Title: Change of vertex amplitude under coarse graining In Euclidean spinfoam model II

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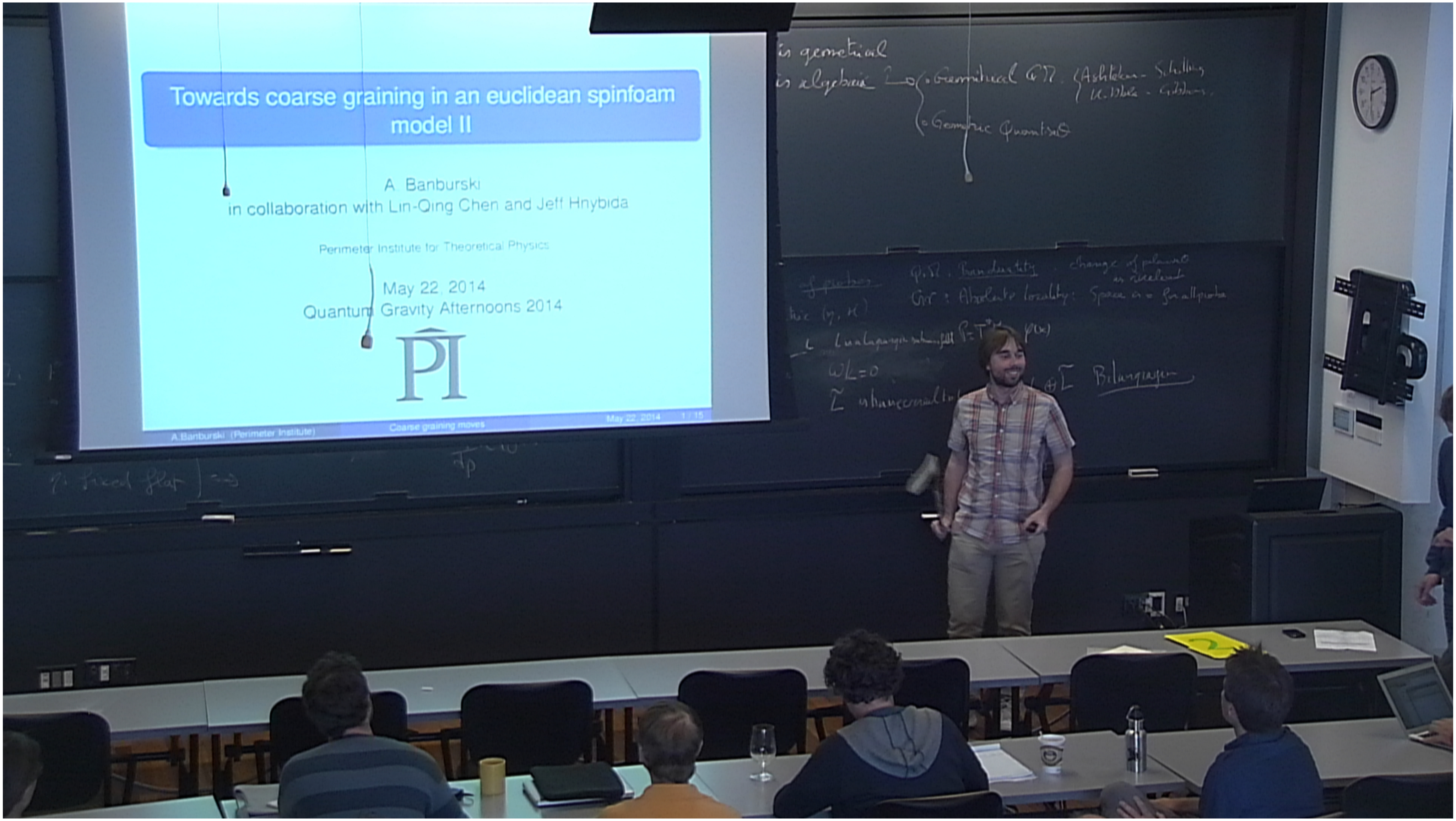
Abstract:


Towards coarse graining in an euclidean spinfoam model II
A. Banburski
in collaboration with Lin-Qing Chen and Jeff Hnybida
Perimeter Institute for Theoretical Physics
May 22, 2014
Quantum Gravity Afternoons 2014
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Coarse graining moves
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In the previous episode...

