Title: Propagation of particles in discrete spacetime: Can high cosmic rays be a result of Quantum Gravity?

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Abstract:

Introduction

Causal set theory is based on the idea that causal structure is the fundamental structure of spacetime. Theorems by Hawking, Malament and Levichev in continuum causal analysis tell us that

$$\text{Causal structure} + \text{Volume} = \text{Geometry.}$$

Causal Set Theory

A causal set C is a partially ordered set with a binary relation $\prec$ satisfying the following properties:

(i) Irreflexivity: $x \not\prec x, \forall x \in C$;

(ii) Transitivity: If $x \prec y$ and $y \prec z$ then $x \prec z$, $\forall x, y, z \in C$;

(iii) Local finiteness: $\forall x, z \in C$ the set $\{y | x \prec y \prec z\}$ is finite.

$$\text{Order} \Leftrightarrow \text{Causal Structure}$$

$$\text{Number} \Leftrightarrow \text{Volume}$$

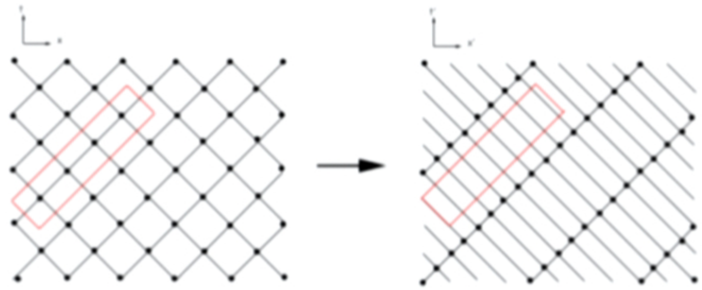
Leads to

$$\text{Order} + \text{Number} \approx \text{Geometry}$$

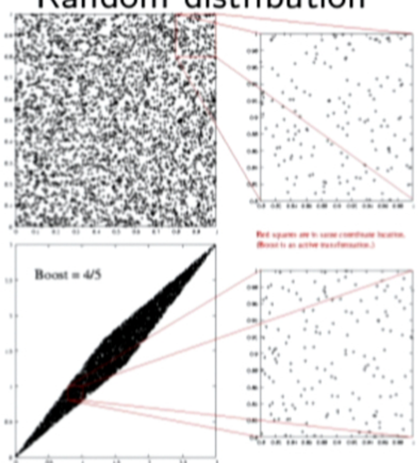
Causal Set Theory

Discrete spacetime that is Lorentz invariant.

Regular lattice



Random distribution[1]



[1] arXiv:gr-gc/0311055v3

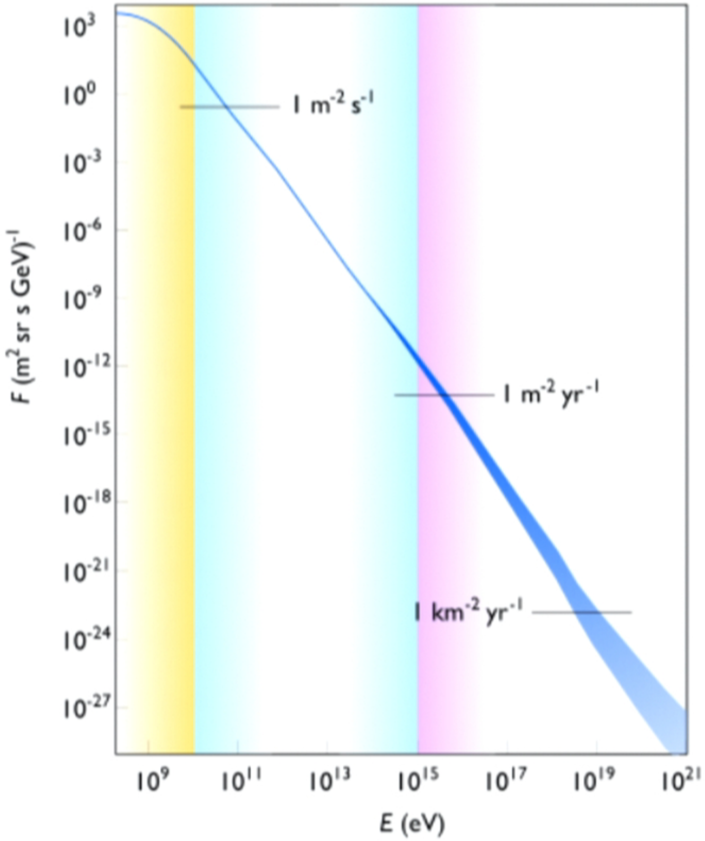
Causal Set Theory

Some predictions of causal set theory:

- Local Lorentz invariance
- Particles will swerve slightly

Propagation of particles in discrete spacetime.

Can swerves explain high energy cosmic rays?[2]



[2] S. Swordy, The energy spectra and anisotropies of cosmic rays, 2001, Space Science Reviews 99, pp85-94.

Outline

- Introduction
  - Causal set review
- Propagation of particles in causal sets
  - Models of propagation of massive particles
    - Simulations
    - Constraints on diffusion constant
- Conclusion

Propagation of massive particles

Any process that undergoes stochastic evolution and is Lorentz- and translation invariant at macroscopic scale can be approximated by a diffusion equation[3] .

The diffusion is on a phase space $\mathbb{H}^3 \times \mathbb{M}^4$, where $\mathbb{H}^3$ is the mass shell (Lobachevsky space) and $\mathbb{M}^4$ is Minkowski spacetime. In terms of proper time

$$\frac{\partial \rho}{\partial \tau} = k\nabla^2 \rho - \frac{1}{m} p^\mu \frac{\partial}{\partial x^\mu} \rho$$

where $\rho \equiv \rho(p^\mu, x^\mu; \tau)$ is a scalar distribution on $\mathbb{H}^3 \times \mathbb{M}^4$, ∇^2 is the Laplacian on $\mathbb{H}^3$, m is the mass of the particle and k is the diffusion constant. In terms of cosmic time

$$\frac{\partial \rho}{\partial t} = k \frac{\partial}{\partial p} \left(\frac{\sqrt{m^2 + p^2}}{m} \frac{\partial \rho}{\partial p} \right) - \frac{p}{\sqrt{m^2 + p^2}} \frac{\partial \rho}{\partial x}$$

where ρ is the probability distribution on $\mathbb{H}^3 \times \mathbb{M}^4$, p is three-momentum.

[3] Proved by Philpott, Dowker and Sorkin, assuming trajectories continuous in momentum

Models

- Distance between two causal elements x and y, $d(x, y)$, is the longest chain between x and y.

$$d(x, y) = 4$$

Models

- Distance between two causal elements x and y, $d(x, y)$, is the longest chain between x and y.
- For Trajectory to be close to geodesic, the particle will have forgetting number n_f, $n_f \gg 1$ so that swerve is minimal.
- $n_f \ll n_{macro}$ process is approximately Markovian $(1 \ll n_f \ll n_{macro},)$.
- Two models intrinsic models and one partially extrinsic model.

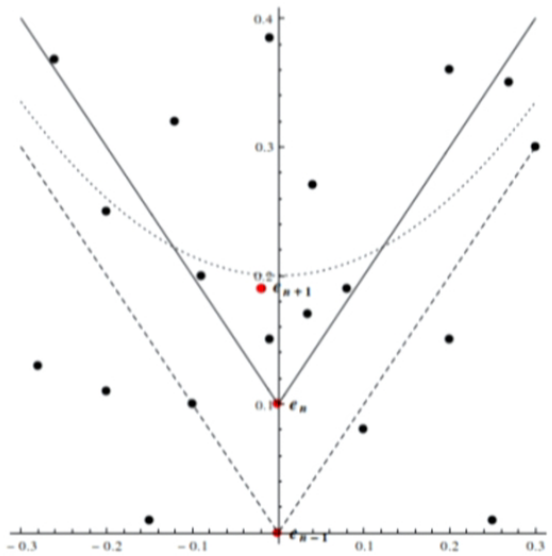
Model 1a (Intrinsic)

Given a segment of the path of the particle $e_1, \ldots, e_{n-1}, e_n$, the next element e_{n+1} is selected using the following rule.

