Title: Positive Energy in Quantum Gravity

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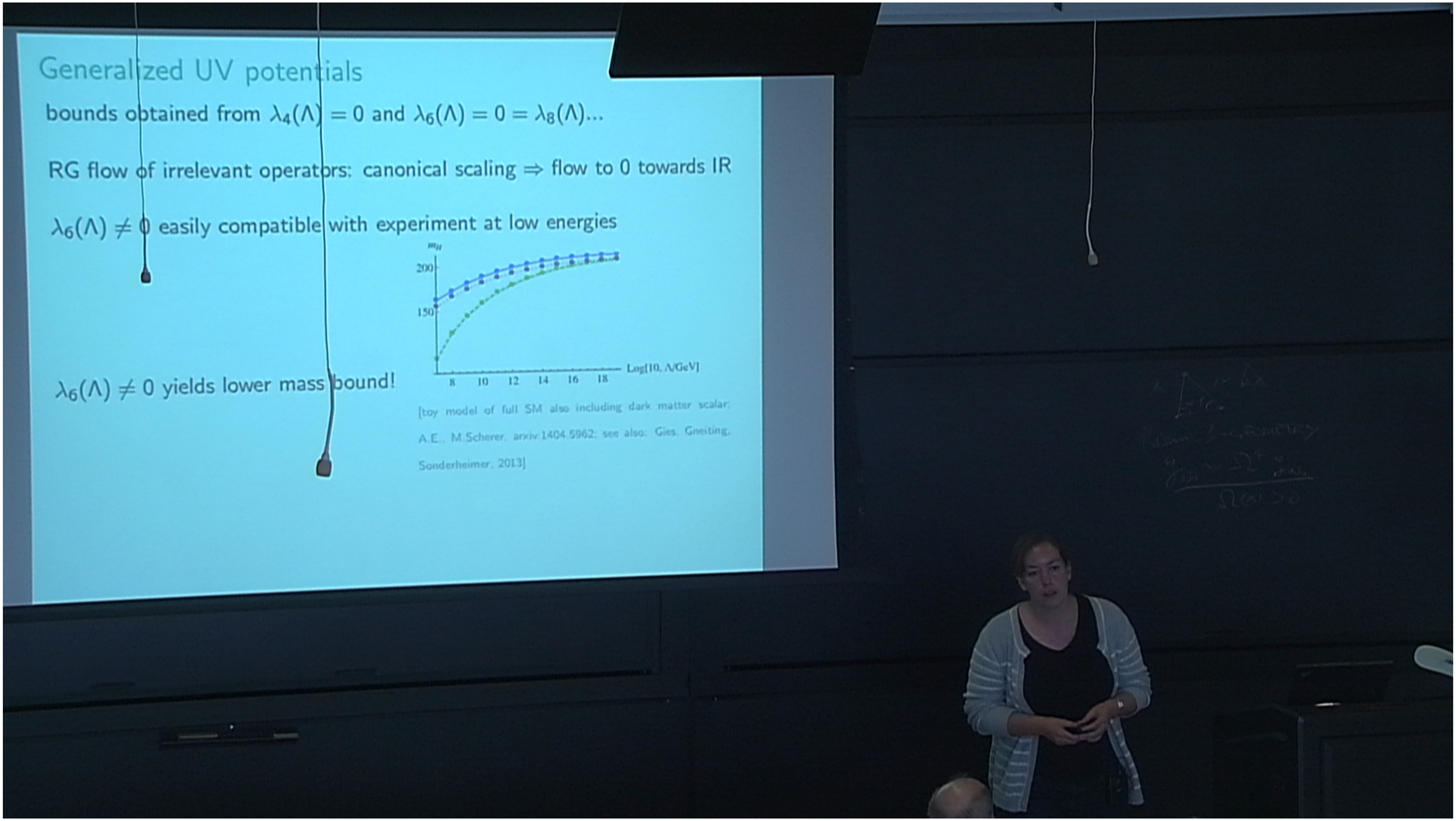
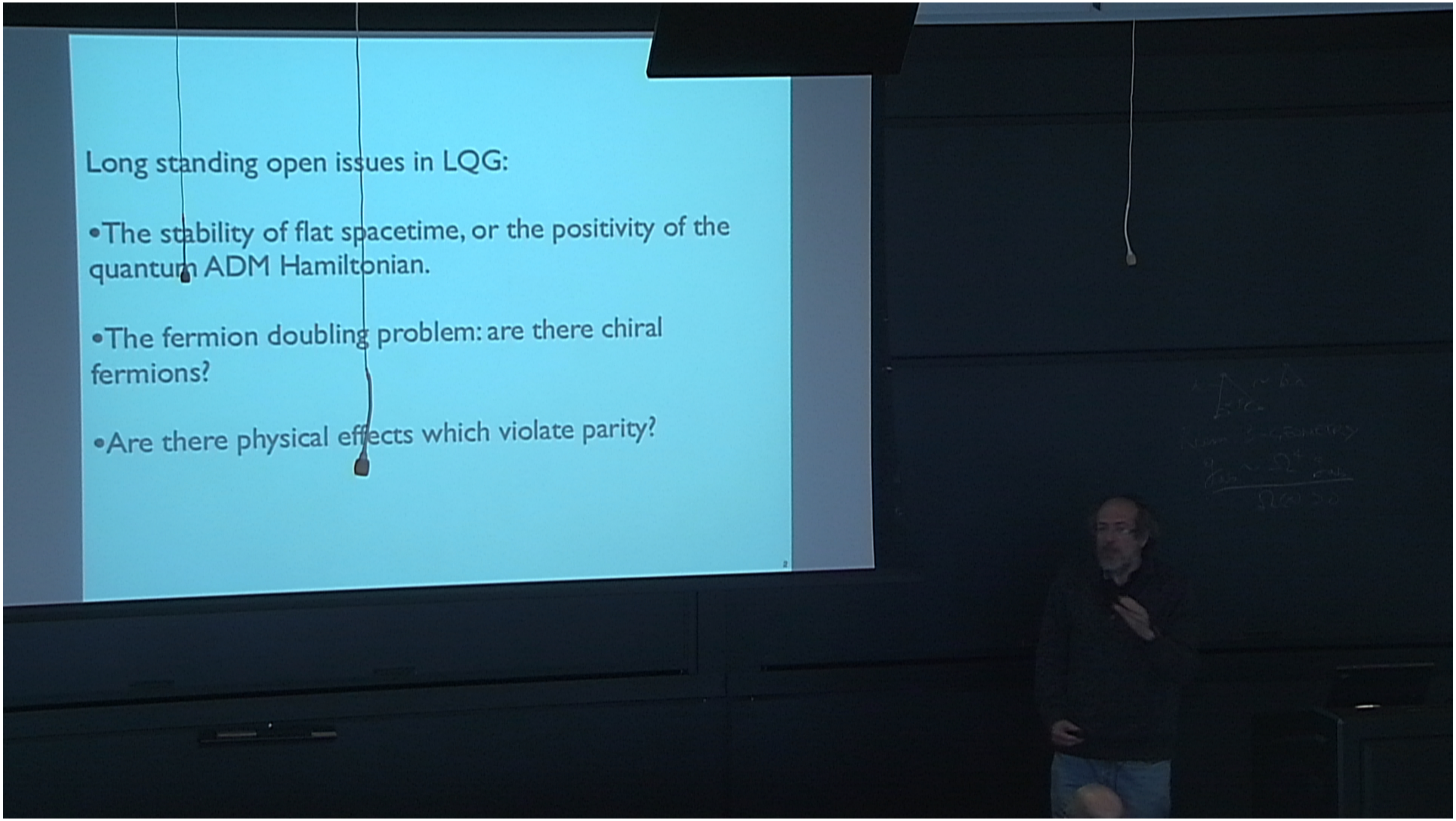
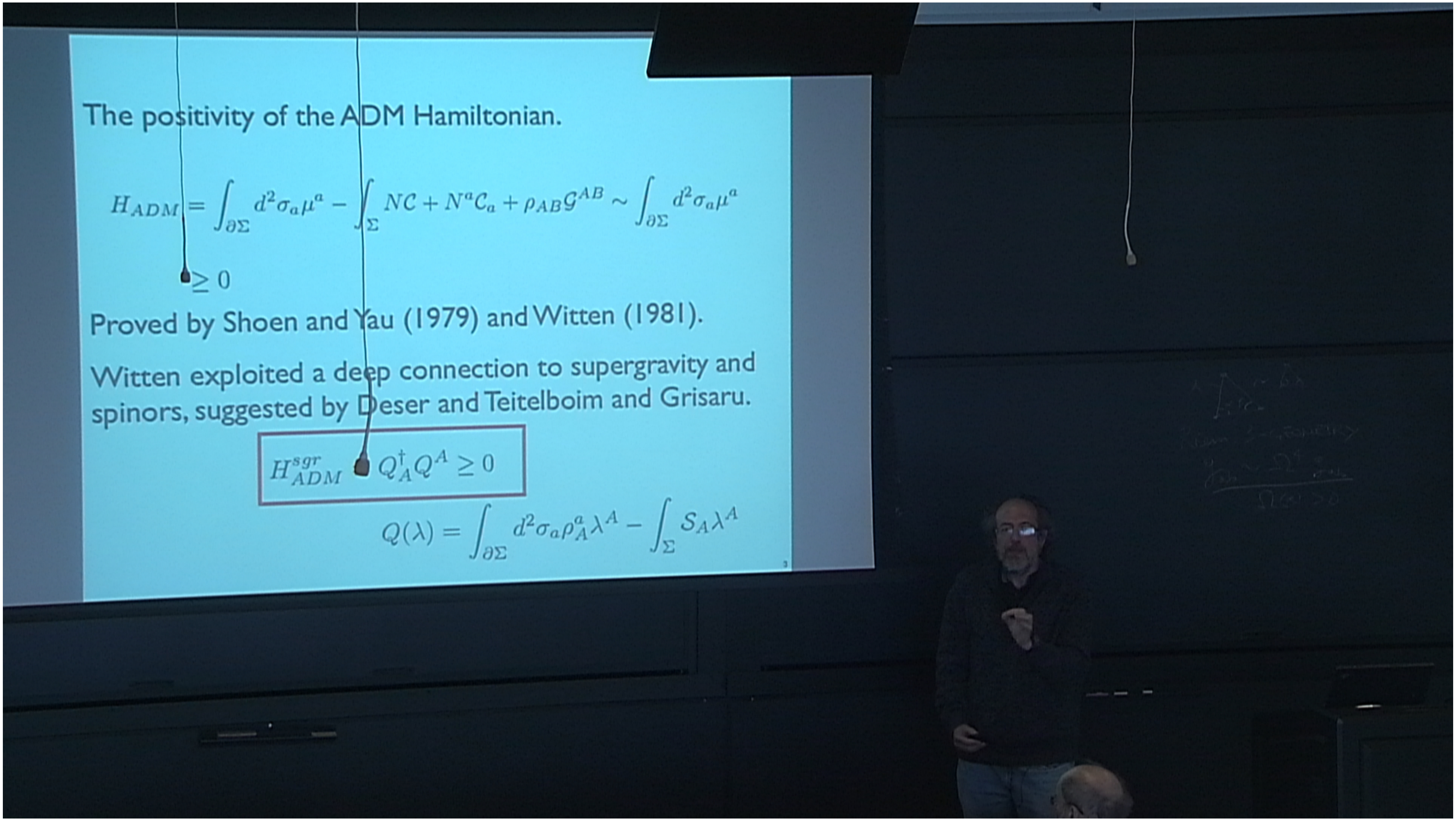
URL: http://pirsa.org/14050126

Abstract:


Generalized UV potentials
bounds obtained from λ4(Λ) = 0 and λ6(Λ) = 0 = λ8(Λ)...
RG flow of irrelevant operators: canonical scaling ⇒ flow to 0 towards IR
λ6(Λ) ≠ 0 easily compatible with experiment at low energies
λ6(Λ) ≠ 0 yields lower mass bound!
mH
200
150
8 10 12 14 16 18
Log[10, Λ/GeV]
[toy model of full SM also including dark matter scalar: A.E., M.Scherer, arxiv:1404.5962; see also: Gies, Gneiting, Sonderheimer, 2013]


Long standing open issues in LQG:
•The stability of flat spacetime, or the positivity of the quantum ADM Hamiltonian.
•The fermion doubling problem: are there chiral fermions?
•Are there physical effects which violate parity?


