Title: Entanglement and Edge Modes in Gauge Theory and Gravity

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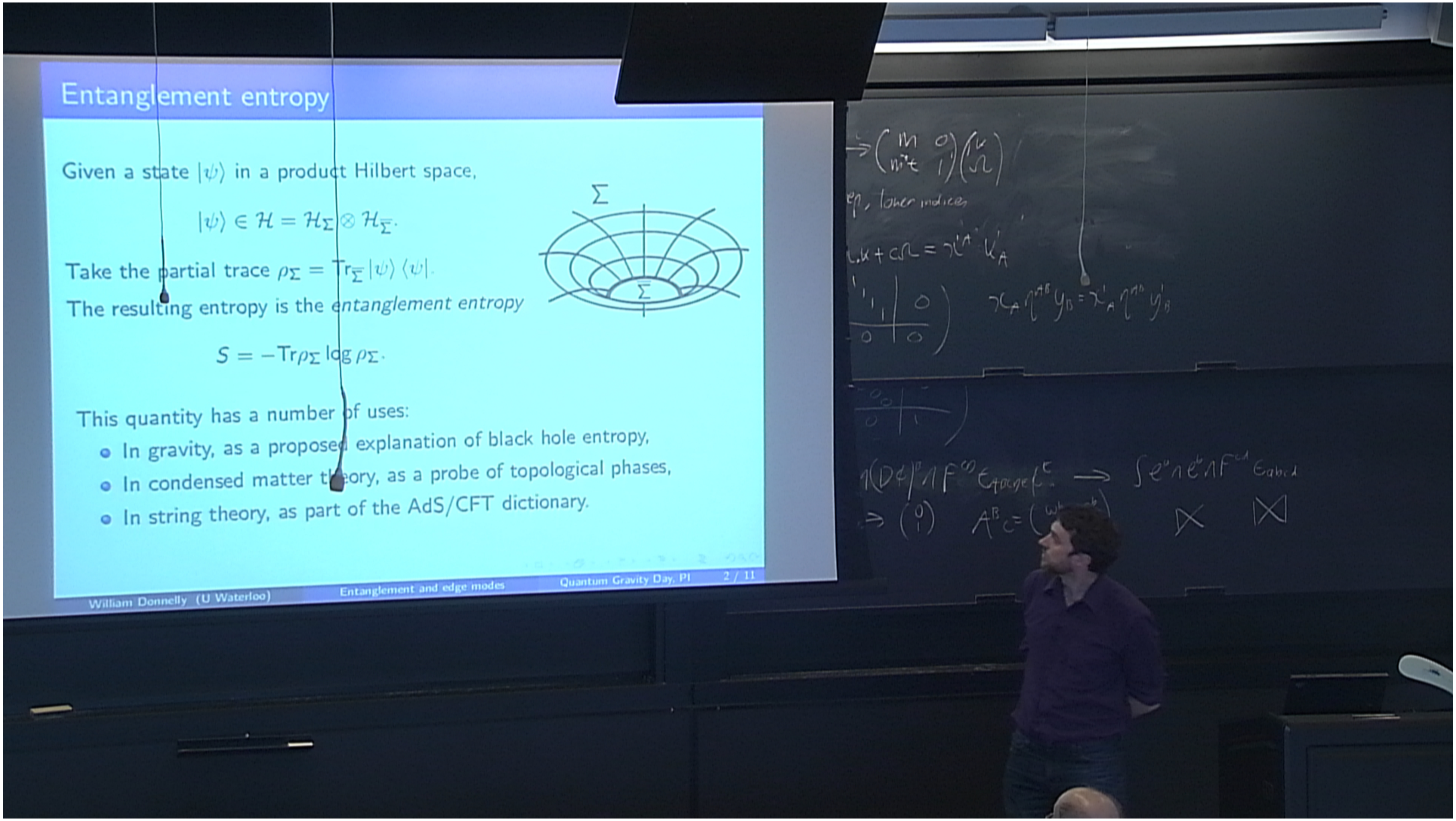
Abstract:


Entanglement entropy
Given a state $|\psi\rangle$ in a product Hilbert space,
$|\psi\rangle \in \mathcal{H} = \mathcal{H}_\Sigma \otimes \mathcal{H}_{\bar{\Sigma}}$.
Take the partial trace $\rho_\Sigma = \mathrm{Tr}_{\bar{\Sigma}} |\psi\rangle \langle\psi|$.
The resulting entropy is the *entanglement entropy*
$S = -\mathrm{Tr}\rho_\Sigma \log \rho_\Sigma$.
This quantity has a number of uses:
In gravity, as a proposed explanation of black hole entropy,
In condensed matter theory, as a probe of topological phases,
In string theory, as part of the AdS/CFT dictionary.
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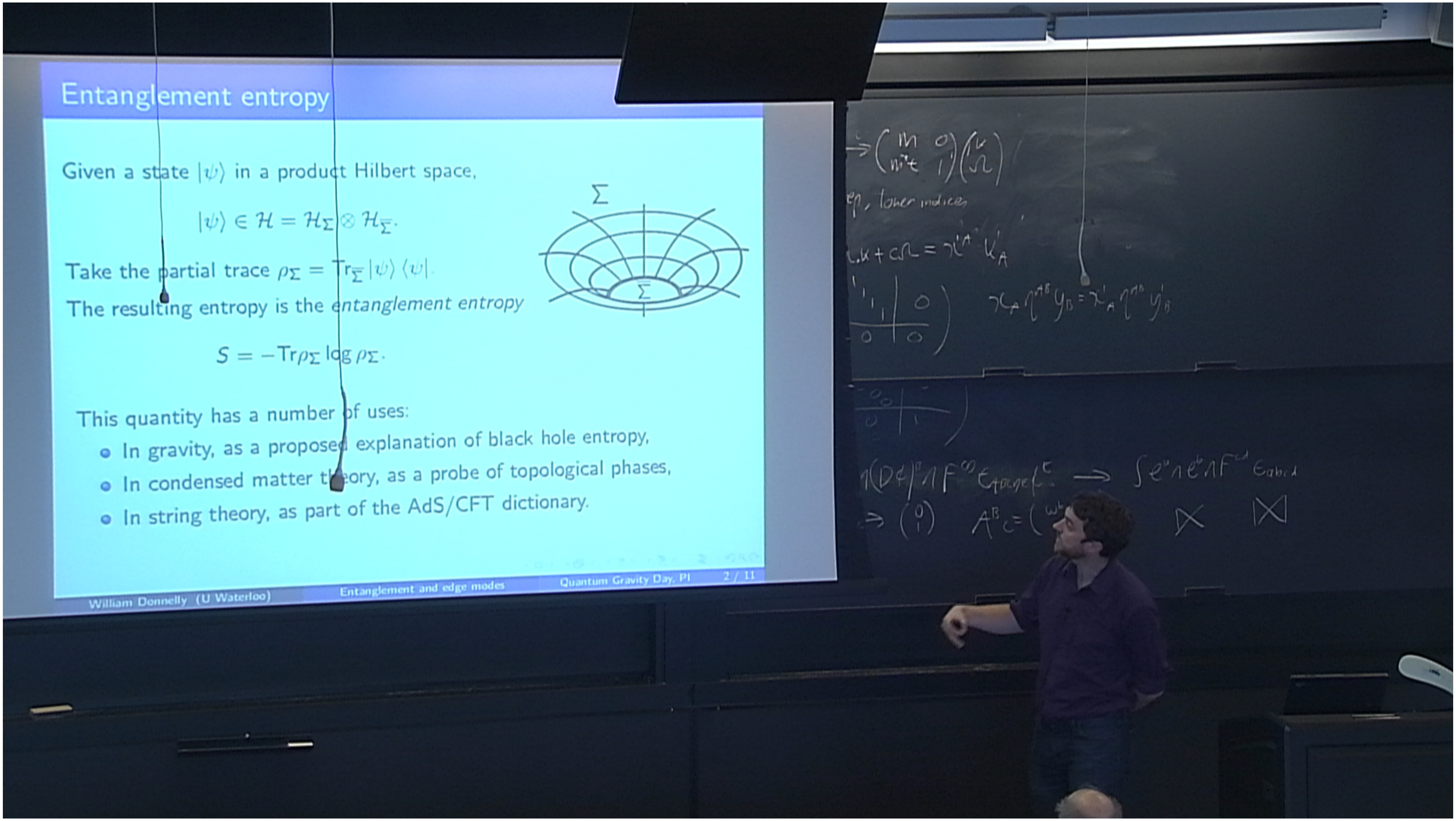
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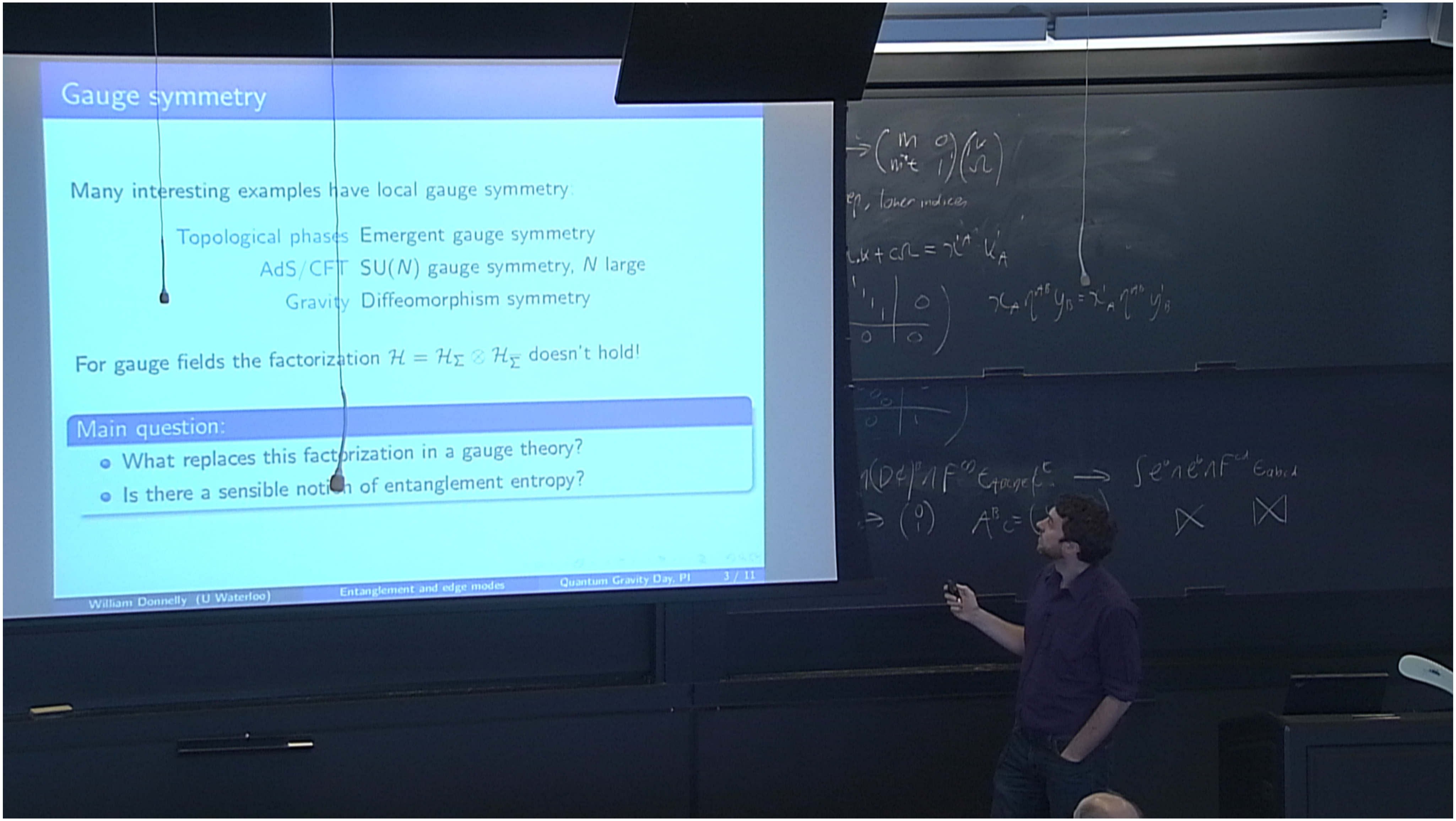
Gauge symmetry
Many interesting examples have local gauge symmetry:
Topological phases Emergent gauge symmetry
AdS/CFT SU(N) gauge symmetry, N large
Gravity Diffeomorphism symmetry
For gauge fields the factorization H = H_Σ ⊗ H_Σ̄ doesn't hold!
