Title: Some Conceptual Problems in General Relativity and Cosmology

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Abstract: This talk will try to highlight some basic problems connected with conclusions uncritically drawn from well known works. These include: 1. The Schwarzschild solution 2. The formation of black holes by gravitational collapse 3. The no hair theorem 4. The principle of equivalence in the very early universe.

Some Conceptual Problems in GR

"I am not aware that relativity is at present regarded by physicists as a theory that may be believed or not, at will"

Clemence, G.M., 1947
[Rev. Mod. Phys. 19, 361]

A personal view on a few 'well understood' problems in GR…

1. **The Schwarzschild Solution**
2. **Black Holes in Astrophysics**
3. **No-hair Theorem**
4. **Strong Principle of Equivalence in the Early Universe**

In other words, the point mass singularity determines the potential uniquely in a self contained manner.

In GR, this does not seem possible! Take the Schwarzschild line element:

$$ds^2 = e^{\nu}\, dt^{2} - e^{\lambda}\, dr^{2} - r^{2}[\, d\,\theta^2 + \sin^2\theta\, d\varphi^2]$$

and substitute in Einstein's equations for $r > 0$. We get the solution:

$$e^{\lambda} = e^{-\nu} = 1 - (D/\,r)$$

How do we determine D ?

At this stage an appeal is made to the Newtonian approximation, and taking the weak field limit at large r, we derive the value of D as $2GM$.

Unlike the Newtonian case, we have not used the information of the point source at the origin. Instead we have gone far away from the origin to derive the value of *D*.

Why not use the field equation for λ ? This gives

$$e^{-\lambda} = 1 - 8\pi G \int_0^r r_1{}^2 \rho_1 dr_1 / r$$

We have a problem if we interpret the integral on the R.H.S. as $M/4\pi r$, because the element of proper volume is not what is given in the integral but has to be multiplied by $e^{\lambda/2}$ evaluated at r_1.

To do this properly we have to write:

$$M = \int_0^R 4\pi r^{2}\, e^{\lambda/2}\, \rho_N\, dr$$

$$+ \int_0^R 4\pi r^{2}\, e^{\lambda/2}\, (\rho - \rho_N)\, dr$$

$$+ \int_0^R 4\pi r^{2}\, e^{\lambda/2}\, (e^{-\lambda/2} - 1)\, \rho\, dr$$

That is,

Gravitational Mass = Nucleonic mass
+ Internal energy
+ Gravitational Potential Energy

[For a more detailed discussion of the problem *see*: Petrov, A. N. and JVN, Found. Phys, *26*, 1201, 1996]

However, our original problem is still not answered!

Try another way…Consider the field equation:

$$R = -\kappa T$$

Which can be written in the above Schwarzschild spacetime as:

$$X'' + 2X'/r + 2X/r^2 = -8\pi G T$$

Where $X = e^{-\lambda} - 1$. This integrates to the required answer provided

$$T = M\,\delta(r)$$

But the static solution requires us to have

$$T^0{}_0 = T^1{}_1 = 0$$

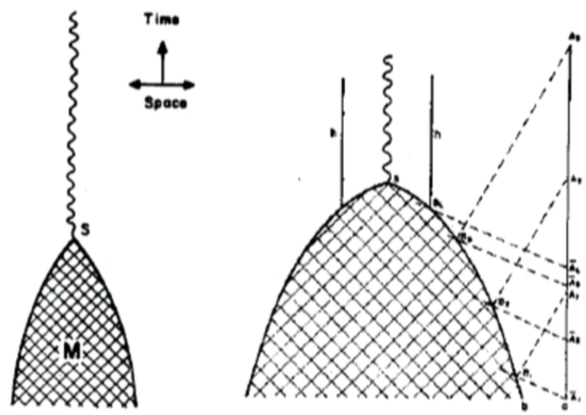
Thus there is a contradiction! The moral is : ***Keep away from the singularity at r = 0.***

Black Holes in Astrophysics

These form from gravitational collapse of massive stars. *But for an external observer, they are never formed!*

The astrophysicist may argue that the collapsing object has become practically black in a finite time as measured by the external observer, and hence may be called a black hole for all practical purposes. However,…

Since the laws of black hole physics require the horizon to have formed – which never happens for an external observer, can these laws have any physical relevance?


Time
Space
S
M

No Hair Theorem

"A collapsing object of any irregular shape will lose all its moments / dynamical information / other physical characteristics except mass, charge and angular momentum."

This is proved by Price for small perturbations from spherical symmetry. Why should one believe it in the most general case ?

Is this an article of faith with the GR community or have I missed a general demonstration of this result?

Strong Principle of Equivalence in the Early Universe

The justification of using standard flat space physics in a covariant way in the presence of gravity lies in the SPC.

In the SPC, we reduce the coordinate system in a local region $\wp$ to a locally inertial one, and then apply flat space physics to it.

Normally this procedure requires the validity of the following kind:

$$L = \text{Radial size of } \wp << 1/\sqrt{R}$$

Where, R is the typical curvature component of spacetime. [This is the 'flat earth approximation]

This condition becomes difficult to satisfy in the early universe.

[- *see* : Padmanabhan, T. and Vasanthi, M.M. Phys. Lett. A, *89*, 327]

There, we have $R \sim 1 / t^2$ and so we require,

$$L / t \equiv \varepsilon << 1$$

But to define temperature of this epoch one needs large enough number of relativistic particles in a region of this size.

The particle number density, *assuming* that it is large enough to apply standard statistical mechanics is

$$n \sim (g / \pi^2) [kT/h]^3$$

where g is the number of spin degrees of freedom. The time (t) – temperature (T) relationship is

$$t/t_P = [45/\ 16\pi^3 g]^{1/2}\ (T_P/T)^2$$

where t_P is the Planck time-scale and T_P is the Planck temperature. From these relations we get the number of particles in a cube of size L as

$$N \cong (1\ /\ 300\ \sqrt{g})\ (\varepsilon\ T_P/T)^3$$

Notice that ε means that N will be small. However, if the temperature under consideration is small compared to the Planck temperature, then N can be large.

Taking $\varepsilon \sim 10^{-2}$, we see that at the GUT epoch, with g ~100, N is only ~ 1/3…!

Thus, the entire calculation is inconsistent. To make it consistent, one must use statistical mechanics in curved space time or use some other technique to deal with the few-body problem.

Conclusion

These are elementary problems; but they require conceptual clarity in resolving them. So far I have not found any unambiguous answers to these queries…