- In the previous talk you've heard about the Spin(4) holomorphic spin foam model. The partition function is defined by contractions of projectors with simplicity constraints ($[z_i^L|z_j^L\rangle = \rho^2[z_i^R|z_j^R\rangle$):

$$\tilde{P}(\{z_i\},\{w_i\}) = \sum_J {}_2F_1\Big(-J,-J-1;2;\rho^4\Big)\,\frac{\Big(\sum_{i<j}[z_i|z_j\rangle[w_i|w_j\rangle\Big)^J}{J!(J+1)!}$$

which graphically is

Amplitude for a 4-simplex

- A part of the partition function for a 4-simplex is then simply

$$A(\{z\}) = \int \left(\prod_{i=1}^{10} \mathrm{d}\mu(w_i) \right) \tilde{P}_1(\{z\}, \{w\}) \ldots \tilde{P}_5(\{z\}, \{w\})$$

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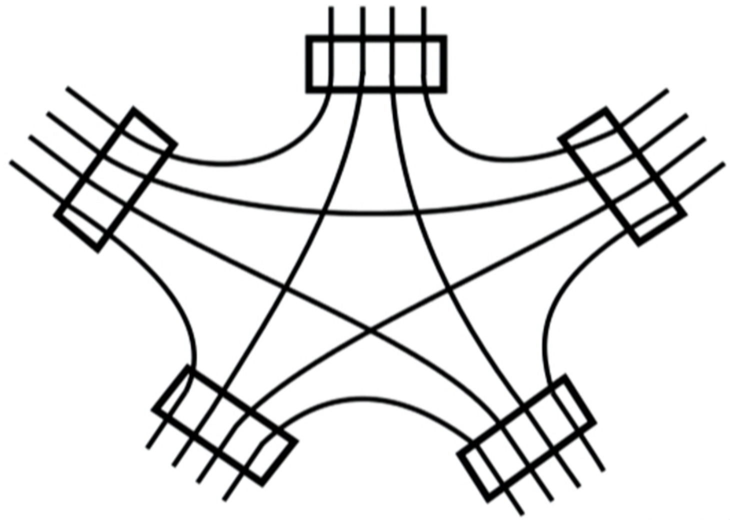

Spinor techniques

- We can perform integrals of this constrained projector by keeping track of homogeneity by a parameter τ. The projector becomes then just

$$e^{\sum_{i<j} \tau [z_i|z_j\rangle [w_i|w_j\rangle}$$

- Upon expanding this in power series, we substitute for each τ^J a factor of ${}_2F_1(-J, -J-1; 2; \rho^4)\,/(J+1)!$ to get back to our constrained projector.

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Spinor techniques II

- For what will follow, it is useful to consider the integral of the constrained projector over one of the spinors, say z_n, with a contraction $w_n = z_n$:

$$\int \mathrm{d}\mu(z_n) e^{\sum_{i<j} \tau [z_i|z_j\rangle [w_i|w_j\rangle} = \frac{e^{\sum_{1\leq i<j<n} \tau [z_i|z_j\rangle [w_i|w_j\rangle}}{\det(\mathbb{1} - \sum_{i\neq n} \tau |w_i\rangle [z_i|)}$$

where $\mathrm{d}\mu(z) = \pi^{-2} e^{-\langle z|z\rangle} \mathrm{d}^4 z$.