Main question:
What replaces this factorization in a gauge theory?
Is there a sensible notion of entanglement entropy?
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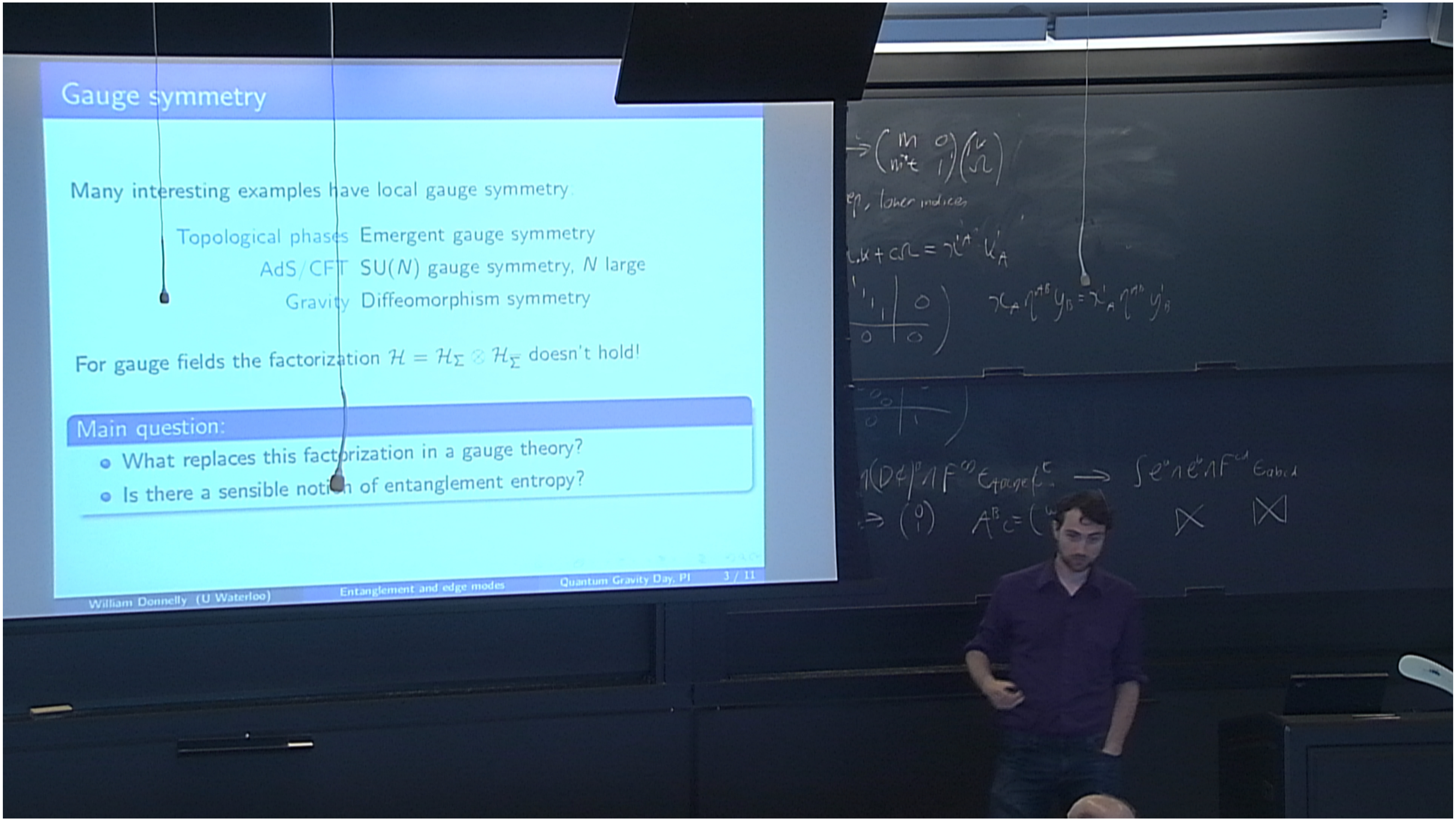
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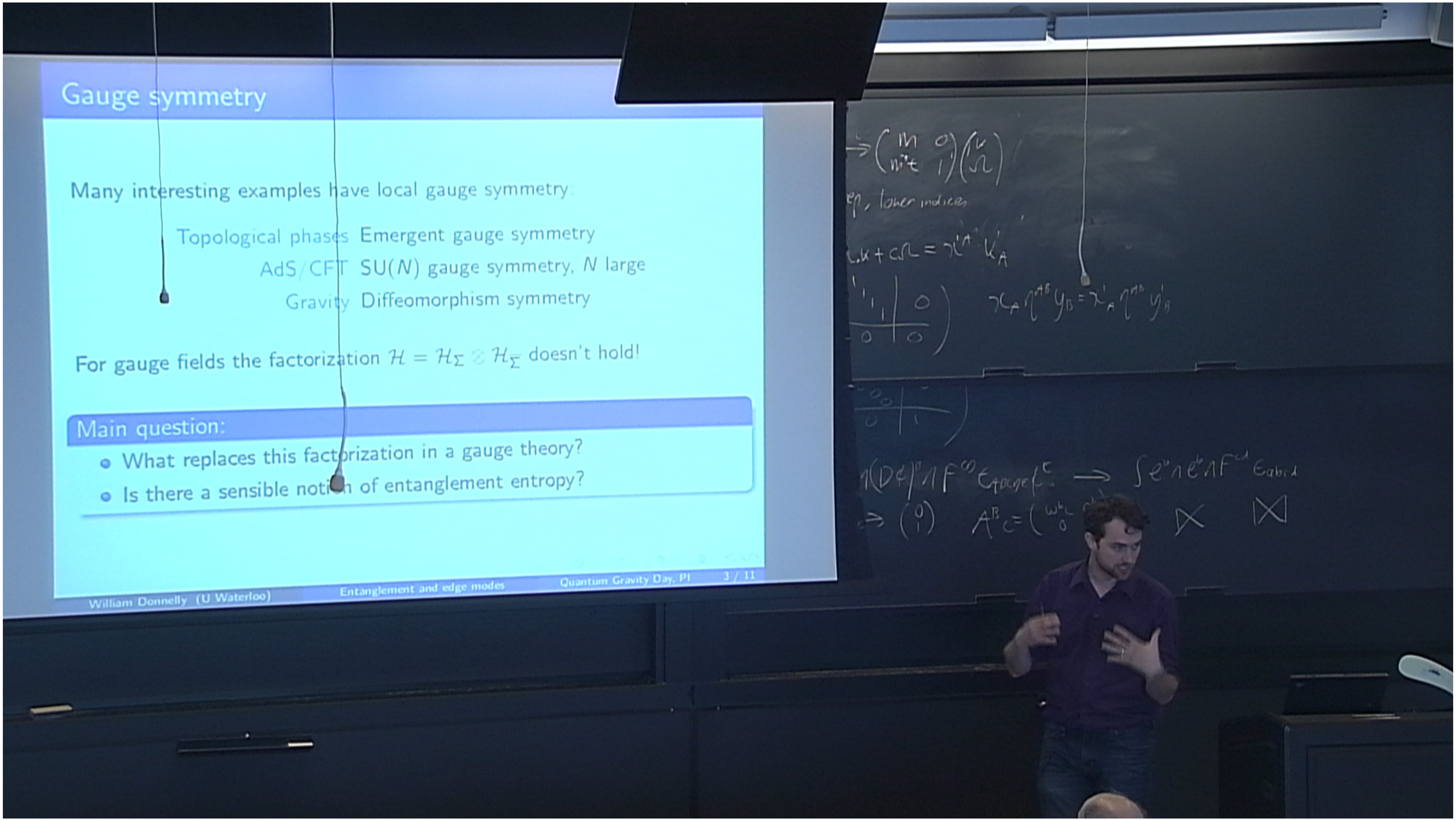
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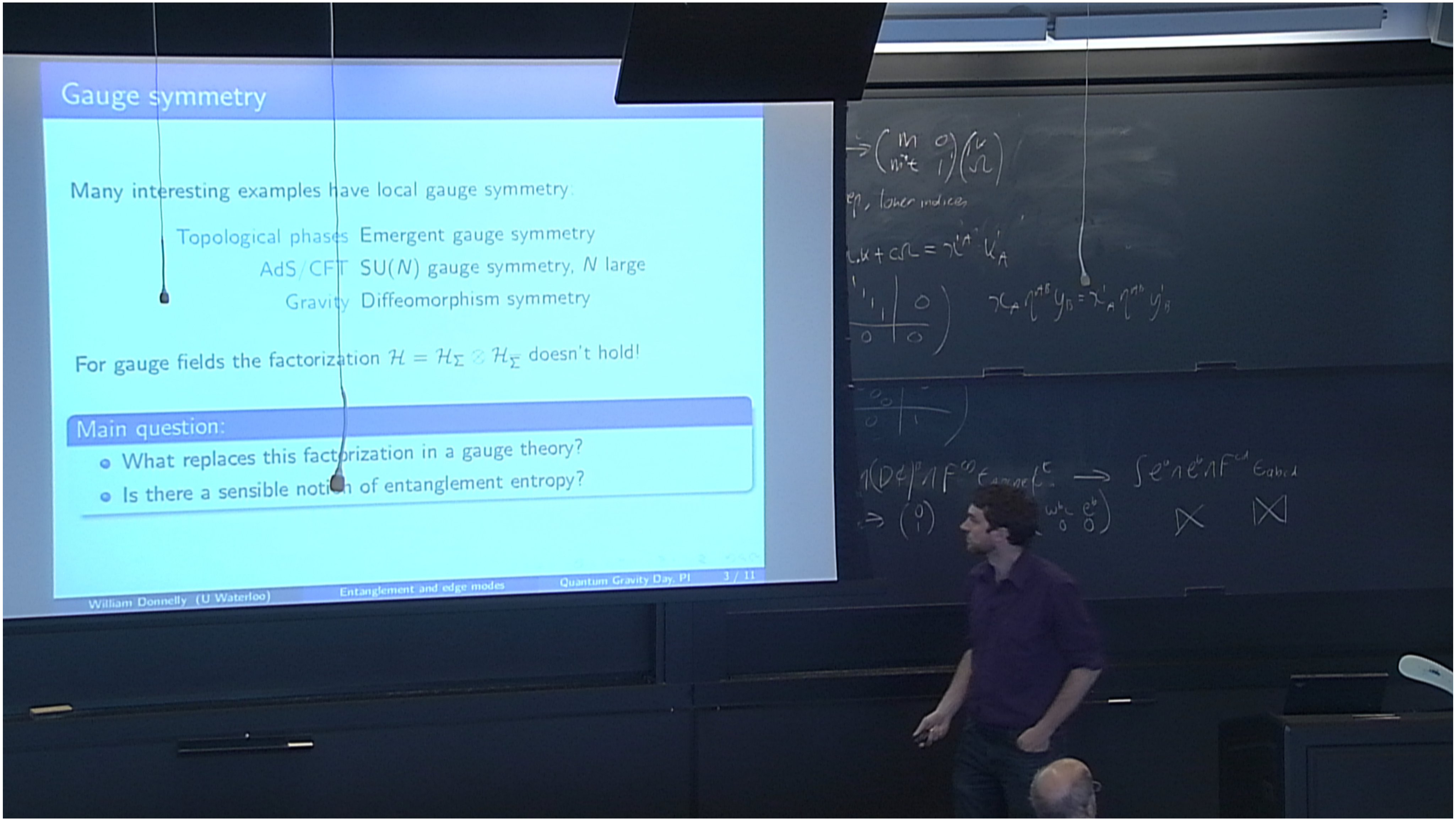
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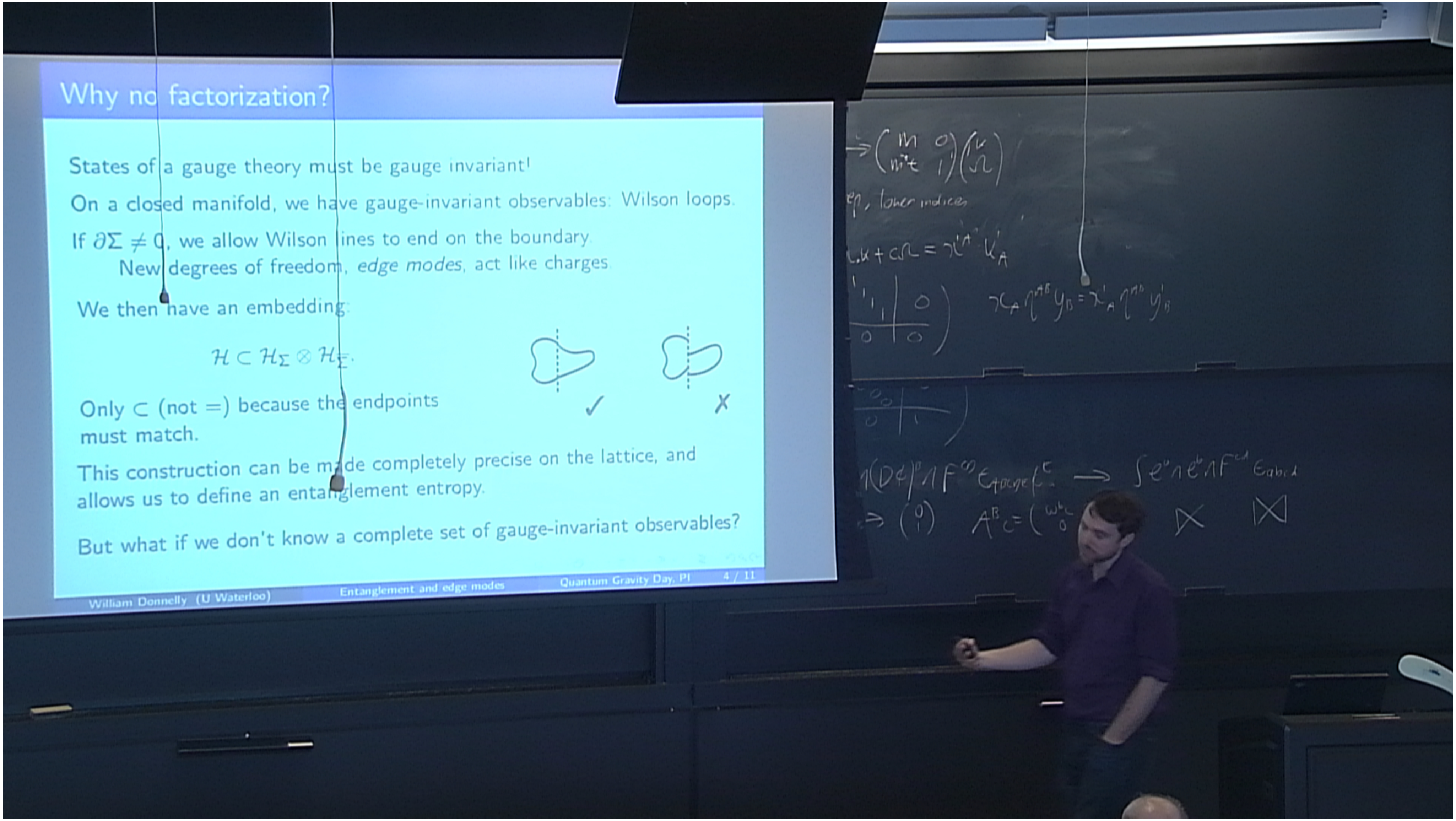

Why no factorization?
States of a gauge theory must be gauge invariant!
On a closed manifold, we have gauge-invariant observables: Wilson loops.
If ∂Σ ≠ 0, we allow Wilson lines to end on the boundary.
New degrees of freedom, edge modes, act like charges.
We then have an embedding:
ℋ ⊂ ℋ_Σ ⊗ ℋ_Σ̄.
