Title: Projective Statistics in Quantum Gravity

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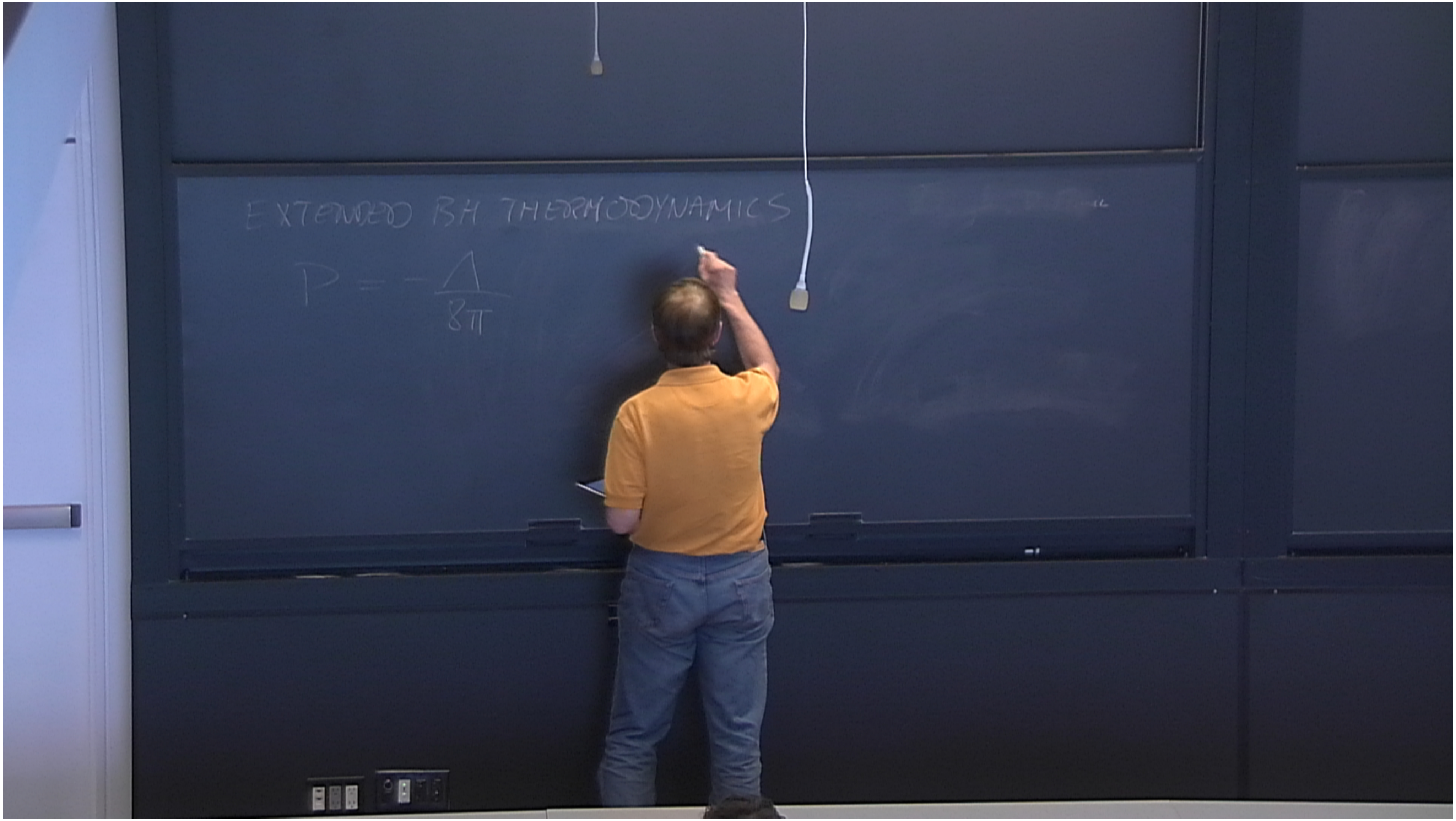
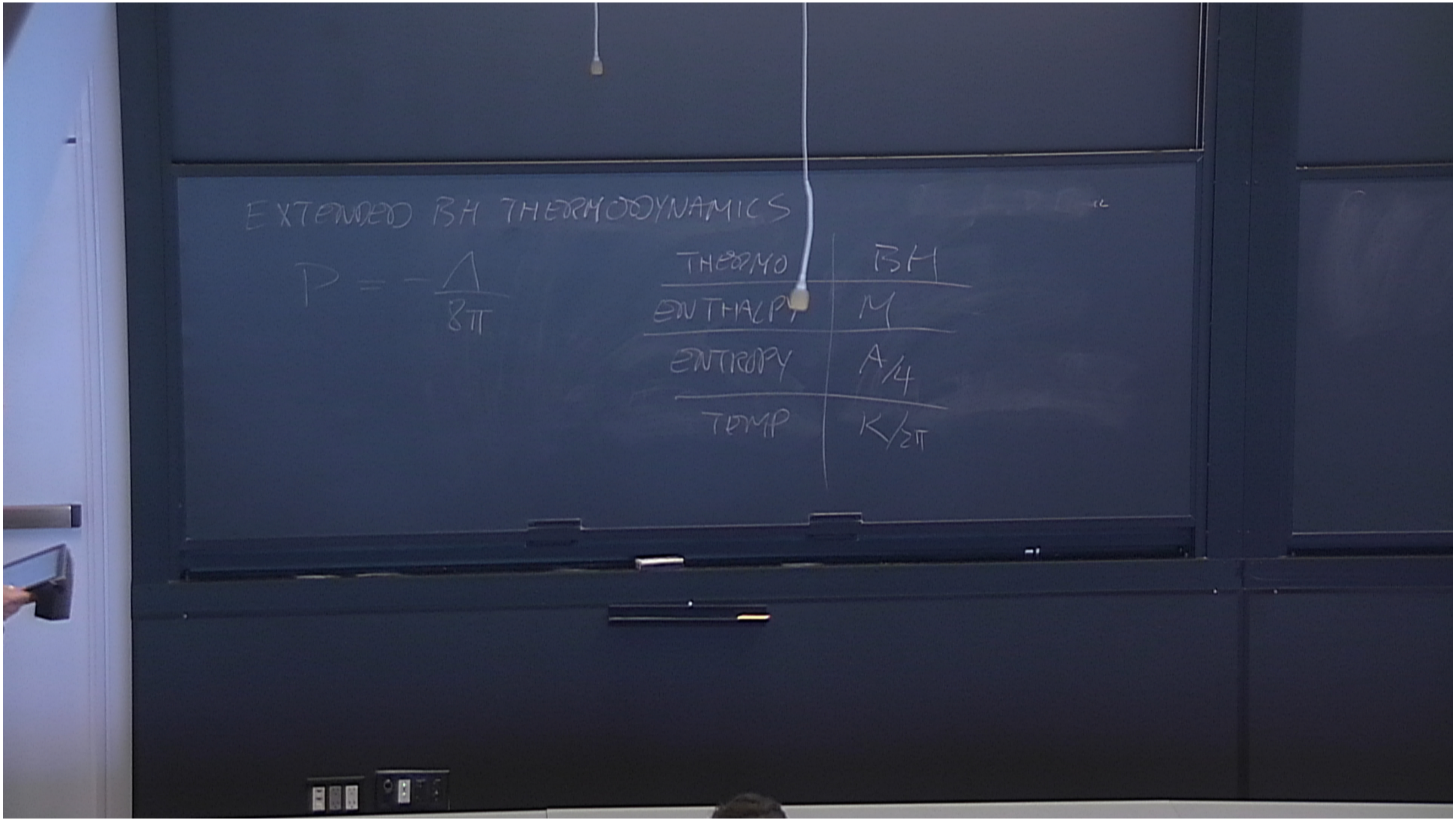
Abstract: To the extent that spacetime remains a manifold M on small scales, excitations of the spatial topology can function as particles called topological geons. In a first quantized theory of topological geons (aka continuum quantum gravity without topology change), different irreducible unitary representations of the mapping-class group G of M, yield different superselection sectors of the theory. In some of these sectors the geons behave as fermions, even though gravitons themselves are of course bosons. A still more exotic possibility is "projective statistics", where the operators that permute identical geons only preserve group multiplication up to a phase-factor that cannot be eliminated. (Such a representation must be nonabelian.) I will describe a simple example of this phenomenon with four RP^3 geons.