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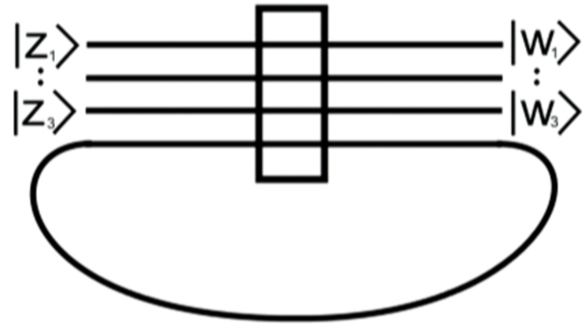
$$\det\left(\mathbb{1} - \sum_i C_i |A_i\rangle [B_i|\right) = 1 - \sum_i C_i [B_i|A_i\rangle + \sum_{i<j} C_i C_j [A_i|A_j\rangle [B_i|B_j\rangle$$

Loop identity

- The integral we computed can be graphically represented as



- In a bigger graph this corresponds to integrating out a face in the interior of a triangulation, but we have to add a face weight of (2j+1) $\Rightarrow$ change measure to $\mathrm{d}\tilde{\mu}(z) = (\langle z|z\rangle - 1)\mathrm{d}\mu(z)$

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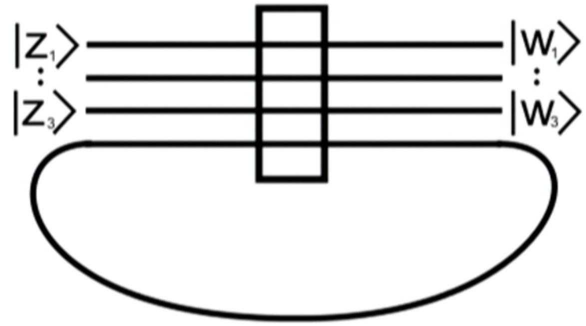

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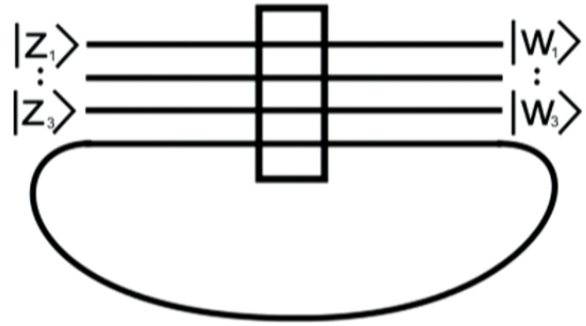

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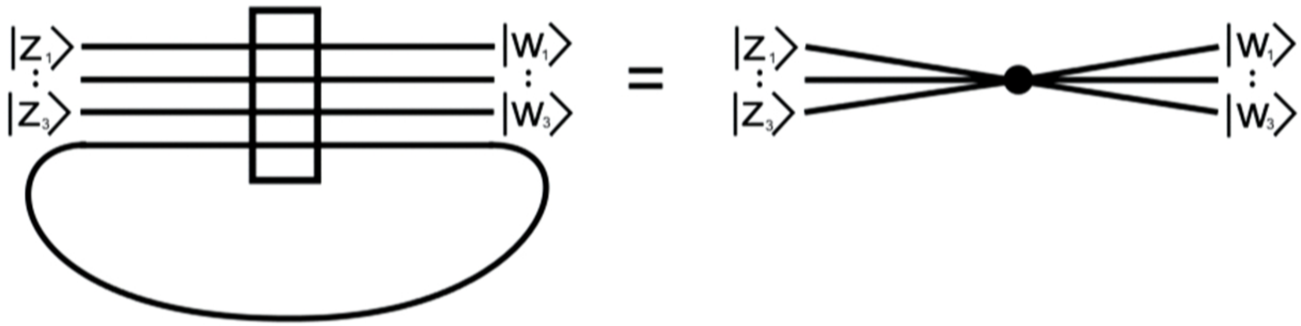

