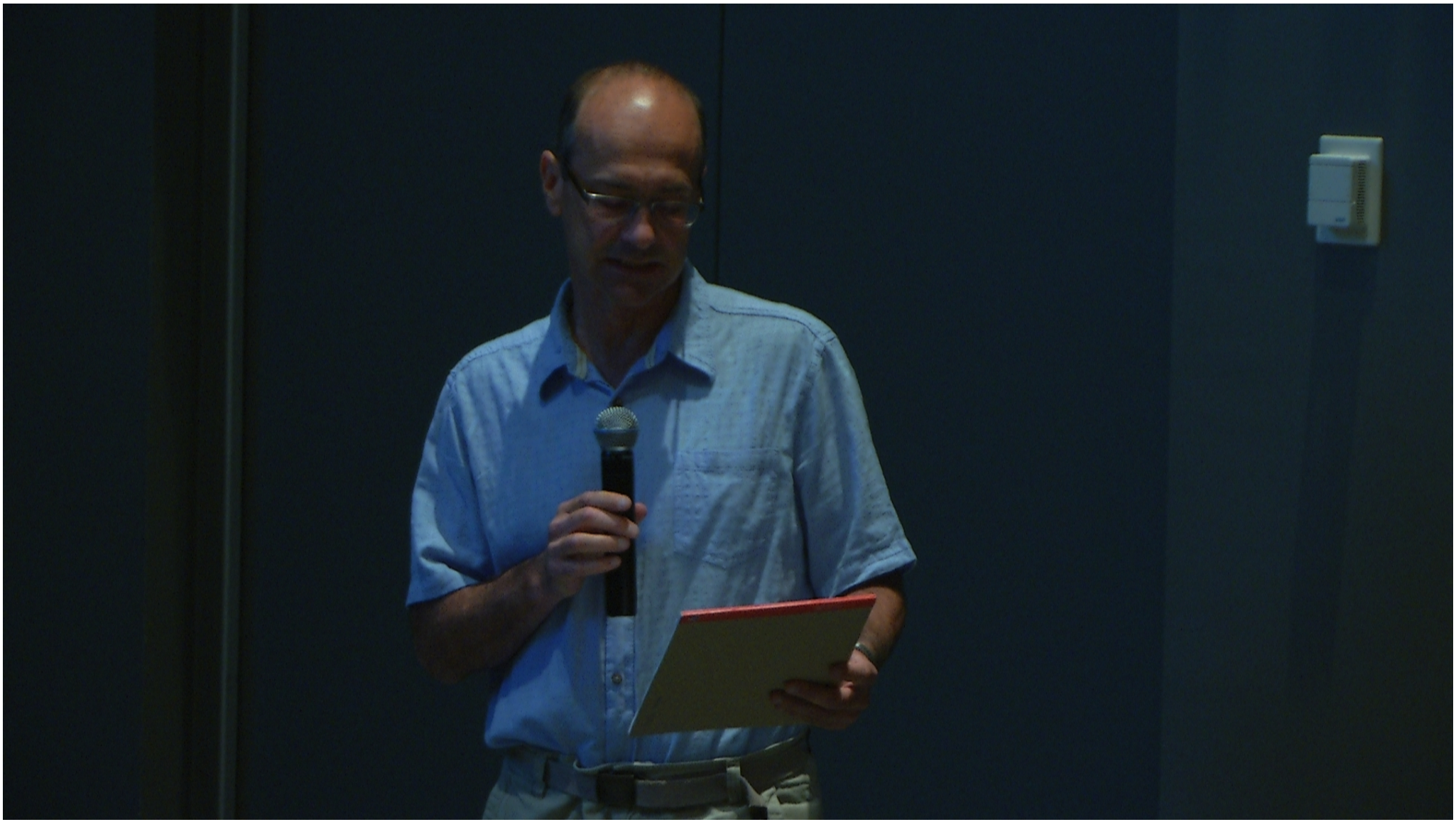
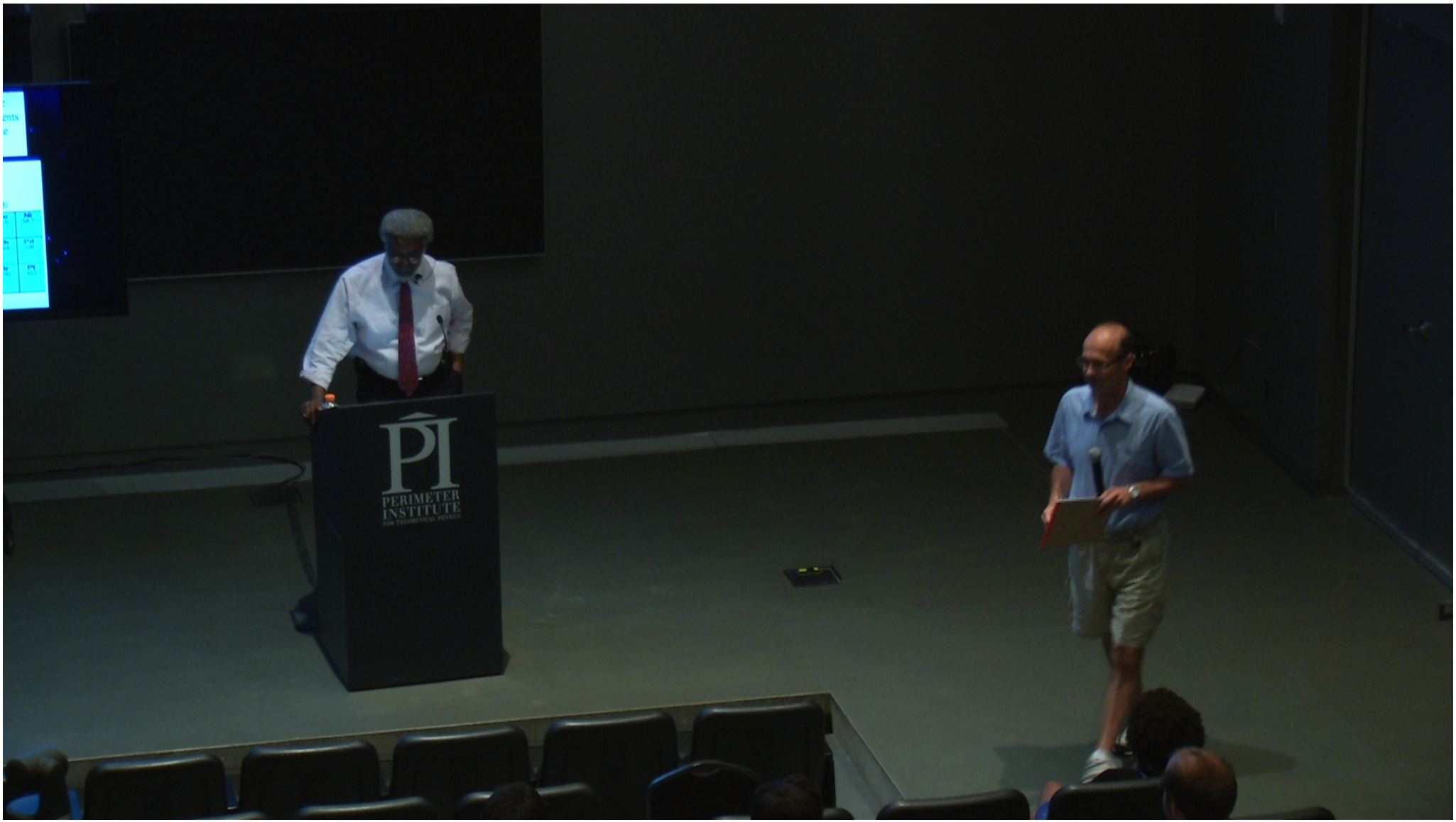
Title: How Much Information Can SUSY QM Conserve About SUSY QFT (Part-2)?