The positivity of the ADM Hamiltonian.
$H_{ADM} = \int_{\partial\Sigma} d^2\sigma_a \mu^a - \int_{\Sigma} N\mathcal{C} + N^a \mathcal{C}_a + \rho_{AB}\mathcal{G}^{AB} \sim \int_{\partial\Sigma} d^2\sigma_a \mu^a$
≥ 0
Proved by Shoen and Yau (1979) and Witten (1981).
Witten exploited a deep connection to supergravity and spinors, suggested by Deser and Teitelboim and Grisaru.
$H^{sgr}_{ADM} = Q^{\dagger}_A Q^A \geq 0$
$Q(\lambda) = \int_{\partial\Sigma} d^2\sigma_a \rho^a_A \lambda^A - \int_{\Sigma} \mathcal{S}_A \lambda^A$
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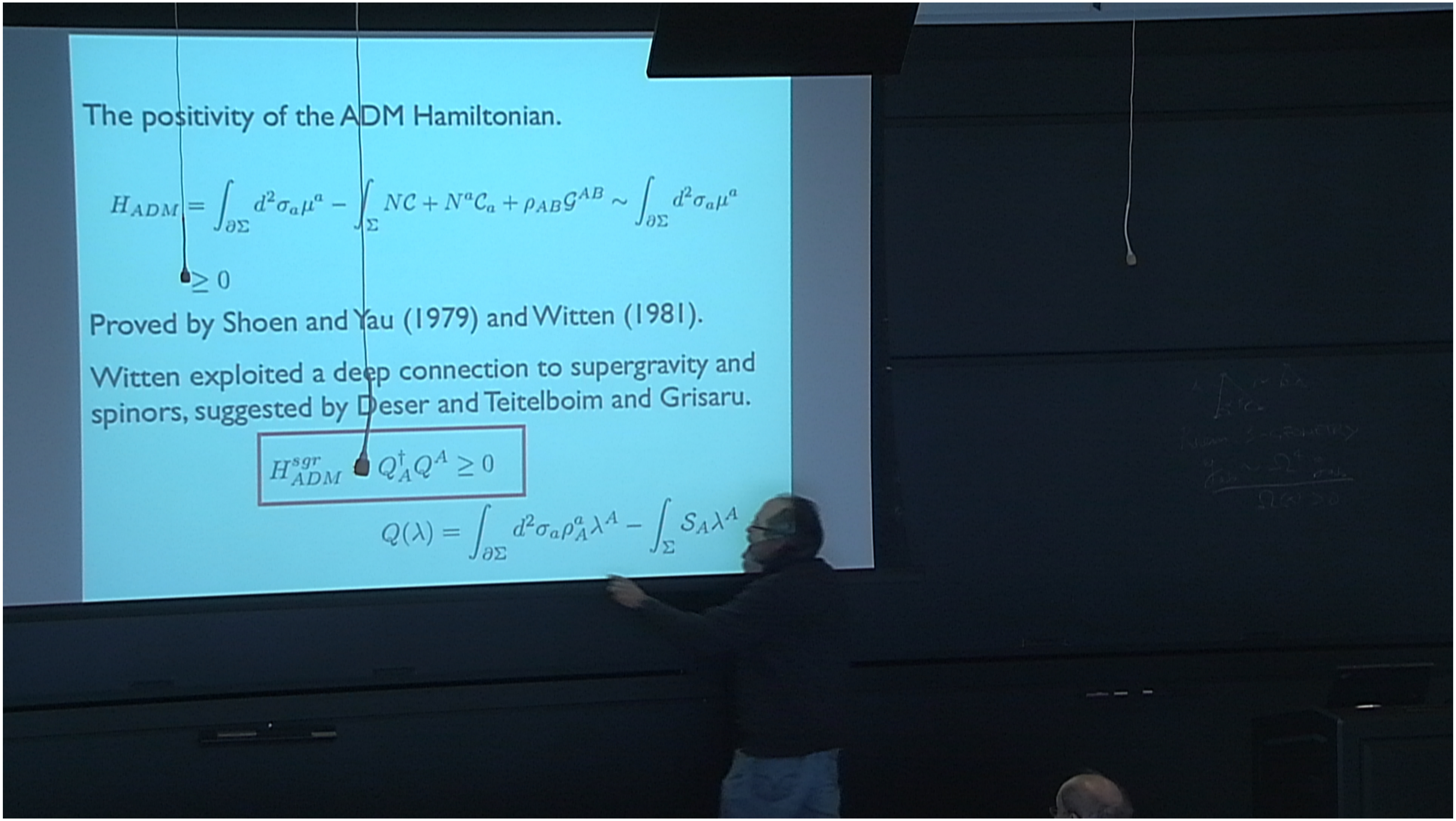
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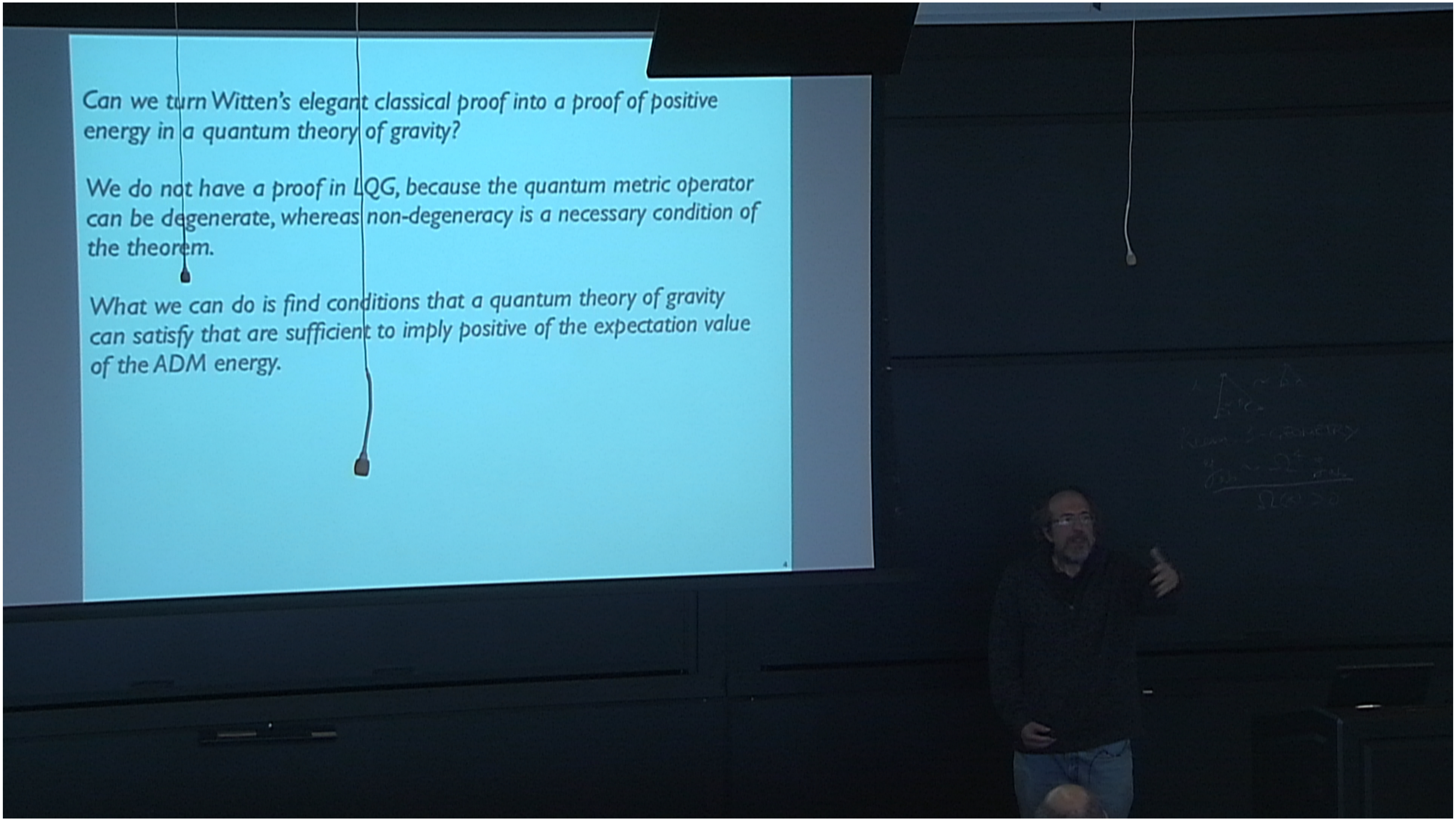
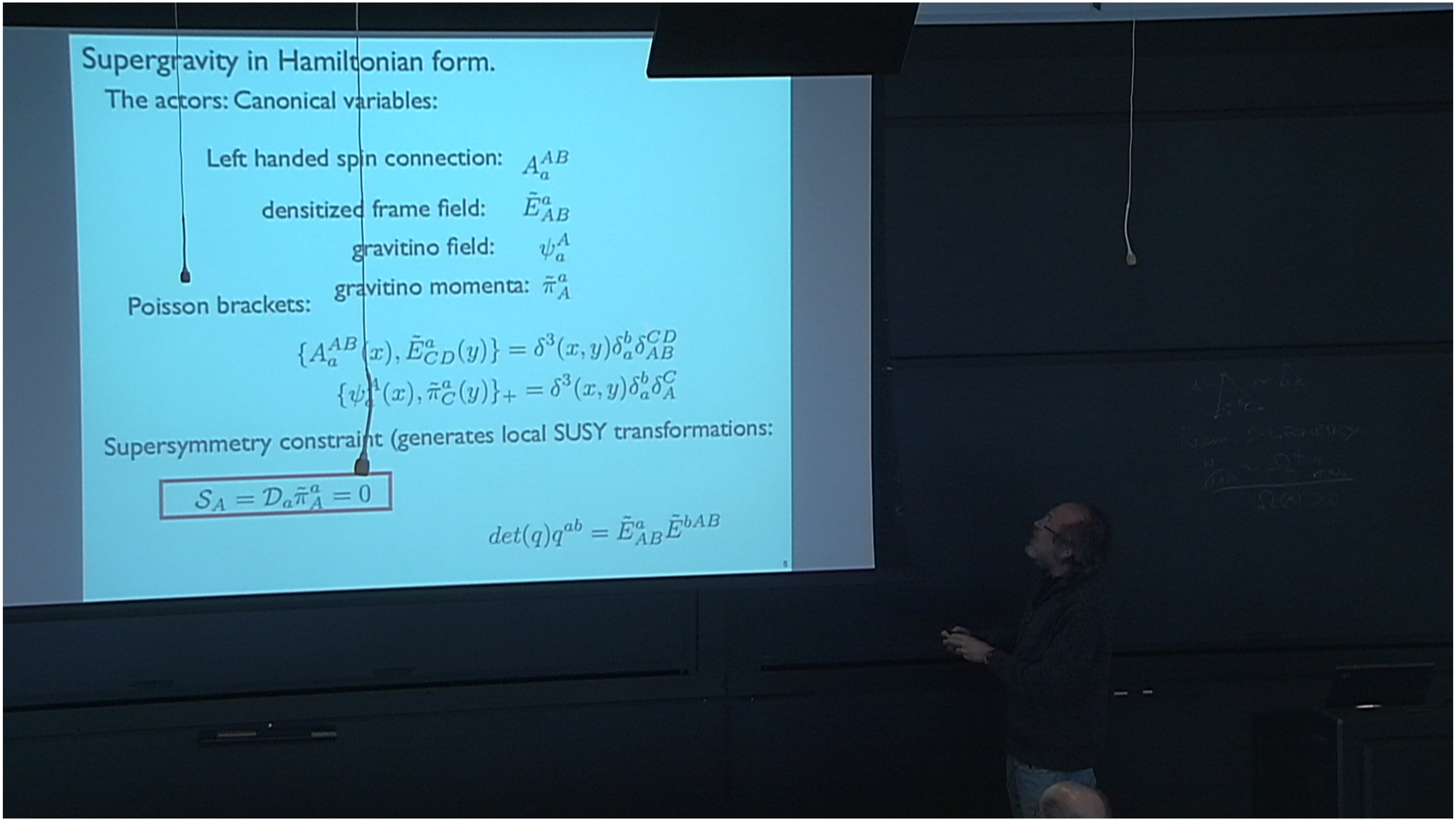
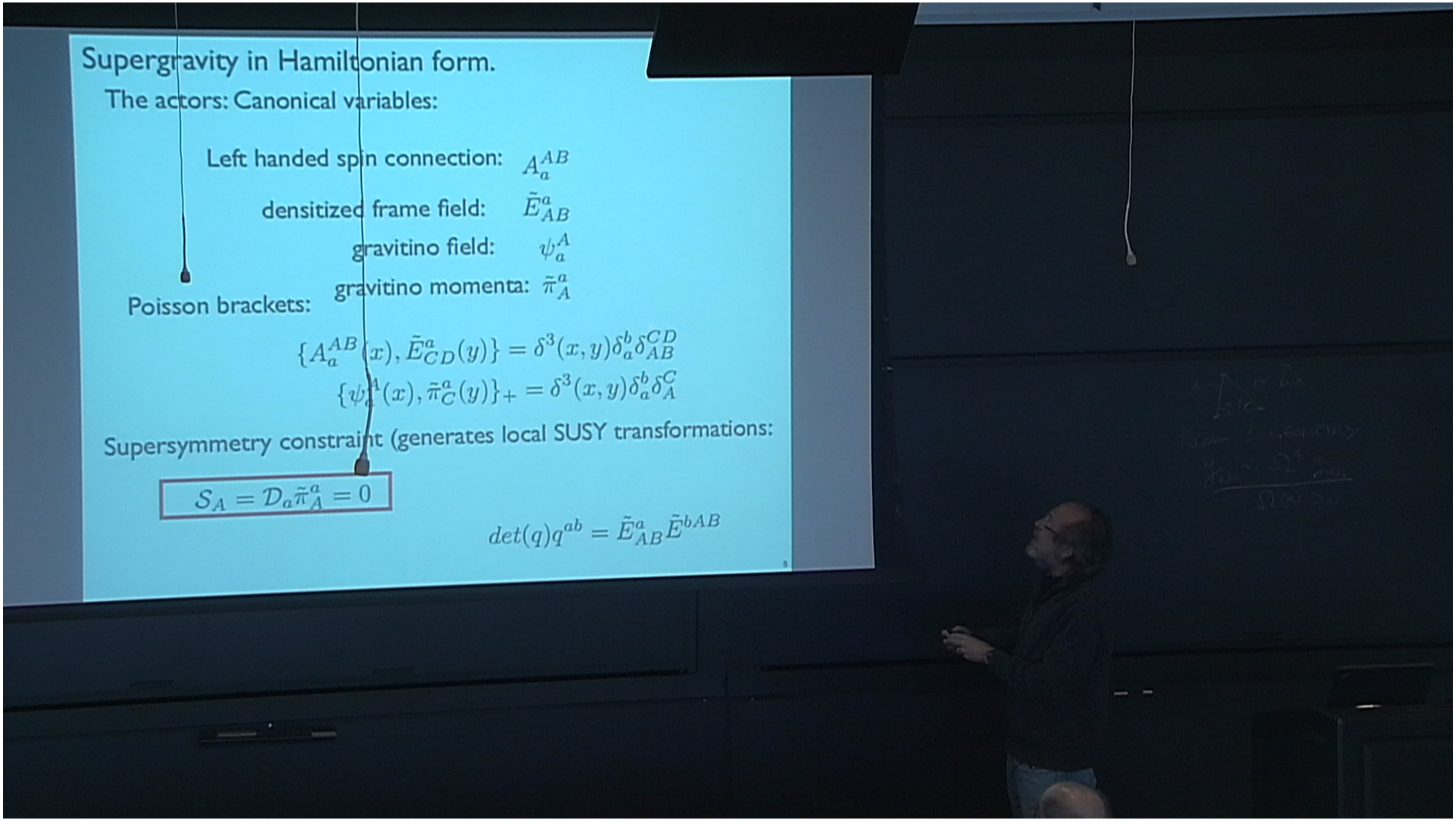

