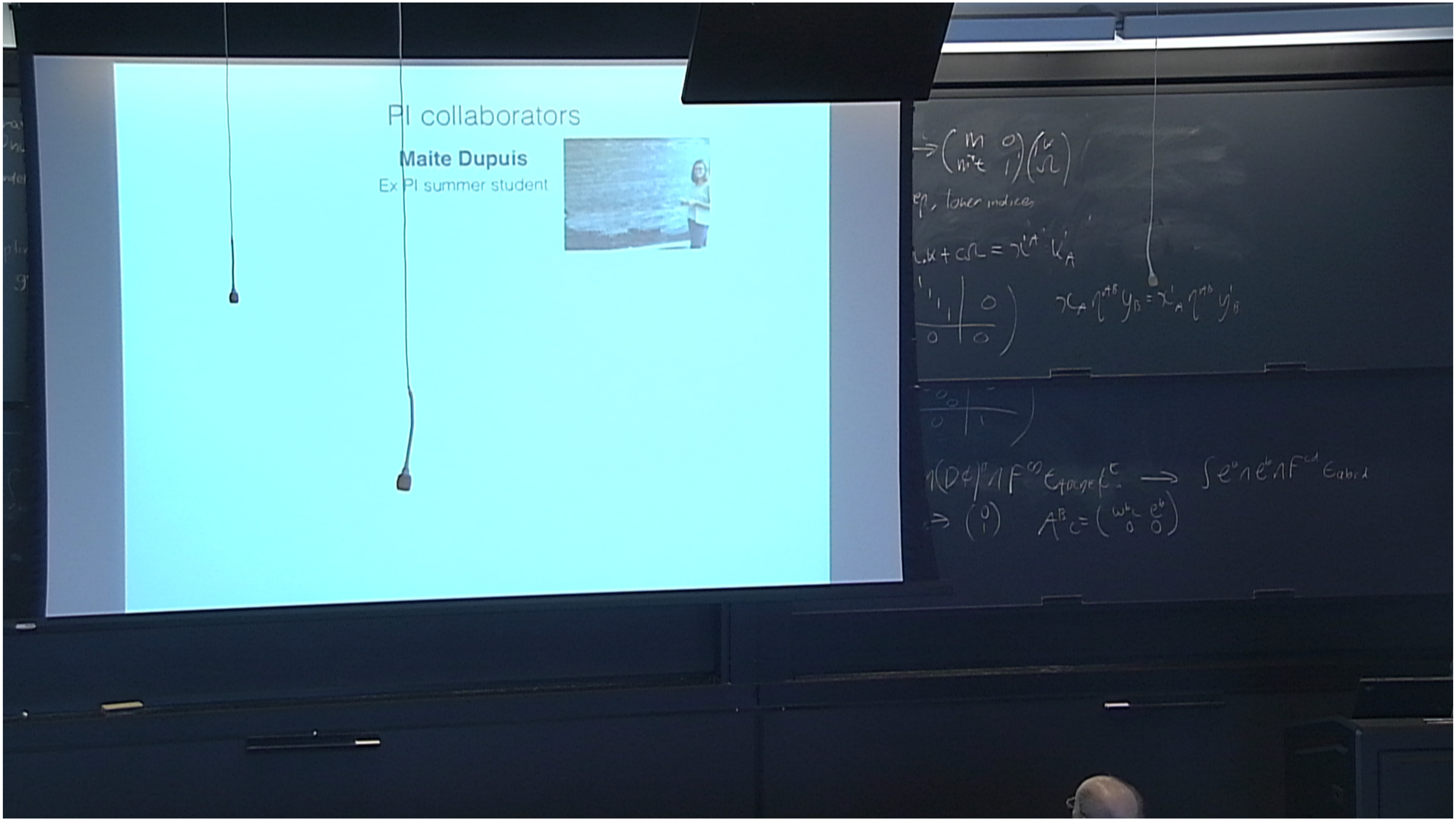
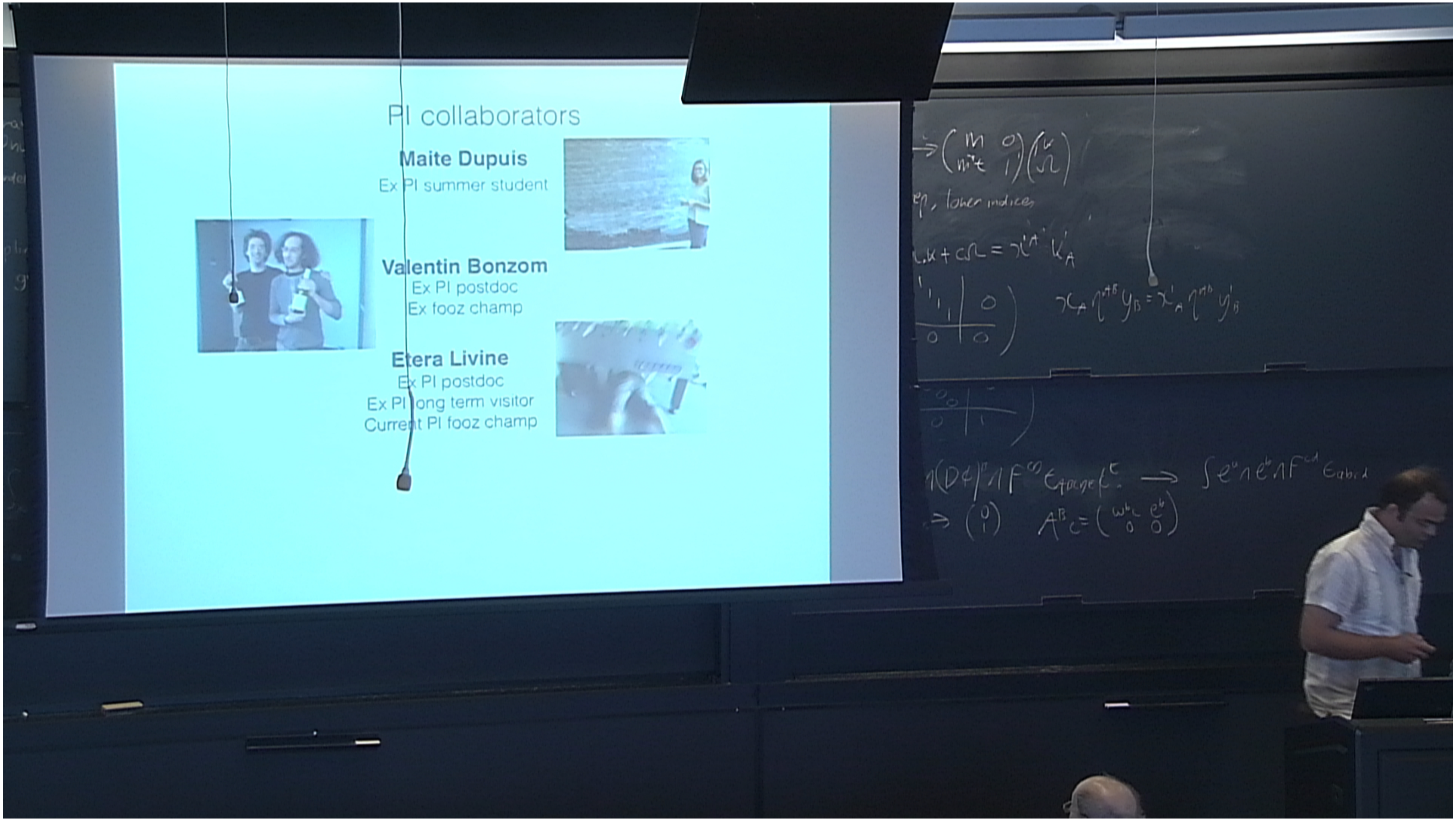
Title: TBA

Date: May 21, 2014 03:50 PM

URL: http://pirsa.org/14050121

Abstract:


PI collaborators
Maite Dupuis
Ex PI summer student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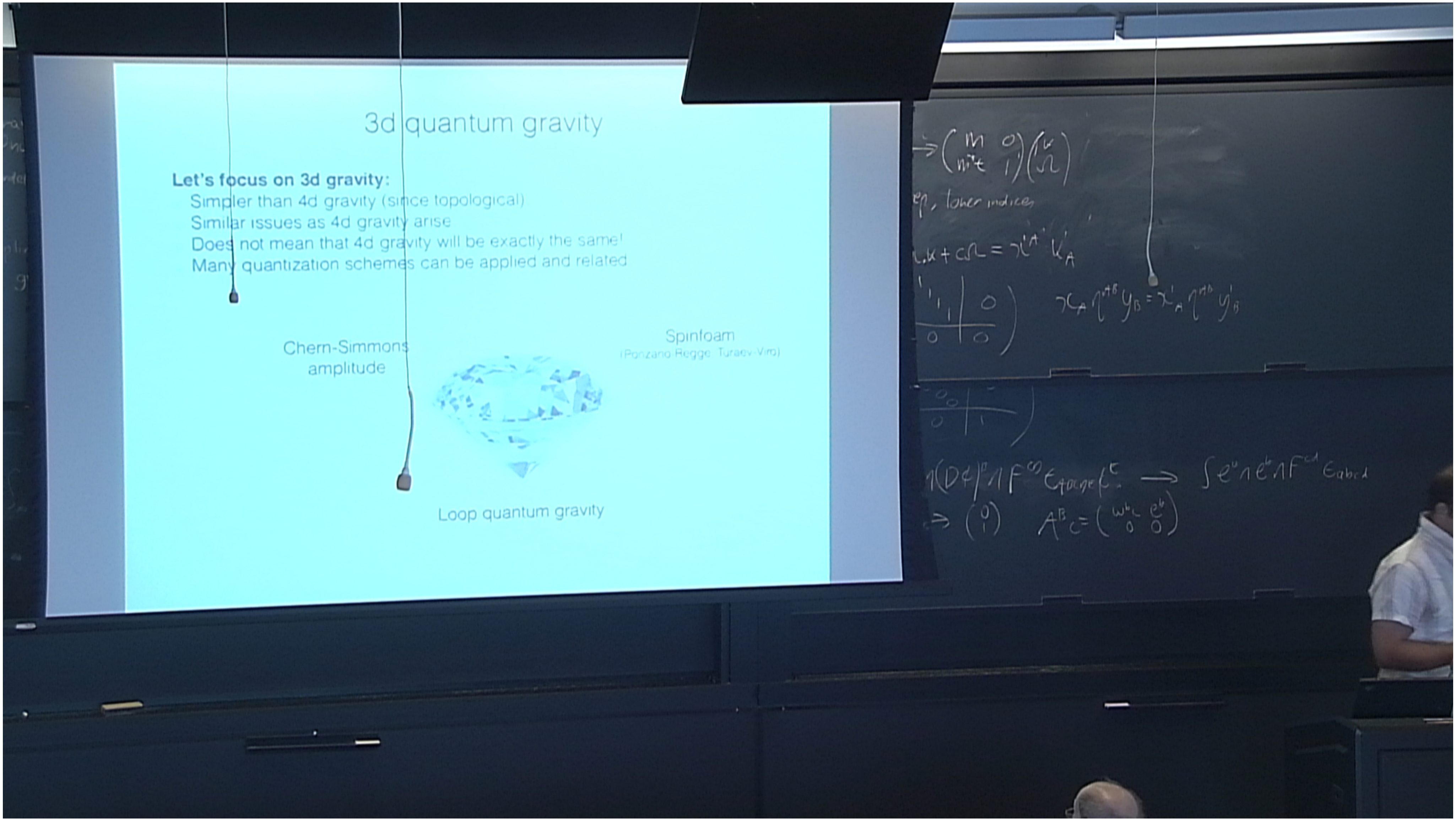
Maite Dupuis
Ex PI summer student
Valentin Bonzom
Ex PI postdoc
Ex fooz champ
Etera Livine
Ex PI postdoc
Ex PI long term visitor
Current PI fooz champ