Date: Jun 04, 2014 02:00 PM

URL: http://pirsa.org/14060005

Abstract: We provide additional evidence that supersymmetrical quantum mechanical systems can contain a remarkable amount of information about supersymmetrical field
 theories in greater than one dimension.

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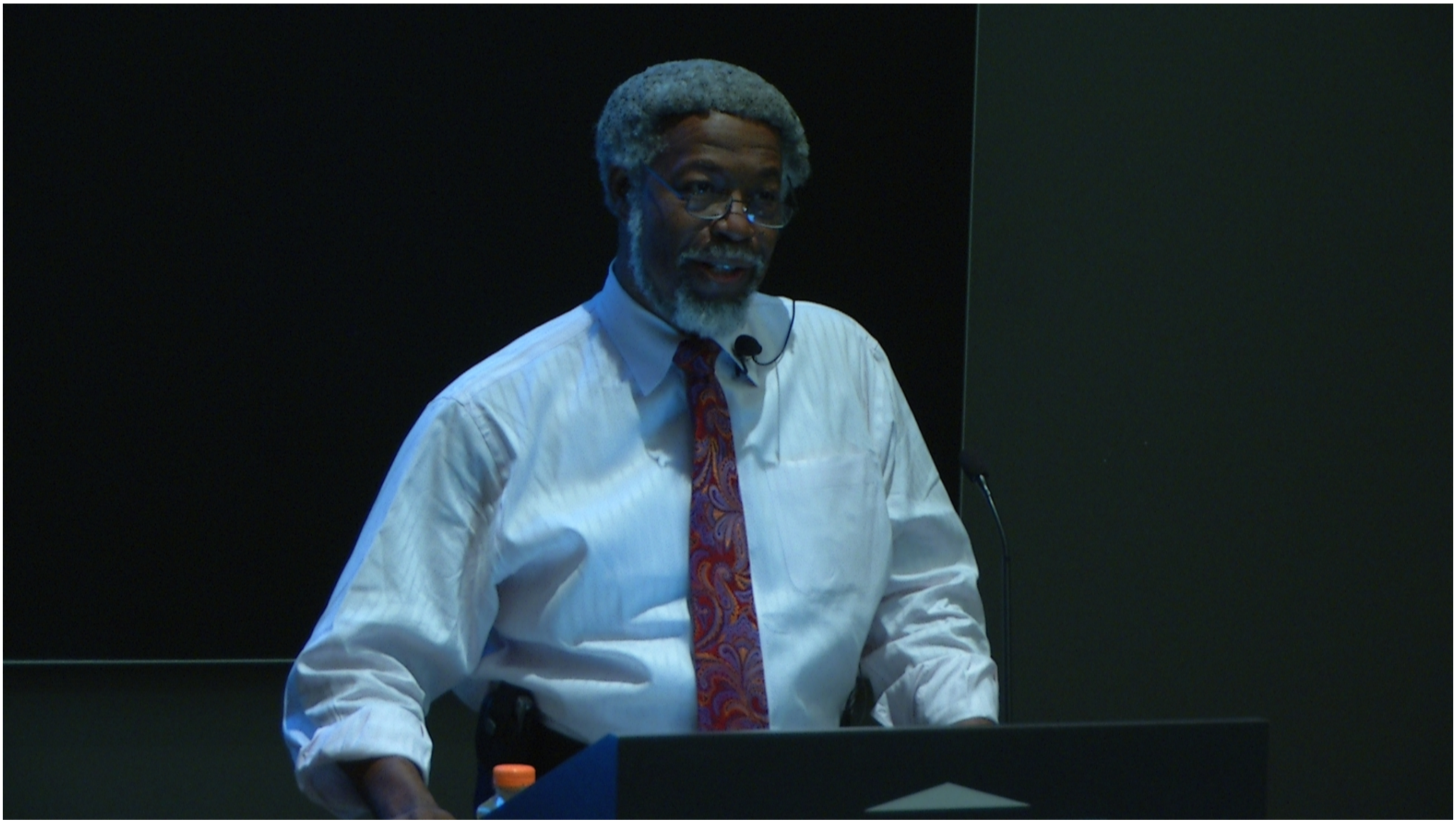