Can we turn Witten's elegant classical proof into a proof of positive energy in a quantum theory of gravity?
We do not have a proof in LQG, because the quantum metric operator can be degenerate, whereas non-degeneracy is a necessary condition of the theorem.
What we can do is find conditions that a quantum theory of gravity can satisfy that are sufficient to imply positive of the expectation value of the ADM energy.


Supergravity in Hamiltonian form.
The actors: Canonical variables:
Left handed spin connection: A_a^{AB}
densitized frame field: $\tilde{E}^a_{AB}$
gravitino field: ψ_a^A
gravitino momenta: $\tilde{\pi}^a_A$
Poisson brackets:
$\{A_a^{AB}(x), \tilde{E}^a_{CD}(y)\} = \delta^3(x,y)\delta^b_a\delta^{CD}_{AB}$
$\{\psi_a^A(x), \tilde{\pi}^a_C(y)\}_+ = \delta^3(x,y)\delta^b_a\delta^C_A$
Supersymmetry constraint (generates local SUSY transformations:
$\mathcal{S}_A = \mathcal{D}_a\tilde{\pi}^a_A = 0$
$det(q)q^{ab} = \tilde{E}^a_{AB}\tilde{E}^{bAB}$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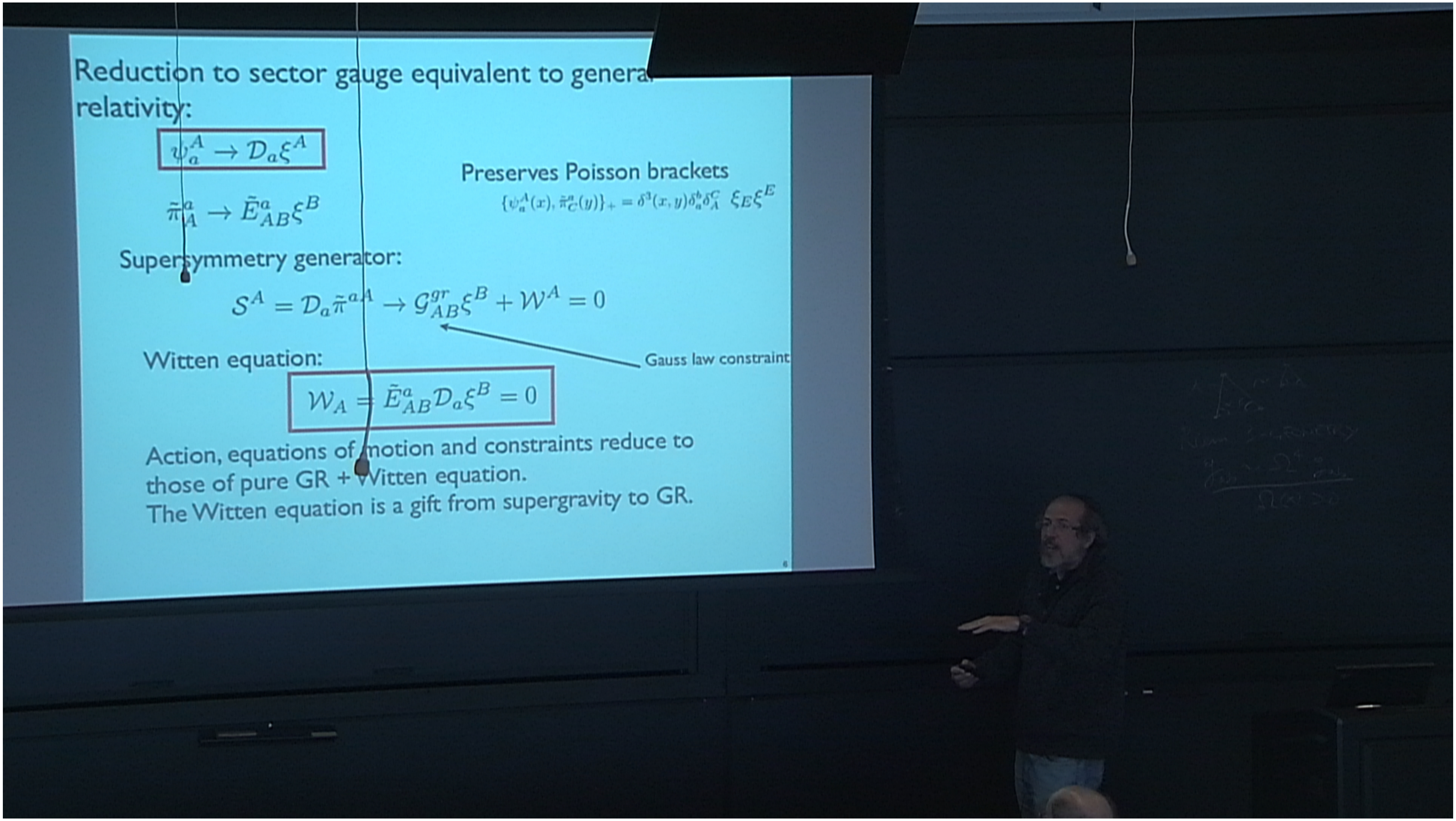
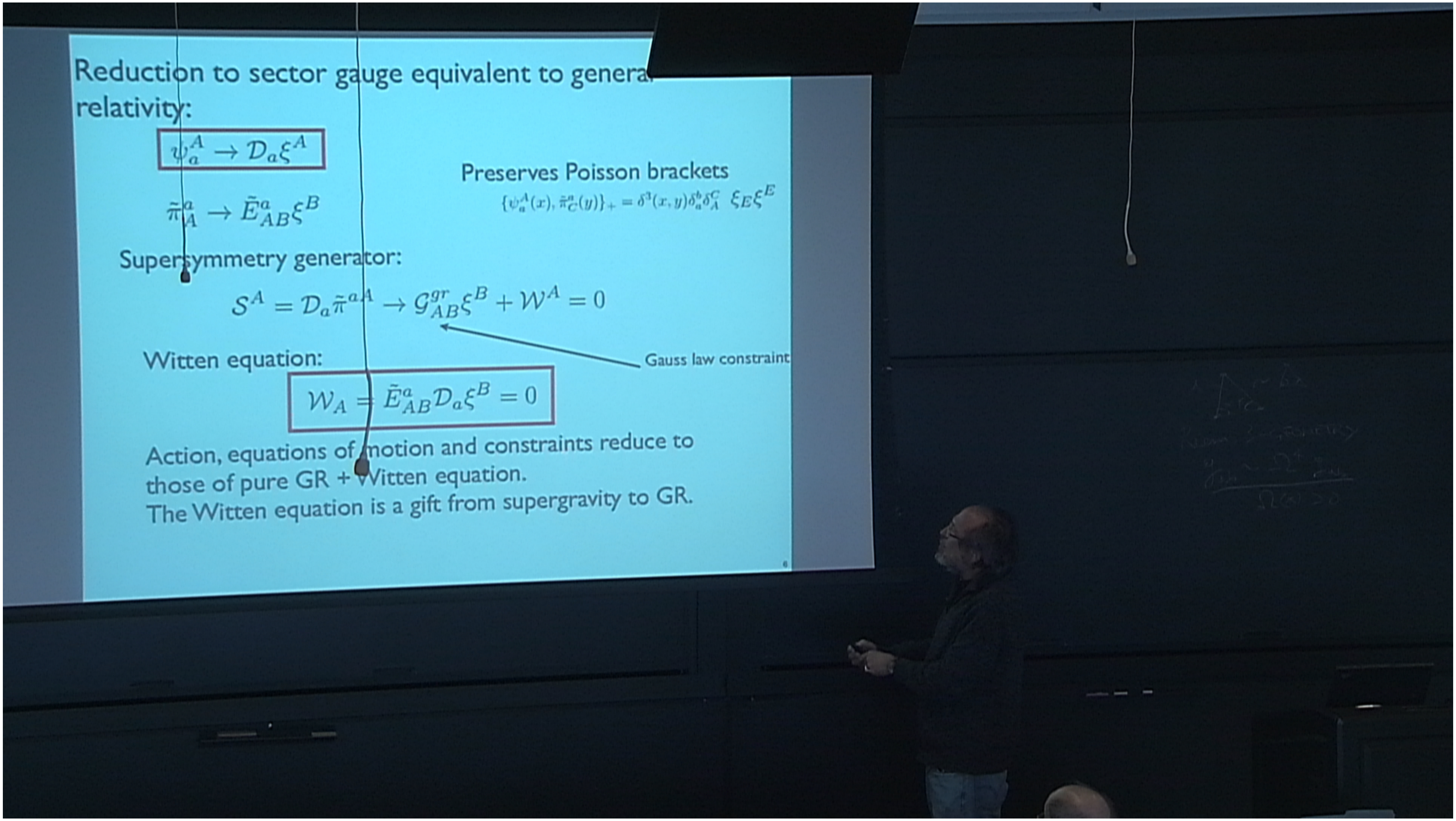

