Title: Phases for (analogue) spin foam models

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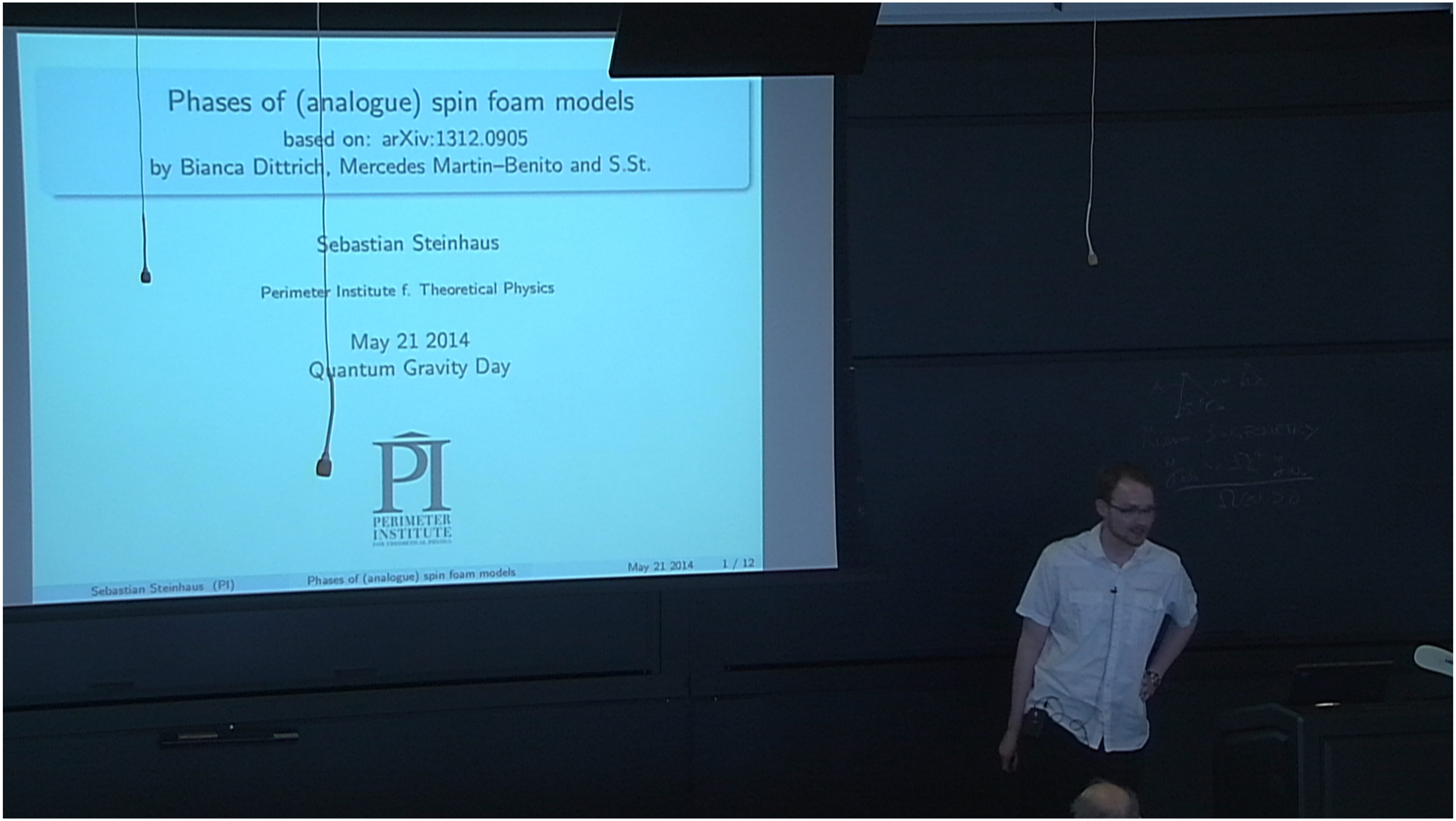
Abstract:


Phases of (analogue) spin foam models
based on: arXiv:1312.0905
by Bianca Dittrich, Mercedes Martin–Benito and S.St.
Sebastian Steinhaus
Perimeter Institute f. Theoretical Physics
May 21 2014
Quantum Gravity Day
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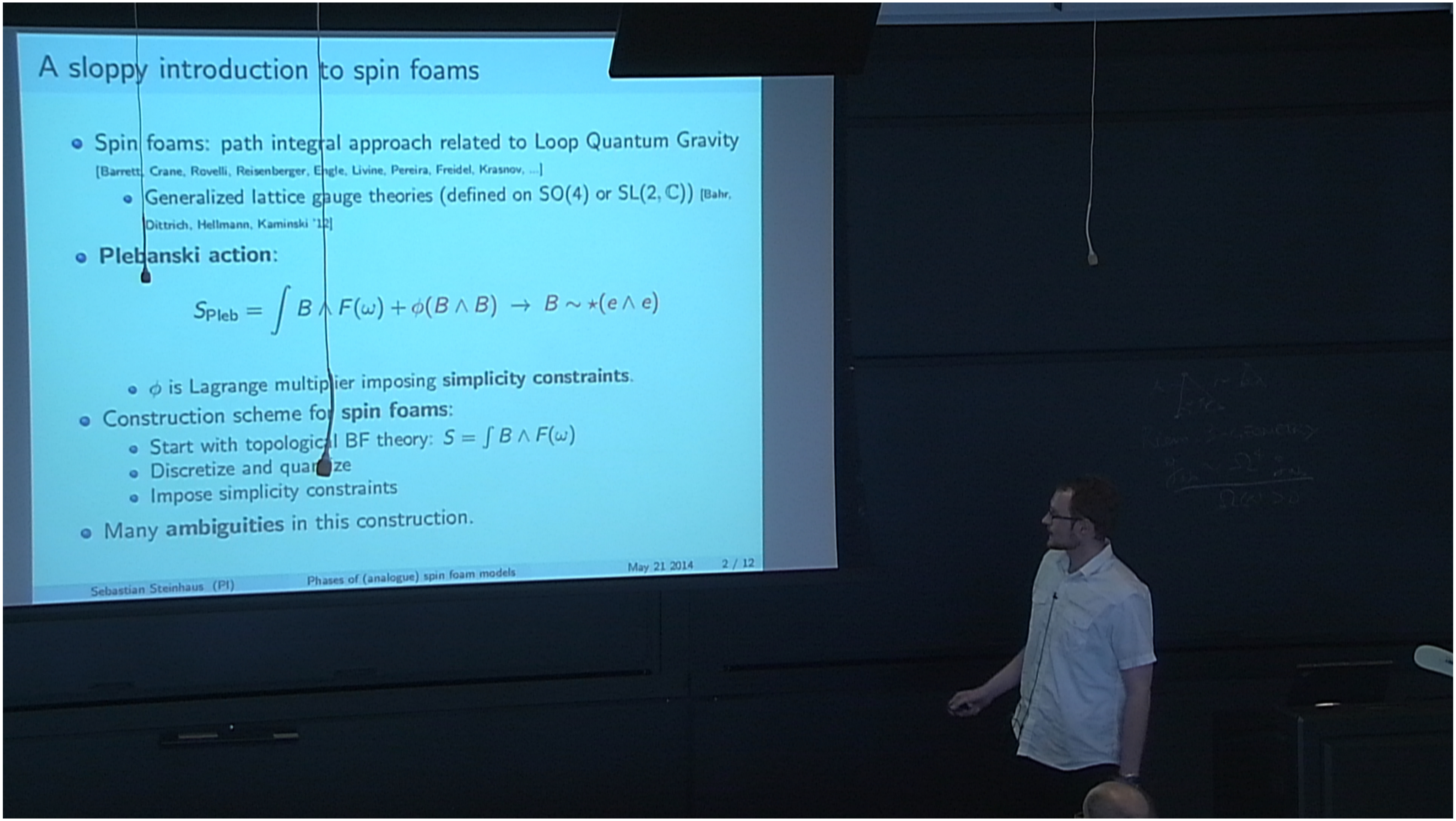
A sloppy introduction to spin foams
Spin foams: path integral approach related to Loop Quantum Gravity
[Barrett, Crane, Rovelli, Reisenberger, Engle, Livine, Pereira, Freidel, Krasnov, ...]
Generalized lattice gauge theories (defined on SO(4) or SL(2, ℂ)) [Bahr, Dittrich, Hellmann, Kaminski '12]
Plebanski action:
$S_{\text{Pleb}} = \int B \wedge F(\omega) + \phi(B \wedge B) \rightarrow B \sim \star(e \wedge e)$
ϕ is Lagrange multiplier imposing simplicity constraints.
Construction scheme for spin foams:
Start with topological BF theory: $S = \int B \wedge F(\omega)$
Discretize and quantize
Impose simplicity constraints
Many ambiguities in this construction.