3d quantum gravity
Let's focus on 3d gravity:
Simpler than 4d gravity (since topological)
Similar issues as 4d gravity arise
Does not mean that 4d gravity will be exactly the same!
Many quantization schemes can be applied and related
Chern-Simmons
amplitude
Spinfoam
(Ponzano-Regge, Turaev-Viro)
Loop quantum gravity

3d quantum gravity

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- Simpler than 4d gravity (since topological).
- Similar issues as 4d gravity arise.
- Does not mean that 4d gravity will be exactly the same!
- Many quantization schemes can be applied and related.

Chern-Simmons
amplitude

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Loop quantum gravity

3d quantum gravity

Treat cosmological constant as a coupling constant, just like *G*

$$S_{CS}(A) = \frac{k}{4\pi}\int_M tr\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) \quad \Leftrightarrow \quad S_{BF}(e,\omega) = \int_M tr\left(e\wedge F(\omega) + \frac{\Lambda}{3}e\wedge e\wedge e\right)$$

$$k = \frac{1}{\ell_P\sqrt{|\Lambda|}}$$

Combinatorial quantization → Chern-Simmons amplitude

Path integral quantization → Spinfoam (Ponzano-Regge, Turaev-Viro)

Canonical quantization → Loop quantum gravity

LQG vs spinfoam when Λ is not zero

Upon Hamiltonian analysis, the cosmological constant appears only in the Hamiltonian constraint, i.e. not in the Gauss constraint.

The quantum Gauss constraint implements the relevant gauge structure.

Spin network based on SU(2) whether Λ is zero or not.

Euclidian case considered here!

LQG vs spinfoam when Λ is not zero

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Spin network based on SU(2) whether Λ is zero or not.

Boundary states of the Turaev-Viro model are defined in terms of a quantum group

Spin network based on $\mathcal{U}_q(\mathfrak{su}(2))$

Euclidian case considered here!

Cosmological constant in LQG

BF theory

Discretization

Flat classical discrete geometries

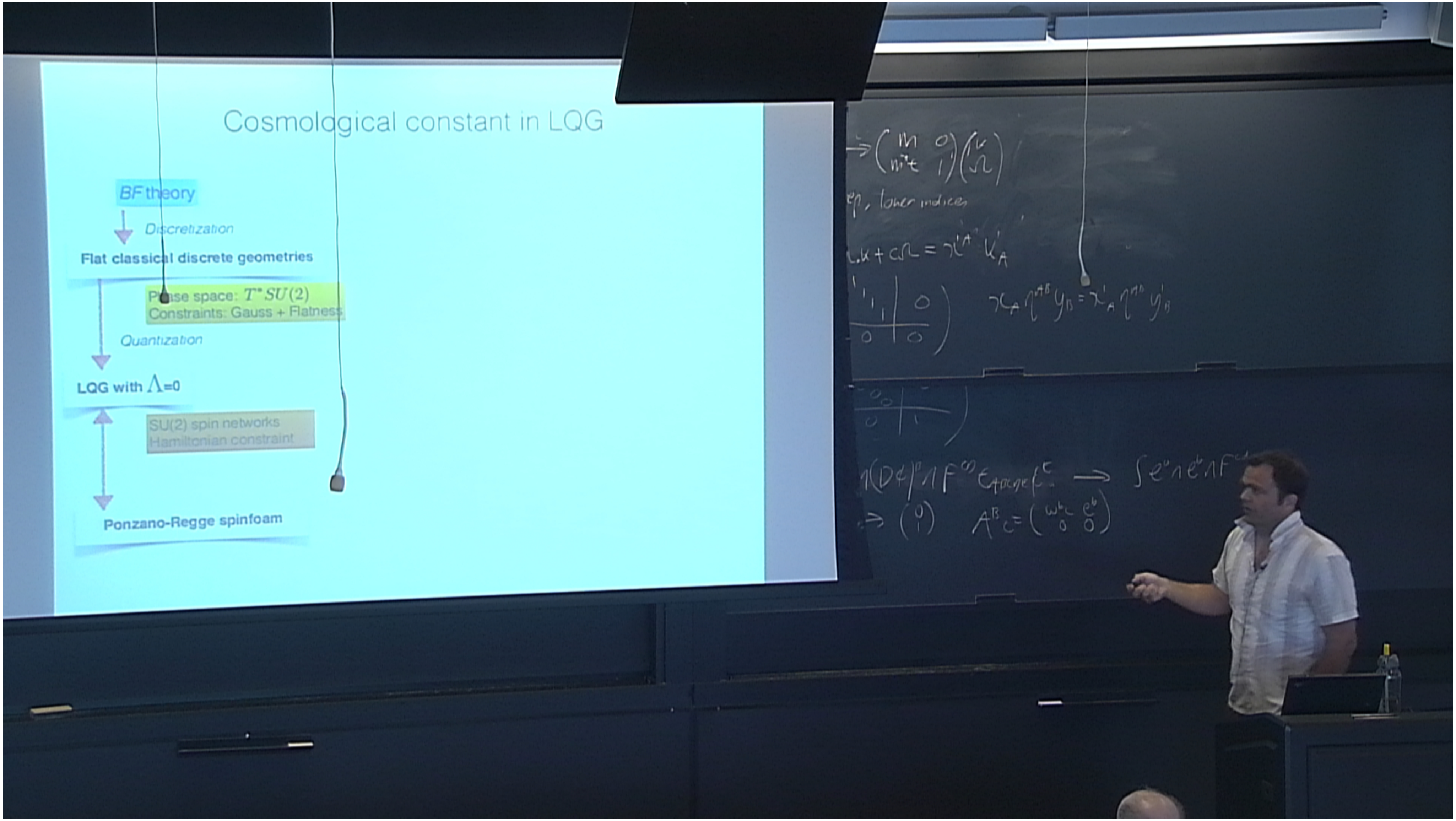
Phase space: $T^* SU(2)$
Constraints: Gauss + Flatness

Quantization

LQG with Λ=0

SU(2) spin networks
Hamiltonian constraint

Ponzano-Regge spinfoam


Cosmological constant in LQG
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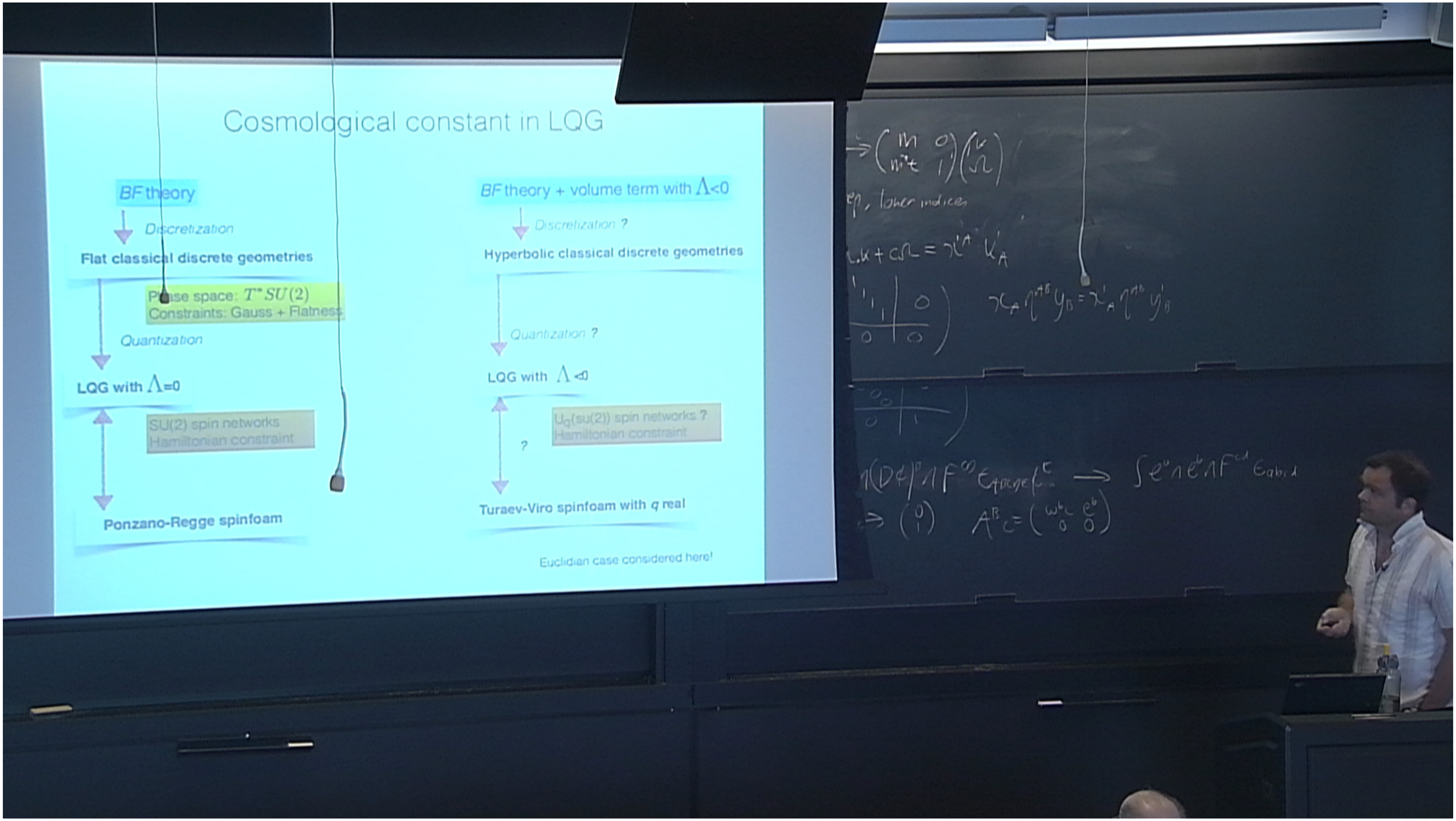
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BF theory + volume term with Λ<0
Discretization ?
Hyperbolic classical discrete geometries
Quantization ?
LQG with Λ<0
$U_q(su(2))$ spin networks ?
Hamiltonian constraint
?
Turaev-Viro spinfoam with q real
Euclidian case considered here!

