Title: A mechanism for vortex annihilation in two dimensional superfluid turbulence

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Abstract: Recent numerical simulations [1] have suggested that two dimensional superfluid turbulence is characterized by a direct cascade of energy to small length scales, in contrast to the inverse cascade of normal fluids, where energy is transported to large length scales. This direct cascade is characterized by many vortex-antivortex annihilation events. Recent experimental work [2] on Bose-Einstein condensates appears to demonstrate qualitatively similar physics. I will discuss recent work in progress towards identifying the physical mechanism underlying this direct cascade, using techniques of effective field theory.

Collaborators

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What is a Superfluid?

- a **superfluid** is a phase of matter in which a global U(1) symmetry is spontaneously broken
- simplest example (for HEP...): complex φ^4 theory: $(\mu, \lambda > 0)$

$$\mathcal{L} = -\partial_\mu \bar{\varphi} \partial^\mu \varphi + \mu |\varphi|^2 - \frac{\lambda}{2} |\varphi|^4$$

- condensed matter: U(1) symmetry = particle conservation. $(a \to \mathrm{e}^{\mathrm{i}\varphi} a,\ a^\dagger \to \mathrm{e}^{-\mathrm{i}\varphi} a^\dagger)$.
- microscopic example: $(n = 1)$ 2d Bose-Hubbard model: superfluid phase for small U/t: [Fisher *et al* (1989)]

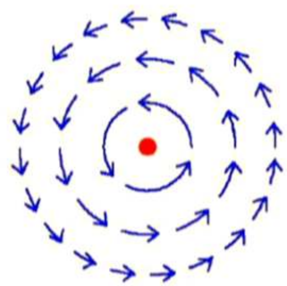
$$H = -t \sum_{i \sim j} a_i^\dagger a_j + U \sum_i a_i^\dagger a_i \left(a_i^\dagger a_i - 1 \right)$$

Vortices

- 2d: defects called **vortices**. order parameter $\varphi(r,\theta) \sim \mathrm{e}^{\mathrm{i}W\theta}$ ($W \in \mathbb{Z}$).
- $W = \pm 1$ vortices are gapped, stable excitations. SF velocity $\mathbf{v} = \nabla\theta/m$:

$$\oint \mathrm{d}\mathbf{s} \cdot \mathbf{v} = \frac{W}{m}\hbar$$

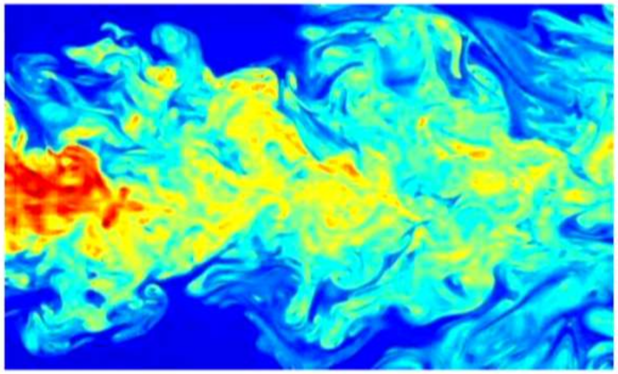
where m is microscopic mass scale



- dynamics of $N \gg 1$ vortices is chaotic: **superfluid turbulence**

Kolmogorov Scaling Theory

- **classical turbulence** describes hydrodynamic flows which are strongly nonlinear and **chaotic**



- characterized by structures on many length and time scales
- drive at length scale L. for wave numbers k such that

$$L^{-1} \ll k \ll \xi^{-1} \equiv \varepsilon^{1/4}\nu^{-3/4}$$

(ν =kinematic viscosity, ε =energy injection rate/mass), energy at scale k: [Kolmogorov (1941)]

