Title: Change of vertex amplitude under coarse graining In Euclidean spinfoam model I

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Abstract:

Towards coarse graining in an Euclidean Spin foam model I

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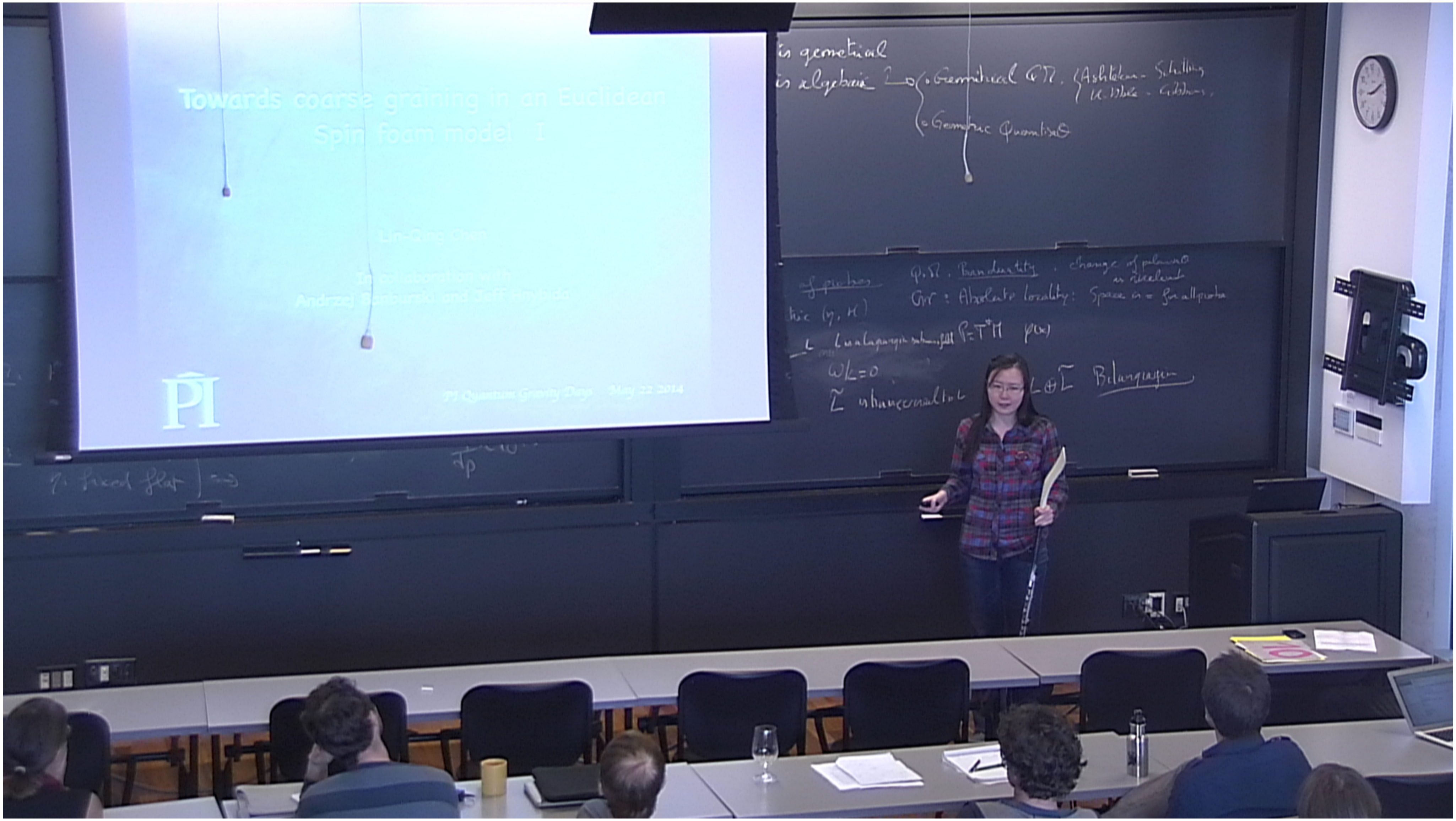
Towards coarse graining in an Euclidean
Spin foam model I