Cosmological constant in LQG

BF theory

↓ *Discretization*

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↕

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↓ *Discretization* **?**

Hyperbolic classical discrete geometries

↓ *Quantization* **?**

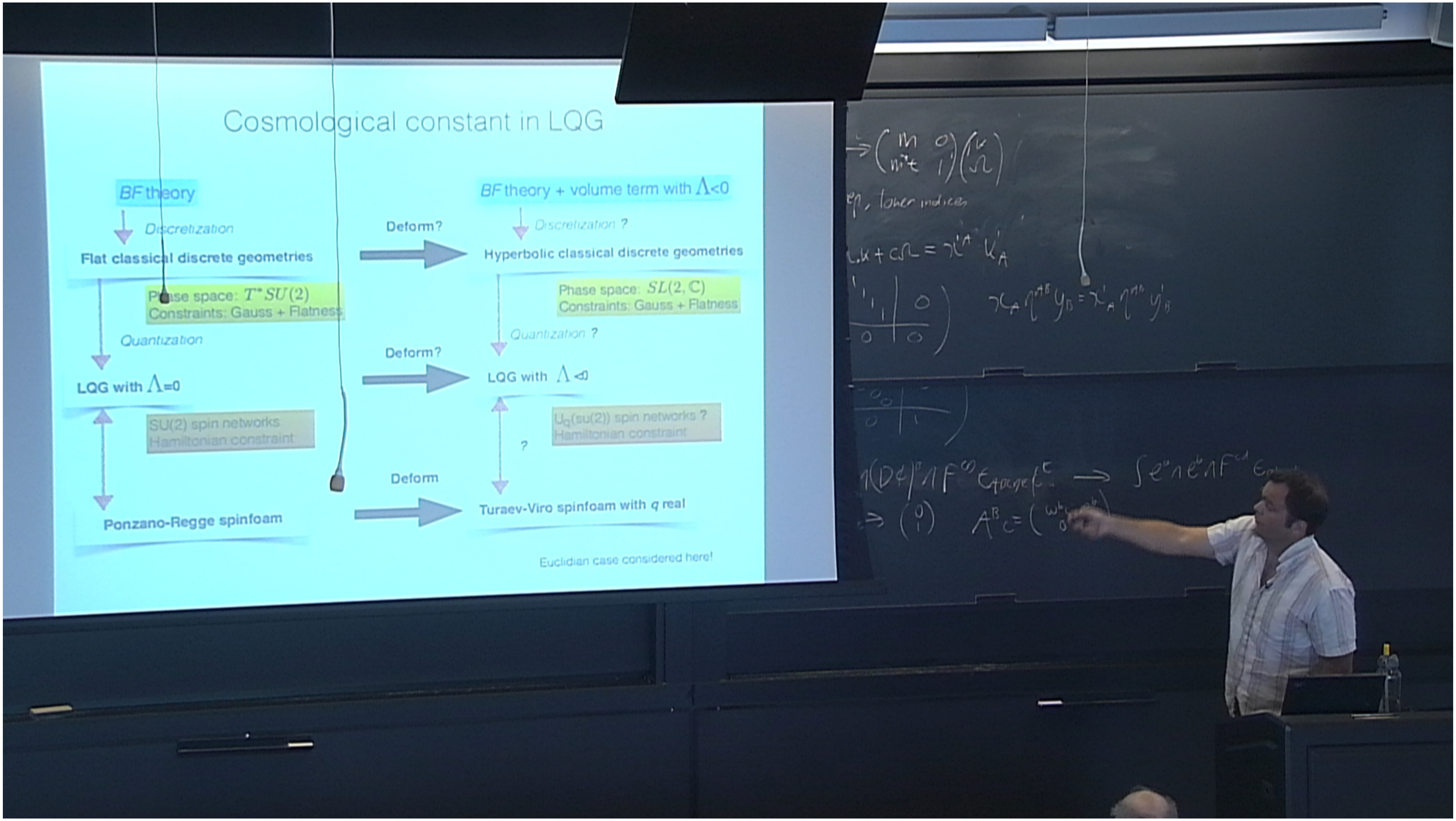
LQG with Λ<0

$U_q(su(2))$ spin networks **?**
Hamiltonian constraint

↕ ***?***

Turaev-Viro spinfoam with *q* real

Euclidian case considered here!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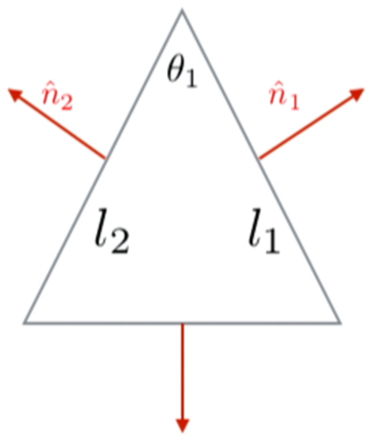
A taste of what's happening



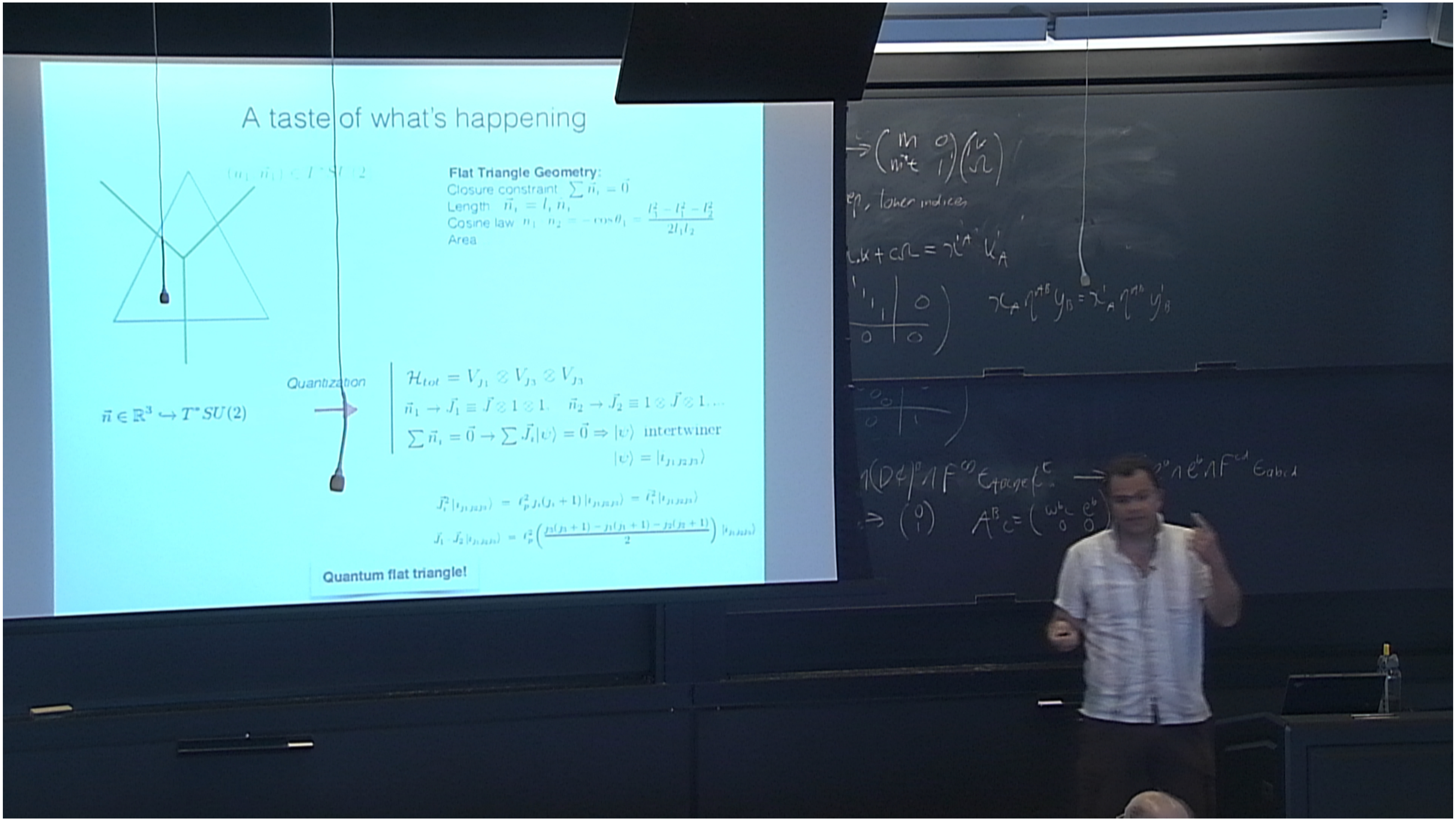
Flat Triangle Geometry:

Closure constraint: $\sum \vec{n}_i = \vec{0}$

Length: $\vec{n}_i = l_i \, \hat{n}_i$

Cosine law: $\hat{n}_1 \cdot \hat{n}_2 = -\cos\theta_1 = \frac{l_3^2 - l_1^2 - l_2^2}{2 l_1 l_2}$

