

Title: Lorentzian non-locality and off-shell modes

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URL: <http://pirsa.org/14050132>

Abstract:

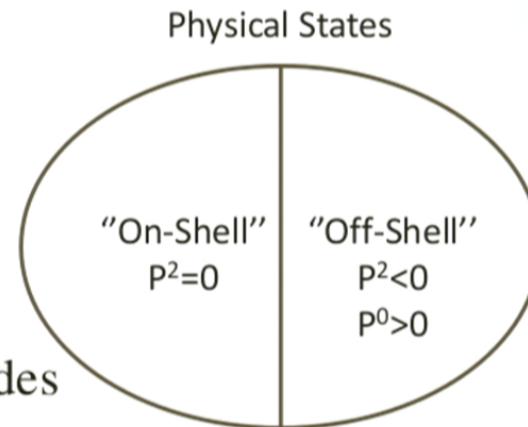
## Summary: Motivations

- Lorentz transformation is a very well tested symmetry of nature.
- Combining LI and LQFT results in a very successful framework in physics.
- Focus only on LI in the context of field theory → non-local evolution
- Causal Lorentzian non-locality
  - New phenomena
  - Constraining non-locality scale
- Fundamental or effective non-locality
- Inspired by Causal set (but not restricted to)
- Attention: field theory in continuum flat space-time (-+++) with modified evolution law

## Summary: Results

Modifying a massless **scalar** field evolution (can also be extended to massive scalar field)

1. Continuum of particles
2. Modification to scattering amplitudes
  - Feynman Green's function
  - Amplitudes assigned to initial or final states
3.  $\sigma(p_1 \dots p_n) \neq 0$  iff  $p_i^2=0$  for  $i=1,\dots,n$



# Outline

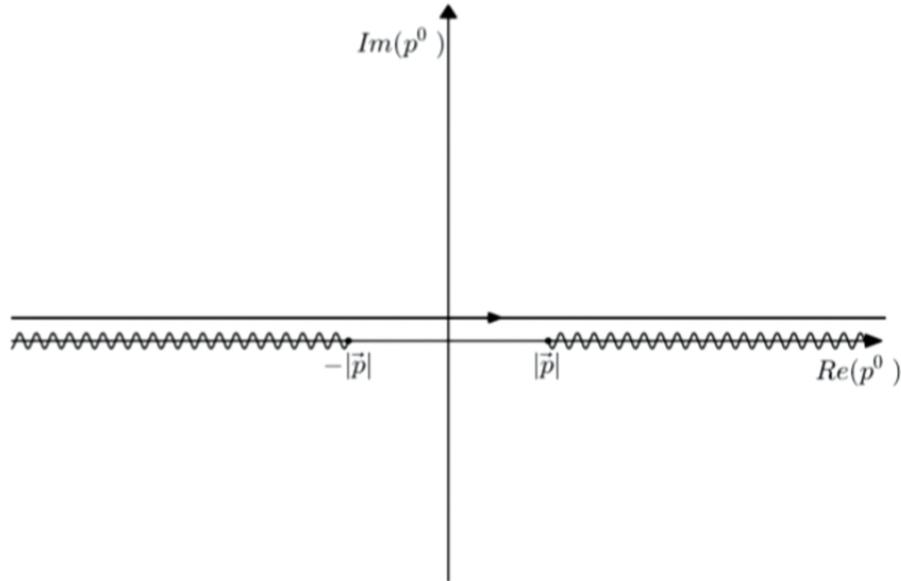
- Summary: motivations and results
- Causal Lorentzian Evolution
- Quantization
- Concluding Remarks

