clock
1 us/day
Quartz clock
Shortt clock
Rlefler clock
1 ms/day
Free pendulum
Harrison's clock
Barometric compensation
Temperature compensation
1 s/day
Huygens pendulum
Graham escapement
Chinese water clock
Cross escapement
1000 s/day
Verge & Foliot escapement
1000
1200
1400
1600
1800
2000
Year (AD)



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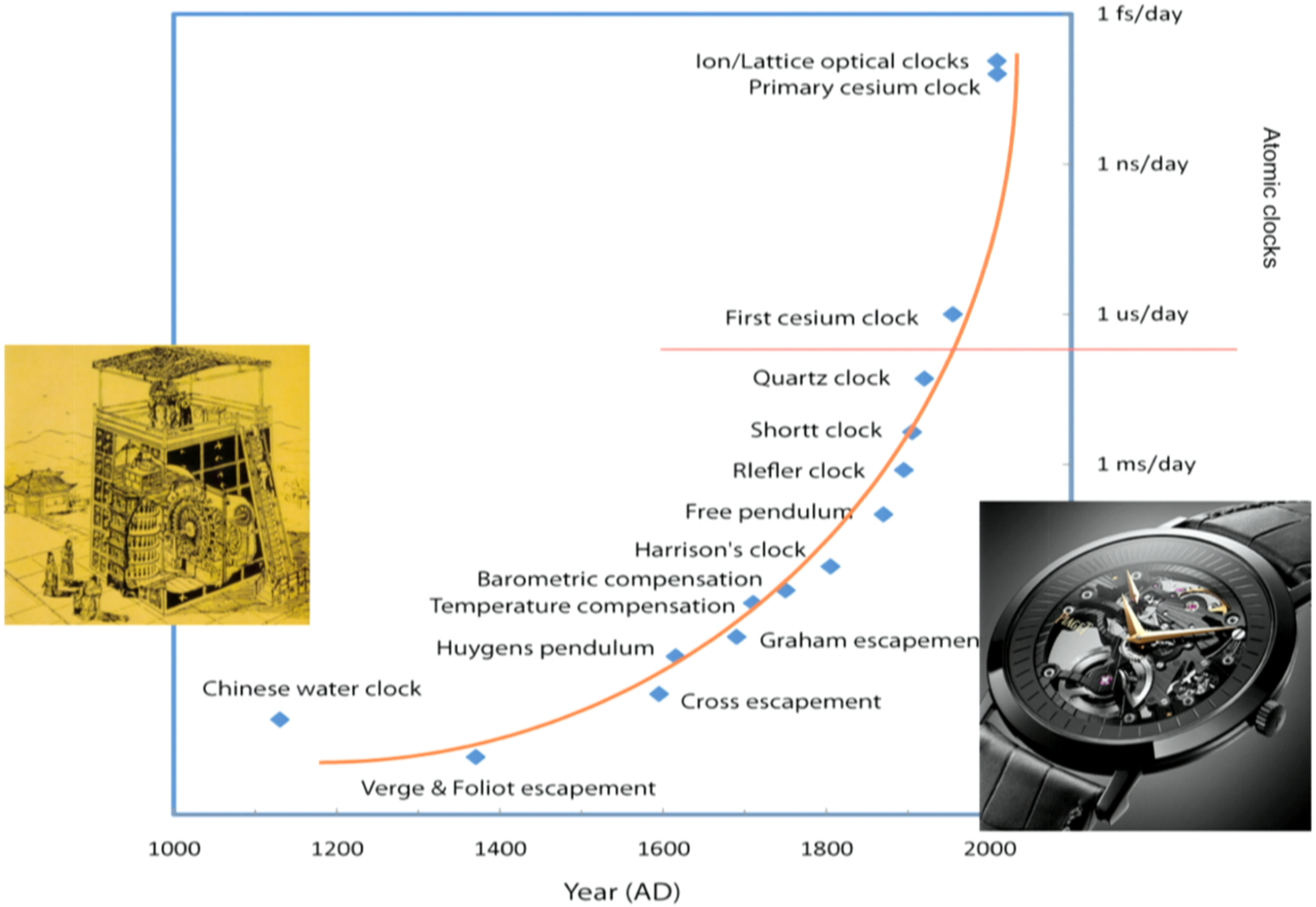
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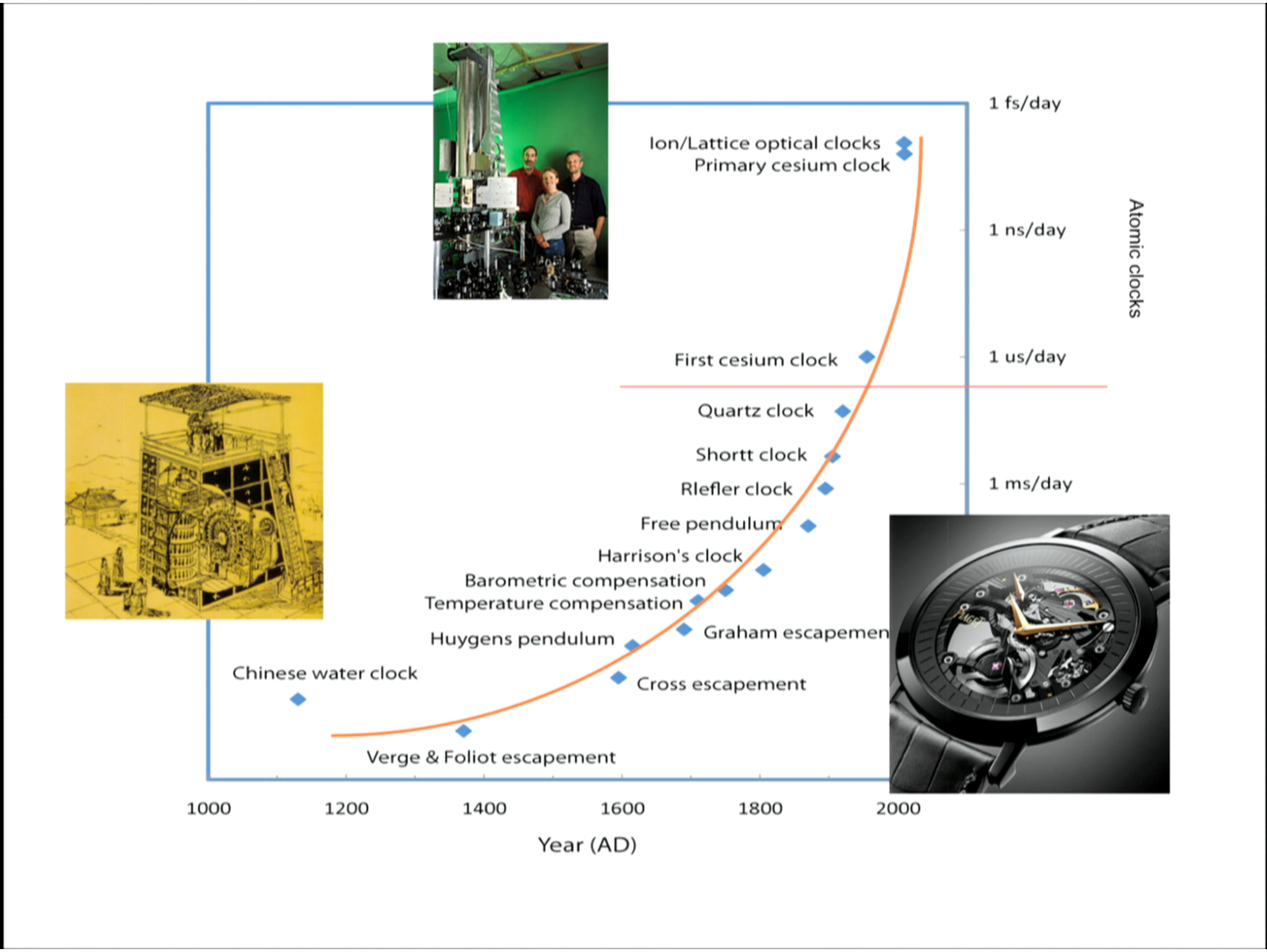