4D Chiral Supermultiplet

$$(\boldsymbol{A},\ \boldsymbol{B},\ \boldsymbol{\psi_a},\ \boldsymbol{F},\ \boldsymbol{G})$$

$$\mathrm{D}_a A = \psi_a \ ,$$

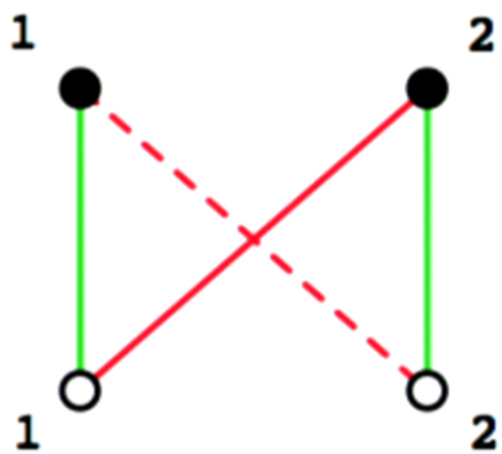
$$\mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \ ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^\mu)_{ab}\,\partial_\mu A - (\gamma^5\gamma^\mu)_{ab}\,\partial_\mu B - i\,C_{ab}\,F + (\gamma^5)_{ab}\,G \ ,$$

$$\mathrm{D}_a F = (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \ ,$$

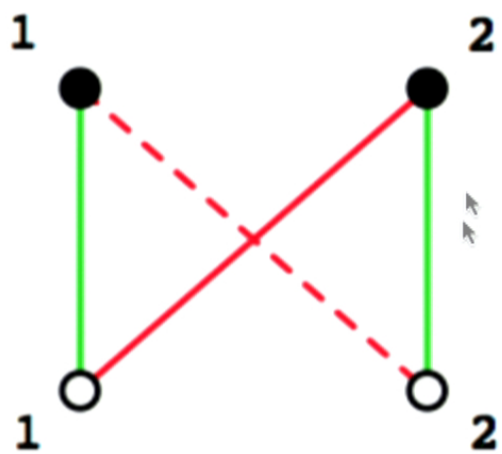
$$\mathrm{D}_a G = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \ .$$

Adinkra



$$
\begin{aligned}
D_1\Phi_1 &= i\,\Psi_1 \quad , \quad D_2\Phi_1 = i\,\Psi_2 \quad , \\
D_1\Phi_2 &= i\,\Psi_2 \quad , \quad D_2\Phi_2 = -\,i\,\Psi_1 \quad , \\
D_1\Psi_1 &= \partial_\tau\Phi_1 \quad , \quad D_2\Psi_1 = -\,\partial_\tau\Phi_2 \quad , \\
D_1\Psi_2 &= \partial_\tau\Phi_2 \quad , \quad D_2\Psi_2 = \partial_\tau\Phi_1 \quad .
\end{aligned}
$$

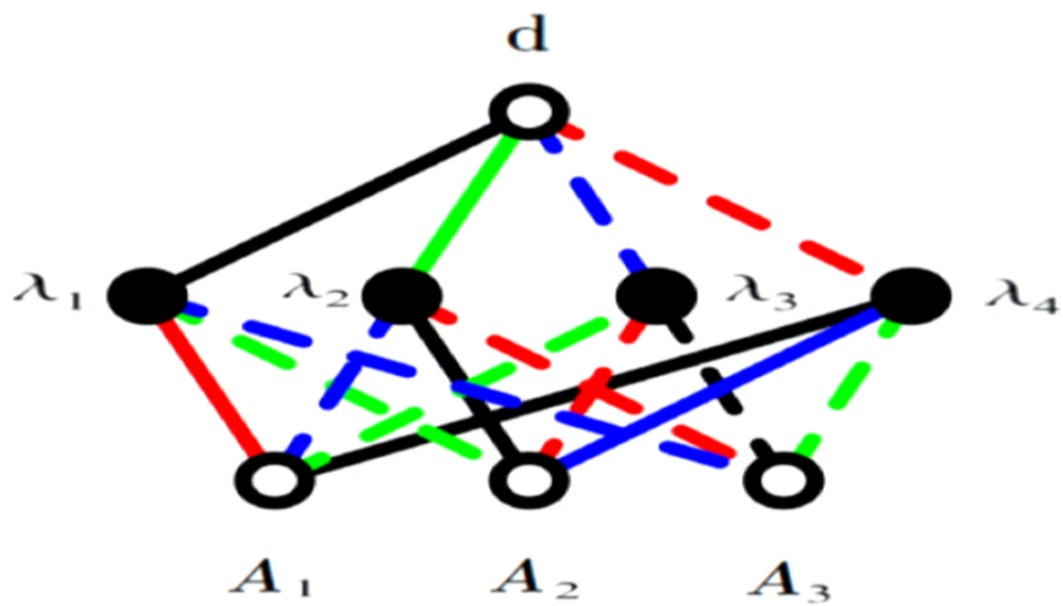
Adinkra



$$
\begin{aligned}
D_1\Phi_1 &= i\,\Psi_1 \quad , \quad D_2\Phi_1 = i\,\Psi_2 \quad , \\
D_1\Phi_2 &= i\,\Psi_2 \quad , \quad D_2\Phi_2 = -\,i\,\Psi_1 \quad , \\
D_1\Psi_1 &= \partial_\tau\Phi_1 \quad , \quad D_2\Psi_1 = -\,\partial_\tau\Phi_2 \quad , \\
D_1\Psi_2 &= \partial_\tau\Phi_2 \quad , \quad D_2\Psi_2 = \partial_\tau\Phi_1 \quad .
\end{aligned}
$$