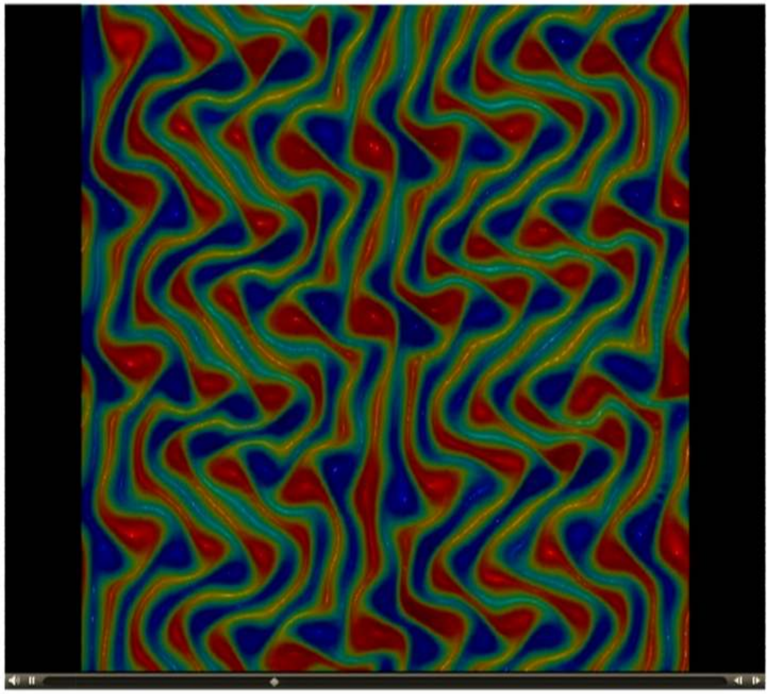
$$E(k) \sim \varepsilon^{2/3}k^{-5/3}$$

Direct vs. Inverse Cascade

- 3+d: energy flows from $k = L^{-1} \to k = \xi^{-1}$: **direct cascade**
- 2d: energy flows to large lengths: **inverse cascade**. follows from enstrophy ($\int d^2x\ \omega^2$) conservation. [Kraichnan (1967)]

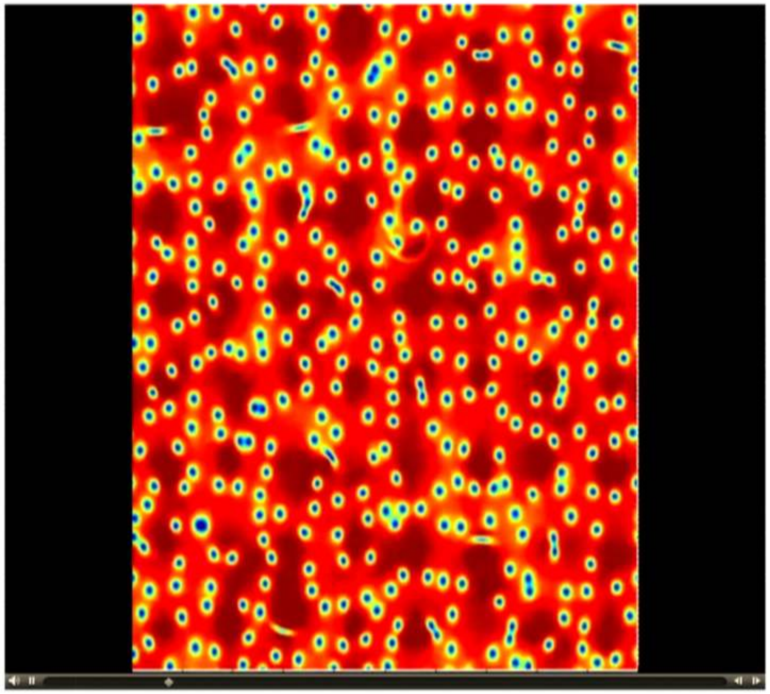
turbulent flow in Jupiter's atmosphere

- inverse cascade also occurs in conformal hydrodynamics [Carrasco et al (2012)]. seen with holography [Adams *et al* (2014), Green *et al* (2014)]. relativistic enstrophy conserved a simulation from [Adams *et al* (2014)] of (conformal) classical 2d turbulence:



Direct Cascades

finite T 2d superfluids have direct cascade, driven by vortex annihilation, instead [Chesler *et al* (2013), Numasoto *et al* (2010)], though some simulations differ [Reeves *et al* (2013)]