EXTENDED BH THERMODYNAMICS
P = - Λ / 8π


EXTENDED BH THERMODYNAMICS
P = −Λ/8π
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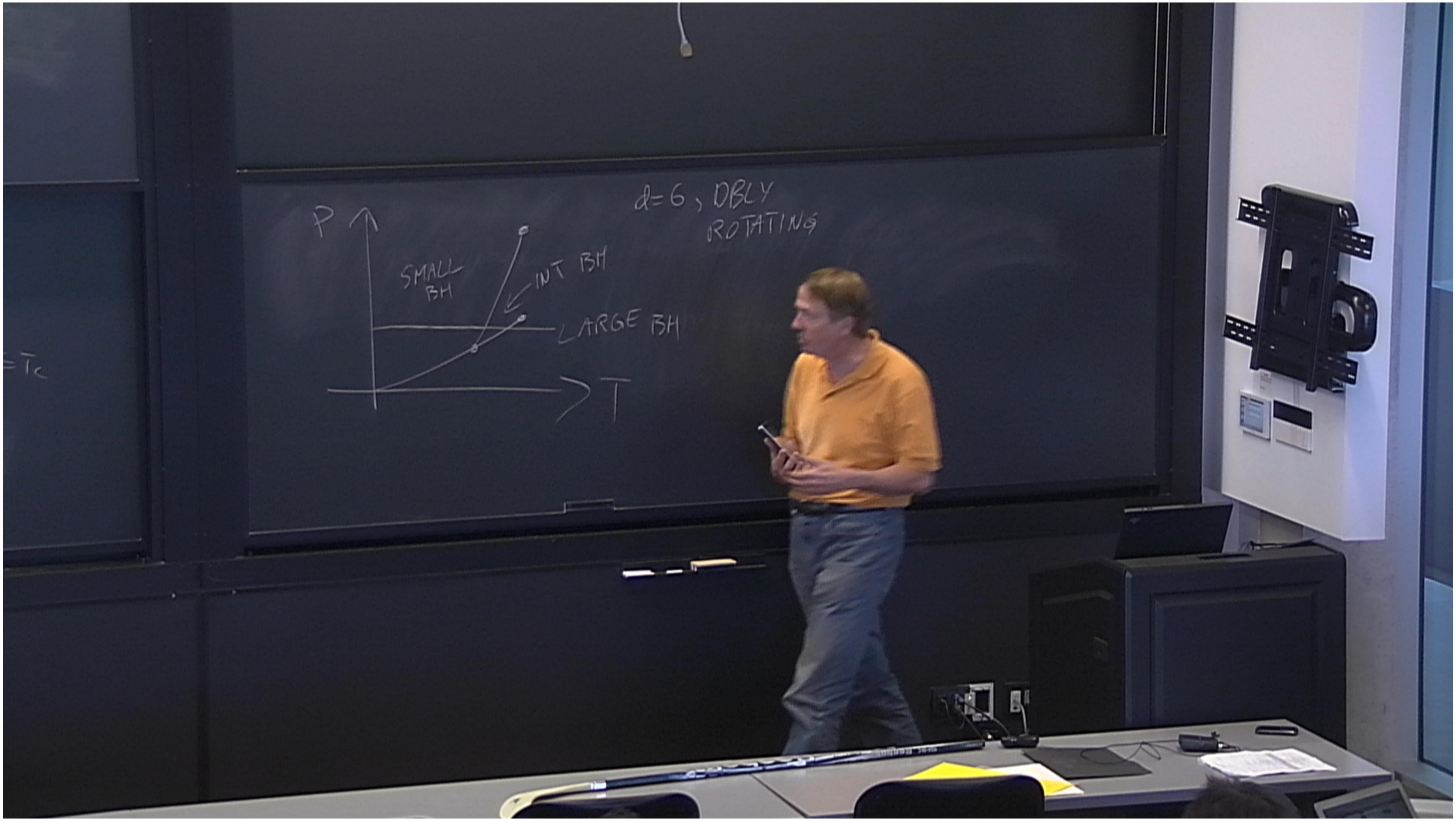

d=6, DBLY
ROTATING
P
T
SMALL
BH
INT BH
LARGE BH

Work with

Sumati Surya, Fay Dowker, John Friedman

References

Spin-1/2 from Gravity
PRL **44** 1100-1103 (1980); and **45** 148(1980)

Particle-Statistics in Three Dimensions
PRD **27** 1787-1797(1983)

An Analysis of the Representations of the Mapping Class Group of a Multi-geon Three-manifold
Int. J. Mod. Phys. A **13** 3749-3790 (1998)
http://www.pitp.ca/personal/rsorkin/some.papers/90.mcg.pdf

What is a projective representation (PUIR)?