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Atomic clocks

- Most precise instruments ever built
- Modern nuclear/atomic clocks aim at 19 significant figures of precision
- Best limits on modern-epoch drift of fundamental constants

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$$\Delta t = \left(\frac{\Delta \omega}{\omega_{\text{clock}}} \right) \times t = 1\,\text{s}$$

fractional inaccuracy 10^{-18}

Age of the universe 10^{18} s

Atomic clocks

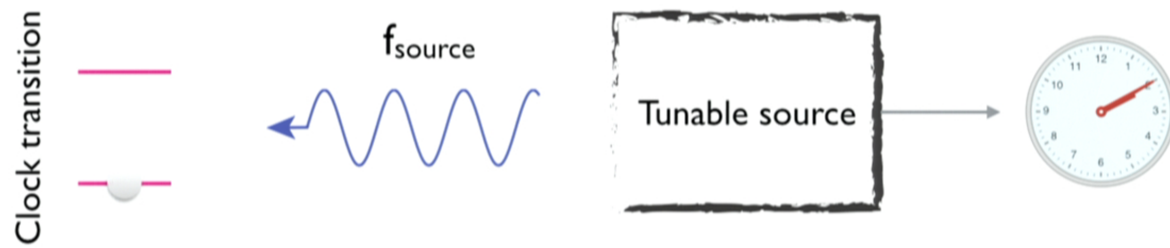
$\Rightarrow$

$$|e\rangle$$
$$|g\rangle$$

$$\nu_{\text{clock}} = \frac{E_e - E_g}{h}$$

How does it work?

Time = (number of oscillations) x ($1/f_{clock}$)



