

Title: TBA

Date: May 21, 2014 03:50 PM

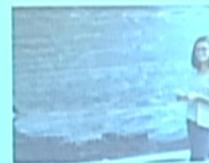
URL: <http://pirsa.org/14050121>

Abstract:

PI collaborators

Maite Dupuis

Ex PI summer student



$$\rightarrow \begin{pmatrix} M & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

ep, lower indices

$$L_k + c\eta = \gamma^A_k \kappa_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} 0 & - \\ 0 & 1 \end{pmatrix}$$

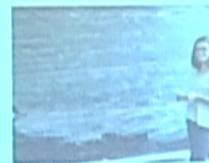
$$\gamma(D\phi)^T \gamma F^C \epsilon_{ABCE} f^E \rightarrow \text{second Faddeev-Popov ghost}$$

$$\rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B_C = \begin{pmatrix} w^B_C & e^B \\ 0 & 0 \end{pmatrix}$$

PI collaborators

Maite Dupuis

Ex PI summer student



Valentin Bonzom

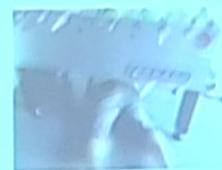
Ex PI postdoc

Ex fooz champ

Etera Livine

Ex PI postdoc

Ex PI long term visitor
Current PI fooz champ



$$\Rightarrow \begin{pmatrix} M & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$\epsilon_{\mu\nu}$, lower indices

$$L_k + c_n = \gamma^A_k K_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} - & - \\ 0 & 1 \end{pmatrix}$$

$$\gamma(D\phi)^T / F^m \epsilon_{\mu\nu\eta\epsilon} f_\nu \rightarrow \text{some } F^{ab} \epsilon_{abc}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B_C = \begin{pmatrix} w^b_c & e^b \\ 0 & 0 \end{pmatrix}$$



3d quantum gravity

Let's focus on 3d gravity:

Simpler than 4d gravity (since topological)

Similar issues as 4d gravity arise

Does not mean that 4d gravity will be exactly the same!

Many quantization schemes can be applied and related

Chern-Simmons
amplitude



Loop quantum gravity

Spinfoam
(Ponzano-Regge, Turaev-Viro)

$$\rightarrow \begin{pmatrix} M & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

ep, lower indices

$$L_K + c_R = \gamma^A K_A$$

$$\begin{pmatrix} 1 & | & 0 \\ 0 & | & 0 \end{pmatrix} \quad \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} 0 & | & - \\ 0 & | & 1 \end{pmatrix}$$

$$\gamma(D\varphi)^a / F^{ab} \epsilon_{abc} e^c \rightarrow \text{some } F^{ab} \epsilon_{abc}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B_C = \begin{pmatrix} \omega^a_b & e^b \\ 0 & 0 \end{pmatrix}$$

3d quantum gravity

Let's focus on 3d gravity:

- Simpler than 4d gravity (since topological).
- Similar issues as 4d gravity arise.
- Does not mean that 4d gravity will be exactly the same!
- Many quantization schemes can be applied and related.

Chern-Simmons
amplitude

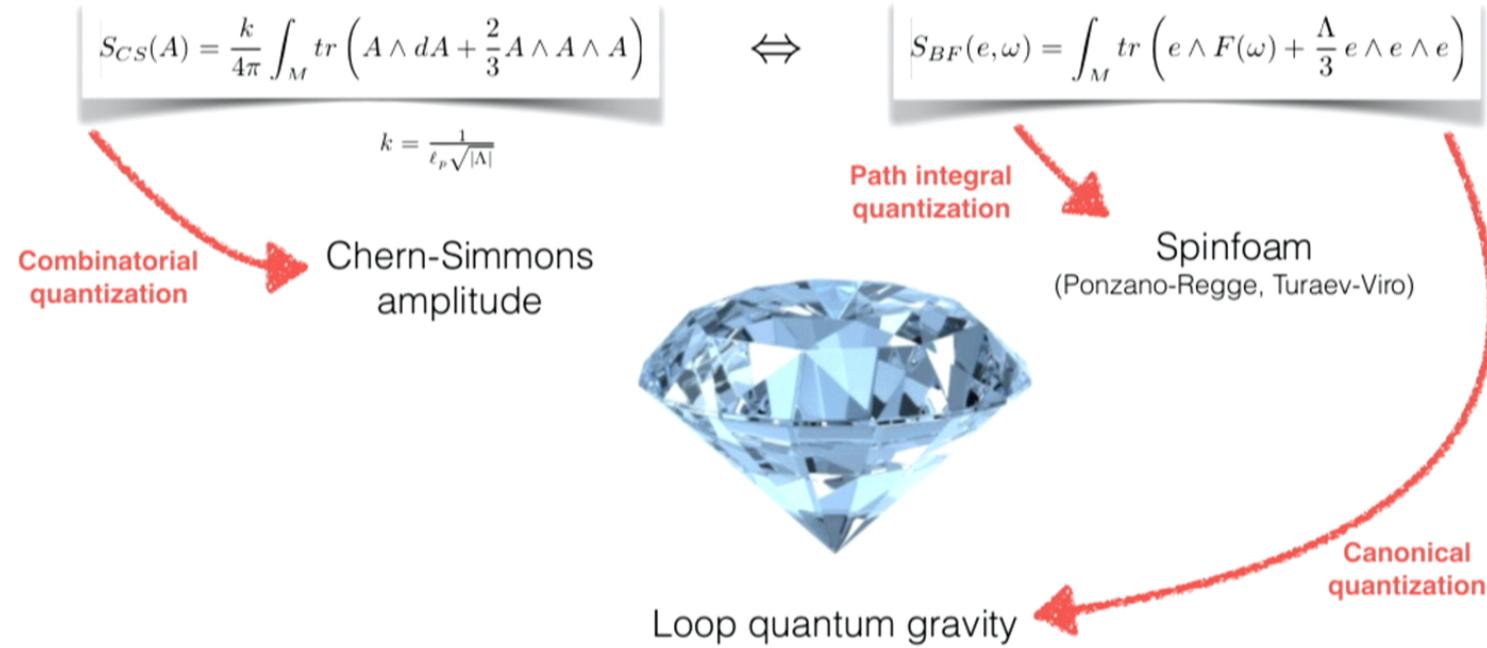


Spinfoam
(Ponzano-Regge, Turaev-Viro)

Loop quantum gravity

3d quantum gravity

Treat cosmological constant as a coupling constant, just like G



LQG vs spinfoam when Λ is not zero

Upon Hamiltonian analysis, the cosmological constant appears only in the Hamiltonian constraint, i.e. not in the Gauss constraint.

The quantum Gauss constraint implements the relevant gauge structure.



Spin network based on SU(2)
whether Λ is zero or not.

Euclidian case considered here!

LQG vs spinfoam when Λ is not zero

Upon Hamiltonian analysis, the cosmological constant appears only in the Hamiltonian constraint, i.e. not in the Gauss constraint.