Only ⊂ (not =) because the endpoints must match.
✓
✗
This construction can be made completely precise on the lattice, and allows us to define an entanglement entropy.
But what if we don't know a complete set of gauge-invariant observables?
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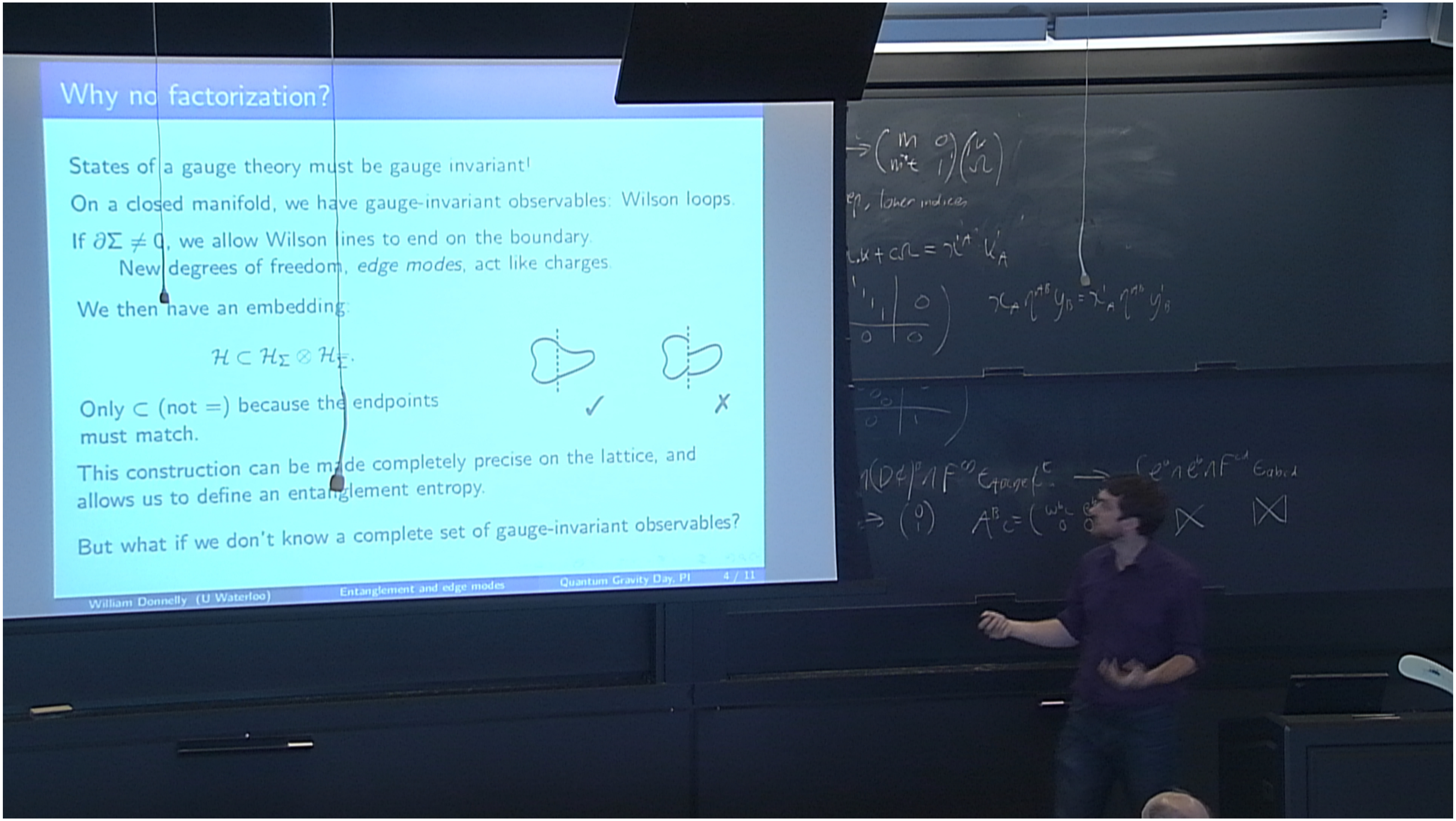
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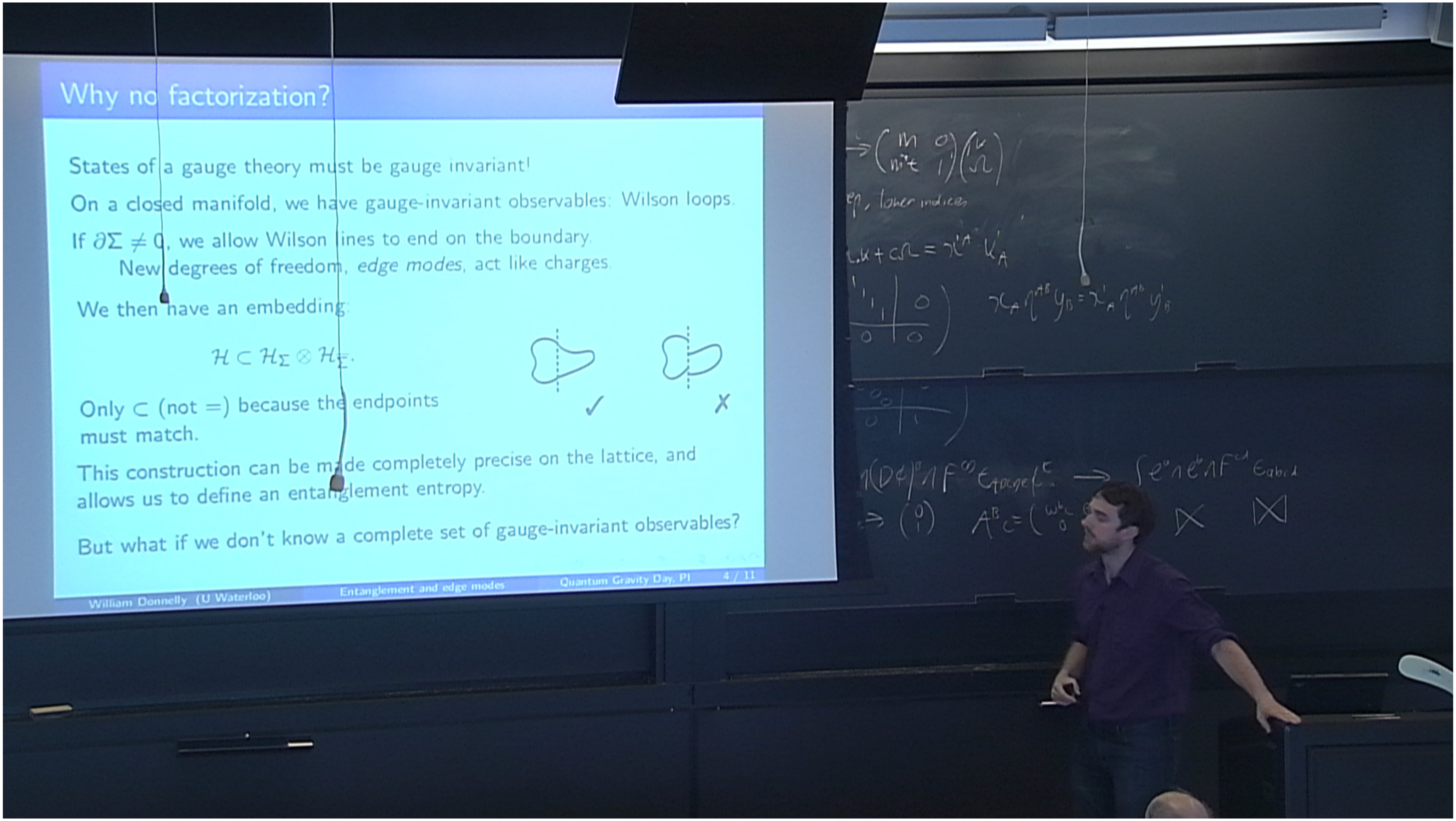
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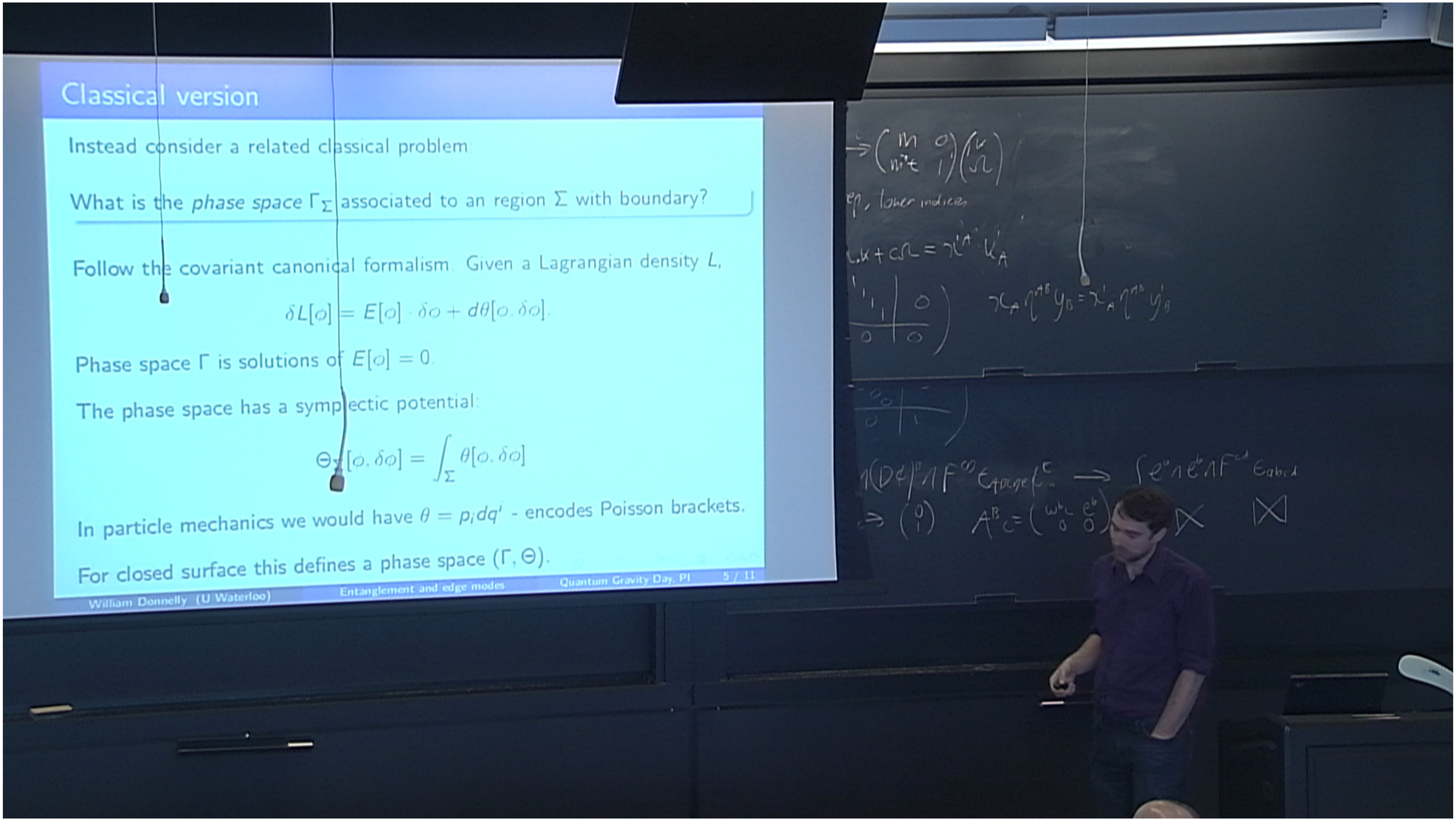
Classical version
Instead consider a related classical problem:
What is the *phase space* Γ_Σ associated to an region Σ with boundary?
Follow the covariant canonical formalism. Given a Lagrangian density L,
$$\delta L[\phi] = E[\phi] \cdot \delta\phi + d\theta[\phi, \delta\phi].$$
Phase space Γ is solutions of $E[\phi] = 0$.