- $d(e_n, e_{n+1}) = 1$,
- $d(e_{n-nf}, e_{n+1}) + \ldots + d(e_{n-1}, e_{n+1}) + d(e_n, e_{n+1})$ is minimized

Model 1b (Intrinsic)

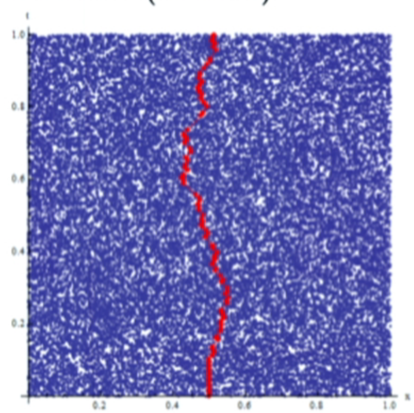
Given a segment of the path of the particle $e_1, \ldots, e_{n-1}, e_n$, the next element e_{n+1} is selected using the following rule.



- It is in the causal future of e_n ,
- $d(e_{n-1}, e_{n+1}) \leq 2n_f$,
- $d(e_n, e_{n+1}) \leq n_f$,
- $Max[d(e_{n-1}, e_{n+1})]$

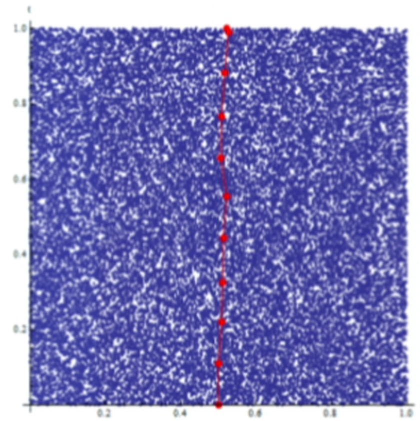
Simulations N=32 768 in 2D

Model 1a (Intrinsic)



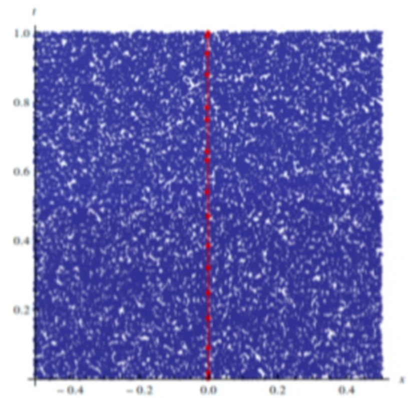
$n_f = 20$

Model 1b (Intrinsic)



$n_f = 20$

Model 2 (Partially Extrinsic)

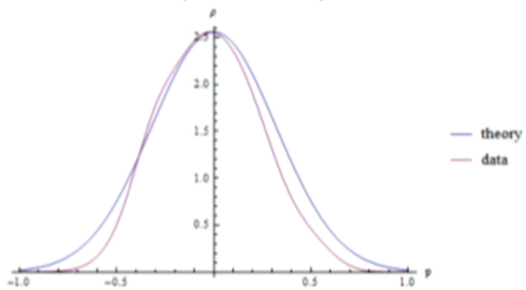


$\tau_f = 0.11$

Simulations N=32 768 in 2D

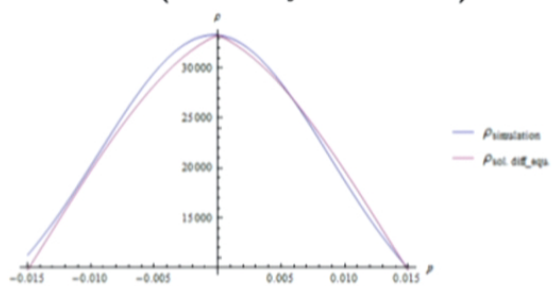
Fitting data from simulations to diffusion equation to getting diffusion constant k.