The quantum Gauss constraint implements the relevant gauge structure.

Boundary states of the Turaev-Viro model are defined in terms of a quantum group



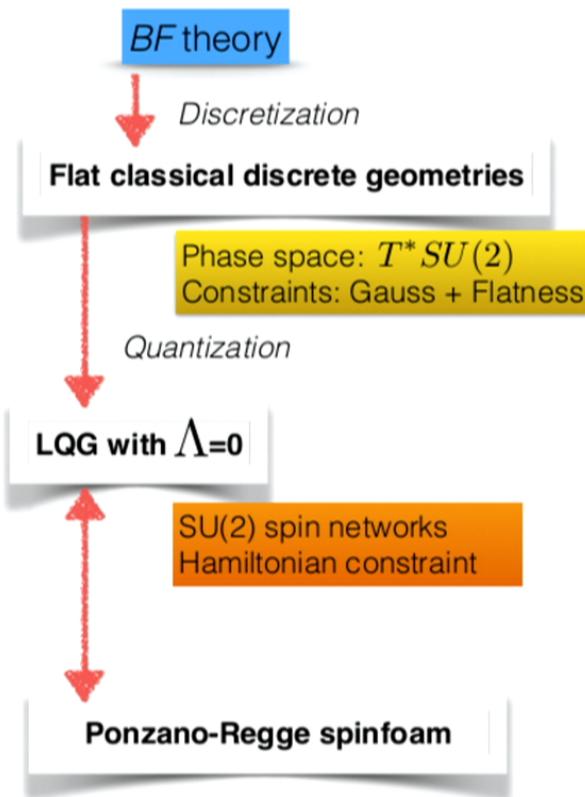
Spin network based on SU(2) whether Λ is zero or not.



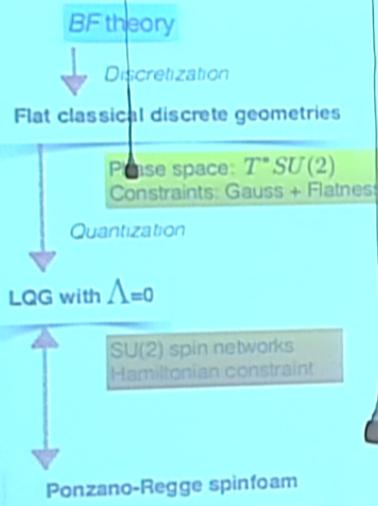
Spin network based on $\mathcal{U}_q(\mathfrak{su}(2))$

Euclidian case considered here!

Cosmological constant in LQG



Cosmological constant in LQG



$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$c_A + c_B = \gamma^{AB} K_A$$

$$\begin{pmatrix} 1 & | & 0 \\ 0 & | & 0 \end{pmatrix} \quad \gamma_A \gamma_B^{AB} y_B = \gamma_A \gamma_B^{AB} y_B$$

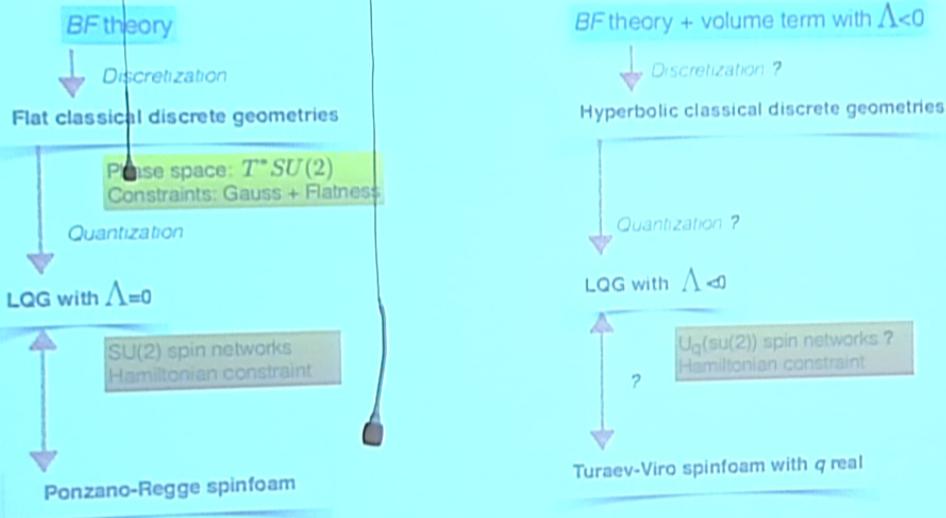
$$\begin{pmatrix} 0 & | & - \\ 0 & | & 1 \end{pmatrix}$$

$$(D\phi)^a / \sqrt{F} e_{ab} e^{bc} \rightarrow \epsilon^{abc} \sqrt{F} e^c$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B{}_C = \begin{pmatrix} \omega^a{}_b & e^b \\ 0 & 0 \end{pmatrix}$$

A man in a white shirt is standing at a chalkboard, gesturing with his right hand. He appears to be explaining the mathematical derivation on the board.

Cosmological constant in LQG



$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$c_A k + c_B l = \gamma_A^\mu k_A^\mu$$