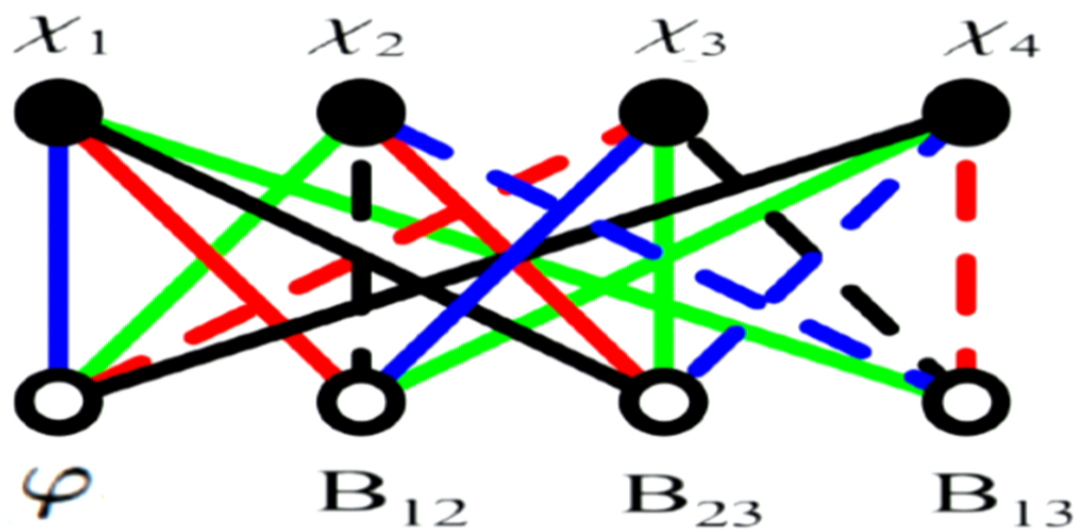
Adinkra graph describing particles and sparticles of non-gravitational energy.

4D Vector Supermultiplet



Adinkra graph describing particles and sparticles of gravitational energy predicted by Superstring/M-Theory.

4D Tensor Supermultiplet

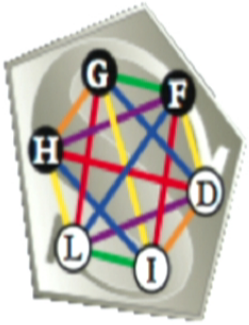


http://www.kaleidoscope.net/greg/math/The%20Adinkramat.html

https://code.google.com/p/adinkramat/

The Adinkramat

COHOMOLOGY.COM BIBDESK SCRIPTS THE LINEAR ALGEBRATOR THE ADINKRAMAT

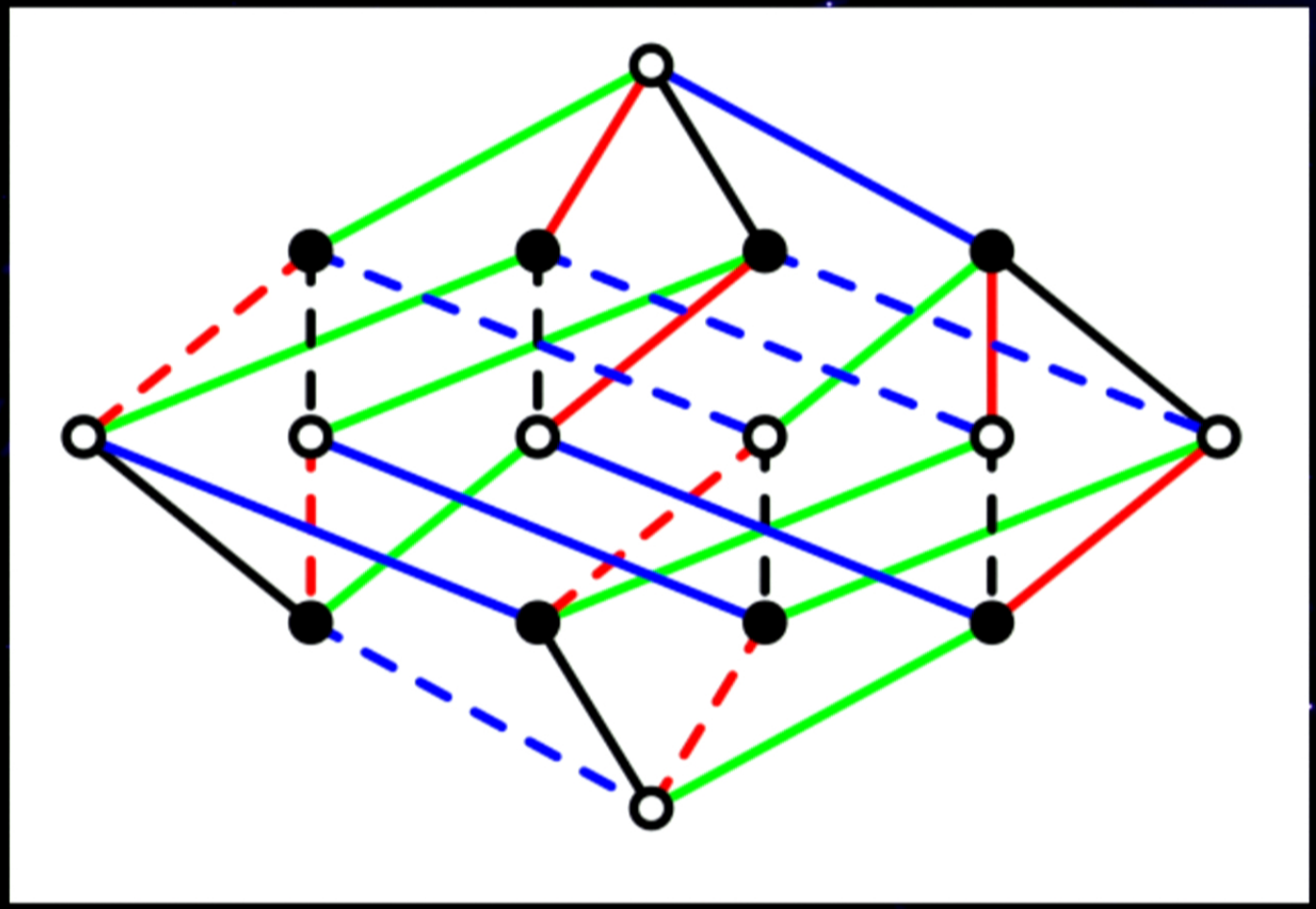


Have you ever tried to understand off-shell supermultiplets and thought that it might be interesting to describe the component fields and supersymmetry actions in terms of diagrams? If so, you're probably one of my research collaborators, Chuck Doran, Mike Faux, Jim Gates, Tristan Hübsch, or Kevin Iga. **Adinkras** are graphs which encapsulate representation of d=1, N-extended off-shell supersymmetry. Such representations are used in supersymmetric quantum mechanics, and they also arise as the dimensional reductions of higher dimensional supersymmetric field theories.