The basic idea of Spin foam models

- Start from BF theory, which is defined on a principle bundle over d dimensional manifold $\mathcal{M}$, with a group G and connection ω . B is a d-2 form in the adjoint representation of G.

$$S = \int_{\mathcal{M}} tr(B \wedge F(\omega))$$

In 4d Euclidean or Minkowski space,
a bivector B is **a simple bivector** $\Leftrightarrow$ $\exists$ a vector n_I s.t. $n_I B^{IJ} = 0$

$$S = \int_{\mathcal{M}} tr(*(e \wedge e) \wedge F + \frac{1}{\gamma} e \wedge e \wedge F)$$

- Discretize BF on simplicial complex $\Rightarrow$ Path integral quantization $\Rightarrow$

  $\Rightarrow$ Impose simplicity constraints

- Check the model: semi-classical limit (large spin), **continuum limit through renormalization (large number of building blocks)**

- Spin foam is a bottom-up approach of Quantum Gravity. Without renormalization, it is difficult to convincingly give predictions about observables which can be related with current/future experiments.

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- Coarse graining on triangulations are made out of elementary moves called "Pachner moves", which are local changes of triangulation.
- In this project we focused on studying how partition function/amplitude changes under a most basic coarse graining move in an Euclidean spin foam model. It is also the only Pachner move which by itself is coarse graining. The truncation method is work in progress.

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Let's start from BF

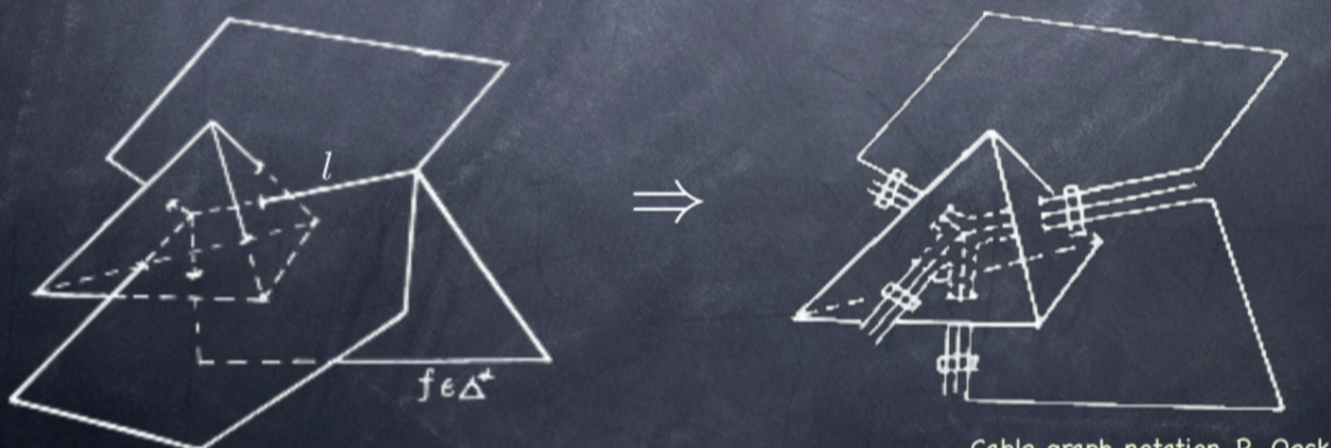
$$\mathcal{Z} = \int \mathcal{D}[B]\mathcal{D}[\omega] exp(i \int_{\mathcal{M}} tr[B \wedge F(\omega)])$$

triangulation Δ , dual Δ^* $\quad g_l = \mathcal{P}exp(-\int_l \omega)$

$$\mathcal{Z}(\Delta) = \sum_{j_f} \int \prod_{l \in \Delta^*} dg_l \prod_{f \in \Delta^*} (2j_f + 1) tr[\rho_f(g_{l_1} ... g_{l_n})]$$

$$= \sum_{j_f} \prod_{f \in \Delta^*} (2j_f + 1) \prod_{l \in \Delta^*} \int dg_l \; \rho_{f1}(g_l) \otimes ... \otimes \rho_{fd}(g_l)$$

project onto $Inv[\rho_1 \otimes .... \otimes \rho_d]$



Cable graph notation, R. Oeckl

Holomorphic Representation

[Dupuis, Livine, Hnybida, Freidel, etc.]