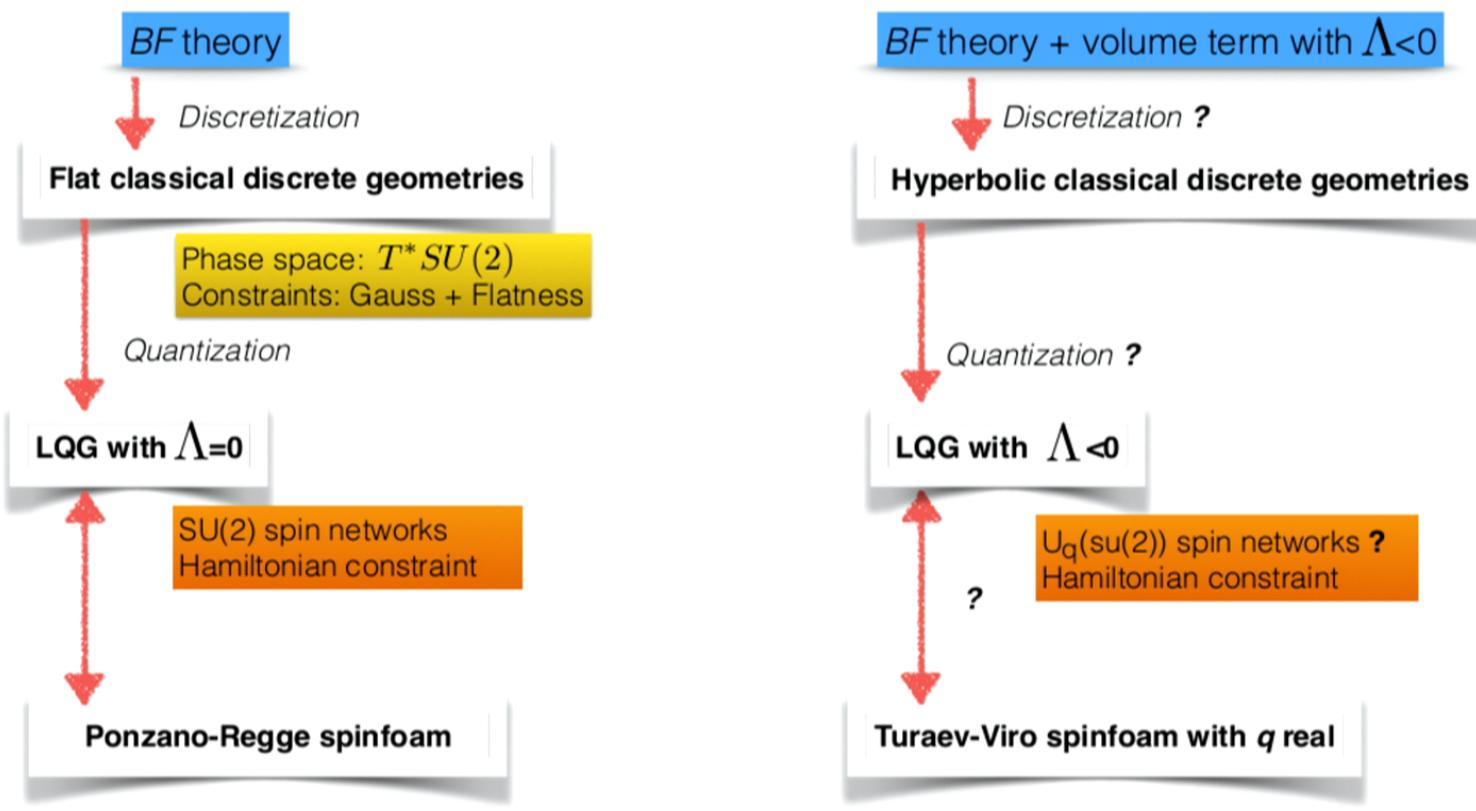
$$\begin{pmatrix} 1 & | & 0 \\ 0 & | & 0 \end{pmatrix} \quad \gamma_A^\mu \gamma_B^{\mu b} y_B = \gamma_A^\mu \gamma_B^{\mu b} y_B$$

$$\begin{pmatrix} - & | & - \\ 0 & | & 1 \end{pmatrix}$$

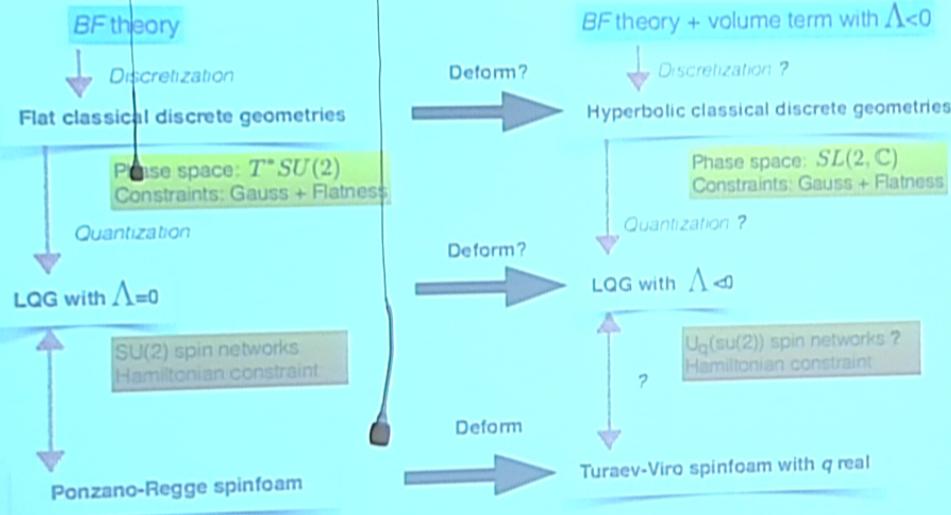
$$1(D\phi)^a / \sqrt{F} \epsilon_{abc} f_c^b \rightarrow \sum e^a e^b F^{ab} \epsilon_{abc}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B_C = \begin{pmatrix} \omega^b_a & e^b \\ 0 & 0 \end{pmatrix}$$

Cosmological constant in LQG



Cosmological constant in LQG



$$\rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

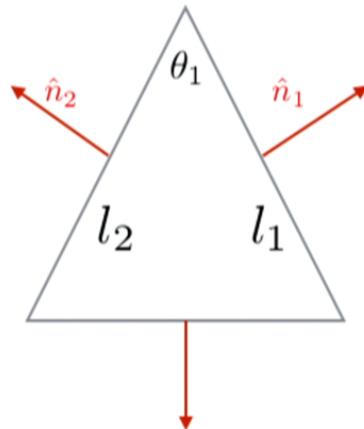
$$c_A + c_B = \gamma^{AB} K_A$$

$$\begin{pmatrix} 1 & | & 0 \\ 0 & | & 0 \end{pmatrix} \quad \gamma_A \gamma_A^{AB} y_B = \gamma_A \gamma_A^{AB} y_B$$

$$1(D\phi)^n / n! F^n \epsilon_{\text{cycle}} \rightarrow S^n e^n F^n \epsilon_{\text{cycle}}$$

$$\rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B{}_C = \begin{pmatrix} \omega_{AB} & 0 \\ 0 & 0 \end{pmatrix}$$

A taste of what's happening



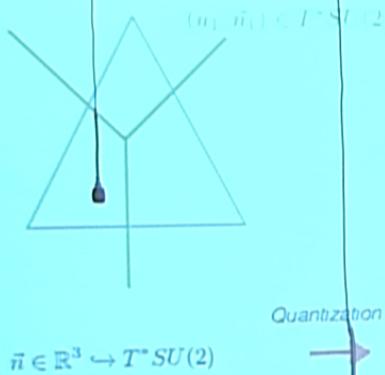
Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = \vec{0}$

Length: $\vec{n}_i = l_i \hat{n}_i$
Cosine law: $\hat{n}_1 \cdot \hat{n}_2 = -\cos \theta_1 = \frac{l_3^2 - l_1^2 - l_2^2}{2l_1 l_2}$

Area...