The phase space has a symplectic potential:
$$\Theta_\Sigma[\phi, \delta\phi] = \int_\Sigma \theta[\phi, \delta\phi]$$
In particle mechanics we would have $\theta = p_i dq^i$ - encodes Poisson brackets.
For closed surface this defines a phase space (Γ, Θ).
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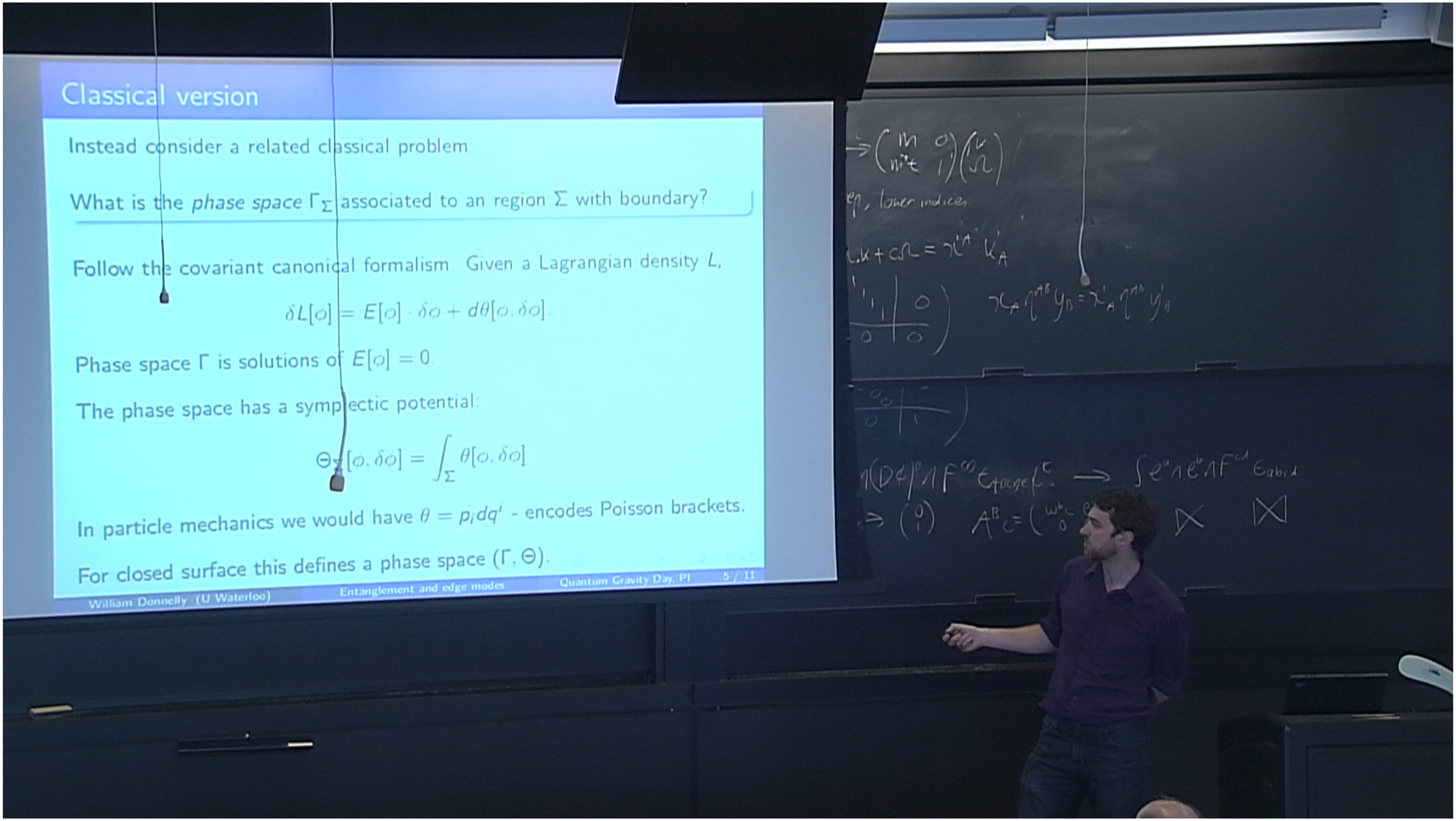
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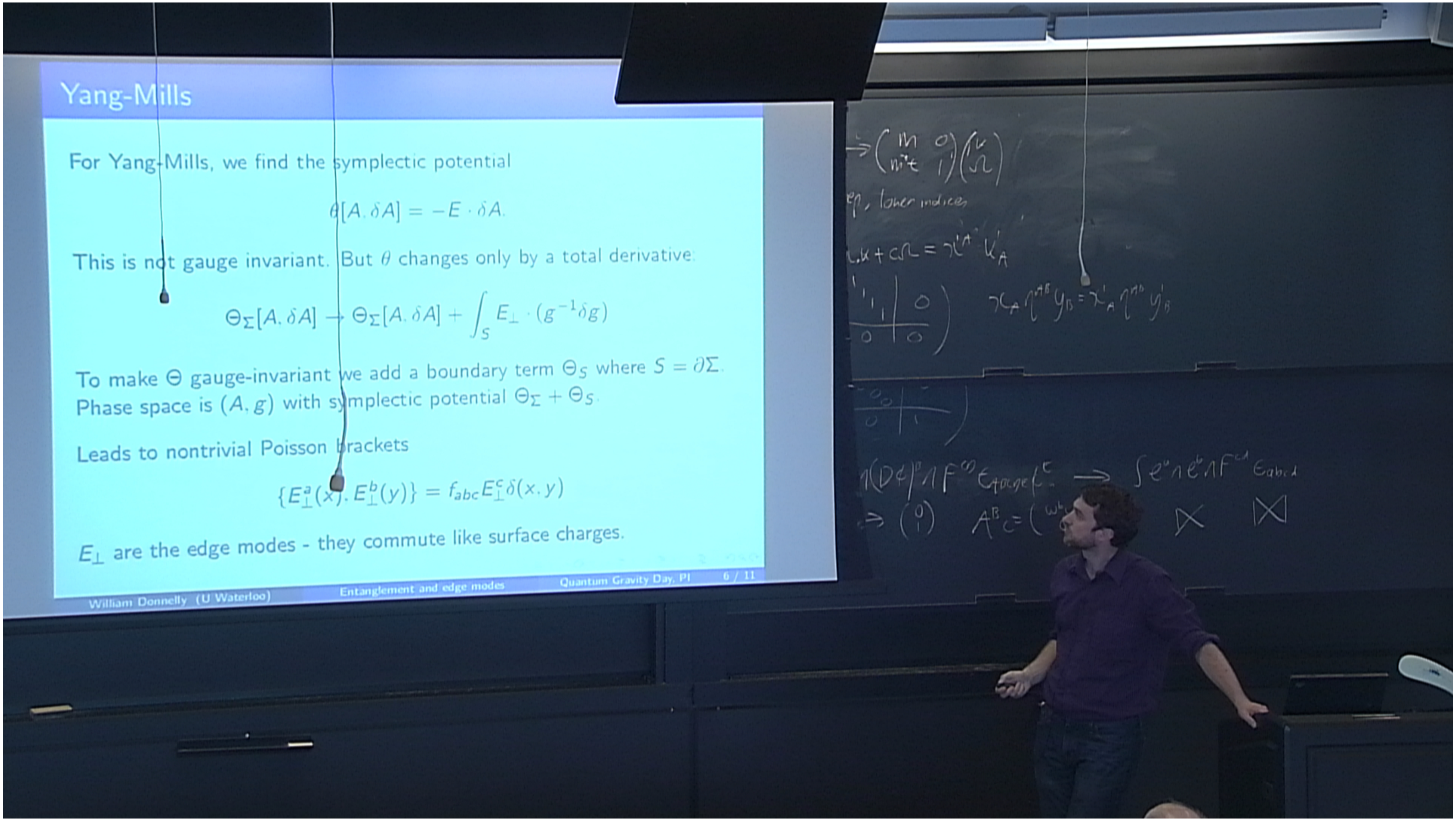
Yang-Mills
For Yang-Mills, we find the symplectic potential
$\theta[A, \delta A] = -E \cdot \delta A.$
This is not gauge invariant. But θ changes only by a total derivative:
$\Theta_\Sigma[A, \delta A] \to \Theta_\Sigma[A, \delta A] + \int_S E_\perp \cdot (g^{-1}\delta g)$
To make Θ gauge-invariant we add a boundary term Θ_S where $S = \partial\Sigma$.