The **Adinkramat** is a document-based multi-threaded Mac OS X application for exploring Adinkras, which automatically generates the standard Adinkras, lets you rearrange the vertices, and exports the results as PDF and Encapsulated PostScript.

Download Adinkramat 1.1 (November 14, 2006)

If you have any questions, comments, or suggestions, please e-mail Greg Landweber.



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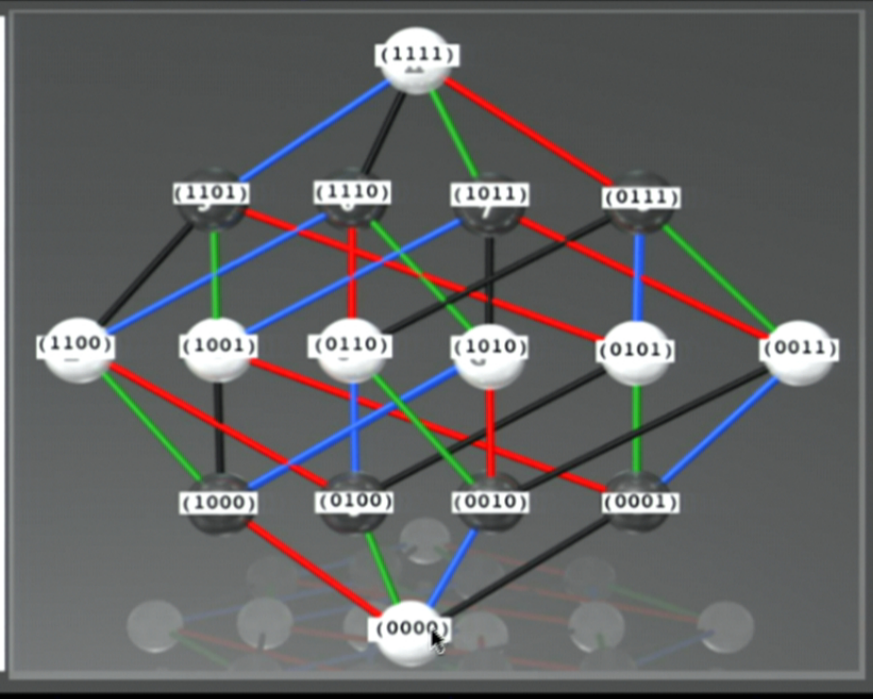


The 'Garden Algebra'

$$\mathrm{D}_{\mathrm{I}}\Phi_i = i\,(\mathrm{L}_{\mathrm{I}})_{i\hat{k}}\,\Psi_{\hat{k}}$$

$$\mathrm{D}_{\mathrm{I}}\Psi_{\hat{k}} = (\mathrm{R}_{\mathrm{I}})_{\hat{k}i}\,(\partial_\tau \Phi_i)$$

$$(\mathrm{L}_{\mathrm{I}})_i{}^{\hat{j}}\,(\mathrm{R}_{\mathrm{J}})_{\hat{j}}{}^{k} + (\mathrm{L}_{\mathrm{J}})_i{}^{\hat{j}}\,(\mathrm{R}_{\mathrm{I}})_{\hat{j}}{}^{k} = 2\,\delta_{\mathrm{IJ}}\,\delta_i{}^{k}\ ,$$

$$(\mathrm{R}_{\mathrm{J}})_{\hat{i}}{}^{j}\,(\mathrm{L}_{\mathrm{I}})_j{}^{\hat{k}} + (\mathrm{R}_{\mathrm{I}})_{\hat{i}}{}^{j}\,(\mathrm{L}_{\mathrm{J}})_j{}^{\hat{k}} = 2\,\delta_{\mathrm{IJ}}\,\delta_{\hat{i}}{}^{\hat{k}}\ ,$$

$$(\mathrm{R}_{\mathrm{I}})_{\hat{j}}{}^{k}\,\delta_{ik} = (\mathrm{L}_{\mathrm{I}})_i{}^{\hat{k}}\,\delta_{\hat{j}\hat{k}}\ .$$

For a fixed value of $\mathcal{N}$ there is a minimum value $d_{\mathcal{N}}$ such that $d_{\mathcal{N}} \times d_{\mathcal{N}}$ matrices faithfully represent this algebra. With $\mathcal{N} = 8\,m + n$, $1 \leq n \leq 8$ and using the definition if $\mathcal{N} = 8\,k \rightarrow m = k - 1$ for $k = 1, 2, 3, \ldots \infty$, this minimum value is shown in the following table

$$\mathrm{d}_{\mathcal{N}} = 16^{m}\, F_{\mathcal{R}\,\mathcal{H}}(n)$$

n	$F_{\mathcal{R}\,\mathcal{H}}(n)$
1	1
2	2
3	4
4	4
5	8
6	8
7	8
8	8

L-Matrix Decomposition

$$(\mathrm{L}_{\mathrm{I}})_i{}^{\hat{k}} = (\mathcal{S}^{(\mathrm{I})})_i{}^{\hat{\ell}} (\mathcal{P}_{(\mathrm{I})})_{\hat{\ell}}{}^{\hat{k}} \quad , \qquad \text{for each fixed } \mathrm{I} = 1, 2, 3, 4.$$

$$(\mathcal{S}^{(\mathrm{I})})_i{}^{\hat{\ell}} = \begin{bmatrix} (-1)^{b_1} & 0 & 0 & 0 \\ 0 & (-1)^{b_2} & 0 & 0 \\ 0 & 0 & (-1)^{b_3} & 0 \\ 0 & 0 & 0 & (-1)^{b_4} \end{bmatrix} \leftrightarrow \left(\mathcal{R}^{\mathrm{I}}\right)_b = \sum_{i=1}^{4} b_i 2^{i-1}$$