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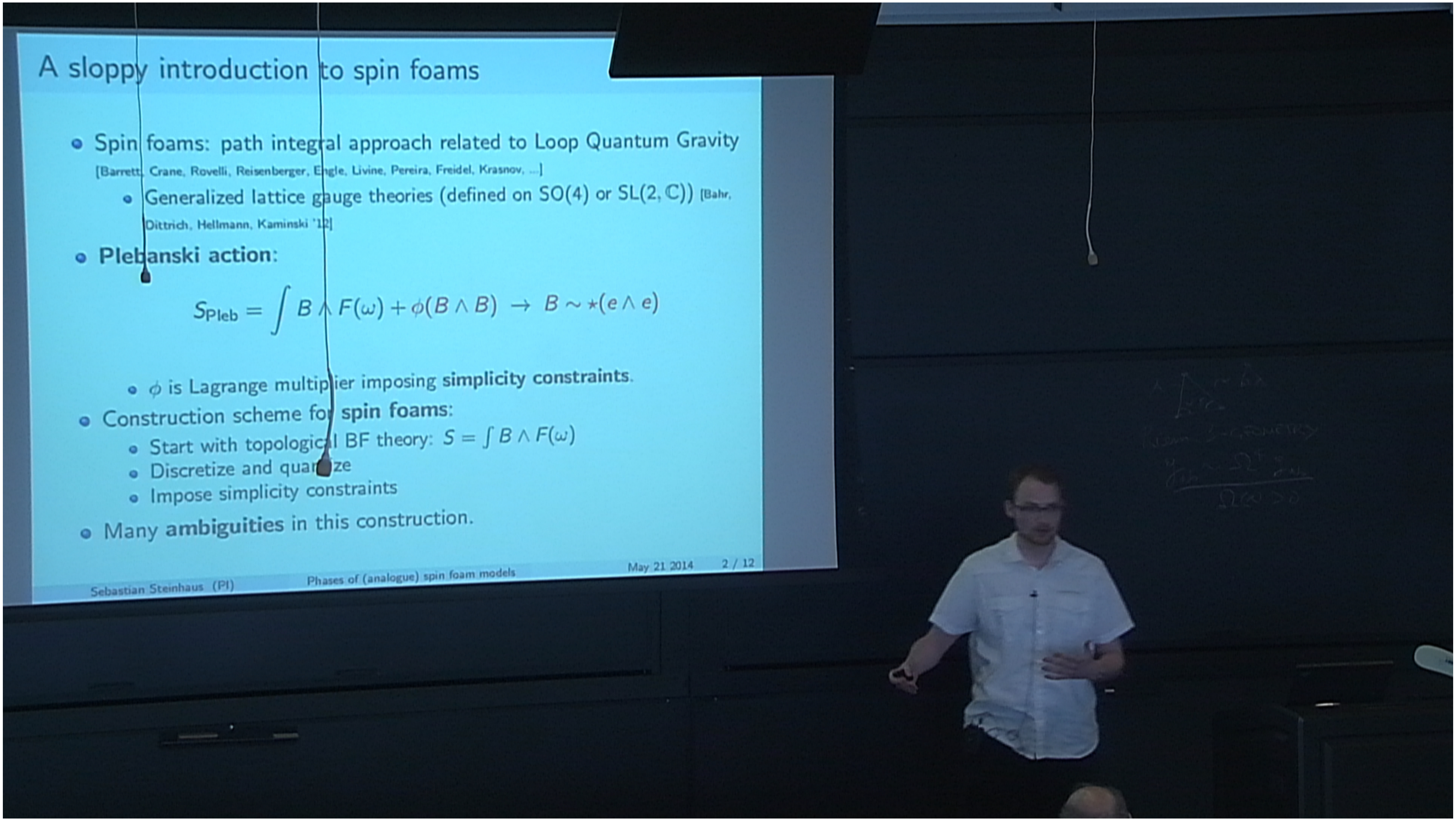
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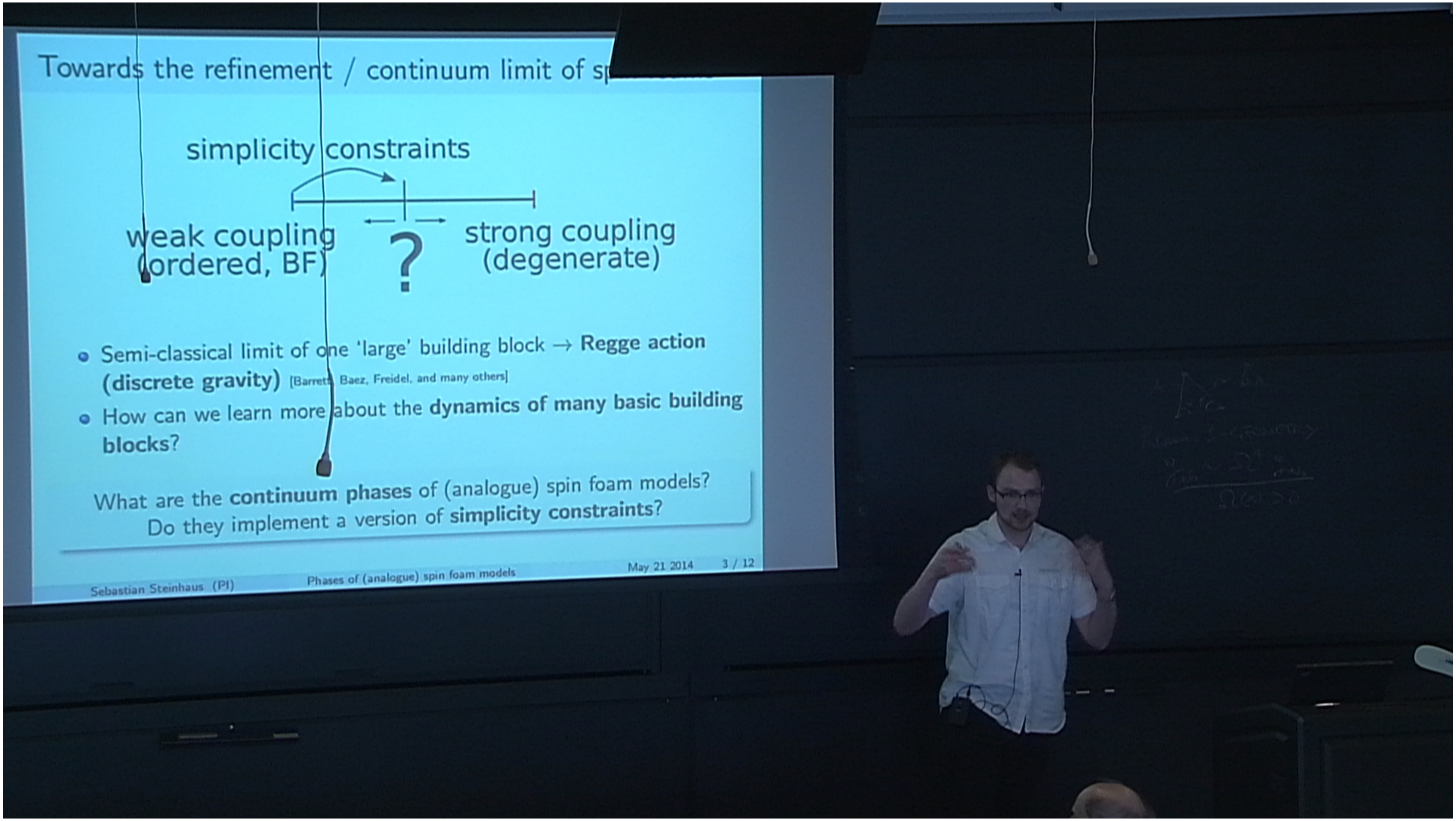
Towards the refinement / continuum limit of sp
simplicity constraints
weak coupling
(ordered, BF)
?
strong coupling
(degenerate)
Semi-classical limit of one 'large' building block → Regge action (discrete gravity) [Barrett, Baez, Freidel, and many others]
How can we learn more about the dynamics of many basic building blocks?
What are the continuum phases of (analogue) spin foam models?
Do they implement a version of simplicity constraints?