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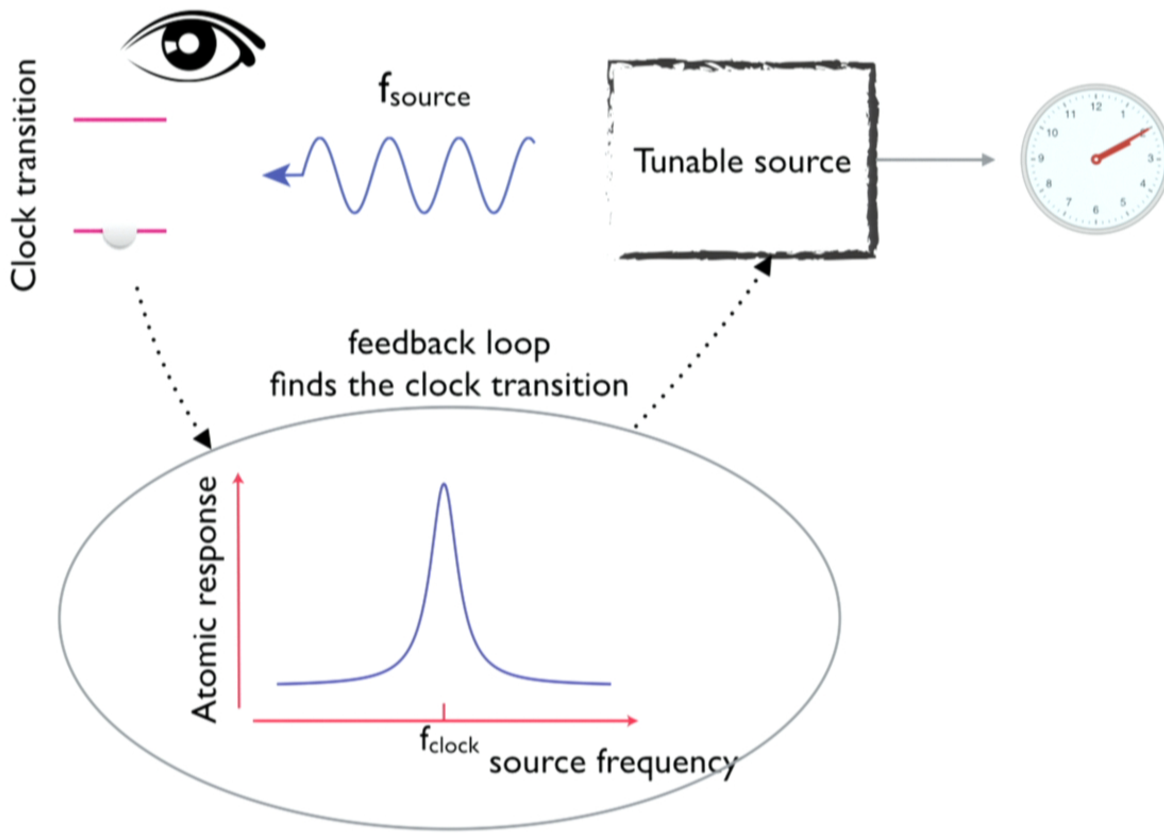
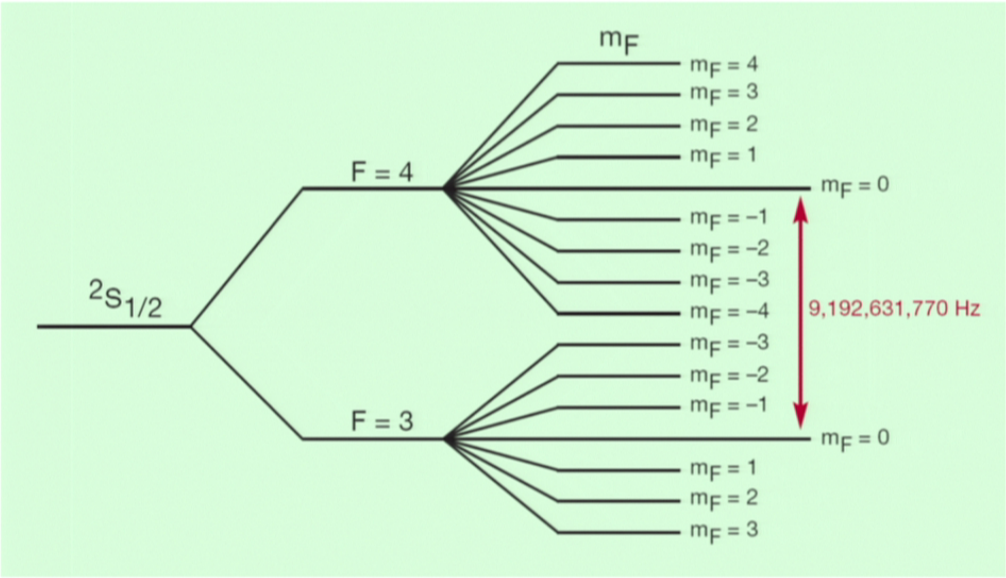
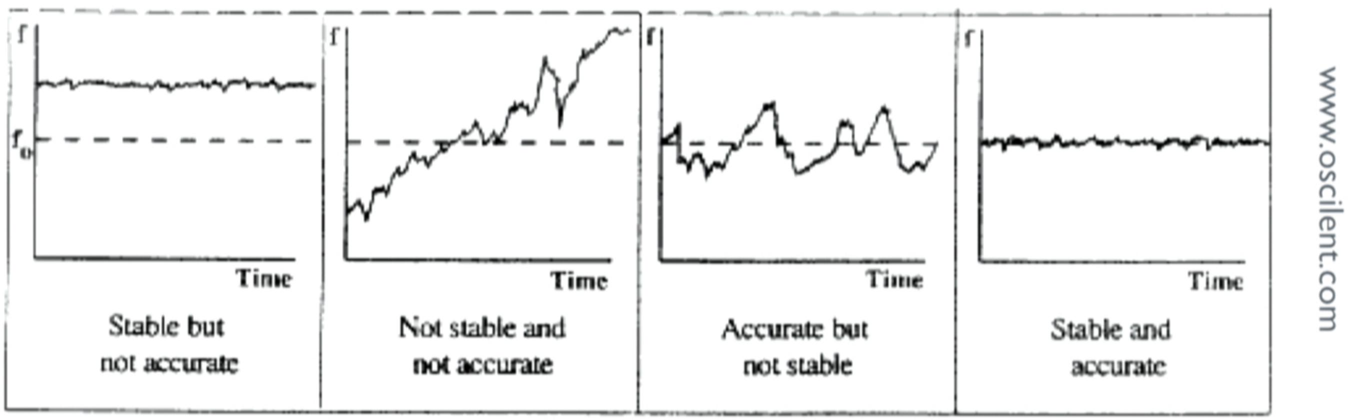


SI definition of the second

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom. This definition refers to a cesium atom at rest at a temperature of 0 K.