Area…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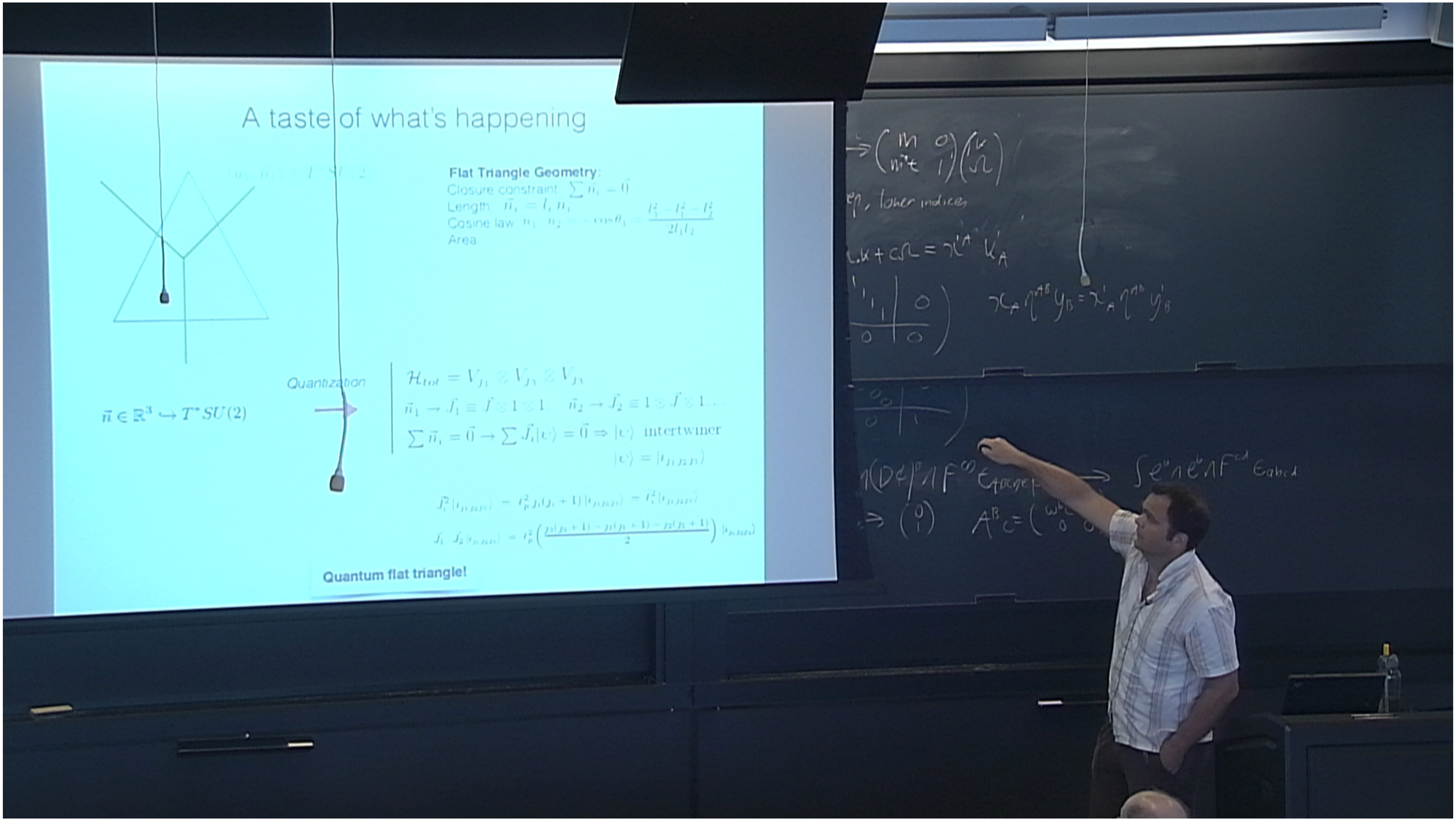
Flat Triangle Geometry:
Closure constraint
Length
Cosine law
Area
Quantization
intertwiner
Quantum flat triangle!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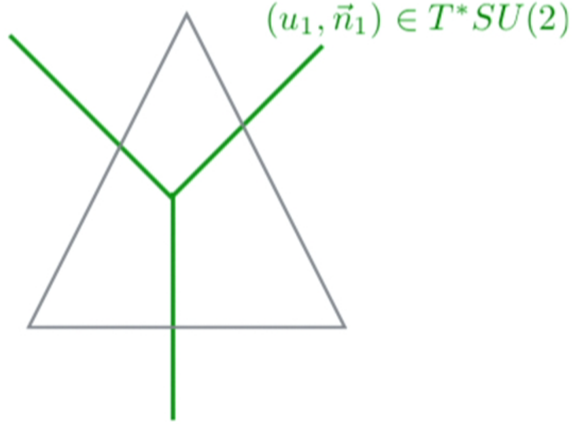

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Area...

$\vec{n} \in \mathbb{R}^3 \hookrightarrow T^*SU(2)$

Quantization →

$$\mathcal{H}_{tot} = V_{j_1} \otimes V_{j_3} \otimes V_{j_3}$$

$$\vec{n}_1 \to \vec{J}_1 \equiv \vec{J} \otimes 1 \otimes 1, \quad \vec{n}_2 \to \vec{J}_2 \equiv 1 \otimes \vec{J} \otimes 1, ...$$

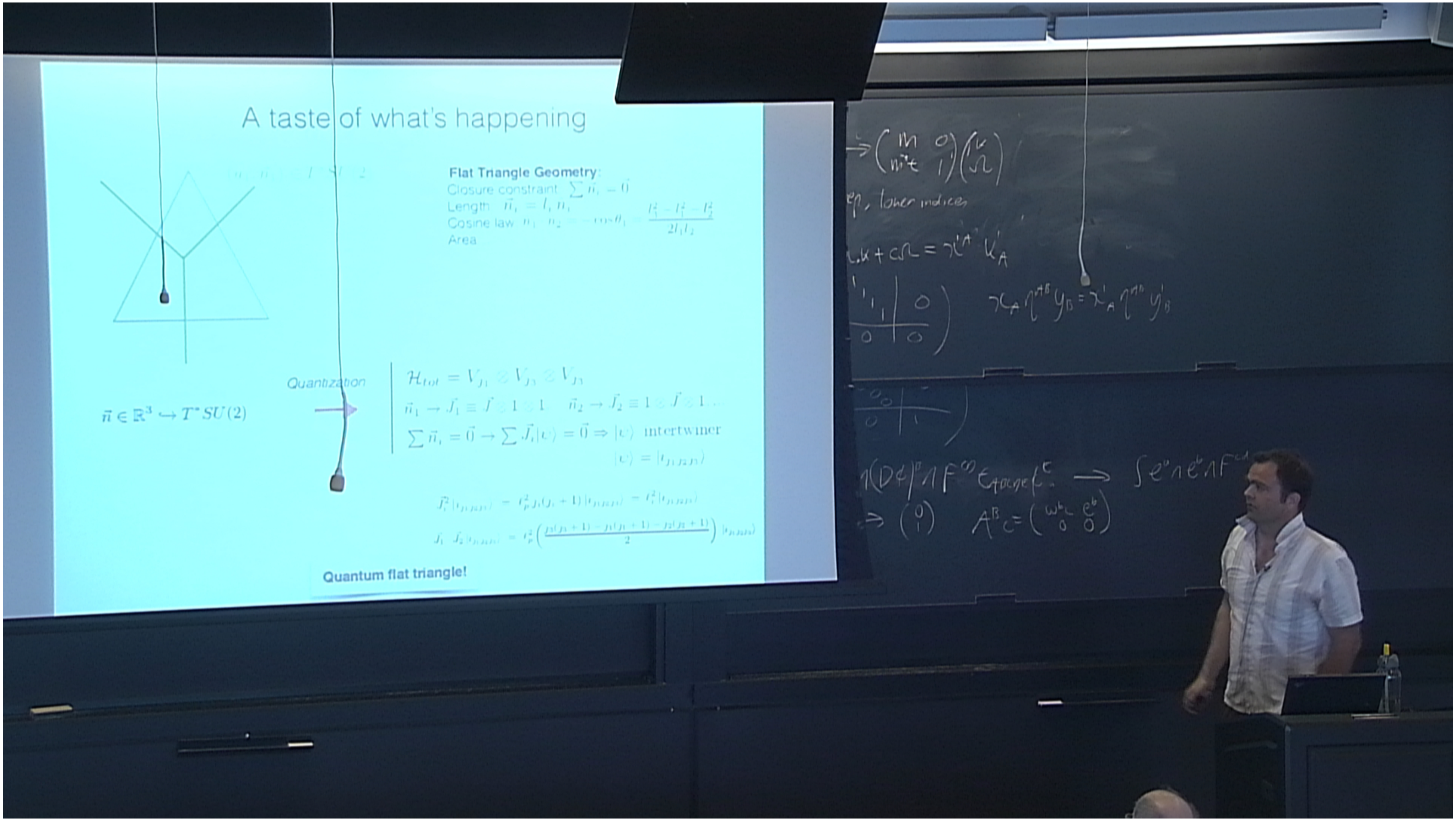
$$\sum \vec{n}_i = \vec{0} \to \sum \vec{J}_i |\psi\rangle = \vec{0} \Rightarrow |\psi\rangle \ \text{intertwiner}$$

$$|\psi\rangle = |i_{j_1 j_2 j_3}\rangle$$

$$\vec{J}_i^2 \, |i_{j_1 j_2 j_3}\rangle = \ell_p^2 \, j_i(j_i+1) \, |i_{j_1 j_2 j_3}\rangle = \hat{\ell}_i^2 \, |i_{j_1 j_2 j_3}\rangle$$

$$\vec{J}_1 \cdot \vec{J}_2 \, |i_{j_1 j_2 j_3}\rangle = \ell_p^2 \left(\frac{j_3(j_3+1) - j_1(j_1+1) - j_2(j_2+1)}{2} \right) |i_{j_1 j_2 j_3}\rangle$$