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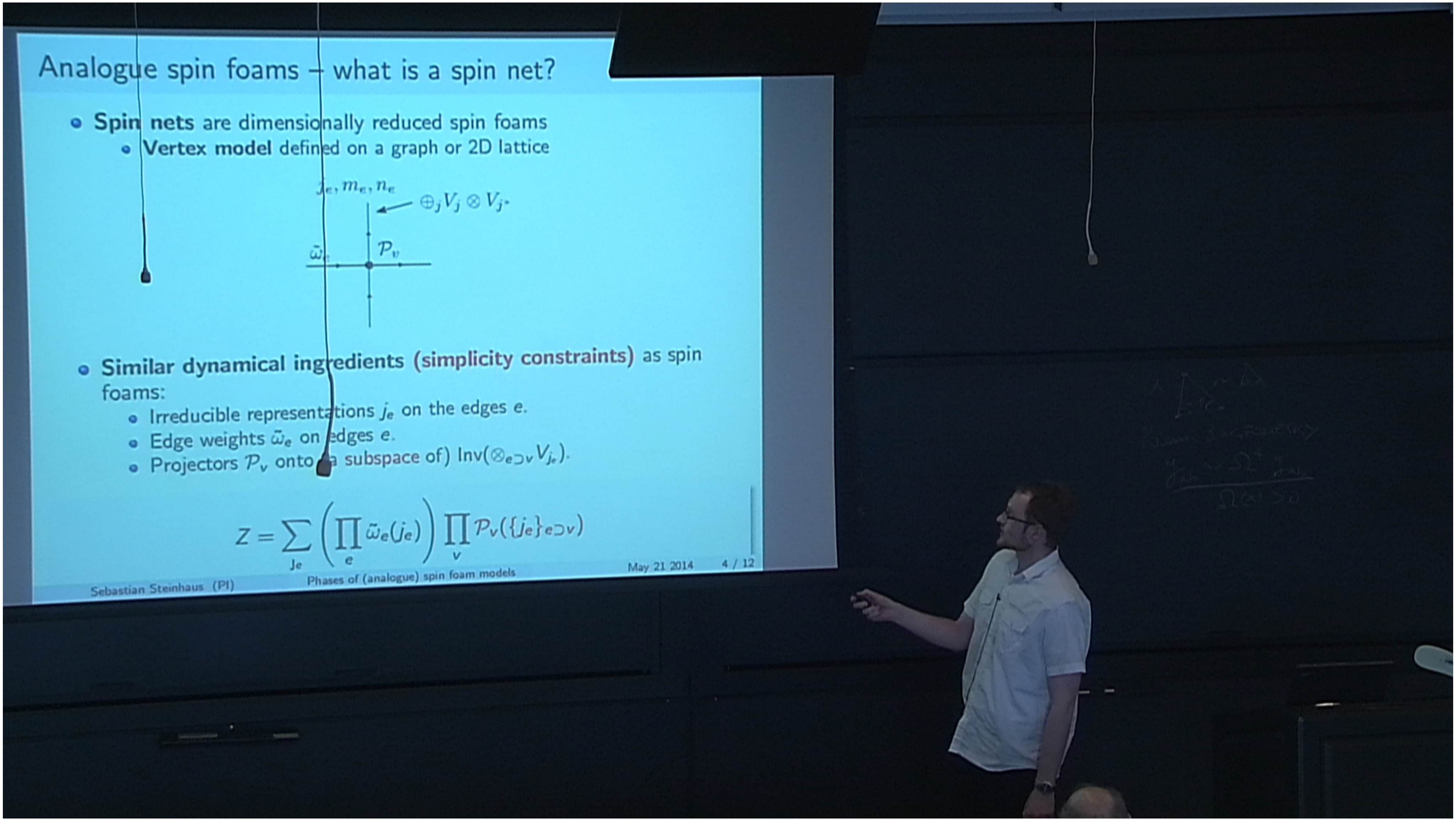
Analogue spin foams – what is a spin net?
Spin nets are dimensionally reduced spin foams
Vertex model defined on a graph or 2D lattice
j_e, m_e, n_e
$\oplus_j V_j \otimes V_{j^*}$
$\tilde{\omega}_e$
$\mathcal{P}_v$
Similar dynamical ingredients (simplicity constraints) as spin foams:
Irreducible representations j_e on the edges e.
Edge weights $\tilde{\omega}_e$ on edges e.
Projectors $\mathcal{P}_v$ onto (a subspace of) $\mathrm{Inv}(\otimes_{e\supset v} V_{j_e})$.
$Z = \sum_{j_e} \left(\prod_e \tilde{\omega}_e(j_e) \right) \prod_v \mathcal{P}_v(\{j_e\}_{e\supset v})$
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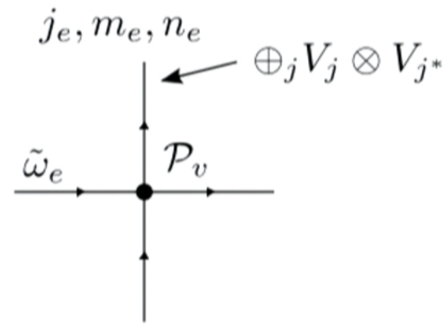
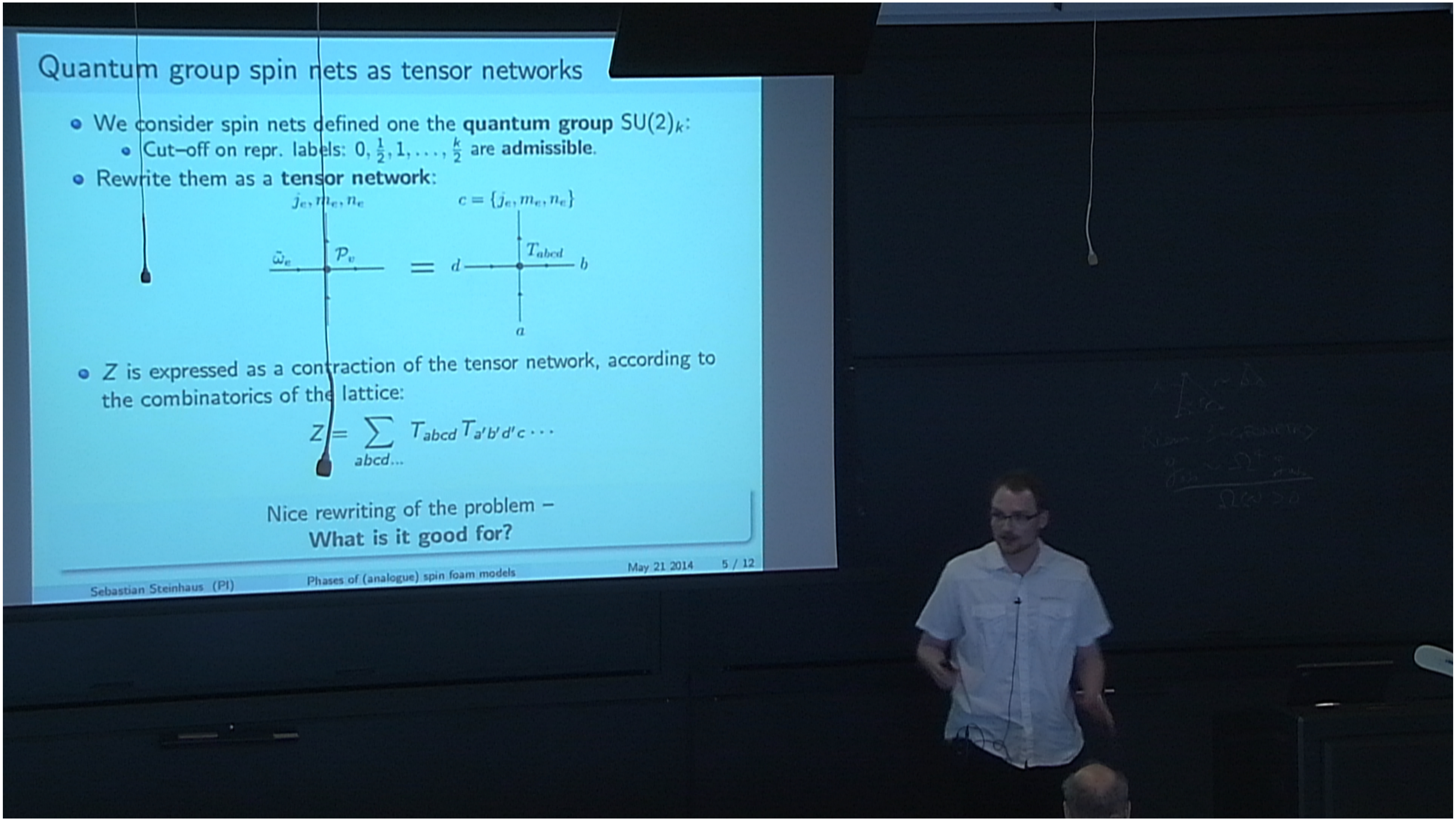
Analogue spin foams – what is a spin net?

- **Spin nets** are dimensionally reduced spin foams
  - **Vertex model** defined on a graph or 2D lattice



- **Similar dynamical ingredients (simplicity constraints)** as spin foams:
  - Irreducible representations j_e on the edges e.