Familiar example of spin-1/2

There is a phase that you cannot eliminate $e^{2\pi i J_z} = -1$

Proof If $e^{i\theta J_x} \to e^{i\theta J_x} e^{i\psi}$ then

$$J_x \to J_x + \psi \qquad (\psi \in \mathbb{R})$$

But then $J_z = -i[J_x, J_y] \to$ self

Thus you cannot change that $e^{2\pi i J_z} = -1$, or more precisely that

$$\exp(2\pi[J_x, J_y]) = -1$$

Can also see this topologically — paths in U1 from $u \to u^{-1}$

The permutation group S_n also has projective representations

Compare parastatistics

picture of young tableau for n=3

This is all there is for $n = 1, 2, 3$

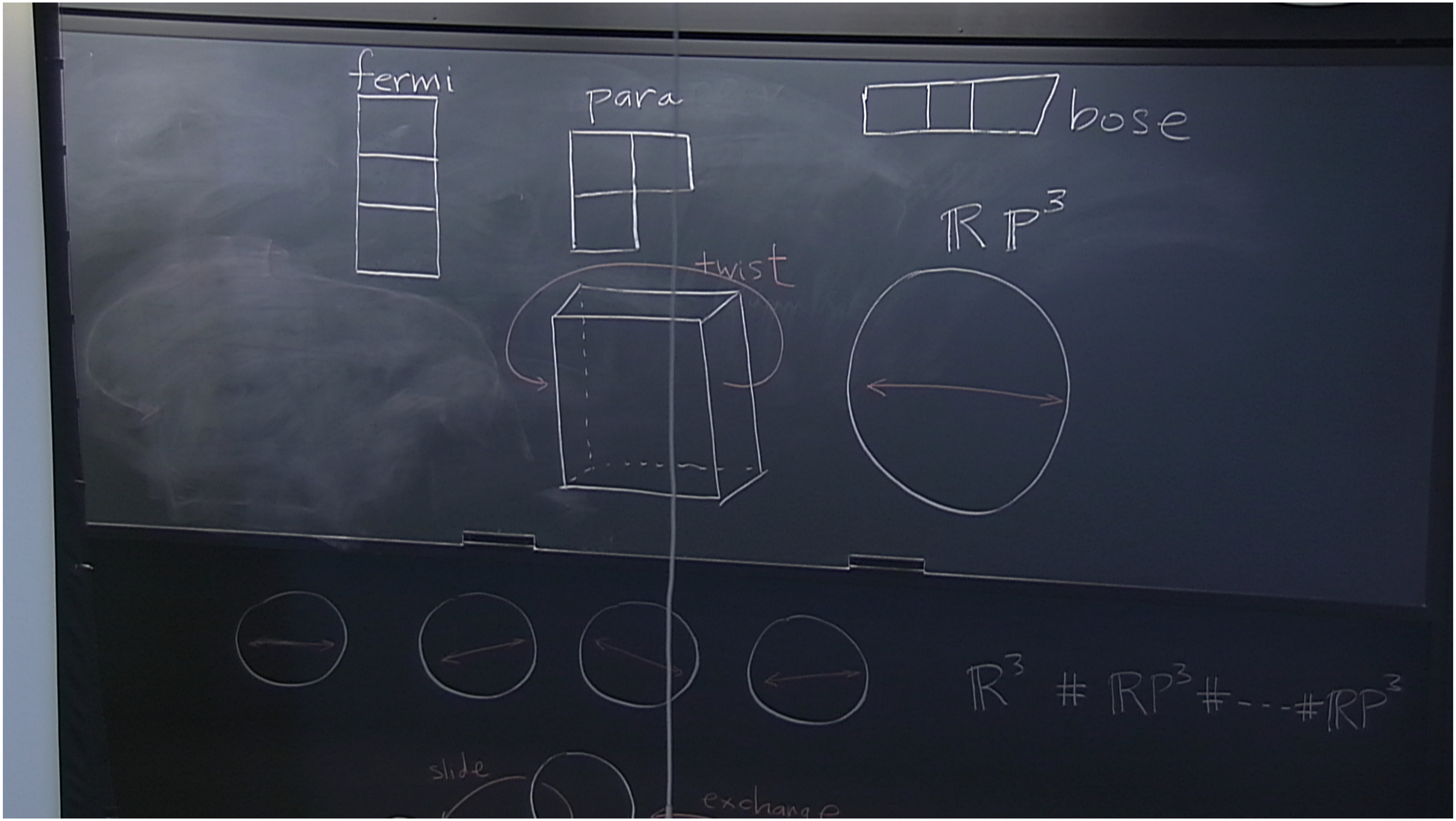
But for $n = 4$ something new happens

$$E_{12}\, E_{34} = -E_{34}\, E_{12}$$

(This is the sure Schur sign: $E_{12}\, E_{34}\, E_{12}^{-1}\, E_{34}^{-1} = -1$)

