

Title: Cosmological Dynamics of Space As a Quantum Gravity Condensate

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Abstract:

# Cosmological Dynamics of Space as a Quantum Gravity Condensate

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Quantum Gravity Afternoons at PI

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SG, D. Oriti, L. Sindoni: 1303.3576 (Phys. Rev. Lett. **111** (2013) 031301),  
1311.1238 (JHEP to appear),  
SG: 1404.2944 & work in progress

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## Introduction and Motivation

It has been a major challenge for various contenders for the theory of quantum gravity (e.g. string theory, loop quantum gravity, causal dynamical triangulations) to connect to early universe cosmology. Here we focus on *loop quantum gravity*, whose dynamics can be defined in terms of a *spin foam model* (defining the sum-over-histories / partition function), or in terms of a standard quantum field theory known as **group field theory** (GFT).

We focus on the GFT formulation of loop quantum gravity in order to use tools and intuition from similar situations in quantum field theory, in particular the use of coherent states and the physics of **Bose-Einstein condensates**.

**Key idea:** Describe a macroscopic, spatially homogeneous universe as a collection of many quantum geometric degrees of freedom (*“atoms of space”*) in a (generalised) *coherent state*, which form a *condensate*.  
(Compare with Bose-Einstein condensation or macroscopic e.m. fields in QED.)

## How to do Quantum Cosmology

Quantum gravity (GFT) model

*Ansatz for state  $|\Psi\rangle$*   $\Downarrow$

Quantum dynamics for  $|\Psi\rangle$   
(e.g. Schwinger-Dyson equations)

*Approximation scheme*  $\Downarrow$

Effective cosmological dynamics

$$\hat{\mathcal{D}}\Psi(g_I) + \text{nonlinearities} = 0$$

Classical theory, e.g. GR +  $V(\phi)$

*Ansatz (for metric,  $\phi$ )*  $\Downarrow$

Classical cosmological dynamics  
(Friedmann equation + pert.)

*"Quantisation" scheme*  $\Downarrow$

Quantum cosmology model

$$\hat{\mathcal{H}}\psi(a, \phi, \dots) = 0$$

$\Leftrightarrow$   
*interpretation*

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*compare!*

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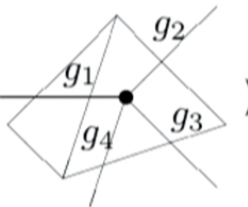
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## Quantum Gravity Condensates

The GFT/LQG vacuum describes a completely degenerate geometry (*i.e.* metric). Excitations over this vacuum are elementary tetrahedra with geometric data:

$$\hat{\varphi}^\dagger(g_1, g_2, g_3, g_4)|\emptyset\rangle = \left| \begin{array}{c} \text{tetrahedron with edges } g_1, g_2, g_3, g_4 \end{array} \right\rangle$$


The interpretation of  $g_I$  is as parallel transports of the gravitational connection,  $g_I \sim \mathcal{P} \exp \int A$ . The dual (“momentum”) variables represent a (discrete) metric.

This tetrahedron is dual to a 4-valent vertex of a loop quantum gravity spin network. Arbitrary (4-valent) spin network states can be viewed as many-particle states in the GFT Fock space; in this sense GFT is a *second quantisation* of LQG [Orti 2013]. The states are not yet physical (do not solve the dynamics).

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## Extracting (Quantum) Cosmology

The interpretation of the quantum dynamics as equations for the “wavefunction”  $\xi(g_I)$  is most straightforward for *linear* differential equations. For the “gas of molecules” introduced above, with (mild) assumptions on the potential of the GFT model, we do indeed obtain a linear equation of the form

$$\int (dg')^4 \hat{\mathcal{K}}(g_I, g'_I) \xi(g'_I) = 0,$$

where  $\mathcal{K}(g_I, g'_I)$  is the kinetic operator of the GFT model. For a local kinetic term, this becomes

$$\hat{\mathcal{K}} \xi(g_I) = 0$$

which now defines a standard quantum cosmology model. Note that this is derived directly from the fundamental dynamics of the quantum gravity (*i.e.* GFT / loop quantum gravity) degrees of freedom.

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## Summary & Outlook

- Spatially homogeneous universes can be described in background-independent quantum gravity in terms of a *condensate* of “atoms” / “molecules” of space.
- The *group field theory* formulation of loop quantum gravity can be used to treat these states with methods similar to those used in Bose-Einstein condensation. An effective dynamics for the “condensate wavefunction” can be computed and interpreted as a quantum cosmology model.
- The semiclassical limit of this effective dynamics, for a specific choice of condensate state, corresponds to a Friedmann equation.
- **Open issues:** *cosmological* (treatment of inhomogeneities, examination of regime in which the approximations used are valid: what happens near Big Bang?, explicit solutions) and *theoretical* (e.g. comparison with other possible LQG vacua [Bianca's talk], analogy with phase transition in BEC)