A taste of what's happening



Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = \vec{0}$

Length: $\vec{n}_1 = l_1 \hat{n}_1$

Cosine law: $n_1 \cdot n_2 = -\cos \theta_1 = \frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}$

Area

$$\begin{aligned} \mathcal{H}_{tot} &= V_{j_1} \otimes V_{j_2} \otimes V_{j_3} \\ \vec{n}_1 \rightarrow \vec{j}_1 &\equiv \vec{j} \otimes 1 \otimes 1, \quad \vec{n}_2 \rightarrow \vec{j}_2 \equiv 1 \otimes \vec{j} \otimes 1, \dots \\ \sum \vec{n}_i = \vec{0} &\rightarrow \sum \vec{j}_i |\psi\rangle = \vec{0} \Rightarrow |\psi\rangle \text{ intertwiner} \\ |\psi\rangle &= |i_{j_1 j_2 j_3}\rangle \end{aligned}$$

$$\begin{aligned} \vec{j}_i^2 |i_{j_1 j_2 j_3}\rangle &= l_p^2 j_i(j_i+1) |i_{j_1 j_2 j_3}\rangle = l_i^2 |i_{j_1 j_2 j_3}\rangle \\ \vec{j}_1 \cdot \vec{j}_2 |i_{j_1 j_2 j_3}\rangle &= l_p^2 \left(\frac{j_1(j_1+1) - j_1(j_1+1) - j_2(j_2+1)}{2} \right) |i_{j_1 j_2 j_3}\rangle \end{aligned}$$

Quantum flat triangle!

$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$k + c\bar{n} = \gamma^A k_A$$

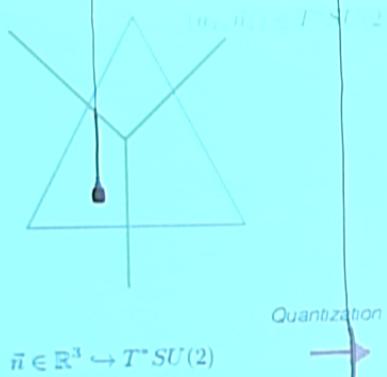
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$1(D\phi)^a / F^{ab} \epsilon_{abc} \rightarrow \gamma^a n^b F^{cd} \epsilon_{abcd}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B C = \begin{pmatrix} \omega_b & e^b \\ 0 & 0 \end{pmatrix}$$

A taste of what's happening



$$\vec{n} \in \mathbb{R}^3 \hookrightarrow T^*SU(2)$$

Quantization

Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = 0$

$$\text{Length: } \vec{n}_1 = l_1 \vec{n}_1$$

$$\text{Cosine law: } \vec{n}_1 \cdot \vec{n}_2 = -\cos \theta_1 = \frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}$$

Area

$$\mathcal{H}_{tot} = V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$$

$$\vec{n}_1 \rightarrow \vec{j}_1 \equiv \vec{j} \otimes 1 \otimes 1, \quad \vec{n}_2 \rightarrow \vec{j}_2 \equiv 1 \otimes \vec{j} \otimes 1, \dots$$

$$\sum \vec{n}_i = \vec{0} \rightarrow \sum \vec{j}_i |\psi\rangle = \vec{0} \Rightarrow |\psi\rangle \text{ intertwiner}$$

$$|\psi\rangle = |i_{j_1 j_2 j_3}\rangle$$

$$\vec{j}_i^2 |i_{j_1 j_2 j_3}\rangle = l_p^2 j_i(j_i+1) |i_{j_1 j_2 j_3}\rangle = l_i^2 |i_{j_1 j_2 j_3}\rangle$$

$$j_1 \cdot j_2 |i_{j_1 j_2 j_3}\rangle = l_p^2 \left(\frac{j_1(j_1+1) + j_2(j_2+1) - j_3(j_3+1)}{2} \right) |i_{j_1 j_2 j_3}\rangle$$

Quantum flat triangle!

$$\xrightarrow{\sim} \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$\omega_k + c\omega_\mu = \gamma^{AB} K_A$$

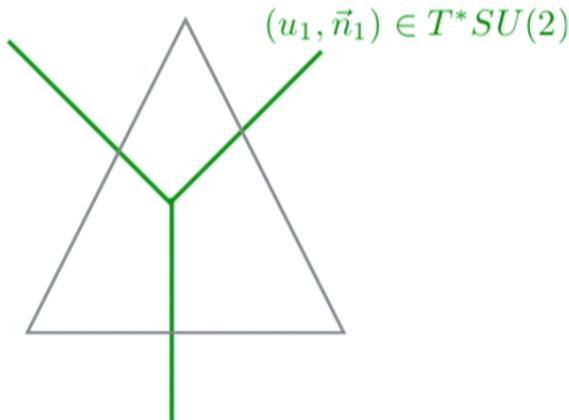
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$$

$$1(D\phi)^a / \Lambda F^{ab} \epsilon_{abc} \rightarrow \text{some } \Lambda F^{ab} \epsilon_{abc}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B C = \begin{pmatrix} \omega_B \\ 0 \end{pmatrix}$$

A taste of what's happening



$$\vec{n} \in \mathbb{R}^3 \hookrightarrow T^*SU(2)$$

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Area...

$$\mathcal{H}_{tot} = V_{j_1} \otimes V_{j_3} \otimes V_{j_3}$$

$$\vec{n}_1 \rightarrow \vec{J}_1 \equiv \vec{J} \otimes 1 \otimes 1, \quad \vec{n}_2 \rightarrow \vec{J}_2 \equiv 1 \otimes \vec{J} \otimes 1, \dots$$

$$\sum \vec{n}_i = \vec{0} \rightarrow \sum \vec{J}_i |\psi\rangle = \vec{0} \Rightarrow |\psi\rangle \text{ intertwiner}$$

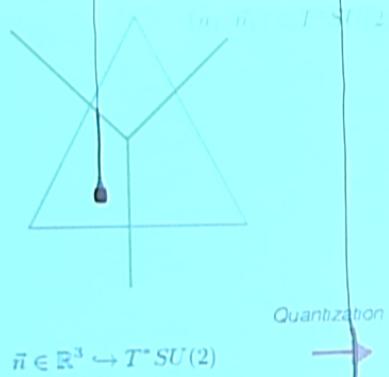
$$|\psi\rangle = |i_{j_1 j_2 j_3}\rangle$$

$$\vec{J}_i^2 |i_{j_1 j_2 j_3}\rangle = \ell_p^2 j_i(j_i + 1) |i_{j_1 j_2 j_3}\rangle = \hat{\ell}_i^2 |i_{j_1 j_2 j_3}\rangle$$

$$\vec{J}_1 \cdot \vec{J}_2 |i_{j_1 j_2 j_3}\rangle = \ell_p^2 \left(\frac{j_3(j_3 + 1) - j_1(j_1 + 1) - j_2(j_2 + 1)}{2} \right) |i_{j_1 j_2 j_3}\rangle$$

Quantum flat triangle!