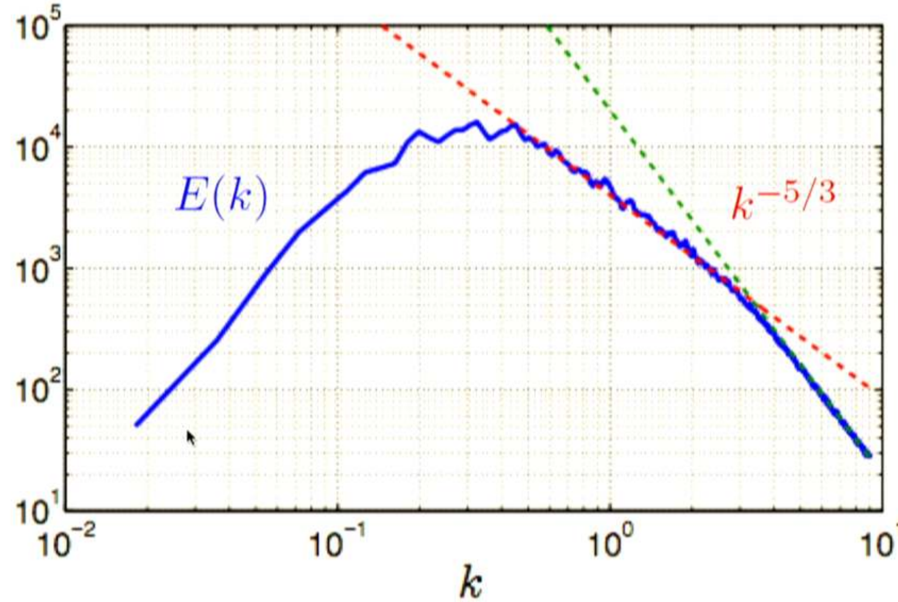


Kolmogorov Scaling

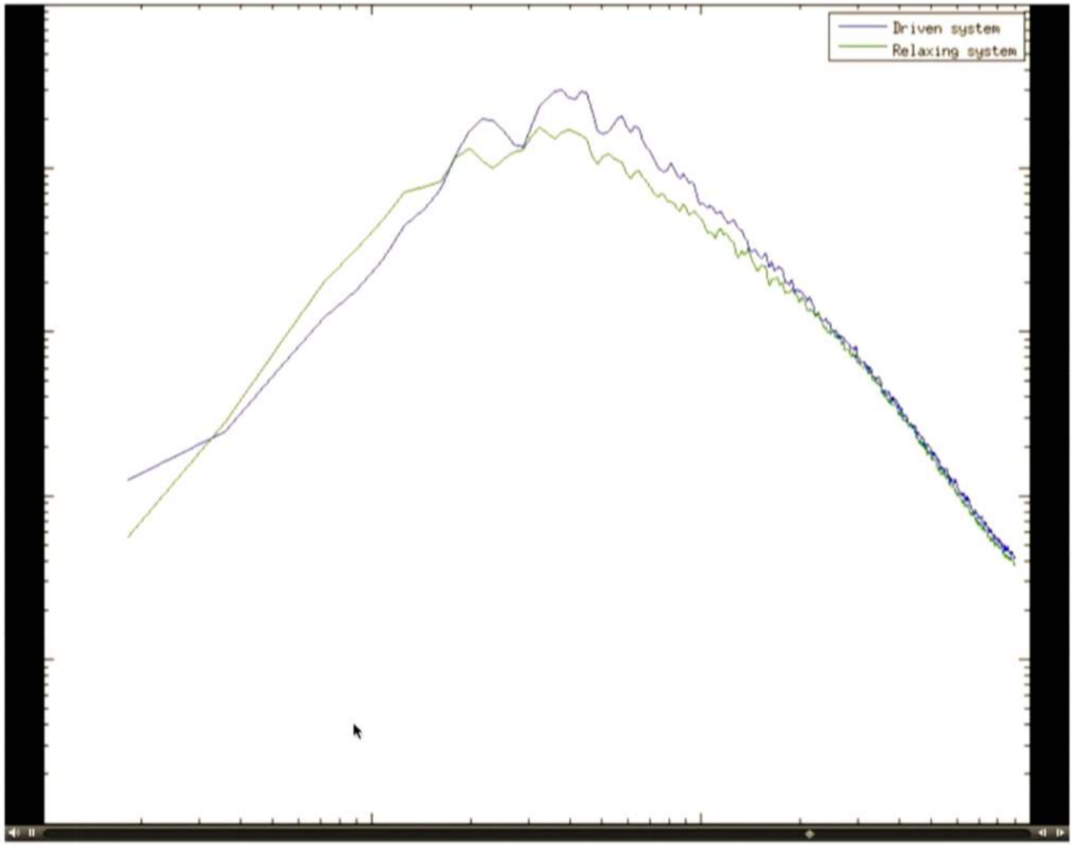
- analogous to classical fluids, can define SF power spectrum

$$E(k) = \int \mathrm{d}\Omega_k \ |(\psi \mathbf{v})(\mathbf{k})|^2$$

- $k^{-5/3}$ scaling in simulations [Chesler *et al* (2013)]



Movie of the Direct Cascade



Experiments

- vortex annihilation events in SF of ^{23}Na BEC [Kwon *et al* (2014)]. experimental evidence for direct cascade?