Accuracy and stability



Systematics and statistics

Accuracy

Quantum oscillator must be well protected from
the environmental perturbations (no systematic shifts)

Atomic clock taxonomy

Clock frequency

Microwave

Hyperfine transitions

- Cs fountains [10^{-16}]
- Rb beam/cell
- Hg+ ion clock
- Hydrogen maser
- ....

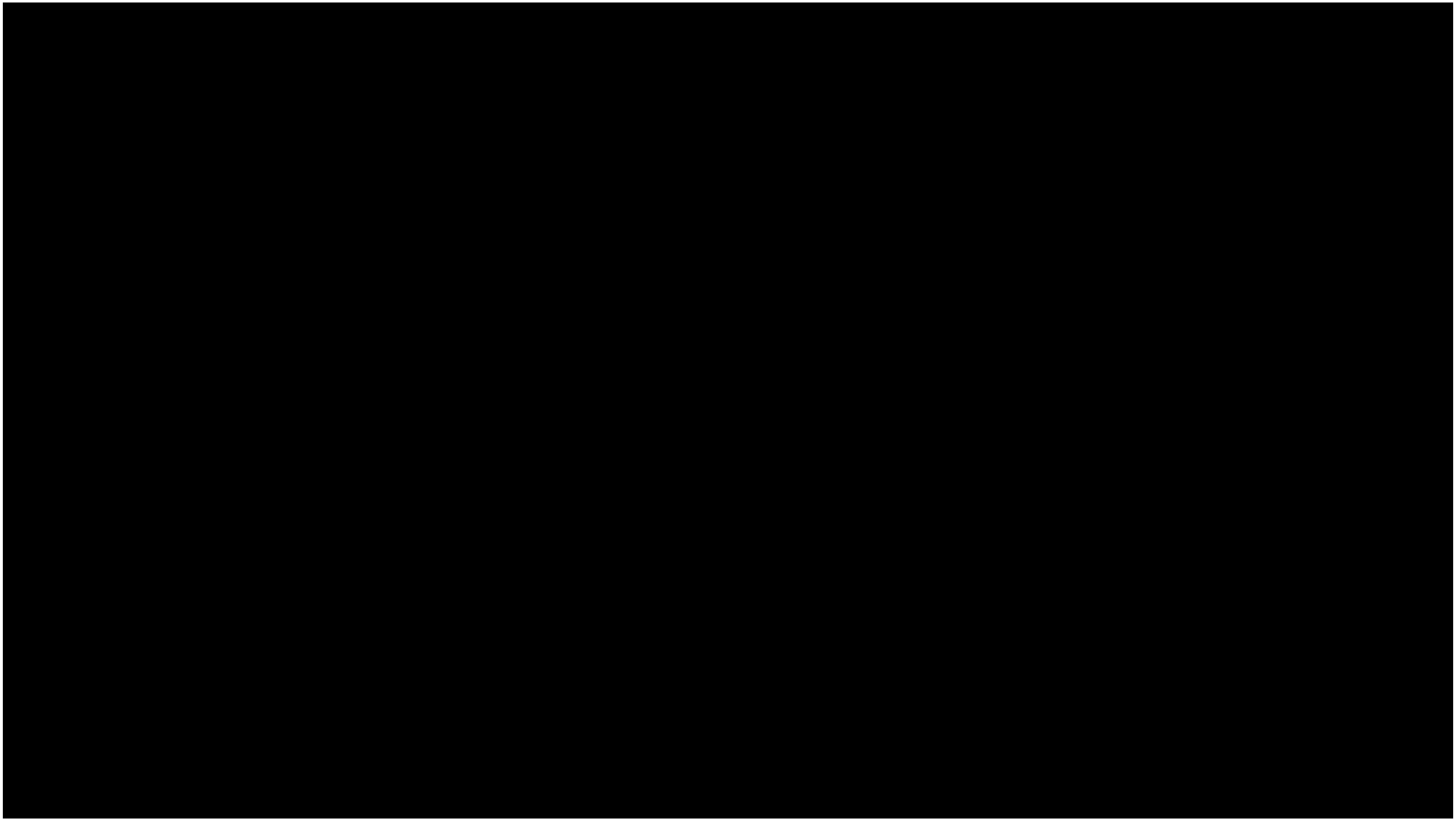
Optical

- Al+,Yb+,... ion [10^{-18}]
- Sr,Yb,.. optical lattice [10^{-18}]
- Th nuclear [10^{-19}]
- Highly-charged ions [10^{-19}]
- ....

XUV

- U nuclear
- Highly-charged ions

[projected fractional accuracy]



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[projected fractional accuracy]

Why higher clock frequency is better?

$|e\rangle$

$|g\rangle$

shifts remain approximately the same

$$\Delta t = \left(\frac{\Delta\omega}{\omega_{\text{clock}}} \right) \times t$$

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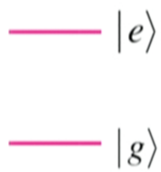

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Why nuclear/HCI clocks would have a better accuracy?