Quantum flat triangle!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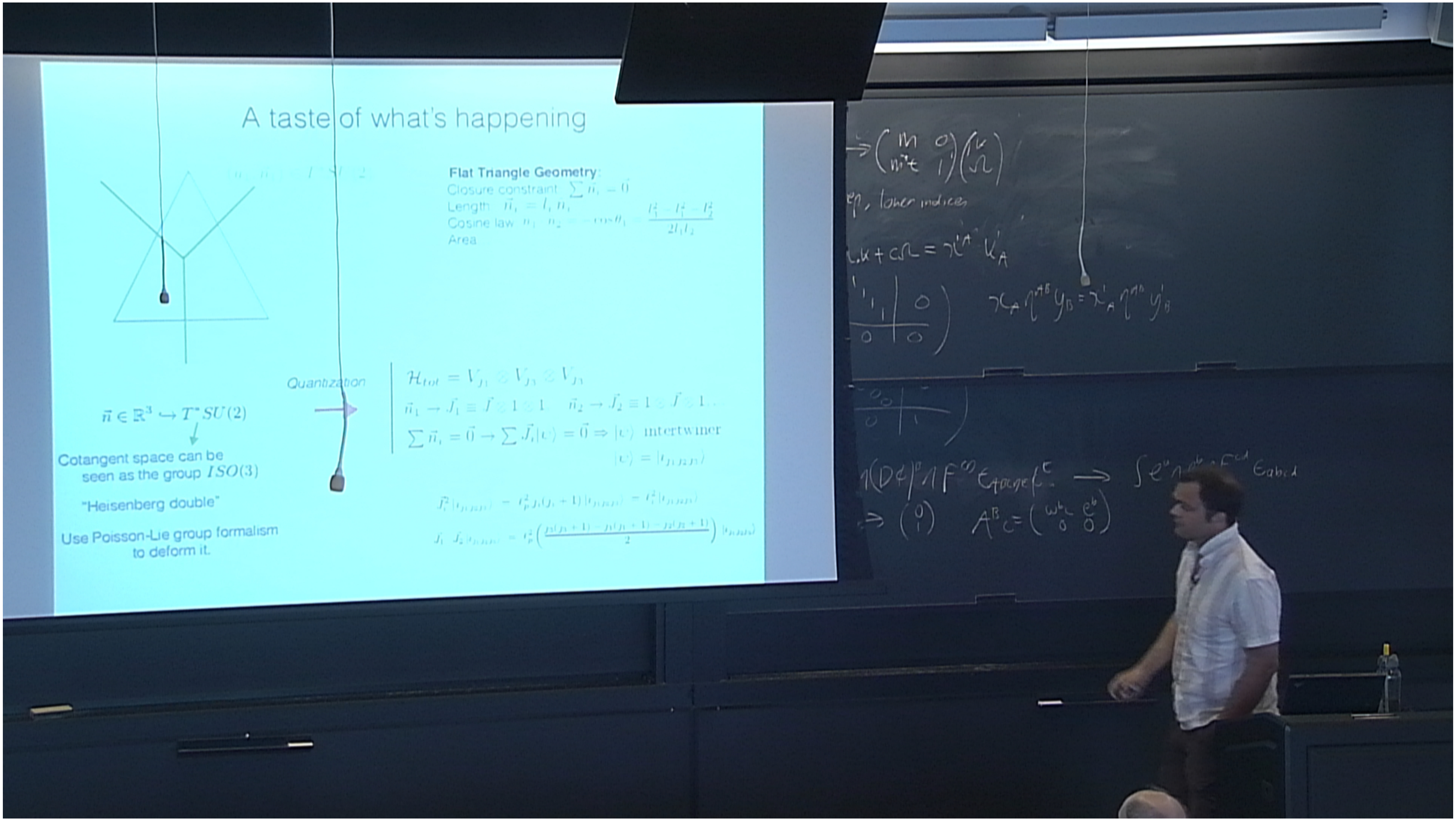

A taste of what's happening
Flat Triangle Geometry
Closure constraint
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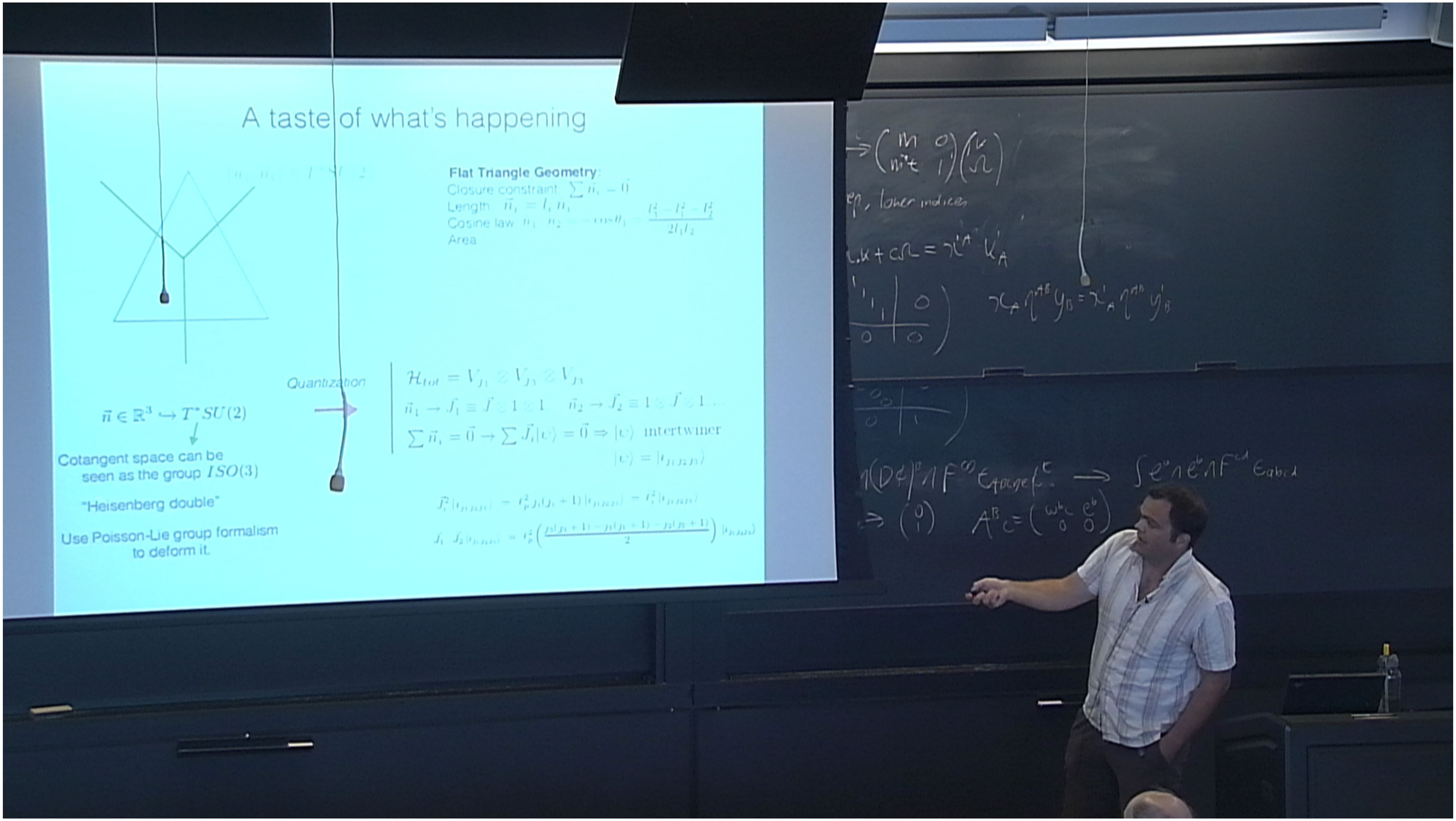
Cotangent space can be seen as the group $ISO(3)$
"Heisenberg double"
Use Poisson-Lie group formalism to deform it.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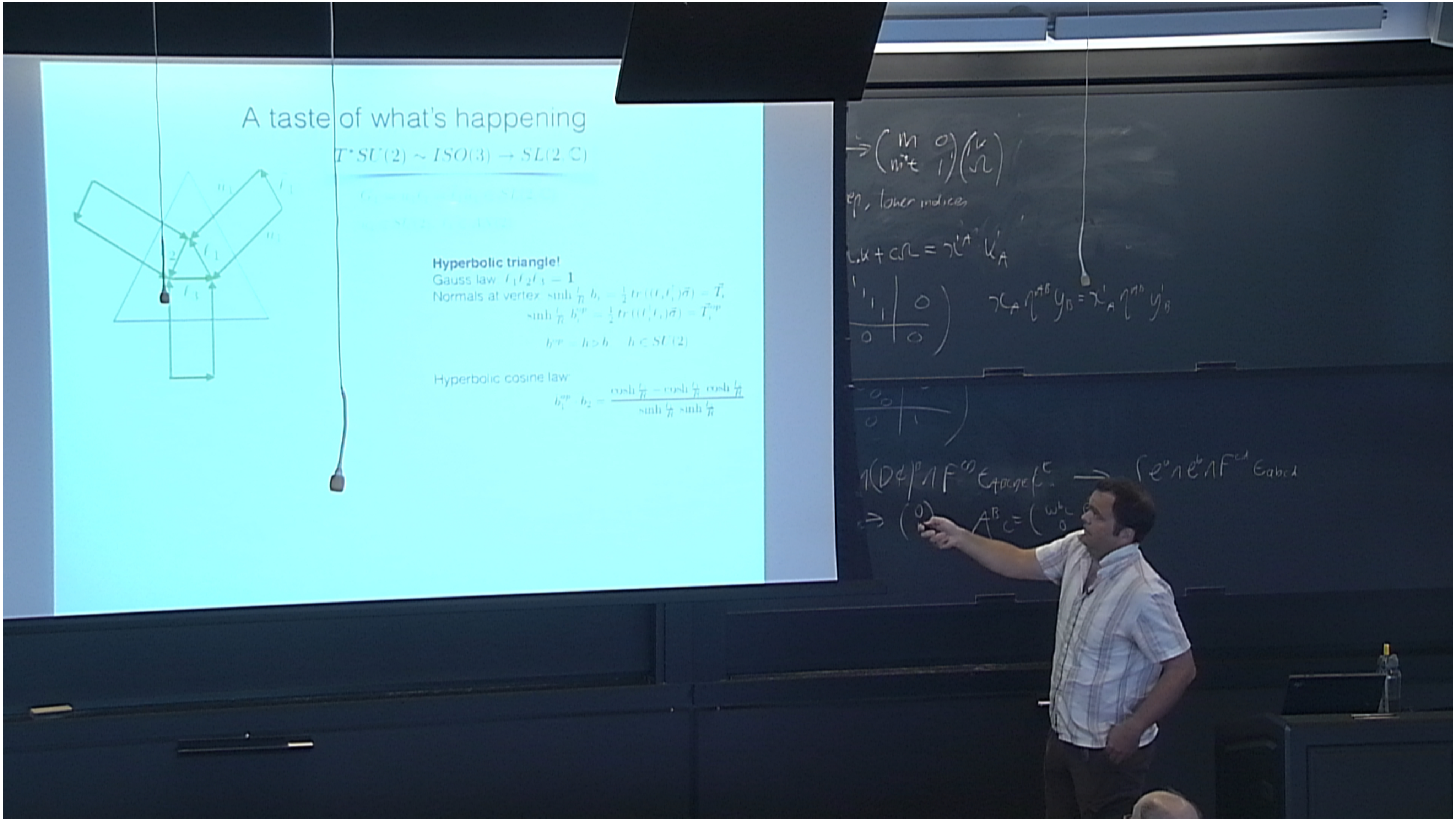
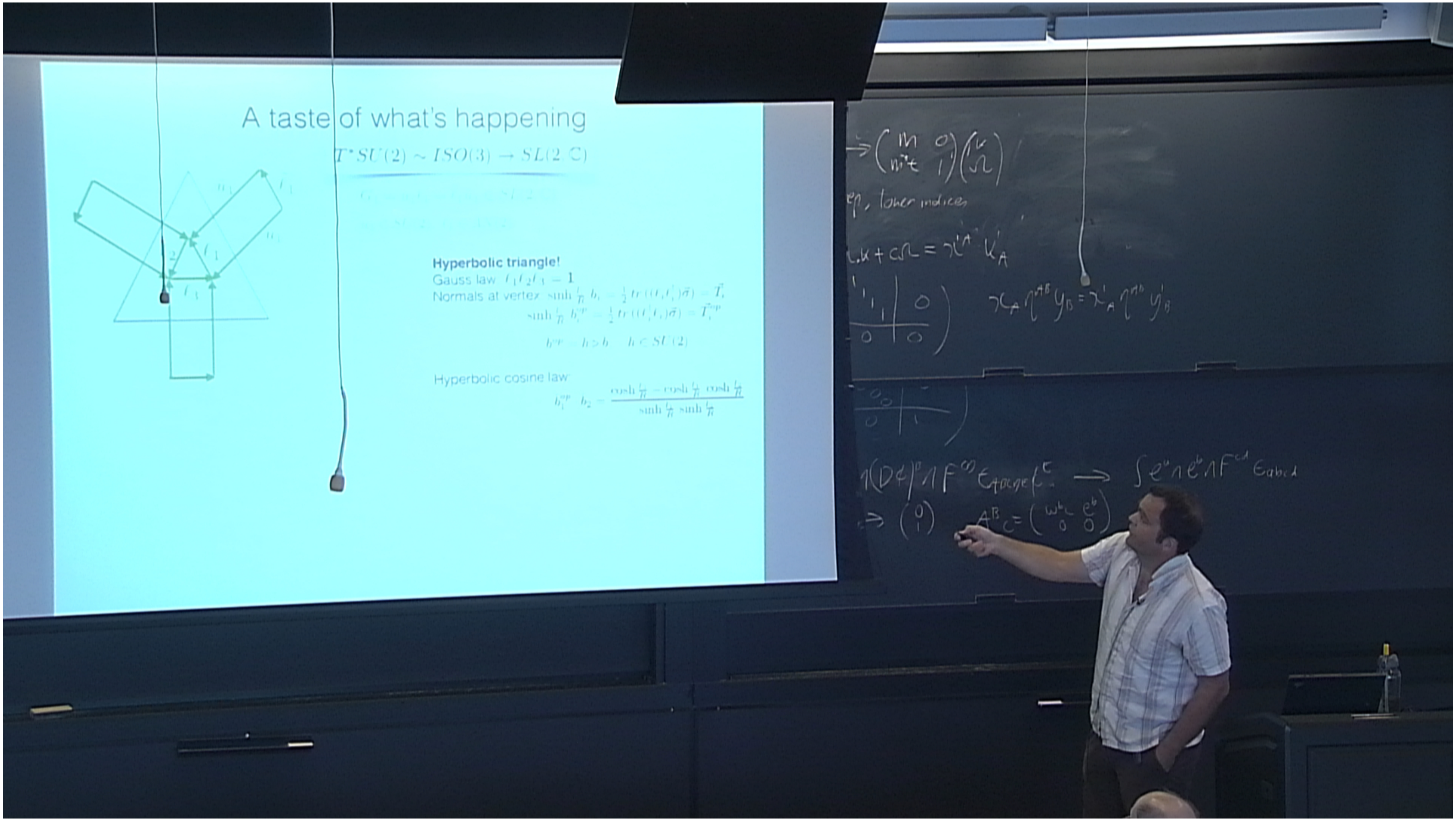