A taste of what's happening



Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = 0$

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Cosine law: $n_1 \cdot n_2 = -\cos \theta_1 = \frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}$

Area

$$\mathcal{H}_{tot} = V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$$

$$\vec{n}_1 \rightarrow \vec{j}_1 \equiv \vec{j} \otimes 1 \otimes 1 \quad \vec{n}_2 \rightarrow \vec{j}_2 \equiv 1 \otimes \vec{j} \otimes 1 \dots$$

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$$\vec{j}_1^2 |i_{j_1 j_2 j_3}\rangle = l_p^2 j_1(j_1+1) |i_{j_1 j_2 j_3}\rangle = l_p^2 |i_{j_1 j_2 j_3}\rangle$$

$$j_1 \cdot j_2 |i_{j_1 j_2 j_3}\rangle = l_p^2 \left(\frac{j_1(j_1+1) - j_1(j_1+1) - j_2(j_2+1)}{2} \right) |i_{j_1 j_2 j_3}\rangle$$

Quantum flat triangle!

$$\Rightarrow \begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$\omega_k + c\mu = \gamma^{kA} K_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_A \gamma^{\mu \nu} y_B = \gamma_A \gamma^{\mu \nu} y_B$$

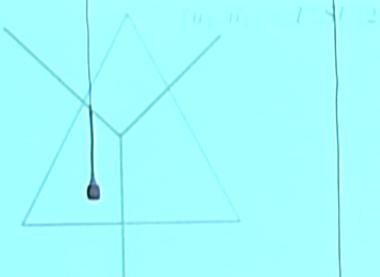
$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma(D\phi)^B A^{\mu \nu} \epsilon_{\mu \nu} \rightarrow \text{some } F^{\mu \nu}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B C = \begin{pmatrix} \omega_b & c^b \\ 0 & 0 \end{pmatrix}$$

A taste of what's happening

$$n \in \mathbb{R}^3 \hookrightarrow T^*SU(2)$$



$\tilde{n} \in \mathbb{R}^3 \hookrightarrow T^*SU(2)$
Cotangent space can be seen as the group $ISO(3)$

"Heisenberg double"

Use Poisson-Lie group formalism to deform it.

Quantization

Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = 0$

Length: $\vec{l}_i \cdot \vec{n}_i = l_i n_i$

Cosine law: $n_1 \cdot n_2 = -\cos \theta_1 = \frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}$

Area

$$\mathcal{H}_{tot} = V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$$

$$\tilde{n}_1 \rightarrow \tilde{J}_1 \equiv \tilde{l} \otimes 1 \otimes 1, \quad \tilde{n}_2 \rightarrow \tilde{J}_2 \equiv 1 \otimes \tilde{l} \otimes 1, \quad$$

$$\sum \tilde{n}_i = \tilde{l} \rightarrow \sum \tilde{J}_i |\psi\rangle = \tilde{l} |\psi\rangle \text{ intertwiner}$$

$$|\psi\rangle = |i_{j_1 j_2 j_3}\rangle$$

$$\tilde{J}_i^2 |i_{j_1 j_2 j_3}\rangle = l_p^2 j_i(j_i+1) |i_{j_1 j_2 j_3}\rangle = l_i^2 |i_{j_1 j_2 j_3}\rangle$$

$$J_1 \cdot J_2 |i_{j_1 j_2 j_3}\rangle = l_p^2 \left(\frac{j_1(j_1+1) + j_2(j_2+1) - j_3(j_3+1)}{2} \right) |i_{j_1 j_2 j_3}\rangle$$

$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \lambda \end{pmatrix}$$

ν, λ , lower indices

$$k + c\lambda = \gamma^A k_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$1(D\phi)^a / F^{\alpha} \epsilon_{\alpha\beta\gamma} \epsilon^{\gamma}_{\text{edge}} \rightarrow \text{second Fad Eabcd}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} A^B C = \begin{pmatrix} \omega_b & e^b \\ 0 & 0 \end{pmatrix}$$

A taste of what's happening

$(n_1, n_2) \in T^*SU(2)$

$\vec{n} \in \mathbb{R}^3 \hookrightarrow T^*SU(2)$

Cotangent space can be seen as the group $ISO(3)$

"Heisenberg double"

Use Poisson-Lie group formalism to deform it.

Quantization

Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = 0$

Length: $\vec{n}_1 = l_1 n_1$

Cosine law: $n_1 \cdot n_2 = -\cos \theta_1 = \frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}$

Area

$$\mathcal{H}_{tot} = V_{j_1} \otimes V_{j_2} \otimes V_{j_3}$$

$$\vec{n}_1 \rightarrow \vec{j}_1 \equiv \vec{j} \otimes 1 \otimes 1 \quad \vec{n}_2 \rightarrow \vec{j}_2 \equiv 1 \otimes \vec{j} \otimes 1 \dots$$

$$\sum \vec{n}_i = 0 \rightarrow \sum \vec{j}_i |\psi\rangle = 0 \Rightarrow |\psi\rangle \text{ intertwiner}$$

$$|\psi\rangle = |i_{j_1 j_2 j_3}\rangle$$

$$\vec{j}_i^2 |i_{j_1 j_2 j_3}\rangle = l_p^2 j_i(j_i+1) |i_{j_1 j_2 j_3}\rangle = l_p^2 |i_{j_1 j_2 j_3}\rangle$$

$$j_1 \cdot j_2 |i_{j_1 j_2 j_3}\rangle = l_p^2 \left(\frac{j_1(j_1+1) - j_2(j_2+1) - j_3(j_3+1)}{2} \right) |i_{j_1 j_2 j_3}\rangle$$

$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$\omega_A + c \alpha = \gamma^{i_A} \kappa_A$$

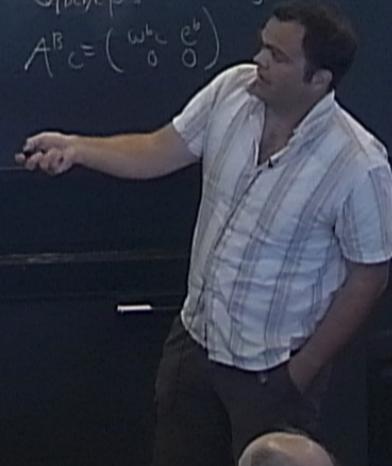
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma \zeta_A \gamma^{i_B} \gamma_B = \gamma_A \gamma^{i_B} \gamma_B$$

$$\gamma(D\phi)^B A^{\alpha} F^{\beta} \epsilon_{\alpha\beta\gamma} \rightarrow \text{some } F^{\alpha} \epsilon_{\alpha\beta\gamma} \epsilon_{\gamma\delta\epsilon} F^{\delta}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B C = \begin{pmatrix} \omega_A & c^B \\ 0 & 0 \end{pmatrix}$$

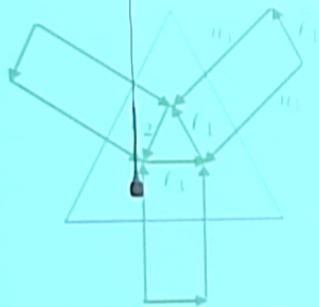


A taste of what's happening

$$T^*SU(2) \cong ISO(3) \rightarrow SL(2, \mathbb{C})$$

$G_2 = \text{Spin}(7) \subset SO(14)$

$\text{Spin}(7) \rightarrow SL(2, \mathbb{C})$



Hyperbolic triangle!