Couplings to external field ~ size of the quantum oscillator

Example of evaluating accuracy

Single-Ion Nuclear Clock for Metrology at the 19th Decimal Place

C. J. Campbell, A. G. Radnaev, A. Kuzmich, V. A. Dzuba, V. V. Flambaum, and A. Derevianko

Phys. Rev. Lett. 108, 120802 (2012)

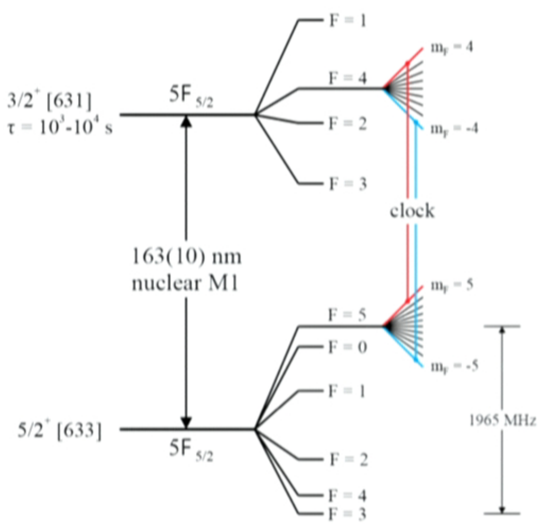
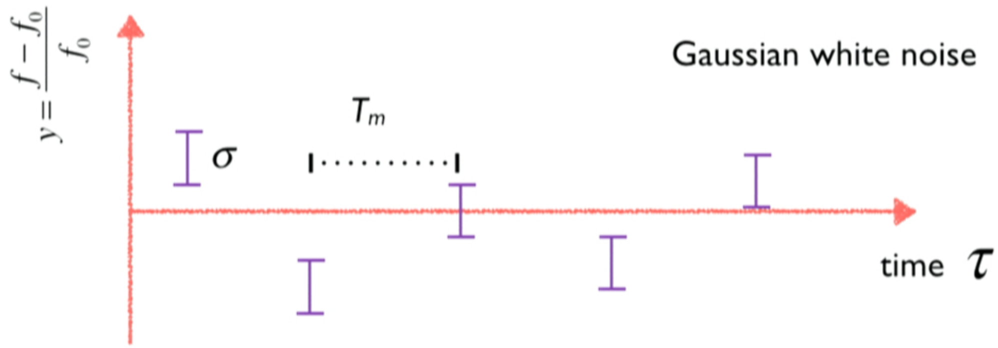


TABLE I. Estimated systematic error budget for a $^{229}Th^{3+}$ clock using realized single-ion clock technologies. Shifts and uncertainties are in fractional frequency units ($\Delta\nu/\nu_{clk}$) where $\nu_{clk} = 1.8$ PHz. See text for discussion.

Effect	$\lvert$Shift$\rvert$ (10^{-20})	Uncertainty (10^{-20})
Excess micromotion	10	10
Gravitational	0	10
Cooling laser Stark	0	5
Electric quadrupole	3	3
Secular motion	5	1
Linear Doppler	0	1
Linear Zeeman	0	1
Background collisions	0	1
Blackbody radiation	0.013	0.013
Clock laser Stark	0	$\ll 0.01$
Trapping field Stark	0	$\ll 0.01$
Quadratic Zeeman	0	0
Total	18	15

Statistical uncertainties depend on how long you can measure



$$\sigma(\tau) = \frac{\sigma}{\sqrt{N_m}} = \frac{\sigma}{\sqrt{\tau / T_m}} \quad \Longrightarrow \quad \sigma(\tau) \propto \frac{1}{\sqrt{\tau}}$$

"Integrating out white noise"

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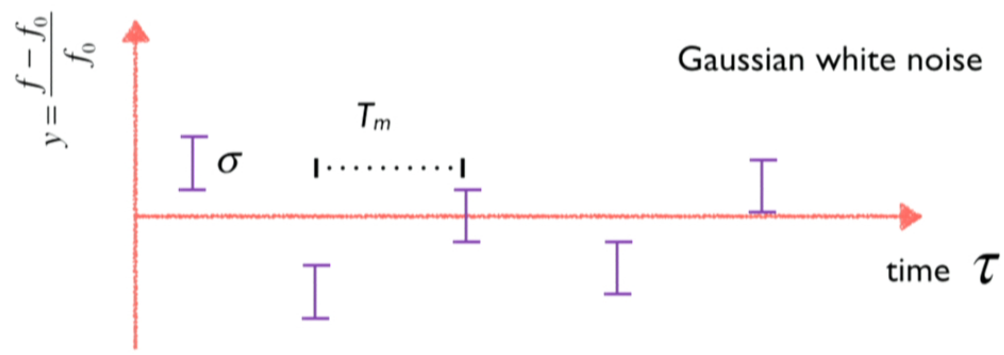
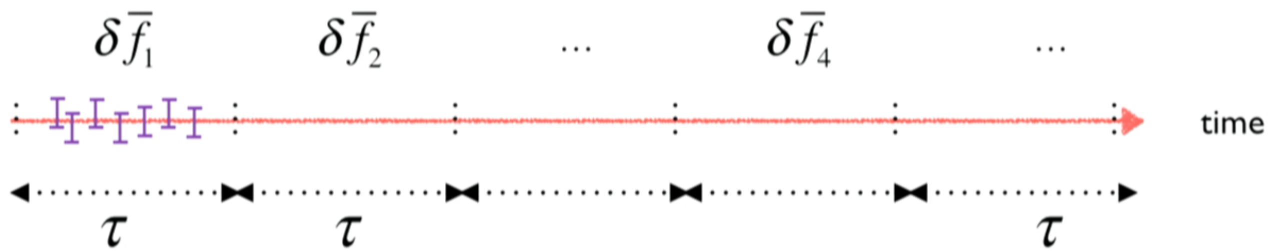


$$\sigma(\tau) = \frac{\sigma}{\sqrt{N_m}} = \frac{\sigma}{\sqrt{\tau / T_m}} \implies \sigma(\tau) \propto \frac{1}{\sqrt{\tau}}$$

"Integrating out white noise"