A taste of what's happening
T*SU(2) ~ ISO(3) → SL(2,C)
Hyperbolic triangle!
Gauss law
Normals at vertex
Hyperbolic cosine law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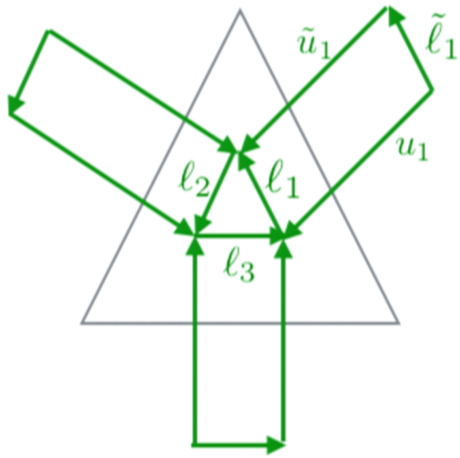


$$T^*SU(2) \sim ISO(3) \to SL(2,\mathbb{C})$$

$$G_1 = u_1\ell_1 = \tilde{\ell}_1\tilde{u}_1 \in SL(2,\mathbb{C})$$

$u_1 \in SU(2),\ \ell_1 \in AN(2)$

Hyperbolic triangle!

Gauss law: $\ell_1\ell_2\ell_3 = \mathbf{1}$

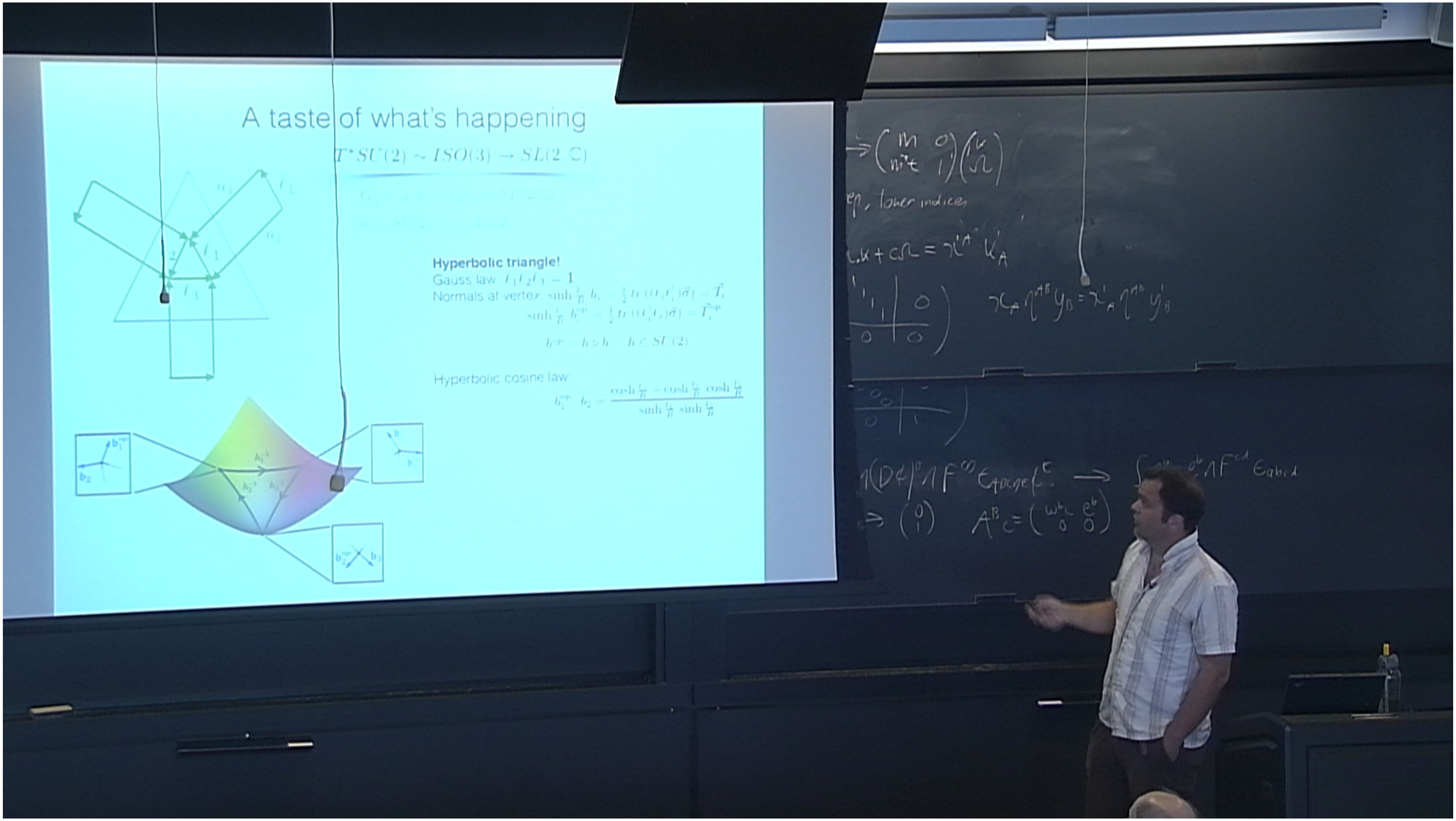
Normals at vertex: $\sinh\frac{l_i}{R}\ \hat{b}_i = \frac{1}{2}\,tr\left((\ell_i\ell_i^\dagger)\vec{\sigma}\right) = \vec{T}_i$