Taking A Second Look

1 Chiral Multiplet Matrices

$$L_1 = (10)_b(243)\ ,\ L_2 = (6)_b(134)\ ,\ L_3 = (0)_b(142)\ ,\ L_4 = (12)_b(123)$$

$$R_1 = (12)_b(234)\ ,\ R_2 = (10)_b(143)\ ,\ R_3 = (0)_b(124)\ ,\ R_4 = (9)_b(132)$$

2 Vector Multiplet Matrices

$$L_1 = (10)_b(1243)\ ,\ L_2 = (12)_b(23)\ ,\ L_3 = (0)_b(14)\ ,\ L_4 = (6)_b(1342)$$

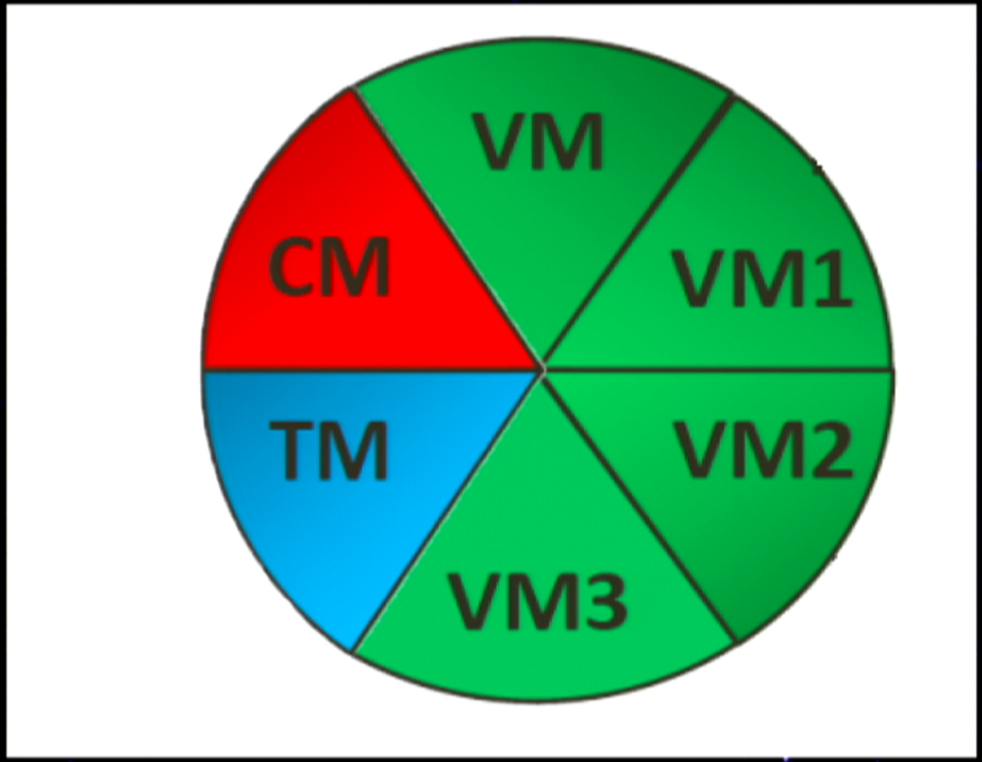
$$R_1 = (12)_b(1342)\ ,\ R_2 = (10)_b(23)\ ,\ R_3 = (0)_b(14)\ ,\ R_4 = (13)_b(1243)$$

3 Tensor Multiplet Matrices

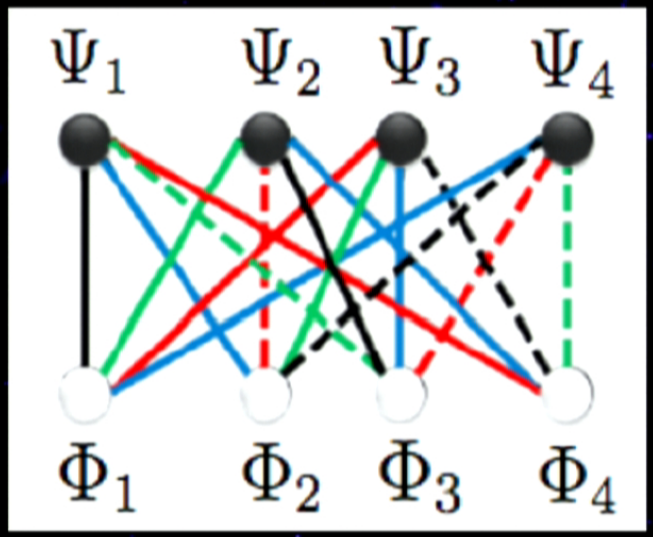
$$L_1 = (14)_b(234)\ ,\ L_2 = (2)_b(143)\ ,\ L_3 = (4)_b(124)\ ,\ L_4 = (8)_b(132)$$

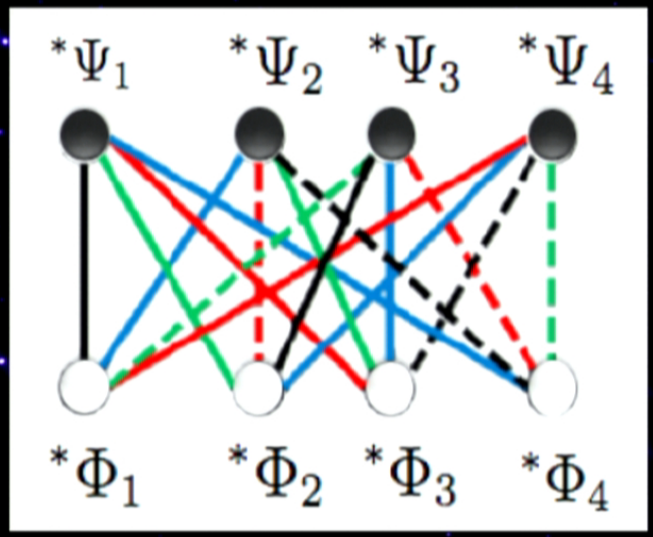
$$R_1 = (14)_b(243)\ ,\ R_2 = (2)_b(134)\ ,\ R_3 = (4)_b(142)\ ,\ R_4 = (8)_b(123)$$