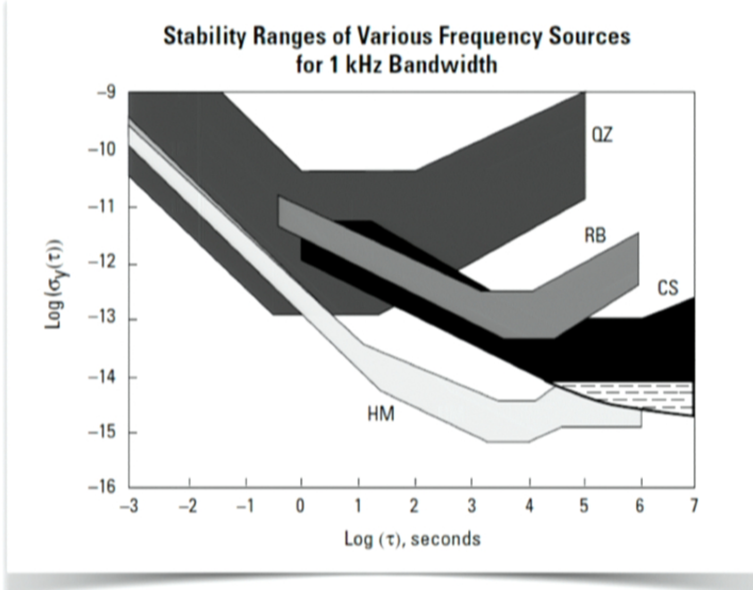
Allan variance as a characteristic of stability

Dealing with random walks and drifts



$$\sigma_y^2(\tau) \equiv \frac{1}{2}\left\langle \left(\frac{\delta\bar{f}_{n+1} - \delta\bar{f}_n}{f_0} \right)^2 \right\rangle_{\text{average over } n}$$

For the white noise still $\sigma_y(\tau) \propto \frac{1}{\sqrt{\tau}}$ flicker noise $\sigma_y(\tau) \propto \text{const}$

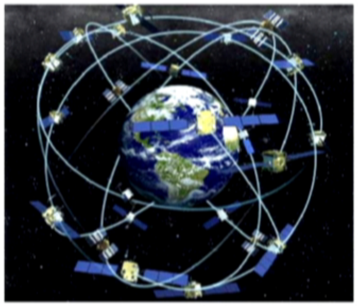


Usual scaling of long-term instability $\sigma_y(T) \propto 1/\sqrt{T}$

Typical values for microwave clocks $\sigma_y(1\ \mathrm{s}) > 10^{-13}$

Projected stability (optical lattice clocks) $\sigma_y(1\ \mathrm{s}) \sim 10^{-18}$

Networks of clocks



Global Positioning System

- Each GPS satellite has four clocks (32 satellites)
- Data are sampled every second
- Vast terrestrial network of monitoring stations (H masers)

Trans-european clock network

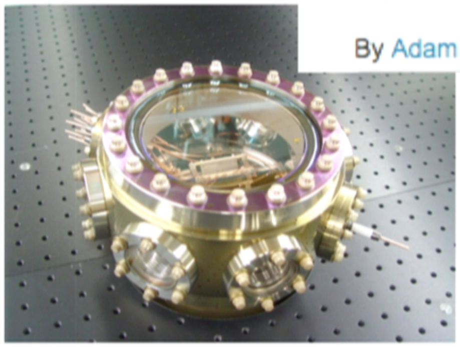
- Optical fiber connects state-of-the art clocks
- Elements were demonstrated
(PTB-MPI Munich 920 km link) *(Predehl et al., Science (2012))*

TAI dissemination network between national labs

WIRED

Laser-Tuned Nuclear Clock Would Be Accurate for Billions of Years

By Adam Mann | March 20, 2012 | 5:28 pm | Categories: Physics



questcequilmanque

You've managed to find the single most depressing scientific endeavor of all time: Spend years of research trying to make an ultra-precise clock more precise. If they succeed, only electrons will notice. What's the suicide rate among these people?

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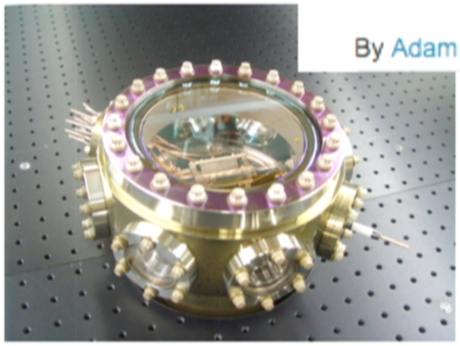

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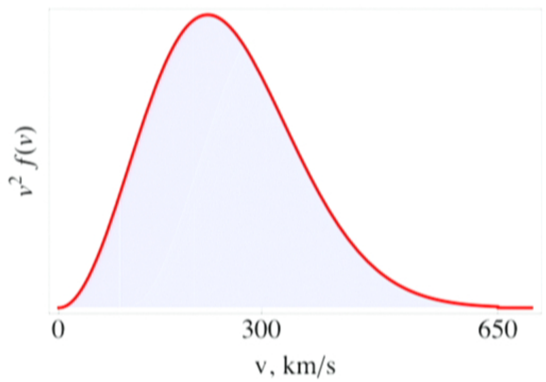
Putting it all together: Evaluating clock sensitivity to transient variations of fundamental constants

Derevianko and Pospelov, arXiv:1311.1244

What do we know about DM?