- or more explicitly

$$\sum_{J,J'} \frac{{}_2F_1(-J,-J-1;2;\rho^4)}{(J+1)!} N(J,J') \left(\sum_{i<j<4}[z_i|z_j\rangle[w_i|w_j\rangle\right)^{J-J'} \left(\sum_{i<4}[z_i|w_i\rangle\right)^{J'}$$

with $N(J,J') = \sum_n \frac{n!(2n-J'+1)}{(J-n)!(n-J')!J'!}$

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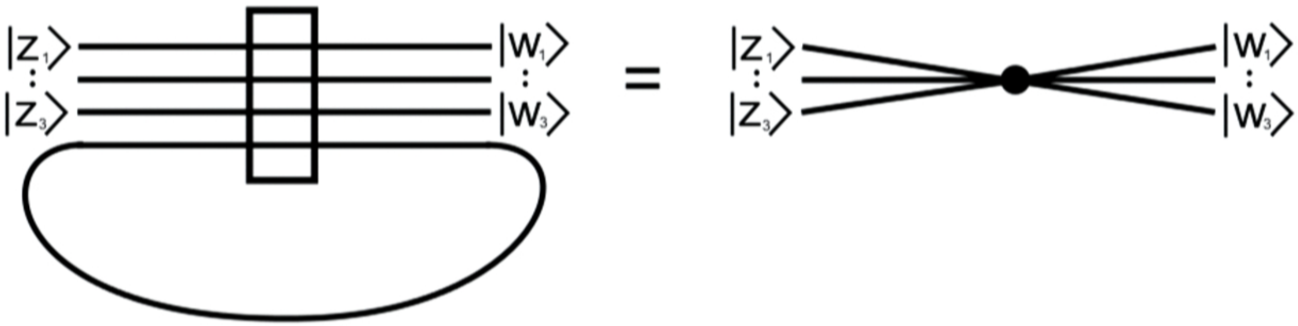

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Pachner moves in a holomorphic spin foam model

- We will now apply the loop identity to calculate Pachner moves. Let us start with 4-2 move:

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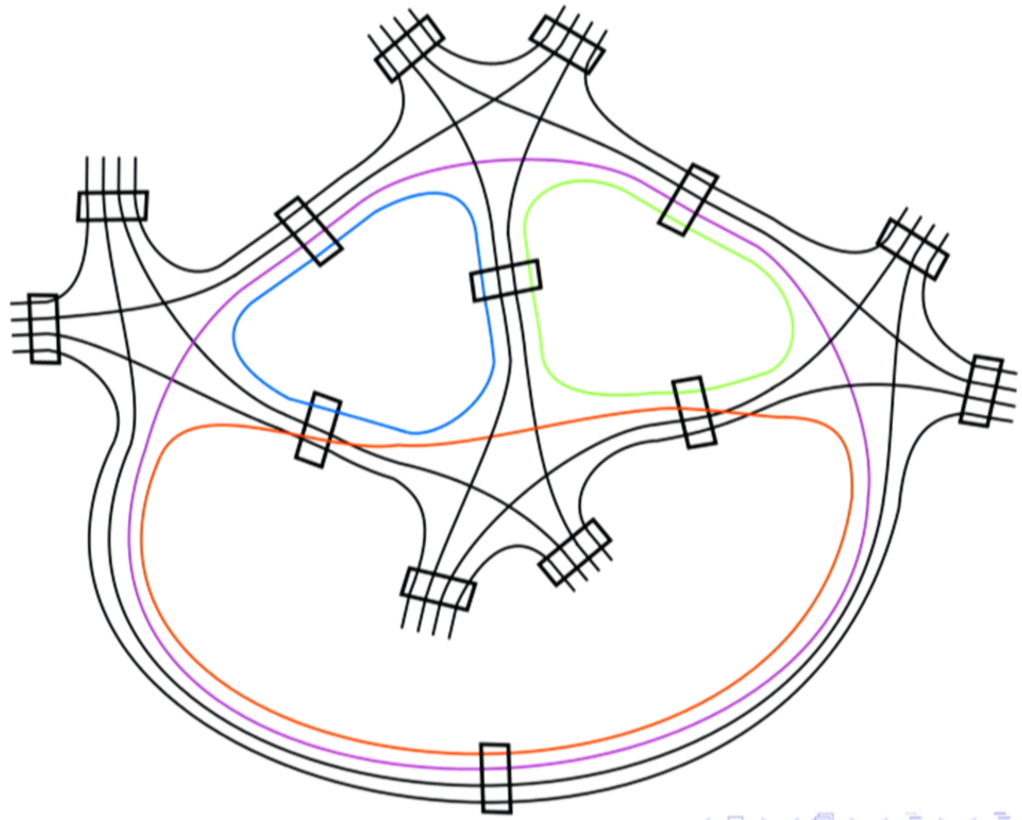