Reduction to sector gauge equivalent to general relativity:
$\psi_a^A \to \mathcal{D}_a \xi^A$
$\tilde{\pi}^a_A \to \tilde{E}^a_{AB} \xi^B$
Preserves Poisson brackets
$\{\psi_a^A(x), \tilde{\pi}^a_C(y)\}_+ = \delta^3(x,y)\delta^b_a\delta^C_A \;\; \xi_E \xi^E$
Supersymmetry generator:
$\mathcal{S}^A = \mathcal{D}_a \tilde{\pi}^{aA} \to \mathcal{G}^{gr}_{AB}\xi^B + \mathcal{W}^A = 0$
Gauss law constraint
Witten equation:
$\mathcal{W}_A = \tilde{E}^a_{AB}\mathcal{D}_a \xi^B = 0$
Action, equations of motion and constraints reduce to those of pure GR + Witten equation.
The Witten equation is a gift from supergravity to GR.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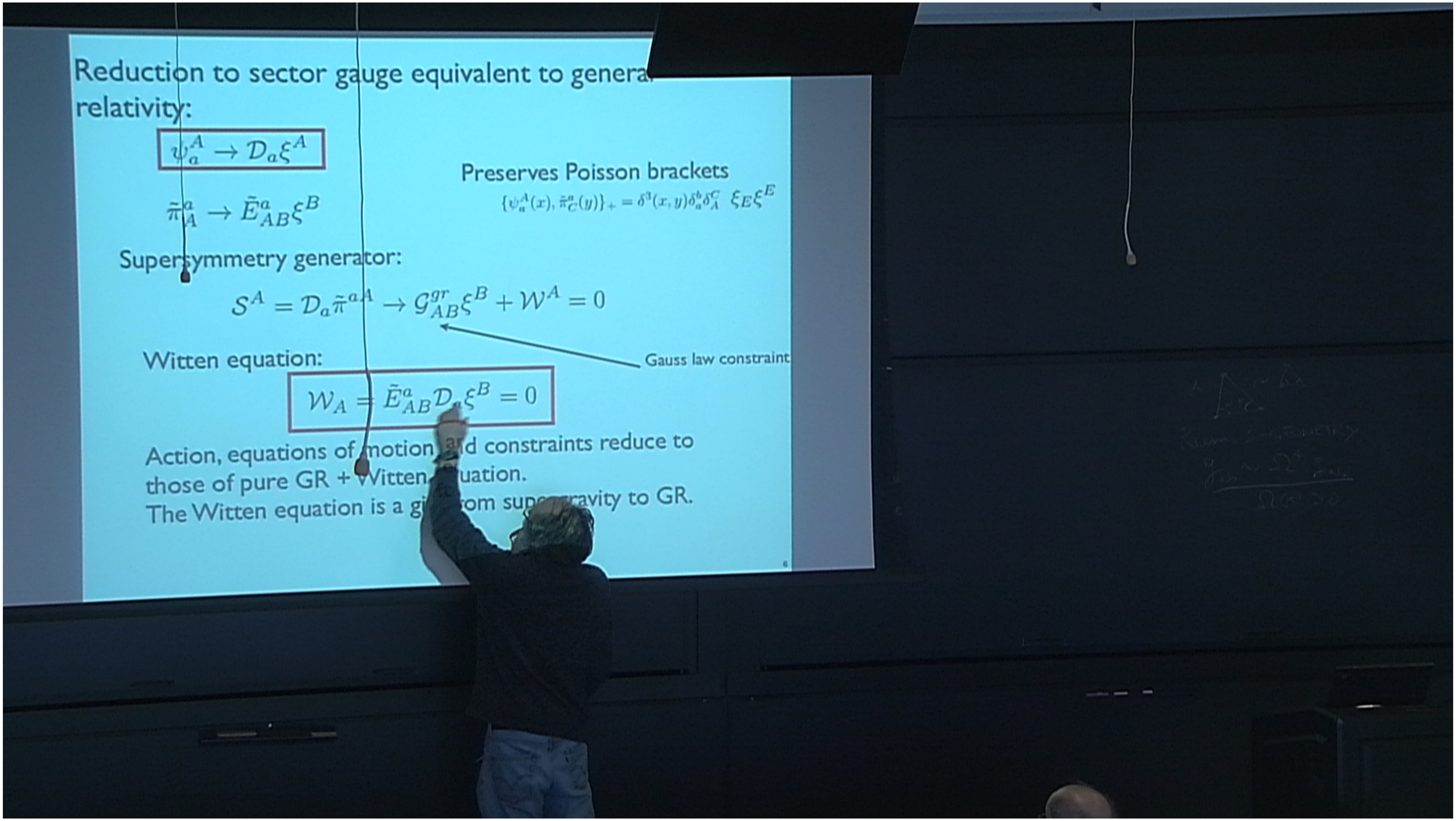
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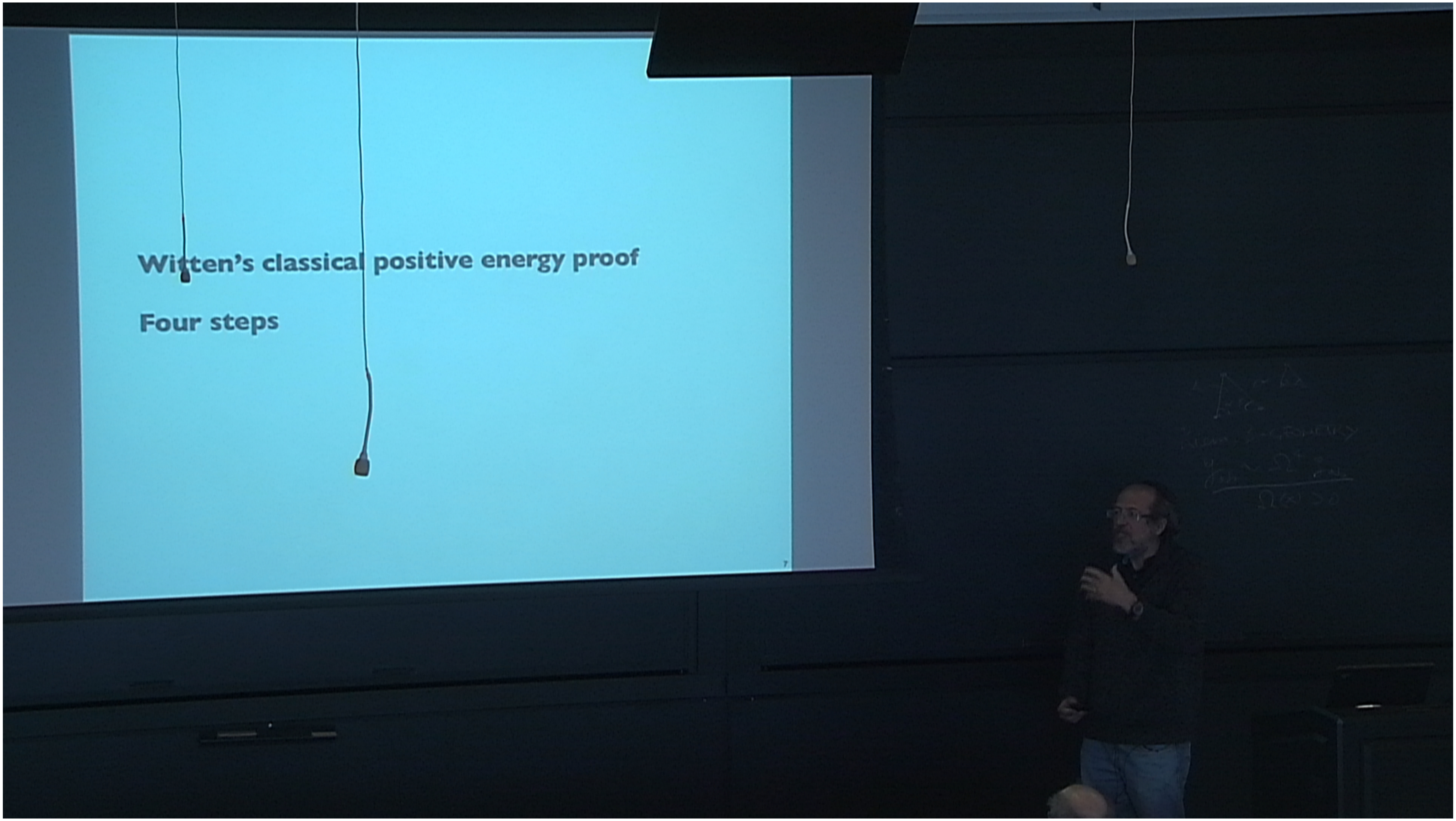
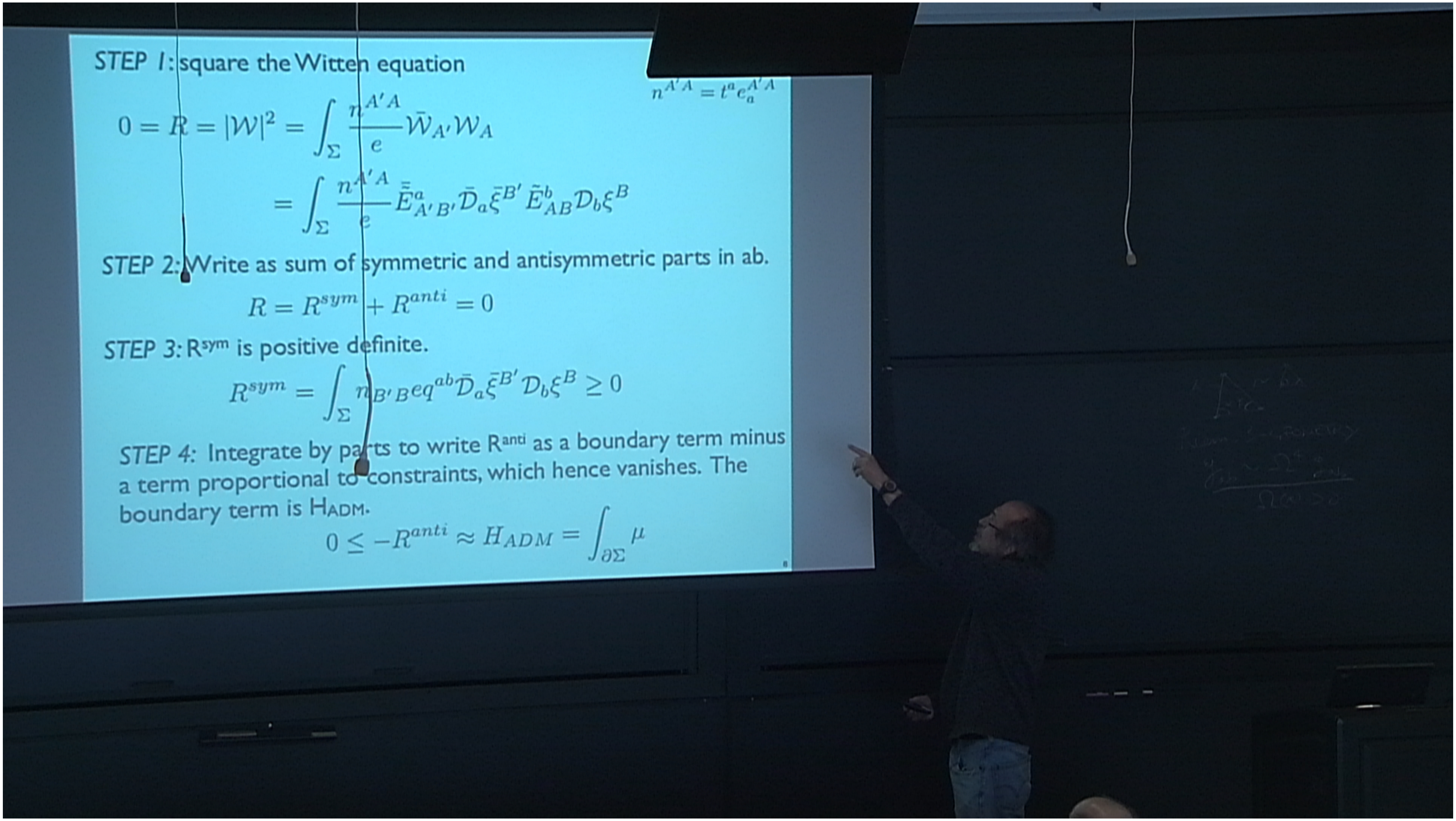