Dark Matter halo

Velocity distribution

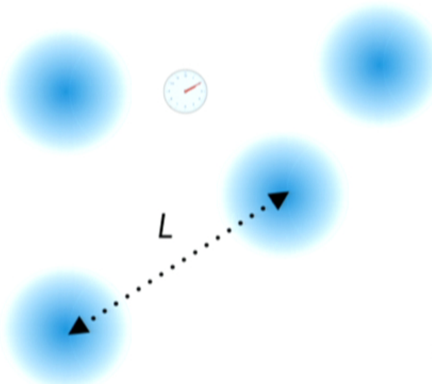


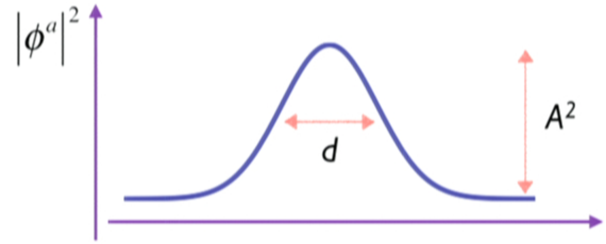
$$v_g \sim 300\,\mathrm{km/s}$$

Energy density

$$\rho_{DM} \sim 0.3\,\mathrm{GeV/cm^3}$$

“Gas of topological defects” DM model





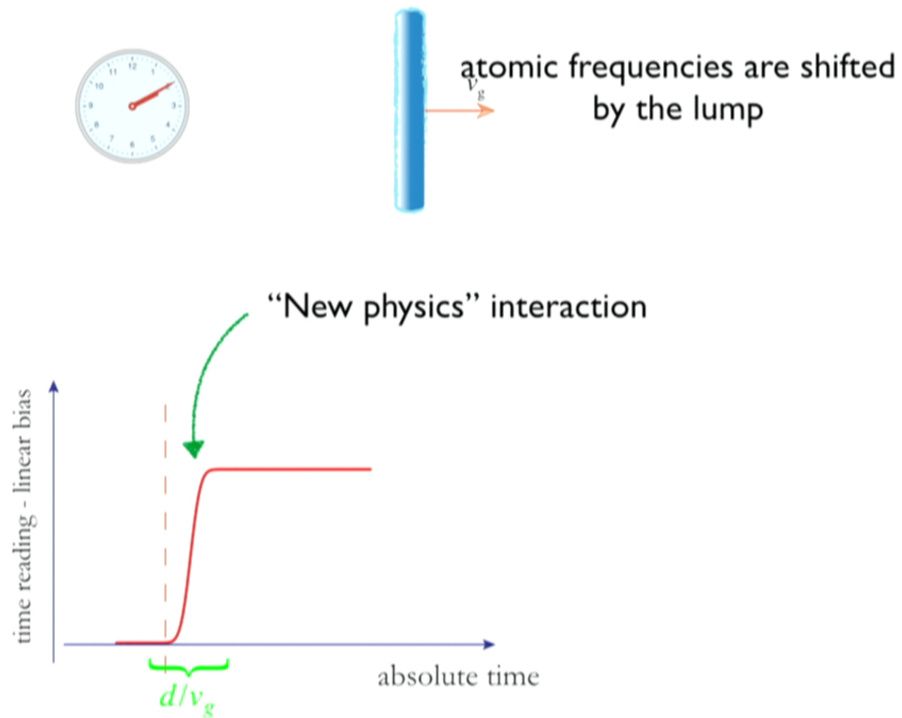
$$\rho_{TDM} \sim \frac{1}{L^3} \times \left(\frac{1}{\hbar c} \frac{A^2}{d^2} d^3 \right)$$ Energy density

$$T_{coll} \sim \frac{1}{n\sigma v} \sim \frac{1}{1/L^3 \times d^2 \times v_g}$$ Time b/w “collisions”

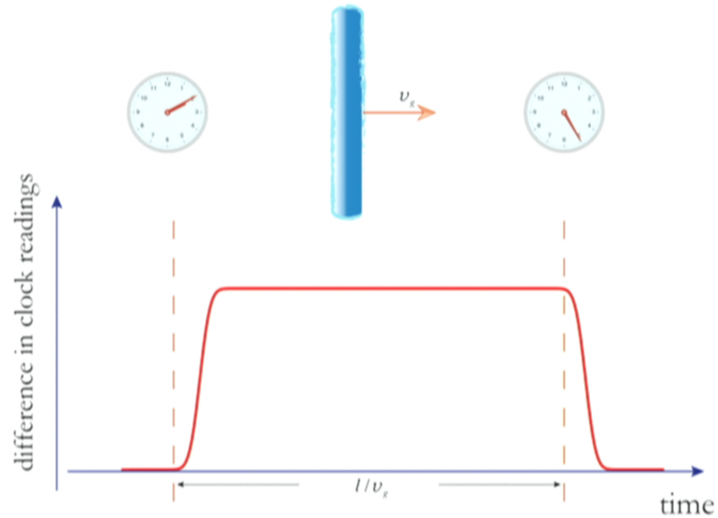
$$\tau \sim \frac{d}{v_g}$$ Interaction time

Setup

Lump of dark matter moving at galactic speed of ~300 km/s



Signature



Monitor time difference b/w two spatially-separated clocks
⇒ persistent clock discrepancy for over time l/v_g

GPS aperture =50,000 km => l/v_g~ 150 sec

Dark-matter portal

$$-L_{\text{int}} = \phi^2(\mathbf{r},t)\left(\frac{m_e \bar{e}e}{\Lambda_e^2} + \frac{m_p \bar{p}p}{\Lambda_p^2} + \frac{1}{4\Lambda_\gamma^2}F_{\mu\nu}^2 + \ldots\right)$$