$$\sinh\frac{l_i}{R}\ \hat{b}_i^{op} = \frac{1}{2}\,tr\left((\ell_i^\dagger\ell_i)\vec{\sigma}\right) = \vec{T}_i^{op}$$

$$\hat{b}^{op} = h \triangleright \hat{b}, \quad h \in SU(2)$$

Hyperbolic cosine law:

$$\hat{b}_1^{op}\cdot\hat{b}_2 = \frac{\cosh\frac{l_3}{R} - \cosh\frac{l_1}{R}\ \cosh\frac{l_2}{R}}{\sinh\frac{l_1}{R}\ \sinh\frac{l_2}{R}}$$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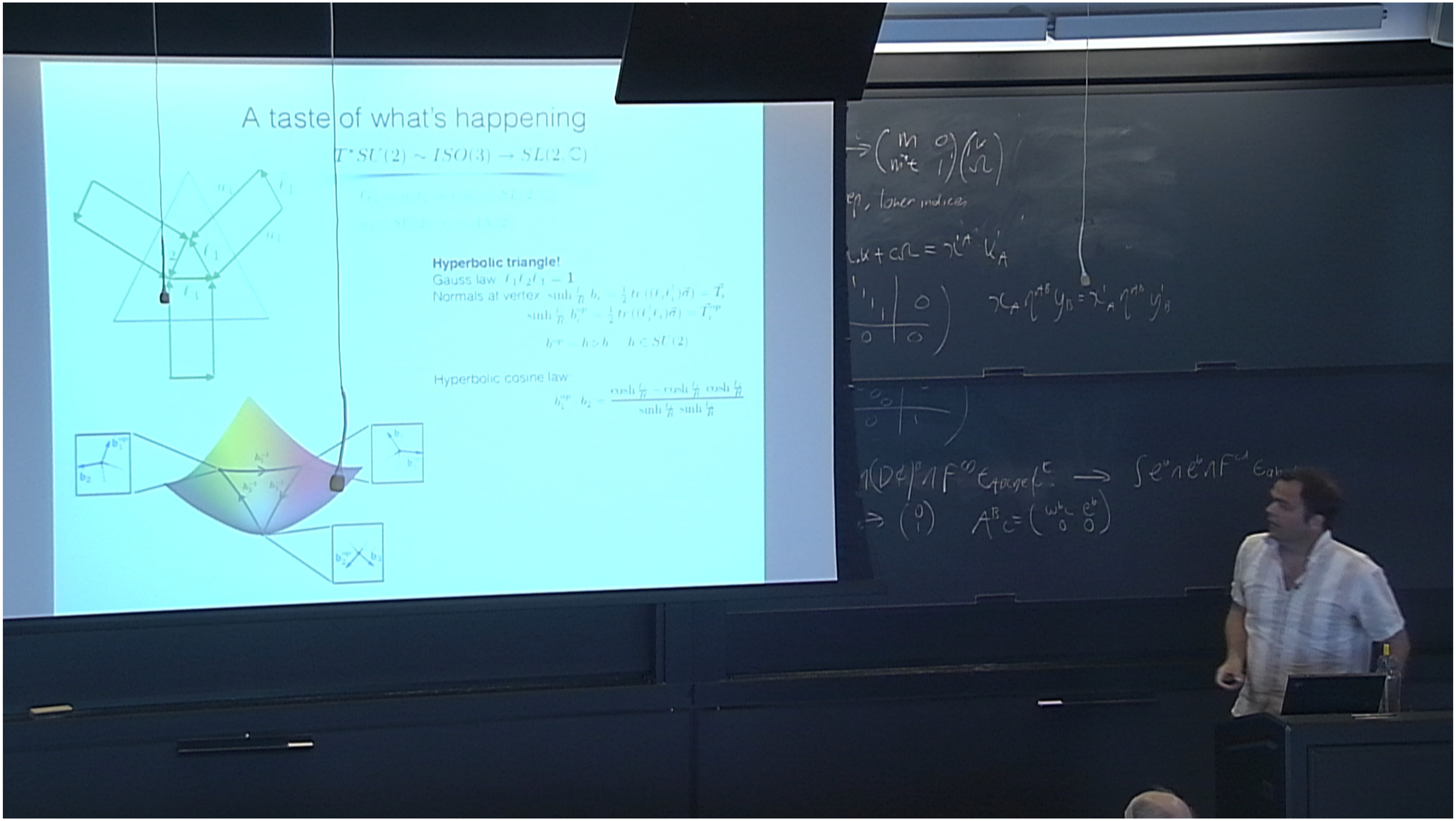
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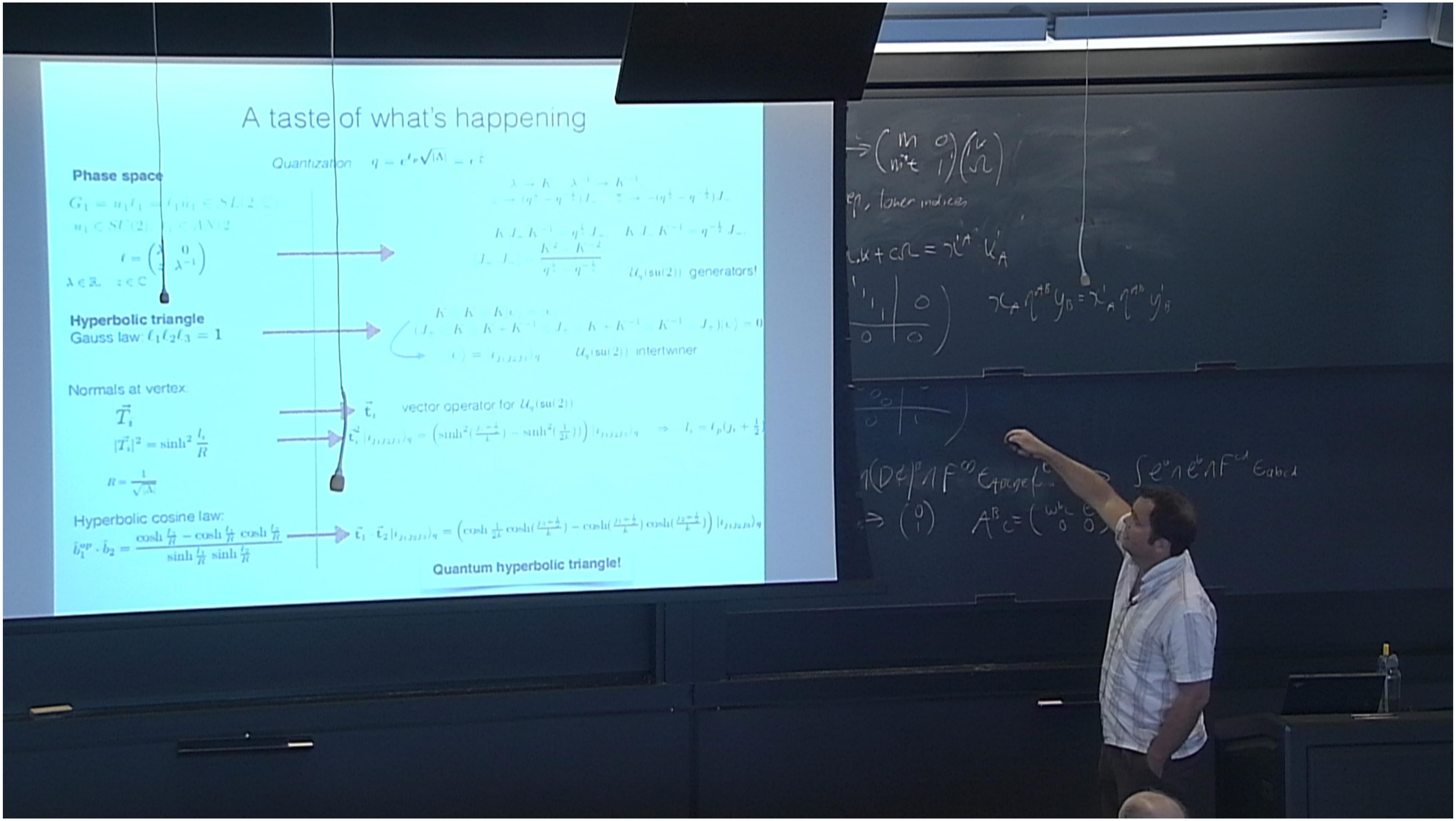
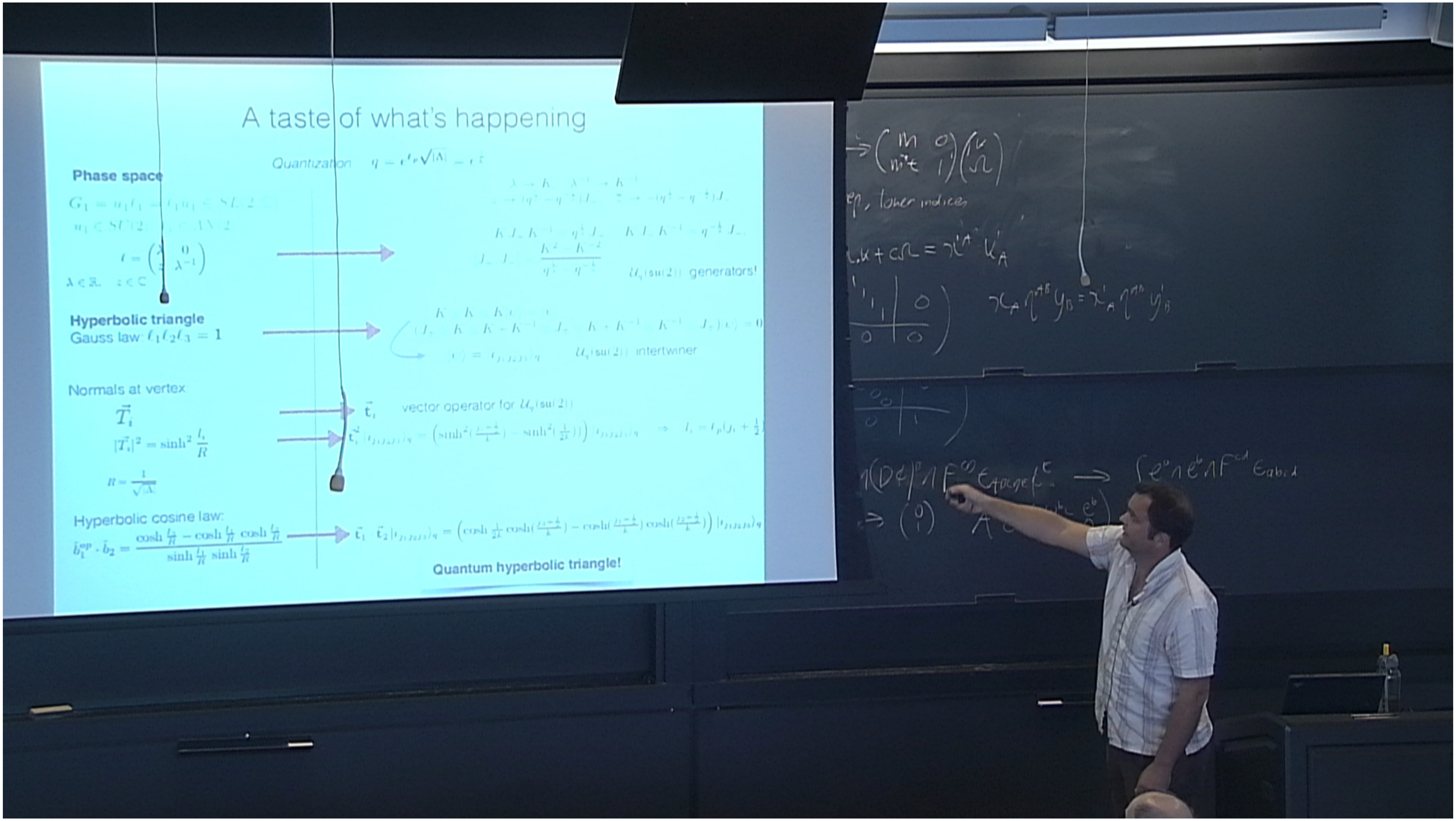