SUSY Permutation Quartets Within A Coxeter Algebra

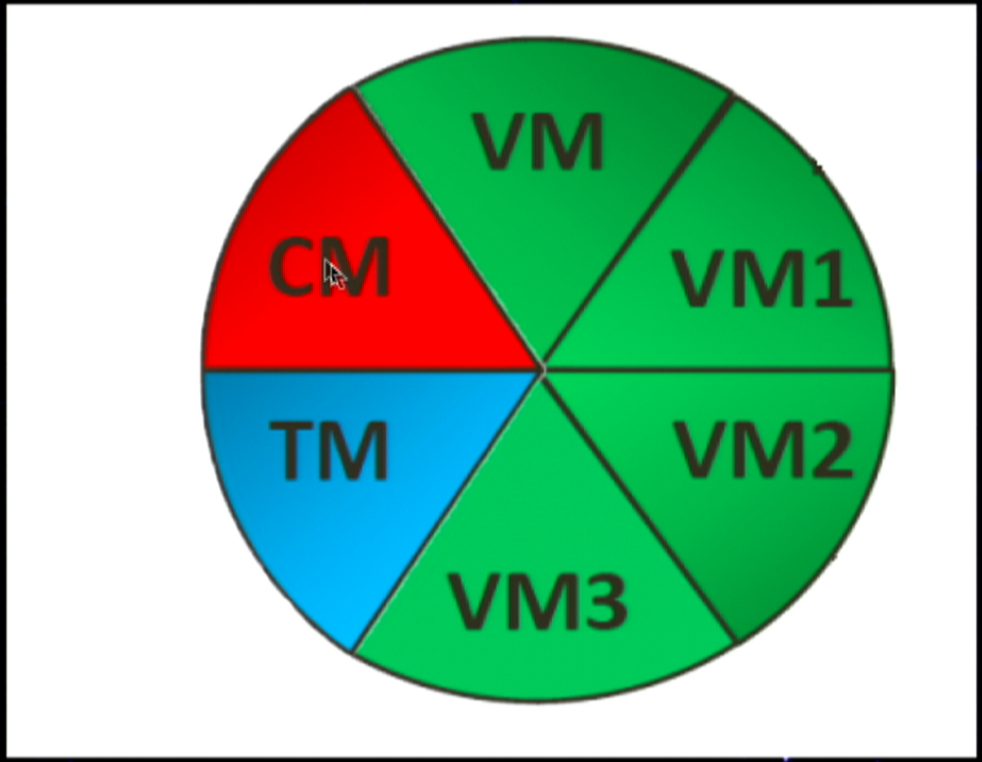


A 1D Hodge Star Map





SUSY Permutation Quartets Within A Coxeter Algebra



Hodge Duality

$$\vec{\nabla}\cdot\vec{\mathcal{E}} = 0 \quad , \quad \vec{\nabla}\cdot\vec{\mathcal{B}} = 0 \quad ,$$

$$\vec{\nabla}\times\vec{\mathcal{B}} - \frac{1}{c}\frac{\partial\vec{\mathcal{E}}}{\partial t} = 0 \quad , \quad \vec{\nabla}\times\vec{\mathcal{E}} + \frac{1}{c}\frac{\partial\vec{\mathcal{B}}}{\partial t} = 0 \quad .$$

$$\vec{\mathcal{E}}' = \vec{\mathcal{E}}\,cos\alpha + \vec{\mathcal{B}}\,sin\alpha \quad ,$$

$$\vec{\mathcal{B}}' = -\,\vec{\mathcal{E}}\,sin\alpha + \vec{\mathcal{B}}\,cos\alpha \quad ,$$

$$\vec{\nabla}\cdot\vec{\mathcal{E}}' = 0 \quad , \quad \vec{\nabla}\cdot\vec{\mathcal{B}}' = 0 \quad ,$$

$$\vec{\nabla}\times\vec{\mathcal{B}}' - \frac{1}{c}\frac{\partial\vec{\mathcal{E}}'}{\partial t} = 0 \quad , \quad \vec{\nabla}\times\vec{\mathcal{E}}' + \frac{1}{c}\frac{\partial\vec{\mathcal{B}}'}{\partial t} = 0 \quad .$$

If $\alpha = \pi/2$ then

$$\vec{\mathcal{E}}' = \vec{\mathcal{B}} \quad , \quad \vec{\mathcal{B}}' = -\,\vec{\mathcal{E}} \, ,$$

which can be written in the manifestly relativistic notation as

$$F_{\mu\nu}{}' = \equiv \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\eta^{\rho\kappa}\eta^{\sigma\lambda}F_{\kappa\lambda} \quad ,$$

Field	ϕ	A_μ	$B_{\mu\nu}\ (B_{\nu\mu} = -B_{\mu\nu})$
Field Strength	$f_\mu = \partial_\mu \phi$	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$	$h_\mu = \frac{1}{3!}\epsilon^{\kappa\lambda\mu\nu}\partial_\lambda B_{\mu\nu}$
Bianchi Identity	$\partial_\mu f_\nu - \partial_\nu f_\mu = 0$	$\epsilon^{\kappa\lambda\mu\nu}\partial_\lambda F_{\mu\nu} = 0$	$\partial^\mu h_\mu = 0$
Action	$S_0 = -\frac{1}{2}\int d^4x\ f^\mu f_\mu$	$S_1 = -\frac{1}{4}\int d^4x\ F^{\mu\nu}F_{\mu\nu}$	$S_2 = \frac{1}{2}\int d^4x\ h^\mu h_\mu$
Equation of Motion	$\frac{\delta S_0}{\delta \phi} = 0 \rightarrow \partial^\mu f_\mu = 0$	$\frac{\delta S_1}{\delta A_\mu} = 0 \rightarrow \partial^\mu F_{\mu\nu} = 0$	$\frac{\delta S_2}{\delta B_{\mu\nu}} = 0 \rightarrow$ $\partial_\mu h_\nu - \partial_\nu h_\mu = 0$
Duality	$f_\mu \leftrightarrow h_\mu$	$F_{\mu\nu} \leftrightarrow \epsilon_{\mu\nu\kappa\lambda}F^{\kappa\lambda}$	$h_\mu \leftrightarrow f_\mu$