- SU(2) spinor and its conjugate

$$|z\rangle \equiv (\alpha, \beta)^T, \qquad |\check{z}\rangle = |z] \equiv (-\overline{\beta}, \overline{\alpha})^T$$

- Representation space: holomorphic functions on spinor space $\mathbb{C}^2$ with Hermitian inner product $\langle f|g\rangle = \int_{\mathbb{C}^2} \overline{f(z)} g(z) d\mu(z)$
  where $d\mu(z) = \pi^{-2} e^{-\langle z|z\rangle} d^4 z$

- Irreducible representations of spin j are given by holomorphic functions homogeneous of degree 2j $f(\lambda z) = \lambda^{2j} f(z)$ in the 2j+1 d subspace.

- A spinor defines a 3 vector $\vec{V}(z)$ on $\mathbb{R}^3$

$$|z\rangle\langle z| = \frac{1}{2}(\langle z|z\rangle \mathbb{1} + \vec{V}(z) \cdot \vec{\sigma})$$

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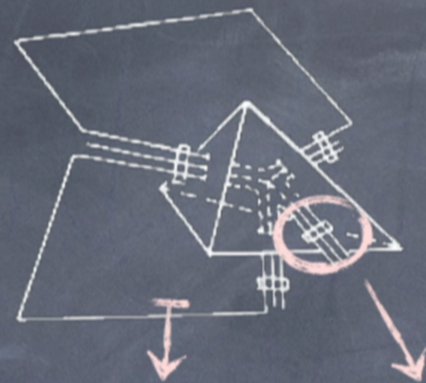
- Thus in the holomorphic representation:

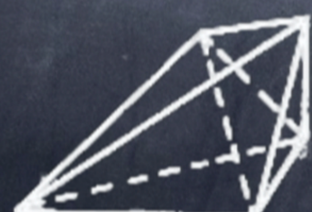


$$\mathcal{Z}_{SU(2)BF} = \int \prod_i d\mu(z_i) d\mu(w_i) \prod_{f\in\Delta^*} \left(\langle z_f|z_f\rangle - 1\right) \prod_{l\in\Delta^*} P_l(z_i;w_i)$$

- To get Euclidean quantum gravity, Spin(4) BF + Simplicity constraint

$$\mathcal{Z}_{spin(4)} = \mathcal{Z}_{SU(2)_L}\mathcal{Z}_{SU(2)_R}$$

- Basic element of the triangulation of the manifold: 4-simplex

 $\Leftrightarrow$ 

4-simplex and its dual cable graph

Holomorphic simplicity constraint [Dupuis&Livine, 2011]

- Euclidean: $Spin(4) = SU(2)_L \times SU(2)_R$

  $z_L \quad z_R$

  ↓

  B field is Bivector: $(\vec{V}(z)_L, \vec{V}(z)_R) = (\vec{J} + \vec{K}, \vec{J} - \vec{K})$

  $\vec{J}$ and $\vec{K}$ are rotation and boost parts of the B field.

- Under closure constraint, which requires around one spin-network vertex:

$$\sum_{i \in v} \vec{V}(z_i)_{L,R} = 0$$

  The following constraints on left and right spinors

$$\boxed{[z_i|z_j\rangle_L = \rho^2 [z_i|z_j\rangle_R,} \quad \rho^2 = |1-\gamma|/(1+\gamma)$$

  are equivalent with existence of a common time normal to B field.

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- Kernel of Spin(4) BF projector

$$[z_i|z_j\rangle_L = \rho^2 [z_i|z_j\rangle_R$$

$$\tilde{P}(z_i;w_i) = \sum_J {}_2F_1(-J-1,-J;2;\rho^4)\frac{(\sum_{i<j}[z_i|z_j\rangle[w_i|w_j\rangle)^J}{J!(J+1)!}$$

$$\Rightarrow \sum_J f(J,\rho)\frac{(\sum_{i<j}[z_i|z_j\rangle[w_i|w_j\rangle)^J}{J!(J+1)!} \qquad (J = J_L + J_R)$$

- Thus the partition function is:

$$\mathcal{Z}_{spinfoam} = \int \prod_i d\mu(z_i,w_i) \prod_{f\in\Delta^*} d_f \prod_{l\in\Delta^*} \tilde{P}_l(z_i;w_i)$$

Not only the G_N and γ are expected to flow during renormalization, but also the face weight and $\tilde{P}(z_i;w_i)$ are expected to be modified before one reaches the fixed point.