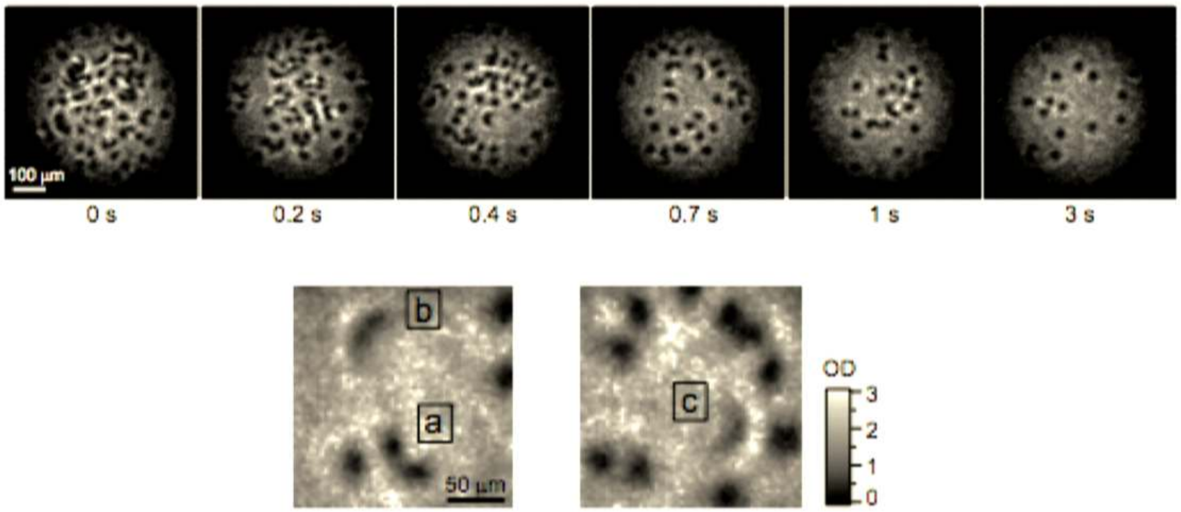


FIG. 3: Vortex pair annihilation in the turbulent superflow.

- proposal: cold atom "gyroscope" distinguishes ± vortices? [Powis *et al* (2014)] ; observation of $k^{-5/3}$ scaling?

1: develop **systematically** an EFT for (dilute) SF turbulence at $T = 0$

2: show that inverse cascades are present at $T = 0$

3: determine the effects of normal fluid on SF vortex dynamics with $T > 0$

4: identify precisely the physical mechanism behind $T > 0$ direct cascade, and understand the transitions between inverse/direct cascades

- most popular SF theory: **Gross-Pitaevskii theory**:

$$\mathcal{L} = \mathrm{i}\bar{\psi}\partial_t\psi - \frac{\nabla\bar{\psi}\cdot\nabla\psi}{2m} + \mu\bar{\psi}\psi - \frac{\lambda}{2}\left(\bar{\psi}\psi\right)^2$$

- the GP equation admits stationary vortex solutions of the form

$$\psi_0(r,\theta) = \sqrt{\rho_0}\mathrm{e}^{\chi(r)\pm\mathrm{i}\theta}$$

  ρ_0 = superfluid density. core size $\xi = 1/\sqrt{2m\mu}$ is finite.

Point Vortex Dynamics

- N vortex ansatz ($\Gamma_n = \pm 1$ denotes winding number)

$$\psi = \sqrt{\rho_0} \exp \left[\sum_{n=1}^{N} \left(\chi(\mathbf{x} - \mathbf{X}_n(t)) + \mathrm{i}\Gamma_n \theta(\mathbf{x} - \mathbf{X}_n(t)) \right) \right]$$

$$+ \delta\psi \text{ (fluctuations due to sound)}$$

- if typical distance btwn vortices $= r_* \gg \xi$, ψ is solution of GPE at leading order if $\mathbf{X}_n$ obeys **point-vortex dynamics**:

$$\dot{\mathbf{X}}_n = \mathbf{V}_n$$

$\mathbf{V}_n$ is SF velocity through core n generated by all other vortices.