$$\text{Gauss law: } \ell_1 \ell_2 \ell_3 = 1$$

$$\text{Normals at vertex: } \sinh \frac{i}{R} b_i = \frac{1}{2} \text{tr}((\ell_i \ell_i^\dagger) \vec{\sigma}) = T_i$$

$$\sinh \frac{i}{R} b_i^{op} = \frac{1}{2} \text{tr}((\ell_i^\dagger \ell_i) \vec{\sigma}) = T_i^{op}$$

$$h^{op} = h \circ h = h \in SL(2)$$

Hyperbolic cosine law:

$$b_1^{op} \cdot b_2 = \frac{\cosh \frac{i}{R} - \cosh \frac{i}{R} \cosh \frac{i}{R}}{\sinh \frac{i}{R} \sinh \frac{i}{R}}$$

$$\Rightarrow \begin{pmatrix} M & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \mu \end{pmatrix}$$

ν, μ , lower indices

$$\omega_A + c \mu = \gamma_A^A \omega_A$$

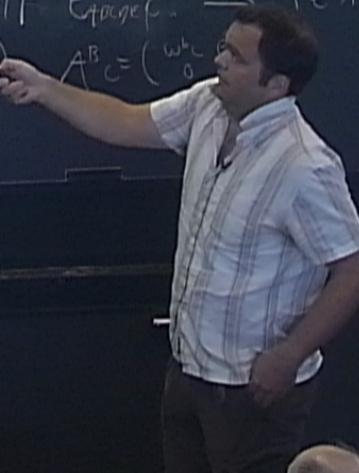
$$\begin{pmatrix} 1 & | & 0 \\ 0 & | & 0 \end{pmatrix}$$

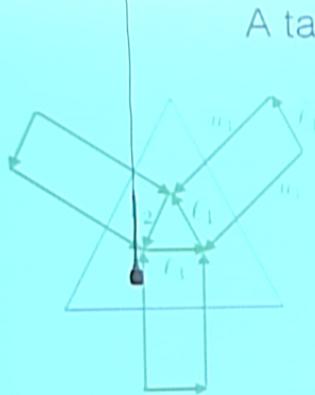
$$\gamma_A \gamma_B^A \gamma_B^B y_B = \gamma_A^A \gamma_B^B y_B$$

$$\begin{pmatrix} 0 & | & - \\ 0 & | & 1 \end{pmatrix}$$

$$1(D\phi)^a / F^a \epsilon_{abc} \epsilon_{abc} \rightarrow e^a e^b F^{ab} \epsilon_{abc}$$

$$\rightarrow \begin{pmatrix} 0 \\ A^B \end{pmatrix} C^B_C = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix}$$





A taste of what's happening

$$T^*SU(2) \cong ISO(3) \rightarrow SL(2, \mathbb{C})$$

$$G = \mathbb{R}^3 \times \mathbb{S}^2 \times \mathbb{S}^1 / \mathbb{Z}_2$$

$$SL(2, \mathbb{C}) = \mathbb{H}^3 / \mathbb{S}^2$$

Hyperbolic triangle!

$$\text{Gauss law: } f_1 f_2 f_3 = 1$$

$$\text{Normals at vertex: } \sinh \frac{i}{h} b_0 = \frac{1}{2} \text{tr} ((f_i f_i^\dagger) \vec{\sigma}) = T_i$$

$$\sinh \frac{i}{h} b_i^{op} = \frac{1}{2} \text{tr} ((f_i f_i) \vec{\sigma}) = T_i^{op}$$

$$b^{op} = h > b = h \in SU(2)$$

Hyperbolic cosine law:

$$b_1^{op} \cdot b_2 = \frac{\cosh \frac{i}{h} - \cosh \frac{i}{h} \cosh \frac{i}{h}}{\sinh \frac{i}{h} \sinh \frac{i}{h}}$$

$$\Rightarrow \begin{pmatrix} M & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

η , lower indices

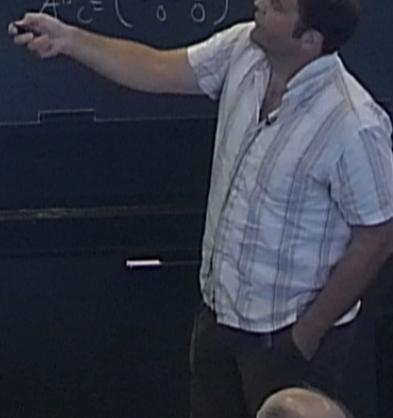
$$u_A + c\eta = \gamma^A \cdot k_A$$

$$\begin{pmatrix} 1 & | & 0 \\ 0 & | & 0 \end{pmatrix} \quad \gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$\begin{pmatrix} 0 & | & - \\ 0 & | & 1 \end{pmatrix}$$

$$1(D\phi)^a \wedge F^{ab} \epsilon_{abc} \rightarrow \star \epsilon^{abc} F^{cd} \epsilon_{abd}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad A^B{}_C = \begin{pmatrix} \omega_C{}^B & e^B \\ 0 & 0 \end{pmatrix}$$

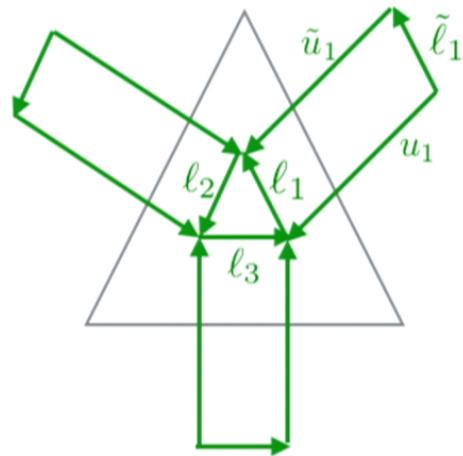


A taste of what's happening

$$T^*SU(2) \sim ISO(3) \rightarrow SL(2, \mathbb{C})$$

$$G_1 = u_1 \ell_1 = \tilde{\ell}_1 \tilde{u}_1 \in SL(2, \mathbb{C})$$

$$u_1 \in SU(2), \ell_1 \in AN(2)$$



Hyperbolic triangle!

$$\text{Gauss law: } \ell_1 \ell_2 \ell_3 = \mathbf{1}$$

$$\begin{aligned} \text{Normals at vertex: } \sinh \frac{l_i}{R} \hat{b}_i &= \frac{1}{2} \operatorname{tr} ((\ell_i \ell_i^\dagger) \vec{\sigma}) = \vec{T}_i \\ \sinh \frac{l_i}{R} \hat{b}_i^{op} &= \frac{1}{2} \operatorname{tr} ((\ell_i^\dagger \ell_i) \vec{\sigma}) = \vec{T}_i^{op} \end{aligned}$$

$$\hat{b}^{op} = h \triangleright \hat{b}, \quad h \in SU(2)$$

Hyperbolic cosine law:

$$\hat{b}_1^{op} \cdot \hat{b}_2 = \frac{\cosh \frac{l_3}{R} - \cosh \frac{l_1}{R} \cosh \frac{l_2}{R}}{\sinh \frac{l_1}{R} \sinh \frac{l_2}{R}}$$