Clearly, rephasing E_{jk} cannot alter this sign

We'll see examples later



fermi
para
bose
$\mathbb{R}P^3$
twist
$\mathbb{R}^3 \# \mathbb{R}P^3 \# \cdots \# \mathbb{R}P^3$
slide
exchange

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Prime manifolds and the connected sum $\sharp$

Let spacetime be $^4M = \mathbb{R} \times {}^3M$

The 3-manifold ^3M has a **prime decomposition**

$$^3M \simeq \mathbb{R}^3 \sharp P_1 \sharp P_2 \sharp P_3 \cdots$$

EXAMPLE "Quaternionic space" (remove cube, identify with twist) **PICTURE**

This spacetime admits half-integer spin

EXAMPLE A single $\mathbb{RP}^3$ geon. Just cut out a 3-ball and identify antipodes.

Henceforth we stick to 4 identical geons $\mathbb{R} \sharp \mathbb{RP}^3 \sharp \mathbb{RP}^3 \sharp \mathbb{RP}^3 \sharp \mathbb{RP}^3$

To construct ^3M, cut out four separate balls and identify antipodes on each

PICTURE of spherical voids with identifications

Conjecture This spacetime supports "projective statistics"

Slides and exchanges

Slides and exchanges are nontrivial loops in the gravitational configuration space, $Q = $ 3-metrics / $Diff^{\infty}$. Equivalently they are large diffeos in $Diff^{\infty} / Diff_0^{\infty}$.
(They are also loops in the spacetime sense — interfering histories)

Let $G = \pi_1(Q) = Diff^{\infty} / Diff_0^{\infty} = \pi_0(Diff^{\infty})$

Condensed matter analogy: "smoke-rings" (closed vortex lines)

PICTURE showing rings with slide and exchange

Let Σ be the group of slides and S_4 the group of permutations of the $\mathbb{RP}^3$ geons. Then Σ is a normal subgroup of S_4 whence

$$G \simeq \Sigma \ltimes S_4$$

With four geons, Σ is generated by 12 slides σ_{jk} modulo some simple relations

Consequences for Quantum gravity

$G = \pi_0(\mathit{Diff}^\infty)$ is called the mapping class group of the manifold

Each UIR of $G \longleftrightarrow$ a different sector of the quantum theory

For identical **point**-particles, G consists of permutations and each sector imparts a different statistics — bose, fermi, or para — one type for each Young tableau

But for extended topological objects like geons the slides make life more interesting!

UIR's of semidirect product groups

Familiar example of the Poincaré group

$G =$ translations $\ltimes$ Lorentz

Build a PUIR of G from a UIR of the translations (momentum k^μ) ...

plus a UIR — or PUIR — of $SO(3)$

$SO(3) \subseteq$ Lorentz is the "Little Group"

What's very different in our geon case is

- The little group will be **all** of S_4
- The overall rep of G is **ordinary** but that of the little group can be **projective**

How to build a UIR of G = Slides $\ltimes$ Permutations

- **Start with** a UIR of the slides

$$\Gamma : \Sigma \to L(\mathbb{C}^d)$$

- Assume for simplicity that the resulting little group includes all of the permutations S_4
- Determine the unique PUIR of S_4 in $\mathbb{C}^d$ that permutes the slides correctly
- **Choose freely** a compatible PUIR T of S_4 in some other vector space $\mathbb{C}^k$

The choice of T is a choice of the particle statistics

If R is properly projective then T must be too — **Projective statistics**

A concrete example from Dirac matrices — almost

Let γ^k be hermitian (Euclidean) γ-matrices $\gamma^j\gamma^k + \gamma^k\gamma^j = \delta^{jk}$

Let the slide of geon j through geon k be represented by $\sigma_{jk} = i\,\gamma^j\gamma^k$

The permutations of S_4 act on this group by permuting the labels (indices)

The following exchange-operators implement this action

$$\sqrt{2}\,E_{12} = (\gamma^1 + \gamma^2)\gamma^3\gamma^4 \qquad \&sym$$

You can check for example that $E_{12}\,\gamma^2 = \gamma^1\,E_{12}$ (i.e. $E_{12}^{-1}\,\gamma^1\,E_{12} = \gamma^2$)

The Schur sign is negative $E_{12}\,E_{34} = -E_{34}\,E_{12}$

But there's one sign wrong in the relations for σ_{jk}

- $\gamma^{12} \natural \gamma^{34}$
- $\gamma^{12} \natural \gamma^{13}\gamma^{23}$ but
- $\gamma^{13} \S \gamma^{23}$ (The slides form a PUIR, not a UIR)

Challenge Represent the slides so as to fix the wrong sign