  - Edge weights $\tilde{\omega}_e$ on edges e.
  - Projectors $\mathcal{P}_v$ onto (a subspace of) $\text{Inv}(\otimes_{e \supset v} V_{j_e})$.

$$Z = \sum_{j_e} \left(\prod_e \tilde{\omega}_e(j_e) \right) \prod_v \mathcal{P}_v(\{j_e\}_{e \supset v})$$


Quantum group spin nets as tensor networks
We consider spin nets defined one the quantum group SU(2)_k:
Cut–off on repr. labels: 0, 1/2, 1, ..., k/2 are admissible.
Rewrite them as a tensor network:
j_e, m_e, n_e
c = {j_e, m_e, n_e}
ω̃_e
P_v
T_abcd
d
b
a
Z is expressed as a contraction of the tensor network, according to the combinatorics of the lattice:
Z = Σ_abcd... T_abcd T_a'b'd'c ···
Nice rewriting of the problem –
What is it good for?
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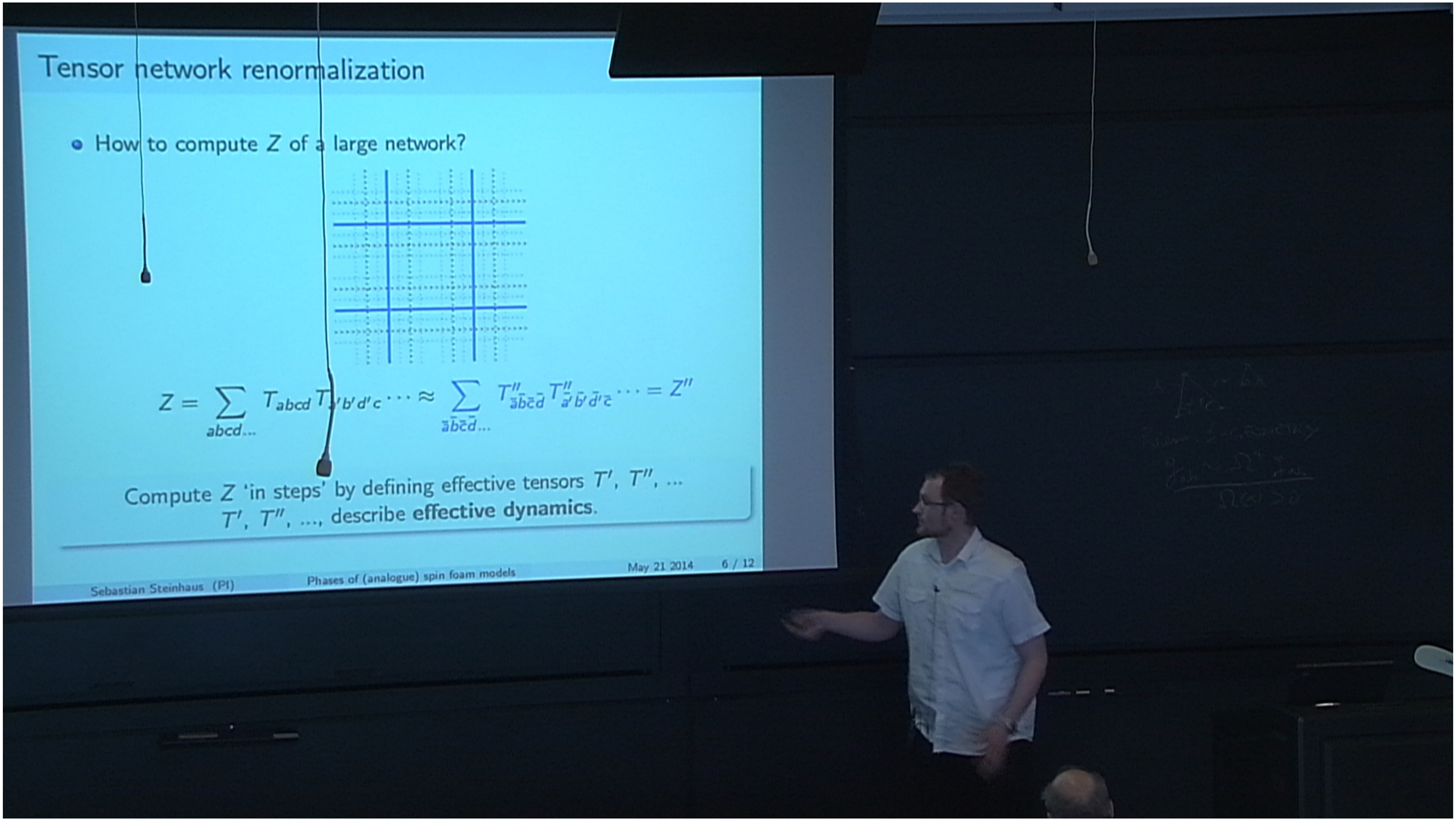
Tensor network renormalization
How to compute Z of a large network?
$Z = \sum_{abcd...} T_{abcd} T_{a'b'd'c} \cdots \approx \sum_{\bar{a}\bar{b}\bar{c}\bar{d}...} T''_{\bar{a}\bar{b}\bar{c}\bar{d}} T''_{\bar{a}'\bar{b}'\bar{d}'\bar{c}} \cdots = Z''$
Compute Z 'in steps' by defining effective tensors T′, T″, ...
T′, T″, ..., describe effective dynamics.
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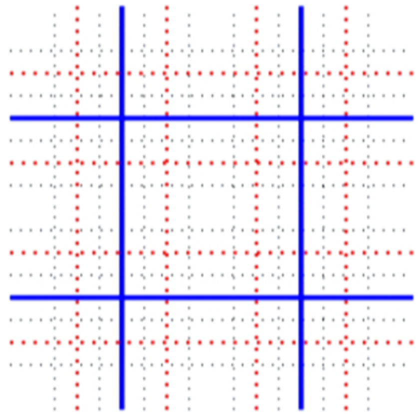
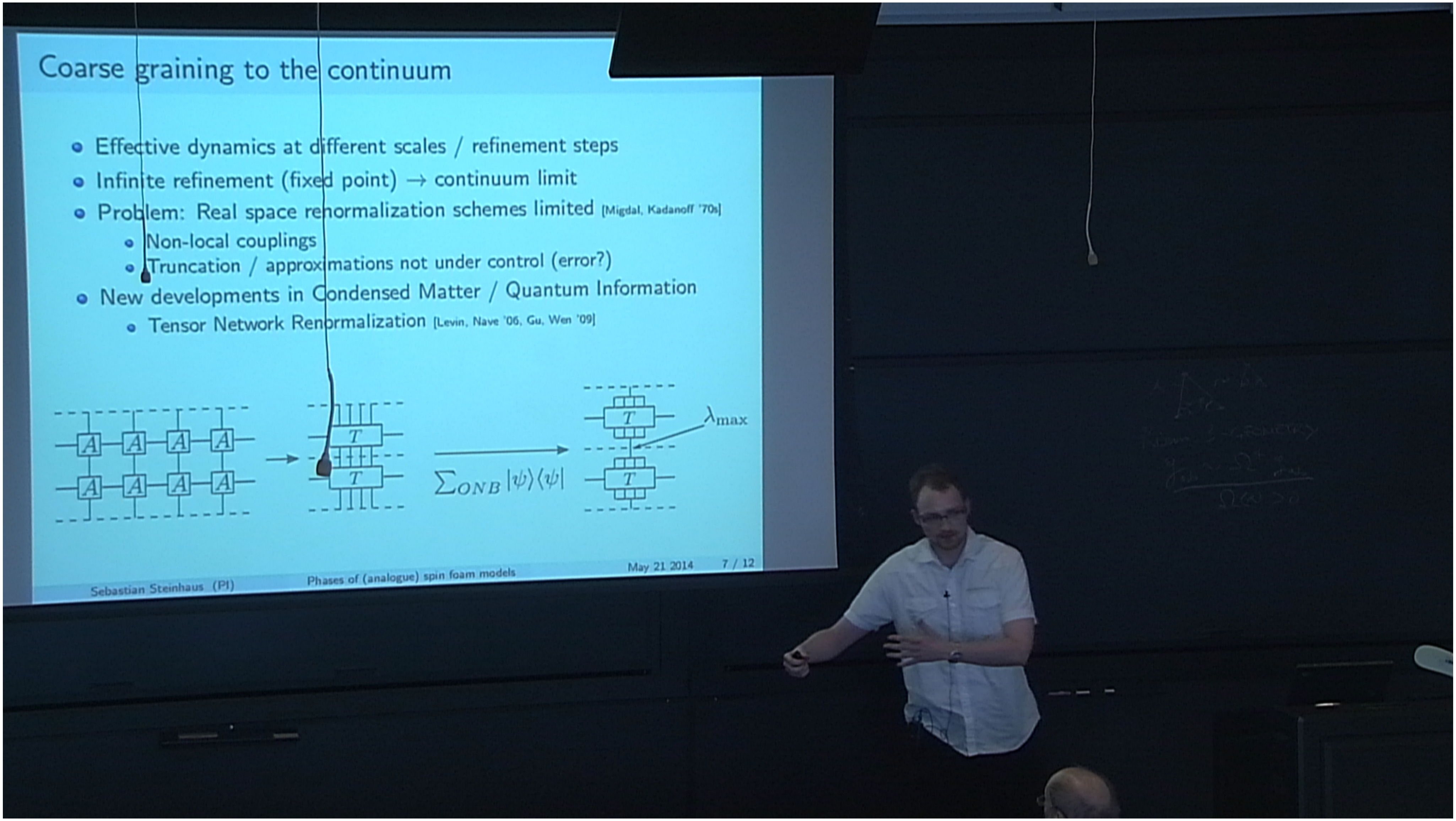
Tensor network renormalization

- How to compute Z of a large network?