DM field, electrons, protons, EM field

Compare to the QED Lagrangian

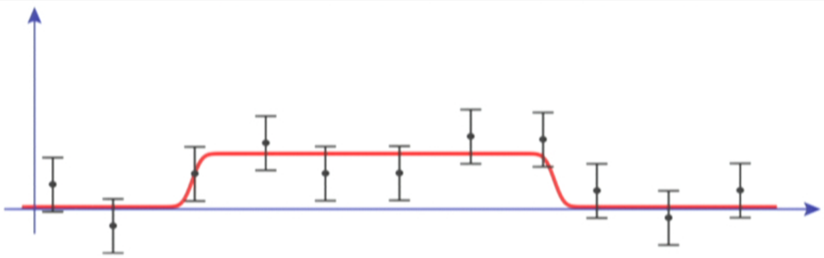
$$L_{\text{QED}} = i\hbar c\bar{e}De - m_e c^2 \bar{e}e - \frac{1}{4\mu_0}F_{\mu\nu}^2$$

$$m_e c^2 \rightarrow m_e c^2\left(1 + \frac{\phi^2(\mathbf{r},t)}{\Lambda_e^2}\right)$$

DM "lump" pulls on the rest masses of electrons, quarks and EM coupling

Energies and frequencies are modulated as the "lump" sweeps through

http://www.dereviankogroup.com/tutorial-on-translating-particle-physics-effective-lagrangians-to-conventional-atomic-physics-and-quantum-chemistry-operators/



Noise

Variance $\langle \Delta\varphi(t)^2 \rangle - \langle \Delta\varphi(t) \rangle^2 = 2R_\varphi(T)$ two independent clocks

phase auto-covariance function and Allan variance

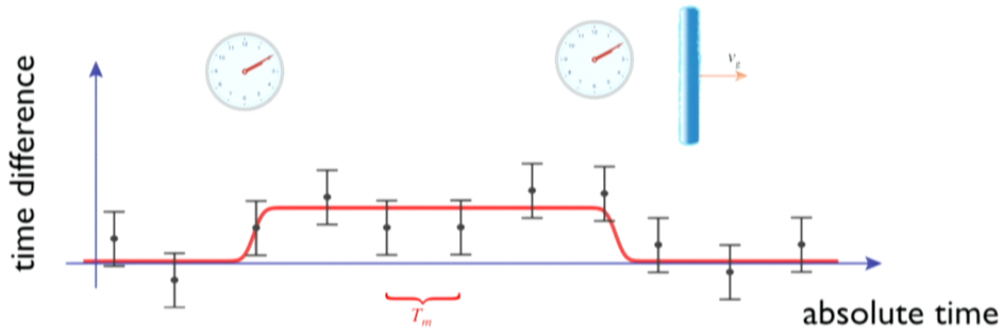
$$R_\varphi(T) \approx (\omega_0 T)^2 \sigma_y^2(T)$$

Noise for a single measurement $\sqrt{2}\omega_0 T \sigma_y(T)$

Noise for N_m measurement

$$\text{Noise} = \frac{\sqrt{2}\omega_0 T \sigma_y(T)}{\sqrt{N_m}} = \frac{\sqrt{2}\omega_0 T \sigma_y(T)}{\sqrt{l/\left(v_g T\right)}}$$

Signal-to-noise ratio



Time b/w events

$$S / N = \frac{c\hbar}{T_m \sigma_y(T_m)\sqrt{2T_m v_g / l}}\, \rho_{\mathrm{DM}}\, d^2\, \mathcal{T}_{\mathrm{coll}} \sum_{\substack{\text{fundametal} \\ \text{constants X}}} \frac{K_X}{\Lambda_X^2}$$

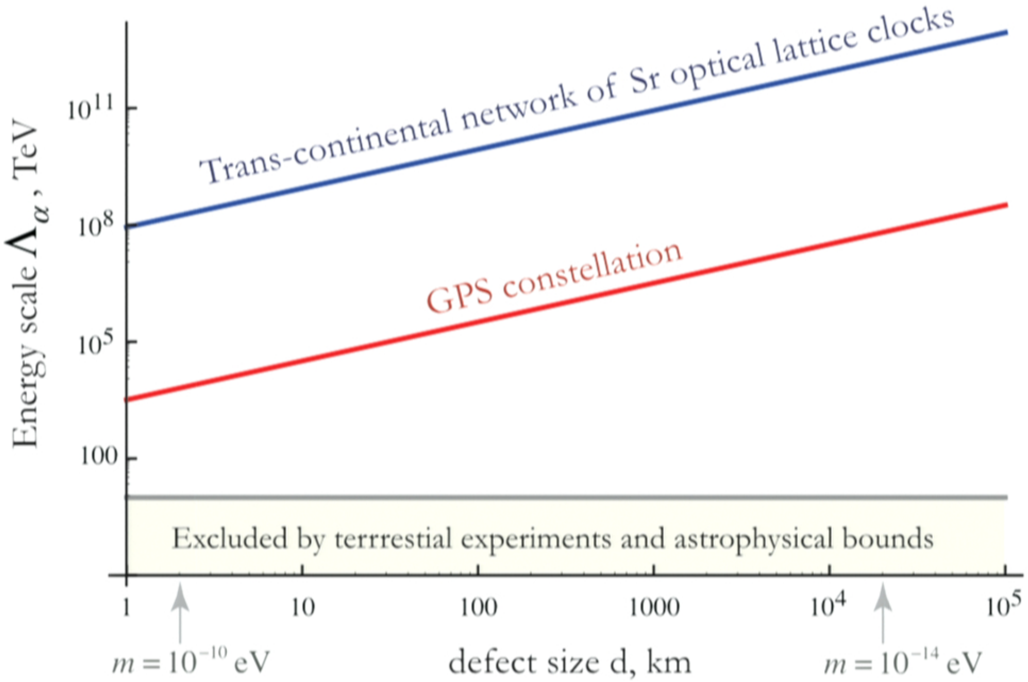
defect size

Allan variance

Dark matter energy density

Projected limits

(if the TDM signature is not observed)



Total monitoring time = 1 year