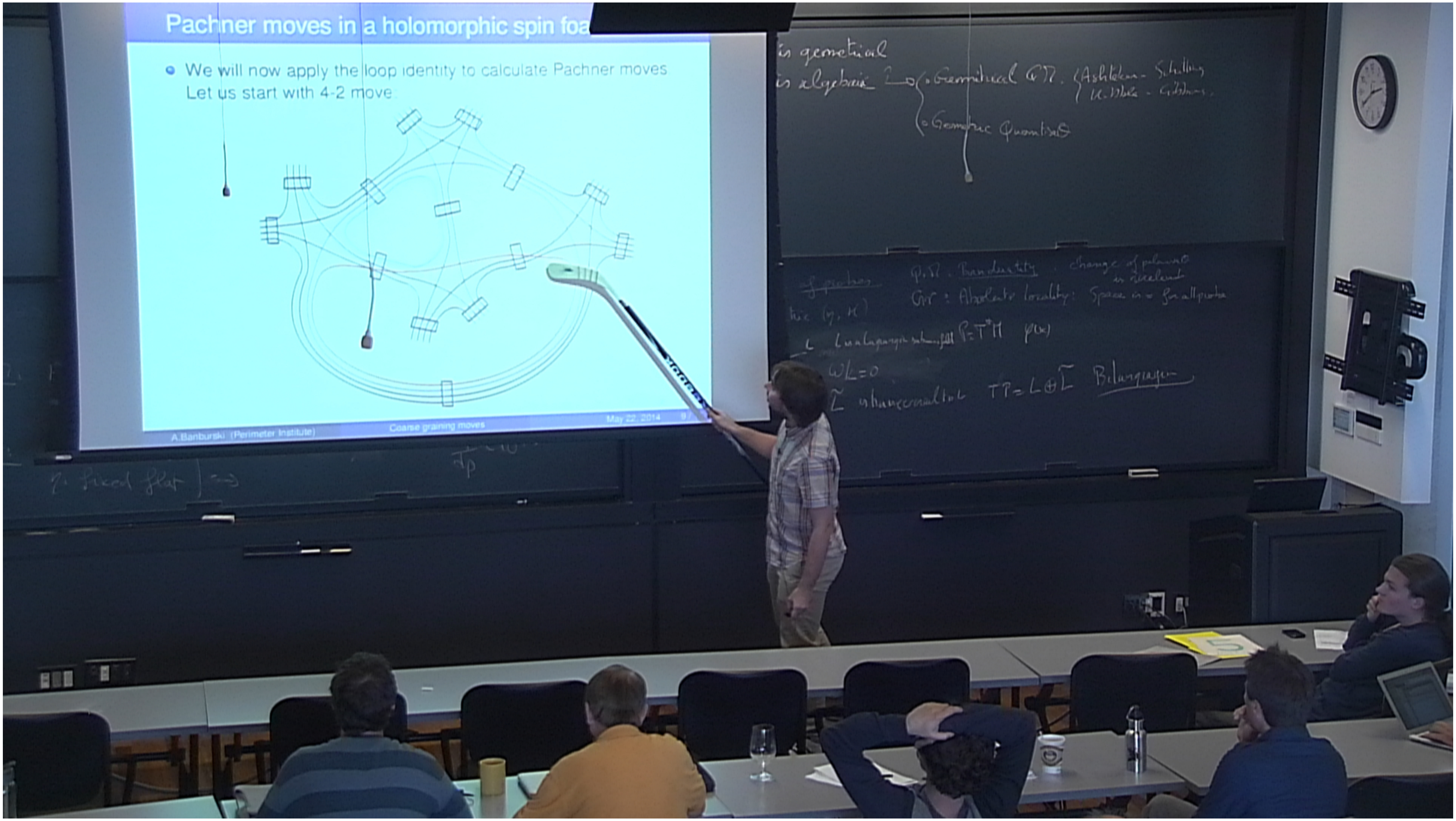
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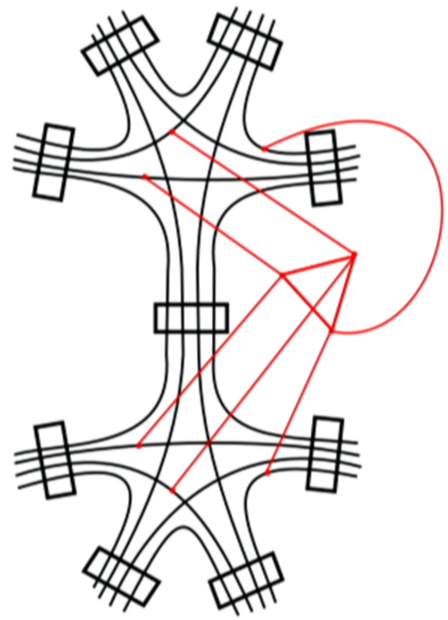
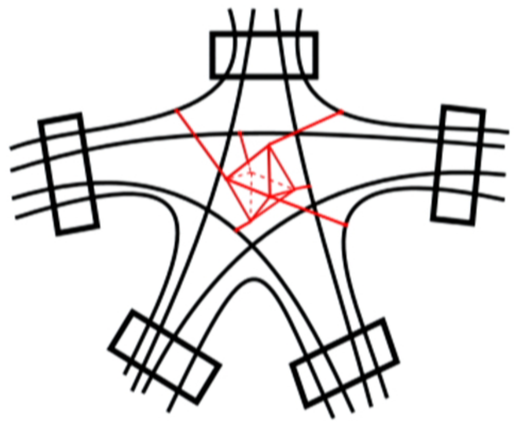
4-2 Pachner move