Integrating out Fluctuations

- Analogous to classical vortex theories [Endlich *et al* (2013)] compute action for fluctuation corrected EOMs with a path integral: $(\psi = \psi_{\mathrm{PV}} + \delta\psi)$

$$S = S[\psi_{\mathrm{PV}}] + \int \mathrm{d}^3x J\delta\psi + \frac{1}{2}\int \mathrm{d}^3x \delta\psi G^{-1}\delta\psi$$

- the effective action (ignoring quantum fluctuations) is

$$S - S[\psi_{\mathrm{PV}}] = \delta S = -\frac{1}{2}\int \mathrm{d}^3x \mathrm{d}^3y J(x) G(x,y) J(y)$$

- $\xi =$ UV cut-off. box size $L =$ IR cutoff. these are *physical*

The Sound-Corrected Equation of Motion

- $T = 0$ Hall-Vinen-Iordanskii equations suggest [Thompson *et al* (2012)]

$$\delta S \sim \sum \log \frac{L}{\xi} \dot{\mathbf{X}}_n^2$$

- our systematic procedure gives *finite corrections to EOMs*:

$$\delta S \sim \log \frac{L}{r_*} \left(\sum \Gamma_n \dot{\mathbf{X}}_n \right)^2 + \log \frac{r_*}{\xi} \sum \left(\dot{\mathbf{X}}_n - \mathbf{V}_n \right)^2 + \left\{ \text{finite } \mathcal{O}\left(r_*^{-2}\right) \text{ terms} : \mathbf{V}_n \cdot \mathbf{V}_n, \dot{\mathbf{X}}_n \cdot \mathbf{V}_n, \dot{\mathbf{X}}_n \cdot \dot{\mathbf{X}}_m \right\}$$

- vortex-antivortex pair generally *not unstable*
- inverse cascade to "$T < 0$ Onsager state": same sign vortices cluster [Siggia *et al* (1981), Simula *et al* (2014)]

Dissipative Effective Theory

- finite $T > 0$: normal fluid exerts dissipative vortex drag on vortex.
- leading order correction to EOM given by momentum transfer from SF vortex n to normal fluid at velocity $\mathbf{U}$:

$$\frac{\mathrm{d}p_n^i}{\mathrm{d}t} = -\eta(\dot{X}_n^i - U_n^i) - \eta'\Gamma_n\epsilon^{ij}(\dot{X}_n^j - U_n^j)$$

- η, η' depend on microscopics. beyond hydro/EFT.

Superfluid Magnus Force

- compute SF momentum flux around vortex at rest in infinitesimal (far zone) background SF velocity $\mathbf{V}$:

$$\frac{\mathrm{d}p_n^i}{\mathrm{d}t} = \oint \mathrm{d}\Sigma_j \Pi_{\mathrm{s}}^{ij}$$

- vortex rest frame: $\Pi_{\mathrm{s}}^{ij} = \rho_{\mathrm{s}}(\mu\delta^{ij} + v^i v^j)$. Landau two-fluid equations imply $\mu = \mu_0 - \frac{1}{2}\mathbf{v}_{\mathrm{s}}^2$. The background SF velocity is

$$v^i = V^i - \frac{\Gamma_n}{m}\epsilon^{ij}\frac{x^j}{x^2}.$$

- Surface integral picks out $l = \pm 1$ $(\sin / \cos\theta)$ modes linear in V. Shift back to frame of moving vortex:

$$\frac{\mathrm{d}p_{\mathrm{s}}^i}{\mathrm{d}t} = \frac{2\pi\Gamma_n}{m}\rho_{\mathrm{s}}\epsilon^{ij}\left(\dot{X}^j - V^j\right).$$

The Equations of Motion

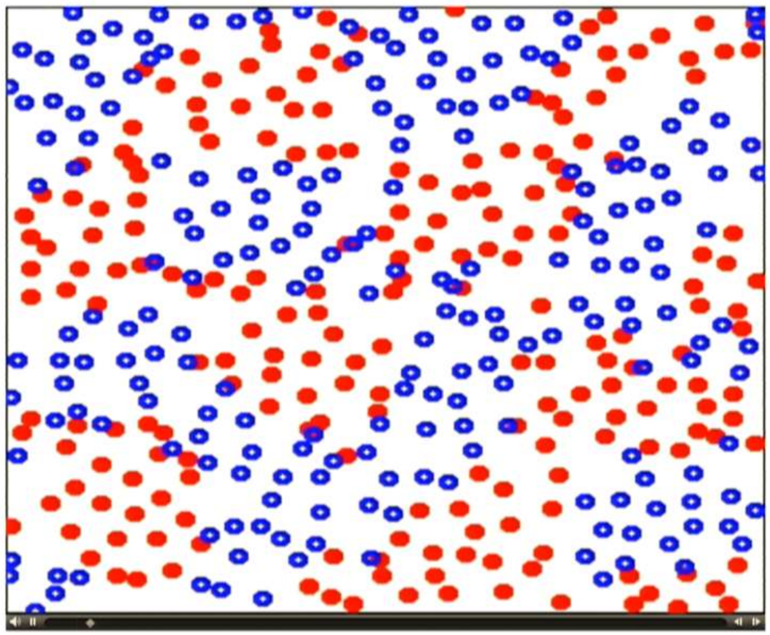
- $\mathbf{V} \approx$ contributions purely from other vortices: SF sound $1/r_*^2$ suppressed
- $\mathbf{U}$: momentum/energy injected into normal fluid by vortex drag leads to $1/r_*^2$ suppressed corrections. subleading
  - consistent to treat normal fluid as static: $\mathbf{U} = \mathbf{0}$
- ultimate EOM: Magnus force balances drag force at core:

$$\frac{2\pi\Gamma_n}{m}\rho_s\epsilon^{ij}\left(\dot{X}_n^j - V_n^j\right) = -\eta\dot{X}_n^i - \eta'\epsilon_{ij}\dot{X}_n^j.$$

Superfluid Turbulence with Vortex Drag

Numerical Comparison to Holographic Superfluid

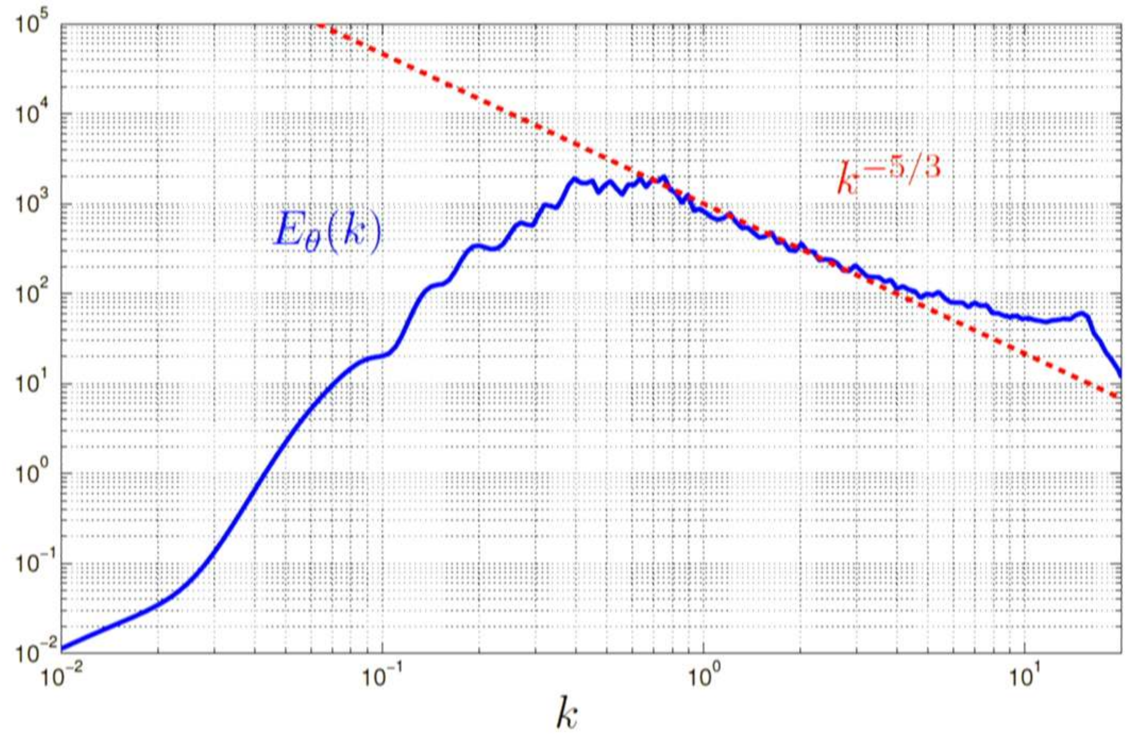
EFT simulations correctly capture qualitative vortex physics of [Chesler *et al* (2013)] holographic simulation



Superfluid Turbulence with Vortex Drag

Kolmogorov Scaling

$k^{-5/3}$ scaling: system drives itself via drag/vortex annihilation.