Phase space is (A, g) with symplectic potential $\Theta_\Sigma + \Theta_S$.
Leads to nontrivial Poisson brackets
$\{E_\perp^a(x), E_\perp^b(y)\} = f_{abc} E_\perp^c \delta(x, y)$
$E_\perp$ are the edge modes - they commute like surface charges.
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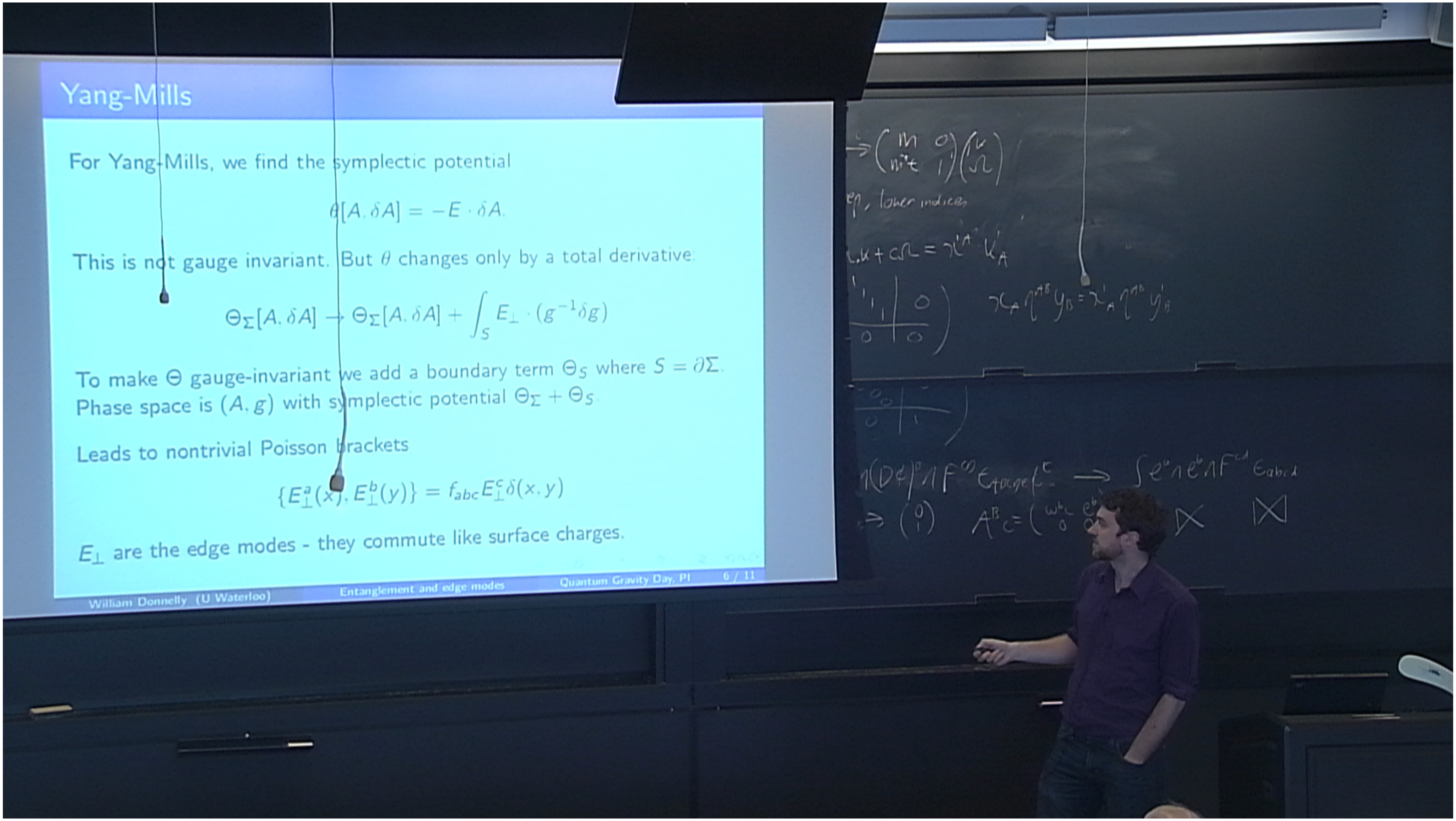
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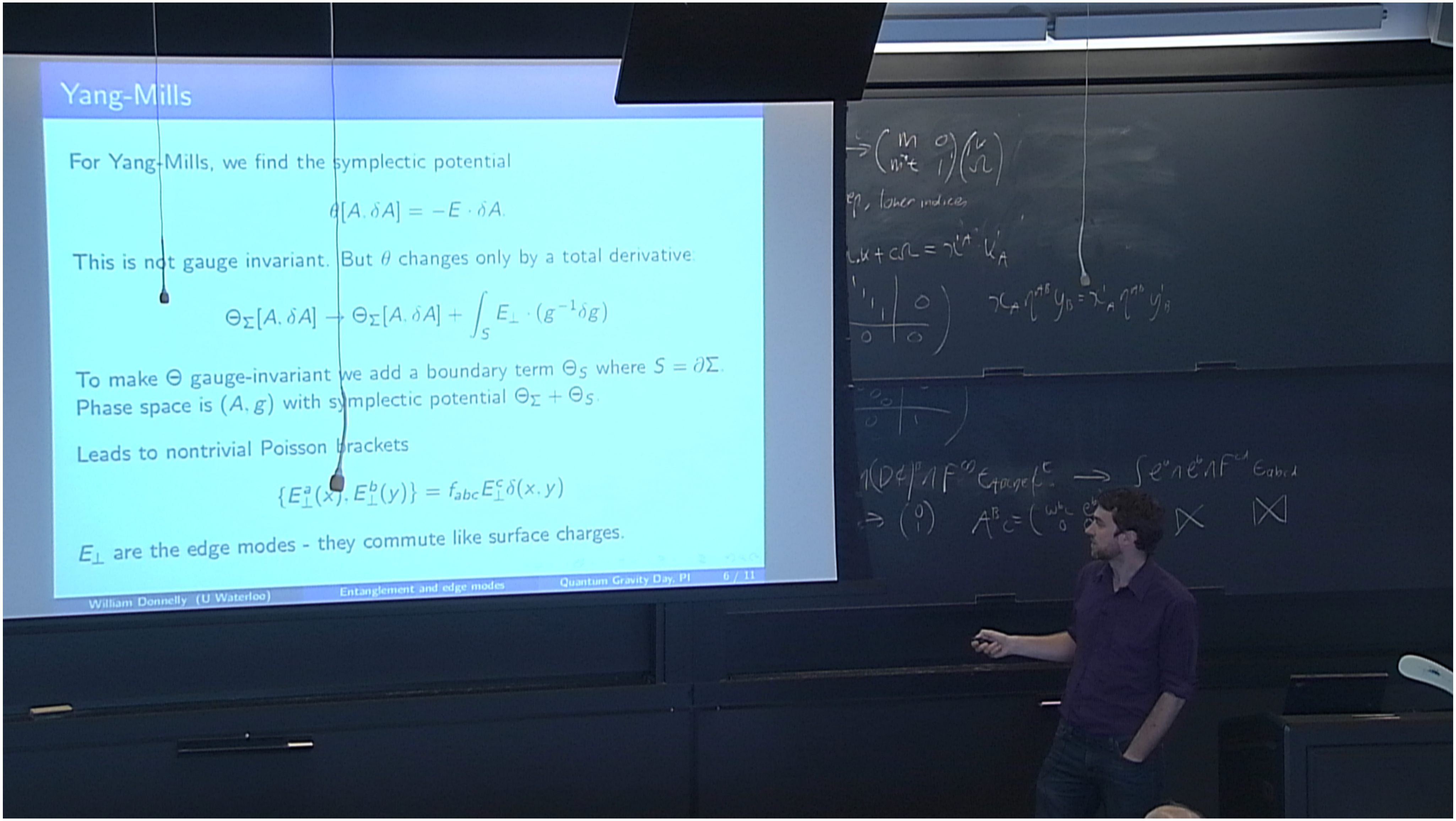
Yang-Mills
For Yang-Mills, we find the symplectic potential
θ[A, δA] = −E · δA.
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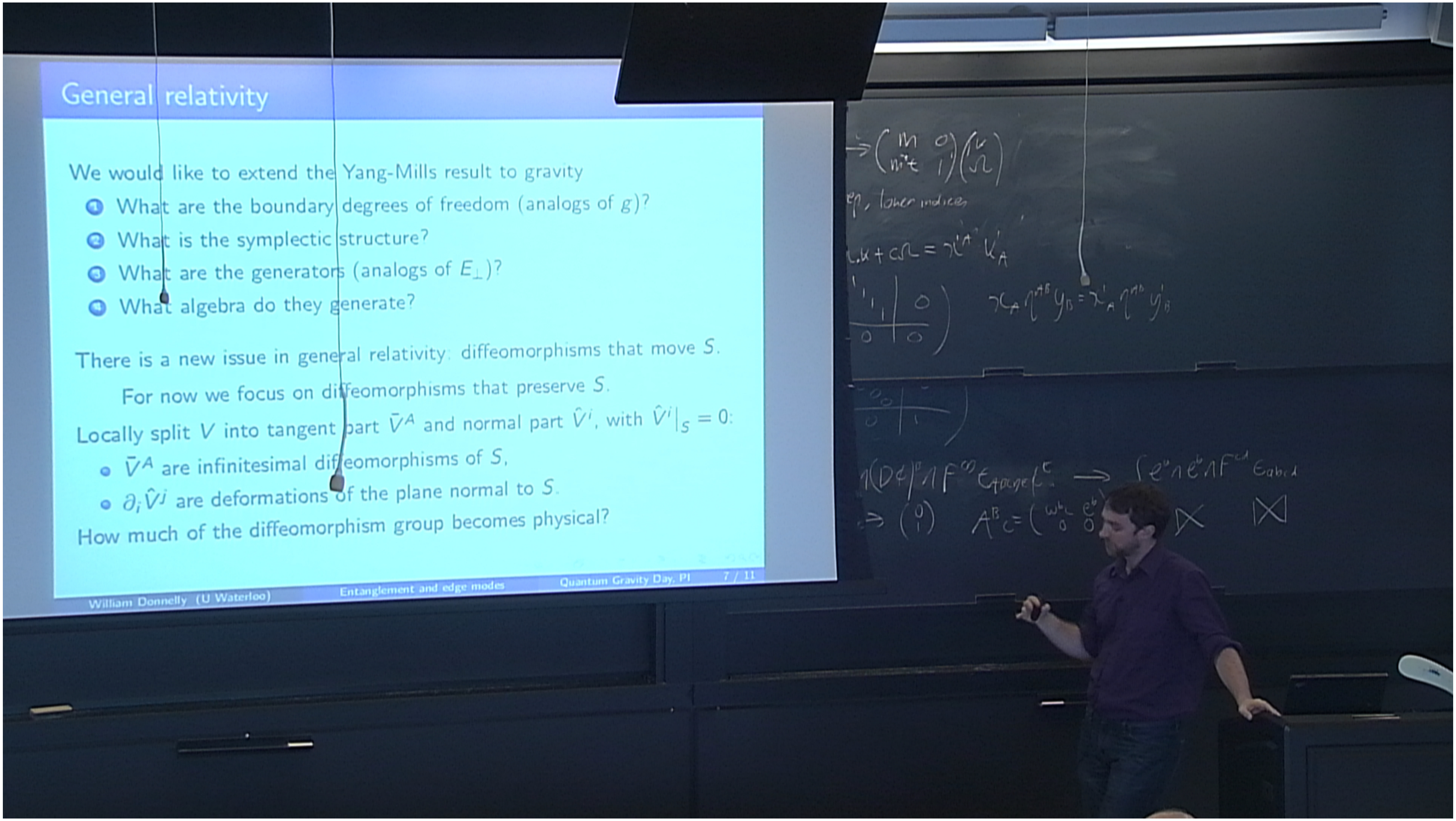
General relativity
We would like to extend the Yang-Mills result to gravity
What are the boundary degrees of freedom (analogs of g)?