Confronting Information Loss

$$\mathcal{L}_{Spin-0} = -\tfrac{1}{2}(\partial_\mu \phi^{\mathcal{I}})(\partial^\mu \phi^{\mathcal{I}}) \quad ,$$

$$\mathcal{L}_{Spin-0} = \tfrac{1}{2}(\partial_\tau \phi^1)(\partial_\tau \phi^1) + \tfrac{1}{2}(\partial_\tau \phi^2)(\partial_\tau \phi^2) + \tfrac{1}{2}(\partial_\tau \phi^3)(\partial_\tau \phi^3) \quad ,$$

$$\mathcal{L}_{Spin-1} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\tfrac{1}{2} F_{0i} F^{0i} \quad .$$

$$\mathcal{L}_{Spin-1} = \tfrac{1}{2}\left[(\partial_\tau A_1)^2 + (\partial_\tau A_2)^2 + (\partial_\tau A_3)^2 \right] \quad .$$

Confronting Information Loss

$$\mathcal{L}_{Spin-0} = -\tfrac{1}{2}(\partial_\mu \phi^{\mathcal{I}})(\partial^\mu \phi^{\mathcal{I}}) \quad ,$$

$$\mathcal{L}_{Spin-0} = \tfrac{1}{2}(\partial_\tau \phi^1)(\partial_\tau \phi^1) + \tfrac{1}{2}(\partial_\tau \phi^2)(\partial_\tau \phi^2) + \tfrac{1}{2}(\partial_\tau \phi^3)(\partial_\tau \phi^3) \quad ,$$

$$\mathcal{L}_{Spin-1} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\tfrac{1}{2} F_{0i} F^{0i} \quad .$$

$$\mathcal{L}_{Spin-1} = \tfrac{1}{2}\left[(\partial_\tau A_1)^2 + (\partial_\tau A_2)^2 + (\partial_\tau A_3)^2 \right] \quad .$$

$$
\begin{aligned}
\mathcal{L} \;=\; & \tfrac{1}{2}(\partial_\tau A^{\mathcal{I}})(\partial_\tau A^{\mathcal{I}}) + \tfrac{1}{2}(\partial_\tau B^{\mathcal{I}})(\partial_\tau B^{\mathcal{I}}) + \tfrac{1}{2}(\partial_\tau F^{\mathcal{I}})(\partial_\tau F^{\mathcal{I}}) + \tfrac{1}{2}(\partial_\tau G^{\mathcal{I}})(\partial_\tau G^{\mathcal{I}}) \\
& + \tfrac{1}{2}(\partial_\tau A_i)(\partial_\tau A_i) + \tfrac{1}{2}(\partial_\tau \mathrm{d})(\partial_\tau \mathrm{d}) + i\tfrac{1}{2}(\gamma\cdot\mathcal{T})^{ab}\psi^{\mathcal{I}}_a\partial_\tau\psi^{\mathcal{I}}_b + i\tfrac{1}{2}(\gamma\cdot\mathcal{T})^{cd}\lambda_c\partial_\tau\lambda_d
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{D}_a A^{\mathcal{J}} &= \psi^{\mathcal{J}}_a \quad , \quad \mathrm{D}_a B^{\mathcal{J}} = i\,(\gamma^5)_a{}^b\,\psi^{\mathcal{J}}_b \quad , \\
\mathrm{D}_a F^{\mathcal{J}} &= (\gamma\cdot\mathcal{T})_a{}^b\,\psi^{\mathcal{J}}_b \quad , \quad \mathrm{D}_a G^{\mathcal{J}} = i\,(\gamma^5\gamma\cdot\mathcal{T})_a{}^b\,\psi^{\mathcal{J}}_b \quad , \\
\mathrm{D}_a \psi^{\mathcal{J}}_b &= i\,(\gamma\cdot\mathcal{T})_{ab}\left(\partial_\tau A^{\mathcal{J}}\right) - (\gamma^5\gamma\cdot\mathcal{T})_{ab}\left(\partial_\tau B^{\mathcal{J}}\right) \\
&\quad - i\,C_{ab}\left(\partial_\tau F^{\mathcal{J}}\right) + (\gamma^5)_{ab}\left(\partial_\tau G^{\mathcal{J}}\right) \quad , \\
\mathrm{D}_a A_i &= (\gamma_i)_a{}^b\lambda_b \quad , \quad D_a\mathrm{d} = i(\gamma^5\gamma\cdot\mathcal{T})_a{}^b\,\lambda_b \quad , \\
\mathrm{D}_a \lambda_b &= -\tfrac{i}{2}([\gamma\cdot\mathcal{T}\,,\,\gamma^i])_{ab}\left(\partial_\tau A_i\right) + (\gamma^5)_{ab}\left(\partial_\tau \mathrm{d}\right) \quad .
\end{aligned}
$$

arXiv.org > hep-th > arXiv:1311.3736

High Energy Physics - Theory

Geometrization of N-Extended 1-Dimensional Supersymmetry Algebras

Charles Doran, Kevin Iga, Greg Landweber, Stefan Mendez-Diez

(Submitted on 15 Nov 2013)

The problem of classifying off-shell representations of the N-extended one-dimensional super Poincar\'{e} algebra is closely related to the study of a class of decorated graphs known as Adinkras. We show that these combinatorial objects possess a form of emergent supergeometry: Adinkras are equivalent to very special super Riemann surfaces with divisors. The method of proof critically involves Grothendieck's theory of "dessins d'enfants", work of Cimasoni-Reshetikhin expressing spin structures on Riemann surfaces via dimer models, and an observation of Donagi-Witten on parabolic structure from ramified coverings of super Riemann surfaces.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