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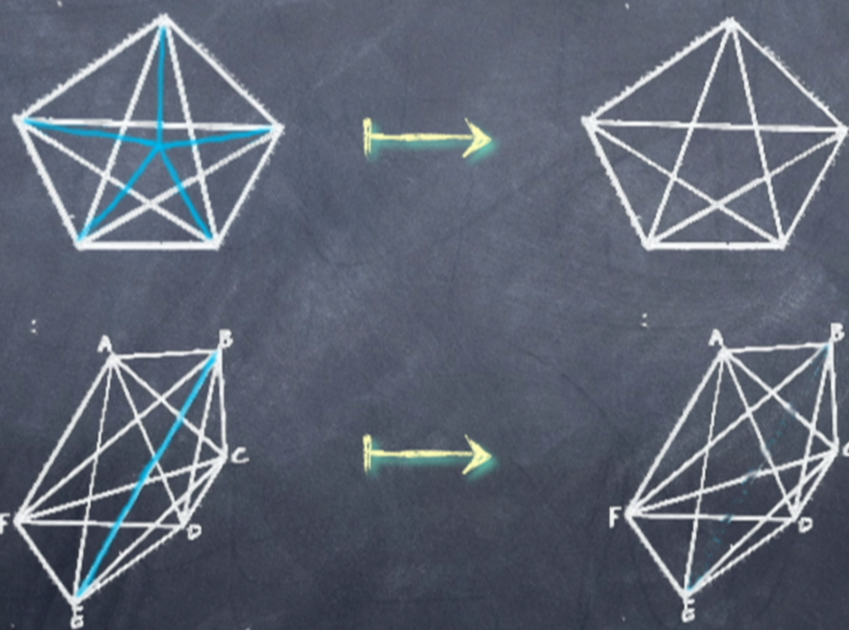
Pachner moves

- Pachner moves are local changes of triangulation. In 4d, through the following 5-1, 4-2, and 3-3 three basic moves, you can go from any triangulation of a manifold to any other triangulation.



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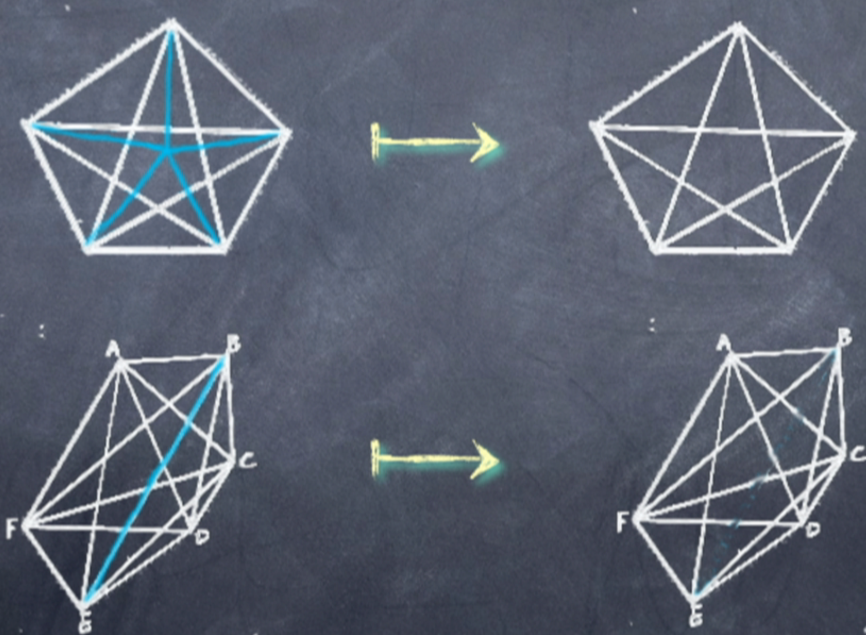