## Causal Lorentzian Evolution

- Massless scalar field  $\square\phi = J(x)$
- The new evolution is obtained by  $\square \rightarrow \tilde{\square}$   
 $\tilde{\square}\phi = J(x)$
- $\tilde{\square}$  required to be:
  1. Linear
  2. Real
  3. Poincare invariant
  4. Local at low energies:  $\tilde{\square}$  is in essence a modification of  $\square$  at high energies.
  5. Stable
  6. Causal (=retarded)
- 1+2+3:  $\tilde{\square}e^{ip\cdot x} = B(p)e^{ip\cdot x}$      $B(p) = B(sgn(p^0), p \cdot p)$   
 $B(-sgn(p^0), p_\mu p^\mu) = B^*(sgn(p^0), p_\mu p^\mu)$
- 4:  $B(p) \xrightarrow{p \cdot p \rightarrow 0} -p \cdot p$

# Causal Lorentzian Evolution

Stability and Causality:



For example:  $G^R(x, y) = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{B(p)} e^{ip \cdot (x-y)}$

# Quantization

The goal is to find a quantum theory which in classical limit reduces to EOM  $\tilde{\square}\phi = J(x)$

Method I: SJ Quantization

- Only needs the retarded Green's function

Method II: Double path integral

- Gives a retarded EOM in the classical limit

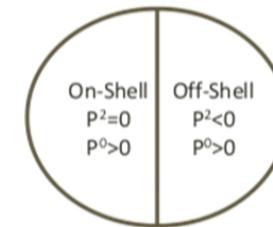
Note: Feynman path integral does not work.

# Physical States

$$\langle \hat{\phi}(x)\hat{\phi}(y) \rangle = \int \frac{d^4 p}{(2\pi)^4} \tilde{W}(p) e^{ip \cdot (x-y)}$$

$$\tilde{W}(p) > 0 \quad \text{iff} \quad p^2 \leq 0 \quad \& \quad p^0 > 0$$

- There is a continuum of massive particles



$$\tilde{W}(p) = 2\pi\delta(p^2)\theta(p^0) + \text{finite terms}$$



originating from the pole

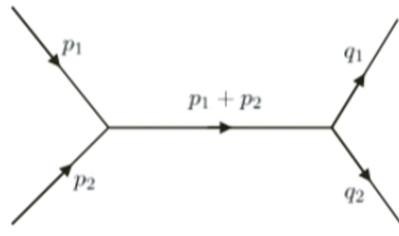


Originating from the cut

# Interacting Theory

Feynman rules with modified Feynman Green's(?) function and amplitudes for external legs

For example  $\frac{\lambda}{3!} \phi^3$ :



Note: p&q are 4-momenta not 3-momenta

$$S_{q_1 q_2, p_1 p_2} = \frac{-i\lambda^2}{(2\pi)^8} \sqrt{\tilde{W}(p_1)\tilde{W}(p_2)\tilde{W}(q_1)\tilde{W}(q_2)} \left[ \tilde{G}^F(p_1 + p_2) + \tilde{G}^F(p_1 - q_1) + \tilde{G}^F(p_1 - q_2) \right] \delta^{(4)}(\sum p - \sum q)$$

$$\tilde{G}^F(p) = \frac{\theta(p^0)}{B(p)} + \frac{\theta(-p^0)}{B^*(p)}$$

## Concluding Remarks

- The resulting theory is unitary with finite total cross section.
- Causal Lorentzian evolution results into a continuum of massive particles. **Causality** (retardedness) is crucial.
- Implementing non-locality in the kinetic term rather than interaction
- $\sigma(p_1 p_2) \neq 0$  iff  $p_i^2 = 0$  for  $i=1,2$

There is no way of detecting off-shell modes through scattering  
→ candidate for dark matter

- In a concrete model, non-locality scale can be constrained using the abundance of dark matter.
- Extension to massive scalar field (✓) and other fields (✗)
- Predictions:
  - no detection of dark matter
  - missing energy in scattering experiments

$$\left( \frac{E_{cm}}{E_{nl}} \right)^{2n}, \quad n = 1, 2, \dots$$

$$\tilde{\square} \phi = \alpha \phi + \int dy f$$

$\bar{J}(x)$