Witten's classical positive energy proof
Four steps
7


STEP 1: square the Witten equation
$n^{A'A} = t^a e_a^{A'A}$
$0 = R = |\mathcal{W}|^2 = \int_\Sigma \frac{n^{A'A}}{e} \bar{\mathcal{W}}_{A'} \mathcal{W}_A$
$= \int_\Sigma \frac{n^{A'A}}{e} \bar{\bar{E}}^a_{A'B'} \bar{\mathcal{D}}_a \bar{\xi}^{B'} \bar{E}^b_{AB} \mathcal{D}_b \xi^B$
STEP 2: Write as sum of symmetric and antisymmetric parts in ab.
$R = R^{sym} + R^{anti} = 0$
STEP 3: R^{sym} is positive definite.
$R^{sym} = \int_\Sigma n_{B'B} e q^{ab} \bar{\mathcal{D}}_a \bar{\xi}^{B'} \mathcal{D}_b \xi^B \geq 0$
STEP 4: Integrate by parts to write R^{anti} as a boundary term minus a term proportional to constraints, which hence vanishes. The boundary term is H_{ADM}.
$0 \leq -R^{anti} \approx H_{ADM} = \int_{\partial\Sigma} \mu$
8

STEP 1: square the Witten equation

$$n^{A'A} = t^a e_a^{A'A}$$

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$$= \int_\Sigma \frac{n^{A'A}}{e} \bar{\tilde{E}}^a_{A'B'} \bar{\mathcal{D}}_a \bar{\xi}^{B'} \tilde{E}^b_{AB} \mathcal{D}_b \xi^B$$

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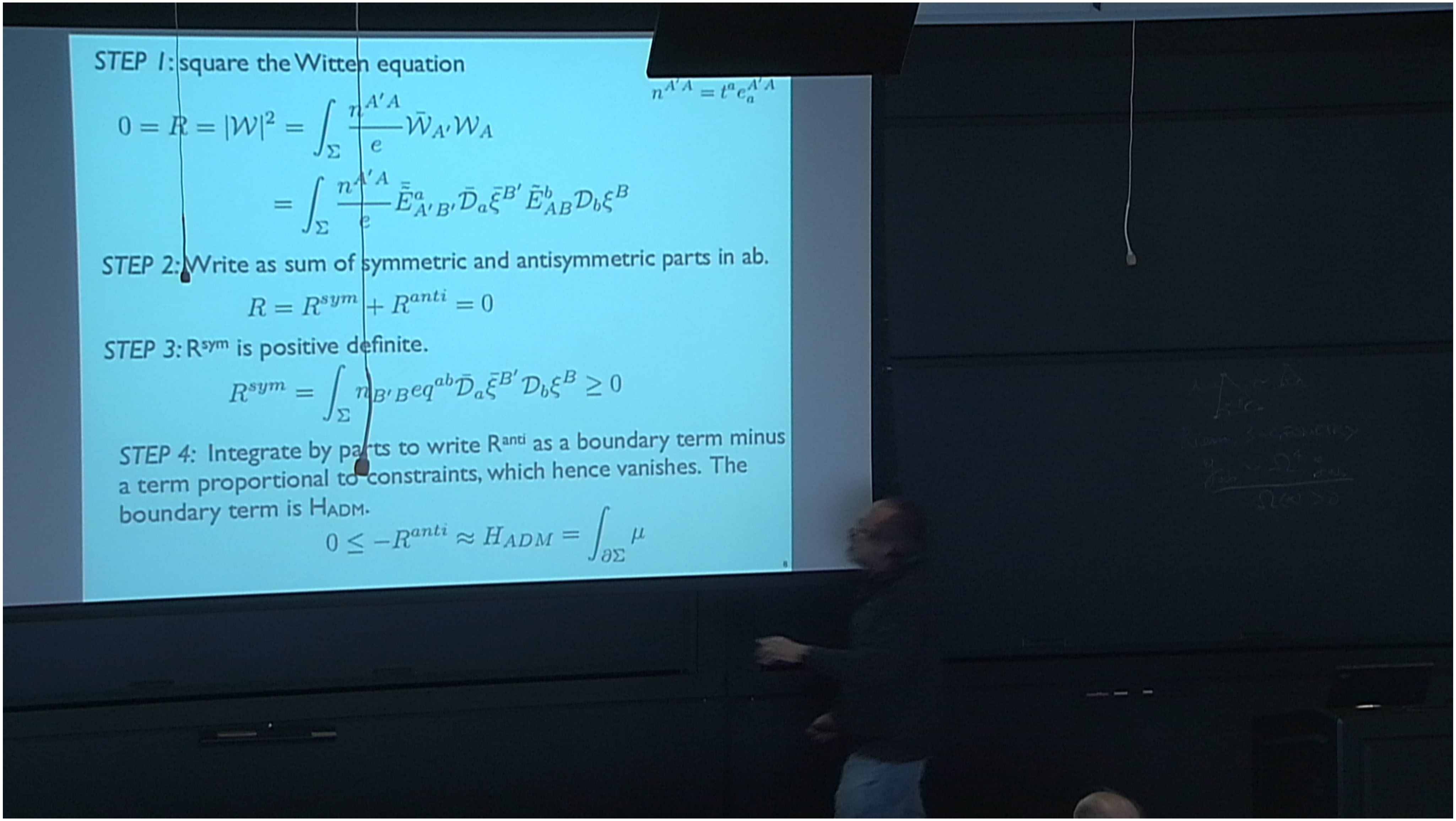
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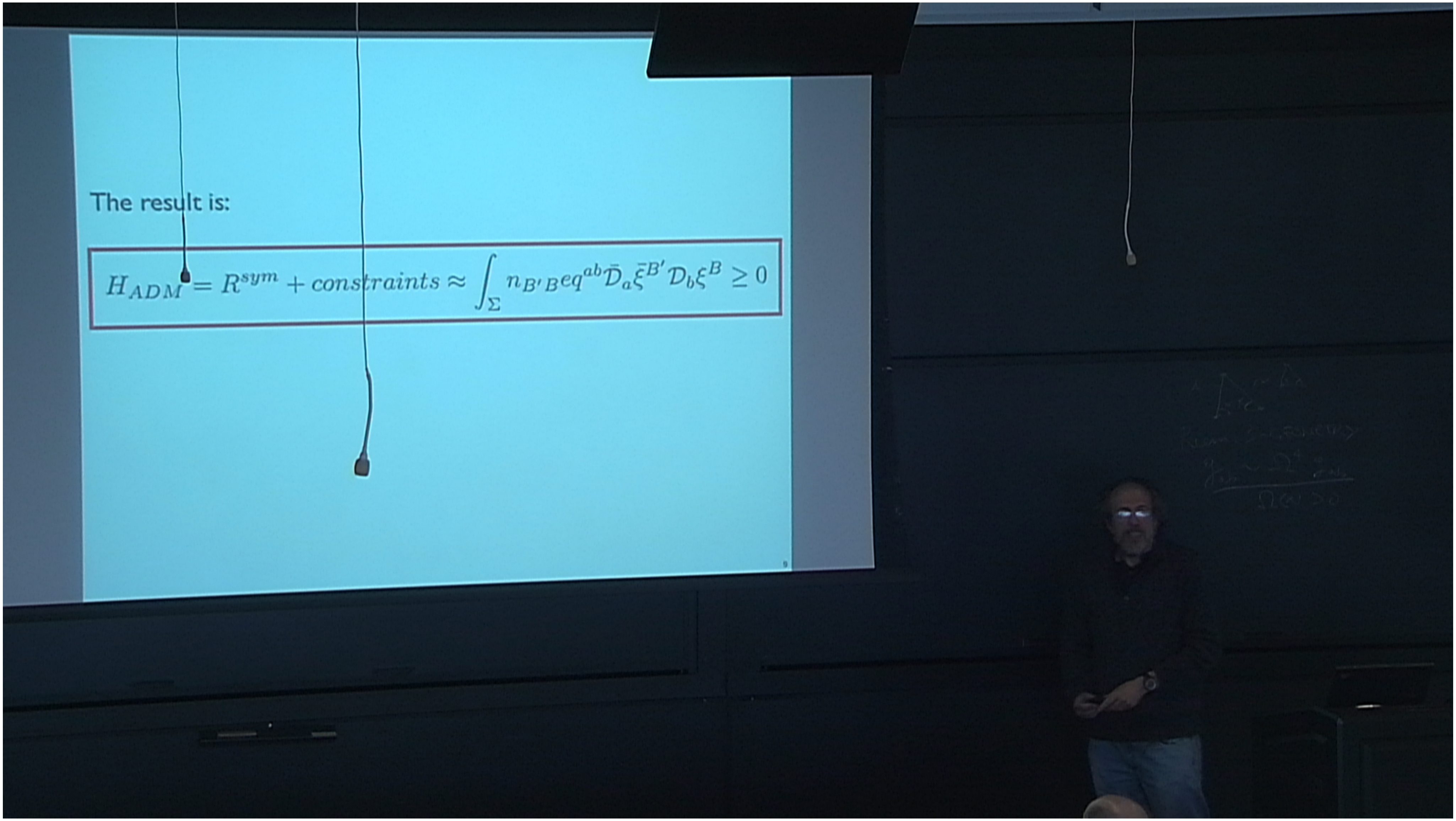
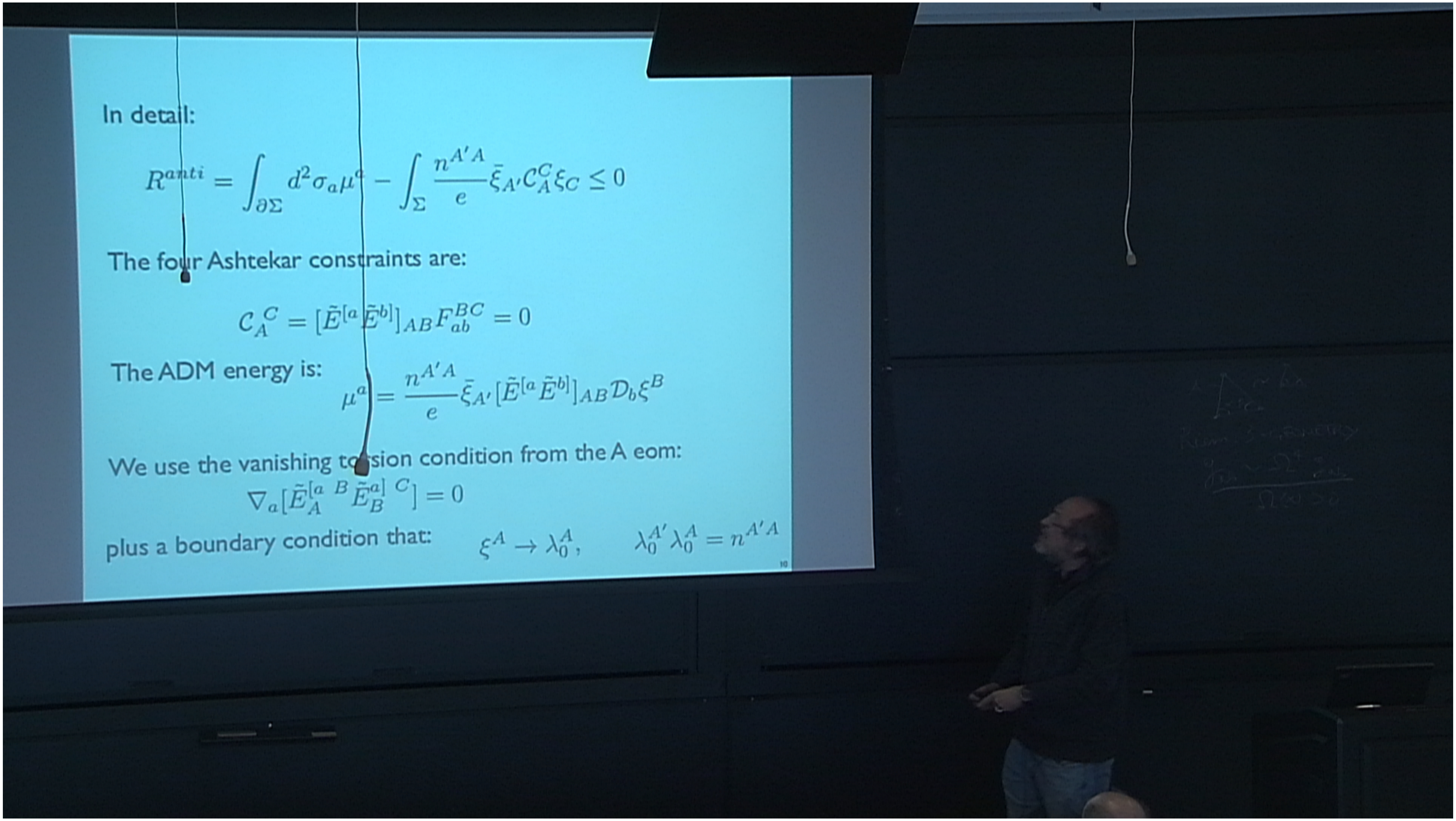
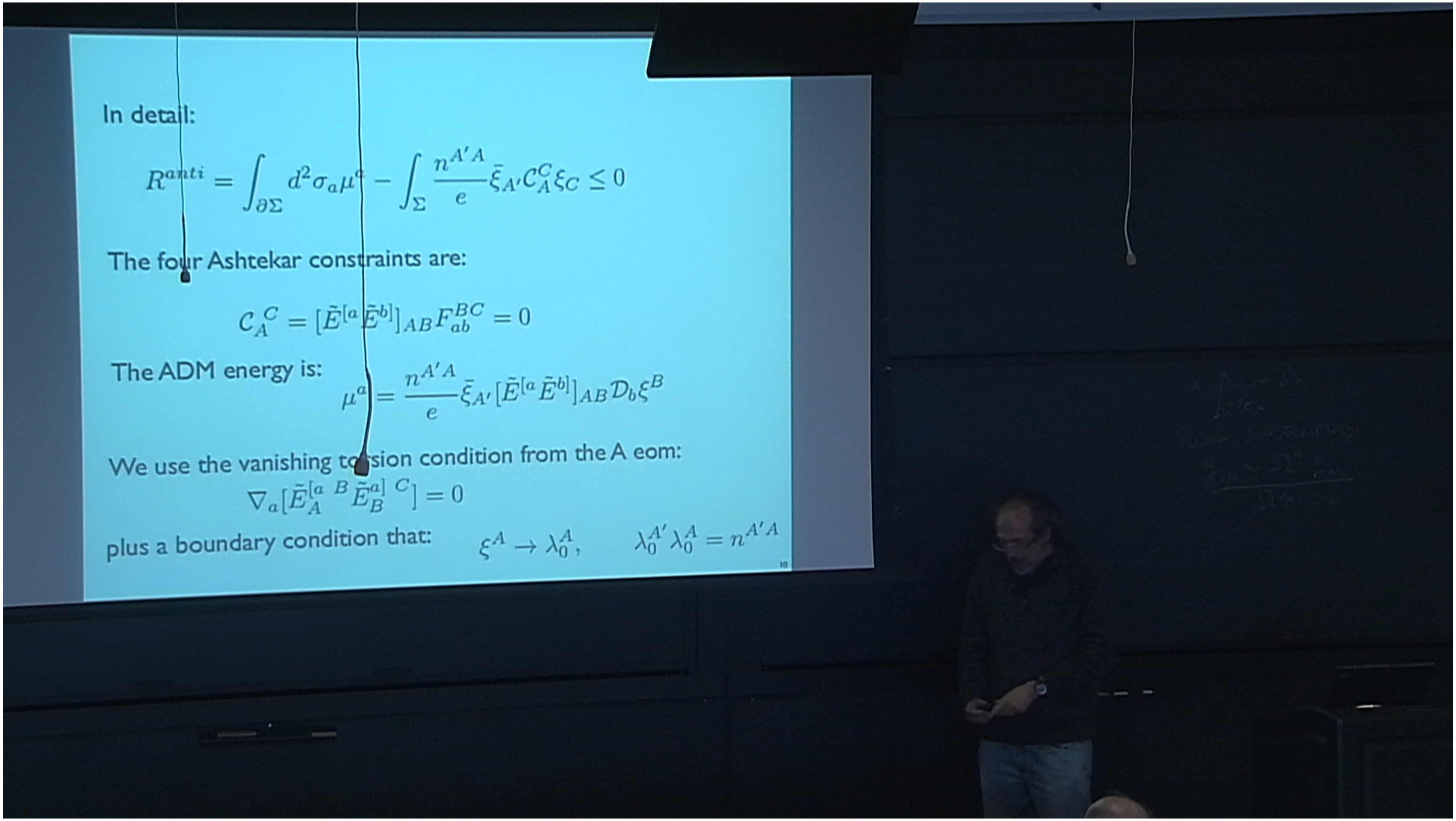