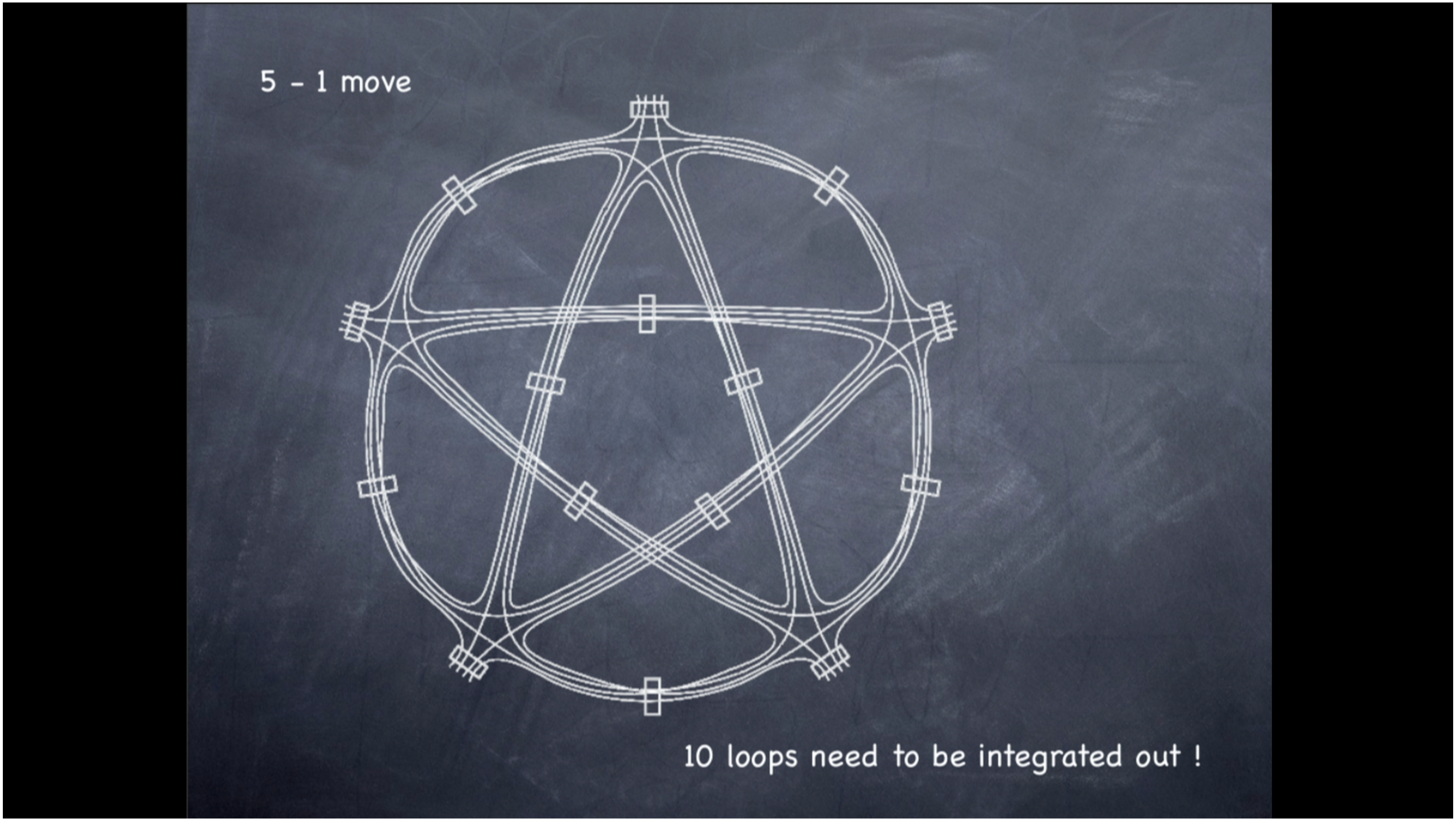
5 - 1 move
10 loops need to be integrated out !

What happens then?......

Watch the next episode!!

Thank you!