Superfluid Turbulence with Vortex Drag

Vortex Annihilation Events

- consider a vortex-antivortex pair separated by distance r_*
- if $\eta = 0$, this pair travels forever at velocity $1/mr_*$.
- if $0 < \eta \ll 1$, they annihilate in a time

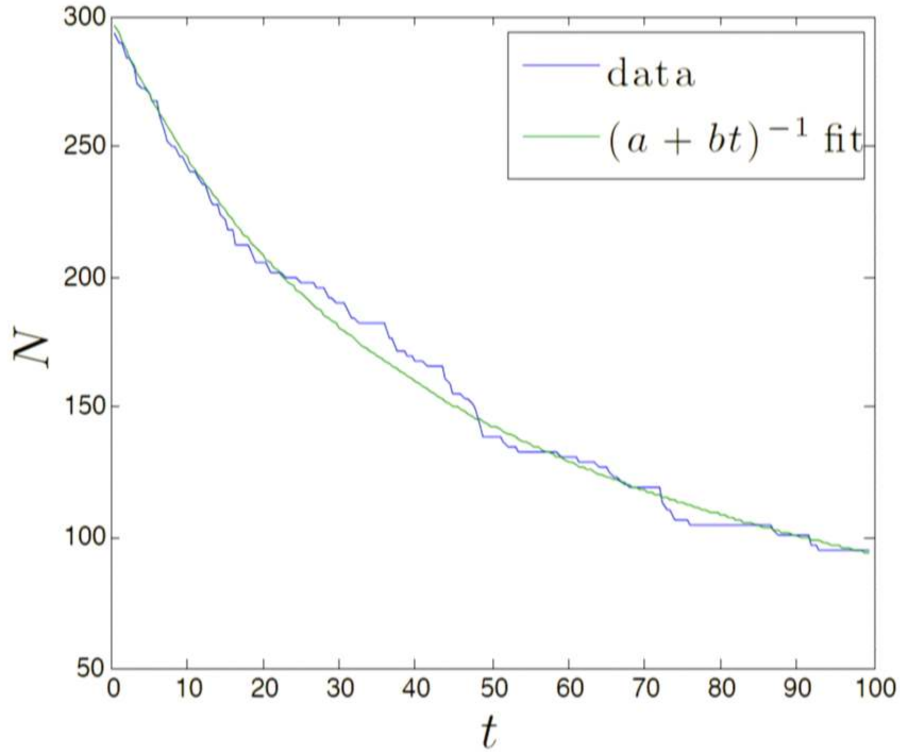
$$t_{\mathrm{ann}} \sim \frac{\rho_{\mathrm{s}} r_*^2}{\eta}$$

- at vortex density n, rate of annihilation $\sim n$, and $\dot{n} \sim -\eta n^2$
- total vortices N should decay as $\dot{N} = -\gamma N^2$ with $\gamma \sim \eta/\rho_{\mathrm{s}} L^2$:

$$N(t) = \frac{N_0}{1 + N_0 \gamma t}$$

Counting Vortices

this is obeyed quite well in our simulations: $\gamma \approx 10\eta/\rho_s L^2$:

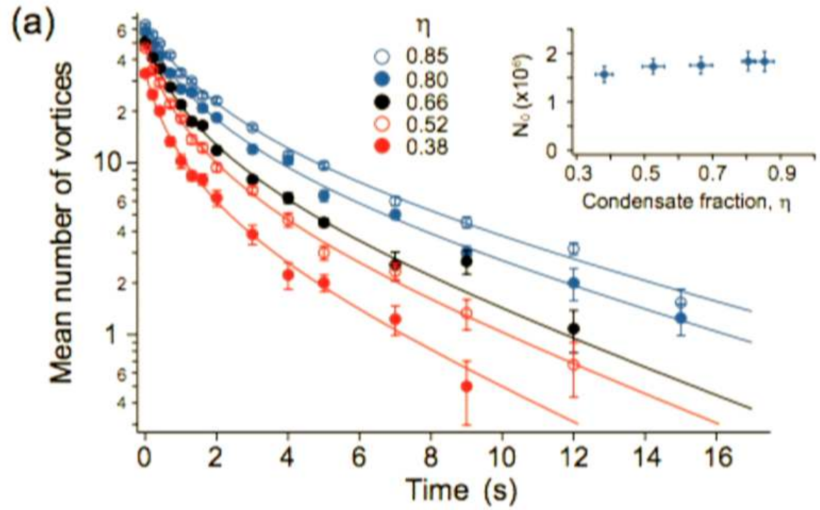


Counting Vortices: Experimental Results

- experiment [Kwon *et al* (2014)]: direct observation of

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\Gamma_1 N - \Gamma_2 N^2.$$