$$Z = \sum_{abcd\ldots} T_{abcd} T_{a'b'd'c} \cdots \approx \sum_{\bar{a}\bar{b}\bar{c}\bar{d}\ldots} T''_{\bar{a}\bar{b}\bar{c}\bar{d}} T''_{\bar{a}'\bar{b}'\bar{d}'\bar{c}} \cdots = Z''$$

Compute Z ‘in steps’ by defining effective tensors T', T'', ...
T', T'', ..., describe **effective dynamics**.


Coarse graining to the continuum
Effective dynamics at different scales / refinement steps
Infinite refinement (fixed point) → continuum limit
Problem: Real space renormalization schemes limited [Migdal, Kadanoff '70s]
Non-local couplings
Truncation / approximations not under control (error?)
New developments in Condensed Matter / Quantum Information
Tensor Network Renormalization [Levin, Nave '06, Gu, Wen '09]
A
T
$\sum_{ONB} |\psi\rangle\langle\psi|$
λ_{max}
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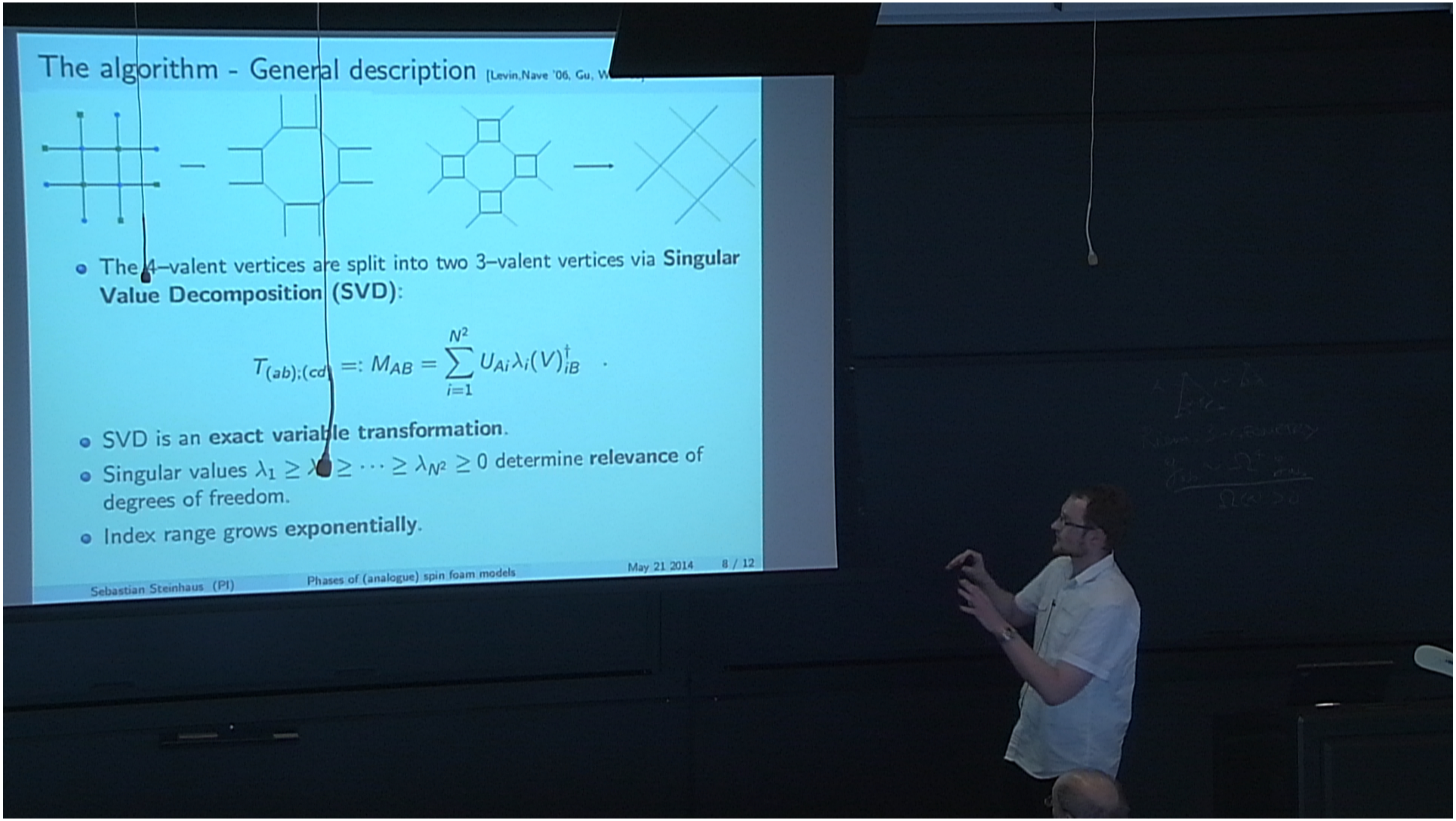
The algorithm - General description [Levin,Nave '06, Gu, W
The 4–valent vertices are split into two 3–valent vertices via Singular Value Decomposition (SVD):
$T_{(ab);(cd)} =: M_{AB} = \sum_{i=1}^{N^2} U_{Ai} \lambda_i (V)^{\dagger}_{iB}$.
SVD is an exact variable transformation.
Singular values $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{N^2} \geq 0$ determine relevance of degrees of freedom.
Index range grows exponentially.
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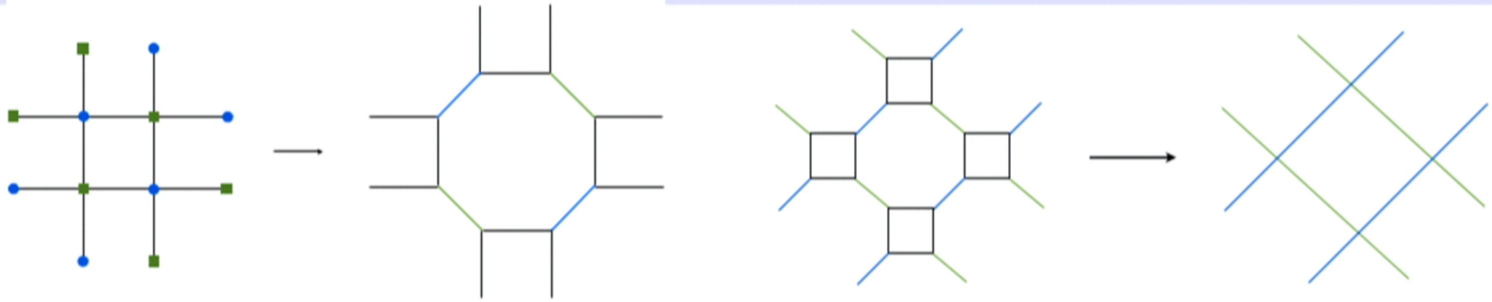
The algorithm - General description [Levin,Nave '06, Gu, Wen '09]



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- SVD is an **exact variable transformation**.