The result is:
$H_{ADM} = R^{sym} + constraints \approx \int_\Sigma n_{B'B} e q^{ab} \bar{\mathcal{D}}_a \bar{\xi}^{B'} \mathcal{D}_b \xi^B \geq 0$
9


In detail:
$R^{anti} = \int_{\partial\Sigma} d^2\sigma_a \mu^a - \int_\Sigma \frac{n^{A'A}}{e} \bar{\xi}_{A'} \mathcal{C}_A^C \xi_C \leq 0$
The four Ashtekar constraints are:
$\mathcal{C}_A^{\ C} = [\tilde{E}^{[a} \tilde{E}^{b]}]_{AB} F_{ab}^{BC} = 0$
The ADM energy is:
$\mu^a = \frac{n^{A'A}}{e} \bar{\xi}_{A'} [\tilde{E}^{[a} \tilde{E}^{b]}]_{AB} \mathcal{D}_b \xi^B$
We use the vanishing torsion condition from the A eom:
$\nabla_a [\tilde{E}_A^{[a \ B} \tilde{E}_B^{a] \ C}] = 0$
plus a boundary condition that: $\xi^A \to \lambda_0^A, \quad \lambda_0^{A'} \lambda_0^A = n^{A'A}$
10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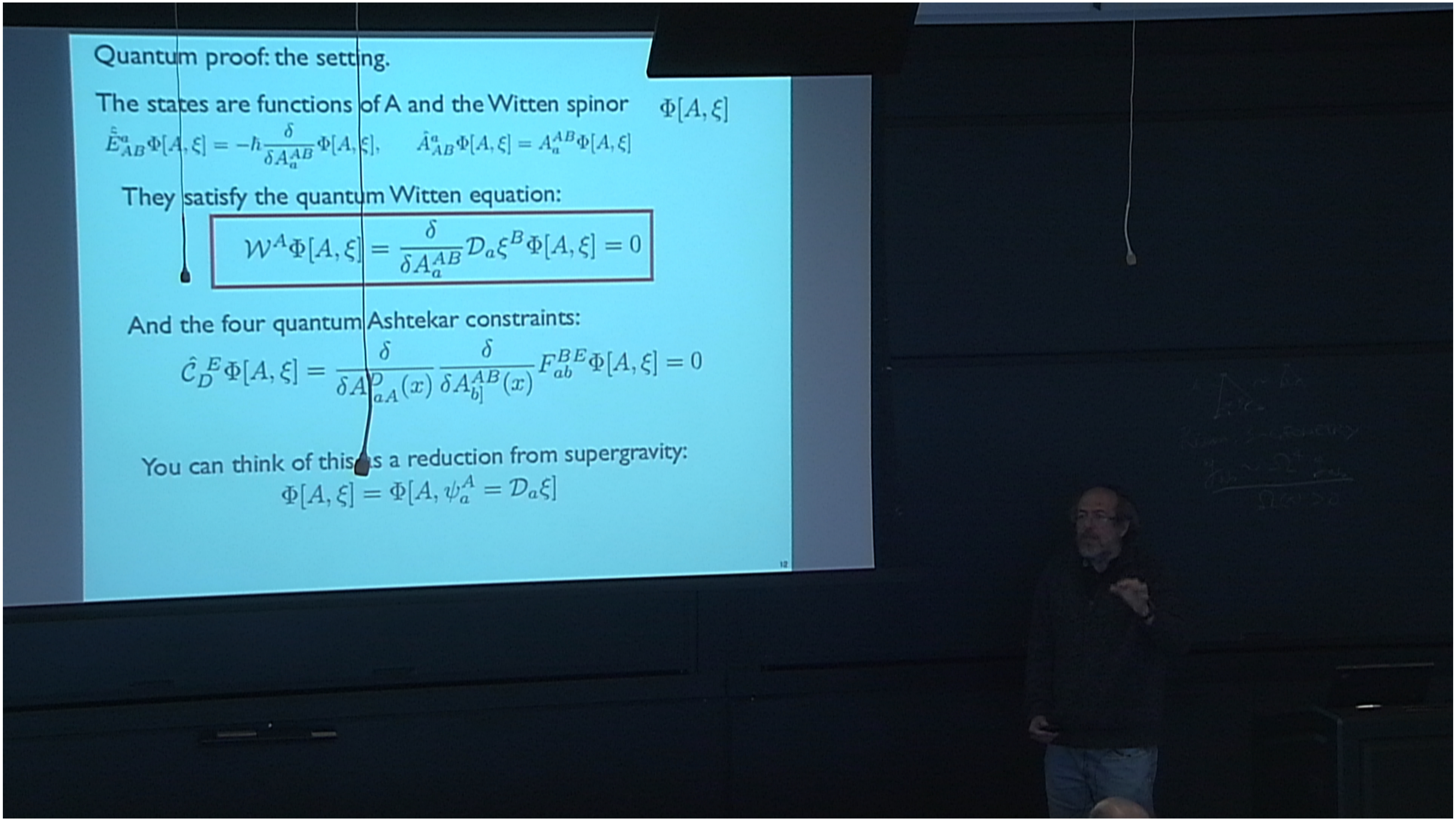
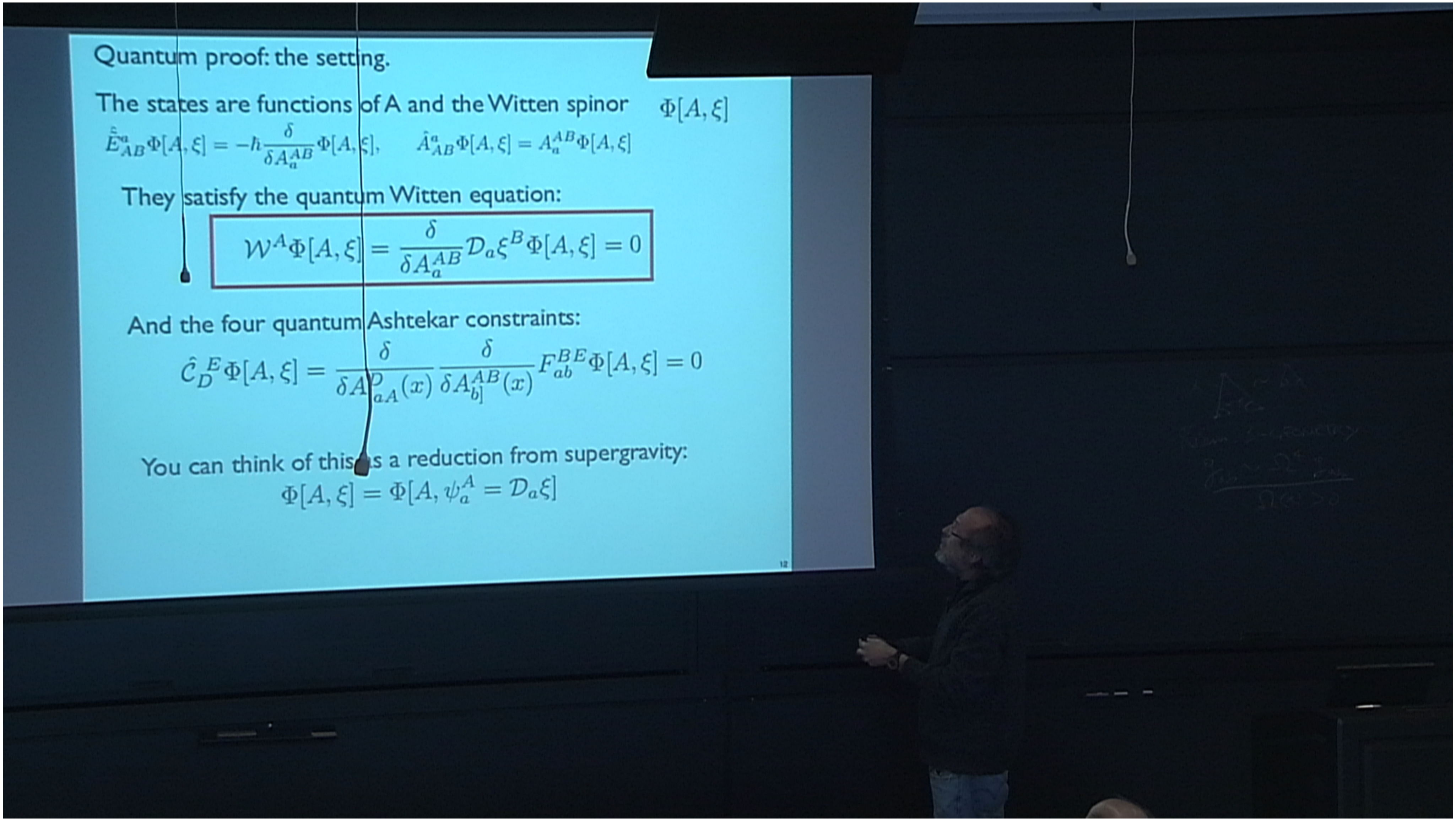
10

Quantum positive energy proof

Note, this is a formal result, we pay attention to operator ordering but not regularization of operator products.