What is the symplectic structure?
What are the generators (analogs of $E_\perp$)?
What algebra do they generate?
There is a new issue in general relativity: diffeomorphisms that move S.
For now we focus on diffeomorphisms that preserve S.
Locally split V into tangent part $\bar{V}^A$ and normal part $\hat{V}^i$, with $\hat{V}^i|_S = 0$:
$\bar{V}^A$ are infinitesimal diffeomorphisms of S,
$\partial_i \hat{V}^j$ are deformations of the plane normal to S.
How much of the diffeomorphism group becomes physical?
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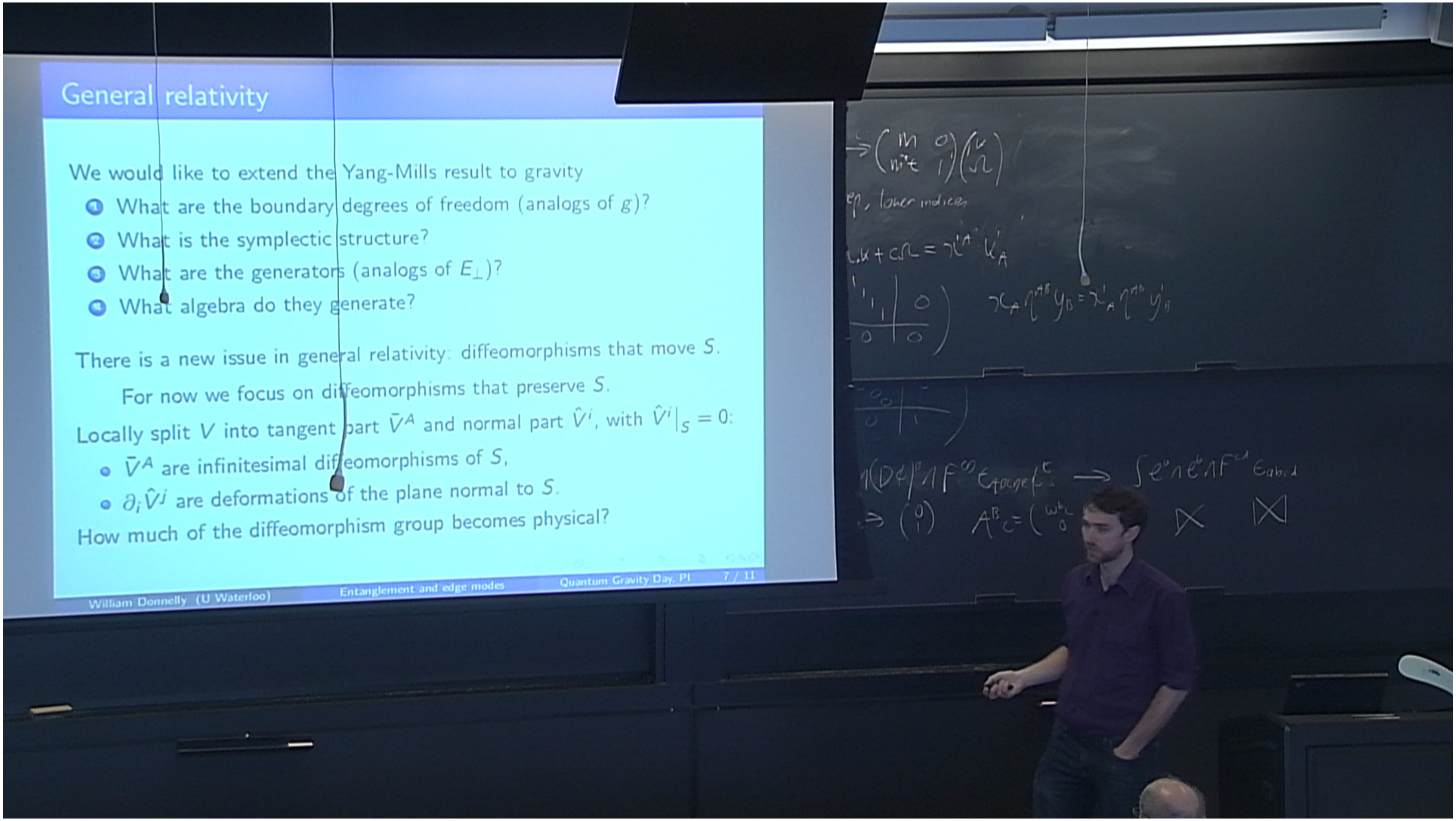
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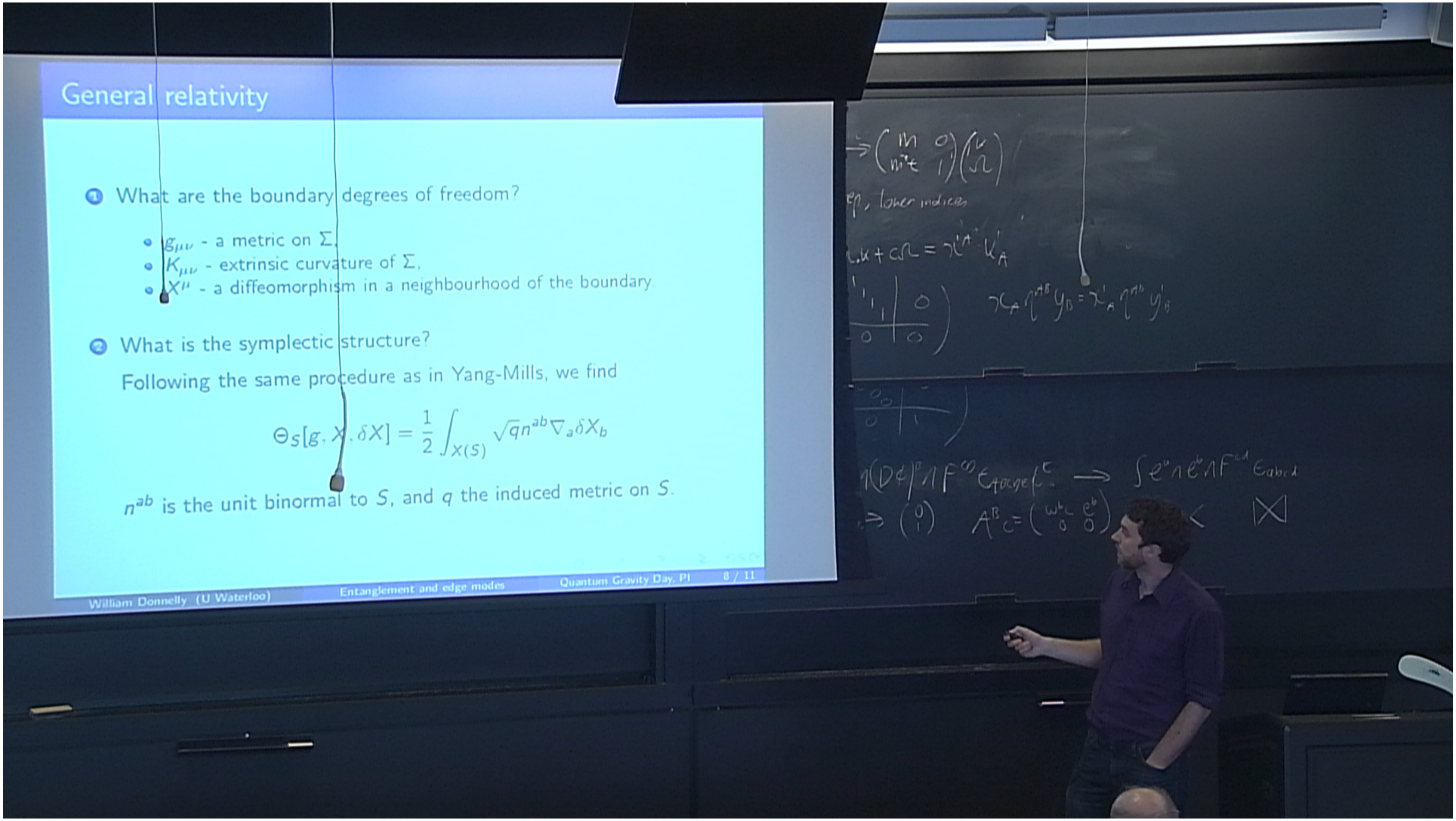
General relativity
What are the boundary degrees of freedom?
$g_{\mu\nu}$ - a metric on Σ,
$K_{\mu\nu}$ - extrinsic curvature of Σ,
X^{μ} - a diffeomorphism in a neighbourhood of the boundary.
What is the symplectic structure?
Following the same procedure as in Yang-Mills, we find
$\Theta_S[g, X, \delta X] = \frac{1}{2}\int_{X(S)} \sqrt{q} n^{ab} \nabla_a \delta X_b$
n^{ab} is the unit binormal to S, and q the induced metric on S.