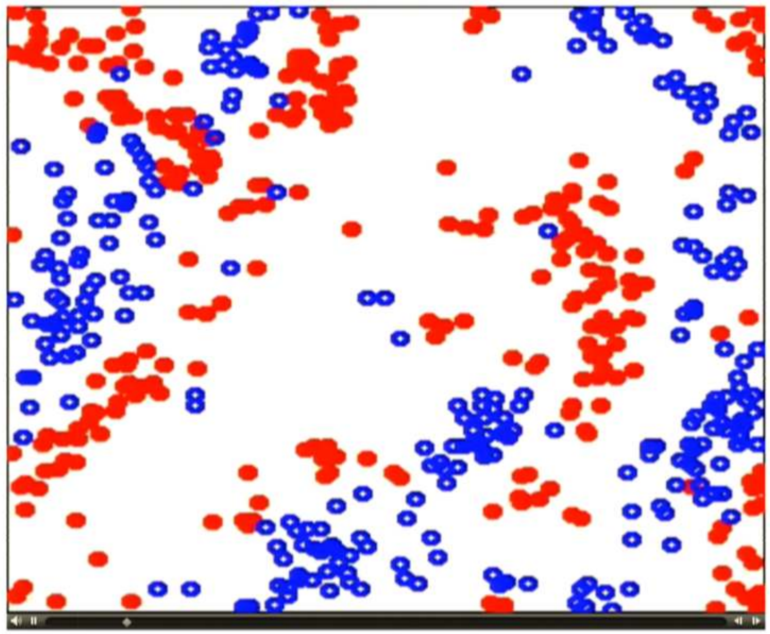
vortices leave imaging region



- $\Gamma_2 \sim T^2/\mu L^2$. *experimental determination of* η!

Tuning the Cascades

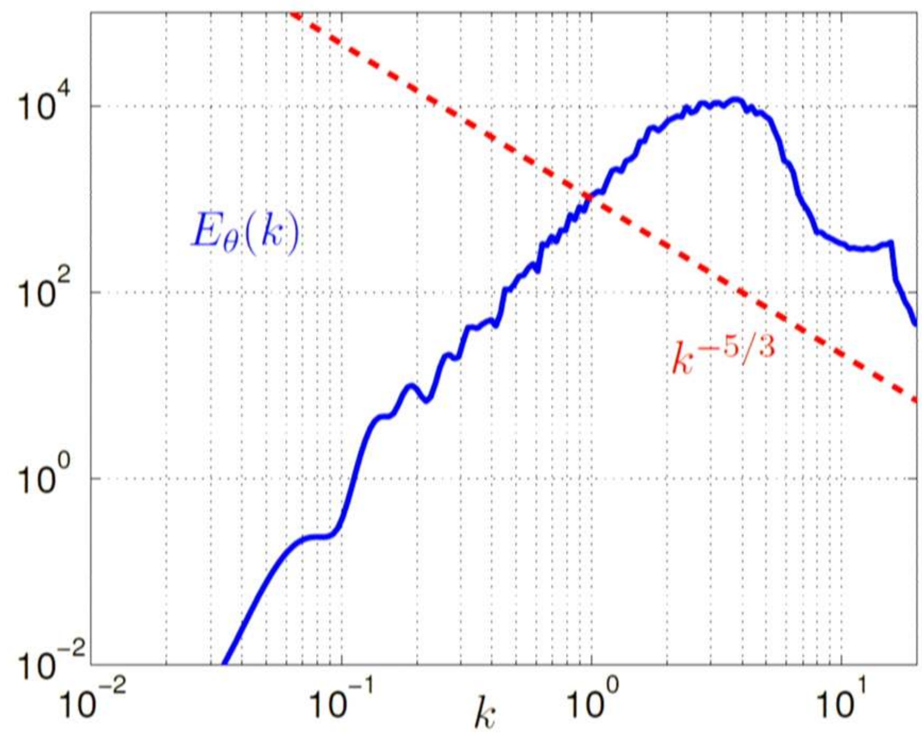
for $\eta > 0$ very small, long lived inverse cascade in numerics:



inverse cascade to length scale $l \sim r_*/\eta$?

Kolmogorov Scaling

inverse cascade: no $k^{-5/3}$ as there is no driving?



Conclusions

What We Know

- there is a chaotic regime of SF vortex dynamics governed by our EFT
- second-order HVI equations not valid for multiple vortices.
- a natural mechanism for the $T > 0$ direct cascade is vortex drag
- consistent with experiment if $\eta \sim T^2$.
- EFT much easier numerics than fluid PDEs.

What We Don't Know

- to what extent are classical/SF turbulence similar phenomena?
- does particle-vortex duality provide a simpler route to $T = 0$ EFT?
- is the scaling of vortex drag coefficient η with parameters like T, μ (somewhat) universal?
- can our theory quantitatively reconcile direct and inverse cascades of previous numerics? transitions btwn regimes?