Model 1b (Intrinsic)



$n_f = 20$

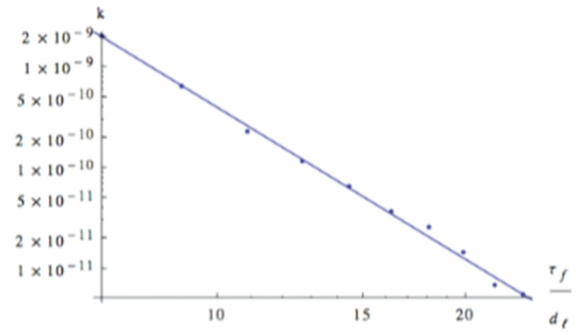
Model 2 (Partially Extrinsic)



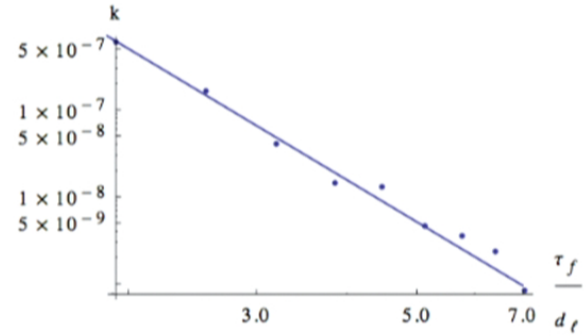
$\tau_f = 0.04$

Simulations

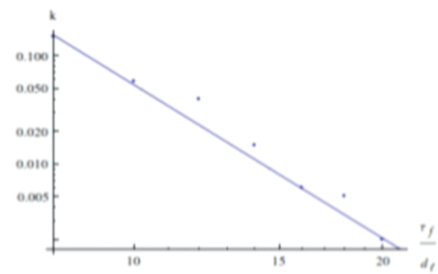
Dependency on forgetting time:



Model 2 (Partially Extrinsic) 2D, $4.8 \times 10^{-3}x^{-5.01}$



Model 2 (Partially Extrinsic) 3D, $1.33 \times 10^{-3}x^{-4.92}$



Model 1b (Intrinsic)2D $2708.46x^{-4.709}$

These results show that in both 2 and 3 dimensions

$$k \sim \frac{m^2 d_{pl}^4}{\tau_f^5}.$$

Bound on k

Bound on the diffusion constant k:

Experiment	bound on k	τ_f (in units of ℓ_p)
Laboratory hydrogen[4]	$k \lesssim 10^{-44} GeV^3$	2×10^{10}
NH_3 in Edge cloud 2[5]	$k \lesssim 10^{-41} GeV^3$	6×10^{9}
Cosmic neutrino[5]	$k \lesssim 10^{-61} GeV^3$	6×10^{13}

Assuming that in 4D the diffusion constant is given by

$$k \sim \frac{m^2 d_\ell^4}{\tau_f^5},$$

from these bounds, we can calculate the forgetting time.

[4] arXiv:0311055v3
[5] arXiv:10.1103/PhysRevD.74.106001

Bound on k

Bound on the diffusion constant k:

Experiment	bound on k	τ_f (in units of ℓ_p)
Laboratory hydrogen[6]	$k \lesssim 10^{-44} GeV^3$	2×10^{10}
NH_3 in Edge cloud 2[7]	$k \lesssim 10^{-41} GeV^3$	6×10^{9}
Cosmic neutrino[5]	$k \lesssim 10^{-61} GeV^3$	6×10^{13}

Can swerves explain high energy cosmic rays?
Unfortunately with these bounds on k, no. Maybe more sophisticated models of swerves can:

- k depends on temperature,
- k depends on density of particles,
- Consider quantum particles.

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[6] arXiv:0311055v3
[7] arXiv:10.1103/PhysRevD.74.106001

Conclusion

- Causal set theory predicts particles will swerve and the phenomenological diffusion constants have now been tightly bounded.
- The simple model of swerving unfortunately can not explain the high energy cosmic rays, but maybe a more sophisticated model could.
- Simulations indicate that the diffusion constant scales like $k \sim \frac{m^2 d_\ell^4}{\tau_f^5}$ in $2D$ and $3D$.

Thank You