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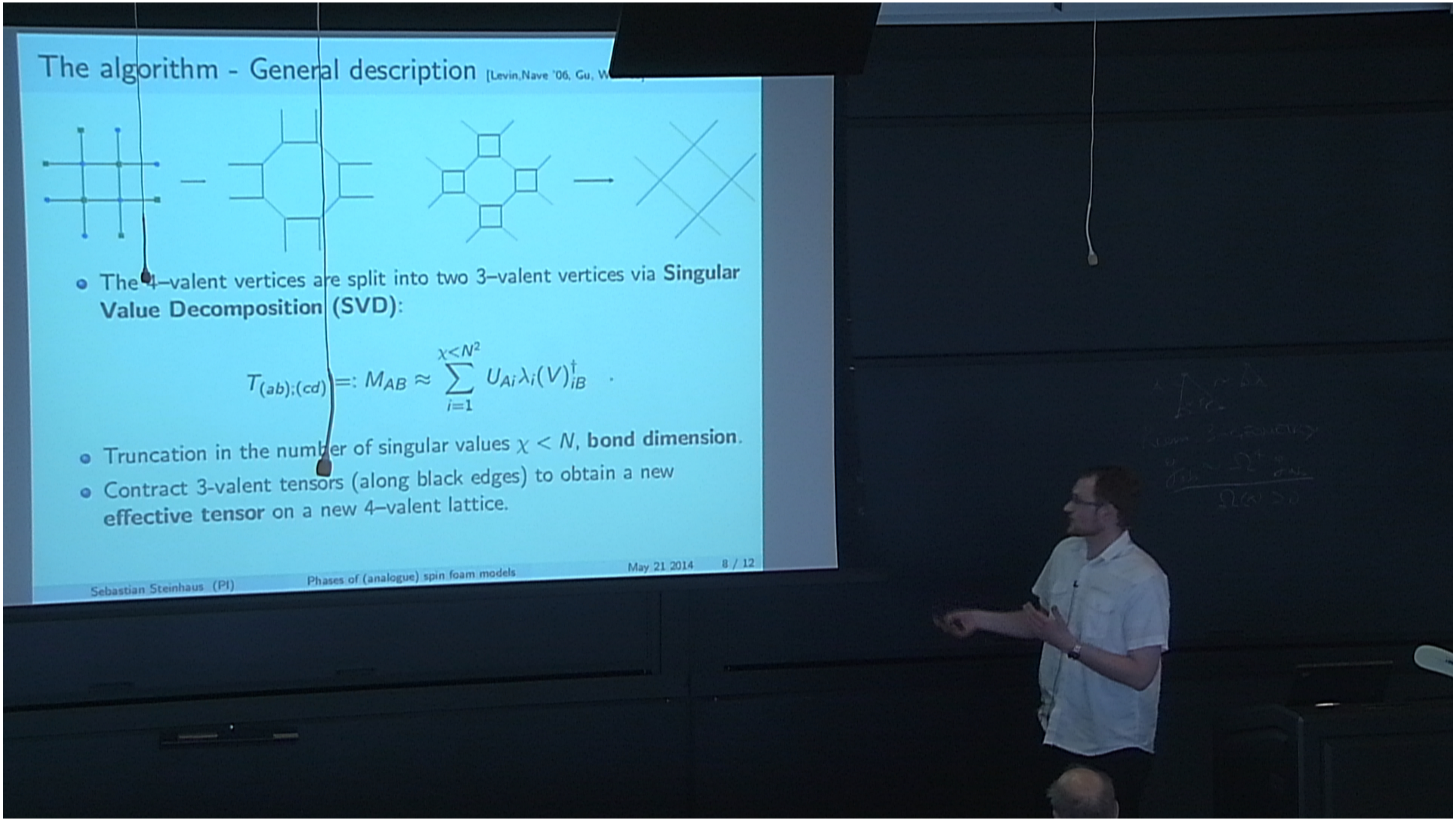
- Index range grows **exponentially**.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The 4–valent vertices are split into two 3–valent vertices via Singular Value Decomposition (SVD):
$T_{(ab);(cd)} =: M_{AB} \approx \sum_{i=1}^{\chi<N^2} U_{Ai}\lambda_i(V)^\dagger_{iB}$.
Truncation in the number of singular values $\chi < N$, bond dimension.
Contract 3-valent tensors (along black edges) to obtain a new effective tensor on a new 4–valent lattice.
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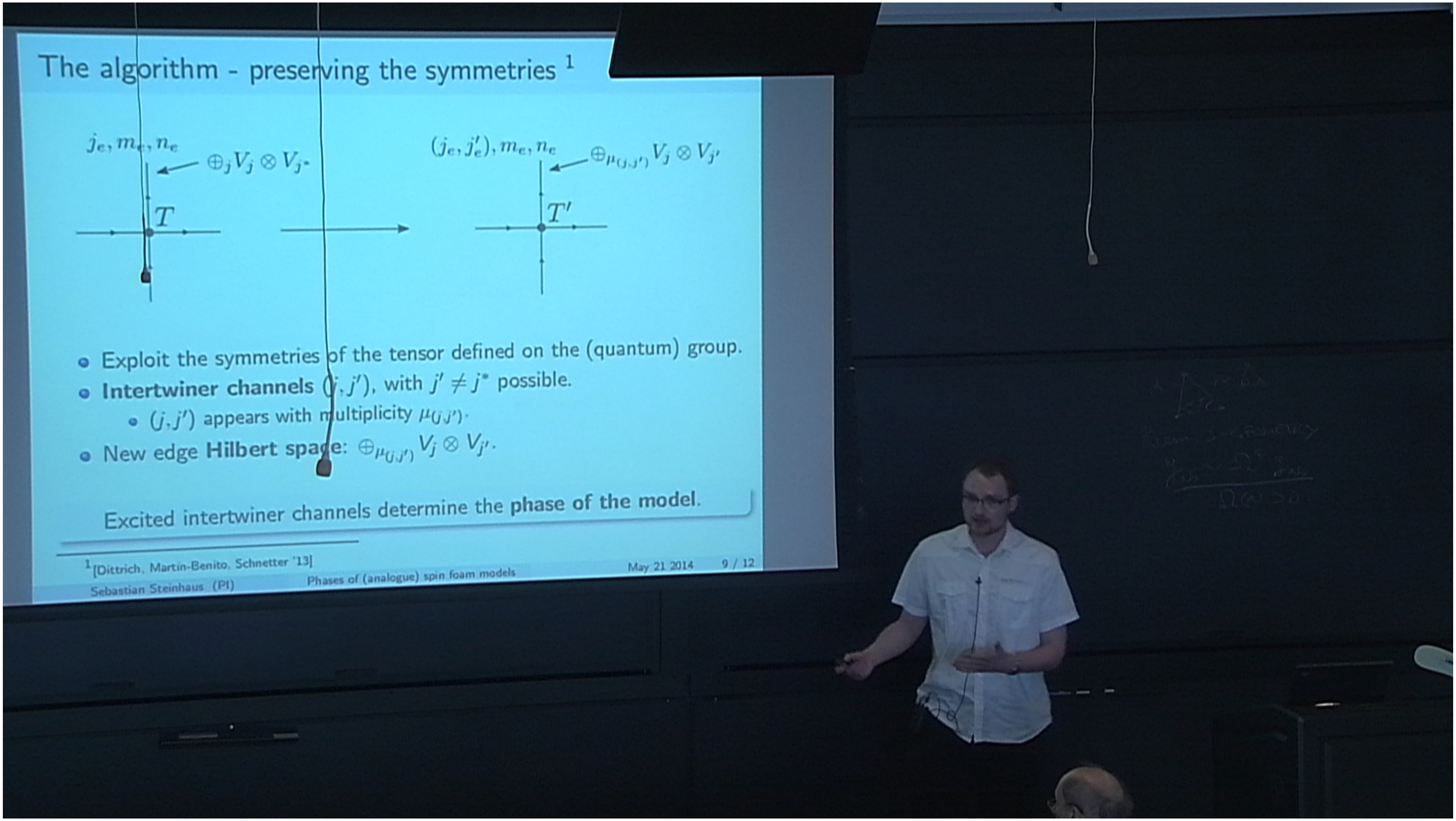
The algorithm - preserving the symmetries [1]
j_e, m_e, n_e
$\oplus_j V_j \otimes V_{j^*}$
T
$(j_e, j'_e), m_e, n_e$
$\oplus_{\mu_{(j,j')}} V_j \otimes V_{j'}$
T'
Exploit the symmetries of the tensor defined on the (quantum) group.
Intertwiner channels (j, j'), with $j' \neq j^*$ possible.
(j, j') appears with multiplicity $\mu_{(j,j')}$.
New edge **Hilbert space**: $\oplus_{\mu_{(j,j')}} V_j \otimes V_{j'}$.
Excited intertwiner channels determine the **phase of the model**.