A taste of what's happening

$$T^*SU(2) \cong ISO(3) \rightarrow SL(2, \mathbb{C})$$

$$G = \text{SO}(3) \cong SU(2)$$

$$\mathfrak{su}(2) \cong \mathfrak{so}(3)$$

Hyperbolic triangle!

$$\text{Gauss law: } \ell_1 \ell_2 \ell_3 = 1$$

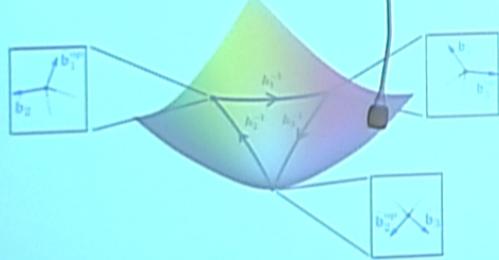
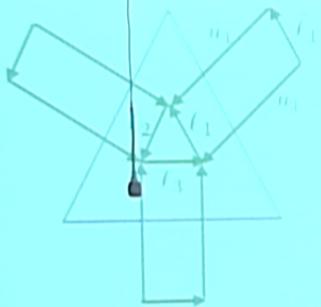
$$\text{Normals at vertex: } \sinh \frac{i}{R} b_0 = \frac{1}{2} \text{tr}((\ell_i \ell_i^\dagger) \vec{\sigma}) = T_i$$

$$\sinh \frac{i}{R} b_i^{op} = \frac{1}{2} \text{tr}((\ell_i \ell_i) \vec{\sigma}) = T_i^{op}$$

$$h^{op} = h \circ h^{-1} = h \in SU(2)$$

Hyperbolic cosine law

$$b_1^{op} \cdot b_2 = \frac{\cosh \frac{i}{R} - \cosh \frac{i}{R} \cosh \frac{i}{R}}{\sinh \frac{i}{R} \sinh \frac{i}{R}}$$



$$\Rightarrow \begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} n \\ 1 \end{pmatrix}$$

\mathfrak{su}_n , lower indices

$$\omega_A + c \alpha = \gamma^A \omega_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & - \\ 0 & 1 \end{pmatrix}$$

$$\gamma_A \gamma^B y_B = \gamma_A \gamma^B y_B$$

$$(D\phi)^a / F^a \epsilon_{edge} \rightarrow$$

$$\int \gamma^a \partial^b A^c F^{ab} \epsilon_{abcd}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} A^B C = \begin{pmatrix} \omega_A & \epsilon^b \\ 0 & 0 \end{pmatrix}$$

Manifolds

A taste of what's happening

$$T^*SU(2) \cong ISO(3) \rightarrow SL(2, \mathbb{C})$$

$G_2 = \text{Lie group} \cong SL(2)$

$SL(2) \cong SU(2)$

Hyperbolic triangle!

$$\text{Gauss law: } \ell_1 \ell_2 \ell_3 = 1$$

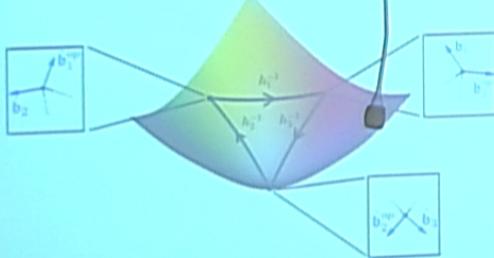
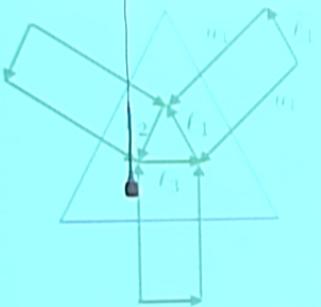
$$\text{Normals at vertex: } \sinh \frac{i}{h} b_i = \frac{1}{2} \text{tr}((\ell_i \ell_i^\dagger) \vec{\sigma}) = T_i$$

$$\sinh \frac{i}{h} b_i^{op} = \frac{1}{2} \text{tr}((\ell_i^\dagger \ell_i) \vec{\sigma}) = T_i^{op}$$

$$h^{op} = h > h = h \in SU(2)$$

Hyperbolic cosine law

$$b_1^{op} \cdot b_2 = \frac{\cosh \frac{i}{h} - \cosh \frac{i}{h_1} \cosh \frac{i}{h_2}}{\sinh \frac{i}{h} \sinh \frac{i}{h}}$$



$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} (\nu)$$

ν , lower indices

$$k + c\nu = \gamma^A k_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_A \gamma^{ab} y_B = \gamma_A \gamma^{ab} y_B$$

$$\begin{pmatrix} 0 & - \\ 0 & 1 \end{pmatrix}$$

$$\gamma(D\phi)^a \gamma^b \epsilon_{abc} \rightarrow \text{some } F^{ab} \epsilon_{abc}$$

$$\Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad A^B C = \begin{pmatrix} \omega_a & e^b \\ 0 & 0 \end{pmatrix}$$



A taste of what's happening

Phase space

$$G_1 = u_1 \ell_1 + \ell_1 u_1 \in SL(2, \mathbb{C})$$

$$u_1 \in SU(2) \quad \ell_1 \in A(N/2)$$

$$\ell = \begin{pmatrix} \lambda & 0 \\ z & \lambda^{-1} \end{pmatrix}$$

Hyperbolic triangle

$$\text{Gauss law: } \ell_1 \ell_2 \ell_3 = 1$$

Normals at vertex.

$$\vec{T}_i$$

$$|\vec{T}_i|^2 = \sinh^2 \frac{l_i}{R}$$

$$R = \frac{1}{\sqrt{|\Delta|}}$$

Hyperbolic cosine law:

$$\tilde{b}_1^{ap} \cdot \tilde{b}_2 = \frac{\cosh \frac{l_1}{R} - \cosh \frac{l_2}{R} \cosh \frac{l_3}{R}}{\sinh \frac{l_1}{R} \sinh \frac{l_2}{R}}$$

Quantization $q = e^{i\kappa\sqrt{|\Delta|}} = e^{\frac{i\pi}{k}}$

$$\lambda \rightarrow K \quad \lambda^{-1} \rightarrow K^{-1}$$

$$z \rightarrow (q^{\frac{1}{2}} - q^{-\frac{1}{2}}) J_z \quad \frac{z}{\lambda} \rightarrow -(q^{\frac{1}{2}} - q^{-\frac{1}{2}}) J_{\pm}$$

$$K J_+ K^{-1} = q^{\frac{1}{2}} J_+ \quad K J_- K^{-1} = q^{-\frac{1}{2}} J_-$$

$$[J_+, J_{\pm}] = \frac{K^2 - K^{-2}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}} \quad U_q(\mathfrak{su}(2)) \text{ generators!}$$

$$(J_+ - K - K^{-1} - J_{\pm} - K + K^{-1} - K^{-1} - J_{\mp})(\psi) = 0$$

$$|\psi\rangle = |\ell_{12323}\rangle_q$$

$U_q(\mathfrak{su}(2))$ intertwiner

$$\vec{t}_i \quad \text{vector operator for } U_q(\mathfrak{su}(2))$$

$$\vec{t}_i^2 |\ell_{12323}\rangle_q = \left(\sinh^2 \left(\frac{l_i + \frac{1}{2}}{k} \right) - \sinh^2 \left(\frac{l_i}{2k} \right) \right) |\ell_{12323}\rangle_q \Rightarrow l_i = l_p(j_i + \frac{1}{2})$$