- After gauge fixing the Spin(4) symmetry, we can integrate 3 of the loops.
- The last loop has no projectors left on it, and we are left with
- In BF theory this would just be a factor of a delta function, resulting from unfixed shift symmetry
- Because of simplicity constraints we get a nonlocal mixing of strands
- The asymmetry is due to gauge fixing



5-1 Pachner move

- From the 10 loops we can integrate 6, which leaves us with



- We get again a nonlocal mixing of strands.

Non-localities

- We can think of these mixings as insertions of operators. Indeed in [E. Livine, J. Tambornino, 2013] it was shown that grasping operators and Wilson loops can be written in terms of such products of spinors.
- We know these are divergent. The expectation in the field is that this corresponds to symmetry, but not necessarily away from fixed point of RG flow.
- It would be interesting to know if this is related to the recently found nonlocal measure for 5-1 and 4-2 moves in linearized quantum Regge calculus [B. Dittrich, W. Kaminski, S. Steinhaus, 2014].
- Question: is such non-locality specific to the model? We expect similar behavior for other models, which should be checked.

Towards renormalization

- These non-localities most likely stem from the same place as non-local couplings in real space RG – coarse graining without truncation.
- The expectation is that a fixed point we would have invariance of the partition function under at least the 5-1 move.
- We need a truncation to get locality – work in progress.
- We cannot apply Tensor Network methods in 4d spin foams, need some optimal truncation - possible ideas are to keep the most divergent part (too crude?) or require asymptotics to be unchanged.