[1] [Dittrich, Martin-Benito, Schnetter '13]
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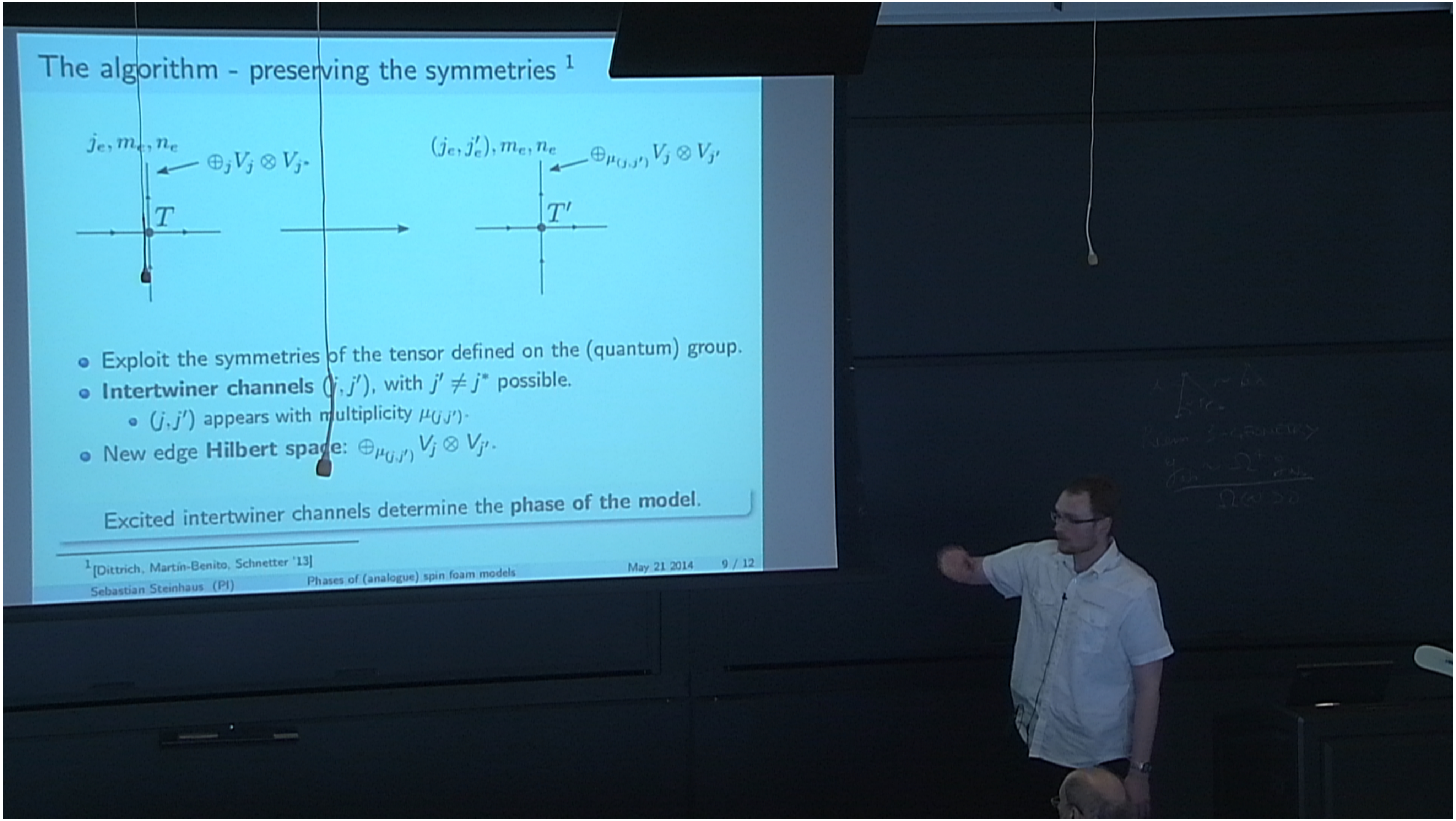
The algorithm - preserving the symmetries 1
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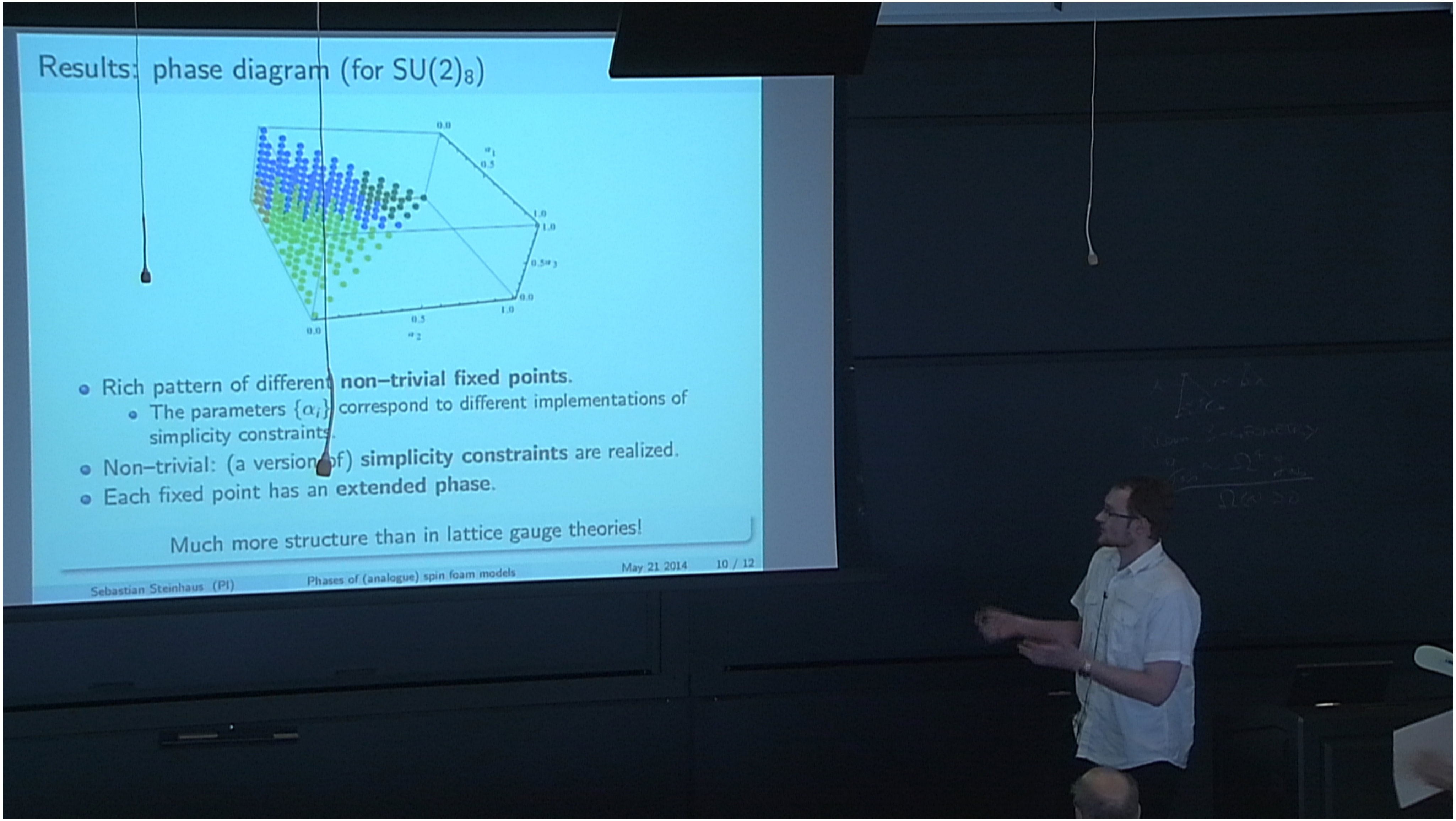
Results: phase diagram (for SU(2)8)
Rich pattern of different non–trivial fixed points.
The parameters {αi} correspond to different implementations of simplicity constraints.
Non–trivial: (a version of) simplicity constraints are realized.
Each fixed point has an extended phase.
Much more structure than in lattice gauge theories!