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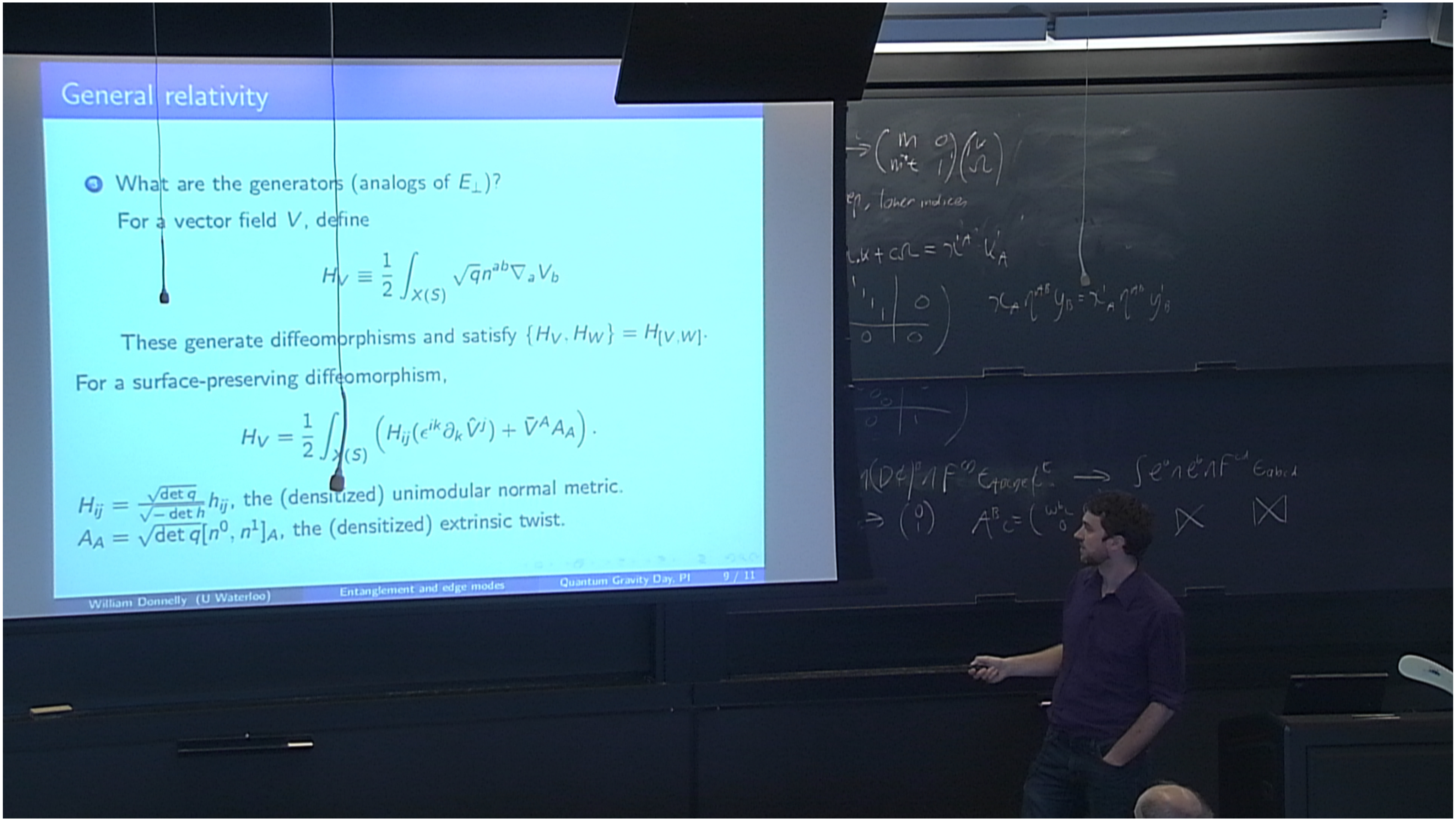
General relativity
What are the generators (analogs of $E_\perp$)?
For a vector field V, define
$$H_V \equiv \frac{1}{2}\int_{X(S)} \sqrt{q}\, n^{ab}\nabla_a V_b$$
These generate diffeomorphisms and satisfy $\{H_V, H_W\} = H_{[V,W]}$.
For a surface-preserving diffeomorphism,
$$H_V = \frac{1}{2}\int_{X(S)} \left(H_{ij}(\epsilon^{ik}\partial_k \hat{V}^j) + \bar{V}^A A_A \right).$$
$H_{ij} = \frac{\sqrt{\det q}}{\sqrt{-\det h}} h_{ij}$, the (densitized) unimodular normal metric.
$A_A = \sqrt{\det q}[n^0, n^1]_A$, the (densitized) extrinsic twist.
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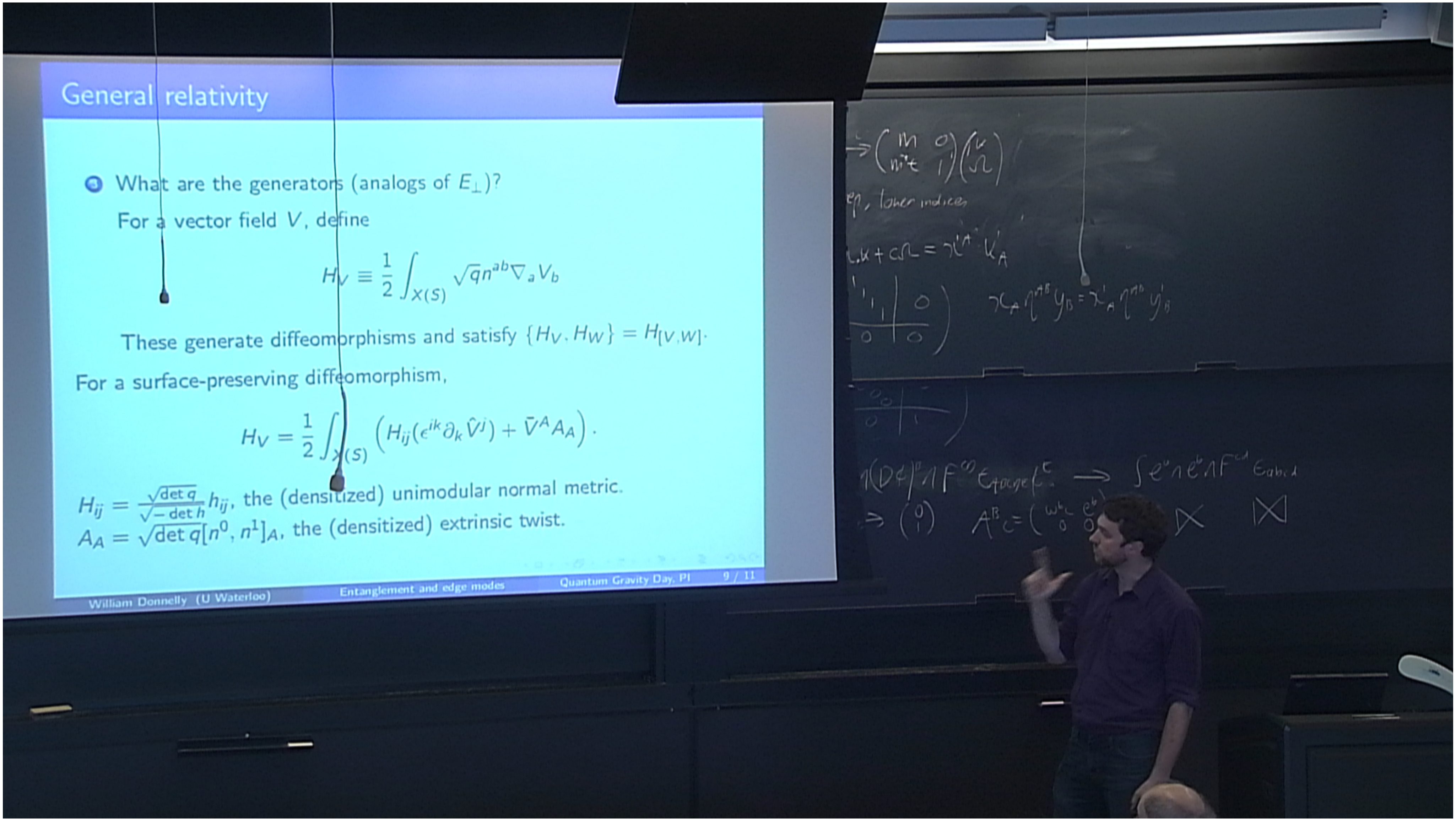

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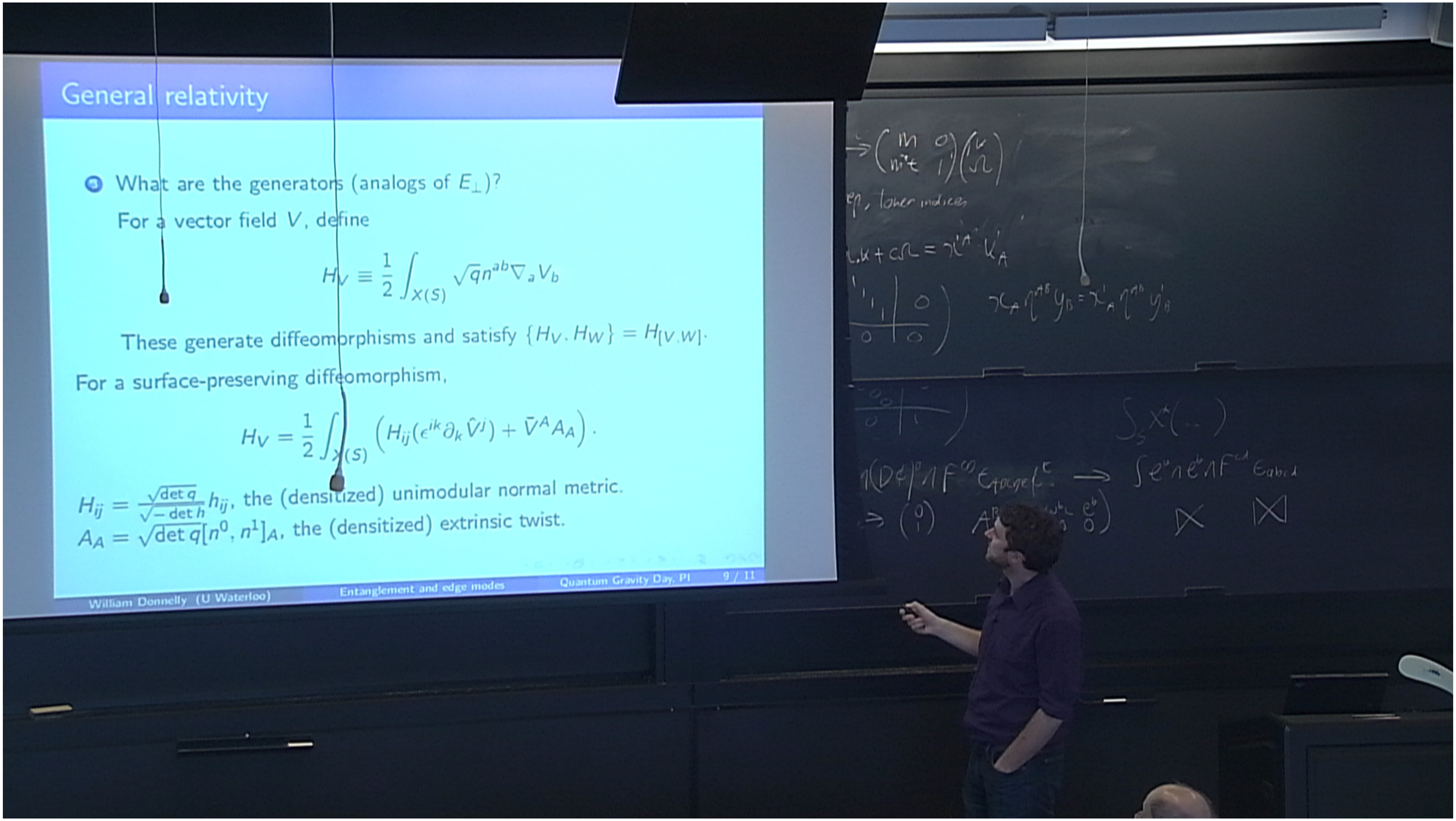
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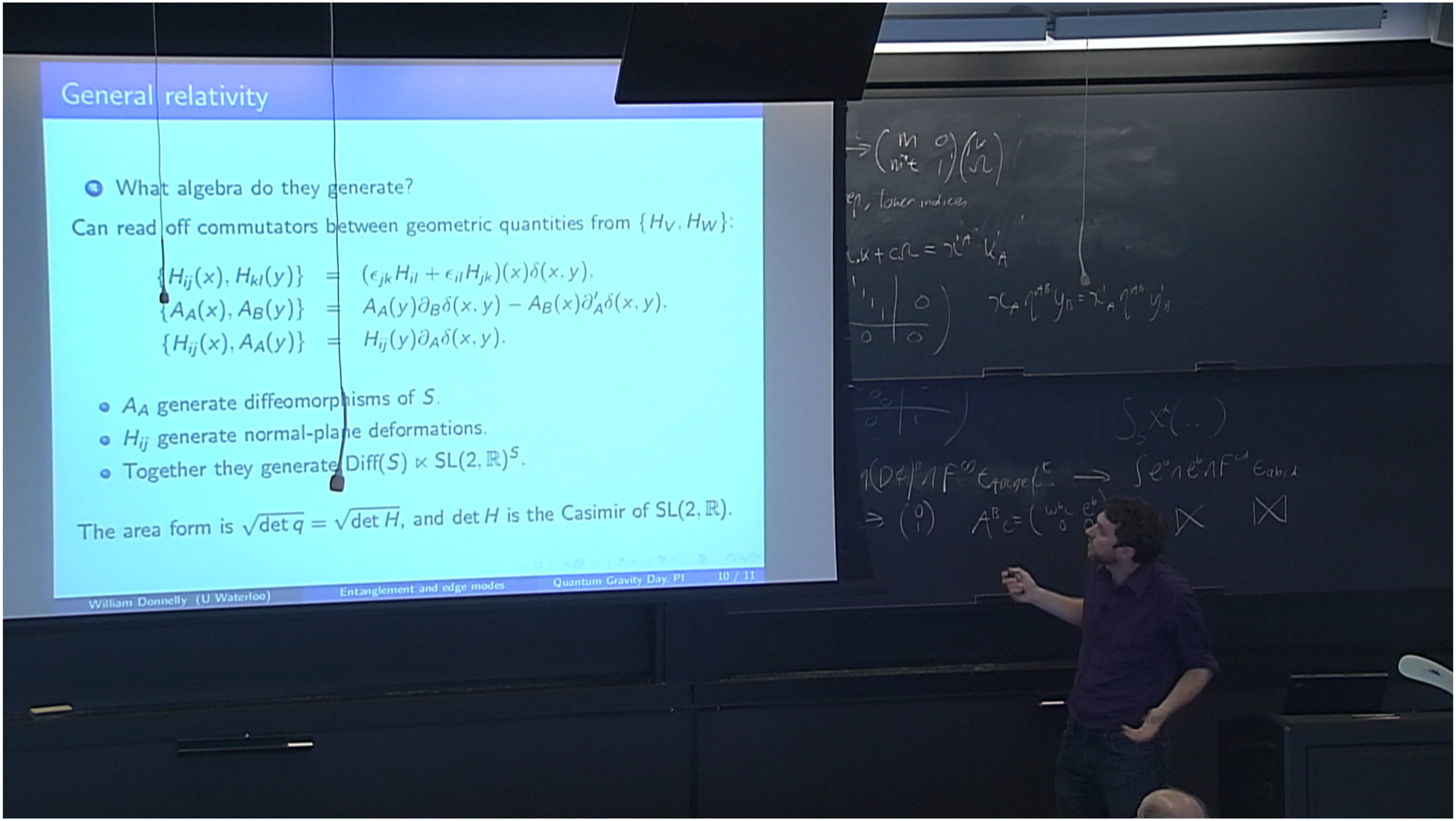
General relativity
What algebra do they generate?