Quantum proof: the setting.
The states are functions of A and the Witten spinor $\Phi[A,\xi]$
$\hat{E}^a_{AB}\Phi[A,\xi] = -\hbar\frac{\delta}{\delta A^{AB}_a}\Phi[A,\xi], \quad \hat{A}^a_{AB}\Phi[A,\xi] = A^{AB}_a\Phi[A,\xi]$
They satisfy the quantum Witten equation:
$\mathcal{W}^A\Phi[A,\xi] = \frac{\delta}{\delta A^{AB}_a}\mathcal{D}_a\xi^B\Phi[A,\xi] = 0$
And the four quantum Ashtekar constraints:
$\hat{\mathcal{C}}_D^E\Phi[A,\xi] = \frac{\delta}{\delta A^D_{aA}(x)}\frac{\delta}{\delta A^{AB}_{b]}(x)}F^{BE}_{ab}\Phi[A,\xi] = 0$
You can think of this as a reduction from supergravity:
$\Phi[A,\xi] = \Phi[A,\psi^A_a = \mathcal{D}_a\xi]$
12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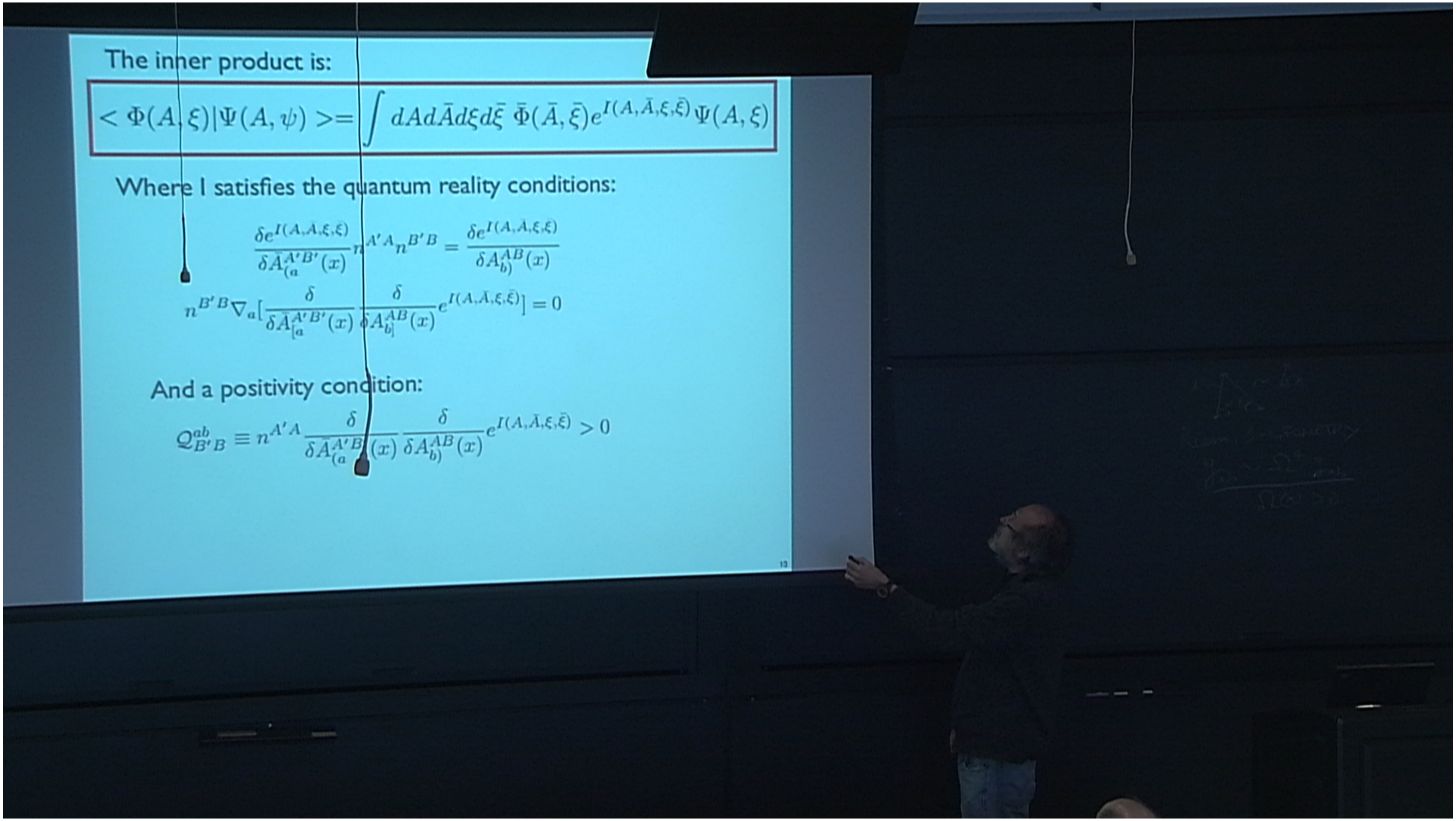
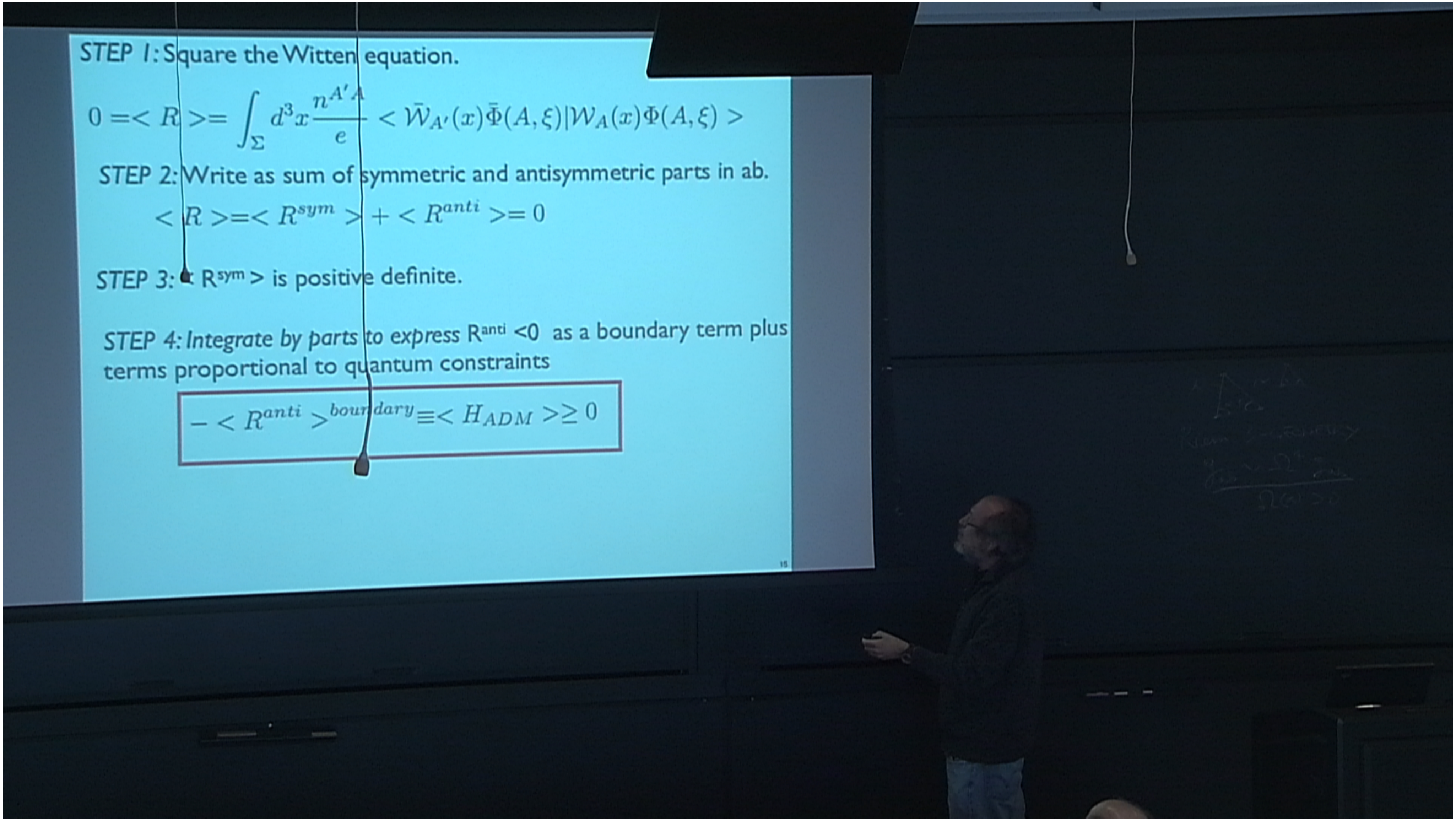
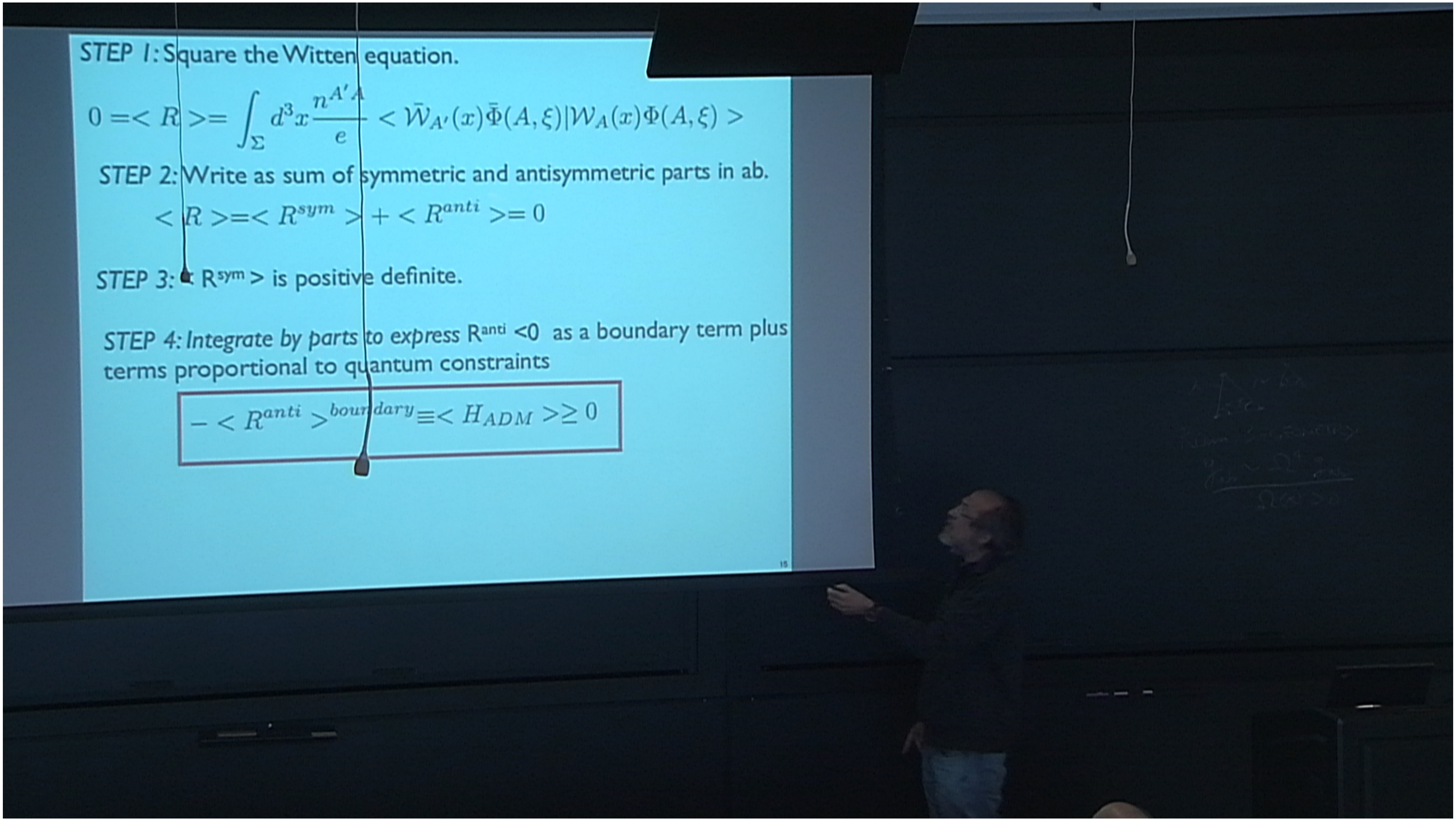