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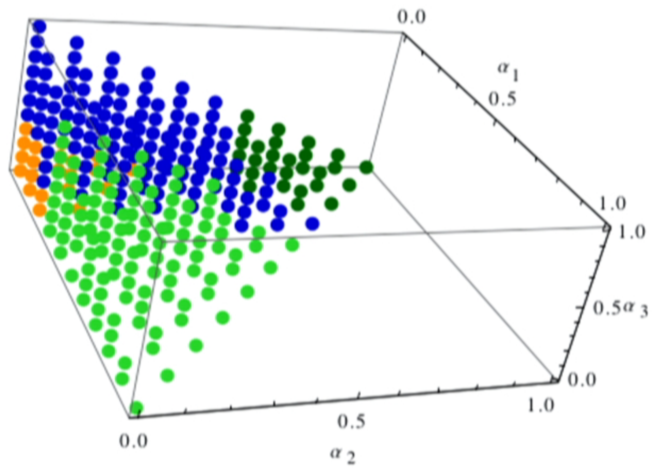
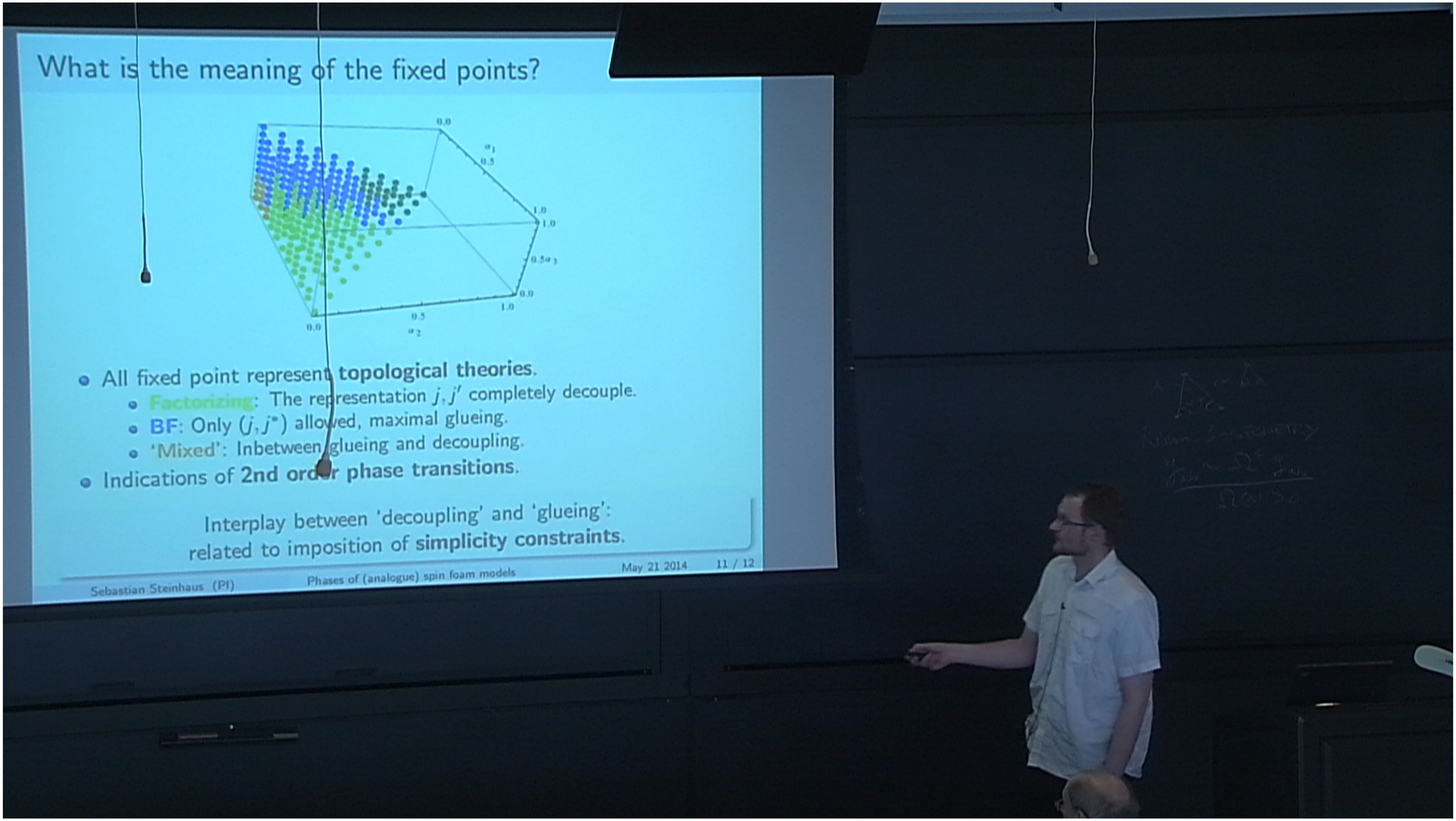
Results: phase diagram (for $SU(2)_8$)



- Rich pattern of different **non–trivial fixed points**.
  - The parameters $\{\alpha_i\}$ correspond to different implementations of simplicity constraints.
- Non–trivial: (a version of) **simplicity constraints** are realized.
- Each fixed point has an **extended phase**.

Much more structure than in lattice gauge theories!


What is the meaning of the fixed points?
0.0
α_1
0.5
1.0
1.0
$0.5\,\alpha_3$
0.0
1.0
0.5
0.0
α_2
All fixed point represent topological theories.
Factorizing: The representation j, j' completely decouple.
BF: Only (j, j^*) allowed, maximal glueing.
'Mixed': Inbetween glueing and decoupling.
Indications of 2nd order phase transitions.
Interplay between 'decoupling' and 'glueing':
related to imposition of simplicity constraints.
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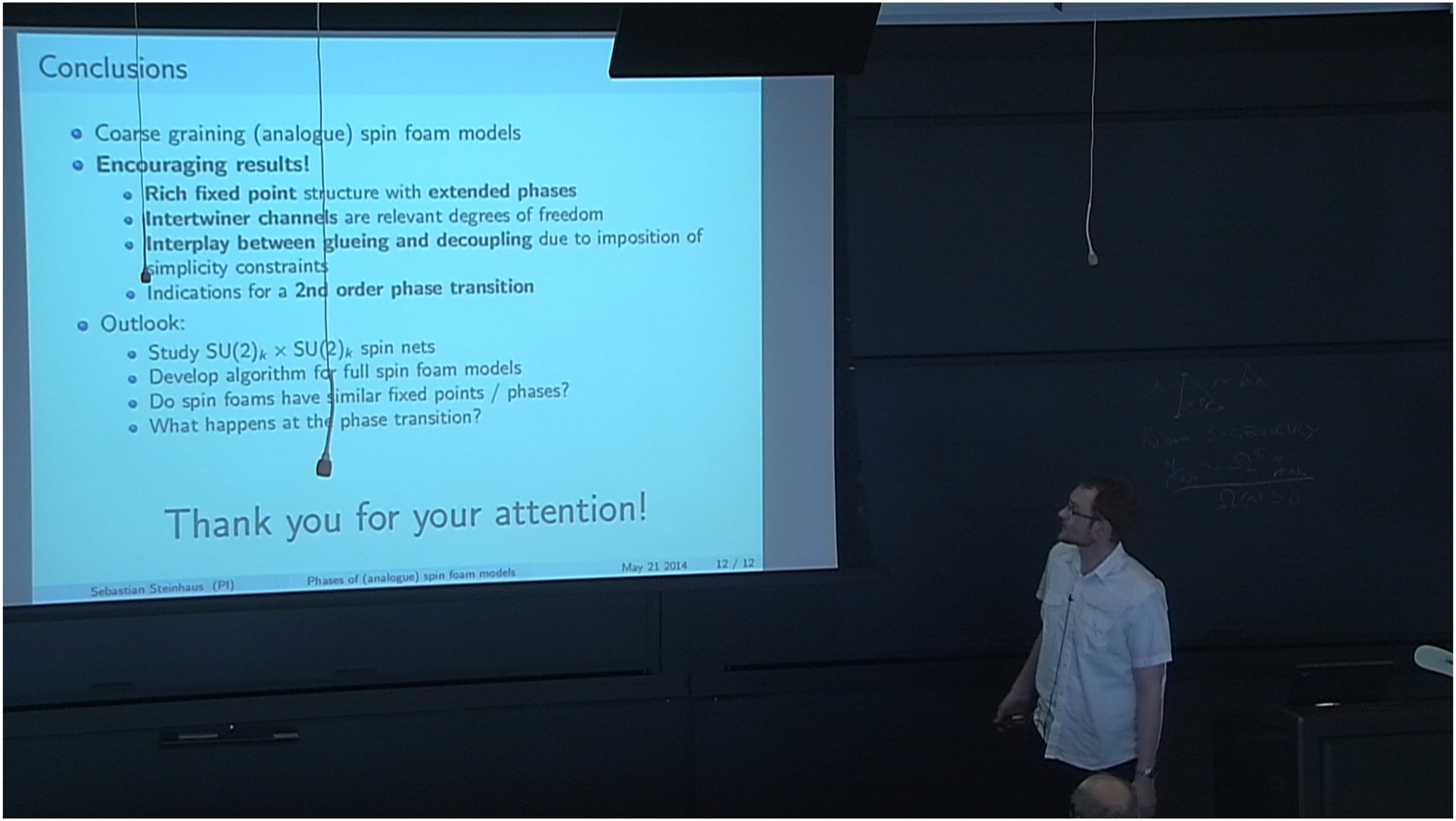
Conclusions
Coarse graining (analogue) spin foam models
Encouraging results!
Rich fixed point structure with extended phases
Intertwiner channels are relevant degrees of freedom
Interplay between glueing and decoupling due to imposition of simplicity constraints
Indications for a 2nd order phase transition
Outlook:
Study SU(2)k × SU(2)k spin nets
Develop algorithm for full spin foam models
Do spin foams have similar fixed points / phases?
What happens at the phase transition?
Thank you for your attention!
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