A taste of what's happening
Phase space
Quantization
Hyperbolic triangle
Gauss law: $\ell_1\ell_2\ell_3 = 1$
generators!
intertwiner
Normals at vertex
vector operator for $\mathcal{U}_q(\mathrm{su}(2))$
Hyperbolic cosine law:
Quantum hyperbolic triangle!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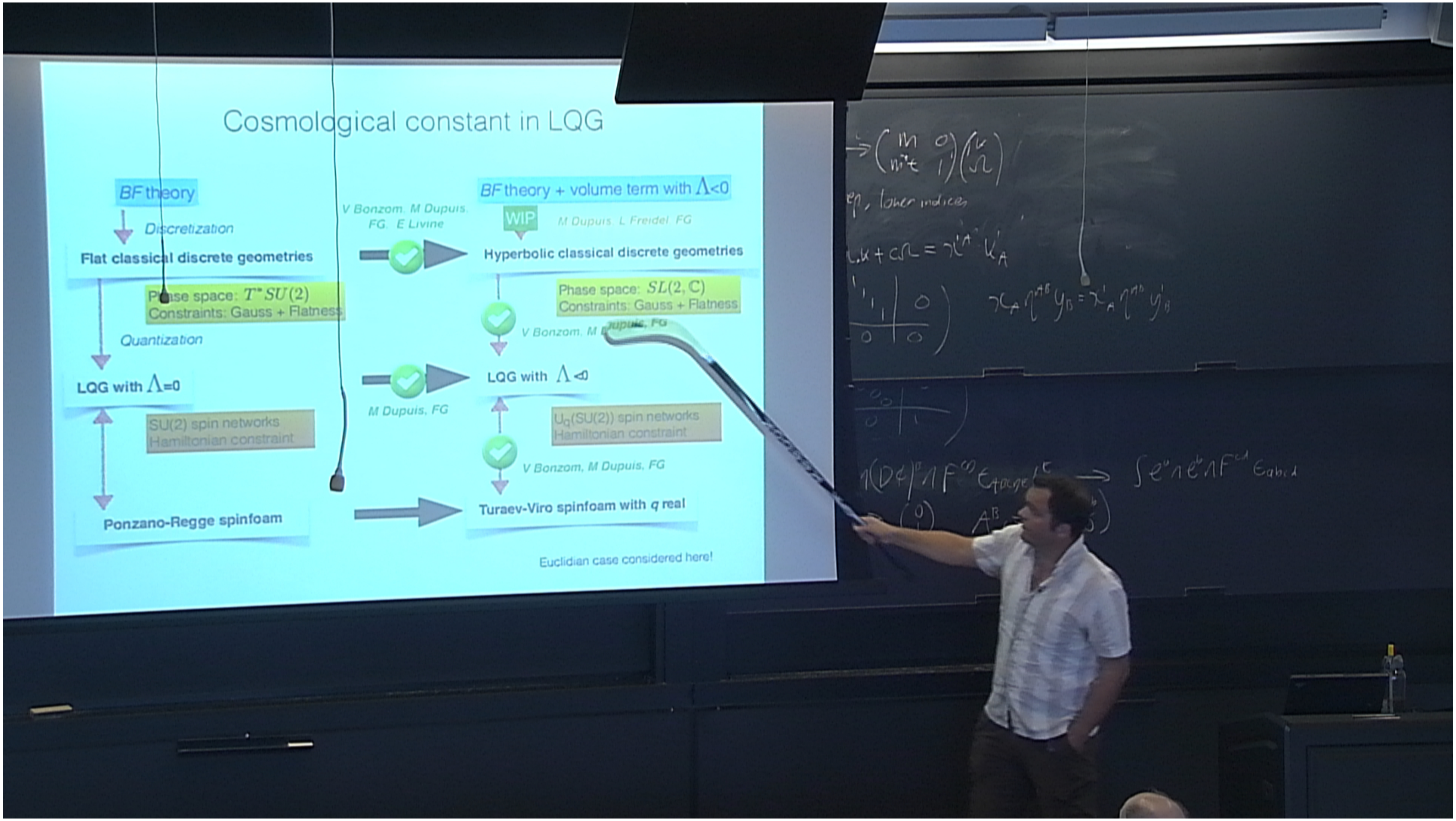
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WIP
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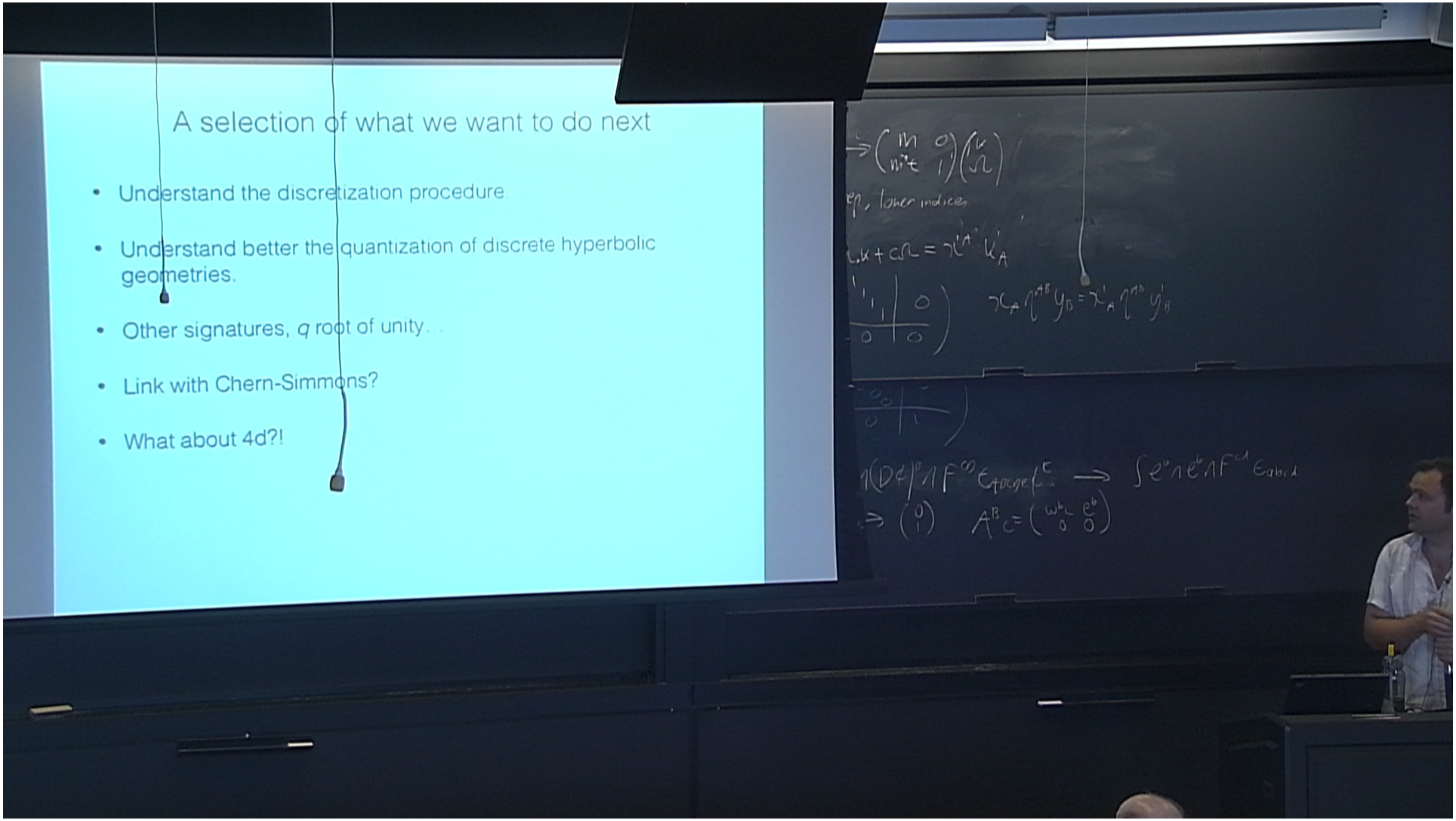
A selection of what we want to do next
Understand the discretization procedure.
Understand better the quantization of discrete hyperbolic geometries.
Other signatures, q root of unity…
Link with Chern-Simmons?
What about 4d?!