The inner product is:
$< \Phi(A,\xi)|\Psi(A,\psi) >= \int dAd\bar{A}d\xi d\bar{\xi}\ \bar{\Phi}(\bar{A},\bar{\xi})e^{I(A,\bar{A},\xi,\bar{\xi})}\Psi(A,\xi)$
Where I satisfies the quantum reality conditions:
$\frac{\delta e^{I(A,\bar{A},\xi,\bar{\xi})}}{\delta \bar{A}^{A'B'}_{(a}(x)} n^{A'A} n^{B'B} = \frac{\delta e^{I(A,\bar{A},\xi,\bar{\xi})}}{\delta A^{AB}_{b)}(x)}$
$n^{B'B}\nabla_a[\frac{\delta}{\delta \bar{A}^{A'B'}_{[a}(x)} \frac{\delta}{\delta A^{AB}_{b]}(x)} e^{I(A,\bar{A},\xi,\bar{\xi})}] = 0$
And a positivity condition:
$Q^{ab}_{B'B} \equiv n^{A'A} \frac{\delta}{\delta \bar{A}^{A'B'}_{(a}(x)} \frac{\delta}{\delta A^{AB}_{b)}(x)} e^{I(A,\bar{A},\xi,\bar{\xi})} > 0$


STEP 1: Square the Witten equation.
$0 =< R >= \int_\Sigma d^3x \frac{n^{A'A}}{e} < \bar{\mathcal{W}}_{A'}(x)\bar{\Phi}(A,\xi)|\mathcal{W}_A(x)\Phi(A,\xi) >$
STEP 2: Write as sum of symmetric and antisymmetric parts in ab.
$< R >=< R^{sym} > + < R^{anti} >= 0$
STEP 3: < R^{sym} > is positive definite.
STEP 4: Integrate by parts to express R^{anti} <0 as a boundary term plus terms proportional to quantum constraints
$- < R^{anti} >^{boundary} \equiv < H_{ADM} > \geq 0$
15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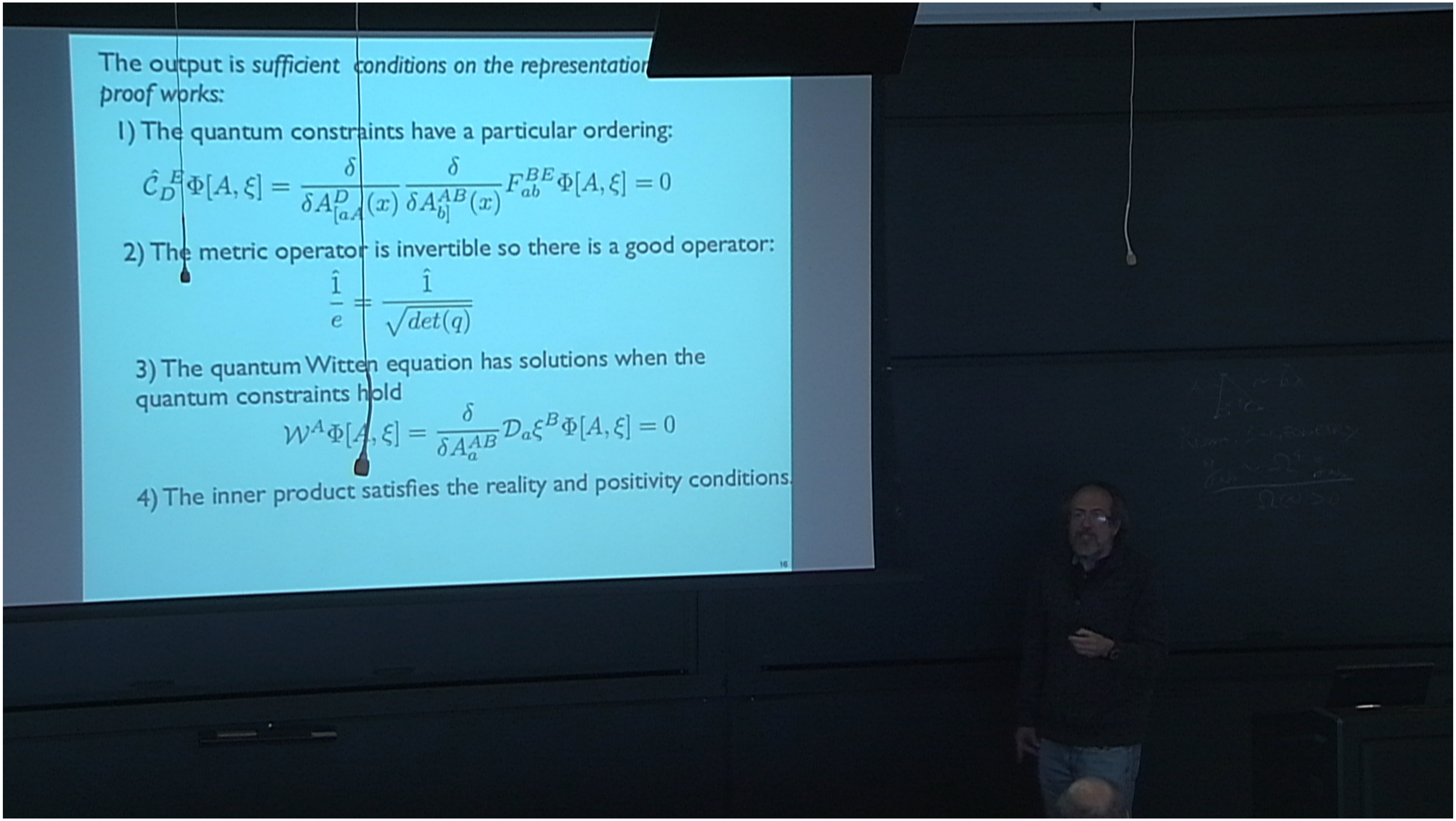
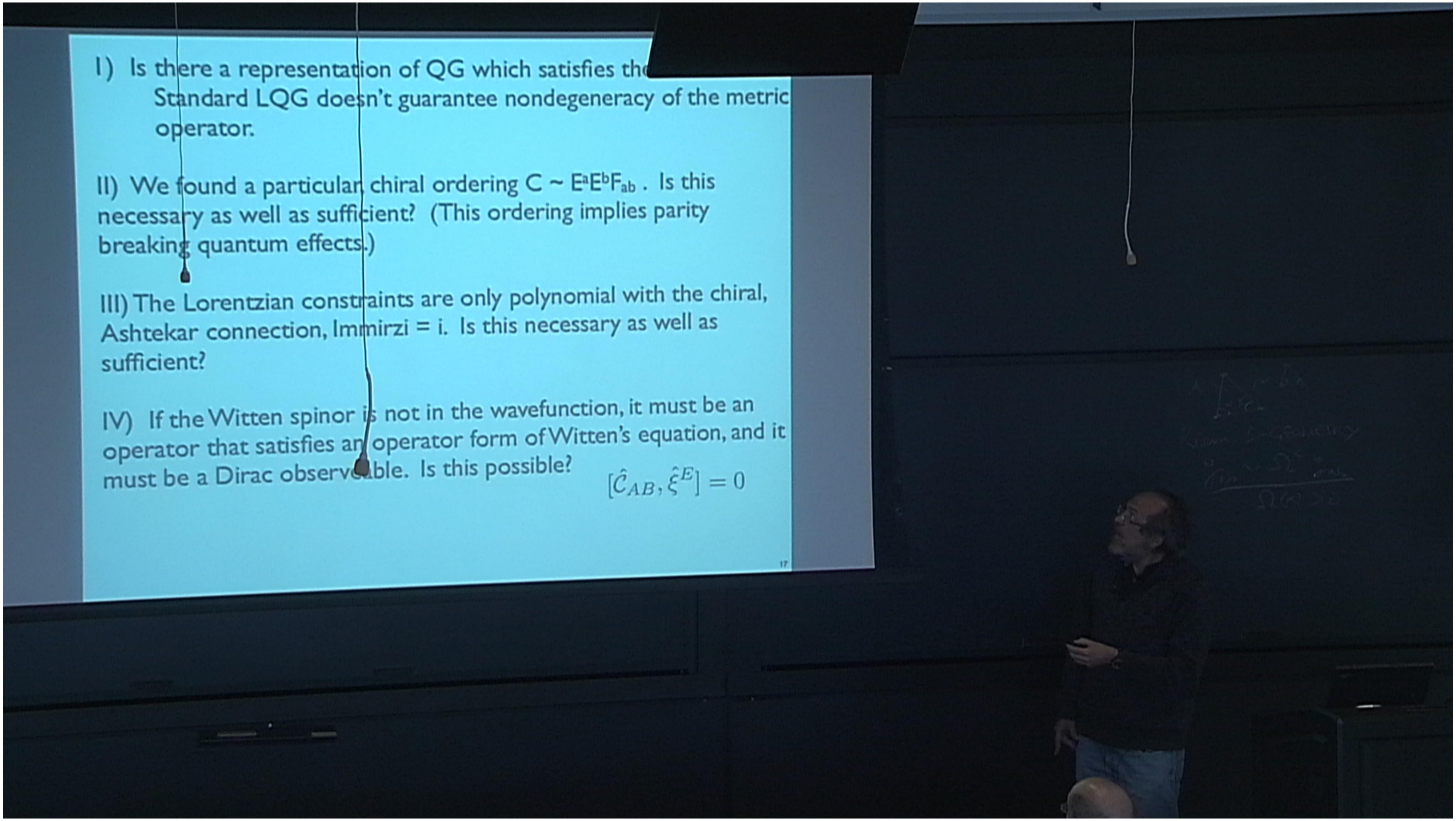

The output is sufficient conditions on the representation
proof works:
1) The quantum constraints have a particular ordering:
$\hat{C}_D^E \Phi[A,\xi] = \frac{\delta}{\delta A^D_{[aA}(x)} \frac{\delta}{\delta A^{AB}_{b]}(x)} F^{BE}_{ab} \Phi[A,\xi] = 0$
2) The metric operator is invertible so there is a good operator:
$\frac{\hat{1}}{e} = \frac{\hat{1}}{\sqrt{det(q)}}$
3) The quantum Witten equation has solutions when the quantum constraints hold
$\mathcal{W}^A \Phi[A,\xi] = \frac{\delta}{\delta A^{AB}_a} \mathcal{D}_a \xi^B \Phi[A,\xi] = 0$
4) The inner product satisfies the reality and positivity conditions.
16


I) Is there a representation of QG which satisfies the
Standard LQG doesn't guarantee nondegeneracy of the metric operator.
II) We found a particular chiral ordering C ~ $E^aE^bF_{ab}$. Is this necessary as well as sufficient? (This ordering implies parity breaking quantum effects.)
III) The Lorentzian constraints are only polynomial with the chiral, Ashtekar connection, Immirzi = i. Is this necessary as well as sufficient?
IV) If the Witten spinor is not in the wavefunction, it must be an operator that satisfies an operator form of Witten's equation, and it must be a Dirac observable. Is this possible?
$[\hat{C}_{AB}, \hat{\xi}^E] = 0$
17

I) Is there a representation of QG which satisfies the conditions? Standard LQG doesn't guarantee nondegeneracy of the metric operator.

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$$[\hat{\mathcal{C}}_{AB}, \hat{\xi}^{E}] = 0$$