Can read off commutators between geometric quantities from $\{H_V, H_W\}$:
$\{H_{ij}(x), H_{kl}(y)\} = (\epsilon_{jk} H_{il} + \epsilon_{il} H_{jk})(x)\delta(x, y)$,
$\{A_A(x), A_B(y)\} = A_A(y)\partial_B \delta(x, y) - A_B(x)\partial'_A \delta(x, y)$,
$\{H_{ij}(x), A_A(y)\} = H_{ij}(y)\partial_A \delta(x, y)$.
A_A generate diffeomorphisms of S.
H_{ij} generate normal-plane deformations.
Together they generate $\mathrm{Diff}(S) \ltimes SL(2, \mathbb{R})^S$.
The area form is $\sqrt{\det q} = \sqrt{\det H}$, and $\det H$ is the Casimir of $SL(2, \mathbb{R})$.
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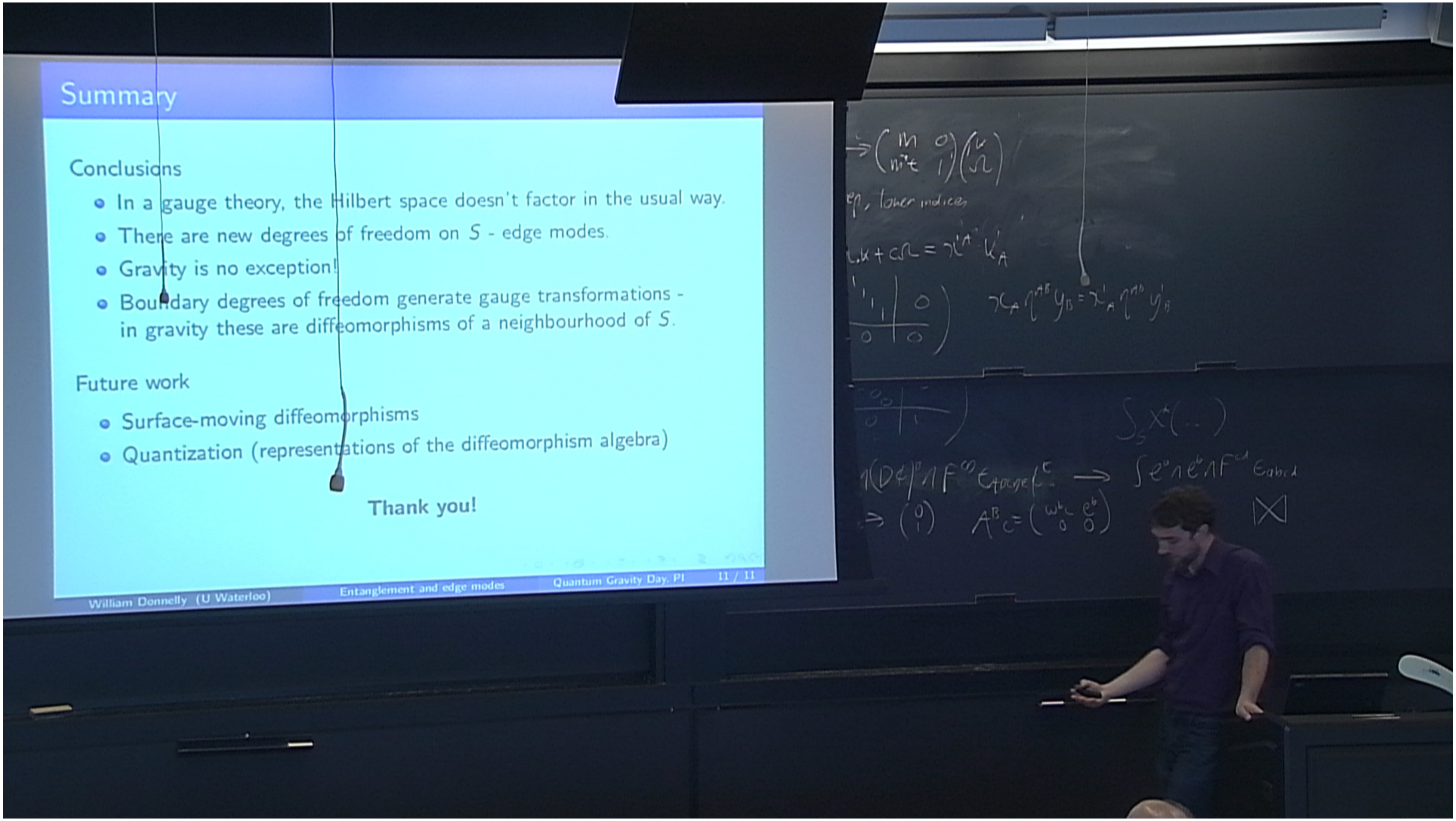
Summary
Conclusions
In a gauge theory, the Hilbert space doesn't factor in the usual way.
There are new degrees of freedom on S - edge modes.
Gravity is no exception!
Boundary degrees of freedom generate gauge transformations - in gravity these are diffeomorphisms of a neighbourhood of S.
Future work
Surface-moving diffeomorphisms
Quantization (representations of the diffeomorphism algebra)
Thank you!
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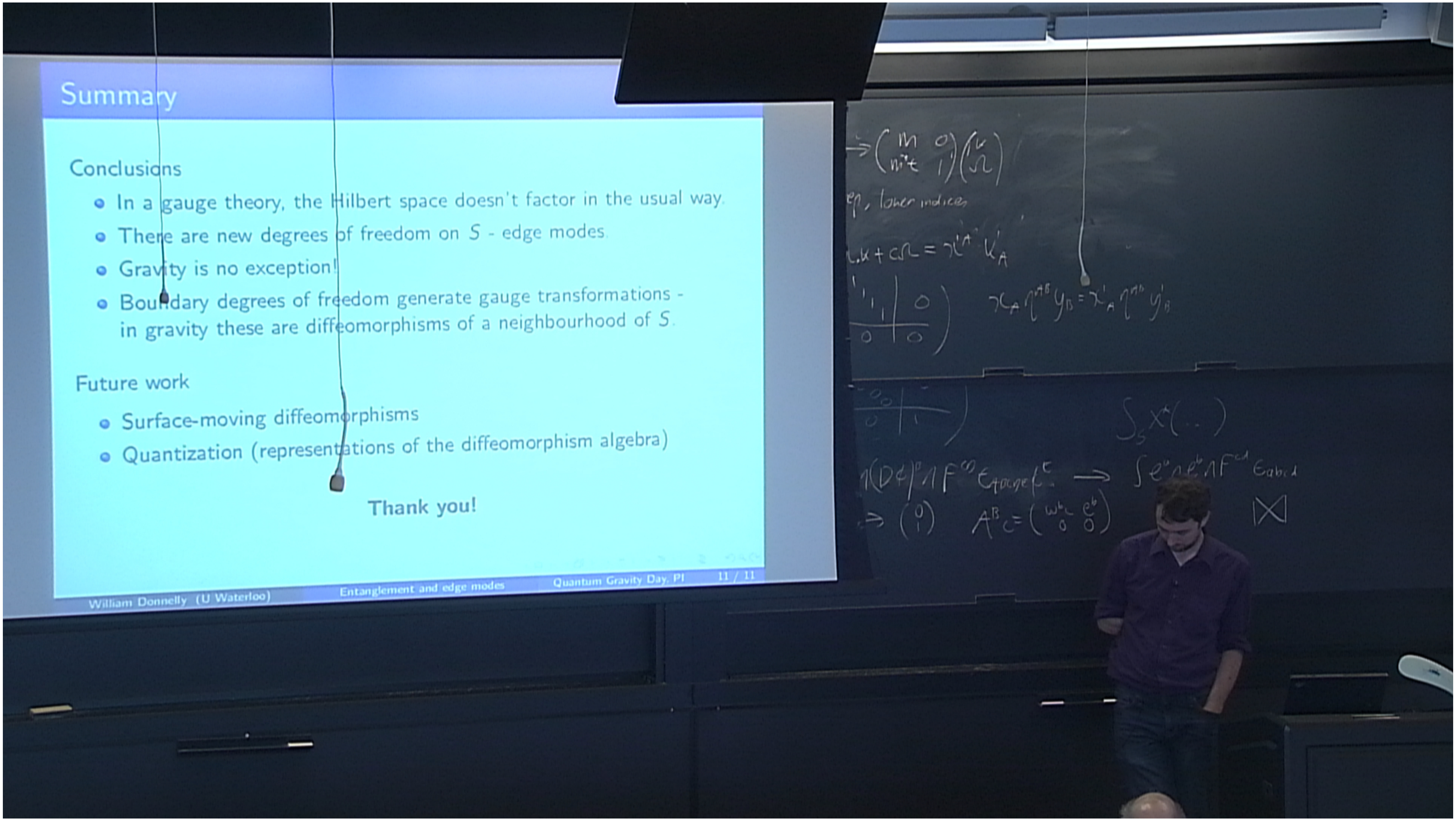
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