Quantum hyperbolic triangle!

$$\xrightarrow{\cong} \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} \nu \\ \eta \end{pmatrix}$$

ν, η , lower indices

$$K + c\eta = \gamma^A K_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \gamma^A \gamma^B y_B = \gamma_A^1 \gamma_B^B y_B$$

$$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$$

$$1(D\phi)^n F^m \epsilon_{\text{edge}} \xrightarrow{\text{several } F^m} \epsilon_{\text{edge}}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A^B C = \begin{pmatrix} \omega_C & 0 \\ 0 & 0 \end{pmatrix}$$

A taste of what's happening

Phase space

$$G_1 = u_1 \ell_1 + \ell_1 u_1 \in SL(2)$$

$$u_1 \in SU(2) \quad \ell_1 \in AN(2)$$

$$\ell = \begin{pmatrix} \lambda & 0 \\ z & \lambda^{-1} \end{pmatrix}$$

$\lambda \in \mathbb{R}, \quad z \in \mathbb{C}$

Hyperbolic triangle

$$\text{Gauss law: } \ell_1 \ell_2 \ell_3 = 1$$

Normals at vertex

$$\vec{T}_i$$

$$|\vec{T}_i|^2 = \sinh^2 \frac{l_i}{R}$$

$$R = \frac{1}{\sqrt{|\Delta|}}$$

Hyperbolic cosine law:

$$\tilde{b}_1^{sp} \cdot \tilde{b}_2 = \frac{\cosh \frac{l_1}{R} - \cosh \frac{l_2}{R} \cosh \frac{l_3}{R}}{\sinh \frac{l_1}{R} \sinh \frac{l_2}{R}}$$

Quantization: $q = e^{i\kappa\sqrt{|\Delta|}} = e^{\frac{i\pi}{k}}$

$$\lambda \rightarrow K, \quad \lambda^{-1} \rightarrow K^{-1}$$

$$z \rightarrow (q^{\frac{1}{2}} - q^{-\frac{1}{2}})J_z, \quad \bar{z} \rightarrow -(q^{\frac{1}{2}} + q^{-\frac{1}{2}})J_{\bar{z}}$$

$$K J_+ K^{-1} = q^{\frac{1}{2}} J_+, \quad K J_- K^{-1} = q^{-\frac{1}{2}} J_-$$

$$J_+ J_- = \frac{K^2 - K^{-2}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}} \quad U_q(\mathfrak{su}(2)) \text{ generators!}$$

$$(J_+ - K - \bar{K} + K^{-1}) J_+ - (J_- - K + K^{-1} - K^{-1}) J_- = 0$$

$$|\psi\rangle = |\ell_{123}\rangle_q \quad U_q(\mathfrak{su}(2)) \text{ intertwiner}$$

\vec{t}_i vector operator for $U_q(\mathfrak{su}(2))$

$$\vec{t}_i^2 |\ell_{123}\rangle_q = \left(\sinh^2 \left(\frac{j_1 + \frac{1}{2}}{k} \right) + \sinh^2 \left(\frac{j_2}{k} \right) \right) |\ell_{123}\rangle_q \Rightarrow l_i = l_p(j_i + \frac{1}{2})$$

Quantum hyperbolic triangle!

$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix}$$

By lower indices

$$v_k + c w_k = \gamma^A v^A \quad K_A$$

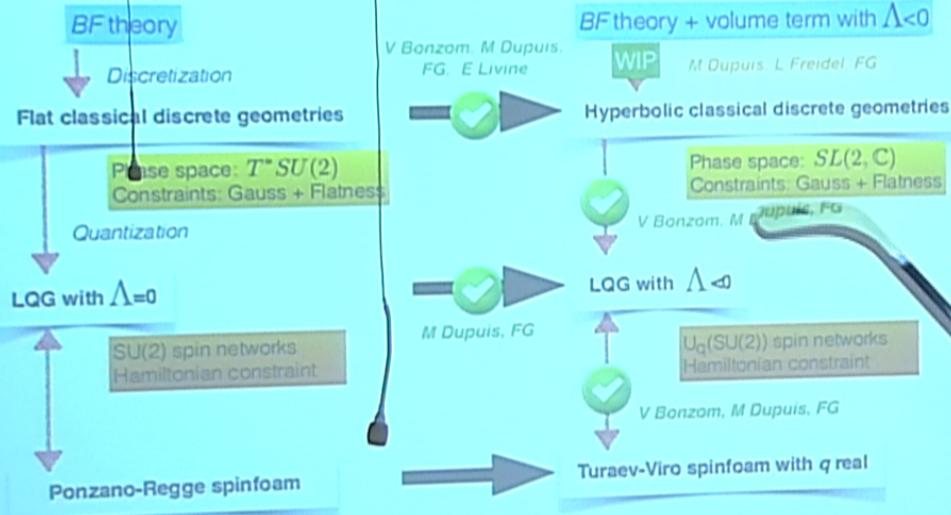
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \gamma_A v^A w_B = \gamma_A^I v^B w_B$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$1(D\phi)^a F^{\mu\nu} \epsilon_{\mu\nu} \rightarrow e^a e^b F^{\mu\nu} \epsilon_{\mu\nu}$$

$$\Rightarrow \begin{pmatrix} 0 \\ A \end{pmatrix} \quad \begin{pmatrix} 0 \\ B \end{pmatrix}$$

Cosmological constant in LQG



$$\Rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

EP, other indices

$$c_A + c_B = \gamma^{AB} K_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \gamma_A^B y_B = \gamma_A^B y_B$$

$$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$$

$$1(D\phi)^B A^A F^B e_{AB} e^{CDE} \rightarrow S \epsilon_{ABC} F^{AB} e_{CDE}$$

(0) A^B γ^B

A selection of what we want to do next

- Understand the discretization procedure.
- Understand better the quantization of discrete hyperbolic geometries.
- Other signatures, q root of unity...
- Link with Chern-Simmons?
- What about 4d?!

$$\rightarrow \begin{pmatrix} m & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

EP, lower indices

$$k + c\ell = \gamma^A k_A$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \gamma_A \gamma^{AB} y_B = \gamma_A \gamma^{AB} y_B$$

$$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$$

$$\gamma(D\phi)^a \gamma F^{ab} \epsilon_{edge} \rightarrow \text{some } F^{ab} \epsilon_{edge}$$

$$\rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} A^B{}_C = \begin{pmatrix} \omega_C{}^B \\ 0 \end{pmatrix}$$